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ABSTRACT

The Affectionate Society: Does Competition for Partners Promote Friendliness?*

We study household formation in a model where collective consumption decisions of a household depend on the strategic choices of its members. The surplus of households is determined by individual choices of levels of friendliness to each other. A strategic conflict arises from a coupling condition that ceteris paribus, a person's friendlier attitude reduces the individual's influence in the household's collective decision on how to divide the ensuing surplus. While partners in an isolated household choose the minimum level of friendliness, competition for partners tends to promote friendliness. We find that affluence does not buy affection, but can lead to withholding of affection by an affluent partner who can afford to do so. In general, the equilibrium degree of friendliness proves sensitive to the socio-economic composition of the population.

JEL Classification: D13, D50, D70 and J10
Keywords: collective decisions, competition for partners, coupling condition, friendliness and socio-economic composition

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1 Introduction

Household formation ranks among the most important decisions a person ever makes. Besides, decision making within the household is a very important recurrent activity for most people. To a varying degree, they seek and find emotional comfort, social identity and material gain in marriage and other socio-economic partnerships. In the current paper, we study household formation and household stability, with the emphasis on two-person partnerships or households and the endogenous choice of personal attributes.

In the typical model of a multi-person household, the welfare of a household member may depend on the composition of the household and the individual consumption of every household member.[1] Such a model allows for consumption externalities within a household. It can also accommodate local public goods via intra-household externalities by having individual welfare solely depend on the aggregate consumption of the good within the household. The model further accommodates pure group externalities that is instances where the identity and personal attributes of fellow household members matter to an individual.

Undoubtedly, there are personal traits over which an individual has some control and which affect others. Our initial motivation for this study stems from the fact that despite its potential descriptive richness, the typical model of a multi-member household rules out the possibility of deliberately chosen personal attributes or attitudes. Even when household formation is endogenous, the personal attributes that a member brings to the household are typically treated as exogenous.

[1]For prominent contributions, see Becker (1978, 1993), Browning, Chiappori, and Lechene (1994), Browning and Chiappori (1998), Chiappori (1988, 1992), Manser and Brown (1980), McElroy and Horney (1981), Lundberg and Pollak (1994), among others. See also Gersbach and Haller (2001) and Haller (2000).

In our model, a particular personal attribute or attitude — which we call "friendliness" — is a strategic choice variable of the individual. An obvious question is how the strategic nature of friendliness and the chosen levels of friendliness affect the stability of households and the allocation of resources within households. Conversely, the question arises how the friendliness decisions are influenced by the availability of resources and outside options to household members. To the extent that household formation, resource allocation and personal attribute selections are interdependent choices, the two questions cannot be answered separately.

The label "friendliness" stands for personal attribute choices like showing a friendly or sour face; choosing a warm or cold tone; paying attention to or ignoring fellow household members. More generally, "friendliness" can serve as a generic term for any personal attribute that can be chosen at different levels; that exerts a positive externality upon others which increases with the level; that is neither marketable nor arrangeable by contract. Consequently, "friendliness" is not a standard commodity and not subject to intra-household bargaining. In particular, it is not merely a household produced commodity, a case already encompassed by the traditional multi-member household model.

Friendliness, in the colloquial sense, towards other people increases their well-being. Although friendliness is observable, it is in general not contractible, and is left fully to the discretion of each individual. There are important exceptions to that. For instance, in many service occupations, the degree of friendliness towards customers is part of the job description. Moreover, some people may have no choice in the matter. They cannot help to be friendly — or obnoxious — either by nature or by habit. Others may be naturally friendly or naturally obnoxious, but are able to act out of character, if they make a conscious effort.

For the sake of simplicity, we assume that the individuals of our model society can effortlessly choose to be friendly or unfriendly. Friendliness is an endogenous choice. We concentrate on affection shown in any two-person partnership that economically constitutes a household. Individuals in those households often face a trade-off between being nice, understanding and friendly to their partner, and an associated reduction of bargaining power when it comes to the allocation of resources, or surplus in a broad sense, within the household. This conflict arises from the fact that more often than not, enhanced bargaining power derives from a stern and tough posture which sabotages the attempt to appear friendly. We call this phenomenon **the coupling condition**:

> *Ceteris paribus*, a friendlier attitude reduces a person's bargaining power.

The coupling condition rules out the possibility that a person grants the pleasure of friendly company to others and conveys the image of a hard and determined negotiator at the same time. It also rules out the possibility that friendliness is reciprocated, so that friendly behavior triggers a friendly response.[2] Finally, it rules out emotional altruism: The partner's direct benefit from friendly behavior does not contribute to one's own welfare. This is not to say that the assumed away possibilities are unimportant or uninteresting. Here we isolate and explore just one plausible and intriguing trade-off which, among other things, implies that the distribution of bargaining power (regarding the allocation of commodities) within households is endogenously

[2] Basu (1999) forwards the idea that while labor supply and consumption of household members are determined by the balance of power within the household (and relative prices), the household's balance of power in turn depends on individually earned income, hence on individual labor supply (in addition to relative prices). In a *household equilibrium*, both the allocation of resources and the balance of power within the household are endogenized simultaneously. Basu (2001) pursues this idea further.

determined. In the last section, the coupling condition is related to concepts in social psychology.

Prima facie, it appears that the individuals in a society where the coupling condition prevails have no reason to be friendly, since all they would get is a worse bargain within their households. They are penalized for their friendliness. This is certainly correct when a household is considered in isolation. However, competition for partners can make these people friendly. We assume that the household structure, that is, the partition of the population into households, is itself an endogenous outcome. A situation in which a friendly household member is taken advantage of by an unfriendly partner is unstable if the friendly person can find better opportunities outside this household, either by going single or by teaming up with another partner. This suggests that in general, a stable household structure requires friendly behavior by all parties.

The questions at hand suggest a bottom-up or inductive approach to model building, in order to progressively study more sophisticated behavior and increasingly complex scenarios as we move along. We begin with an instructive benchmark case: an isolated household. We then turn to a very simple and amenable version of the model where individuals compete for partners. We keep adding new features and develop richer model variants as we proceed.

Absence of friendliness prevails in the isolated household analyzed in section 2. In section 3 we introduce competition for partners and find that it does promote friendliness. In a homogeneous population (with asexual partnerships), extreme friendliness can result where each individual is matched with a partner and chooses the maximum level of friendliness. We further find that in our context, affluence does not buy affection, but can lead to withholding of affection by an affluent partner who can afford to do so. While competition for partners tends to promote friendliness, it can also have a destabilizing effect on households if there is an unmatched individual around.

In section 4, we turn to sexual partnerships. In a society with an equal number of heterosexual males and females, there is a continuum of equilibria which can be ranked from the point of view of male welfare — and in reverse order from the female perspective. If there is a majority of men, then the worst outcome for males (and the best for females) occurs — and the opposite obtains, if there is a majority of women. In section 5, we draw the parallels and divides between our model and models of multilateral bargaining, matching, and assignment games. In section 6, we offer concluding remarks and point out parallels between the coupling condition and some theories from social psychology. We also relate our theoretical results to recent empirical findings of Stevenson and Wolfers (2000).

2 The Isolated Household

We first consider an isolated and fixed household or partnership h where two persons, $i = 1, 2$, choose their friendliness and then bargain over the utility allocation. One can think of two persons who consider each other the only adequate partners. One can also think of a snapshot of a society where separation (divorce) is not an option, so that people are locked into their partnerships. Or one can simply regard this as a benchmark case. We assume that there is a single private good and that the household is endowed with the quantity $\omega_h = 2$ of that good. There is no need for trade with the outside world, so the household is isolated both socially and economically. The welfare of household member i depends on his consumption of the private good, $x_i \geq 0$ and on the group externality g_{ji} individual i receives. The value of g_{ji} is chosen by the other individual j, who selects a particular personality profile that is associated with a certain level of friendliness. His preferences are represented by the utility function

$$U_i(x_i, g_{ji}) = \ell n\, x_i + g_{ji}.$$

We assume that $g_{ji} \in [0, \overline{g}]$ with $\overline{g} > 0$. While we assume that selecting g_{ji} has no direct costs or benefits for individual j, there are indirect costs in terms of bargaining power. In particular, we assume that the utilitarian weight of the first individual, denoted by $\alpha \in [0, 1]$, is a differentiable function $f(g_{21}, g_{12})$ with the following properties:

(CC) $f_1 > 0$, $f(\overline{g}, 0) = 1$, $f(g_{21}, g_{12}) = 1 - f(g_{12}, g_{21})$

where f_t, $t = 1, 2$, denotes the partial derivative of f with respect to its tth argument.

Two more properties follow: $f_2 < 0$ and $f(g, g) = 1/2$. An example is $f(g_{21}, g_{12}) = (\overline{g} + g_{21} - g_{12})/(2\overline{g})$. The assumptions on f simply capture the coupling condition, the previously postulated effect that a higher level of friendliness generated by an individual *ceteris paribus* decreases his bargaining power, because he is more accommodating to his partner.

The allocation in the household is determined by the following two-step procedure:

(i) Individuals choose their levels of friendliness, g_{12} and g_{21}.

(ii) The household takes a collective decision based on utilitarian weights $\alpha = f(g_{21}, g_{12})$ for member 1 and $1 - \alpha = 1 - f(g_{21}, g_{12}) = f(g_{12}, g_{21})$ for member 2.

We solve the household's allocation problem by working backwards. Given g_{12} and g_{21}, the household solves the following maximization problem in the second step:

$$(M_h): \quad \max_{x_1,x_2} \ \alpha \cdot (\ell n\, x_1 + g_{21}) + (1-\alpha)\cdot(\ell n\, x_2 + g_{12})$$
$$s.t.\, x_1 + x_2 \leq \omega_h$$

With $\omega_h = 2$, we immediately obtain $x_1 = 2\alpha$ and $x_2 = 2 - 2\alpha$. We assume that 1 and 2 behave strategically in the first step, correctly anticipating the implied outcome of the second step. Looking at a Nash equilibrium outcome of the overall allocation process we obtain:

Proposition 1 *There exists a unique Nash equilibrium with:*

$$\begin{aligned} g_{12} &= g_{21} = 0; \\ \alpha &= 1-\alpha = \frac{1}{2}; \\ U_1 &= U_2 = 0. \end{aligned}$$

The proposition follows immediately from the observation that given any level of g_{21}, the best reply of the first individual is to set $g_{12} = 0$ in order to maximize α, and thus the utility from consumption. The reason is that g_{12} itself has no effect on the group externality received by the first individual for given g_{21}. It is obvious that the equilibrium outcome is Pareto-inefficient since there is an allocation with

$$\begin{aligned} g_{12} &= g_{21} = \overline{g}, \\ \alpha &= 1-a = \frac{1}{2}, \\ U_1 &= U_2 = \overline{g}. \end{aligned}$$

But should one include α, which is not an argument of individual utility functions, in the description of a Pareto-improvement? When considering alternative allocations, ought households to be restricted to commodity allocations which are determined, via the coupling condition, by the chosen levels of friendliness as we have presumed so far? This restriction amounts to a concept of constrained Pareto-efficiency. In the absence of the restriction, we shall use the term unconstrained Pareto-efficiency or simply Pareto-efficiency.

The above Nash equilibrium outcome is not even constrained Pareto-efficient. Since the creation of friendliness is costless and increases the utility of other individuals, it is obvious that a constrained Pareto-efficient allocation requires that friendliness by at least one individual be maximal. For otherwise, one can increase g_{12} and g_{21} so that α and the resulting commodity allocation remain the same, but the positive externalities increase. A constrained Pareto-efficient allocation does not require maximal friendliness of both individuals. Indeed, $g_{12} = 0$, $g_{21} = \overline{g}$, $x_1 = 2$, $x_2 = 0$ constitutes the best constrained Pareto-efficient allocation from the point of view of individual 1. In contrast, unconstrained Pareto-efficiency does require maximal friendliness on the part of both household members.

3 Competing Partnerships: A Simple Social Equilibrium Model

Without competition, unfriendliness prevails in the isolated household. In this section, we embed the previously isolated partnership in a society where different partnerships compete with each other. For that purpose, we amend and modify the previous model as follows:

Let there be a population of N people, with $N \geq 3$, represented by $I = \{1, 2, \ldots, N\}$. A household is a non-empty subset of the population. A **household structure** is a partition of the population into households. We assume again that there is a single private good. Household h, if it is formed, is endowed with the quantity $\omega_h > 0$ of that good. We assume that forming a household of three or more persons creates enormous negative group externalities and will never be considered. Therefore, we can restrict ourselves to the formation of two- or one-person households. In a single-person household, an individual consumes his endowment. In a two-person household $h = \{i, j\}$, each individual i chooses a level of friendliness g_{ij} towards the other household member j. As before, his utility is $U_i(x_i, g_{ji}) = \ell n\, x_i + g_{ji}$, depending on his own consumption of the private good, x_i, and the friendliness received, g_{ji}. The utilitarian weights within household h are determined by a function f satisfying the analogue of the coupling condition (CC).

In this and subsequent versions of the model, the household structure is not a *fait accompli.* People are free to leave and will leave a household if they can get a better deal elsewhere. A household structure is stable if everyone has an incentive to leave a household. Stability is an endogenous property. Whether a household structure is stable depends on what happens in the corresponding households and what a member can expect when he leaves his household. We consider an equilibrium of the following multi-stage allocative process:

Stage 1: Partnerships are formed. A household structure $\mathbb{P}$ consisting of two-person or one-person households emerges.

Stage 2: In two-person households $h \in \mathbb{P}$, individuals decide on the group externalities $g_{ij}, i, j \in h, i \neq j$.

Stage 3: Collective consumption decisions in two-person households $h \in \mathbb{P}$ take place, with the utilitarian weights in the analogue of problem M_h determined by the choices made in stage 2.

Stage 4: Individuals leave to form new households.

An **equilibrium** of this process is a tuple $(\mathbb{P}; \mathbf{x}; g_{ij}, i, j \in h \in \mathbb{P}, |h| = 2)$ in which $\mathbb{P}$ is a household structure, $\mathbf{x} = (x_i)_{i \in I}$ is an allocation of the private consumption good and the g_{ij} are levels of friendliness in two-person households such that

- given $\mathbb{P}$ and the chosen group externalities g_{ij}, $\mathbf{x}$ is the allocation resulting from stage 3 of the process and therefore feasible, i.e. $\sum_i x_i = \sum_{h \in \mathbf{P}} \omega_h$;
- no individual has an incentive to leave a two-person household and go single, consuming his own endowment;
- no two individuals in different partnerships, say $i \in h$ and $k \in h'$, have an incentive to offer each other group externalities g_{ik} and g_{ki} such that both individuals would be (strictly) better off in the newly created household $\{i, k\}$.
- no member of a two-person household can change the group externality decision in stage 2 and achieve a higher utility without the partner feeling compelled to leave in stage 4.

Notice that we apply our fundamental postulate that household members cannot decouple friendliness and bargaining power to both actual and potential households. Notice further that the employed equilibrium concept is static.

3.1 An Extremely Friendly Society

We assume $N = 2n$ with $n \geq 2$ and further $y > 0$ such that for each household h, $\omega_h = |h| \cdot y$. In this society, fierce competition for partners leads people to be extremely friendly to their partners: everybody receives the maximal level of friendliness, $\overline{g}$. More precisely, we obtain:

Proposition 2 *Up to permutation of individuals, there exists a unique equilibrium:*

- $\mathbb{P} = \{h_1, \ldots, h_n\}$ *where* $h_\nu = \{2\nu - 1, 2\nu\}$ *for* $\nu = 1, \ldots, n$;
- $g_{ij} = \overline{g}, \ \forall\, i, j \in h \in \mathbb{P}, \ i \neq j$;
- $x_i = y, \ \forall\, i \in I$.

Proof. See appendix.

Observe that the equilibrium of this extremely friendly society is Pareto-efficient. However, this welfare conclusion is not robust. Whereas competition for partners tends to enhance equilibrium welfare, it does not always lead to Pareto-efficient equilibrium outcomes, in some cases not even to constrained Pareto-efficient ones. The latter outcome occurs in the next subsection when there are two affluent and four normally endowed individuals.

3.2 Affluence and Affection

The exchange of wealth for care or affection has been examined in the context of intergenerational transfers, notably bequests. Here we explore the possibility of this kind of exchange between spouses or partners belonging to the same generation. We find that in our context, affluence does not buy affection, but can cause the withholding of affection by an affluent partner.

We modify the model from the previous section. We assume again that $N = 2n$ with $n \geq 2$ and $y > 0$. But now there are two types of individuals, normally endowed individuals and affluent individuals. Normally endowed individuals have an individual endowment of $\omega_i = y$ of the consumption good. We assume that there are at least four more of them than there are affluent individuals. Moreover, there exists a number $Y > y$ so that every affluent individual a has an individual endowment $\omega_a = Y$ of the consumption good. For a household h, $\omega_h = \sum_{i \in h} \omega_i$. For convenience, let us further assume $\overline{g} \geq \ell n\, 2$.

One Affluent Person

Let a denote the affluent individual. In an equilibrium, individual a will be paired with some normally endowed individual, say b. The rest of society forms 2-person households consisting of normally endowed individuals. Since there are at least two of those households, we can apply the uniqueness argument of Proposition 2 to this subpopulation and find that in equilibrium, everybody must choose $\overline{g}$ in those households, with resulting utility $\ell n\, y + \overline{g}$. Consequently, the equilibrium choice in household $\{a, b\}$ is $g_{b,a} = \overline{g}$, whereas $g_{a,b}$ satisfies

$$g_{a,b} + \ell n \left[f(g_{a,b}, \overline{g}) \cdot (y + Y) \right] = \ell n\, y + \overline{g} \tag{1}$$

This makes individual b indifferent between being in this household and forming a household with another normally endowed individual. Let us check that this is, indeed, the equilibrium choice. First, an individual i from another household would not want to form a new household with a. In order to lure a away from b, a's utility has to increase, which is only possible if i's utility falls. Similarly, i cannot lure away b from a. Second, given $g_{b,a} = \overline{g}$, (1) describes a's best choice subject to the constraint that b not be driven away. Third, if b chose less than $\overline{g}$, a would have an incentive to form another partnership. It remains to be verified that a does not prefer to remain single. This is the case if, and only if,

$$\ell n[(1 - f(g_{ab}, \overline{g})) \cdot (y + Y)] + \overline{g} \geq \ell n \, Y$$

or, equivalently,

$$\ell n(1 - f(g_{a,b}, \overline{g})) + \overline{g} \geq \ell n(Y/(y + Y)).$$

Since $1 - f(g_{ab}, \overline{g}) > 1/2$ and $\overline{g} \geq \ell n \, 2$, the left-hand side is positive, while the right-hand side is negative. Hence the inequality holds. If we had not made the above assumption on $\overline{g}$, the inequality would still hold for Y sufficiently close to y. It would also hold for sufficiently large Y. But without the assumption, we were unable to verify it for intermediate values of Y.

We conclude that affluence does not buy affection here, but rather induces the affluent person to show less than maximal affection. Clearly, the affluent individual fares better than when single, in either an isolated household, or normally endowed. But interestingly enough, the lack of other affluent people does not give her an added competitive advantage. To see this, let us compare the current situation with an alternative society consisting only of affluent people. To be precise, let us take the previous model with $y = Y$. For the affluent individual to find the current situation at least as good as the alternative, it must be the case that

$$\ell n[(1 - f(g_{ab}, \overline{g})) \cdot (y + Y)] + \overline{g} \geq \ell n \, Y + \overline{g}$$

which reduces to

$$f(g_{ab}, \overline{g}) \leq y/(y + Y). \tag{2}$$

But (2) and $g_{ab} < \overline{g}$ imply $g_{a,b} + \ell n(f(g_{a,b}, \overline{g})) < \overline{g} + \ell n(y/(y+Y))$, a violation of (1).

Two Affluent Persons

Let a and c denote the two affluent individuals. Again, only 2-person households are formed in equilibrium and at least two of those households consist entirely of normally endowed individuals. Therefore, the uniqueness argument of Proposition 2 applies once more: The members of two-person households composed of normally endowed individuals all choose $\overline{g}$. Moreover, the household $\{a, c\}$ is formed. For suppose not. Then a is in a household with a normally endowed individual b. But the maximal utility a can obtain in this household is implicitly given by (1). An analogous statement holds for c. But a and c could form a new household and both choose $\overline{g}$, making both better off since (1) and (2) have been shown to be inconsistent. This shows that only the formation of household $\{a, c\}$ is compatible with equilibrium. This does not mean, however, that both affluent individuals actually choose $\overline{g}$ in equilibrium. Each will drive down the other's utility to the maximal level they can obtain when paired with a normally endowed individual. Evidently, this constitutes an equilibrium. Thus, the two affluent individuals find each other, but neither one gains or loses from the other's presence. Incidentally, this constitutes an example where there is competition for partners, but the equilibrium outcome fails to be constrained Pareto-efficient.

Three Affluent Persons

By the same reasoning as in the case of two affluent persons, equilibrium requires the following. Only two-person households form. Two of the affluent individuals form a two-person household. All normally endowed individuals choose $\overline{g}$. All affluent individuals attain the same utility, namely the maximal utility they can achieve when paired with a normally endowed individual. However, such a constellation does not constitute an equilibrium. For one affluent individual, say a, is paired with a normally endowed individual, say b, and as we have already argued, such a household is bound to break up. Therefore, no equilibrium and, consequently, no stable household structure exists.

Four Affluent Persons

By the previous arguments, only two-person households form in equilibrium. Every mixed household will break up. Considering the individuals of the same type separately, we can conclude from the former analysis of the extremely friendly society that each individual has to choose $\overline{g}$ in equilibrium and that this constitutes an equilibrium for each of the two subpopulations. By the inconsistency of (1) and (2), no two individuals of different type can both benefit from breaking up their partnerships and forming a mixed household. Hence, an equilibrium exists and equilibrium is characterized by the following two properties. Each individual is paired with an individual of the same type. Each individual chooses the maximal level of friendliness. Thus an extremely friendly society emerges.

More Than Four Affluent Persons

With an odd number of affluent persons, instability prevails. With an even number of affluent persons, an extremely friendly society results.

3.3 The Destabilizing Effect of Free Agents

As in subsection 3.1, let there be an *ex ante* homogeneous population. But now let $N = 2n + 1$ with $n \geq 1$. Then the equilibrium still requires that as many two-person households as possible are formed. In a two-person household, it is impossible for the utility of both household members to exceed $\ell n\, y + \overline{g}$. Further, because of the odd number of individuals, there always remains one free agent i, an individual who is currently single. Now take any two-person household and pick a member j of this household whose utility does not exceed $\ell n\, y + \overline{g}$. Then i and j can form a new household and choose $g_{ij} = \overline{g}$ and $g_{ji} = \overline{g} - \epsilon$ with $\epsilon > 0$ being sufficiently small so that both are better off. This shows that any household structure that conceivably might emerge in equilibrium is destabilized by the presence of one free agent. Thus, equilibrium does not exist.

4 Sexual Partnerships

So far we have dealt with a model of a society where sex and sexual orientation do not matter for the formation of households. This could represent a genuinely asexual society. It could also reflect a society of unbiased bisexuals.

We now consider the case where individuals are distinguished by sex and sexual orientation. Individuals are identical in all other respects, with the characteristics introduced in subsection 3.1. But in addition to impeding the formation of households with more than two members, we rule out two-person households where one person's sex does not match the other's sexual orientation. It turns out that this additional assumption makes a significant difference.

To describe equilibrium outcomes for a heterosexual population, we introduce a threshold level of friendliness $\underline{g} \in (0, \overline{g})$ given by

$$\underline{g} + \ell n[f(\underline{g}, \overline{g}) \cdot y] = \ell n\, y$$

or $\underline{g} + \ell n\, f(\underline{g}, \overline{g}) = 0$. This threshold $\underline{g}$ is the level of friendliness of the other household member, who makes an individual choosing $\overline{g}$ indifferent between staying in the household or going single. We begin with a society where the numbers of males and females match.

Proposition 3 *Let $N = 2n$, with $n \geq 2$, n males and n females. Then for any $\hat{g} \in [\underline{g}, \overline{g}]$, the following constitutes an equilibrium:*

- *The household structure $\mathbb{P}$ consists of two-person households each composed of a male and a female.*
- *All males choose $\hat{g}$ and all females choose $\overline{g}$.*
- *The commodity allocation in each two-person household h is determined by solving M_h.*

The corresponding outcome with the respective role of males and females reversed also constitutes an equilibrium.

The proof of the proposition is straightforward. This result suggests that the current society may settle for one of many conventions favoring one or the other sex. This finding is quite different from what we found in Proposition 2 for the extremely friendly society, a homogeneous asexual population with an even number of people. Moreover, we obtained instability for a homogeneous asexual population with an odd number of people, which is not the case here:

Proposition 4 *Let there be m males and n females. If there is a majority of males, $m > n \geq 1$, or a majority of females, $n > m \geq 1$, then an equilibrium exists and the following properties hold:*

- *Each member of the minority forms a two-person household with a member of the majority. The remaining members of the majority form one-person households.*
- *Minority members choose $\underline{g}$ and majority members choose $\overline{g}$.*
- *The commodity allocation in each two-person household h is determined by solving M_h.*

Again, the proof is straightforward. The minority benefits from the fact that majority members are pitted against each other in their quest for partners. In the following, we draw the parallels and divides between our model and models of multilateral bargaining, matching, and assignment games.

5 Common Features in Models of Group Formation

This research originates from our work on general equilibrium models of an economy where households operate in a competitive market environment, can have several members and make efficient collective consumption decisions in the sense of the collective rationality postulate of Chiappori (1988, 1992). In several papers [Haller (2000), Gersbach and Haller (1999, 2000, 2001)] we have investigated the interaction between collective household decisions and collective markets. Here we have trivialized the role of the market by assuming a single consumption good. Moreover, we abandoned the collective rationality postulate for the purpose of the present paper, notwithstanding our intellectual debt to Chiappori and others. Our novel element is the endogenous creation of a pure group externality in two-person households that stems from individually chosen levels of friendliness towards the partner. Moreover, we stipulate the *coupling condition*: that one's friendliness negatively affects one's internal bargaining power. As a consequence, while the amount of surplus generated within a household is shared efficiently, this endogenously determined amount need not be optimal — which constitutes a departure from collective rationality.

In the following we discuss how our model and the results relate to the literature on outside options in multilateral bargaining problems, to two-sided matching and assignment games.

5.1 Outside Options and Multilateral Bargaining

Household formation, household decisions and household stability in our model hinge on the outside options available to household members. The idea of which households are actually formed and which decisions they make might depend on the decisions made in other existing or potential households is also central to a small strand of literature on multilateral bargaining problems represented by Rochford (1984), Crawford and Rochford (1986), and Bennett (1988, 1997).[3] In this literature, a "solution" specifies an agreement for each eligible coalition that is consistent with the bargaining process in every other coalition. More specifically, bargaining functions and potential surpluses are exogenously given for all conceivable coalitions. The agreement for a coalition is then determined by its bargaining function, its potential sur-

[3]Binmore (1985) and Houba and Bennett (1997) take a non-cooperative approach to the special case of three-player/three-cake problems.

plus, and the outside options offered to coalition members by the (feasible or infeasible) agreements in other coalitions, as suggested by the solution.

Our model shares the basic interdependence of household formation and household decisions with the multilateral bargaining literature, but differs from it in several important aspects: In the present setting, for each two-person household, a bargaining function is endogenously selected from a parametric family instead of being exogenously given. Furthermore, for each two-person household, the available surplus is endogenously and possibly inefficiently determined — instead of being exogenously given. And finally, alternative households are not bound by the "solution" in the choice of their surplus and bargaining function. Because of the last feature, that outside options are not restricted by a solution, our model is close in spirit to models of two-sided matching and assignment games. For the sake of comparison, we briefly report on some of the pertinent results obtained for those two models in the next two subsections. In the final subsection, we relate these results to ours.

5.2 Two-sided Matching

The matching literature dates back at least to a combinatorial lemma by Hall (1935) and Maak (1936) that later became known as the "marriage theorem". The seminal contribution to two-sided matching is Gale and Shapley (1962). The main results are surveyed in Roth and Sotomayor (1990). The standard model assumes two finite disjoint populations of men, M, and women, W. Each $m \in M$ has a strict preference relation P_m on $W \cup \{m\}$. Each $w \in W$ has a strict preference relation P_w on $M \cup \{w\}$. For example, $w' P_m w$ means that man m prefers marrying woman w' to marrying woman w. In particular, $m P_m w$ means that m prefers remaining single (a self-match) to marrying

w. A **matching** is a partition $\mathbb{P}$ of $M \cup W$ such that for each $h \in \mathbb{P}$: either $|h| = 1$ or [$|h \cap M| = 1$ and $|h \cap W| = 1$]. In our terminology, a matching is a household structure with only one-person and heterosexual two-person households. A matching can be represented by the bijective mapping $\mu : M \cup W \to M \cup W$ that satisfies $\mu(i) = i$ for the individuals which are single in $\mathbb{P}$ and $\mu(i) = j, \mu(j) = i$, if i and j form a two-person household (marriage) in $\mathbb{P}$. The matching is called *stable* if (i) there is no individual i that prefers being single to his or her status in $\mathbb{P}$, i.e. $\neg i P_i \mu(i) \, \forall i$; and (ii) there is no couple $(m, w) \in M \times W$ where both prefer being married to each other to their status in $\mathbb{P}$, i.e. $\neg[w P_m \mu(m) \wedge m P_w \mu(w)] \, \forall m \forall w$. Notice that each preference relation P_i induces a weak preference relation $\succsim_i$ on the set of stable matchings. Some of the main results are the following (see Roth and Sotomayor (1990), Ch. 2 and 3):

1. The common preferences of men on the set of stable matchings are opposed to the common preferences of women. If $\mathbb{P}$ and $\mathbb{P}'$ are stable matchings and all men like $\mathbb{P}$ at least as well as $\mathbb{P}'$, then all women like $\mathbb{P}'$ at least as well as $\mathbb{P}$. If in addition, some man prefers $\mathbb{P}$ to $\mathbb{P}'$, then some woman prefers $\mathbb{P}'$ to $\mathbb{P}$.

2. There exists a unique M-optimal stable matching that all men like at least as well as any other stable matching and that is the worst stable matching for women, giving each woman her least preferred choice. Similarly, there is a unique W-optimal stable matching.

3. The set of stable matchings forms a distributive lattice with respect to the common preferences of men. It forms a dual distributive lattice with respect to the common preferences of women.

4. The addition of a woman does not harm any man and does not help any woman in the M-optimal stable matching and in the W-optimal stable matching.

We select these four results because of the strikingly parallel results for assignment games and for our model of heterosexual societies.

5.3 Assignment Games

The people in the two-sided matching model care about with whom they are matched and nothing else. In contrast, in standard assignment games an individual only cares about consumption or disposable income. Nevertheless, the sets of stable outcomes of both models possess strikingly similar structures. The seminal paper on assignment games is Shapley and Shubik (1972). A survey is provided by Roth and Sotomayor (1990). The standard model again assumes two finite disjoint populations, M and W. As a single person, an individual i can produce a profit or surplus of $\sigma_i = \sigma_{\{i\}} = 0$, whereas a couple $(i, j) \in M \times W$ can produce the amount $\sigma_{\{i,j\}} \geq 0$. An **outcome** is a triple $(\mathbf{u}, \mathbf{v}; \mathbb{P})$ where $\mathbf{u} = (u_i)_{i \in M} \in \mathbb{R}^M$ is a utility allocation for the members of M; $\mathbf{v} = (v_j)_{j \in W} \in \mathbb{R}^W$ is a utility allocation for the members of W; $\mathbb{P}$ is a matching (also called an "assignment"). The outcome $(\mathbf{u}, \mathbf{v}; \mathbb{P})$ is **feasible** if

$$\sum_{i \in M} u_i + \sum_{j \in W} v_j = \sum_{h \in \mathbf{P}} \sigma_h.$$

The latter condition presumes that individual welfare depends only on the dividend distribution received by the individual. The outcome $(\mathbf{u}, \mathbf{v}; \mathbb{P})$ is **stable** if it is feasible and (i) $\mathbf{u} \geq 0$, $\mathbf{v} \geq 0$ and (ii) $u_i + v_j \geq \sigma_{\{i,j\}}$ for all $(i, j) \in M \times W$. That is, no individual or couple can fare better on their own. Notice that (i) and (ii) combined with feasibility imply that the surplus σ_h

is distributed among the members of h for each $h \in \mathbb{P}$. Several of the main results for assignment games parallel, by and large, those for the two-sided matching model and are as follows (see Roth and Sotomayor (1990), Ch. 8):

1. The common preferences of men on the set of stable outcomes are almost opposed to the common preferences of women. Namely, if $(\mathbf{u}, \mathbf{v}; \mathbb{P})$ and $(\mathbf{u}', \mathbf{v}'; \mathbb{P}')$ are stable outcomes and $\mathbf{u} > \mathbf{u}'$, then $\mathbf{v} \leq \mathbf{v}'$. Similarly, if $(\mathbf{u}, \mathbf{v}; \mathbb{P})$ and $(\mathbf{u}', \mathbf{v}'; \mathbb{P}')$ are stable outcomes and $\mathbf{v} > \mathbf{v}'$, then $\mathbf{u} \leq \mathbf{u}'$.

2. There exists an M-optimal stable outcome, $(\bar{\mathbf{u}}, \bar{\mathbf{v}}; \bar{\mathbb{P}})$, which is liked by all men at least as much as any other stable outcome. The vector $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$ is uniquely determined and satisfies $\bar{\mathbf{u}} \geq \mathbf{u}$ and $\bar{\mathbf{v}} \leq \mathbf{v}$ for all stable outcomes $(\mathbf{u}, \mathbf{v}; \mathbb{P})$. Similarly, there exists a W-optimal stable outcome with corresponding properties.

3. The set of stable outcomes forms a complete lattice with respect to the common preferences of members of M. It forms a dual complete lattice with respect to the common preferences of members of W.

4. Addition of a woman does not harm any man and does not help any woman in the M-optimal stable outcome and in the W-optimal stable outcome.

Although our analysis does not, and in fact cannot, rely on the previous literature on matching and assignment games, it turns out that the similarity of results extends to our model of heterosexual societies as well.

5.4 Equilibria of Heterosexual Societies Reconsidered

To develop the common features in more detail, the heterosexual population of section 4 is divided into a finite set of m males, M, and a finite set of n females, W, as in the two-sided matching model and in standard assignment games. Similar to assignment games, a single individual i achieves an exogenously given utility level, $\sigma_i = \sigma_{\{i\}} = \ell n\, y$ when consuming the endowment with the private good. As in non-trivial assignment games, a couple has the potential to generate a surplus in excess of the sum of their σ_i. But in contrast to assignment games, the total surplus that can be produced and shared by a couple $(i, j) \in M \times W$ cannot be summarized by a single number $\sigma_{\{i,j\}}$. Rather, the strategically chosen levels of friendliness g_{ij} and g_{ji} determine both the size and the division of surplus in the partnership. Moreover, the constrained Pareto frontier for the couple is non-linear. In spite of this apparent major difference, the structure of the equilibrium set bears a strong resemblance to the features of the equilibrium sets for two-sided matching models and for assignment games which we have highlighted above. The common structural properties are easy to identify, provided that the list of equilibria described in Proposition 3 proves exhaustive. In a preliminary step, we establish an "equal treatment property" which constitutes yet another instance of a heuristic principle pointed out by Shapley and Shubik (1972), page 121 and footnote 1 *ibid.*: *intergroup allocations are relatively indeterminate, intragroup allocations are relatively precise.*

Lemma 1 (Equal Treatment Property)
If $m = n \geq 2$, then only male-female households form in equilibrium, and members of the same sex choose the same level of friendliness.

The proof is relegated to the appendix. As an immediate consequence, we obtain:

Corollary 1 *In case $m = n \geq 2$, all equilibria are of the form given in Proposition 3.*

In view of the lemma, the crucial step in the proof is to show that it cannot occur in equilibrium that all men choose a level of friendliness $g^* < \overline{g}$ and all women choose a level of friendliness $g^{**} < \overline{g}$. The key argument is that then a man and a woman could form a new household and both increase their level of friendliness in such a way that their relative intra-household bargaining power does not change. Details are given in the appendix.

Now the common features of the equilibrium set of our model of heterosexual partnerships and the stable outcomes of two-sided matching models and assignment games can be easily seen from Propositions 3 and 4 and the corollary:

1. The common preferences of men on the set of equilibrium allocations are opposed to the common preferences of women.

2. There exists a unique equilibrium that all men prefer to all other equilibria. There exists a unique equilibrium that all women prefer to all other equilibria.

3. If there is an equal number of men and women, then there is a continuum of equilibria which can be totally ranked from the point of view of the men — and in reverse order from the point of view of the women. In each equilibrium, the equal treatment property holds. In the best equilibrium for men, all men choose $\underline{g}$ and all women choose $\overline{g}$. The opposite choices are made in the worst equilibrium for men. If there is

a majority of men, then equilibrium is unique and corresponds to the worst possible outcome for men in the equal numbers case. If there is a majority of women, then equilibrium is unique and corresponds to the worst possible outcome for women in the equal numbers case.

4. The addition of a woman does not harm any man and does not help any woman.

The three models have in common that the population is split into two groups and that the agents of each group compete for partners from the other group. Our model is distinguished by the fact that individuals can make strategic choices that determine the group externalities two partners exercise upon each other. Moreover, via the coupling condition, these strategic choices also fix a bargaining function (welfare weights) within a partnership.

In the case of section 2 of a homogeneous population, or asexual partnerships, either extreme friendliness or non-existence of equilibrium prevailed. The latter is perhaps not too surprising. For it is well known (although less well understood and investigated) that one-sided matching problems are significantly different from two-sided matching problems. For example, in the "problem of roommates" there need not exist a stable matching, even if the number of individuals is even and each has strict preferences, as demonstrated by Gale and Shapley (1962).

6 Conclusion

Household formation is an integral part of economic and social activities. The general equilibrium perspective leads to very different conclusions about the degrees of friendliness prevailing in a society than in an isolated household. One of our main findings suggests that an exit option causes individuals to behave less badly to one other. Such a finding has some empirical support. Examining state panel data, Stevenson and Wolfers (2000) "find a striking decline in female suicide and domestic violence rates arising from the advent of unilateral divorce. Total female suicide declined by around 20% in states that adopted unilateral divorce. There is no discernable effect on male suicide." The data suggest an asymmetry with respect to male and female suicide. One explanation could be that an unbearable domestic situation is less of a factor for male than female suicide. An alternative explanation could be that the absence of unilateral divorce constitutes less of a barrier to exit for males than for females. Notice that our model yields asymmetric equilibrium outcomes (social conventions) with equal numbers of males and females.

As regards the significant behavioral response to the change of divorce laws, one might suspect that behavior in persisting marriages did not actually change, that unsatisfied spouses simply took the escape route of unilateral divorce. However, this does not seem to be the case. Stevenson and Wolfer conclude: "Unilateral divorce changed the bargaining power in marriages, and therefore impacted many marriages — not simply the extra few divorces enabled by unilateral divorce." This confirms our finding that exit options increase the degree of friendliness in households.

Lest the reader considers our theoretical result (that an exit option can improve welfare) unsurprising, let us hasten to observe that added outside options for household members need not necessarily increase equilibrium welfare. In an economy with several tradeable commodities, the value of an outside option may depend on relative prices. In Gersbach and Haller (2000) we show the following possibility:[4] Without the particular outside option, there can be two equilibria one of which weakly Pareto-dominates the other one. When the outside option is introduced, its high value to some individuals in the Pareto-dominant equilibrium destabilizes certain households. As a consequence, the outside option leads to the elimination of the better equilibrium while the Pareto-inferior equilibrium continues to exist.

A key assumption for our analysis is the coupling condition which has parallels in social psychology. Within interdependence theory, social value orientations have been found to affect individuals' behavior in negotiations [e.g. De Dreu and Van Lange (1995), O'Connor and Carnevale (1997) and De Dreu, Weingart and Kwon (2000)]. Prosocial negotiators who develop positive attitudes and seek to understand one another's perspective have placed lower demands and made greater concessions in negotiations than others. Prosocial behavior can be interpreted as choosing a high level of friendliness (strategically prosocial) or exhibiting emotional concern for others (genuinely prosocial). Others have pointed out that negotiators try to build a positive climate in order to increase the joint and individual surplus [Lewicki, Litterer, Minton and Saunders (1994)]. A further, somewhat remote parallel is the perception of justice in terms of entitlement beliefs [see the survey of Mikula and Wenzel (2000)]. Entitlement beliefs are categorizations about what an individual and others deserve or are entitled to. Entitlement beliefs can influence resistance and concession making in negotiations, because the violation of somebody's entitlement constitutes an incident of perceived in-

[4] In Gersbach and Haller (2000) as in the rest of the literature, personal attributes are assumed exogenous.

justice. A higher level of friendliness could tend to increase the entitlement beliefs of the partner, and thus could increase his demands for concessions. This would provide a foundation for the coupling condition.

While we think that the strategic role of the coupling condition captures some important aspects of household behavior, it would be unwise to neglect other stabilizing elements. Future research should aim at combining affection and affluence with other forces which bring people into durable relationships, like trust and commitment to support one another, insurance, and caring for each other. A dynamic approach could explicitly deal with the formation as well as the dissolution of households and allow a thorough analysis of situations where a static equilibrium fails to exist.

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APPENDIX

Proof of Proposition 2

We have to show existence and uniqueness of the equilibrium.

Existence. We show that the given tuple $(\mathbb{P}; \mathbf{x}; g_{ij}, i, j \in h \in \mathbb{P}, |h| = 2)$ is indeed an equilibrium. The consumption allocation $\mathbf{x} = (x_i)_{i \in I}$ is feasible and $(x_{2\nu-1}, x_{2\nu})$ maximizes the partnership welfare function of household h_ν, $\nu = 1, \ldots, n$. For instance, consider partnership h_1, which solves

$$\begin{aligned} &\max_{x_1, x_2} = f(g_{12}, g_{21}) \cdot ln\, x_1 + g_{21} + (1 - f(g_{12}, g_{21})) \cdot ln\, x_2 + g_{12} \\ &s.t.\ x_1 + x_2 = 2y. \end{aligned}$$

Since $g_{12} = g_{21} = \overline{g}$ we have $f(g_{12}, g_{21}) = 1/2$ and therefore $x_1 = x_2 = y$.

Nobody can leave a partnership and benefit relative to the status quo: If an individual goes single, he foregoes the positive group externality. If two individuals form a new partnership and each chooses $\overline{g}$, neither one gains; if one of them chooses less than $\overline{g}$, at least one of them loses. Finally, if in an existing household, $\{i, j\}$, one member, say i, tried to improve his bargaining power by choosing $\hat{g}_{ij} < \overline{g}$, he would induce his partner j to leave and form a new partnership with some $k \notin \{i, j\}$ where they choose, for instance, $g_{jk} = \overline{g}$ and $g_{kj} = (\hat{g}_{ij} + \overline{g})/2$. Hence all the equilibrium conditions are met.

Uniqueness. Consider first a household structure where (at least) two individuals, say 1 and 2, remain single. Then they can do better by forming h_1 and choosing $g_{12} = g_{21} = \overline{g}$. Hence, all individuals have a partner in equilibrium.

Suppose second that $\mathbb{P} = \{h_1, \ldots, h_n\}$ is the prevailing household structure, but not everybody chooses $g_{ij} = \overline{g}$. Let g^* denote the highest choice made and I^* denote the set of individuals who made this choice.

If $I^* = I$, then $g_{ij} = g^* < \overline{g}$ for all ij and individuals 2 and 3 can both benefit from forming a new household and choosing $g_{23} = g_{32} = \overline{g}$.

If $I^* \neq I$, we distinguish two cases.

Case 1. At least one member of I^* is paired with a member of I^*, say $g_{12} = g_{21} = g^*$. Then there exists at least one other household, say h_2, with a member not belonging to I^*. Without loss of generality, let $g_{34} = \min\{g_{34}, g_{43}\} < g^*$. Then individuals 1 and 4 can both benefit from forming a new household and choosing $g_{41} = \overline{g}, g_{14} = \overline{g} - \epsilon$, with $\epsilon > 0$ sufficiently small.

Case 2. No member of I^* is paired with a member of I^*. If $|I^*| \geq 2$, then two members of i and j of I^* can form a new partnership, set $g_{ij} = g_{ji} = \overline{g}$ and both be better off. If $|I^*| = 1$, $I^* = \{k\}$, then there exists a household, say h_1, none of whose members belongs to I^*. Without loss of generality, let $g_{12} \leq g_{21} < g^*$. Then individuals 2 and k can both benefit from forming a new partnership and choosing $g_{2k} = g_{k2} = \overline{g}$.

We have shown that there is always a pair of individuals who can benefit from forming a new partnership if $g_{ij} = \overline{g}$ does not hold for some g_{ij}. Hence $g_{ij} = \overline{g}$ for all ij has to hold in equilibrium. This completes the proof. q.e.d.

Proof of Lemma 1

First of all, an equilibrium household structure consists of n two-person households, each consisting of a male and a female. For if there were a single person left, then there would also exist a single person of the opposite sex, since we ruled out same sex partnerships. These two individuals could do better by forming a two-person household where both choose $\overline{g}$.

Second, in equilibrium individuals of the same sex choose the same level of friendliness. Consider two two-person households $h_1 = \{a, b\}$ and $h_2 = \{i, j\}$ which belong to the equilibrium household structure. Let a and i be male and b and j be female. We have to show that $g_{ab} = g_{ij}$ and $g_{ba} = g_{ji}$. Suppose not. Without loss of generality, assume $g_{ab} > g_{ij}$. We shall show that this is inconsistent with equilibrium. We consider two cases.

If $g_{ji} \geq g_{ba}$, then a and j can form a new household, choose $g_{aj} = (g_{ab}+g_{ij})/2$ and $g_{ja} = g_{ji}$, and both be better off.

If $g_{ji} < g_{ba}$, i.e. $g_{ba} > g_{ji}$, then several subcases can be distinguished. In case $g_{ij} = 0$, equilibrium requires $g_{ji} = 0$. For otherwise j could fare better by going single. Without loss of generality, assume $g_{ab} \geq g_{ba} > 0$. Then a and j can benefit from forming a new household and choosing $g_{ja} = g_{ba}$ and $g_{aj} = g_{ab} - \epsilon$ with $\epsilon > 0$ sufficiently small. Similarly, $g_{ji} = 0$ would require $g_{ij} = 0$, and again at least one couple could benefit from forming a new household and choosing suitable levels of friendliness.

From now on let us assume $g_{ab} > g_{ij} > 0$ and $g_{ba} > g_{ji} > 0$. When $g_{ab}/g_{ij} \geq g_{ba}/g_{ji}$, a and j can benefit from forming a new household and choosing $g_{ja} = g_{ba}$ and $g_{aj} = g_{ab} - \epsilon$ with $\epsilon > 0$ sufficiently small. When $g_{ab}/g_{ij} < g_{ba}/g_{ji}$, b and i benefit from forming a new household and choosing $g_{ib} = g_{ab}$ and $g_{bi} = g_{ba} - \epsilon$ with $\epsilon > 0$ sufficiently small. This exhausts all possible subcases. We always found a pair of individuals who could benefit from forming a new household and choosing suitable levels of friendliness. Thus, we have demonstrated that $g_{ab} > g_{ij}$ is always inconsistent with equilibrium.

Hence, we have shown that in equilibrium, only male-female households are formed. Moreover, all males choose the same level of friendliness. An analogous argument shows that all females are equally friendly as well. This proves the assertion. q.e.d.

Proof of Corollary 1

The equal treatment property holds by the lemma. We can rule out that a group chooses zero friendliness in equilibrium by an argument developed in the proof of the lemma. Suppose now that all males choose $0 < g^* < \overline{g}$ in equilibrium and all females choose $0 < g^{**} < \overline{g}$. Without loss of generality, let $g^* \leq g^{**}$ and denote $\alpha^* = f(g^{**}, g^*)$. In case $g^* = g^{**}$, set $g^m = g^f = \overline{g}$. Then $f(g^f, g^m) = f(\overline{g}, \overline{g}) = \alpha^*$. In case $g^* < g^{**}$, $f(\overline{g}, g^*) > \alpha^* > 1/2 = f(\overline{g}, \overline{g})$. By the intermediate value theorem, we can pick $g^m \in (g^*, \overline{g})$ with $f(\overline{g}, g^m) = \alpha^*$. Further, let us set $g^f = \overline{g}$. In any case, there exist $g^m > g^*$ and $g^f > g^{**}$ such that $f(g^f, g^m) = \alpha^*$. Now take any male i and any female j who are not paired in the candidate equilibrium household structure. Then i and j can both benefit from forming a new household $\{i, j\}$ and choosing $g_{ij} = g^m, g_{ji} = g^f$, upsetting the equilibrium. Hence, to the contrary, in equilibrium at least one group has to choose the maximum level of friendliness, $\overline{g}$. But in order to prevent the members of this group from going single, the other group has to choose at least the level $\underline{g}$. Thus the equilibria described in Proposition 3 are the only ones. q.e.d.