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ABSTRACT

Real Exchange Rate Overshooting RBC Style

This paper establishes the ability of a Real Business Cycle model to account for real exchange rate (RXR) behaviour, using UK experience as empirical focus. We show that a productivity burst simulation is capable of explaining the appreciation of RXR and its cyclical pattern observed in the data. We then test if our model is consistent with the facts. We bootstrap our model to generate pseudo RXR series and check if the ARIMA parameters estimated for the data lie within 95% confidence limits implied by our model. We find that RXR behaviour is explicable within an RBC framework.

JEL Classification: E32, F31 and F41
Keywords: productivity, real business cycle and real exchange rate

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1 Introduction

The continuous strength of the dollar over the 1990s fuelled interest in the relationship between productivity and exchange rates. As US productivity surged in the second half of the 1990s, the dollar began its climb against all the major currencies of the world. This has led to a large body of literature analysing the links between the real exchange rate (RXR) and productivity. The 'conventional' view of the impact of a productivity shock on an economy is that the real exchange rate depreciates. However, this is completely at odds with the empirical findings of currency appreciation after a productivity spurt. It also fails to explain the cyclical pattern observed in the RXR data. In this paper we explore the ability of a Real Business Cycle (RBC) model to account for the behaviour of the RXR, using UK experience as our empirical focus. First, we find that a one percent deterministic productivity growth shock, shows clearly that the RXR appreciates on impact and then goes back to equilibrium[1], producing a business cycle - giving us the simulation properties that we are after. Second, we show that the RBC alone can reproduce the univariate properties of the RXR - by implication there is no necessary case here to add nominal rigidity.

A commonplace observation is that exchange rates appear to behave in seemingly inexplicable ways. A large number of studies have examined movements in the real exchange rate (RXR) [2] and found that they exhibit swings away from various definitions of 'purchasing power parity' (PPP) by which is meant the longer-run equilibrium value of RXR. Such an equilibrium is akin to the 'natural rate' of output or unemployment in a general equilibrium macroeconomic model and it may move over time for a variety of reasons - one commonly used model is that of Balassa and Samuelson based on differing productivity trends. Many studies have found definite evidence of reversion to PPP but very slow reversion. More

[1] This equilibrium however represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price.

[2] We define real exchange rate (Q_t) as the nominal exchange rate (S_t) adjusted for the ratio of foreign prices to domestic prices.

$$Q_t = \frac{S_t P_t^f}{P_t}$$

recently studies that have allowed for non-linear adjustment (such that as the real exchange rate moves further away from PPP the pressures of goods market arbitrage become stronger) have found that the speed of reversion is much greater, and becomes of similar order to that for other macro variables such as output and inflation.

One can think of these studies as final form equations of the RXR, where unspecified shocks to the economy, from demand and supply, stochastically disturb the RXR away from some smoothly-moving trend. Macroeconomic models that could in principle produce such a final form range from on the one hand models with a high degree of nominal rigidity to at the other extreme real business cycle models. However models relying on nominal rigidity - with a high implicit elasticity of output to shocks - have a problem in reproducing the considerable variability of the RXR exhibited by the data. It has been often remarked that these models to take a recent example have great difficulty in accounting for the very large swings of the dollar against the DM/Euro since 1995, first upwards and then downwards back to its 1995 starting point by early 2004.

In this paper we explore the ability of a Real Business Cycle (RBC) model to account for the behaviour of the RXR, using UK experience as our empirical focus. Our argument will be that the RBC alone can reproduce the univariate properties of the RXR. We do not rule out the possibility that adding a degree of nominal rigidity, in a sort of 'stretched-RBC' framework, could also be useful. However our concern is to establish the basic ability of the RBC alone to provide explanatory power.

We begin by looking at the empirical evidence on the sterling's real exchange rate. Figure 1 in the appendix shows the behaviour of the pound's RXR from 1986. While it appears to exhibit some sort of classically cyclical, mean reversion to a smoothly-moving trend or indeed even to a constant, the univariate final form equation is in fact best described by a $ARIMA(3,1,3)$ process; the series is therefore not actually mean-reverting but integrated of order 1 with pronounced serial correlation. Our main aim is to see whether our calibrated RBC model can generate the same univariate behaviour.

We begin by running a deterministic shock through a calibrated RBC model of the UK. We build and use a micro-founded stochastic general equilibrium open economy model based on optimising decision of rational agents; in this version we have assumed a real business cycle set-up in which money is irrelevant. The first order conditions of households' and firms' optimisation problems are used to derive the behavioural equations of the model. As we are modelling the UK, an economy of only modest size with minor effects on the rest of the world, we have a full blown model only for the domestic economy, taking the world economy as given. The interaction with the rest of the world comes in the form of Uncovered Real Interest Rate Parity (URIP) and the current account, both of which are explicitly micro-founded.

This deterministic simulation is done in order to establish the basic order of magnitude and shape of the response function of the RXR to the workhorse RBC technology shock. In the RBC world the workhorse shock is a burst of unanticipated productivity growth that raises the level of productivity in steady state well above its previous path. Figure 2 in the appendix shows the model simulation of such a burst: a 5-year rise of the productivity growth rate by 1% p.a. Note that the chart plots percentage deviation from the baseline path. The response profile is attractive in exhibiting a pronounced and persistent cycle.

The logic behind the RXR behaviour pattern can explained as follows. The productivity burst raises permanent income and also stimulates a stream of investments to raise the capital stock in line. Output however cannot be increased without increased labour supply and extra capital, which is slow to arrive. Thus the real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates Uncovered Real Interest Parity (URIP) which must be restored by a rise in the RXR relative to its expected future value. This rise is made possible by the expectation that the RXR will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival on stream of sufficient capital and so output, the RXR also moves back to equilibrium. This equilibrium however represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price.

Ultimately we can only settle whether our model could be consistent with the facts by asking whether it could have generated the patterns of RXR we find in the actual data. What we now do is to calibrate the model to our available data for the UK, and derive from this calibration the behaviour of the productivity and preference shocks. We then generate the sampling variability within the model by the method of bootstrapping the model's estimated residuals; this allows us to generate a large number of pseudo-samples of the RXR. We then run an $ARIMA$ for the RXR on all these pseudo-samples to generate the distribution of the $ARIMA$ parameters. In our final step we compare the estimated parameters for the UK RXR with this distribution, testing whether we can reject the RBC model at the 95% level of confidence; we would do this if the $ARIMA$ parameters lay outside the 95% confidence limits generated by the bootstrap process.

1.1 Real Exchange Rate & Productivity - background

The continuous strength of the dollar over the 1990s fuelled interest in the relationship between productivity and exchange rates. As US productivity surged in the second half of the 1990s, the dollar began its climb against all the major currencies of the world. Tille et al. (2001) point out that between 1995 and 1999, the dollar appreciated 4.8 percent against the yen and 5.8 percent against the euro on an average annual basis. The fact that these two trends were happening together tended to suggest that productivity gains were driving the appreciation of the dollar. Further, Alquist and Chinn (2002) find that the real exchange rate is cointegrated with a broad productivity differential. Using a number of different specifications, sample periods and estimation techniques, they find that each one percentage point increase in the US-Euro area productivity differential results in between a four to five percent appreciation in the dollar/euro exchange rate.

The 'conventional' view of the impact of a productivity shock on an economy is that a spurt in productivity leads to an expansion of output; if this extra output has to be sold in the world markets the price of the good must fall i.e. higher world supply of its goods should reduce their relative price. The country should experience a depreciation of the real exchange rate and a worsening of its terms of trade. However, the data is at odds with the theory. Further this view also fails to explain the cyclical patter that we observe in actual real exchange rate data[3].

Corsetti et al. (2004) point out that the repeated good news about growth in the US in 1999-2000, led to a dollar appreciation with upward revision of the growth gap relative to the euro. According to them, this can be interpreted in terms of 'crowding out', to borrow the terminology of the Mundell-Fleming model. The expectations of persistent productivity growth raise domestic consumption and investment much more than domestic supply. Forward looking consumers increase consumption due to expectations of higher future income and higher productivity increases expected future profits, raising investment demand. Now, in order for the markets to clear, a higher international price is needed to 'crowd-out' net exports. This would explain the appreciation of the dollar.

Bailey, Millard and Wells (2001) make explicit the theoretical link between profitability and the exchange rate - they argue that an increase in productivity raises future expected profits, raising equity prices and stimulating investment. This additional investment can be financed by capital inflows, enabling the domestic residents to finance the additional investment without forgoing any current consumption. These effects are consistent with other aspects of the 'New Economy' explanation of the appreciation of the dollar against the euro, as put forward by Meredith (2001). This line of explanation argues that high returns on the US investments attracted large foreign inflows of capital, which in turn appreciated the dollar through the capital account.

According to the Harrod-Balassa-Samuelson (HBS) Hypothesis, the real exchange rate does not respond to productivity differentials across countries; but to differences in the productivity gap between tradable and nontradable sectors of an economy. Suppose there is faster productivity growth in the tradable sector than the nontradable sector; the price of tradable goods will not change as they are tied to the world

[3]Fig. 1 in the appendix plots the real value of the pound sterling from 1986 till date.

market. Productivity gains will however lead to higher wages for workers in the tradable sector. Wages will also rise in the nontradable sector as employers seek to retain their workers. However, because of lower productivity gains the firms in this sector will be unable to absorb the wage increase and will pass it on to consumers in the form of higher prices. As a result the overall price index will increase.[4] This implies an appreciation of the real exchange rate. Such an appreciation can be intensified in the medium term by demand effects. Schnatz, Vijselaar and Osbat (2003) point out that the increased productivity raises expected future income, leading to an increased demand for goods. The increase in demand for traded goods can be satisfied by running a trade balance deficit. However, the increased demand in non-traded goods will lead to an increase in their prices. Thus, demand effects lead to a relative price shift and thereby a real appreciation.

Although this explanation is quite popular, it is unable to fully explain the appreciation of the dollar. Tille et al. (2001) find that at most this hypothesis can account for 2/3 of the dollar real appreciation in the 1990s. Further Corsetti et al. (2004) point out that there are many other studies which find that nontraded goods prices explain very little about exchange rate movements; in the 1990s the US real appreciation appears to be driven mostly by improvements in the US terms of trade, rather than nontraded prices. Furthermore, in today's age of information technology the nontradable sector itself is shrinking as most of the services can now be bought from across borders.

It is quite clear that there is no single factor model to determine the exchange rate. In general equilibrium, the exchange rate responds to many shocks - including productivity.

In the current paper we explore the ability of a Real Business Cycle (RBC) model to account for the behaviour of the RXR, calibrating the model to UK quarterly data. The objective of the paper is to establish that RBC alone can reproduce the univariate properties of the RXR. We however, do not rule out the possibility that adding a degree of nominal rigidity, in a sort of 'stretched-RBC' framework, could

[4]The finding that productivity gains lead to higher prices sounds puzzling. Note, however, that because the productivity increase boosts wages by more than prices, the purchasing power (wages deflated by price index) of the worker increases.

also be useful. We hope to test the impact of the explanatory power of nominal rigidity in forthcoming research.

The paper is organised as follows. In section II we set out the real business cycle model - in this version treating money as irrelevant. In section III we calibrate the model to UK quarterly data and show the results of a 1 percent deterministic productivity growth shock, which is very encouraging to the idea that the behaviour of real exchange rates is explicable within the RBC context. Section IV establishes the facts of RXR; it is integrated of order 1 and is highly persistent, the best fitting univariate process being an $ARIMA$(3,1,3). In section V we formally test our model and evaluate statistically whether our calibrated model is seriously consistent with the RXR data, using bootstrapping procedure. Section VI we conclude that RXR behaviour in fact can be explained using a pure RBC model with no nominal rigidity.

2 The Model

Consider an economy populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment[5]. We assume that there are no market imperfections. At the beginning of each period 't', the representative agent chooses (a) the commodity bundle necessary for consumption, (b) the total amount of leisure that she would like to enjoy, and (c) the total amount of factor inputs necessary to carry out production. All of these choices are constrained by the fixed amount of time available and the aggregate resource constraint that agents face. During the period 't', the model economy is influenced by various random shocks.

In an open economy goods can be traded but for simplicity it is assumed that these do not enter in the production process but are only exchanged as final goods. The consumption, C_t in the utility function below is composite per capita consumption, made up of agents consumption of domestic goods, C_t^d and their consumption of imported goods, C_t^f. [6] The composite consumption function can be represented as an Armington aggregator of the form

$$C_t = \left[\omega \left(C_t^d \right)^{-\rho} + (1 - \omega) \left(C_t^f \right)^{-\rho} \right]^{\left(\frac{-1}{\rho} \right)} \tag{1}$$

where ω is the weight of home goods in the consumption function and σ, the elasticity of substitution is equal to $\frac{1}{1+\rho}$.

The consumption-based price index that corresponds to the above specification of preference[7], denoted P_t is derived as

[5]To simplify the notation we abstract from population growth and represent all variables in per capita terms.

[6]It is to be noted C_t^f is the same as IM_tused later.

[7]The consumption-based price index P_t is defined as the minimum expenditure that is necessary to buy one unit of the composite good C_t, given the price of the domestic good and foreign good.

$$P_t = \left[\omega^{\frac{1}{1+\rho}} \left(P_t^d\right)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}} \left(P_t^F\right)^{\frac{\rho}{1+\rho}}\right] \tag{2}$$

where P_t^d is the domestic price level and P_t^F [8] is the foreign price level in domestic currency.

Given the specification of the consumption basket, the agent's demand for home and foreign goods are a function of their respective relative price and the composite consumption

$$C_t^d = \left(\frac{P_t^d}{\omega P_t}\right)^{-\left(\frac{1}{1+\rho}\right)} C_t \tag{3}$$

$$C_t^f = \left(\frac{P_t^F}{(1-\omega)\, P_t}\right)^{-\left(\frac{1}{1+\rho}\right)} C_t \tag{4}$$

In a stochastic environment a consumer is expected to maximise her expected utility subject to her budget constraint. Each agent's preferences are given by

$$U = MaxE_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right)\right], \qquad 0 < \beta < 1 \tag{5}$$

where β is the discount factor, C_t is consumption in period 't'[9], L_t is the amount of leisure time consumed in period 't' and E_0 is the mathematical expectations operator. The essential feature of this structure is that the agent's tastes are assumed to be constant over time.

The objective of this paper is to specify a fully articulated model of an open economy which we propose

[8] $P_t^F = S_t P_t^f$ where S_t is the nominal exchange rtae and P_t^f is the foreign price level in foreign prices.

[9] For the sake of convenience we shall use consumption in place of composite consumption through out the paper.

to calibrate/estimate using data for the UK[10]. We use this model to explain the behaviour of real exchange rate and also evaluate the impact of various demand and supply shocks.

[10] The model presented here is an enriched variant of a prototype RBC model embodying a representative agent framework as in McCallum (1989).

2.1 The Representative Household

The model economy is populated by a large number of identical household's who make consumption, investment, and labour supply decisions overtime. Each households objective is to choose sequences of consumption and hours of leisure that maximise its expected discounted stream of utility.[11] We assume a time-separable utility function of the form

$$U\left(C_t, 1-N_t\right) = \theta_0 \left(1-\rho_0\right)^{-1} C_t^{(1-\rho_0)} + \left(1-\theta_0\right)\left(1-\rho_2\right)^{-1}\left(1-N_t\right)^{(1-\rho_2)} \tag{6}$$

where $0 < \theta_0 < 1$, and ρ_0, $\rho_2 > 0$ are the substitution parameters. This sort of functional form is common in the literature for example McCallum and Nelson, (1999a).The advantage of using this specification is that it does not restrict elasticity of substitution between consumption and leisure to unity.[12]

Individual economic agents view themselves as playing a dynamic stochastic game. Changes in expectations about future events would generally affect current decisions. Each agent in our model is endowed with a fixed amount of time which she spends on leisure L_t and/or work N_t. If H_t, total endowment of time is normalised to unity, then it follows that

$$N_t + L_t = 1 \text{ or } L_t = 1 - N_t \tag{7}$$

Let us assume that $(\bar{l}\,)$ is the normal amount of leisure which is necessary for an agent to sustain her

[11]The utility function is assumed to possess the following properties. The representative agent is assumed to derive positive, but diminishing marginal utility from the consumption of goods and leisure. The utility function is further assumed to be strictly concave in its arguments i.e., consumption and leisure. In addition we postulate that consumption and leisure are normal goods, meaning that they both increase with wealth.

[12]The Cobb-Douglas utility function is a special case of the CES utility function when $\rho_0 = \rho_2 = 0$.

productivity over a period of time. If an agent prefers more than normal amount of leisure say 'U_t' she is assumed to be unemployed $\left(U_t = (1 - N_t) - \bar{l}\right)$ in this framework. An agent who chooses U_t is entitled to an unemployment benefit 'μ_t' from the state. It is assumed that $\mu_t < v_t$ (i.e., the consumer real wage as defined below) so that there is an incentive for the agent to search for a job. With the introduction of unemployment benefits substitution between work and leisure is higher.

The representative agents budget constraint is

$$\begin{aligned} (1+\phi_t)C_t + \tfrac{b_{t+1}}{1+r_t} + \tfrac{Q_t b^f_{t+1}}{\left(1+r^f_t\right)} + \tfrac{p_t S^p_t}{P_t} = \\ (1-\tau_{t-1})\, v_{t-1} N_{t-1} + \mu_{t-1}\left[(1-N_{t-1}) - \bar{l}\right] + b_t + Q_t b^f_t + \tfrac{(p_t+d_t)S^p_t}{P_t} \end{aligned} \tag{8}$$

where p_t denotes the present value of share, $v_t = \frac{W_t}{P_t}$[13] is the real consumer wage, $w_t = \frac{W_t}{P^d_t}$ is the producer real wage[14]. Consumption and labour income are taxed at rates ϕ_t and τ_t respectively, both of which are assumed to be stochastic processes. Also, $(1+\phi_t)\, C^d_t = \frac{M^{dp}_t}{P^d_t}$ i.e., representative agent's real demand for domestic money is equal to consumption of domestic goods inclusive of sales tax. In a similar way, the agent's real demand for foreign money is equal to consumption of foreign goods inclusive of sales tax, $(1+\phi_t)\, C^f_t = \frac{M^{Fp}_t}{P^F_t}$.[15] This follows from the fact that consumption in this framework is treated as a 'cash good' i.e., the cash-in-advance constraint is binding only in the case of consumption. Investment is treated as a credit good. b^f_t denotes foreign bonds, b_t domestic bonds, S^p_t demand for domestic shares and Q_t is the real exchange rate.

In a stochastic environment the representative agent maximizes her expected discounted stream of

[13] $P_t = \left[\omega^{\frac{1}{1+\rho}} \left(P^d_t\right)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}} \left(P^F_t\right)^{\frac{\rho}{1+\rho}}\right]$ and W_t is nominal wage.

[14] Please note that consumers take into account domestic and foreign prices while evaluating their real wages. However, producers do not, this is because they do not use imported intermediate goods.

[15] $M^{Fp}_t = S_t M^{fp}_t$, where M^{fp}_t is foreign money in foreign currency value.

utility subject to her budget constraint. The first order conditions with respect to C_t, N_t, b_t, b_t^f and S_t^p are:

$$(1-\rho_0)\,\theta_0\,(1-\rho_0)^{-1}\,C_t^{-\rho_0} = \lambda_t\,(1+\phi_t) \tag{9}$$

$$(1-\rho_2)\,(1-\theta_0)\,(1-\rho_2)^{-1}\,(1-N_t)^{-\rho_2} = \beta E_t \lambda_{t+1}\left[(1-\tau_t)\,v_t - \mu_t\right] \tag{10}$$

$$\frac{\lambda_t}{1+r_t} = \beta E_t \lambda_{t+1} \tag{11}$$

$$\frac{\lambda_t Q_t}{(1+r_t^f)} = \beta E_t \lambda_{t+1} Q_{t+1} \tag{12}$$

$$\frac{\lambda_t p_t}{P_t} = \beta E_t \lambda_{t+1}\left(\frac{p_{t+1}+d_{t+1}}{P_{t+1}}\right) \tag{13}$$

The first of the above equations equates the marginal utility of domestic consumption to the shadow price of output. Note that sales tax impinges on this equation. The second equates the marginal disutility of labour to labour's marginal product - the real wage. The marginal product of labour is affected both by tax on labour and the unemployment benefit. From the representative household's first-order condition we know that supply of labour is positively related to the net-of-tax real wage and negatively related to the unemployment benefit. If the after-tax real wage is temporarily high, substitution effect overpowers the income effect. The increase in work effort raises employment and output. On the other hand unemployment benefits negatively impinge upon supply of work effort. These equations which are the stochastic analogue of the well known Euler equations which characterizes the expected behavior of the economy, determine the time path of the economy's values of labour, consumption, and investments (in financial assets).

Substituting equation (11) in (9) yields [16] :

$$(1 + r_t) = \left(\frac{1}{\beta}\right)\left(\frac{C_t}{C_{t+1}}\right)^{-\rho_0}\left(\frac{1 + \phi_{t+1}}{1 + \phi_t}\right) \tag{14}$$

Now substituting (9) and (11) in (10) yields

$$(1 - N_t) = \left\{\frac{\theta_0 C_t^{-\rho_0}\left[(1 - \tau_t)\, v_t - \mu_t\right]}{(1 - \theta_0)(1 + \phi_t)(1 + r_t)}\right\}^{\frac{-1}{\rho_2}} \tag{15}$$

Substituting out for v_t equation (15) becomes[17]

$$(1 - N_t) = \left\{\frac{\theta_0 C_t^{-\rho_0}\left[(1 - \tau_t)\ \ \exp\left(\log w_t^* - \frac{(1-\omega)^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}}\log Q_t\right) - \mu_t\right]}{(1 - \theta_0)(1 + \phi_t)(1 + r_t)}\right\}^{\frac{-1}{\rho_2}} \tag{16}$$

Substituting (11) in (13) yields

$$p_t = \left(\frac{p_{t+1} + d_{t+1}}{(1 + r_t)}\right)\frac{P_t}{P_{t+1}} \tag{17}$$

[16]All future values are expected - for convenience expectations operator is dropped.

[17]Using $v_t = \frac{W_t}{P_t}$, $w_t = \frac{W_t}{P_t^d}$ and $Q_t = \frac{S_t P_t^f}{P_t}$.

Using the arbitrage condition and by forward substitution the above yields

$$p_t = \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1+r_t)^i} \left(\frac{P_t}{P_{t+i}} \right) \tag{18}$$

The above equation states that the present value of a share is simply discounted future dividends.

In small open economy models the domestic real interest rate is equal to the world real interest rate, which is taken as given. Further, it is assumed that the economy has basically no effect on the world rate because, being a small part of the world, its affect on the world savings and investment is negligible. These assumptions imply that the real exchange rate for the small open economy is constant. However, we are modelling a medium sized economy. In our set up the economy is small enough to continue with the assumption that world interest rates are exogenous but large enough for the domestic rate to deviate from the world rate. Hence, in our model real exchange rates are constantly varying.

To derive the uncovered interest parity condition equation (11) is substituted into (12)

$$\left(\frac{1+r_t}{1+r_t^f} \right) = \frac{Q_{t+1}}{Q_t} \tag{19}$$

In logs this yields to

$$r_t = r_t^f + E_t \Delta \log Q_{t+1} \tag{20}$$

2.2 The Government

In this framework it is assumed that the government spends current output according to a non-negative stochastic process that satisfies $G_t \leq Y_t$ for all 't'[18]. In the case of equilibrium business cycle models embodying rational expectations, output is always at its 'desired' level. Given the information set, agents are maximizing their welfare subject to their constraints. Since there are no distortions in this set-up government expenditure may not improve welfare through its stabilization program. As stated above the state also pays out unemployments benefits μ_t which leads to higher substitution between work and leisure.

The government finances its expenditure by collecting taxes on labour income, τ_t, and taxes on consumption, ϕ_t, which are assumed to be stochastic processes. Also, it issues debt, bonds (b_t) each period which pays a return next period. Then, it collects seigniorage, i.e., $\frac{M^d_{t+1}-M^d_t}{P^d_t}$ which is assumed to act as a lump-sum tax, leaving real asset prices and allocation unaltered and is assumed to be a stochastic process.

The government budget constraint is:

$$G_t + b_t + \mu_t \left[(1 - N_t) - \bar{l}\right] = \tau_{t-1} v_{t-1} N_{t-1} + \phi_{t-1} C_{t-1} + \frac{b_{t+1}}{1 + r_t} + \frac{M^d_{t+1} - M^d_t}{P^d_t} \tag{21}$$

where b_t is real bonds and P^d_t is the domestic price level. Note that $\tau_{t-1} v_{t-1} N_{t-1} + \phi_{t-1} C_{t-1}$ is the total tax revenue collected by the state. Also, the government faces a cash-in advance constraint, i.e.:

$$P^d_t G_t \leq M^{dg}_t \tag{22}$$

where M^{dg}_t is government's demand for domestic money. Here we assume that the government has home bias, i.e. it consumes only domestic goods.

[18]The variable G_t denotes per capita government expenditure at 't'.

2.3 The Representative Firm

Firms rent labour and buy capital inputs from households[19] and transform them into output according to a production technology and sell consumption and investment goods to households and government. The technology available to the economy is described by a constant-returns-to scale production function:

$$Y_t = Z_t N_t^{\alpha} K_t^{1-\alpha} \tag{23}$$

where $0 \leq \alpha \leq 1$, Y_t is aggregate output per capita, K_t is capital carried over from previous period $(t-1)$, and Z_t reflects the state of technology.

It is assumed that $f(N, K)$ is smooth and concave and it satisfies *Inada-type* conditions i.e., the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to 0 and approaches 0 as capital (or labour) goes to infinity.

$$\begin{aligned} \lim_{K \longrightarrow 0} (F_K) &= \lim_{N \longrightarrow 0} (F_N) = \infty \\ \lim_{K \longrightarrow \infty} (F_K) &= \lim_{N \longrightarrow \infty} (F_N) = 0 \end{aligned} \tag{24}$$

The capital stock evolves according to:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{25}$$

where δ is the depreciation rate and I_t is gross investment.

In a stochastic environment the firm maximizes present discounted stream, V, of cash flows, subject to

[19] Households own shares in the firms and therefore own them.

the constant-returns-to-scale production technology,

$$MaxV = E_t \sum_{i=0}^{T} d_{it}^{i} \left(Y_t - K_t(r_t + \delta) - w_t N_t^d\right) \tag{26}$$

subject to the evolution of the capital stock in the economy, equation (25). Here r_t and w_t are the rental rates of capital and labour inputs used by the firm, both of which are taken as given by the firm. Output of the firm depends not only on capital and labour inputs but also on Z_t. The firm optimally chooses capital and labour so that marginal products are equal to the price per unit of input. The first order conditions with respect to K_t and N_t^d are as follows:

$$K_t = \frac{(1-\alpha) Y_t}{r_t + \delta} \tag{28}$$

$$N_t^d = \left(\frac{w_t}{\alpha Z_t}\right)^{\left(\frac{1}{\alpha - 1}\right)} K_t \tag{29}$$

The non-negativity constraint applies i.e., $K_t \geq 0$.

2.4 The Foreign Sector

As Obstfeld and Rogoff (1996) argue, relative prices are a central feature of open economy macroeconomics. In particular the response of the trade balance to shocks on the terms of trade has preoccupied trade theorists for decades. In open economies a country's investment and consumption plans are no longer constrained by its own production frontier.

In a stochastic environment the representative agent maximizes her expected discounted stream of utility subject to her budget constraint. In order to derive the real exchange rate and hence the balance of payments explicitly from micro-foundations we take into account the consumption constraint on the agent

$$P_t C_t = P_t^d C_t^d + P_t^F C_t^f \tag{30}$$

Now the first order conditions with respect to C_t^d and C_t^f associated with the agent's maximisation subject to the budget as well as consumption constraint are :

$$\theta_0 C_t^{-\rho_0} \frac{\partial C_t}{\partial C_t^d} - \lambda_t (1 + \phi_t) \frac{\partial C_t}{\partial C_t^d} - \lambda_t^c P_t^d + \lambda_t^c P_t \frac{\partial C_t}{\partial C_t^d} \tag{31}$$

$$\theta_0 C_t^{-\rho_0} \frac{\partial C_t}{\partial C_t^f} - \lambda_t (1 + \phi_t) \frac{\partial C_t}{\partial C_t^f} - \lambda_t^c P_t^F + \lambda_t^c P_t \frac{\partial C_t}{\partial C_t^f} \tag{32}$$

Dividing equation (32) by equation (31), we have the real exchange rate[20]

[20] In equilibrium, terms of trade can be computed from the intra-temporal marginal rate of substitution between goods in the Armington aggregator function, see Backus, Kehoe and Kydland (1994). The marginal rate of substitution, i.e., the slope of the indifference curve is given by

$$Q_t = \left(\frac{1-\omega}{\omega}\right)\left(\frac{C_t^d}{C_t^f}\right)^{\left(\frac{1}{\sigma}\right)} \tag{33}$$

To the extent that home and imported goods are not perfect substitutes, σ will take some finite value. The lower the estimated σ means the less the substitution between the two goods. In other words the greater the degree of product differentiation, the smaller the elasticity of substitution between the products.

From the real exchange rate equation we can derive import equation for our economy[21]

$$\log IM_t = \sigma \log(1-\omega) + \log C_t - \sigma A \log Q_t \tag{34}$$

$$A = \frac{\omega^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}} \tag{35}$$

The equation states that imports into the country are positively related to the total consumption in the home country and negatively related to the real exchange rate, i.e. as Q_t increases that is the currency depreciates, import demand falls.

Now there exists a corresponding real exchange rate equation for the foreign country.

$$Q_t^f = \left(\frac{1-\omega^f}{\omega^f}\right)\left(\frac{C_t^{df}}{C_t^{ff}}\right)^{\left(\frac{1}{\sigma_1}\right)} \tag{36}$$

$$\frac{P_t^F}{P_t^d} = \frac{\frac{\partial C_t}{\partial C_t^f}}{\frac{\partial C_t}{C_t^d}} = \left(\frac{1-\omega}{\omega}\right)\left(\frac{C_t^d}{C_t^f}\right)^{1+\rho}$$

[21]Note that $IM_t = C_t^f$. Full derivation is available from the authors on request.

where C_t^F is the composite consumption of the foreign country, C_t^{df} is the foreign country's consumption of own goods, C_t^{ff} is the foreign country's consumption of home goods[22], ω^f is the weight of foreign country's own goods in its composite consumption function, Q_t^f is the real exchange rate for the foreign country[23], $\sigma_1 = \frac{1}{1+\rho_3}$ is the elasticity of substitution between home goods i.e. home exports and foreign country's own goods.

From the foreign real exchange rate equation we can derive export equation for our economy

$$\log EX_t = \sigma_1 \log\left(1 - \omega^f\right) + \log C_t^F + \sigma_1 A^f \log Q_t \tag{37}$$

$$A^f = \frac{\left(\omega^f\right)^{\frac{1}{1+\rho}}}{\left(\omega^f\right)^{\frac{1}{1+\rho}} + \left(1 - \omega^f\right)^{\frac{1}{1+\rho}}} \tag{38}$$

The equation states that exports of the home country are a positive function of the total consumption in the foreign country and also a positive function of the real exchange rate. If Q_t increases, i.e. the home currency depreciates then exports will increase.

In the model home and foreign agents need foreign and home money respectively, in order to transact with each other. The foreign agents need home money to buy our exports, but get home money for imports as well as our purchase of foreign bonds. So their net supply of foreign money is equal to net exports plus sales of foreign bonds i.e. the balance of payments surplus. This surplus is equal to the home agents net demand for foreign money, who get foreign money from firms exporting to foreign agents and need foreign money for imports and purchases of foreign bonds. So if home agents adjust their sales of foreign bonds

[22]Note that $EX_t = C_t^{ff}$
[23]Please note $Q_t^f = \frac{1}{Q_t}$

then all balances. In equilibrium it is assumed that exports and imports are equal and hence the agents would have no tendency to change their asset position. In disequilibrium the changes between domestic and foreign bonds will depend upon net exports.

$$NX_t = EX_t - IM_t \tag{39}$$

Foreign bonds thus evolve over time according to the following equation

$$b_{t+1}^{f} = (1 + r_t^{f}) b_t^{f} + NX_t \tag{40}$$

3 Calibration & Model Simulation

3.1 Calibration

In order to carry out model simulations, numerical values should be assigned to the structural parameters of the models.[24] These values, such as for example the output elasticity of the production factors, the degree of risk-aversion or the elasticity of intertemporal substitution, are taken from micro data estimates or from some casual empirical characteristics for the economy which is to be studied. For instance Kydland and Prescott (1982) derive values for some of the remaining structural parameters so that their steady-state levels match sample averages observed in actual time series data.

The exogenous stochastic processes should also be calibrated. However, it is hard to find information from a real economy concerning the stochastic structure of technology shocks, shocks to preferences, error of controlling money growth or tax revenues, or the correlations among them. For this purpose, persistence properties in actual time series data can be used to calibrate some aspects of the model. For instance, in the simplest business cycle model, an $AR(1)$ model is assumed for productivity shocks, with the coefficient generally chosen so that the simulated output series exhibits persistence similar to the GNP series in actual economies.

3.2 Model Simulation

Once the model has been solved numerically, one can analyse the characteristics of the transition of the model to its steady-state. This may arise either because initially the economy is outside steady-state or because some structural change is introduced (it could be a policy intervention) altering the steady-state. This type of analysis is crucial, among other things, to evaluate the possible effects of change in policy rules i.e., of policy interventions and to assess the overall properties of the model.

[24]See appendix for values of parameters used.

Our simulations start in 1986 (quarter 3) and end in 2000 using quarterly UK data. Results of our simulation exercise are reported in graphical form in the appendix. The charts show the percentage deviation of a particular variable - real output, price level, and so on - from the baseline path except in the case of interest rates where it shows percentage point deviations from the baseline.

3.2.1 Results

The effects of both demand and supply shocks on the behaviour of output, consumption, capital stock, investment, employment, price level, real wage, real interest rate, imports, exports, and real exchange rate can be examined by deterministically simulating the calibrated model. However, in the current paper we are concentrate on the explanation of real exchange rate behaviour[25].

For the *baseline* simulation - that is, the simulation with no change in policy instruments - the endogenous variables are set so as to track the actual historical values perfectly. This is done by adding residuals to each equation. The residuals are computed as if the future expectations of the endogenous variables that appear in the model are equal to the actual values. These residuals therefore include not only the shocks to the equations, but also the forecast errors.

1% p.a. Productivity Growth Shock

Consider the case of an unanticipated 1% per annum growth shock to productivity till year five and then 5% permanent increase in the level of productivity relative to the base from the fifth year onwards. So productivity grows at 1% in the first year, 2% in the second year and so on till the fifth year when it grows at 5%. After that it is permanently 5% above the base. Although unanticipated at the time of the initial increase, the entire path of productivity spurt is assumed to be incorporated into agent's forecasts as of the first quarter of the simulation. The predictions of the model for the case of an increase in productivity are shown in Fig 2.

The productivity burst raises permanent income and also stimulates a stream of investments to raise

[25]The interested reader can obtain simulation results for other demand and supply shocks directly from the authors

the capital stock in line. Output however cannot be increased without increased labour supply and extra capital. The increased demand for labour pushes up the real wage. Now extra capital is slow to arrive. Thus the real interest rate must rise to reduce demand of capital to the available supply. The rising real interest rate violates Uncovered Real Interest Parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that the real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival on stream of sufficient capital and so output, the real exchange rate also moves back to equilibrium. It must be noted however that this new equilibrium represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price.

4 Data Patterns

In this section we estimate univariate processes for the real exchange rate. The path of the sterling real exchange rate (RXR) is presented in Fig 1. The RXR data used is the same as in the Liverpool model; the ratio of UK to other OECD consumer prices adjusted for nominal exchange rate, where the nominal exchange rate is the sterling effective exchange rate.

Our first observation is that there is no apparent trend in the real exchange rate and it has a non-zero mean. There is a large body of literature that finds the real exchange rate to be non-stationary[26]. In order to estimate non-spurious univariate processes we first check for non-stationarity.

Using both the Augmented Dickey Fuller test and Phillips-Perron test, we find that the sterling's RXR is an $I(1)$ series. Table 1 reports the results[27]. The RXR series in levels fails to reject the null hypothesis of non-stationarity at 1 percent level of significance, using both the ADF and the PP test statistic. When we test with the first difference form of the series we can easily reject the null, again at 1 percent.

Having established the non-stationarity of the series we now proceed to estimate the best fitting $ARIMA$ process to the real exchange rate, using data from 1986:1 till 2000:4. We take into account seasonal dummies as it is a well-known fact that price series are seasonal in nature. Table 3 summarises our results. Clearly the results below indicate that RXR is a highly persistent series. An $ARIMA$(3,1,3) best describes the data.

[26] See for example Alquist and Chinn (2002).

[27] For detailed results please see Table 2

5 Bootstrapping

Comparison of our model with the $ARIMA$ we have estimated on the actual data cannot be done via deterministic simulation because the estimated equations depend on the distributions of all the shocks, each of them with rather different impulse response functions because of different MA processes. What we wish to do is to replicate the stochastic environment to see whether within it our estimated $ARIMA$ equations could have been generated. This we do via bootstrapping the models above with their error processes.

To do this we generate the sampling variability within the model by the method of bootstrapping the model's estimated residuals; this permits us to find the 95% confidence limits around the RXR $ARIMA$ regression parameters. The idea is to create pseudo data samples (here 500) for the real exchange rate. We draw the vectors of iid shocks in our error processes with replacement, by drawing vectors for the same time period we preserve their contemporaneous cross-correlations; we then input them into their error processes and these in turn into the model to solve for the implied path of RXR over the sample period. In Fig 3 we plot a random selection of the pseudo RXR series (shaded/dashed lines) generated by bootstrapping the model's errors with the historical RXR data for the UK (solid line). Note that actual data starts from 1986:1 while the model generated series start from 1986:3. As can be seen from the graph the model is capable of generating series with lots of cycles, very much like the true RXR series.

We run $ARIMA$ regressions on all the samples to derive the implied 95% confidence intervals for all the coefficients. Finally we compare the $ARIMA$ coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals The comparison both guides us on whether our model is moving the parameters in the right direction; and informs us more formally whether the data rejects the model. The Table 5 summarises the results of this exercise.

The above results clearly validate our hypothesis that real exchange rate behaviour is explicable within the RBC framework. Five out of the six $ARIMA$ parameters, comfortably lie within the 95% confidence intervals. $MA(3)$ lies only marginally outside the upper limit. The model also captures the direction of movement of the parameters.

6 Conclusion

In this paper we have built a micro founded general equilibrium open economy model based on optimising decisions of rational agents; in this version we have assumed a real business cycle set-up in which money is irrelevant. The first order conditions of the household and firm optimisation problem are used to derive the behavioural equations of the model. We are modelling a medium-sized open economy, so we have a full blown model only for the domestic economy taking the world economy as given. The interaction with the rest of the world comes in the form of uncovered real interest rate parity and the current account, both of which are explicitly micro-founded.

It is a well established empirical fact that a burst in productivity leads to an appreciation of the currency. However, according to the 'conventional' view, if a country becomes more productive, a higher world supply of its good should result in a relative price reduction. The country should experience a deterioration of the terms of trade. This is completely at odds with the data. The Harrod-Balassa-Samuelson (HBS) Hypothesis, can explain the appreciation following a burst in productivity in the tradable sector. However, it still fails to explain the cycles that we observe in actual real exchange rate data.

The objective of this paper is to specify a fully articulated model of an open economy which we calibrate using data for the UK and use to explain the behaviour of the real exchange rate; both the appreciation following a productivity burst and the cyclical pattern.

Our deterministic simulation of a 1 percent productivity growth shock clearly shows that the real exchange appreciates on impact and then goes back to its new depreciated equilibrium. The reasoning is quite simple, yet it has an intuitive appeal. As productivity increases, permanent income expands. However, for output to increase extra labour and capital is required. Capital, is slow to arrive, which pushes up the real interest rate in the economy so that the capital market clears. The increase in the real interest rate violates the uncovered real interest rate parity. UIRP is established by an appreciation of the pound with the expectation that it will depreciate in the future back to its equilibrium value. This equilibrium however represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price. This gives us a 'business cycle' in the real exchange rate.

Ultimately we can only settle whether our model could be consistent with the facts by asking whether it could have generated the patterns we find in the actual real exchange rate data. To do this we generate the sampling variability within the model under each regime by the method of bootstrapping the model's estimated residuals; this permits us to find the 95% confidence limits around the real exchange rate $ARIMA$ regression parameters. This tells us what the standard errors of these regressions are *under the null hypothesis of our model*- the relevant standard errors for us, rather than the usual ones which tell us whether the regression, viewed atheoretically, can reject a zero null hypothesis, a fairly uninteresting one for an economist.

We find that our model tells quite a good story, the gyrations of the real exchange rate can be explained within a RBC context. We do not rule out the possibility that adding a degree of nominal rigidity, in a sort of 'stretched-RBC' framework, could also be useful. However our concern here has been to establish the basic ability of the RBC alone to provide explanatory power.

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Appendix: Behavioural Equations of the RBC Model - Steady State Equations - Exogenous Processes - Values of Coefficients

Behavioural Equations

(1) Consumption C_t ; solves for r_t:

$$\begin{aligned}(1+r_t) &= \frac{1}{\beta}\left(\frac{C_t}{E_t\left[C_{t+1}\right]}\right)^{-\rho_0}\left(\frac{1+\phi_{t+1}}{1+\phi_t}\right)\\ r_t &= \frac{1}{\beta}\left(\frac{C_t}{E_t\left[C_{t+1}\right]}\right)^{-\rho_0}\left(\frac{1+\phi_{t+1}}{1+\phi_t}\right)-1\end{aligned}$$

where $C_t = \left[\omega\left(C_t^d\right)^{-\rho} + (1-\omega)\left(C_t^f\right)^{-\rho}\right]^{\left(\frac{-1}{\rho}\right)}$

(2) Money supply $\overline{M}_t^d$; solves for P_t^d:

$$\begin{aligned}\overline{M}_t^d &= (1+\phi_t)C_t^d P_t^d + \overline{G}_t P_t^d\\ P_t^d &= \frac{\overline{M}_t^d}{(1+\phi_t)C_t^d + \overline{G}_t}\end{aligned}$$

where $P_t = \left[\omega^{\frac{1}{1+\rho}}\left(P_t^d\right)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}\left(P_t^F\right)^{\frac{\rho}{1+\rho}}\right]$

(3) Demand for shares, S_{t+1}^p :

$$S_{t+1}^p = \overline{S}_t \text{ ; } b_{t+1} = b_{t+1}^p \text{ implied.}$$

(4) Present value of share :

$$p_t = E_t\sum_{i=1}^{\infty}\frac{d_{t+i}}{(1+r_t)^i}\left(\frac{P_t}{P_{t+i}}\right)$$

where d_t (dividend per share), p_t (present value of shares in nominal terms).

(5) Production function Y_t:

$$Y_t = Z_t N_t^{\alpha} K_t^{(1-\alpha)}$$

(6) Demand for labour :

$$N_t^d = \left(\frac{\alpha Z_t}{w_t}\right)^{\left(\frac{1}{1-\alpha}\right)} K_t$$

(7) Capital :

$$K_t = (1-\alpha)\frac{Y_t}{r_t + \delta}$$

(8) GDP identity, Y_t; solves for C_t:

$$Y_t = C_t + I_t + G_t + NX_t$$

where NX_t is net exports.

(9) Investment :

$$K_{t+1} = (1-\delta)K_t + I_t$$

(10) W_t ; currently this variable is not defined.

(11) Wage w_t :

$$w_t = w_t^*$$

(12) Evolution of b_t ; government budget constraint:

$$b_{t+1} = (1 + r_t)b_t + PD_t - \frac{\Delta \overline{M}_t}{P_t}$$

(13) Equilibrium wage, w_t^*; w_t^* is derived by equating demand for labour, N_t^d, to the supply of labour N_t^s, where

$$(1 - N_t^s) = \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[(1 - \tau_t) \ \ \exp\left(\log w_t^* - \frac{(1-\omega)^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}} \log Q_t \right) - \mu_t \right]}{(1 - \theta_0)\,(1 + \phi_t)\,(1 + r_t)} \right\}^{\frac{-1}{\rho_2}}$$

where Q_t is the real exchange rate, $(1-\omega)^{\frac{1}{1+\rho}}$ is the weight of domestic prices in the CPI index.

(14) Dividends are surplus corporate cash flow :

$$\begin{aligned} d_t \overline{S_t} &= Y_t - N_t^s w_t - K_t(r_t + \delta) \\ d_t &= \frac{Y_t - N_t^s w_t - K_t(r_t + \delta)}{\overline{S}_t} \end{aligned}$$

(15) Primary deficit PD_t :

$$PD_t = G_t + \mu_t \left(1 - N_t^s - \overline{l}\right) - \tau_{t-1} v_{t-1} N_{t-1}^s - \phi_{t-1} C_{t-1} - T_{t-1}$$

(16) Tax T_t :

$$T_t = T_{t-1} + \gamma^G \left(PD_{t-1} + b_t r_t\right) + \varepsilon_t$$

(17) Exports EX_t:

$$\log EX_t = \sigma_1 \log\left(1-\omega^f\right) + \log C_t^F + \sigma_1 A^f \log Q_t$$

where $A^f = \frac{\left(\omega^f\right)^{\frac{1}{1+\rho}}}{\left(\omega^f\right)^{\frac{1}{1+\rho}} + \left(1-\omega^f\right)^{\frac{1}{1+\rho}}}$

(18) Imports IM_t:

$$\log IM_t = \sigma \log(1-\omega) + \log C_t - \sigma A \log Q_t$$

where $A = \frac{\omega^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}}$

(19) UIP condition:

$$r_t = r_t^f + E_t \bigtriangleup \log Q_{t+1} + \varepsilon_{UIP}$$

where r^f defined the foreign real interest rate.

(20) Net exports:

$$NX_t = EX_t - IM_t$$

(21) Evolution of foreign bonds b_t^f:

$$b_{t+1}^f = (1 + r_t^f) b_t^f + NX_t$$

(22) Nominal exchange rate, S_t:

$$\log S_t = \log Q_t - \log P_t^f + \log P_t$$

where P_t^f is foreign price and S_t the nominal exchange rate.

(23) Evolution of household debt D_{t+1}:

$$D_{t+1} = (1 + r_t)D_t - Y_{t-1} + (1 + \phi_t)C_t + \tau_t v_t N_t^s + T_t$$

(24) Household transversality condition:

$$Y_{T-1} - r_T D_T - \phi_T C_T - \tau_T v_T N_T^s - T_T = C_T$$

(25) Government transversality condition:

$$G = 0.30$$

Steady State Equations of the model

$\beta = \frac{1}{1+r_t}$

$\overline{M} = (1+\phi)CP + GP$

$p = \frac{d}{1+r}$

$Y = ZN^{\alpha}K^{(1-\alpha)}$

$N = \frac{\alpha Y}{w^{*}}$

$K = \frac{(1-\alpha)Y}{r+\delta}$

$Y = C + I + G + NX$

$I = \delta K$

$w = w^{\star}$

$rb = -PD$

$1 - N = \left(\frac{\theta_0 C^{-\rho_0}[(w^{*}-(1-h)Q)(1-\tau)-\mu]}{(1-\theta_0)(1+\phi)(1+r)} \right)^{-\frac{1}{\rho_2}}$

$d = \frac{Y - w^{*}N - (r+\delta)K}{\overline{S}}$

$PD = G + \mu(1 - N - \overline{l}) - \tau\nu N - \phi C - T$

$r = r^{f}$

$r^{f} b^{f} = NX$

$\log\ Q = \log S$

$Y - (1+\phi)C - \tau\nu N - T = rD$

Exogenous processes

(1) $\Delta \ln Z_t = \varepsilon_{1,t}$

(2) $\Delta \tau_t = \varepsilon_{2,t}$

(3) $\Delta \phi_t = \varepsilon_{3,t}$

(4) $\Delta \mu_t = \varepsilon_{4,t}$

(5) $\Delta \ln \overline{M}_t = \varepsilon_{5,t}$

(6) $\Delta \ln P_t^f = \varepsilon_{6,t}$

(7) $\Delta \ln C_t^F = \varepsilon_{7,t}$

(8) $\Delta \ln r_t^f = \varepsilon_{8,t}$

Values of coefficients

Note: the values of the coefficients used in the model have been calibrated from the recent literature.

Coefficient	Value - Single equation
α	0.70
β	0.97
δ	0.0125
ρ_0	1.20
θ_0	0.50
γ^G	0.05
ρ_2	1.00
ω	0.70
ρ	-0.50
ω^f	0.70
h	0.80
ρ_3	-0.50
σ	2
σ_1	2

References

[1] Alquist, R. and M. D. Chinn (2002), 'The Euro and the Productivity Puzzle: An Alternative Interpretation' mimeo.

[2] Armington, P.S. (1969), 'A theory of demand for products distinguished by place of production', IMF Staff Paper, Vol. 16, 159-178.

[3] Backus, D., and P. Kehoe (1989), 'International evidence on the historical properties of business cycles', Federal Reserve Bank of Minneapolis, Research Department, Working Paper No. 402R.

[4] Backus, D., Kehoe, and F. Kydland (1994), 'Dynamics of the trade balance and the terms of trade: The J-curve?', American Economic Review, Vol. 84, 84-103.

[5] Bailey, A., S. Millard and S. Wells (2001), 'Capital Flows and Exchange Rates', Bank of England Quarterly Bulletin, Autumn 2001.

[6] Benigno, G. and P. Benigno (2001), 'Monetary Policy Rules and the Exchange Rate', CEPR Discussion Paper 2807.

[7] Braun, R.A. (1994), 'Tax Disturbances and real economic activity in the postwar United States', Journal of Monetary Economics, Vol. 33, 441-462.

[8] Christiano, L.J. and M. Eichembaum. (1992), 'Current Real-Business-Cycle Theories and Aggregate Labour-Market Fluctuations', The American Economic Review, Vol.82, No.3, 430-450.

[9] Cooley, T.F. and G.D. Hansen. (1989), 'The Inflation Tax in a Real Business Cycle Model', The American Economic Review, Vol.7, 4, 733-748.

[10] Cooper, R.W. (1997), 'Business Cycles: Theory, Evidence and Implications', NBER Working Paper No.5994, 1-44.

[11] Correia, I., S. Rabelo and J.C. Naves (1995), 'Business Cycles in a Small Open Economy', CEPR Discussion Paper No. 996.

[12] Corsetti, G. (2004), 'Productivity and the Euro-dollar Exchange Rate', Five Years of the Euro: Successes and New Challenges, Federal Reserve Bank of Dallas, May 2004.

[13] Corsetti, G., L. Dedola and S. Leduc (2004), 'International Risk Sharing and the Transmission of Productivity Shocks', ECB Working Paper No. 308.

[14] Dellas, H. and G. Tavlas (2002), 'Wage rigidity and monetary union', CEPR Discussion Paper No. 3679.

[15] Hansen, G.D. (1985), 'Indivisible Labour and the Business Cycle', Journal of Monetary Economics, Vol.16, 309-327.

[16] King, R.G. and C.I. Plosser. (1984), 'Money, Credit, and Prices in a Real Business Cycle', The American Economic Review, Vol.74, 3, 363-380.

[17] Kydland, F. and E.C. Prescott. (1982), 'Time to Build and Aggregate Fluctuations', Econometrica, 1345-1370.

[18] Long, B.L and C.I. Plosser. (1983), 'Real Business Cycles', Journal of Political Economy, Vol.91, 1, 39-69.

[19] Lucas, R.E. Jr. (1977), 'Understanding Business Cycles', in K. Brunner and A.H. Meltzer (eds), 'In stabilization of the domestic and International Economy', Vol. 5 of Carnegie-Rochester Conference Series on Public Policy. Amsterdam: North Holland Company.

[20] Lundvik, P. , (1992), 'Foreign demand and domestic business cycles: Sweden 1871-1988', Institute for International Economic Studies.

[21] Mankiw, N.G. (1989), 'Real Business Cycles: A New Keynesian Perspective', Journal of Economic Perspectives, Vol.3, 3, 79-90.

[22] McCallum, B.T. (1989), 'Real Business Cycles Models', in R.J. Barro (ed), 'Modern Business Cycles Theory'. Basil Blackwell, Oxford.

[23] McCallum, B.T., and E. Nelson. (1999a), 'An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis', Journal of Money Credit and Banking, Vol. 31, 296-316.

[24] McGrattan, E.R. (1994), 'The macroeconomic effects of distortionary taxation', Journal of Monetary Economics, Vol. 33, 573-601.

[25] Mendoza, E., (1991), 'Real business cycles in a small open economy', The American Economic Review, Vol. 81, 797-818.

[26] Meredith, G. (2001), 'Why Has the Euro Been So Weak?', IMF Working Paper 01/155.

[27] Obstfeld, M. and K. Rogoff (2001), 'Global Implications of Self-oriented Monetary Policy Rules', mimeo.

[28] Obstfeld, M. and K. Rogoff (1996), 'Foundations of International Macroeconomics', Cambridge, MA: MIT Press.

[29] Obstfeld, M. and K. Rogoff (1995), 'Exchange Rate Dynamics Redux', Journal of Political Economy, Vol. 103, 624-660.

[30] Schnatz, B., F. Vijselaar and C. Osbat, (2003), 'Productivity snd the "Synthetic" Euro-Dollar Exchange Rate', ECB Working Paper No. 225

[31] Tille, C., N. Stoffels and O. Gorbachev (2001), 'To What Extent Productivity Drive the Dollar?', Federal Reserve Bank of New York, Current Issues in Economics and Finance.

Fig. 1 **Historical Real Exchange Rate in the UK**

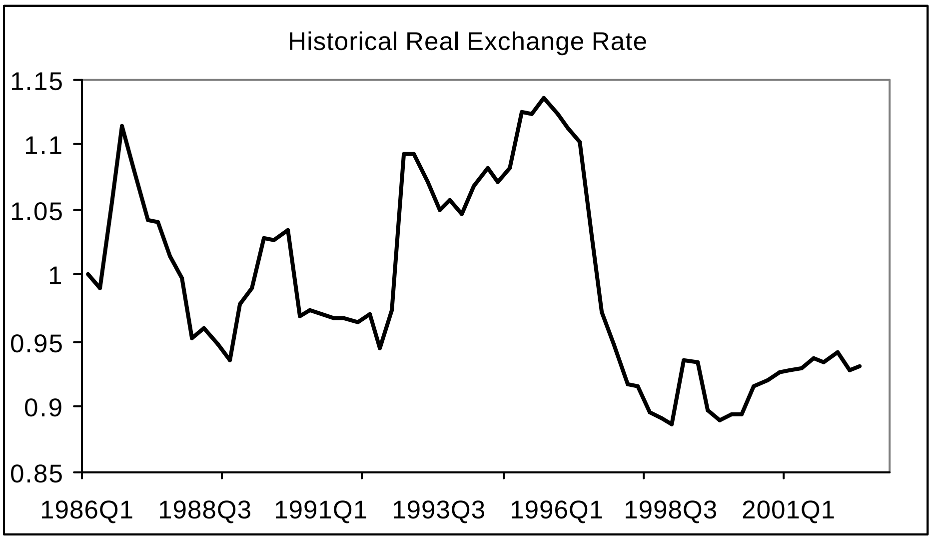


Fig 2. **Real Exchange Rate After 1%p.a. Productivity Growth Shock**

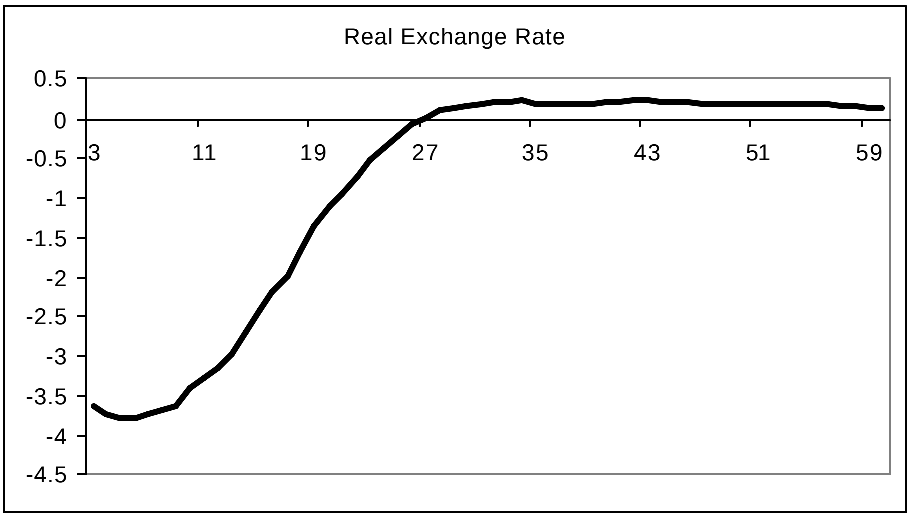
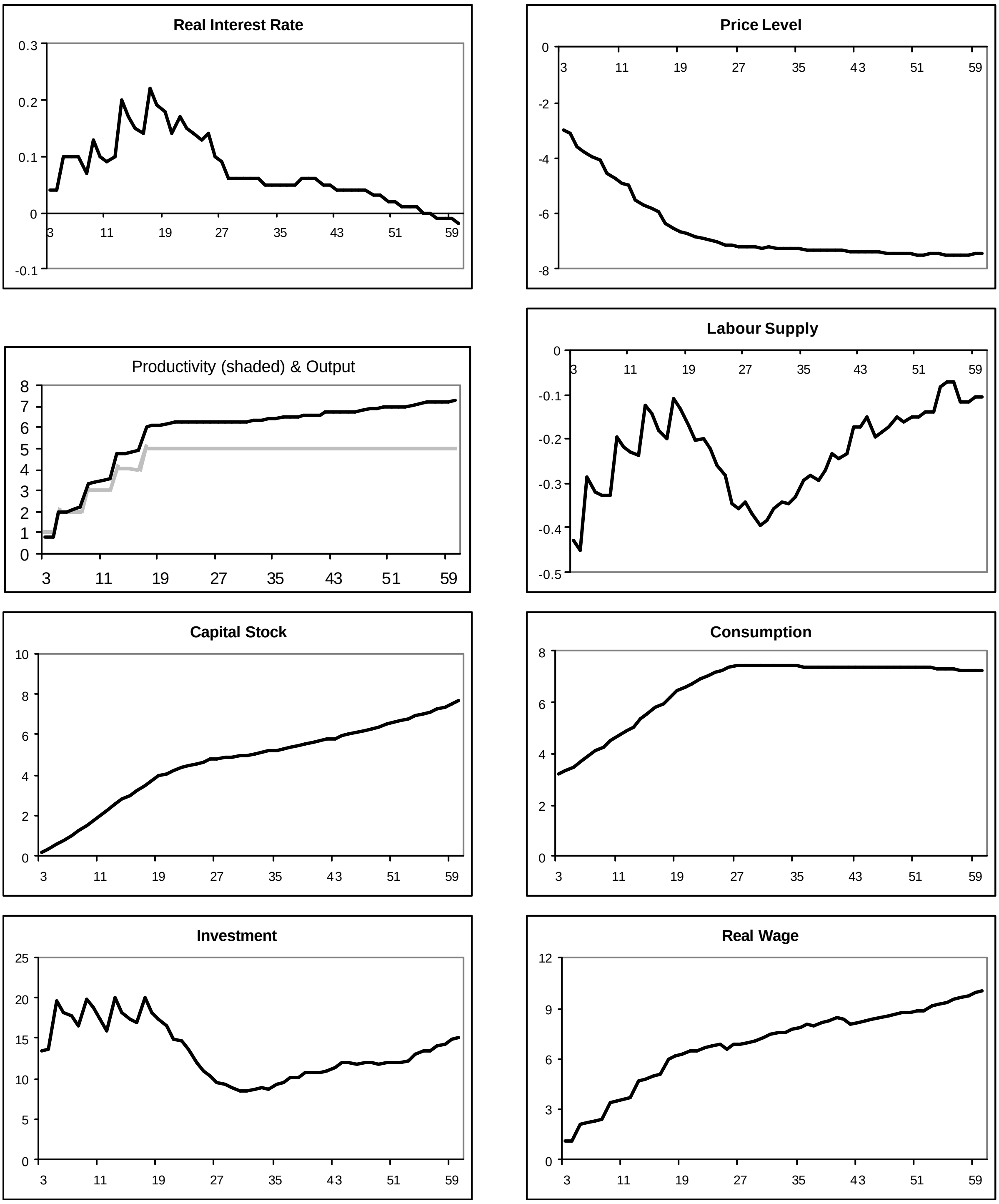
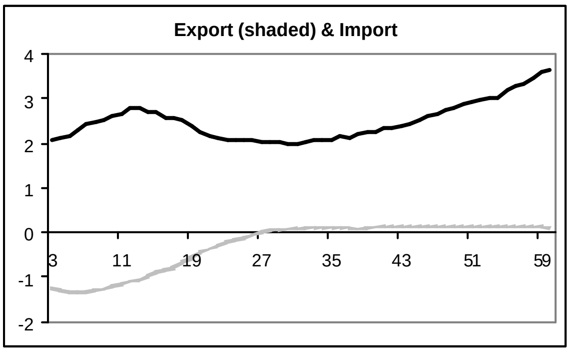


Fig 3. **1% p.a.Growth Shock to Productivity**




Export (shaded) & Import
4
3
2
1
0
-1
-2
3
11
19
27
35
43
51
59

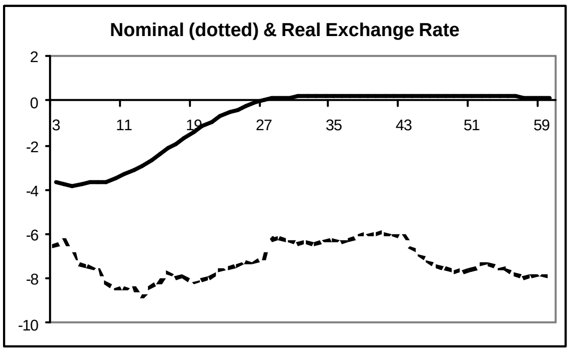
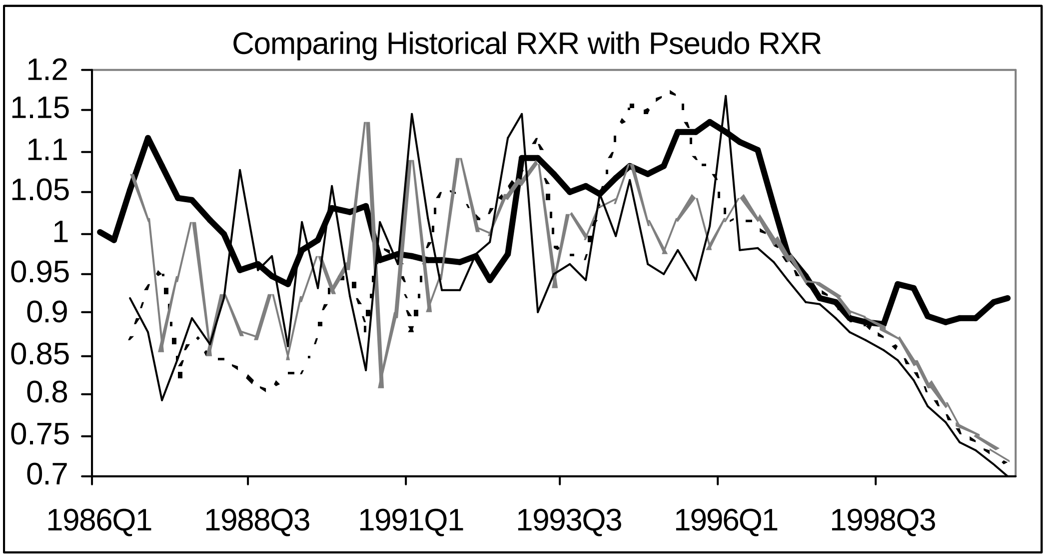

Nominal (dotted) & Real Exchange Rate
2
0
-2
-4
-6
-8
-10
3
11
19
27
35
43
51
59

Fig 4. **Comparing Historical RXR with Model Generated Pseudo RXR**



Tables

Table 1. **Test for Non-stationarity of the Real Exchange Rate**

Unit Root Tests (with intercept)		
	Levels	First Difference
ADF Test Statistic	-2.029676*	-5.258587
PP Test Statistic	-1.785482*	-5.786253

Table 2. **Unit root tests for RXR**

(1) ADF test: with intercept no trend

(i) Levels

ADF Test Statistic	-2.029676	1% Critical Value*	-3.5457
		5% Critical Value	-2.9118
		10% Critical Value	-2.5932

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Q)
Method: Least Squares
Date: 02/06/05 Time: 16:11
Sample(adjusted): 1986:3 2000:4
Included observations: 58 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Q(-1)	-0.111369	0.054870	-2.029676	0.0472
D(Q(-1))	0.307323	0.129739	2.368783	0.0214
C	0.110813	0.055121	2.010355	0.0493

R-squared	0.125570	Mean dependent var	-0.001121
Adjusted R-squared	0.093772	S.D. dependent var	0.031369
S.E. of regression	0.029862	Akaike info criterion	-4.134125
Sum squared resid	0.049045	Schwarz criterion	-4.027550
Log likelihood	122.8896	F-statistic	3.949054
Durbin-Watson stat	1.860878	Prob(F-statistic)	0.024971

(ii) First difference

ADF Test Statistic	-5.258587	1% Critical Value*	-3.5478
		5% Critical Value	-2.9127
		10% Critical Value	-2.5937

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Q,2)
Method: Least Squares
Date: 02/06/05 Time: 16:13
Sample(adjusted): 1986:4 2000:4
Included observations: 57 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(Q(-1))	-0.833594	0.158521	-5.258587	0.0000
D(Q(-1),2)	0.118001	0.128942	0.915153	0.3642
C	-0.002096	0.003928	-0.533688	0.5957

R-squared	0.401815	Mean dependent var	-0.001032
Adjusted R-squared	0.379660	S.D. dependent var	0.037589
S.E. of regression	0.029606	Akaike info criterion	-4.150500
Sum squared resid	0.047331	Schwarz criterion	-4.042971
Log likelihood	121.2893	F-statistic	18.13656
Durbin-Watson stat	2.080531	Prob(F-statistic)	0.000001

(2) Phillips Perron test: with intercept no trend

(i) Levels

PP Test Statistic	-1.785482	1% Critical Value*	-3.5437
		5% Critical Value	-2.9109
		10% Critical Value	-2.5928

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3	(Newey-West suggests: 3)	
Residual variance with no correction		0.000917
Residual variance with correction		0.001294

Phillips-Perron Test Equation
Dependent Variable: D(Q)
Method: Least Squares
Date: 02/06/05 Time: 16:14
Sample(adjusted): 1986:2 2000:4

Included observations: 59 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Q(-1)	-0.080614	0.055015	-1.465303	0.1483
C	0.079443	0.055229	1.438418	0.1558

R-squared	0.036301	Mean dependent var	-0.001271
Adjusted R-squared	0.019394	S.D. dependent var	0.031119
S.E. of regression	0.030815	Akaike info criterion	-4.088296
Sum squared resid	0.054126	Schwarz criterion	-4.017871
Log likelihood	122.6047	F-statistic	2.147112
Durbin-Watson stat	1.445507	Prob(F-statistic)	0.148333

(ii) First Difference

PP Test Statistic	-5.786253	1% Critical Value*	-3.5457
		5% Critical Value	-2.9118
		10% Critical Value	-2.5932

*MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: 3	(Newey-West suggests: 3)	
Residual variance with no correction		0.000909
Residual variance with correction		0.000858

Phillips-Perron Test Equation
Dependent Variable: D(Q,2)
Method: Least Squares
Date: 02/06/05 Time: 16:14
Sample(adjusted): 1986:3 2000:4
Included observations: 58 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(Q(-1))	-0.754965	0.129520	-5.828929	0.0000
C	-0.000781	0.004033	-0.193665	0.8471

R-squared	0.377615	Mean dependent var	0.000267
Adjusted R-squared	0.366501	S.D. dependent var	0.038549
S.E. of regression	0.030682	Akaike info criterion	-4.096379
Sum squared resid	0.052719	Schwarz criterion	-4.025329
Log likelihood	120.7950	F-statistic	33.97641
Durbin-Watson stat	1.833816	Prob(F-statistic)	0.000000

Table 3. **Best Fitting ARIMA**

Dependent Variable: D(Q)
Method: Least Squares
Date: 02/02/05 Time: 19:15
Sample(adjusted): 1987:1 2000:4
Included observations: 56 after adjusting endpoints
Convergence achieved after 35 iterations
Backcast: 1986:2 1986:4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
@SEAS(1)	-0.010030	0.006358	-1.577398	0.1216
@SEAS(2)	-0.000548	0.006172	-0.088796	0.9296
@SEAS(3)	-0.006772	0.006270	-1.080044	0.2858
@SEAS(4)	0.008130	0.006121	1.328153	0.1907
AR(1)	-0.350984	0.132100	-2.656958	0.0108
AR(2)	-0.631597	0.120298	-5.250260	0.0000
AR(3)	-0.469182	0.123157	-3.809622	0.0004
MA(1)	0.793836	0.028323	28.02764	0.0000
MA(2)	0.794252	0.055741	14.24902	0.0000
MA(3)	0.990501	0.050620	19.56734	0.0000

R-squared	0.367220	Mean dependent var	-0.003378
Adjusted R-squared	0.243415	S.D. dependent var	0.029483
S.E. of regression	0.025645	Akaike info criterion	-4.328506
Sum squared resid	0.030253	Schwarz criterion	-3.966836
Log likelihood	131.1982	Durbin-Watson stat	1.898058

Inverted AR Roots	.12+.88i	.12 -.88i	-.60
Inverted MA Roots	.10 -.99i	.10+.99i	-1.00

Table 4. **Confidence Limits from our Model for ARIMA**

Autoregressive Integrated Moving Average			
ARIMA(3,1,3)		95% Confidence Limits	
	Estimated	Lower	Upper
AR(1)	-0.350984	-1.618262	0.935524
AR(2)	-0.631597	-1.208567	0.748745
AR(3)	-0.469182	-0.664775	0.575872
MA(1)	0.793836	-1.667825	1.319203
MA(2)	0.794252	-0.862156	1.331956
MA(3)	0.990501	-0.764474	0.96245