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Laurian Lungu and Patrick Minford

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Laurian Lungu, Cardiff Business School
Patrick Minford, Cardiff Business School and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Explaining The Equity Risk Premium

We develop a simple overlapping generations model in which the young have a choice in investing in equities and index-linked bonds. Projections of share price uncertainty over a 30-year period show that the risk associated with such a long-term investment predicts an equity premium that matches historical values.

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Laurian Lungu
Cardiff Business School
Cardiff University
Aberconway Building
Colum Drive
Cardiff
CF1 3EU
Tel: (44 29) 2087 5558
Fax: (44 29) 2087 4419
Email: lungul@cf.ac.uk

Patrick Minford
Cardiff Business School
Cardiff University
Aberconway Building
Colum Drive
Cardiff
CF1 3EU
Tel: (44 29) 2087 5728
Fax: (44 29) 287 4419
Email: minfordp@cf.ac.uk

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I. Introduction

It is widely agreed that general equilibrium consumption based asset pricing models cannot explain the huge observed average rate of return differential between stocks and short-term Treasury Bills. In a seminal article, Mehra and Prescott [1985] find, using annual 1889-1978 U.S. data, that the average real annual return on the Standard and Poor's 500 Index was almost 7 percent while the average real return on short-term government bonds was less than 1 percent. Kocherlakota [1994] shows that adding ten more years of data to the Mehra-Prescott period does not change the result.

Mehra and Prescott [1985] show that the difference in the covariances of these returns with consumption growth is only large enough to explain the difference in the average returns if the representative consumer is either extremely risk averse or not rational. This implies a coefficient of relative risk aversion around 40, which is much too high to be reasonable. This is what they called the "equity premium puzzle", in other words stocks are not riskier enough compared to Treasury Bills to explain the spread in their returns¹.

Because inflation uncertainty is low at a 3-month horizon and the return is riskless in nominal terms, 3-month Treasury Bills have been traditionally used in the literature to approximate a risk free return². Mehra and Prescott [1985] found that the average real return on a relatively riskless security for the 1889-1978 period was 0.8 percent per year. For the

¹ Alternatively if one views investors as indeed highly risk averse, as implied by the large observed equity premium, standard models of preferences would suggest that investors have an overpowering predisposition for a flat consumption path, transferring wealth from periods with high consumption to periods with low consumption. Given the fact that average per capita consumption growth has tended to grow steadily over time, this implies an excessive strong desire to borrow from the future. And, unless investors are extremely patient, that is, their rate of time preference is very low, or even negative, this should drive up the equilibrium real rate of interest. Instead, the real rate of interest has been scarcely positive over long periods of time. This is what Weil [1989] calls "the risk free rate puzzle".

² Treasury Bills were first issued in 1920, before that date commercial paper was used for short rates. For a more detailed account on this issue see Siegel (1991).

U.K., during the same period the return was 0.75 percent³ [Siegel 1991]. The hyperinflation in Germany in the 1920s and Japan after the Second World War wiped out bondholders altogether. While in both these cases equities managed to regain most of their real value, this suggests that equity premium was actually greater for Japan and Germany during this century than for the U.S. Campbell [1996] reports values for the Treasury Bill real returns in the range of 0.5 percent – 3 percent for the same 12 countries during 1970-1994.

Real stock returns for period 1926-1992 is 5.71 percent for the U.K. [Siegel 1991]. Gregor Gielen [1994] and Hirose and Tso [1995] have found that, despite the abrupt decline in the stock prices of the Axis powers at the end of the war, the average real compound annual return on stocks from 1926 to 1995 was 9.5 percent in Germany and 4 percent in Japan. Campbell [1996] uses the Morgan Stanley Capital International (MSCI) stock market data to construct an international⁴ quarterly data set for the period 1970-1994. He finds that, with a few exceptions⁵, average real returns ranged from 5.83 percent for Japan to 11.9 percent for the Netherlands.

As it can be seen from Table I the risk premium depends on the period under consideration. However, between 1929-2000 it averages around 7 percent, which is close to the value reported in most of the studies⁶.

³ Siegel (1991) extends the time period back to 1800. He found that between 1800 and 1888 the average annual arithmetic real return for a risk free asset was 5.19 percent for the US and 4.84 percent for the UK. However, data reliability questions the robustness of his results.

⁴ Campbell works with data from 12 countries: Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

⁵ Exceptions are Italy (0.38 percent), Spain (-1.54 percent) - both these markets have a very small stock market capitalisation to the GDP ratio - Australia (2.53 percent), and Canada (3.89 percent).

⁶ Small differences in the estimation of the risk premium might arise due to data differences or the method employed to derive the risk premium. Jagannathan et al. (2000), for example, apply the Gordon (1962) stock valuation model to compute the equity risk premium by using several different stock portfolios and bonds of different maturities.

While the United States has been the most successful large economy in the past century, high annualised equity returns have been documented in other markets. In their book, Dimson et al. [2002] estimate historical equity premium for 16 developed countries for the period 1900-2000. According to their research the equity premium was the highest in Germany, averaging 6.8 percent in the last century, while for the U.S. it was 5 percent, and for the U.K. it was only 4.4 percent.

To form an idea about the magnitudes of the mean and standard errors for the returns on financial assets and consumption growth in different periods we have recalculated the U.S. equity premium for the period 1929-2000. The results are also shown in Table I. A plot of these series are presented in Figures I-IV.

The magnitude of the risk premium is not merely an academic debate. As pointed out in the 'The Economist' issue from February 2nd 2002 (pages 80-81), pension funds have to consider it. Because most of the pension funds' liabilities are in general offset by risky shares, the expected returns on equity could fall short of expectations. The problem is aggravated by the anticipated surge in the number of retired people from the developed countries – the so-called baby boomers – which are expected to start withdrawing their investments they previously made in shares for consumption. There is also the risk faced by the participants in pension schemes with defined contributions rather than defined benefits, such as the U.S.'s 401(k) plans, where a lower return on equity could reduce their retirement income.

The premises of this literature, which we review in more details in the next section, is that the portfolio holding period is a relatively short period, perhaps a quarter or a year. Thus each period the utility-maximising household decides on its equity strategy for the coming

period; at the end of it he re-optimises and so on. The familiar first-order conditions are obtained, from which the risk-premium puzzle is derived.

Our argument in this paper is that investment in equities should be viewed in the framework of overlapping generations. The young generation invests in equities and bonds for its old age. The old generation will be selling its stocks of equities and bonds- perforce to the young. Thus in terms of market clearing the young must take up the required capital stock and the government bond issue. As a group in effect the young cannot dispose of either; should they try to do so in reaction to shocks the market price will adjust to force them to retain both. Thus effectively when a young person buys an equity he knows that he will be acquiring it until old age⁷.

This fact is reinforced socially by government regulations and taxes, designed to encourage the long-term holding of stocks. There are in most countries strong tax incentives to buy private pensions which in turn will hold equities and bonds for long periods to match their long-term liabilities. Thus governments encourage a culture of ‘long termism’ which matches the generational set-up we have described.

If so then the risk associated with an equity is over a long period, not merely a year or even less but over a ‘generation’ from average youth to average old age. In this paper we use

⁷ We imagine that the young wish to save for retirement; hence the assets they buy are assessed in terms of their capacity to fund retirement, say in 30 years’ time. The only people buying equities in a market with ‘2n’ generations are those in the ‘n’ young generations. The youngest generation is the marginal one, with the longest period to retirement. They will calculate the return on equity over that period. They know that they will have the option to sell the equity in a year’s time for example; but also that they will be better off then holding onto their existing equity, since, should they sell, the price then will reflect whatever the then-youngest generation would pay, which again will reflect the risks over the succeeding 30 years- these are greater than for only 29 years and so the price will be higher than necessary to induce the intra-marginal generations to continue to hold. Thus the

a 2-generation model and assume the ‘period’ to be 30 years. We calibrate the risk over this holding period from the returns on equity; since these are close to a random walk, the accumulated variance is roughly equal to 30 times the 1-year variance. This more or less accounts for the equity risk-premium at normal levels of risk-aversion. Thus in brief our idea is that the risk premium can be accounted for by extending the implied holding period for the standard portfolio model.

The plan of the paper is as follows. In the next section we review the literature. In section III we derive an expression for the risk premium in a two-period overlapping generations model. Section IV presents results for the U.K. and U.S. economy, matched to a calibrated version of this model. We conclude finally that the model can account for the equity risk premium.

II Literature Review

The equity premium puzzle has generated a huge literature that seeks to explain the puzzle. There are a number of papers that summarise different theoretical frameworks employed to solve the puzzle and their results (see for example Kocherlakota [1996], Heaton and Lucas [1995], Campbell [2000]).

One response to the equity premium puzzle is to accept the possibility that there is no puzzle after all, that is, the risk aversion might be higher than was traditionally considered reasonable. Some authors (Cecchetti and Mark 1993 or Hansen et al. 1994) believe that individuals are more risk averse than we thought and, therefore, the size of the risk premium is justified. However, the general view is that values for the coefficient of risk aversion higher than five are difficult to rationalise, generating a higher implausible behaviour on the part of

youngest generation will price the equity on the assumption that they hold it for the full 30 years. In effect,

individuals. Another strand of research has focused on alterations of preferences considered by Mehra and Prescott [1985]. Epstein and Zin [1989, 1991] use a generalised expected utility of isoelastic preferences where the intertemporal substitution and risk aversion can be partially disentangled, thus allowing individual's attitudes towards risk and growth to be no longer governed by the same parameter.

A number of authors have attempted to explain the equity risk premium using a non-expected utility framework⁸ (Weil 1989b and Hung 1994). Another way in which preferences have been modified is by considering the effect of habit formation (Abel 1990, Campbell and Cochrane 1999, Constantinides 1990, Ferson and Constantinides 1991, Heaton 1995 or Sundersan 1989). The basic idea that motivates the assumption of habit is that the risk aversion cannot be rigidly linked to the level of consumption and wealth, since that increases over time while equity premium has not declined. By developing a habit for either higher or lower consumption, the individual's risk aversion will depend on the level of his consumption relative to some trend or the recent past.

Several researchers (Constantinides and Duffie 1996, Weil 1992 or Aiyagari and Gertler 1991) have been trying to relax the assumption of complete markets, typically by postulating a non-insurable source of risk, usually taken to be labour income risk. For instance, if individuals cannot fully insure against spells of unemployment due to moral hazard or other contractual problems, individual consumption will fluctuate with employment status. As a result, while the covariance of per capita consumption growth with stock returns is small, individual consumption growth may co-vary enough with stock returns to explain the equity premium.

However, as long as investors do not face any kind of market frictions, they could use part of their accumulated stock of wealth to self-insure against idiosyncratic risk (if shocks

though free in principle to disinvest, they are 'locked in' to any asset they decide to hold for retirement.

are not remarkably persistent⁹). Therefore, usually, the assumption of incomplete markets is used in conjunction with some sort of market frictions such as transaction costs or borrowing constraints. Aiyagari and Gertler [1991] and Heaton and Lucas [1995], among others, assert that trading costs across stock and bond markets are large enough in order to generate the observed equity premium. Although appealing, their assumption is not fully validated by the empirical evidence. More has to be done in order to document the sizes and sources of trading costs, especially where pension funds and institutional investors are concerned.

Constantinides et al. [1998] explore the implications of the borrowing constraint using an overlapping generations model in two versions of the economy: borrowing-constrained and borrowing-unconstrained. Although their results are in line with what theory predicts, i.e. the borrowing constraint has the effect of lowering the interest rate and raising the equity premium, their model could not completely solve the puzzle. Moreover, Heaton and Lucas [1995] show that borrowing constraints do not seem to have much impact on the size of the equity premium. The constraint should apply to both equity and bond market otherwise individuals could simply shift their resources from one market to another. Thus, if the risk free rate falls so should the equity return in order to clear the constrained markets¹⁰.

A number of authors have suggested that equities could have suffered from a so-called *peso problem*¹¹. For example Rietz [1988] and Danthine and Donaldson [1998] attempt to explain the equity premium puzzle in the context of a model that has a peso problem environment. Miller et al. [2001] stress, with the aide of a simple formula, the fact that the

⁸ The class of non-expected utility preferences has been used in various other contexts such as studying puzzles in permanent income theory or other asset pricing anomalies (Weil, 1990, Koskiewicz, 1999).

⁹ Heaton and Lucas (1996) find, using microdata from the Panel Study of Income Dynamics, that idiosyncratic income variation is, indeed, largely transitory.

¹⁰ Cochrane and Hansen (1992) proves this reasoning is legitimate using a model where individuals face a market wealth constraint.

added risk premium could be as high as 4 percent if the agents expect a loss of 1 percent per annum and the assumed degree of risk aversion is two. The main obstacle in studying peso problems is how to model agents' expectations in an unstable environment. Given the fact that historical data is not of much use when dealing with peso problems a test for the presence of these is relatively difficult to perform.

A series of authors have studied the equity premium in the context of a real business cycle models (Danthine and Donaldson 1998, Jermann 1998, Boldrin et al. 1999 or Guvenen 2003). The idea in these papers has been to explain simultaneously both business cycle facts as well as the observed asset returns. By introducing some sort of market frictions such as limited stock market participation or capital adjustment costs modelled in a framework with habit formation preferences, these models have gone some way in providing answers to the questions addressed. However, shortcomings still remain. For example, the model in Jermann [1998] predicts a risk premia for bonds which is too high relative to the equity premium whereas the model of Boldrin et al. [1999] yields a higher volatility of the risk free rate.

In this paper we adopt a different approach. Our suggestion here is that an important feature of equities is being overlooked: that they are used very heavily in retirement plans, either explicitly or implicitly, implying that investors have very long-term horizons. When this fact is combined with the well-known random walk behaviour of share prices it follows that equity investments have very high risk over long-term horizons.

III The Model

¹¹ Peso problems can arise when the likelihood of some infrequent or unprecedented event that may occur could affect asset prices. In fact, there is a growing literature that documents a number of interesting applications for the peso problem in the context of asset prices. See, for example, Sill (2000) or Evans (1996).

Our modelling framework is an overlapping generations model. Its attractiveness comes from the fact that it allows the study of aggregate implications of life-cycle savings by individuals. The life-cycle model is particularly appealing because it allows households to choose optimal paths of consumption and labour supply over their lifetime, given their preferences and lifetime budget constraint. The main building block of the life-cycle model is the saving decision, i.e., the division of income between consumption and saving, which is being driven by preferences between present and future consumption. In turn, individuals' savings during their working lives generate the capital stock.

In order to make up for the lower income during retirement, implied by a low pension income, and to avoid a sharp drop in utility at the point of retirement, individuals will save some fraction of their income during their working life and dis-save during retirement. Thus, the planning period is finite: people save only for themselves. From the postulate of utility maximisation it follows that consumption is evenly distributed over time and this, in turn, implies that the individual during his active period builds up a stock of wealth, which he consumes during his old age.

By allowing the young and old coexisting at any time, overlapping generations models have been fruitful in depicting the equilibrium pattern of growth in an economy over time and in bringing into sharp relief the role of interest rates.

Given its descriptive appeal, the overlapping generations framework seems to be a suitable choice in our attempt to explain the equity premium puzzle. If individuals take a long-term view when they make an investment decision then it is reasonable to assume that the riskiness of such a decision requires a premium that, as we are going to show below, could match historical values. In other words, what we suggest here is that an important feature of equities is being overlooked: that they are used very heavily in retirement plans, either explicitly or implicitly, implying that investors have very long-term horizons. When this fact

is combined with the well-known random walk behaviour of share prices¹² it follows that equity investments have very high risk over long-term horizons. So, although shares are much more likely to outperform bonds over long horizons than over short horizons [Malkiel 1996], the riskiness of the shares demands a risk premium. However, the risk premium is not only influenced by the coefficient of relative risk aversion¹³ but also by the expected variance of productivity shocks for the period in which the investment is made and by the share of wealth invested in equity.

We assume here that the predominant motive for saving is to provide for old age. In our set-up this is the main act of ‘consumption-smoothing’ a household makes since it will have no income at all otherwise in old age. We also assume that private savings for old age are effectively irreversible, made in long term investment schemes, either in government bonds or in equity, taken to be a broad spread across the index. This corresponds to practical settings, whether because governments encourage it via tax incentives or regulation, or because pension funds keep rather stable ratios of these assets for prudential reasons. Public pension schemes are assumed to be compulsory and Pay-As-You-Go. Hence our households face three classes of asset: a risk-free short-term deposit, primarily used to smooth consumption within youth, and a long-term bond and an equity which are both held once purchased until liquidated in old age. Thus, households consider the very long-term risks of their investment decision instead of just looking one period ahead to the possible capital gain/loss.

¹² The random walk hypothesis of share prices is motivated by the fact that these reflect all available information. Since only news changes their behaviour, the unpredictability of share prices justifies a first-order approximation.

¹³ Jagannathan and Kocherlakota (1996) argue that, with the households having no possibility of rebalancing their portfolios, the higher the coefficient of relative risk aversion the lower the percentage of wealth invested in equity.

In the model households are assumed neither to leave bequests nor to receive inheritances. A number of authors have suggested that an accurate description of aggregate saving behaviour should treat the bequest motive¹⁴. Although this consideration may be significant our aim here is to construct a benchmark model that can explain the equity risk premium. For our purpose the basic life-cycle model will suffice.

The model does not include money. However, although money is excluded, financial variables are determined in the model. In behaving competitively firms set their marginal product of capital equal to the interest rate.

We use a simple overlapping generations model in which the representative agent lives for two periods. In the first period, consumers (the young) have a choice of investing in indexed bonds or fixed capital (equity).

We assume that the labour of the young is in fixed supply, with a constant marginal product, and that they receive a fixed income from it. The young's savings get transformed into capital which lasts one period (which we take empirically to be 30 years) and is used in conjunction with the young's labour next period to produce extra output enjoyed by the old. In the second period, the representative agent (the old) simply consumes the value of what they have. All variables are expressed per capita. Thus, output is given by:

$$(1) \quad Y_t = \bar{Y} + A_t K_{t-1}^\alpha$$

where A_t represents technology. The natural logarithm of A_t obeys a random walk:

$$(2) \quad A_t = A_{t-1}(1 + \varepsilon_t) \quad \text{or} \quad \ln A_t \cong \ln A_{t-1} + \varepsilon_t$$

where ε_t is white noise.

Each generation (t) maximises its expected lifetime utility function at t :

¹⁴ The motivation of bequests affects their implications for intertemporal efficiency. Bequests can be either voluntary, in which case they reflect concern for the welfare of the future generations or involuntary. Kotlikoff and Summers (1981) estimate that at least 70 percent of wealth in the U.S. is due to bequests while Modigliani (1988) puts this estimate at 30 percent. See also Kotlikoff (1984).

$$(3) \quad E_t[u_t'] = u(C_t) + \beta E_t[u(C_{t+1})]$$

The government issues indexed bonds, B_t , with a real interest rate, r_t , to pay for real government spending, G_t , less lump sum taxes, T_t , on the young:

$$(4) \quad B_t = (1 + r_{t-1})B_{t-1} + G_t - T_t$$

So, the t-generation budget constraint is:

$$(5) \quad \bar{Y} = C_t + K_t + B_t + T_t \quad (\text{when young})$$

$$(6) \quad A_{t+1}K_t^\alpha + B_t(1 + r_t) = C_{t+1} \quad (\text{when old})$$

Market equilibrium is characterised by:

$$(7) \quad \bar{Y} + A_t K_{t-1}^\alpha = C_t + C_{t-1} + G_t + K_t$$

The t-generation first order conditions are familiarly:

$$(8) \quad u'(C_t) = \beta E_t[u'(C_{t+1})(1 + r_t)] = \beta E_t[u'(C_{t+1})\alpha A_{t+1}K_t^{\alpha-1}]$$

To solve for the rate of interest and other endogenous variables we could substitute equation (8) into (7) to obtain their equilibrium values¹⁵ as a function of the forcing processes, A_t , G_t , and T_t . However, here our concern is the risk premium conditional on the economy's general state, and for this the first order conditions are sufficient.

Having derived the first order conditions we can now use these to get an expression for the risk premium. Using equation (2), the expected gross return on capital (equities) is:

$\alpha E_t(A_{t+1}K_t^{\alpha-1}) = \alpha K_t^{\alpha-1} A_t$. So we can write the risk premium (RP) as:

$$(9) \quad RP_t = \alpha A_t K_t^{\alpha-1} - (1 + r_t) = \frac{-\alpha A_t K_t^{\alpha-1} E_t[u'(C_{t+1})\varepsilon_{t+1}]}{E_t[u'(C_{t+1})]}$$

since from (8) the gross return on bonds is:

$$(10) \quad 1 + r_t = \frac{\alpha A_t K_t^{\alpha-1} [E_t\{u'(C_{t+1})\} + E_t\{u'(C_{t+1})\varepsilon_{t+1}\}]}{E_t[u'(C_{t+1})]}$$

Expanding $u'(C_{t+1}^t)$ around $u'[E_t(C_{t+1}^t)]$ in a Taylor series we get:

$$(11) \quad u'(C_{t+1}^t) = u'[E_t(C_{t+1}^t)] + u''[E_t(C_{t+1}^t)][c_{t+1}^t - E_t(C_{t+1}^t)] + \dots$$

Ignoring higher order terms¹⁶, the expectation of this is:

$$(12) \quad E_t[u'(C_{t+1}^t)] = u'[E_t(C_{t+1}^t)]$$

Similarly,

$$(13) \quad E_t[u'(C_{t+1}^t)\varepsilon_{t+1}] = u''[E_t(C_{t+1}^t)]\text{cov}(\varepsilon_{t+1}, C_{t+1}^t)$$

Using equations (12) and (13) together with (6) and (2), we can write:

$$(14) \quad \frac{E_t[u'(C_{t+1}^t)\varepsilon_{t+1}]}{E_t[u'(C_{t+1}^t)]} = \frac{u''[E_t(C_{t+1}^t)]}{u'[E_t(C_{t+1}^t)]} A_t K_t^\alpha \text{var}(\varepsilon_{t+1})$$

$$\text{since } \text{cov}(\varepsilon_{t+1}, C_{t+1}^t) = A_t K_t^\alpha \text{var}(\varepsilon_{t+1}).$$

In order to obtain an expression for the risk premium it is necessary to specify a form for the utility function. We follow the tradition in the risk premium literature and assume a utility function of a constant relative risk aversion class (CRRA):

$$u(C_t, \rho) = \frac{C_t^{1-\rho} - 1}{1-\rho}, \quad 0 < \rho < \infty$$

where ρ is the Arrow-Pratt coefficient of relative risk aversion.

Because the consumer has a choice of investing only between equity and bonds, as

given by equation (2), the ratio $\psi_t = \frac{A_t K_t^\alpha}{E_t(C_{t+1}^t)}$ is nothing but the share of equity income in the

income of the old (expected at t). Thus, the risk premium can be written as¹⁷:

¹⁵ We will solve for the real rate of interest in the next section.

¹⁶ By using this approximation we ignore all non-linearities. Although there is a good case for taking into consideration at least the second order term of the Taylor expansion in equation (11), such an inclusion would make the analytical solution intractable.

¹⁷ We have used the fact that $\frac{u''[E_t(C_{t+1}^t)]}{u'[E_t(C_{t+1}^t)]} = \frac{-\rho}{E_t(C_{t+1}^t)}$

$$(15) \quad RP_t = \alpha A_t K_t^{\alpha-1} \rho \psi_t \text{var}(\varepsilon_{t+1})$$

The risk premium depends on the expected gross return on equity, $\alpha A_t K_t^{\alpha-1}$, the risk-aversion, ρ , the equity share of savings ψ_t , and the variance of the technology shocks. The first and the third variables are plainly endogenous in general equilibrium. Here we wish to see whether our model could predict the risk-premium, conditional on correct predictions of these- the issue of predicting these is a much broader one, which, given the model's free parameters we assume for now can be satisfactorily solved.

IV The Results

The data we use is the average of FTSE 100 Stock Price Index divided by the U.K. consumption deflator for two time periods, 1963 – 1997 (quarterly data) and 1899 – 1996 (annual data). This enables us to get a reasonable range for the 30-year-ahead variance of real equity return. To see if a diversified portfolio can influence our results we also used a value weighted Stock Market Index comprised of four national representative indexes: the U.K., Japan, the U.S., and Germany. The data is from Datastream and covers the 1973Q1 -1997Q3 period. The stock market indexes used are: S&P100 for the U.S., NIK225 for Japan, FT30 for the U.K., and CZBK for Germany.

To estimate the risk premium we need to calibrate each of the variables on the right hand side of equation (15) i.e. the variance of productivity shocks, $\text{var}(\varepsilon_t)$, the coefficient of relative risk aversion, ρ , and the share of equity in the income of the old, ψ .

Estimation of the Variance of Productivity Shocks

We estimated, to begin with, a random walk model¹⁸ for the quarterly average of FTSE 100 Stock Price Index divided by the U.K. consumption deflator. We obtained the following equation for the period 1963Q1 - 1997Q3 (using ordinary least squares):

$$\Delta Y_t = -0.0002 + \varepsilon_t$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000206	0.007566	-0.027254	0.9783
Mean dependent var	-0.000206	S.D. dependent var	0.088877	
S.E. of regression	0.088877	Akaike info criterion	-1.995916	
Sum squared resid	1.082169	Schwarz criterion	-1.974704	
Log likelihood	138.7182	Durbin-Watson stat	1.543342	

The constant term in the regression is negative, however it is not significant. The properties of the residuals are listed below:

Mean	-1.14E-17;	Median	0.004296;	Std. Dev.	0.088877;
Var(ε_t)	0.00789 ;	Skewness	-0.558248;	Kurtosis	4.347428;
Jarque-Bera	17.60724;	Probability	0.000150.		

Here, $\Delta Y_t = Y_t - Y_{t-1}$, and Y_t represents the natural logarithm of the quarterly average Stock Price Index. The equation variance is 0.00789 which should give us a 30-year ahead variance of $120 \times 0.0079 = 0.948$ (120 quarters).

For good measure (the Jarque-Bera statistic rejects the hypothesis that the residuals are normally distributed) we check this via a bootstrap¹⁹ (generating 1,000 draws from a distribution with the characteristics listed above) and obtain 0.906. The frequency

¹⁸ Here we assume that the dividend yield is a completely predictable additional source of equity income, which we set at zero percent per annum for purposes of illustration. Of course any chosen amount can be added into the estimated constant trend of share price growth. Some authors (see for example Barberis, 2000) argue that stocks have long-horizon predictability and exhibit slow mean reversion. To test this we used the FTSE Total Return Index for the 1963Q2-1999Q4 period and were not able to reject the hypothesis of a unit root. The results suggest that even if there is mean reversion it is quantitatively pretty weak over a 30-year horizon.

¹⁹ All bootstraps performed in this section used distributions that were similar to the error distributions obtained from the estimation of the corresponding equation.

distributions of final values 30 years ahead for the 1963 - 1997 period - all expressed as a percentage of the average value at that date - are presented in Figure V.

So, for example, if on average an investor would get a certain amount in 30 years time, the Figure shows the distribution of possible returns, as a percentage of this, that he might get given the uncertainty he faces. (Each bar is a probability of getting a final value in the range shown along the bottom; the probability heights add up to 1,000). Figure V shows the huge uncertainty an investors faces when investing in the stockmarket. For example, after 30 years there is a 10 percent probability of having less than a fifth of the average projected capital value.

For the period 1899–1996, the estimated equation (using Ordinary Least Squares) was:

$$\Delta Y_t = 0.0057 + \varepsilon_t$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005722	0.015494	0.369324	0.7127
Mean dependent var	0.005722	S.D. dependent var	0.152599	
S.E. of regression	0.152599	Akaike info criterion	-0.911750	
Sum squared resid	2.235502	Schwarz criterion	-0.885206	
Log likelihood	45.21986	Durbin-Watson stat	1.510624	

And the proprieties of the residuals were:

Mean	-1.67E-17;	Median	0.013729;	Std. Dev.	0.152599
Var(ε_t)	0.023286;	Skewness	-1.115451;	Kurtosis	7.049329
Jarque-Bera	86.38655;	Probability	0.000000		

The 30-year ahead variance would be in this case: $30 \times 0.023286 = 0.69859$

To check for the sensitivity of our results we also estimate an ARIMA (1,1,1) model. Using Box and Jenkins procedure, the following equation is estimated (1963 – 1997 period):

$$\Delta Y_t = -0.0003 - 0.3785 \Delta Y_{t-1} + \varepsilon_t + 0.6642 \varepsilon_{t-1}$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000345	0.008840	-0.038991	0.9690

AR(1)	-0.378495	0.207813	-1.821322	0.0708
MA(1)	0.664239	0.167879	3.956643	0.0001
Mean dependent var	-0.000491	S.D. dependent var	0.089139	
S.E. of regression	0.085820	Akaike info criterion	-2.051476	
Sum squared resid	0.986919	Schwarz criterion	-1.987534	
Log likelihood	143.5261	F-statistic	6.361994	
Durbin-Watson stat	1.996739	Prob(F-statistic)	0.002293	

The properties of the residuals are:

Mean	-1.49E-05;	Median	0.00531;	Std. Dev.	0.08582
Var(ϵ_t)	0.007365;	Skewness	-0.34809;	Kurtosis	4.66922
Jarque-Bera	18.6719;	Probability	0.000088		

Again the constant term is not significant. The Durbin-Watson statistic shows that there is no serial correlation. Also, comparing the Akaike and Schwarz criterion values of the random walk and the ARIMA(1,1,1) models it can be noticed that both marginally select the later over the former. The estimated AR coefficient is statistically significant at 10 percent level but not at 5 percent level. Here the equation variance is 0.0073. The bootstrap variance 30 years ahead is 1.210 and the frequency distribution of the final values are presented in Figure VI.

Using annual data from 1899 – 1996 the estimated equation is:

$$\Delta Y_t = 0.0070 - 0.1395 \Delta Y_{t-1} + \epsilon_t + 0.4392 \epsilon_{t-1}$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006976	0.019035	0.366473	0.7148
AR(1)	-0.139542	0.311784	-0.447558	0.6555
MA(1)	0.439226	0.283518	1.549204	0.1247
Mean dependent var	0.006930	S.D. dependent var	0.152934	
S.E. of regression	0.147947	Akaike info criterion	-0.953176	
Sum squared resid	2.035609	Schwarz criterion	-0.873040	
Log likelihood	48.75246	F-statistic	4.256144	
Durbin-Watson stat	2.010807	Prob(F-statistic)	0.017035	

The proprieties of the residuals are:

Mean	-1.65E-06;	Median	0.001719;	Std. Dev.	0.146381
Var(ϵ_t)	0.021427;	Skewness	-0.966957;	Kurtosis	5.848355;
Jarque-Bera	47.41261;	Probability	0.000000		

In this case the bootstrap variance 30 years ahead is 0.99.

To construct our diversified portfolio we use a weighted Stock Market Index with the weights assigned based on the correlation coefficients. In principle if two different stock markets with the same variance are combined and they are uncorrelated, their joint variance is half that of each alone. However, as it can be seen in Table II shocks are quite heavily correlated across equity markets with the four series having correlation coefficients in the region of 0.7-0.9. Based on these correlation coefficients²⁰, the computed weights are: 0.28 for the UK(FT 30), 0.22 for Japan (NIK225), 0.25 for the US (S&P100), and 0.25 for Germany (CZBK). To illustrate the gain of diversification we have assumed a portfolio diversified across these four markets with the same trend growth as the U.K., assuming again a zero dividend yield. Table III summarises the results. From Table III it can be noticed that the longer the time horizon, the higher the degree of uncertainty an investor faces. Also in the case of a diversified portfolio there is only a relatively modest reduction in variance. Such a modest reduction in variance is to be expected since the share price correlations across the major stock markets are high.

Estimation of the Coefficient of Relative Risk Aversion, ρ

Arrow [1971] argues on theoretical grounds that ρ should be approximately one. Other empirical studies (see Mehra and Prescott [1985]) estimate it between one and two. Kandel and Stambaugh [1991] suggest that a coefficient of relative risk aversion as high as 30 could be justified. However, even such a high value would be insufficient to match the model to post-war data. Here we assume that ρ lies between one and two.

Estimation of the Share of Equity Income in the Income of the Old, ψ

To estimate the share of equity in the income of the old, variable ψ in our model, we used two approaches. The first approach was to get an estimate of the share of equity in total pensions (private and state). This was obtained multiplying the share of equity in the private pension portfolios by the share of private pensions in total pensions. The 1997 values for the UK, using data from the Office for National Statistics, were 0.70 and 0.67 respectively. These estimates give $\psi = 0.47$.

The second approach was to get a historical estimate of ψ to see if its value changed over time. An estimate of ψ would be the ratio of total private holdings of net capital assets to total private net wealth (which is estimated as total private holdings of net capital assets plus domestically held public debt).

Hargreaves [1966] estimates the domestic public debt value for the UK in 1925 at £6,525 million, while the total UK net capital stock is estimated by Feinstein [1972] at £8,700 million. The year 1925 was chosen because the state pensions system in the UK was in its infancy at that time, non-universal and limited in size, and we assume it can be ignored. Before 1925 private capital formation represented more than 80 per cent of total capital formation [Feinstein 1972]. In estimating the total private holdings of net capital assets we added net private capital stock (taken to be 80 percent of the net capital stock) and net overseas assets (the government's share was negligible). Matthews et al. [1982] estimate net overseas assets at 125 percent of the GDP, that is £5,800 Million. Thus, our second approach in estimating ψ yields 0.66. The methods of evaluating ψ could be challenged on a number of grounds, but our aim here has been to provide a reasonable range for ψ . Small variations in the data used are likely to have little impact on ψ .

The Implied Risk Premium

²⁰ The weights are based on an individual country share price correlation with the UK share price. They were set at 0.25 for S&P100 and CZBK, 0.28 for the FT30 and 0.22 for the NIK225.

The period of our overlapping generations model being assumed to be 30 years, in equation (15) the gross return (on equity and bonds) represents the 30-year holding period return which is defined as (gross fractional return per annum)³⁰. For the U.K. gross real returns on equity and short term government bonds are estimated by Siegel [1991] at 1.0571 percent per annum and 1.0075 percent per annum respectively. Therefore, for the U.K., the risk premium we try to match is given by the left-hand side of equation (16):

$$(16) \quad (1 + 0.0571)^{30} - (1 + 0.0075)^{30} = (1 + 0.0571)^{30} \rho \psi \text{var}(\varepsilon_{t+1})$$

For the U.S., gross real returns on equity and short term government bonds are estimated by Mehra and Prescott [1985] at 1.0698% p.a. and 1.008% p.a. respectively. In this case the risk premium we try to match is:

$$(17) \quad (1 + 0.0698)^{30} - (1 + 0.008)^{30} = (1 + 0.0698)^{30} \rho \psi \text{var}(\varepsilon_{t+1})$$

The risk premium is given by the expression on the right hand side of equation (16) and (17) respectively - each number is shown as percentage per annum, that is, its value to the (1/30)th power minus one, times 100; the left hand side to be matched is thus 4.04 percent for the U.K. and 6.30 percent for the U.S.

Tables IV and V show the implied risk premium for the U.K. and U.S respectively for different plausible values for ρ^{21} . The random walk model on U.K. equity real returns and the ARIMA (1,1,1) process estimated for both time periods suggest that the variance of the log of the equity's real value 30 years ahead lies between 0.70 – 1.21. The results presented in Tables IV and V are reported under the assumption that the variance of technology shocks is one. The share of equity income in the income of the old, ψ , ranges, according to empirical estimates for the U.K., from 0.47 – which is the share of total pension income generated by

²¹ For the U.S we assumed the same range as in the U.K case for calibrated values.

equities in 1990s, to 0.66 – which represents the share of capital in total private wealth in the 1920s. In Tables IV and V we consider three possible values for ψ , 0.47, 0.57, and 0.66.

It can be seen from Tables IV and V that the values for the risk premium we get are very close to the value we try to match. For central values of $\psi = 0.56$, $\rho = 1.5$ the predicted number is 4.44% against the actual 4.04% for the U.K. and 6.36% against the actual 6.30% for the U.S. Of course the risk premium could vary over time, as history suggests, but the results presented above show that it can be explained using values for the coefficient of the relative risk aversion that are supported on both theoretical and empirical grounds.

The Real Rate of Interest

Given our predicted values of risk premium we would like to see whether the model above could predict a real rate of interest that also matches historical values. In order to do that we start from equation (10) which can be written as:

$$(18) \quad 1 + r_t = \alpha A_t K_t^{\alpha-1} \left(1 + \frac{E_t[u'(C_{t+1}^t)\varepsilon_{t+1}]}{E_t[u'(C_{t+1}^t)]} \right)$$

Using the result in equation (14) this becomes:

$$(19) \quad 1 + r_t = \alpha A_t K_t^{\alpha-1} \left[1 - \frac{\rho A_t K_t^\alpha \text{var}(\varepsilon_{t+1})}{E_t(C_{t+1}^t)} \right]$$

Taking expectations of equation (6) and substituting for $E_t(C_{t+1}^t)$ in equation (19) yields:

$$(20) \quad 1 + r_t = \alpha A_t K_t^{\alpha-1} \left[1 - \frac{\rho A_t K_t^\alpha \text{var}(\varepsilon_{t+1})}{A_t K_t^\alpha + B_t(1 + r_t)} \right]$$

Dividing by $A_t K_t^\alpha$ and rearranging one obtains:

$$(21) \quad 1 + r_t = \alpha A_t K_t^{\alpha-1} \left[1 - \frac{\rho \text{var}(\varepsilon_{t+1})}{1 + \alpha \frac{B_t}{K_t} \frac{(1+r_t)}{\alpha A_t K_t^{\alpha-1}}} \right]$$

The real risk-free interest rate depends on the return on equities, the coefficient of relative risk aversion and the ratio of government debt to capital. The above equation is a quadratic in $1 + r_t$, the return on short term government bonds over 30 years. Re-write this as:

$$(22) \quad \alpha \frac{B_t}{K_t} (1 + r_t)^2 + \alpha A_t K_t^{\alpha-1} \left(1 - \alpha \frac{B_t}{K_t} \right) (1 + r_t) + (\alpha A_t K_t^{\alpha-1})^2 [\rho \text{var}(\varepsilon_{t+1}) - 1] = 0$$

The existence of two distinctive real roots is assured if the determinant is positive. Given that the return on equity, $\alpha A_t K_t^{\alpha-1}$, over 30 years is 7.57, calibrating the remaining parameters as

$\alpha = 0.3$, $\frac{B_t}{K_t} = 0.2$, $\rho = 1$ and $\text{var}(\varepsilon_{t+1}) = 0.84$ would imply a gross return on bonds over 30

years of 1.27– essentially corresponding to the observed value of 1.008% per annum²². One

can easily notice that, given the observed return on equity, it is largely the value of ρ that

drives the result. This happens because plausible variations in the other calibrated parameters

are, in general, relatively small. If the determinant is positive and the coefficient

$[\rho \text{var}(\varepsilon_{t+1}) - 1]$ is less than zero one root will be positive and the other one negative²³.

Our model seems to suggest that, when one takes into account the long-term risk of equities,

in order to match the historical return on equity and bonds, the coefficient of relative risk

aversion has to be one or very close to one. This conforms to economic theory and other

empirical studies.

Matching Higher Moments of the Risk Premium and the Real Rate of Interest

²² The second root in this case would be -119.7, which does not make any economic sense.

²³ Essentially, with the above calibrated values, equation (22) has complex roots for values of the coefficient of relative risk aversion greater than 5 and two real negative roots for the coefficient of relative risk aversion greater than 1.4.

Next we would like to check whether our model can predict higher moments of the observed risk premium and the real rate of interest. In order to do this we solve the model by 1st order Taylor series expansion and then linearise around steady state assuming:

$$C_t^t = C_{t+1}^t; A_{t+1} = A_t = A_{t-1}(1 + \varepsilon_t).$$

First, re-write the market clearing (using equations 6 and 7) as:

$$(23) \quad \bar{Y} = C_t^t + K_t + G_t + B_{t-1}(1 + r_{t-1})$$

Using equations (5) and (6) the consumer budget constraint is:

$$(24) \quad C_{t+1}^t = A_{t+1}K_t^\alpha + (\bar{Y} - C_t^t - K_t - T_t)(1 + r_t)$$

And, from equation (8), with a CRRA utility function the t-generation first order conditions are:

$$(25) \quad (C_t^t)^{-\rho} = \beta E_t[(C_{t+1}^t)^{-\rho}](1 + r_t)$$

$$(26) \quad \beta E_t[(C_{t+1}^t)^{-\rho}](1 + r_t) = \beta E_t[(C_{t+1}^t)^{-\rho}] \alpha A_{t+1} K_t^{\alpha-1}$$

Differencing equations 23-26 yields respectively:

$$(27) \quad d(C_t^t) + d(K_t) = 0$$

$$(28) \quad d(C_{t+1}^t) = RP_t d(K_t) + A_t K_t^\alpha d[\ln(A_t)] - (1 + r_t) d(C_t^t) + (\bar{Y} - C_t^t - K_t - T_t) d(r_t)$$

$$(29) \quad d(C_t^t) = \frac{-\beta C_t^t}{\rho} d(r_t) + \beta(1 + r_t) d(C_{t+1}^t)$$

$$(30) \quad \frac{d(K_t)}{K_t} = \frac{-K_t}{\alpha(1-\alpha)(A_t K_t^\alpha)} d(r_t) - \frac{\rho RP_t K_t}{\alpha(1-\alpha)C_{t+1}^t (A_t K_t^\alpha)} d(C_{t+1}^t) + \frac{1}{(1-\alpha)} d[\ln(A_t)]$$

Substituting for $d(C_{t+1}^t)$ from equation (28) into (29) and (30) yields respectively:

$$(31) \quad \frac{d(K_t)}{K_t} = \frac{-K_t}{\alpha(1-\alpha)(A_t K_t^\alpha)} d(r_t) - a_{11} \{ RP_t d(K_t) + A_t K_t^\alpha d[\ln(A_t)] - (1 + r_t) d(C_t^t) + (\bar{Y} - C_t^t - K_t - T_t) d(r_t) \} + \frac{1}{1-\alpha} d[\ln(A_t)]$$

$$(32) d(C_t^t) = \frac{\beta(1+r_t)}{1+\beta(1+r_t)^2} \left[\left(\bar{Y} - C_t^t - K_t - T_t - \frac{C_t^t}{\rho(1+r_t)} \right) d(r_t) + RP_t d(K_t) + A_t K_t^\alpha d[\ln(A_t)] \right]$$

$$\text{where } a_{11} = \frac{\rho RP_t K_t}{(A_t K_t^\alpha) C_{t+1}^t \alpha (1-\alpha)}.$$

After substituting for $d(C_t^t)$, given by equation (32), into (27) and (31) and then eliminating the terms in $d(K_t)$ from the resulting expressions, one can obtain:

$$(33) \quad d(r_t) = \frac{a_{12}}{a_{13}} d[\ln(A_t)], \text{ where}$$

$$a_{12} = \frac{K_t}{1-\alpha} - \frac{\rho RP_t K_t^2}{C_{t+1}^t \alpha (1-\alpha)} + \frac{\rho RP_t K_t^2 \beta (1+r_t)^2}{C_{t+1}^t \alpha (1-\alpha) [1+\beta(1+r_t)^2]} +$$

$$\left[1 + \frac{\rho RP_t^2 K_t^2}{C_{t+1}^t (A_t K_t^\alpha) \alpha (1-\alpha)} - \frac{\rho RP_t^2 K_t^2 \beta (1+r_t)^2}{C_{t+1}^t (A_t K_t^\alpha) \alpha (1-\alpha) [1+\beta(1+r_t)^2]} \right] \frac{\beta(1+r_t)(A_t K_t^\alpha)}{1+\beta(1+r_t)^2 + \beta(1+r_t)RP_t}$$

and

$$a_{13} = \frac{\beta}{\rho} \left[1 + \frac{\rho RP_t^2 K_t^2}{C_{t+1}^t (A_t K_t^\alpha) \alpha (1-\alpha)} - \frac{\rho RP_t^2 K_t^2 \beta (1+r_t)^2}{C_{t+1}^t (A_t K_t^\alpha) \alpha (1-\alpha) [1+\beta(1+r_t)^2]} \right] \frac{C_t^t - \beta(1+r_t)}{1+\beta(1+r_t)^2 + \beta(1+r_t)RP_t} +$$

$$\frac{K_t^2}{\alpha(1-\alpha)(A_t K_t^\alpha)} \left[1 + \frac{RP_t C_t^t \beta (1+r_t)^2 + \rho RP_t (\bar{Y} - C_t^t - K_t - T_t)}{C_{t+1}^t [1+\beta(1+r_t)^2]} \right]$$

Taking logs of our RP expression (equation 15) yields:

$$(34) \quad \ln(RP_t) = \ln[\alpha \rho (\text{var } \varepsilon_{t+1})] + \ln(K_t^{\alpha-1} \psi_t A_t)$$

Differencing equation (34) and noting that the first term on the right hand side can be treated as a constant gives in steady state²⁴:

$$(35) \quad \frac{d[\ln(RP_t)]}{dt} = \frac{d[\ln(A_t)]}{dt} + (\alpha-1) \frac{d[\ln(K_t)]}{dt} + \frac{d[\ln(A_t)]}{dt} + \alpha \frac{d[\ln(K_t)]}{dt} - \frac{d[\ln(C_{t+1}^t)]}{dt}$$

²⁴ Using the fact that $d[\ln(RP_t)] = (\alpha-1) \ln \frac{K_t}{K_{t-1}} + \ln \frac{\psi_t}{\psi_{t-1}} + (\ln A_t - \ln A_{t-1})$ and substituting explicitly for ψ_t .

Noting that, in general, for any variable X_t , $\frac{d[\ln(X_t)]}{dt} = \frac{1}{X_t} d(X_t)$ it is useful to re-write

equations (28)-(30) respectively as:

$$(36) \quad \frac{d[\ln(C'_{t+1})]}{dt} = \frac{RP_t K_t}{C'_{t+1}} \frac{d[\ln(K_t)]}{dt} + \frac{A_t K_t^\alpha}{C'_{t+1}} \frac{d[\ln(A_t)]}{dt} - (1+r_t) \frac{C'_t}{C'_{t+1}} \frac{d[\ln(C'_t)]}{dt} + \frac{(\bar{Y} - C'_t - K_t - T_t)}{C'_{t+1}} \frac{d(r_t)}{dt}$$

$$(37) \quad \frac{d[\ln(C'_t)]}{dt} = \frac{-\beta}{\rho} \frac{d(r_t)}{dt} + \frac{C'_t \beta (1+r_t)}{C'_{t+1}} \frac{d[\ln(C'_{t+1})]}{dt}$$

$$(38) \quad \frac{d[\ln(K_t)]}{dt} = \frac{-K_t}{\alpha(1-\alpha)(A_t K_t^\alpha)} \frac{d(r_t)}{dt} - a_{11} C'_{t+1} \frac{d[\ln(C'_{t+1})]}{dt} + \frac{1}{(1-\alpha)} d[\ln(A_t)]$$

Then, using equations (33) and (36)-(38) substitute out for the change in log consumption,

capital, and real interest rate. Noting that $\frac{d[\ln(A_t)]}{dt} = \varepsilon_t$, equation (35) becomes

$$(39) \quad \frac{d[\ln(RP_t)]}{dt} = z \varepsilon_t, \text{ where}$$

$$z = 2 - \frac{(A_t K_t^\alpha)}{[1 + \beta(1+r_t)^2] C'_{t+1}} - \frac{a_{12}}{a_{13}} \frac{1}{1 + \beta(1+r_t)^2} \left[\frac{\bar{Y} - C'_t - K_t - T_t}{C'_{t+1}} + \frac{\beta(1+r_t)}{\rho} \right] + \frac{1 + \beta(1+r_t)^2}{1 + \beta(1+r_t)^2 + a_{11} K_t RP_t} \left[2\alpha - 1 - \frac{K_t RP_t}{C'_{t+1} [1 + \beta(1+r_t)^2]} \right] \left[\frac{a_{11} (A_t K_t^\alpha)}{(1-\alpha)[1 + \beta(1+r_t)^2]} - \frac{a_{12} K_t}{a_{13} \alpha(1-\alpha)(A_t K_t^\alpha)} - \frac{a_{12}}{a_{13}} \frac{a_{11} C'_{t+1}}{1 + \beta(1+r_t)^2} \left(\frac{\bar{Y} - C'_t - K_t - T_t}{C'_{t+1}} + \frac{\beta(1+r_t)}{\rho} \right) \right]$$

Equation (39) says that the risk premium RP_t is a function of the state variable A_t . In period 't' the only people free to make a new decision are generation 't', the others are locked in by there previous decisions. Hence the price of capital at time 't' must be such as to persuade the marginal holder of capital to hold it, ie to satisfy the t-generation's first order

condition²⁵. At first, we treat ‘z’ in equation (39) as a free parameter to see what its value should be in order to match the observed variance of the risk premium. After that, we compare the actual 3rd and 4th moments of the change in the logarithm of risk premium with the values predicted by using the ‘z’ already obtained – which we compute with some standard error by bootstrapping. Finally, we calibrate the model to see what its prediction for ‘z’ would be in this case.

The return on equities is basically an expected return. This is constructed from the S&P index (which also includes dividends) and, for each year, is taken to be the average of the 20 years following. The data is annual and covers the period from 1933 to 1980. The risk premium is then obtained by subtracting from this the current nominal interest rate. Figure VII shows the properties for the $\Delta[\ln(RP_t)]$ series²⁶. The properties of the residuals, depicted in Figure VIII, are from an OLS regression of a random walk model for the S&P index over the same period. Then, in order to match the observed variances for the risk premium and our measure of technology shocks, ε_t , ‘z’ should be (from equation 39):

$$z = \sqrt{\frac{\text{variance}[\Delta \ln(RP_t)]}{\text{variance}(\varepsilon_t)}} = \frac{0.175554}{0.138123} = 1.271$$

Next, we substitute this value of ‘z’ in equation (39) and generate 1,000 pseudo-samples for the $\Delta[\ln(RP_t)]$ series from which we can get a distribution for the skewness and kurtosis of $\Delta[\ln(RP_t)]$. The bootstrap results for these are presented in Tables VI and VII respectively. As it can be seen, the actual values of the skewness (-0.30084) and kurtosis (3.2413) of $\Delta[\ln(RP_t)]$, are well within the 95% confidence interval limit.

²⁵ There would be a new A_t but the only people ‘pricing it’ in the market – free not to buy or buy – would be the t-generation people.

²⁶ For measurement reason we assumed that $\frac{d}{dt} = \Delta$, where Δ is a discrete time change.

Our final step is to calibrate the model to see what the model predicted value for ‘z’ would be.

Assuming $\alpha = 0.3$, $C_{t+1}^t = 0.33$, $C_t^t = 0.34$, $T_t \approx G_t = 0.17$, $\bar{Y} = 0.78$, $A_t K_t^\alpha = 0.22$,

$\beta(1 + r_t) = 1$, $RP_t = 6.18$, $K_t = 0.17$ the value of z turns out to be 1.27.

For the real rate of interest we follow the same approach and check whether the model can predict higher moments that would match the observed ones. Figure IX shows the properties of $d(r_t)$ for the same time period, 1933-1980. Equation (33) gives the model prediction for the change in the real interest rate. Using the above calibrated values, the ratio a_{12} / a_{13} turns out to be 0.3. Thus, our model predicts a standard deviation for $d(r_t)$ of 0.04^{27} which is pretty close to 0.035, the observed s.d. for $d(r_t)$ from Figure IX. Bootstrapping results for the skewness and kurtosis of the $d(r_t)$ series, depicted in Table VIII and IX respectively, also show that the model prediction is within the 95% confidence interval limit.

V Conclusions

In the context of a simple overlapping generations model we have attempted to explain the observed equity premium by stressing the importance of the high level of risk implied in holding equities for retirement. If private savings for old age are effectively irreversible and are made in long-term investment schemes, either in government bonds or in equity, then random changes in productivity will affect the value of their equity holdings, and hence their wealth. The uncertainty in equity investment is large: the reason is that, when shocks occur they cannot be relied upon to disappear in subsequent years. Although average equity growth is highly impressive, over particular periods it can be dramatically lower or

²⁷ $s.d.[d(r_t)] = 0.3 * s.d.(\varepsilon_t) = 0.3 * 0.138 = 0.04$.

higher, depending on these shocks. Our model fits not only the average risk premium but also the average interest rate and also furthermore the second and higher moments of the annual changes in both risk-premium and interest rate. Our conclusion is that the neglect of the long time-horizon over which risk is assessed may well be a major constituent of the risk-premium puzzle.

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Table I. Historical Returns on U.S. Financial Assets²⁸

Time periods	Percent growth rate of personal consumption		percent real return on a relatively riskless security		percent premium risk		percent real return on S&P composite index	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
1929-2000	3.30	3.19	0.54	3.92	7.11	17.04	7.66	16.75
1931-1940	2.20	5.88	1.96	5.19	3.83	26.81	5.79	26.44
1941-1950	4.24	3.97	-5.09	4.98	14.58	15.71	9.49	16.75
1951-1960	3.33	1.98	0.12	2.52	15.13	14.71	15.25	14.17
1961-1970	4.38	1.58	1.59	0.53	3.05	10.92	4.64	10.98
1971-1980	3.28	2.38	-1.03	1.44	0.54	12.82	-0.49	11.94
1981-1990	3.44	1.62	3.71	1.17	8.18	14.59	11.89	14.83
1991-2000	3.42	1.49	1.77	1.07	10.57	13.14	12.34	13.76

Table II. Stockmarket Indexes, Correlation Matrix.

	S&P100	NIK225	FT 30	CZBK
S&P100	1			
NIK225	0.4676	1		
FT 30	0.8495	0.7250	1	
CZBK	0.8404	0.6869	0.8602	1

²⁸ The time series used to generate the summary statistics provided in Table I are from the following sources. Consumption is real personal expenditures in chained 1996 dollars and it is downloaded from the Bureau of Economic Analysis website. The relatively riskless security is taken to be the 3-month T-Bill rate from the Federal Reserve Economic Data website. In order to obtain the real return we subtract from this inflation. The S&P composite index has been downloaded from Robert Schiller's website.

Table III. Bootstrap Variances (various periods ahead)

	Random Walk Model			ARIMA (1,1,1) Model	
	UK	US	All*	UK	All*
Residuals Variance	0.0079	0.0144	0.0063	0.0072	0.0064
10 years ahead	0.316	0.456	0.241	0.444	0.329
20 years ahead	0.632	0.912	0.492	0.856	0.780
30 years ahead	0.948	1.368	0.754	1.210	1.055

* Refers to the weighted stock market index.

Table IV. Estimated Risk Premium (percent per annum) – the U.K. case

	CRRA Coefficient (ρ)			
		1	1.5	2
Share of Equity Income in	0.47	2.49	3.74	4.98
the income of the old (ψ)	0.56	2.96	4.44	5.92
	0.66	3.49	5.24	6.98

The values in Table IV are obtained for $\text{Var}(\varepsilon_t) = 1$.

Table V. Estimated Risk Premium (percent per annum) – the U.S. case.

	CRRA Coefficient (ρ)			
		1	1.5	2
Share of Equity Income in	0.47	3.56	5.34	7.12
the income of the old (ψ)	0.56	4.24	6.36	8.48
	0.66	5.00	7.49	9.99

The values in Table V are obtained for $\text{Var}(\varepsilon_t) = 1$

Table VI. Bootstrap results for the distribution of the skewness of $\Delta[\ln(RP_t)]$ series (1,000 replications).

Tabulation of SKEWNESS
 Sample: 1001 2000
 Included observations: 1000
 Number of categories: 21

Value	Count	Percent	Cumulative	
			Count	Percent
[-1.2, -1.1)	5	0.50	5	0.50
[-1.1, -1)	8	0.80	13	1.30
[-1, -0.9)	10	1.00	23	2.30
[-0.9, -0.8)	17	1.70	40	4.00
[-0.8, -0.7)	28	2.80	68	6.80
[-0.7, -0.6)	57	5.70	125	12.50
[-0.6, -0.5)	76	7.60	201	20.10
[-0.5, -0.4)	79	7.90	280	28.00
[-0.4, -0.3)	113	11.30	393	39.30
[-0.3, -0.2)	117	11.70	510	51.00
[-0.2, -0.1)	119	11.90	629	62.90
[-0.1, 0)	101	10.10	730	73.00
[0, 0.1)	84	8.40	814	81.40
[0.1, 0.2)	54	5.40	868	86.80
[0.2, 0.3)	36	3.60	904	90.40
[0.3, 0.4)	46	4.60	950	95.00
[0.4, 0.5)	24	2.40	974	97.40
[0.5, 0.6)	11	1.10	985	98.50
[0.6, 0.7)	11	1.10	996	99.60
[0.7, 0.8)	3	0.30	999	99.90
[0.8, 0.9)	1	0.10	1000	100.00
Total	1000	100.00	1000	100.00

Table VII. Bootstrap results for the distribution of the kurtosis of $\Delta[\ln(RP_t)]$ series (1,000 replications).

Tabulation of KURTOSIS

Sample: 1001 2000

Included observations: 1000

Number of categories: 19

Value	Count	Percent	Cumulative	Cumulative
			Count	Percent
[1.6, 1.8)	1	0.10	1	0.10
[2, 2.2)	13	1.30	14	1.40
[2.2, 2.4)	26	2.60	40	4.00
[2.4, 2.6)	54	5.40	94	9.40
[2.6, 2.8)	85	8.50	179	17.90
[2.8, 3)	132	13.20	311	31.10
[3, 3.2)	166	16.60	477	47.70
[3.2, 3.4)	154	15.40	631	63.10
[3.4, 3.6)	127	12.70	758	75.80
[3.6, 3.8)	88	8.80	846	84.60
[3.8, 4)	63	6.30	909	90.90
[4, 4.2)	42	4.20	951	95.10
[4.2, 4.4)	17	1.70	968	96.80
[4.4, 4.6)	12	1.20	980	98.00
[4.6, 4.8)	9	0.90	989	98.90
[4.8, 5)	4	0.40	993	99.30
[5.2, 5.4)	3	0.30	996	99.60
[5.4, 5.6)	3	0.30	999	99.90
[6.8, 7)	1	0.10	1000	100.00
Total	1000	100.00	1000	100.00

Table VIII. Bootstrap results for the distribution of the skewness of $d(r_t)$ series (1,000 replications).

Tabulation of SKEW_R
 Sample: 1001 2000
 Included observations: 1000
 Number of categories: 13

Value	Count	Percent	Cumulative	Cumulative
			Count	Percent
[-1.6, -1.4)	1	0.10	1	0.10
[-1.4, -1.2)	2	0.20	3	0.30
[-1.2, -1)	6	0.60	9	0.90
[-1, -0.8)	38	3.80	47	4.70
[-0.8, -0.6)	83	8.30	130	13.00
[-0.6, -0.4)	183	18.30	313	31.30
[-0.4, -0.2)	226	22.60	539	53.90
[-0.2, 0)	224	22.40	763	76.30
[0, 0.2)	139	13.90	902	90.20
[0.2, 0.4)	59	5.90	961	96.10
[0.4, 0.6)	24	2.40	985	98.50
[0.6, 0.8)	12	1.20	997	99.70
[0.8, 1)	3	0.30	1000	100.00
Total	1000	100.00	1000	100.00

Table IX. Bootstrap results for the distribution of the kurtosis of $d(r_t)$ series (1,000 replications).

Tabulation of KURT_R
 Sample: 1001 2000
 Included observations: 1000
 Number of categories: 9

Value	Count	Percent	Cumulative	Cumulative
			Count	Percent
[1.5, 2)	2	0.20	2	0.20
[2, 2.5)	60	6.00	62	6.20
[2.5, 3)	271	27.10	333	33.30
[3, 3.5)	346	34.60	679	67.90
[3.5, 4)	222	22.20	901	90.10
[4, 4.5)	72	7.20	973	97.30
[4.5, 5)	21	2.10	994	99.40
[5, 5.5)	4	0.40	998	99.80
[5.5, 6)	2	0.20	1000	100.00
Total	1000	100.00	1000	100.00

Figure I. Real Annual Return on a Relatively Riskless Security (percent)

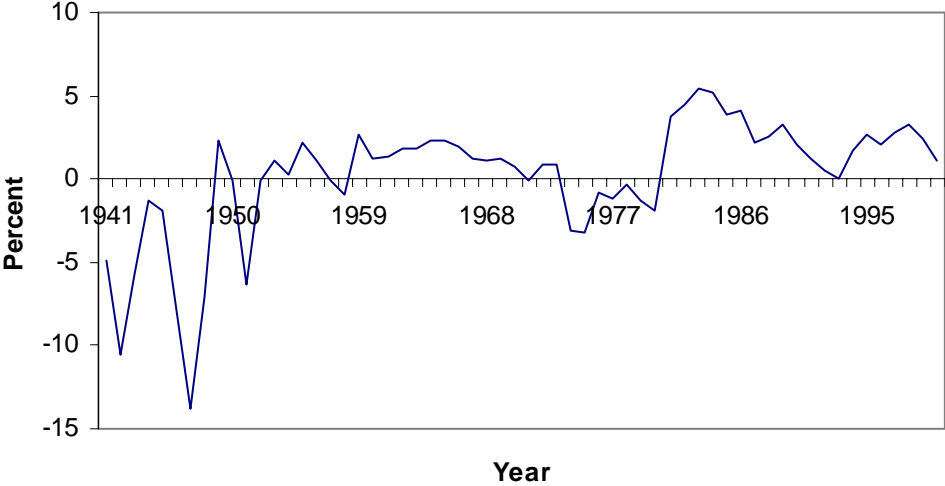


Figure II. Real Return on S&P Composite Index (percent)

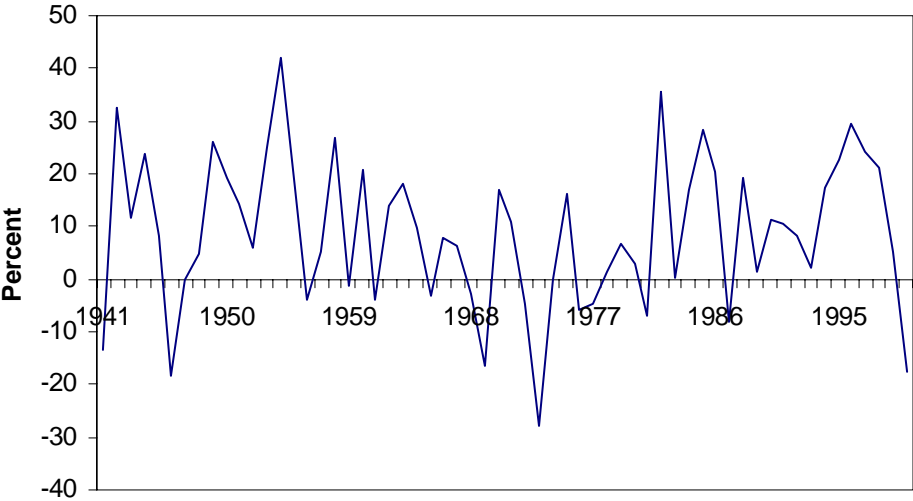


Figure III. Risk Premium (percent)

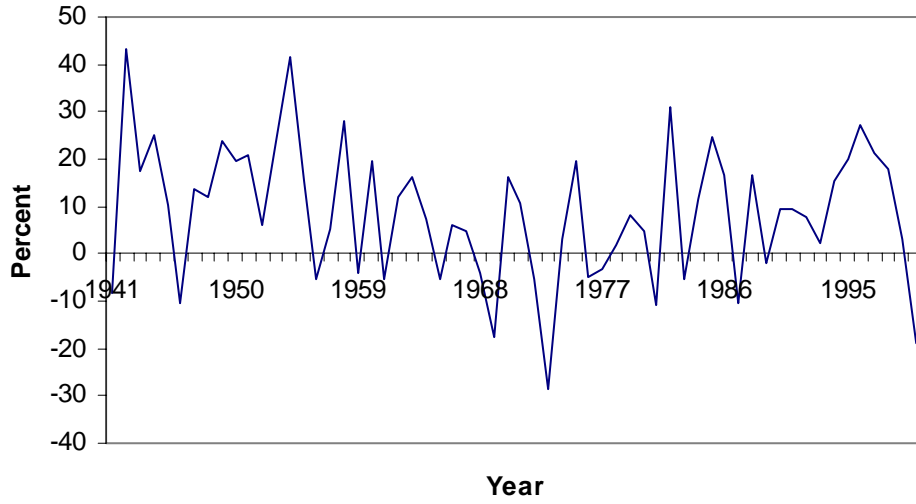


Figure IV. Growth Rate of Real Average Consumption (percent)

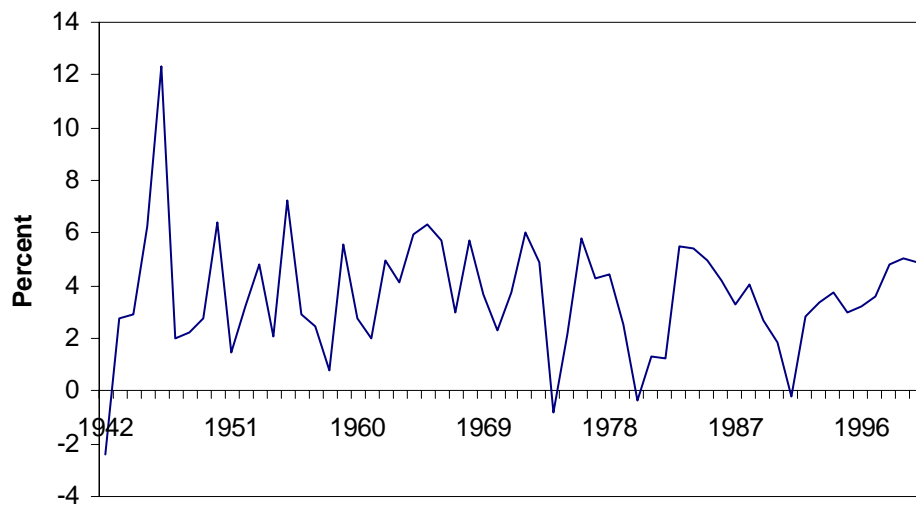


Figure V. Uncertainty 30 years ahead. Random Walk Model.

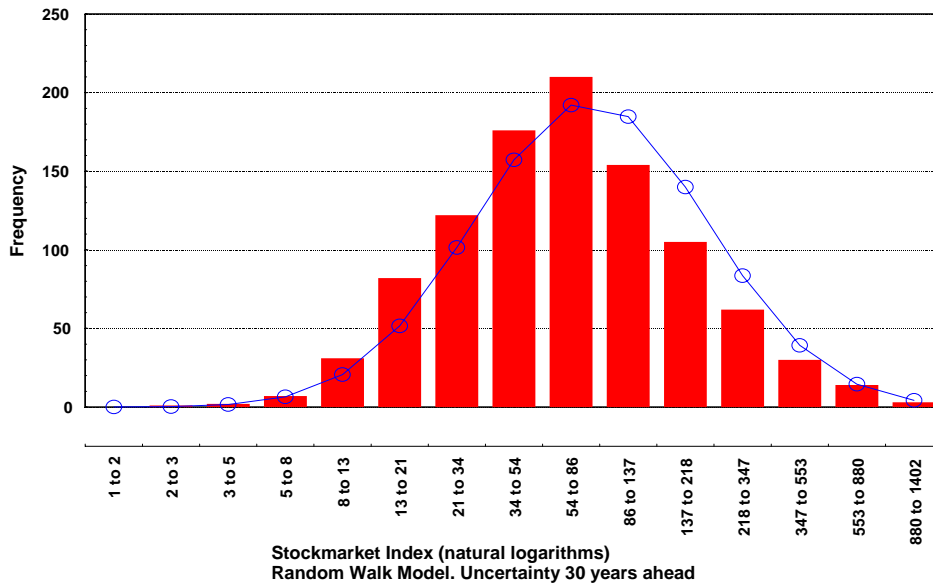


Figure VI. Uncertainty 30 years ahead. ARIMA (1,1,1) Model.

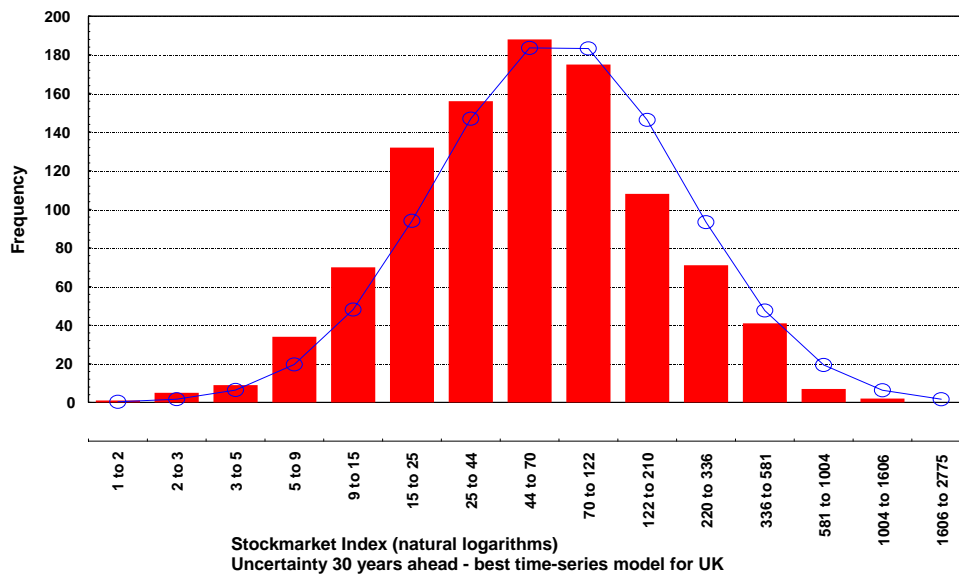


Figure VII. Properties of the $\Delta[\ln(RP_t)]$ series.

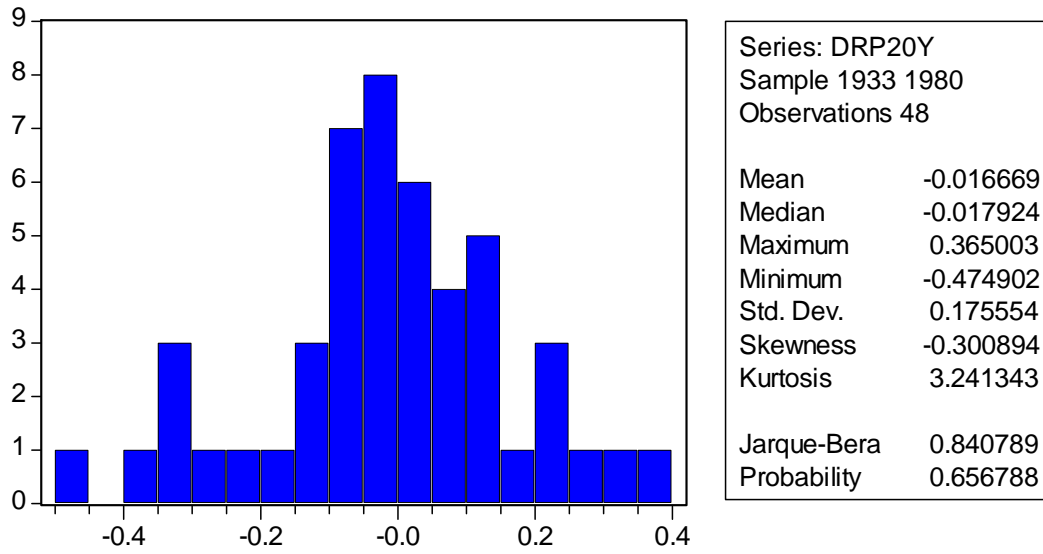


Figure VIII. Properties of the residuals, ε_t .

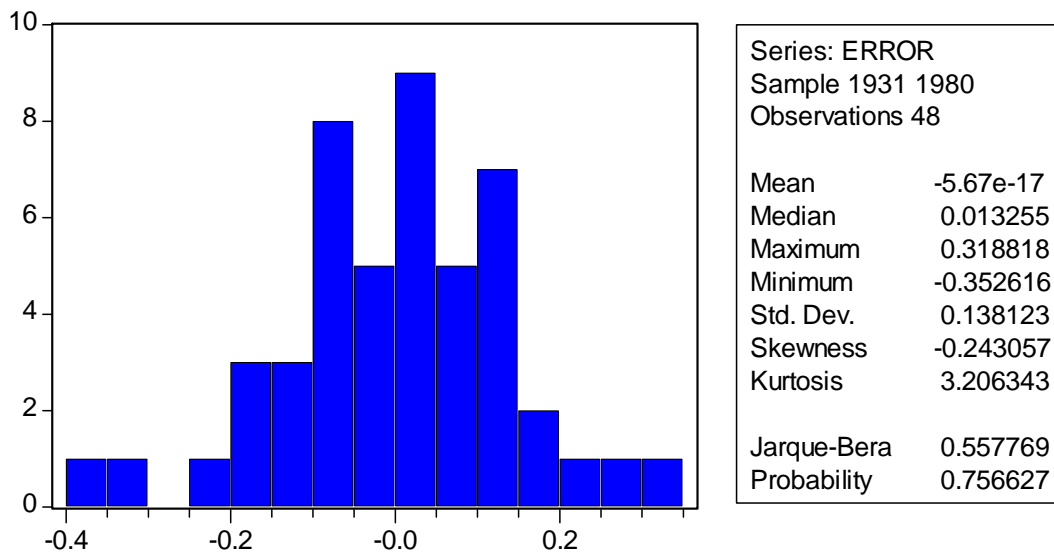


Figure IX. Properties of the $d(r_t)$ series

