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# INEQUALITY, MALNUTRITION AND UNEMPLOYMENT: A CRITIQUE OF THE COMPETITIVE MARKET MECHANISM*

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## ABSTRACT

This essay presents a rigorous theory of involuntary unemployment in less developed countries based on the observation that at low consumption levels a person's overall ability to work is impaired. The theory links the incidence of involuntary unemployment in a market economy to the incidence of malnutrition and this in turn to inequality in the distribution of physical assets. The theory is a classical one and attention is deliberately concentrated on situations where there is no aggregate demand deficiency. It is shown that certain patterns of inequality-reducing asset redistribution reduce the incidence of mal-nourishment and the volume of employment and thus increase the aggregate level of output in a market economy. In particular, this means that in such economies there is no necessary conflict between the goal of equality (in the distribution of assets and consumption) and the goal of increasing the aggregate level of output and employment.

JEL classification: 021, 022, 024, 112, 113, 717, 821, 822, 914.

Keywords: inequality, malnutrition, unemployment, competitive, general equilibrum.

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# NON-TECHNICAL SUMMARY

Received theories of economic development are oblivious to the phenomenon of malnutrition and its possible relationship to involuntary employment, the production and distribution of income and the distribution of assets, in less developed countries. In this essay the authors develop a rigorous theory that can be used to explain the links. The theory is based on the observation that at low consumption levels a person's overall ability to work is impaired. The theory is otherwise classical and attention is deliberately concentrated on situations where there are no missing markest and where all markets are competitive. In particular the hypotheses ensure that there is no aggregate demand deficiency. The model used to illustrate the theory is a fully general equilibrium one. Involuntary unemployment is shown to exist in the construct, not assumed; that is, wage rigidities are explained, not hypothesised.

It is shown that if the economy is moderately endowed with physical assets (in a sense that is made precise) the incidence of malnutrition and involuntary unemployment can be traced directly to inequalities in the distribution of these assets, and in particular, that certain patterns of inequality-**reducing** asset redistributions can **reduce** (and possibly eliminate) the extent of malnourishment and unemployment and **increase** the level of aggregate output. Indeed, the model is so constructed that asset (or consumption) redistribution is the only policy open to the government for eliminating unemployment in the short and medium run. The theory incorporates 'trickle-down' as a phenomon, but draws attention to the fact that there are policies that can more speedily reduce the extend of malnourishment and unemployment

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## 1. The Issues

Give or take a hundred million, some four hundred million people in the world are estimated to be undernourished.[1] International data on the incidence of malnutrition are in large parts only sketchy. Moreover, those that are available aren't always readily interpretable, for the science of nutrition is relatively new.[2] It is of course recognized that a person's nutritional needs depend not only on the sorts of activities he is engaged in, but also on his location and on his personal characteristics, of which the last includes his past history. But in order to assess these needs one wants to know not only whether he is malnourished, but also the type and extent of his undernourishment.[3] This is what makes the subject very difficult.[4]

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The general effects of malnutrition vary widely. In children, they are particularly severe. It can cause muscle wastage, growth retardation, and increased illness and vulnerability to infection. There is evidence that it can affect brain growth and development.[5/] Chronic malnutrition in adults diminishes their muscular strength and the capacity to do work. Persons suffering thus are readily fatigued. There are also marked psychological changes, manifested by mental apathy, depression, introversion and lower intellectual capacity (see e.g., Read [1977]). In expectant mothers there is the additional effect on the fetus, and therefore the quality of the next generation, (see e.g., Morgane, et.al. [1979]). Life expectancy among the malnourished is low, but not nil. Such people don't face immediate death. Malnutrition is this side of starvation. And, as the evidence suggests, the world can indefinitely carry a stock of undernourished people, living and breeding in impaired circumstances.

Not surprisingly, an overwhelming majority of the world's undernourished live in developing countries, (see FAO [1974], p. 66). Not a negligible number of economists have gone on to emphasize that it is the absolute poor who go hungry.[6/] But then who <u>are</u> the absolute poor? The available evidence suggests that they are among the landless, or near-landless people (see DaCosta [1971], Reutlinger and Selowsky [1976], and Fields ([1980], p. 161), for example). Presumably this is because they have no non-wage income, or if they do it is precious little. But then why don't they get employed and earn a wage? One answer is that, because the economy is resource-poor, the low level of prevailing wages doesn't provide the necessary escape from absolute poverty (Leibenstein [1957] and Fei and Chiang [1966]).

Another answer is that these people can't find jobs. But how can that be? It can, if the labour market doesn't clear at the "going wage" and the non-clearance manifests itself in involuntary unemployment. But this begs the question, for why doesn't the labour market clear? In particular, why don't frustrated job-seekers undercut the employed?

To this, received development theory offers scattered answers. Reference is made, on occasion, to union pressure when in most such countries the proportion of those unionized, even among the industrial working-class, is negligible. Or the accusing finger is pointed at minimum wage legislation, when in many such economies the informal production sector is so large that legislation may well have little bearing on the matter. Or reference is made to the deliberate attempt of capitalists as a class to purchase employee loyalties by keeping wages high (Marglin [1976]). But this, too, begs the question, for one must then ask why the price of loyalty is rigid downwards.[7/] Then again, an occasional appeal is made to the "efficiency wage" hypothesis to explain rigid wages, (see Leibenstein [1957] and Mirrlees [1975]), but the idea hasn't been incorporated fully in development theories.

In this essay, we will be concerned with involuntary unemployment and its relation with malnutrition. The particular concept we use is made clear below. It is to be distinghished from surplus labour, the incidence and policy implications of which have been extensively studied. In its broadest sense, surplus labour is said to exist in a dual economy if the wage in the industrial sector exceeds the direct opportunity cost - as measured in terms of foregone output - of labour in the rural sector (see, e.g. Marglin [1976], Chapter 2). Now, one can have surplus labour without involuntary unemployment. For one may imagine an economy carrying surplus labour and in which the industrial wage equals the average product of labour in the rural sector and where rural people working in family farms, consume their average product. The concept of involuntary unemployment we use is somewhat sharper. We will say that there is involuntary unemployment among a particular class of (identical) workers if such workers are rationed in the labour market, so that while a fraction finds employment in the market the remainder doesn't, and those who don't are worse off than those who do.

Now, at the theoretical level this is a satisfactory notion, but it may be asked whether it is operationally useful, since no two persons are in fact identical, (Goodhart [1983]), and indeed, in the framework that we shall study in this essay not all persons are identical. So an extension of the concept is needed, and the obvious one is this: a particular worker is involuntarily unemployed if he can't find employment in a market which does employ a worker very similar to him and if the latter worker, by virtue of his employment in this market, is distinctly better off than him.[8/]

Our objective is to construct a theory of malnutrition and involuntary unemployment; one that lays bare the relationship between these concepts and the ownership distribution of assets.

Received theories of economic development are oblivious to the phenomenon of malnutrition and its possible relationship to involuntary unemployment, the production and distribution of income, and the distribution of assets. And it seems to us that the reason for this is to be found in the model that has so circumscribed the subject's contours, namely Professor Arthur Lewis' construction postulating an unlimited supply of labour (see Lewis [1954]). The point is that if the consumption level of the representative member of the reserve army of labour meets his nutritional needs, malnutrition doesn't exist: the problem is not then there. On the other hand, if it doesn't, then the incidence of undernourishment is very very great--the reserve army being huge--but by the same token, not much can be done about it; at least not for a long while. All these people are undernourished because they are poor, and they are poor because the (rural) economy is incapable of supporting them adequately: food is supply-constrained.[9] For this reason the distinction between surplus labour and involuntary unemployment isn't, at the analytical level, of great moment in the Lewis theory: if malnutrition exists it preys on a great number of people. Wage flexibility wouldn't alleviate the problem. Growth is the only way out.

The paucity of theoretical frameworks linking poverty and malnutrition to unemployment, and more fundamentally, inequality (in the distribution of assets), is particularly glaring, since there is now an increased interest among development economists in these matters and in the state of development economics in particular. There is a considerable body of both time-series and cross-section data regarding

output, employment, inequality and poverty in developing countries.[10/]
There are also a great many commentaries on them. Furthermore, the literature on the axiomatic foundations of inequality and poverty measures is not negligible. (Fields [1980] and Sen [1983] are good references for both such commentaries and welfare axiomatics.) But neither curve-fitting nor welfare axiomatics is a substitute for theory. Neither is designed to cope with counterfactuals, an essential ingredient in *policy* debates, and it is here that theory matters.

In this essay we will develop a theory that links poverty and malnutrition to involuntary unemployment, output and the distribution of income and assets. Involuntary unemployment will be shown to exist in the construct, not assumed. The model we will present to illustrate the theory is a general equilibrium one. The theory will be developed in sections 3.1-3.7. In order to highlight some of the more novel features we will be thinking, in Section 3.8, of an economy which is moderately, but sufficiently, endowed with physical assets. We will argue that theoretical reasons can be provided for the observation that if in such an economy the distribution of assets (which we will, for concreteness, think of as *land*) is unequal it is the assetless and the near-assetless who are poor and malnourished and unemployed. In particular, the market will be shown to treat identical people non-identically and near-identical persons distinctly differently. Since the hypothesis in Section 3.8 will be that the model economy is productive enough to employ all and to feed all, the accusing finger will point directly at the inequality in land distribution as the cause of the misery. We also establish the result that such equilibria are Pareto-efficient, an observation that only underscores the (well-known) uselessness of that criterion in evaluating situations of inequality and malnutrition.

We study, too, the effects of land redistribution, (or equivalently, consumption redistribution). We show that aggregate output is increased by a shift towards a more egalitarian land (or consumption) distribution. While the effect on the set of employed people is less clear (for reasons discussed below), we exhibit explicitly cases where unequal land distributions yield involuntary unemployment, and where an improvement in land distribution yields full employment (and higher output). There is no conflict here between the goals of equal asset (or consumption) distribution and raising output and employment.[11/]

The model that we will develop in this essay postulates frictionless markets for all capital assets and a flawless competitive spirit among employers and workers. We wish to emphasize this point, because at the level of theoretical discourse it won't do to explain poverty, malnutrition and unemployment by an appeal to monopsonistic landlords, or predatory capitalists, or a tradition-bound working class and leave it at that. That's far too easy, but more to the point, one is left vulnerable to the argument that this merely shows that governments should concentrate their attention in freeing markets from restrictive practices. It doesn't provide an immediate argument as to why governments, if they are able to, should intervene to ensure directly that people aren't malnourished. In a penetrating study

Professor P.T. Bauer discusses the role of the government in developing a framework within which material progress can best occur, (Bauer [1971], pp. 90-92). Direct intervention on the problem of malnourishement isn't one of the government tasks he lists. He goes on to say, "It is probable that in many under-developed countries significant numbers of people would respond to a widening of opportunities, especially to opportunities offered by a widening of markets, and by the emergence of external contacts." (Bauer [1971], p. 91). But none of the government services Bauer lists is a substitute for assured food and nutrition. What is a person to do when he wants to avail himself of the opportunities opened up by the widening of markets and external contacts but simply can't because he is malnourished, and thus not capable of providing the quality of labour service the market bears? The market denies such a person what Berlin [1958] calls in a wider context, "positive freedom". "Market failure" is, of course, only a polite way of describing the phenomenon. But the point to emphasize is that such "failure" isn't due to an absence of markets or restrictive practices. The failure can be corrected only through direct central intervention. (See Section 3.8.)

It is usual to argue that an unemployed person is poor because he is unemployed. The causal chain which we will suggest here runs in the opposite direction: a person is unemployed because he is malnourished, and he is malnourished because he is poor, and he is poor because he has little or no assets, not even the appropriate quality of labour power.

## 2. The Consumption-Ability Relationship

Except for the literature on human capital formation, received economic theory takes the ability of an individual to perform tasks as exogenously given. In the concept of human capital acquisition current consumption is traded for future ability and thus future consumption. The clinical literature on malnutrition, on the other hand, draws our attention to the fact that there is—at low consumption levels—a positive relationship between current consumption and current ability.[12/] The relationship was introduced into the development literature in a pioneering work by Leibenstein [1957], and in subsequent contributions Prasad [1970], Mirrlees [1975], Rodgers [1975] and Stiglitz [1976] used it for an explanation of wage rigidity in poor countries. The basic idea is as follows.

One begins by distinguishing labour-time from labour-power and observes that it is the latter which is an input in production. Consider then a person who works for a fixed number of hours a day. Denote the labour power he supplies over the day by $\lambda$ and suppose that it is functionally related to his daily consumption, I, in the manner of the bold-faced curve in Diagram 1a. (We should emphasize that we are thinking of labour power as an aggregate concept, capturing not only power in the thermodynamic sense, but also mental concentration, cognitive faculty, morbidity etc.)

The key features of the functional relationship are that is increasing in the region of interest, and that at low consumption levels

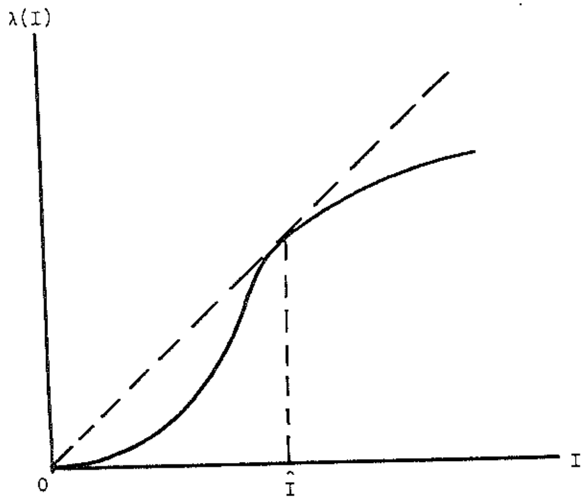



Diagram 1a

it increases at an increasing rate followed eventually by diminishing returns to further consumption.[13/]

An alternative specification of the functional relationship, used, for example, by Bliss and Stern [1978b], is drawn in Diagram 1b. Here, $\lambda$ is nil until a threshold level of consumption, $I^*$. $\lambda(I)$ is an increasing function beyond $I^*$, but it increases at a diminishing rate.

It is only fair to point out at this stage that it is difficult to conceive of the nutrition - labour-power relation as one that exists in a strict quantitative sense. Short term shortfalls in nutrition may be adjusted for by the body (at the cost, to be sure, of medium or

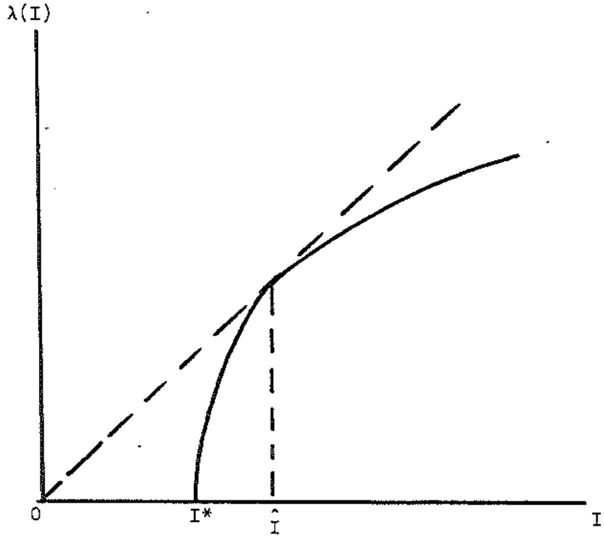



Diagram 1b

longer-run debilitating effects, perceivable, say, in lower body weight[14]). Our postulate of a precise relationship is not be to taken literally - the necessarily simplified analysis only mirrors the implications of a somewhat more fuzzy relationship between nutrition and work efficiency.

Returning to the model, consider a labourer with no non-wage income. Assume that he consumes his entire wage. An enterprise concerned with its profits is interested in the wage it pays the worker per unit of labour power he supplies. In each of Diagrams 1a and 1b this is minimized at a unique value of I. Let us label such a wage $\hat{I}$. Paying him a lower wage would be self-defeating: in effect the labour power would be more expensive![15] This is the source of the wage rigidity identified by Mirrlees [1975], Rodgers [1975], and Stiglitz [1976]. $\hat{I}$ is often referred to as the "efficiency wage".

Mirrlees and Stiglitz were for the greater part uninterested in a market economy. They concentrated their attention on a smaller problem, namely labour allocation in a farm with a small amount of land.[16] A result which they highlighted is that in such an enterprise the members, even when ex-ante identical would not--and in a Utilitarian enterprise, should not--ex-post be given equal consumption; the reason is the region of increasing returns in the individual labour power function $\lambda(I)$ in Diagrams 1a and 1b.[17]

Consider next a person who does have a small source of non-wage income and suppose this amounts to $z$. For concreteness suppose that the consumption-ability curve is as in Diagram 1b. If his wage is $w$ his total income, which by hypothesis equals his consumption, is $z + w$, and so the labour-power he is capable of providing is $\lambda(z + w)$. For such a person wage per unit of labour power is $w/\lambda(z + w)$ and quite obviously the wage at which this is minimized is less than $\hat{I}$. Indeed, if $z$ is larger than $I^*$ in diagram 1b, the wage at which $w/\lambda(z + w)$ is minimized is zero. Rather less obvious is the fact that the resulting total income of the person is also less than the efficiency wage.[18] The implication is that a monopsonist landlord would wish to pay a lower wage to a worker who has an outside income (say from land) than to one who doesn't, and provide the former with a smaller total income. In the case where both types of workers are simultaneously available, the monopsonist would "use up" the landowning type before turning to the set of the landless. Because he equates the marginal labour power across all worker types, total incomes would be equalized across types, too.[19/] This is the central set of results in the theoretical part of the valuable two-part essay by Bliss and Stern [1978a, 1978b]. As we shall see below, however, the effect of competition among employers is to widen income disparities between the landed and the landless, not narrow them.

This rather small number of essays on what is narrowly called the efficiency-wage hypothesis doesn't appear to have been absorbed in the development literature. The consumption-ability relationship should patently be an essential ingredient in any debate concerning growth, inequality and the incidence of poverty and malnutrition, for as we have argued earlier, malnutrition is defined via a person's impaired capacities. Yet it seems to have been bypassed. However, that it may have interesting implications on income distribution was noted long ago. Here is what Alfred Marshall had to say:

> "But it was only in the last generation that a careful study was begun to be made of the effects that high wages have in increasing the efficiency not only of those who receive them, but also of their children and grandchildren ... the application of the comparative method of study to the industrial problems of different countries of the old and new worlds is forcing constantly more and more attention to the fact that highly paid labour is generally efficient and therefore not dear labour; a fact which, though it is more

> full of hope for the future of the human race than any other that is known to us, will be found to exercise a very complicating influence on the theory of distribution." (Marshall [1920], p. 510.)[20/]

In what follows we will identify what some of the more immediate complicating influences might be.

## 3. The Argument

### 3.1 The Basic Model

We will begin by considering a timeless economy. The implications of accumulation we leave for the end, (Section 3.7). For simplicity of exposition, we suppose that two factors, land and labour-power, are involved in the production of a single output , which may be thought of as rice.[21/] Land is homogeneous, workers aren't. Denoting by $T$ the quantity of land and by $E$ the aggregate labour-power employed in production (i.e., the sum of individual labour powers employed) let $F(T,E)$ be the output of rice, where the aggregate production function $F(T,E)$ is assume to be concave, twice differentiable, constant-returns-to-scale, increasing in $E$ and $T$, and displaying diminishing marginal products.[22/] Total land in the economy is fixed, and is $\hat{T}$. Aggregate labour power in the economy is, of course, endogeneous.

Total population, assumed without loss of generality to be equal to the potential work force, is $N$. We take it that $N$ is large, and thus ripe for pure competition. We can therefore approximate and suppose that people can be numbered along the unit interval $[0,1]$.

Each person has a label, $\alpha$, where $\alpha$ is a real number between 0 and 1. In this interval the population density is constant and equal to N. We may therefore normalize and set $N = 1$ so as not to have to refer to the population size again. A person with label $\alpha$ is called an $\alpha$-person. The proportion of land he owns is $t(\alpha)$, so that $\hat{T}t(\alpha)$ is the total amount of land he owns ($t(\alpha)$ is thus a density function). Without loss of generality we label people in such a way that $t(\alpha)$ is non-decreasing in $\alpha$. So $t(\alpha)$ is the land distribution in the economy and is assumed to be continuous. In Diagram 2 a typical distribution is drawn. All persons labelled 0 to $\underline{\alpha}$ are landless. (Thus the land distribution has an atom at zero land.) From $\underline{\alpha}$ the $t(\alpha)$ function is increasing. Thus all persons numbered in excess of $\underline{\alpha}$ own land, the higher the $\alpha$ value of a person the greater the amount of land owned by him.

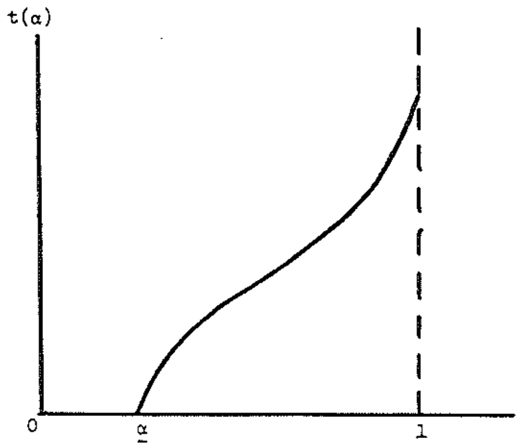
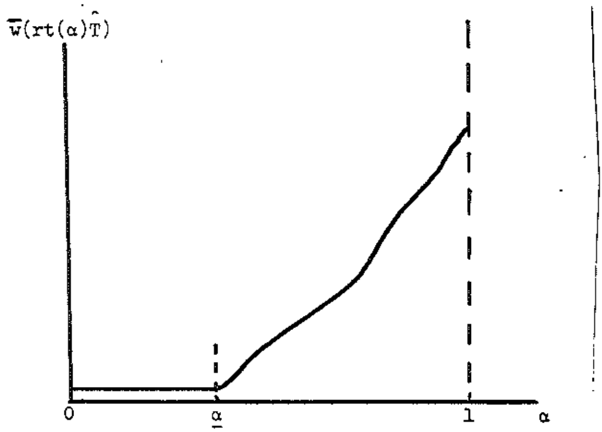



Diagram 2

We will suppose that a person either doesn't work in the production sector or works for one unit of time.[23/] There are competitive markets for both land and labour. Let $r(> 0)$ denote the rental on land. Then $\alpha$-person's non-wage income is $r\hat{T}t(\alpha)$. Each person has a reservation wage which must as a minimum be offered if he is to accept a job in the competitive labour market. For high $\alpha$-types this reservation wage will be high because he receives a high rental income. (His utility of leisure is high.) For low $\alpha$-types, most especially the landless, the reservation wage is low, though possibly not nil. We are concerned with malnutrition, not starvation. In other words, we are supposing that these are normal times that are being modelled. The landless don't starve if they don't find jobs in the competitive labour market. They beg, or at best do odd jobs outside the economy under review, which keep them undernourished. But they don't die.[24/] Thus the reservation wage of even the landless is not necessarily nil. All we assume is that it is less than the efficiency wage, $\hat{I}$, in diagrams 1a or 1b, and in particular that at this reservation wage a person is malnourished.

Denote by $\bar{w}(R)$ the reservation wage function, where the argument $R$ denotes nonwage income. We are supposing here that the $\bar{w}(\cdot)$ function is exogeneously given (continuous and nondecreasing), though of course, nonwage income is endogeneous to the model. For a given rental rate on land, $r > 0$, $\bar{w}(rt(\alpha)\hat{T})$ is constant for all $\alpha$ in the range 0 to $\underline{\alpha}$ (since all these people are identical). Thereafter, $\bar{w}(rt(\alpha)\hat{T})$ increases in $\alpha$ (see Diagram 3)




$\bar{w}(rt(\alpha)\hat{T})$ is the reservation wage of $\alpha$-person at land rental rate $r$

Diagram 3

For our purpose a precise definition of malnutrition isn't required, even for the model economy under study. But for concreteness we are going to choose $\hat{I}$ as the cut-off consumption level below which a person will be said to be undernourished. $\hat{I}$ is then the poverty line. Nothing of analytical consequence depends on this choice. But since the choice of $\hat{I}$ does have a rationale we may as well adopt it. All we need, for our purpose, is the assumption that the reservation wage of a landless person is one at which a person is undernourished.

### 3.2 Definition of an Equilibrium Outcome

We return to the private-ownership, competitive economy with land distribution $t(\alpha)$. Production enterprises are profit maximizing and each person aims to maximize his income given the opportunities he faces.[25/] The aggregate production function, $F(T,E)$, being constant-returns-to-scale, the number of production enterprises isn't well defined. Anyone can become an entrepreneur by exploiting the commonly known production technology. There are no informational gaps in the economy, and so each person is recognized by his label and thus by his landholding.[26/] We would then expect the competitive labour market to sustain a <u>wage schedule</u>; that is, a wage rate per worker type.[27/] Let $w(\alpha)$ denote a wage schedule, the wage that $\alpha$-person is offered in the market for a unit of his labour <u>time</u>.[28/] We may now state the following

<u>Definition:</u> A <u>competitive equilibrium</u> for the private ownership economy is a land rental rate, $\bar{r}$, a set $\bar{G}$ of employed persons, and a wage schedule, $\bar{w}(\alpha)$ defined on $\bar{G}$, such that (i) all agents find it in their interests to abide by $\bar{r}$, $\bar{G}$ and $\bar{w}(\alpha)$, (ii) the supply of land equals the demand for land at these factor prices, and (iii) there is no excess <u>demand</u> for the labour time of any $\alpha$-type at these factor prices, (excess supply of the labour time of some types is, of course, possible).

Observe that an equilibrium announces no wages for the unemployed $\alpha$-types. In other words, firms simply do not deal with those $\alpha$-types that they choose not to employ. No market opens for the labour <u>time</u> of these types. In a conventional translation of this statement: there is a going wage per "efficiency unit" of labour implicit in the equilibrium wage schedule, and unemployed types either cannot, or will not (if their reservation wages are too high) supply the required level of labour power, (see Proposition 1 below).[29]

This definition, while meeting one's intuitive idea of an equilibrium outcome under pure competition, isn't in a useable form. In the following section we shall arrive at an equivalent statement which is very useful.

### 3.3 Characteristics of Equilibria

Aggregate labour power, E, which enters into the production of rice is the sum of the labour powers supplied by all who are employed. It follows, therefore, that at a competitive equilibrium the wage per unit of labour power supplied by all who find employment is the same. Moreover, we should note that the wage per unit of labour power supplied by all who find employment being equal at an equilibrium, a production enterprise is indifferent between all the persons who are employed.

We wish to develop the idea of a competitive equilibrium in our model economy. In order to keep the exposition simple we will specialize and suppose that the consumption - ability curve is of the form given in Diagram 1b and is, barring $I^*$, continuously differentiable at all points.[30] Now define $w^*(\alpha,r)$ as:

$$w^*(\alpha,r) \equiv \arg \min_{w \geq \bar{w}(rt(\alpha)\hat{T})} w/\lambda(w + r\hat{T}t(\alpha)) \quad . \tag{1}$$

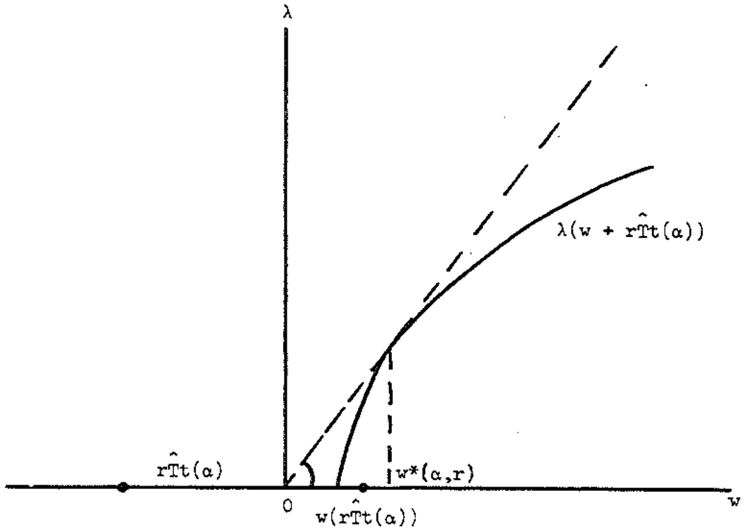
Thus, $w^*(\alpha,r)$ is that wage (per unit of labour time) which, at the land-rental rate $r$, minimizes the wage per unit of labour power of $\alpha$-person, conditional on his being willing to work at this wage rate.[31] Since by hypothesis $\hat{I}$ exceeds the reservation wage of the landless, $w^*(\alpha,r) = \hat{I}$ for the landless. For one who owns a tiny amount of land, $\bar{w}(rt(\alpha)\hat{T}) < w^*(\alpha,r) < \hat{I}$, (see Diagram 4a).

For one with considerable amount of land, $w^*(\alpha,r) = \bar{w}(rt(\alpha)\hat{T})$. finally, for one who owns a great deal of land, $w^*(\alpha,r) = \bar{w}(rt(\alpha)\hat{T}) > \hat{I}$. (See Diagram 4b).[32]

Next, define $\nu(\alpha,r)$ as:

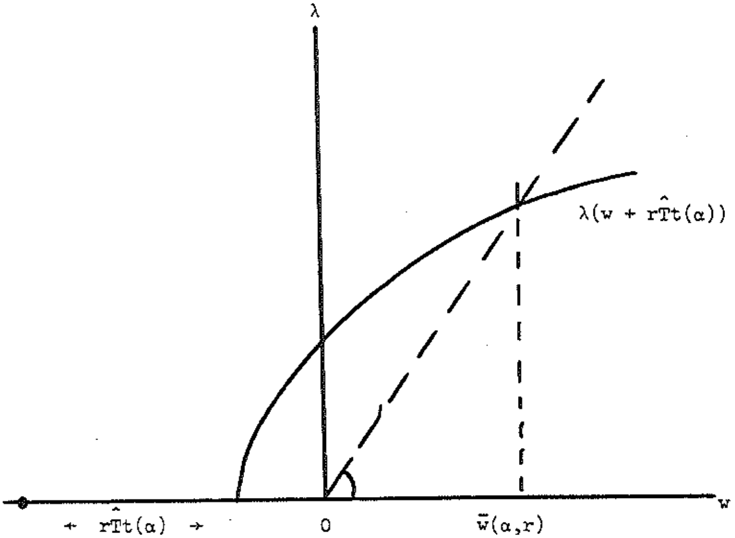
$$\nu(\alpha,r) \equiv w^*(\alpha,r)/\lambda(w^*(\alpha,r) + r\hat{T}t(\alpha)) \quad . \tag{2}$$

Given $r$, $\nu(\alpha,r)$ is therefore the minimum wage per unit of labour power for $\alpha$-person, subject to the constraint that he is willing to work. In diagram 5a a typical shape of $\nu(\alpha,r)$, based on diagrams 4a and 4b, is drawn. $\nu(\alpha,r)$ is "high" for the landless because they have no non-wage income. (In fact, for such people $\nu(\alpha,r) = \hat{I}/\lambda(\hat{I})$.) It is relatively "low" for "smallish" landowners because they do have some non-wage income and because their reservation wage is not too high. $\nu(\alpha,r)$ is "high" for the big land-owners because their reservation wages are very "high".




$w^*(\alpha,r)$ for an $\alpha$-person with small landholding, such that $r\hat{T}t(\alpha) < I^*$ (where $I^*$ is defined in diagram 1b). Note that $\nu^*(\alpha,r) > \ddot{w}(rt(\alpha)\hat{T})$.

Diagram 4a




$w^*(\alpha,r)$ $(= \bar{w}(\alpha,r))$ for an α-person with large landholdings. Here $r\hat{T}t(\alpha) > I^*$.

<u>Diagram 4b</u>

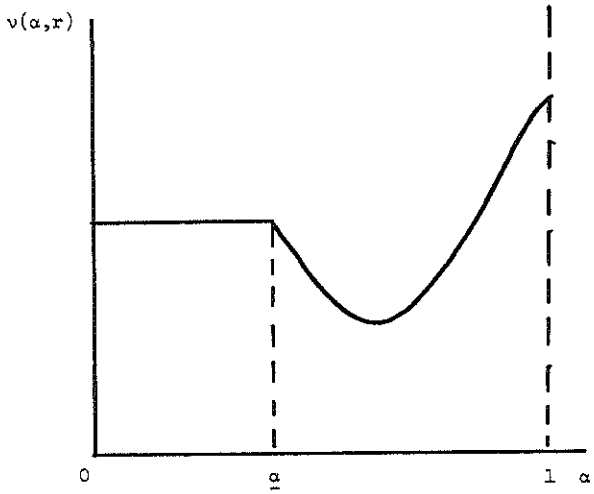



Diagram 5a

While a "typical" shape of $\nu(\alpha,r)$, as in diagram 5a, is used to illustrate the arguments in the main body of the paper, it must be pointed out our assumptions do not, in general, generate this "U-shaped" curve. Diagram 5b illustrates other possible configurations of the $\nu(\alpha,r)$ function, which are perfectly consistent with the assumptions we have made.[33]

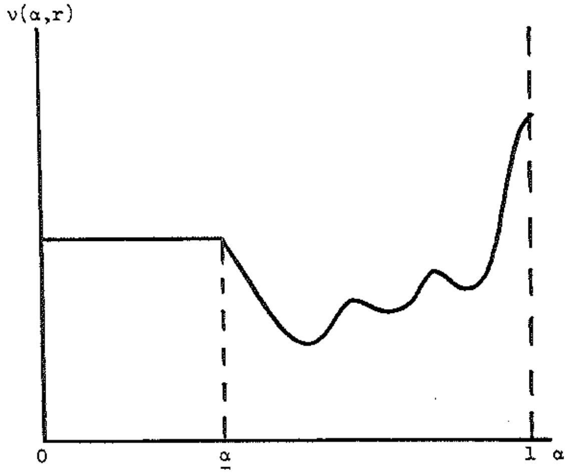



Diagram 5b

What is common to all $\nu(\alpha,r)$ functions are these features:

(a) $\nu(\alpha,r)$ is constant for all landless $\alpha$-types and falls immediately thereafter.

(b) $\nu(\alpha,r)$ continues to decrease in $\alpha$ as long as the reservation wage is not binding in equation (1).

(c) Once the reservation wage binds for some $\alpha$-type, it continues to do so for all $\alpha$-types with more land.[34/]

(d) $\nu(\alpha,r)$ "finally" rises as the effect of the increasing reservation wage ultimately outweighs the (diminishing) increments to labour power associated with greater land-ownership.

Having said this, though, we shall continue to use the simpler diagram 5a for the purpose of exposition. The interested reader is referred to the Appendix for the more rigorous arguments.

We can now begin to characterize a competitive equilibrium for the peasant economy under review. We have already noted that in equilibrium the wage per unit of labour power must be the same for all who are employed. Denote this by $\bar{\nu}$. As earlier, let $\bar{r}$ denote the equilibrium land rental rate. Since production enterprises are competitive, $\bar{r}$ must, in equilibrium equal the marginal product of land and $\bar{\nu}$ must equal the marginal product of aggregate labour-power. We may conclude at once from the latter equality that market demand for the labour time of any $\alpha$-person for whom $\nu(\alpha,\bar{r})$ exceeds $\bar{\nu}$ must be nil. But what of an $\alpha$-person whose $\nu(\alpha,\bar{r})$ is less than $\bar{\nu}$? Every enterprise wants his service: he is not ill-fed and is willing to work! Speaking metaphorically, his wage is bid up by competition to the point where the wage that he is paid per unit of labour power he offers equals $\bar{\nu}$. Demand for his labour time is thus positive. Since the wage he is paid exceeds his reservation wage, he supplies his unit of labour time which, in equilibrium, is what is demanded. Finally, what of an $\alpha$-person whose $\nu(\alpha,\bar{r})$ equals $\bar{\nu}$? Enterprises are indifferent between employing such a worker and not employing him. He is, of course, willing to supply his unit of labour time: with eagerness if the wage he receives in equilibrium exceeds his reservation wage, and as a matter of indifference if it equals it.

We may as well formalize all this. Let $D(\alpha)$ be the market demand for the labour time of an $\alpha$-person. Labour time supply by him is of course zero if the wage he can command is less than his reservation wage, and is unity if it exceeds it. We have therefore proved

Proposition 1: A rental rate $\bar{r}$, a subset $\bar{G}$ of $[0,1]$, and a function $\bar{w}(\alpha)$ on $\bar{G}$, sustain a competitive equilibrium if (and only if) there is a $\bar{\nu} > 0$ such that:

(i) for all $\alpha$-persons such that $\bar{\nu} > \nu(\alpha,\bar{r})$, we have $D(\alpha) = 1$;

(ii) for all $\alpha$-persons for whom $\nu(\alpha,\bar{r}) > \bar{\nu}$, we have $D(\alpha) = 0$;

(iii) $D(\alpha)$ is either zero or unity for all $\alpha$-persons for whom $\bar{\nu} = \nu(\alpha,\bar{r})$;

(iv) $\bar{G} = \{\alpha | D(\alpha) = 1\}$ and $\bar{w}(\alpha)/\lambda(\bar{w}(\alpha) + \bar{r}t(\alpha)\hat{T}) = \bar{\nu}$, with $\bar{w}(\alpha) > w^*(\alpha,\bar{r})$, for all $\alpha$ with $D(\alpha) = 1$. [35]

(v) $\bar{\nu} = \partial F(\bar{E},\hat{T})/\partial E$ where $\bar{E}$ is the aggregate labour-power of all who are employed; (i.e., $\bar{E} = \int_{\bar{G}} \lambda(\bar{\omega}(\alpha) + \bar{r}\hat{T}t(\alpha)d\mu(\alpha)$,

(vi) $\bar{r} = \partial F(\bar{E},\hat{T})/\partial T$.

These six conditions are worth reinterpreting. Note first that $\tilde{\nu}$ is the equilibrium wage per unit of labour power. An $\alpha$-person for whom $\nu(\alpha,\tilde{r}) < \tilde{\nu}$ will obviously wish to supply his unit of labour time, since the equilibrium wage he receives exceeds his reservation wage. (One may verify this by using Diagrams 4a and 4b). Condition (i) says that demand equals supply for such a person's labour time. Now consider an $\alpha$-person for whom $\nu(\alpha,\tilde{r}) > \tilde{\nu}$. He is too expensive for an enterprise to hire. Consequently, (ii) says that there is no demand for such a person's labour. Why doesn't such a person try to undercut the market? He can't, and the reason he can't can be readily seen if one recalls how $\nu(\alpha,r)$ was defined, (see equation (2)). No production enterprise will employ him. Indeed, he cannot afford to be "self employed" in this sector. Condition (iii) says that for an $\alpha$-person for whom $\nu(\alpha,\tilde{r}) = \tilde{\nu}$, demand is either nil or unity. (iv) identifies the equilibrium set and wages for each $\alpha$-type in this set, and (v) and (vi) say that the prices of the two factors of production, labour power and land, equal their respective marginal products. Since the production function is constant-returns-to-scale, production enterprises earn no profits after factor payments have been made. Finally, it is clear that aggregate demand and supply of rice are equal.

We have defined an equilibrium outcome under pure competition in our peasant economy and Proposition 1 provides a characterization. But does an equilibrium exist? This is answered in the affirmative in

Proposition 2: Under the conditions postulated, there exists a competitive equilibrium.

Proof: See Appendix. Q.E.D.

In what follows we will characterize the equilibria diagramatically. To do this we merely superimpose the horizontal curve $\nu = \bar{\nu}$ onto Diagram 5a. There are three different types of equilibria, or regimes, depending on the size of $\hat{T}$, the parameter we vary in the next three sub-sections. We first summarize the three regimes in the form of Proposition 3 and then describe them successively in Sections 3.4-3.6.

Proposition 3: A competitive equilibrium is in one of three possible regimes, depending on the total size of land, $\hat{T}$ and the distribution of land. Given the latter:

(1) If $\hat{T}$ is sufficiently small $\tilde{\nu} < \hat{I}/\lambda(\hat{I})$, and the economy is characterized by malnourishment and unemployment among all the landless and some of the near-landless. (Diagram 6a).

(2) If $\hat{T}$ is neither too small nor too large, $\bar{\nu} = \hat{I}/\lambda(\hat{I})$, and the economy is characterized by malnourishment and involuntary unemployment among a fraction of the landless. (Diagram 6b).

(3) If $\hat{T}$ is sufficiently large, $\tilde{\nu} > \hat{I}/\lambda(\hat{I})$ and the economy is characterized by full employment and an absence of malnourishment. (Diagram 6c).

A formal proof is omitted, but see Sections 3.4-3.6.

The regimes of Proposition 3 will be discussed in more detail below. First, we establish an important property of the equilibrium wage schedule, on the equilibrium set.

<u>Proposition 4</u>: <u>Pick</u> $\alpha_1, \alpha_2$ <u>in</u> $\tilde{G}$ <u>with</u> $t(\alpha_1) < t(\alpha_2)$. <u>Then</u> $\bar{w}(\alpha_1) < \bar{w}(\alpha_2)$

The proof of this rests on the simple observation that wage cost per efficiency unit equates itself across all employed $\alpha$-persons. It is quite intuitive, so we present the argument in the main body of the paper.

<u>Proof</u>: Note that for all $\alpha$ $\bar{G}$, by Proposition 1,

$$\tilde{\nu} = \frac{\tilde{w}(\alpha)}{\lambda(\tilde{w}(\alpha) + \tilde{r}t(\alpha)\hat{T})} \tag{3}$$

Defining $\tilde{I}(\alpha) = \tilde{w}(\alpha) + \bar{r}t(\alpha)\hat{T}$, we find, or using strict concavity of $\lambda$ and by noting that $\lambda(I(\alpha)) > 0$ for an employed person that:

$$\begin{aligned} \tilde{w}(\alpha_2) - \tilde{w}(\alpha_1) &= \tilde{\nu}[\lambda(\tilde{I}(\alpha_2)) - \lambda(\tilde{I}(\alpha_1))] \\ &> \bar{\nu}[\tilde{I}(\alpha_2) - \tilde{I}(\alpha_1)]\lambda'(\tilde{I}(\alpha_2)) \quad . \end{aligned} \tag{4}$$

A rearrangement of (4) yields

$$[\tilde{w}(\alpha_2) - \tilde{w}(\alpha_1)][1 - \tilde{\nu}\lambda'(\tilde{I}(\alpha_2))] = \hat{T}\tilde{\nu}\lambda'(\tilde{I}(\alpha_2))[t(\alpha_2) - t(\alpha_1)]\tilde{r} \tag{5}$$

Now observe that by the first order condition to the solution of the problem in (1), and the fact that $\bar{w}(\alpha_2) > w^*(\alpha_2, \tilde{r})$,

$$\lambda(\tilde{I}(\alpha_2)) > \tilde{w}(\alpha_2)\lambda'(\tilde{I}(\alpha_2)) \quad . \tag{6}$$

It is possible to check from characteristics (b) and (c) of the $\nu(\cdot,\cdot)$ function, mentioned above, that if $\alpha_1, \alpha_2 \quad \bar{G}$ and $t(\alpha_2) > t(\alpha_1)$, then either the reservation wage for $\alpha_2$ is strictly binding in the solution to (1), or $\tilde{w}(\alpha_2) > w^*(\alpha_2, \tilde{r})$. In either case, (6) holds with <u>strict</u> inequality. Using this information and equation (3), it follows that $[1 - \tilde{\nu}\lambda'(\tilde{I}(\alpha_2))] > 0$ in equation (5). Observing that the right-hand side of (5) is positive, the desired result follows.

Q.E.D.

A strong implication of this result is that competition, in some sense, <u>widens</u> the initial disparities in asset ownership by offering higher (employed) asset-owners a higher <u>wage</u> income. Contrast this with the results of Bliss and Stern [1978a], discussed earlier. There, a monopsonist landlord narrows initial asset disparities in his quest to equalize marginal labour power across all labour types. Competition, by placing productive asset-holders at a premium in the job market, has exactly the opposite effect.

One more result before we turn to a more detailed analysis of the various regimes, and this concerns the Pareto-efficiency of our competitive equilibrium. The limitations of Pareto-efficiency in evaluating distribution-sensitive situations is well-known; we underscore the poverty of the Pareto-criterion in

<u>Proposition 4</u>: <u>Under the conditions postulated, a competitive equilibrium is Pareto efficient.</u>

<u>Proof</u>: See Appendix. Q.E.D.

It is worth noting, perhaps, that the result in Proposition 4 is not a standard one (as summarized in the so-called "first fundamental theorem" of welfare economics). This is because our equilibrium corresponds to a conventional (infinite dimensional) Walrasian equilibrium only in some circumstances (the technicalities we relegate to a footnote)[36]. The appendix, in addition, provides an example of a competitive equilibrium which is not Pareto-efficient, when our postulate of diminishing marginal product of labour is amended to allow for possibly constant marginal product in certain regions of the production function. Such an example is, if anything, a further example of competitive market failure in the economies that we are describing. But, in general, under our assumptions, involuntary unemployment and malnourishment coexist with Pareto-efficiency.

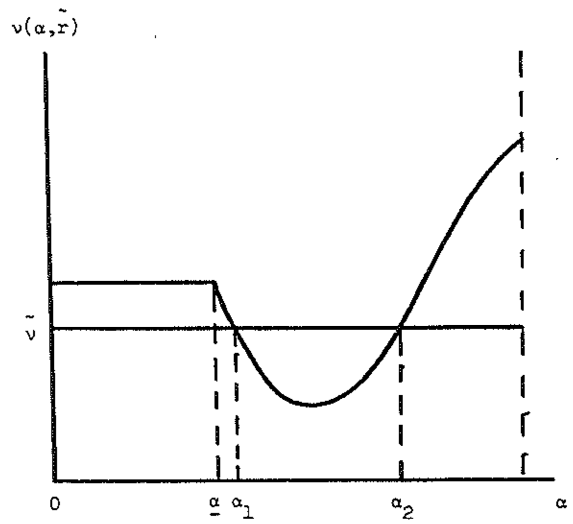
### 3.4 Malnourishment and Unemployment among the Landless and Near-landless (Regime 1)

Diagram 6a depicts a typical equilibrium under regime 1. ($\hat{T}$ is small and so from the first part of Proposition 3, $\tilde{\nu} < \hat{I}/\lambda(\hat{I})$). From condition (i) of Proposition 1, one notes that all $\alpha$-persons between $\alpha_1$ and $\alpha_2$ are employed in the production of rice. Typically, the borderline $\alpha_1$-person will be one for whom the market wage $\tilde{w}(\alpha_1)$ will exceed his reservation wage $\bar{w}(\tilde{r}t(\alpha)\hat{T})$. We will assume this in the exposition.[37] From condition (ii) of Proposition 1, we observe that all $\alpha$-types below $\alpha_1$ and above $\alpha_2$ are out of the market: the former because their labour power is too expensive—they are too poor, and therefore malnourished—, the latter because their reservation wages are too high--they are too rich.

It should also be noted that in this regime, all the landless are malnourished and not worth employing by the market. Moreover, it can be verified that (if, as we are assuming in this essay, malnourishment incomes are defined to be those below the efficiency wage), all the unemployed to the left of $\alpha_1$ are malnourished, too - their rental income is too meagre. And finally, some of the employed are also malnourished, which is verifiable by noting that employed persons "close to" $\alpha_1$ earn less than the efficiency wage.

To be sure, there are no job queues in the labour market; nevertheless, there is involuntary unemployment in the extended sense of the term defined in Section 1. To see this note first that $\tilde{w}(\alpha_1) > \bar{w}(\tilde{r}t(\alpha_1)\hat{T})$. This implies that $\tilde{w}(\alpha) > \bar{w}(\tilde{r}t(\alpha)\hat{T})$ for all $\alpha$ in a neighbourhood to the right of $\alpha_1$. Such people are employed. They are therefore distinctly better off than $\alpha$-persons in a neighbourhood to the left of $\alpha_1$, who are unemployed and who therefore suffer their reservation wage. This means that the equilibrium income schedule is discontinuous at $\alpha_1$, and this is what is meant by involuntary unemployment in a market economy with heterogeneous labour-types. Such a discontinuity is at odds with the Arrow-Debreu theory with convex structures.

Finally, observe that $\alpha$-types above $\alpha_2$ are voluntarily unemployed. Call them the pure rentiers, or the landed gentry. They are capable of supplying labour at the "cost-efficiency" ratio $(\tilde{\nu})$ called for by the market, but choose not to because their reservation



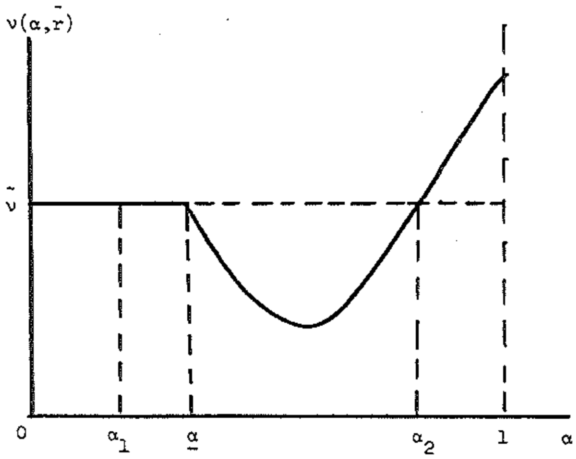

Land poor economy. Incidence of unemployment and malnutrition

Diagram 6a

wages are too high. They are to be contrasted with the unemployed below $\alpha_1$, who are incapable of supplying labour at the market cost-efficiency ratio, because of their general state of malnourishment.

### 3.5 Malnourishment and Involuntary Unemployment among the Landless (Regime 2)

Consider now an intermediate range of values of $\hat{T}$. Within this range the relevant curves are as drawn in Diagram 6b. By Proposition 3, part 2, $\tilde{\nu} = \hat{I}/\lambda(\hat{I})$. It isn't a fluke case: it pertains




Moderate endowment of land. Incidence of malnourishment and involuntary unemployment.

Diagram 6b

to a range of moderate values for $\hat{T}$. From Proposition 1, part (i) we note immediately that all $\alpha$-people between $\underline{\alpha}$ and $\alpha_2$ are employed, and from part (ii) that all above $\alpha_2$ are out of the labour market because their reservation wage is too high. The economy equilibrates via the "indifference" condition (iii) of Proposition 1. A fraction of the landless, $\alpha_1/\underline{\alpha}$, is unemployed, the remaining fraction, $1 - \alpha_1/\underline{\alpha}$, is employed. The size of the fraction depends on $\hat{T}$. There is involuntary unemployment among the landless: the unemployed suffer their reservation wage, and this is below the efficiency-wage, $\hat{I}$, which is what the landless-employed earn. These malnourished unemployed are knocking at the farm entrances for work, but they aren't hired: no production enterprise will hire them, even at a lower wage. (Indeed, they can't afford to be self-employed in this sector.) Their identical brethren, however, are being hired, and they are better off. This is entirely at odds with a Walrasian equilibrium.

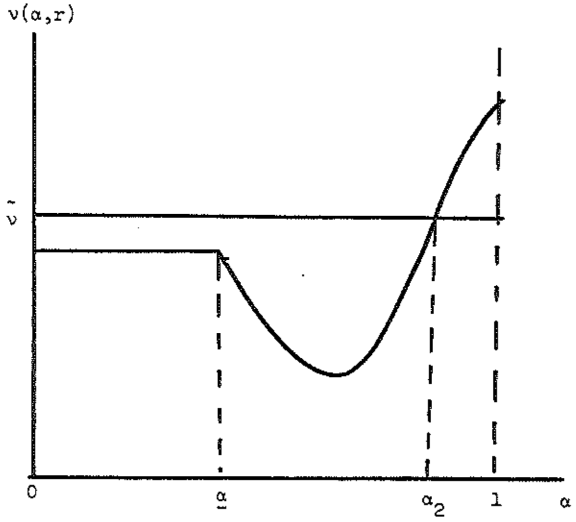
Finally, we observe that, by our definition of malnourishment incomes as being those below $\hat{I}$, the group of unemployed and malnourished coincide under this regime. This is to be contrasted with Regime 1.

### 3.6 The Full Employment Equilibrium (Regime 3)

Diagram 6c presents the third and final regime, pertinent for large values of $\hat{T}$. Here, $\tilde{v} > \hat{I}/\lambda(\hat{I})$. From part (i) of Proposition 1 we conclude that all persons from zero to $\alpha_2$ are employed. From (ii) we note that those above $\alpha_2$ aren't employed. But, as before, they aren't involuntarily unemployed: they are the landed gentry. Thus this regime is characterized by full-employment, and no one is undernourished. This corresponds to a standard Walrasian equilibrium

### 3.7 Growth as a Means of Reducing Malnourishment and Unemployment

It is difficult to resist extending the conclusions of the timeless structure and introducing time. So we won't try. One can




Land rich economy. Full-employment and no incidence of absolute poverty.

Diagram 6c

imagine an economy with a small $\hat{T}$ and a given distribution of land, $t(\alpha)$. An equilibrium is characterized by Diagram 6a. If the propertied class, which is well-to-do at the equilibrium, accumulate in land improvement--that is, in capital that improves the productivity of land-- $\hat{T}$ will increase. Assuming that land distribution, $t(\alpha)$, remains approximately the same, it would follow that with $\hat{T}$ increasing more and more, the economy will after some time enter the regime depicted by

Diagram 6b, and eventually the final regime of Diagram 6c. It is only in the final regime that no one is undernourished. We take it that this is what "trickle-down" amounts to.[38]

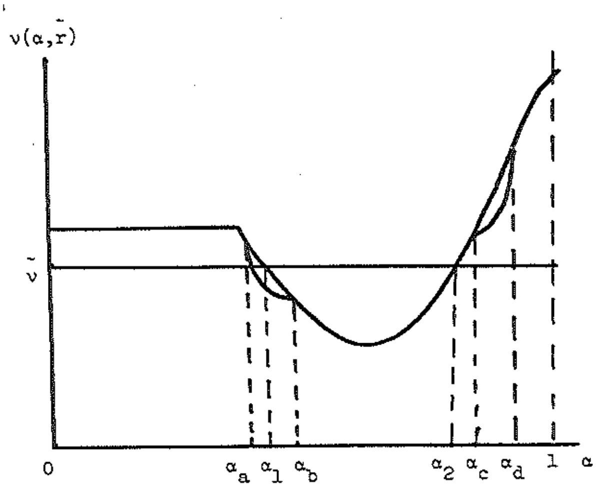
### 3.8 Inequality of Land Ownership as a Cause of Malnutrition, Unemployment, and Low Output

Growth, seen as a means of removing poverty and unemployment, has long dominated the development literature. We want now to argue that in certain circumstances it is the inequality in the distribution of assets which is the cause of poverty and malnutrition and thus in turn involuntary unemployment. To analyze this, we will hold the aggregate quantity of land fixed, and change the land distribution.

A variety of different redistribution schemes can be studied. For ease of exposition, we will first look at a simple, though important case: that of land transfers from the landed gentry to those who are involuntarily unemployed. Such redistributions need not (and, in general, will not) lead to full equalization of asset holdings. To distinguish them from full land redistributions (to perfect equality) which we shall discuss below, call them partial land reforms.

Below, in Diagram 7, a partial land reform, where land is transferred to some of the unemployed as well those "on the margin" of being unemployed, is depicted.[39] The diagram displays the changes evaluated at the original equilibrium $(\tilde{v},\tilde{r})$. People between $\alpha_a$ and $\alpha_b$ gain land; for them, the $\nu(\cdot,\tilde{r})$ function shifts downward, as they are now able to supply labour at a lower cost-to-labour-power ratio. The losers, between $\alpha_c$ and $\alpha_d$, also experience a downward shift in $\nu(\cdot,\tilde{r})$, but for entirely different reasons--their reservation wages have been lowered.

Of course, a new equilibrium will now be established, one with a different wage schedule and rental rate. Can the two be compared? partial answer is given in




A partial land reform: α-types between $\alpha_a$ and $\alpha_b$ gain land, and rentiers between $\alpha_c$ and $\alpha_d$ lose land.

<u>Diagram 7</u>

Proposition 5: Suppose that for each parametric specification, the competitive equilibrium is unique.[40] Then a partial land reform of the kind just described necessarily leads to at least as much output in the economy (strictly more, if $\nu(\alpha,\bar{r})$ is of the form in Diagram 7[41])

Proof: See Appendix. Q.E.D.

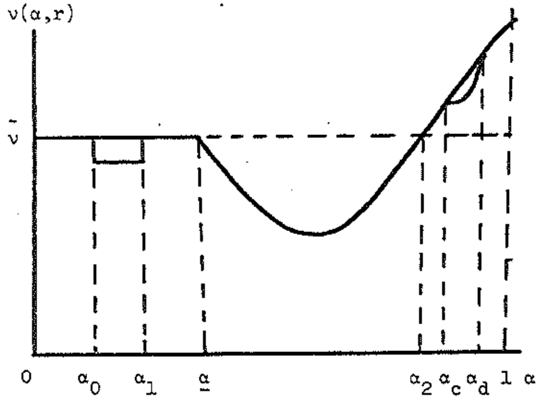
The result implies that there is no necessary conflict between equality and output in a resource-poor economy. Redistributions have three effects. First, the unemployed become more productive, as their extent of malnourishment decreases. Second, the already employed are more productive to the extent that they, too, receive land. Finally, by taking land away from the landed gentry, their reservation wages are lowered, and if this effect is strong enough, this could induce them to forsake their state of voluntary unemployment and enter the labour market. For all these reasons, the number of employed efficiency units in the economy rises, pushing it to a higher-output equilibrium.

Note, however, that Proposition 5 is silent on a description of how the set of employed persons changes. Do previously unemployed persons necessarily find employment? Does the number of involuntarily unemployed fall?

Unfortunately, the answer to this question can go either way. There is a natural tendency for employment to rise, because of the features mentioned above. However--and this is characteristic of all partial (as opposed to full) land reforms--there is a "displacement effect" at work, whereby newly productive workers are capable of displacing previously employed, less productive workers in the labour market.

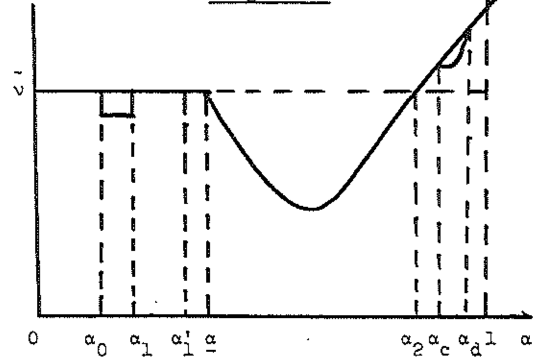
An example of this is given in Diagrams 8a-c. The economy is in Regime 2. Now suppose that a partial reform transfers land from pure rentiers to a fraction of the landless unemployed.[42/] (Diagram 8a) What happens? Well, it is impossible for all the previously employed landless workers to remain employed. If the new equilibrium continues to exhibit the same $(\bar{\nu},\bar{r})$, as it well might (Diagram 8b), it will be associated with the same output. But now the new landowners will displace some previously employed landless workers, and in fact will displace a number larger than the number of new employees, since the new workers are more productive, and the economy-wide aggregate of efficiency units is constant. In the case where $\bar{\nu}$ falls, as the result of more efficiency units in the system, output will rise, but all the previous landless workers will be laid off (Diagram 8c).

This displacement effect cannot exist in the case of full land reform, our final object of analysis. To discuss this, we will assume that the economy is productive enough to feed everyone adequately, in principle. We assume this so as to explicitly highlight the detrimental effects of an unequal land distribution. To explain the argument, some additional machinery needs to be set up. First, in order to give precision to the intuitive idea that there is enough (in principle) to go around, suppose that there is a planner, who is capable of making everyone work on the land, is capable of collecting the rice output and dividing it equally among all. If I denotes the consumption level of



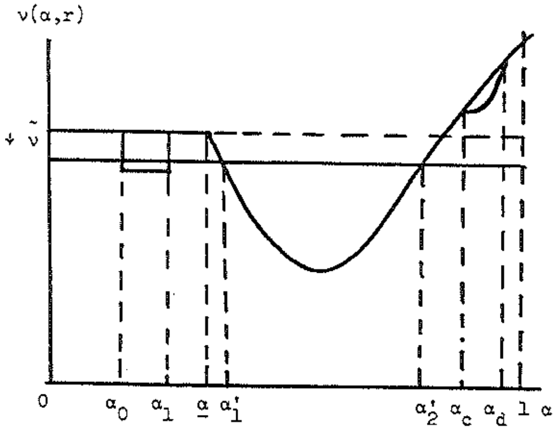

All between $\alpha_1$ and $\alpha_2$ are employed. People between $\alpha_0$ and $\alpha_1$ receive land (from $[\alpha_c,\alpha_d]$, the rentiers).

Diagram 8a




Output remains constant and $\tilde{\nu}$ doesn't fall. The new set of employed are $[\alpha_0,\alpha_1]$ and $[\alpha_1',\alpha_2]$. The displaced set of previously employed, $[\alpha_1,\alpha_1']$ has a larger measure than $[\alpha_0,\alpha_1]$.

Diagram 8b




Here, $\tilde{\nu}$ falls. The new set of employed is smaller than in 8b, with $[\alpha_0, \alpha_1]$ and $[\alpha_1', \alpha_2']$ being the employed set. All previously employed workers are displaced, and a few landed ones besides.

Diagram 8c

each person under such a scheme, $\lambda(I)$ is the labour-power of the representative person, and aggregate output is $F(\hat{T}, \lambda(I))$. (Recall that by normalization, $N = 1$). For such a solution to be viable, there must be a solution (in $I$) to the equation

$$I = F(\hat{T}, \lambda(I)) \tag{7}$$

It is easy to see that if there is a solution to (7), in general there are two.[43/] Concentrate on the larger of the two solutions; call it

$I(\hat{T})$, and let $\hat{T}_1$ be the smallest value of $\hat{T}$ such that $I(\hat{T}) = \hat{I}$. Thus at $\hat{T}_1$, we have a formalization of the idea that the economy is productive enough (just about) to feed all adequately (i.e., at the level of the poverty line $\hat{I}$).

To set the stage, we first state

Proposition 6: Let $(\hat{T}, I(\hat{T}))$ be an equal division solution. Then, if reservation wages are low enough,[44] this is achievable as a competitive equilibrium under full equality of land distribution.

Proof: See Appendix. Q.E.D.

To complete our analysis of full land redistributions, we will show that for each size of land in some range above $\hat{T}_1$, there are unequal distributions of that land that create involuntary unemployment and malnourishment (of the kind in either Regimes 1 or 2), even though full redistributions are associated with full employment and no malnourishment.

Proposition 7: There exists an interval $[\hat{T}_1, \hat{T}_2)$ such that if $\hat{T}$ is in this interval, full redistributions yield competitive equilibria with full employment, and no malnourishment. Moreover for each such $\hat{T}$, there are unequal distributions which give rise to involuntary unemployment and malnourishment.

Proof: See Appendix. The Appendix also contains an explicit example which illustrates some of the general results of this Proposition. Q.E.D.

In other words, we have identified a class of cases, namely, a range of moderate land endowments, where inequality of asset ownership can be pinpointed as the basic cause of involuntary unemployment and malnourishment. The implications for poorly or moderately endowed less developed countries are obvious.

With a large landless population the competitive market mechanism simply can't "afford" to employ all: the landless are too expensive, and they are too expensive because they are hungry. But it is the land distribution in conjunction with the competitive market mechanism which is at fault, not the intrinsic productive capacity of the economy. In such circumstances judicious land reforms, or consumption transfers, can increase output and reduce unemployment and the incidence of poverty. Indeed, if land were equally distributed the market mechanism would sustain this economy in regime 3 (Diagram 6c) in which undernourishment is a thing of the past.

Finally, note that it is perfectly possible that unequal distribution of "adequate" aggregate land (in the sense of Proposition 7) sets us in a position represented, say, by Regime 2 unemployment. And in this case, as we have observed, partial land reforms may well have perverse effects on employment. At the same time, full land redistributions lead to full employment. This observation suggests that in some cases partial reform movements may not serve the desired purpose as well as a more aggressive, total, redistributive policy.

Our last result deals with "rich" economies, for the sake of completeness. Proposition 8, below, states that for all land endowments greater than or equal to $\hat{T}_2$ (see the statement of Proposition 7), inequality in asset holdings cannot lead to malnourishment and involuntary unemployment, through the mechanism highlighted in this paper.[45/]

Proposition 8: For all $\hat{T} > \hat{T}_2$, there is no land distribution which involves involuntary unemployment or malnutrition.

Proof: See Appendix. Q.E.D.

Two remarks. First, observe that we have an exact characteriza tion of the "borderline" aggregate endowment (a comparision of Propositions 7 and 8 will reveal that there are no "fuzzy zones" in between). $\hat{T}_2$ can be explicitly defined in terms of the parameters of the system. (See Appendix, proof of Proposition 7).

Our second and final remark deals with the fact that an economy comes into a conventional Arrow-Debreu equilibrium only if is resource-rich. We comment further on this point in the concluding section of the paper.

## 4. Commentary

In this essay we have argued (Section 3.8) that even if an economy is in principle productive enough to employ all and to feed all adequately, pure competition can't be relied upon to do so, and the problem is accentuated if the distribution of land is very unequal. Market forces under such conditions inflict poverty, malnourishment and thus unemployment on a fraction of its population even when they are unhampered by restrictive practices or incomplete and asymmetric information. The reason is that because a large fraction of the population is landless the market can't "afford" to employ all. Landless persons are too costly for the market to bear in their entirety. While it is true that if accumulation--e.g., via an improvement in land--proceeds, unemployment, and thus malnutrition, will be eradicated in the model economy in time. However, it may be a long while coming. For the immediate future the "quantity" of land can't be altered much. But the extreme inequality in food consumption which the market inflicts _can_ be countered. For economies not generously endowed with physical assets the competitive market mechanism must be judged an unmitigated disaster. The policy implications in the model economy are clear enough.

At the mathematical level it is easy to see why, despite pure competition, there is involuntary unemployment when the number of landless people is large. It is because of the inherent increasing-returns-to-scale in the consumption-ability relation of a person at low consumption levels (diagrams 1a and 1b).[46] It is because of this that the theory outlined here is so different from the Arrow-Debreu theory of perfect competition with convex structures. We have argued that given the land distribution function $t(\alpha)$, it is only when the total quantity of land is large (when $\hat{T}$ is large) that pure competition in the economy in question merges with the standard Arrow-Debreu theory. (See Sections 3.4-3.6). Or to put it another way, if there is sufficient land to feed all but it isn't a land rich country then pure competition in our model economy merges with the standard Arrow-Debreu equilibrium only if land distribution is sufficiently equal, (Section 3.8). However, we have also shown that if the aggregate quantity of land is very large land distribution doesn't matter as regards employment and malnutrition: an equilibrium is a conventional Arrow-Debreu one. (See Proposition 8 and the example in the Appendix). We take this to mean that the conventional Arrow-Debreu theory pertains only to an economy which is asset-rich. This is not to say that an Arrow-Debreu equilibrium has much to commend it from the point of view of the distribution of welfares. There is, however, nothing new in this point and it isn't the one we want to make here. The point we are making here is that the Arrow-Debreu theory can't sustain involuntary unemployment.[47/] Furthermore, the Arrow-Debreu theory doesn't have a vocabulary for malnutrition. Moreover, we have argued (Section 1), that the Lewis theory too has, for the overwhelming part, avoided the issue of malnutrition. And it is the relationship between inequality, malnutrition and unemployment in a market economy that has been the focus of analysis of this essay.

> "Critics of the market", writes Professor Bauer, "often dispute the reality and significance of the freedoms and choices protected by decentralized decisions, on the ground that such choices are of little value to many people, notably the poor ... . It is a criticism which is radically misconceived. The ability to use their resources to their own best advantage, in particular to choose their employment freely and to have different employers competing for their services, is especially important to the poor ..." (Bauer [1984], p. 26.)

Just so. But it is the inability of the market to offer these choices to the poor that has been the source of the criticism in this essay.

## Appendix

1. Proof of Proposition 2: Define $E(r)$, for each $r > 0$, by

$$r \equiv F_T(\hat{T}, E(r)) \tag{8}$$

By our assumptions on $F$, $E(r)$ is well defined and unique for each $r$, with $E(r) \to 0$ as $r \to 0$, and $E(r) \to \infty$ as $r \to \infty$.

Now we construct a correspondence $\mathcal{E}(r)$ in the following way. First, define for each $r > 0$, $\nu(r)$ by

$$\nu(r) \equiv F_E(\hat{T}, E(r)) \tag{9}$$

Again, $\nu(r)$ is uniquely defined, with $\nu(r) \to \infty$ as $r \to 0$, and $\nu(r) \to 0$ as $r \to \infty$.

Let $B(r) = \{\alpha / \nu^*(\alpha, r) < \nu(r)\}$, and $G(r) = \{\alpha / \nu^*(\alpha, r) \leq \nu(r)\}$, (note that $G(r)$ is not, in general, the closure of $B(r)$), and

$$\mathcal{G}(r) = \{G \subseteq [0,1] / G \text{ is closed and } B(r) \subseteq G \subseteq G(r)\} \tag{10}$$

If $G(r) \neq \Phi$, then for each $\alpha \in G(r)$, it is possible to define $w(\alpha, r)$ uniquely by the pair of conditions

$$w(\alpha,r)/\lambda(w(\alpha,r) + rt(\alpha)\hat{T}) \equiv \nu(r) \tag{11}$$

and

$$w(\alpha,r) > w^*(\alpha,r)$$

Note that $w(\alpha, r)$ is continuous in $\alpha$ and $r$.

Finally, define $\mathcal{E}(r)$ as follows:

(12) $$E(r) = \{E \in \mathbb{R}_+ / E = \int_G \lambda(w(\alpha,r) + rt(\alpha)\hat{T})d\mu(\alpha), G \in G(r)\} \quad \text{if } G(r) \neq \Phi ,$$
$$= \{0\} \quad , \text{ if } G(r) = \Phi$$

<u>Lemma 1:</u> <u>For each</u> $r > 0$, $E(r)$ <u>is an interval (possibly singleton).</u>

<u>Proof:</u> It suffices to consider $r > 0$ with $G(r) \neq \Phi$. Define $M \equiv \int_{B(r)} \lambda(w(\alpha,r) + rt(\alpha)\hat{T})d\mu(\alpha)$ and $N \equiv \int_{G(r)} \lambda(w(\alpha,r) + rt(\alpha)\hat{T})d\mu(\alpha)$. Let $H(r) \equiv G(r)\setminus B(r)$, and for $x \in [0,1]$, define

(13) $$F(x) \equiv \int_{[0,x]} \lambda(w(\alpha,r) + rt(\alpha)\hat{T})\chi_{H(r)}d\mu(\alpha) + M$$

where $\chi_{H(r)}$ is the indicator function of $H(r)$. Observe that $\min_x F(x) = M$, and $\max_x F(x) = N$, and that $F(x)$ is continuous. Therefore $F(x)$ attains all intermediate values between $M$ and $N$.

Observe, now, that since $B(r)$ is an open set and $\mu(\cdot)$ is Lebesgue measure, we have

(14) $$M = \int_{\bar{B}(r)} \lambda(w(\alpha,r) + rt(\alpha)\hat{T})d\mu(\alpha)$$

where $\bar{B}(r)$ is the closure of $B(r)$.

So using (13) and (14), $F(x)$ is expressible as

(15) $$F(x) = \int_{\{H(r)\cap[0,x]\}\cup\bar{B}(r)} \lambda(w(\alpha,r) + rt(\alpha)\hat{T})d\mu(\alpha)$$

Note that $\{H(r) \cap [0,x]\} \cup \bar{B}(r) \in G(r)$. Also, since $\min E(r) = M$, $\max E(r) = N$ (for fixed $r$) by continuity of $\lambda(\cdot)$, $w(\cdot,r)$ and $t(\cdot)$, it is proved that $\mathcal{E}(r)$ must be the interval $[M,N]$. Q.E.D.

__Lemma 2:__ __Let__ $\{r^n\}$ __be a positive sequence with__ $\lim_n r^n = r > 0$, __and suppose__ $E^n \in \mathcal{E}(r^n)$, __with__ $\lim_n E^n = E$. __Then__ $E \in \mathcal{E}(r)$.

__Proof:__ First, if $E^n > 0$ for only finitely many $n$, there is $N$ such that $n > N$ implies

$$\nu^*(\alpha,r^n) > \nu(r^n) \quad \text{for all} \quad \alpha \in [0,1]$$

Passing to the limit and using continuity,

$$\nu^*(\alpha,r) > \nu(r) \quad \text{for all} \quad \alpha \in [0,1]$$

which implies that $0 \in \mathcal{E}(r)$.

So, without loss of generality, assume $E^n > 0$ for all $n$. Then there is $\langle G^n \rangle$, $G^n \in \mathcal{G}(r^n)$ for all $n$, such that

$$(16) \qquad E^n = \int_{G^n} \lambda(w(\alpha,r^n) + r^n t(\alpha)\hat{T})d\mu(\alpha)$$

$$= \int_{[0,1]} \lambda(w(\alpha,r^n) + r^n t(\alpha)\hat{T})\chi_{G^n} d\mu(\alpha)$$

Observe, now, that by the continuity of $\nu(\alpha,r)$, $G(r)$ is actually an at most countable collection of closed intervals (some possibly singleton). So it is easy to pick $G^n \in \mathcal{G}(r^n)$ with this property, i.e., we pick $G^n$ to be an at most countable collection of __nonsingleton__ closed intervals. There is a subsequence $n_k$ such that $\lim_k G^{n_k} = G$ in the Hausdorff metric (see, e.g. Hildenbrand [1974], and it is easy to verify that (a) $G \in \mathcal{G}(r)$, and (b) $G$ is an at most countable collection of closed intervals (some possibly singleton). It can then be checked that <u>because of the choice of</u> $G^n$ <u>as unions of nondegenerate closed intervals</u>, $\chi_{G^{n_k}}(\alpha) \to \chi_G(\alpha)$ for $\alpha$ a.e. in $[0,1]$.

Define, for $\alpha \in [0,1]$

$$q^k(\alpha) = \lambda(w(\alpha,r^{n_k}) + r^{n_k}t(\alpha)\hat{T})\chi_{G^{n_k}}(\alpha)$$

Then, observing that $\lambda(\cdot)$, $w(\alpha,\cdot)$ are continuous, and invoking the previous argument,

(17) $$q^k(\alpha) \to q(\alpha) \quad \text{for} \quad \alpha \text{ a.e. in } [0.1]$$

where

(18) $$q(a) = \lambda(w(\alpha,r) + rt(\alpha)\hat{T})\chi_G(\alpha)$$

Since $q^k(\alpha) < Q(\alpha)$ for each $\alpha$, where

(19) $$Q(\alpha) \equiv \lambda(\max_n w(\alpha,r^n) + (\max r^n)t(\alpha)\hat{T}) \quad , \text{ and}$$

$$\int_{[0,1]} Q(\alpha)d\mu(\alpha) < \infty \quad ,$$

it follows from (16)-(19) and Lebesgue's dominated convergence theorem that $E \in E(r)$. Q.E.D.

Return, now, to the proof of Proposition 1. Note that as $r \to 0$, $\nu(r) \to \infty$, and for all sufficiently small $r$, we thus have

(20) $$\nu^*(\alpha,r) < \nu^*(\alpha,0) < \nu(r) \quad , \quad \alpha \in [0,1] \quad .$$

Using (20), for all $r$ sufficiently small, $\min \mathcal{E}(r)$ is bounded away from zero, so that near $0$, $\min \mathcal{E}(r) > E(r)$ [recall that $E(r) \to 0$ as $r \to 0$]. Because $\lambda(\cdot)$ is bounded, so is $\max \mathcal{E}(r)$ for all $r$. So, for large $r$, $\max \mathcal{E}(r) < E(r)$, since $E(r) \to \infty$ as $r \to \infty$.

Finally, combine this argument with the structure of the $(r)$ curve as described in Lemmas 1 and 2. It is easy to verify that there is $\tilde{r}$ and $\bar{E} \in \mathcal{E}(\tilde{r})$ such that

$$\bar{E} = E(\tilde{r}) \quad . \tag{21}$$

Since $\tilde{r} > 0$, $E(\tilde{r}) > 0$, we have $\bar{E} > 0$, so $\mathcal{G}(\tilde{r}) \neq \Phi$. Pick $\bar{G} \in \mathcal{G}(\tilde{r})$ such that

$$\bar{E} = \int_{\bar{G}} \lambda(w(\alpha,\tilde{r}) + \tilde{r}t(\alpha)\hat{T})d\mu(\alpha) \quad , \tag{22}$$

and define

$$\tilde{w}(\alpha) \equiv w(\alpha,\tilde{r}) \quad \text{for all} \quad \alpha \in \bar{G} \tag{23}$$

The triplet $(\tilde{r},\bar{G},\tilde{w}(\alpha))$ is now easily seen to be a competitive equilibrium, by Proposition 1. Q.E.D.

2. <u>Proof of Proposition 4</u>: An equilibrium $(\tilde{r},\bar{G},\tilde{w}(\alpha))$ generates a "utility" schedule for all $\alpha \in [0,1]$, given by

$$\begin{aligned} \tilde{U}(\alpha) &\equiv \tilde{I}(\alpha) \equiv \tilde{R}(\alpha) + \tilde{w}(\alpha) \quad , \quad \text{if} \quad \alpha \in \bar{G} \\ &\equiv \tilde{R}(\alpha) + \bar{w}(\tilde{R}(\alpha)) \quad \text{if} \quad \alpha \notin \bar{G} \end{aligned} \tag{24}$$

where $\tilde{R}(\alpha) \equiv \tilde{r}t(\alpha)\hat{T}$ .

Suppose $(\bar{r},\bar{G},\bar{w}(\alpha))$ is not Pareto-efficient. Then there is a set $A \subset [0,1]$ with $\mu(A) = 1$, and a feasible allocation $\{U(\alpha)\}$ such that $U(\alpha) \geq \tilde{U}(\alpha)$ on $A$ and a set $B \subseteq A$, with $\mu(B) > 0$, and with $U(\alpha) > \tilde{U}(\alpha)$ on $B$.

Let $C = \{\alpha \in \bar{G} \cap A/\ \alpha$ doesn't work under $\{U(\alpha)\}\}$. Let $D = \{\alpha \in A\backslash\bar{G}/\alpha$ works under $U(\alpha)\}\}$, and $E = \{\alpha \in \bar{G} \cap A/\lambda(I(\alpha)) > \lambda(\tilde{I}(\alpha))\}$, where $\{I(\alpha)\}$ denotes the income allocation under $\{U(\alpha)\}$. It is easy to see that $C \cup D \cup E = A$.

<u>Case I:</u> $C = \Phi$.

In this case, it's clear that under the new allocation, the number of efficiency units $E$ strictly exceeds $\bar{E}$.

Now pick $\alpha \in E$. Then $\lambda(\tilde{I}(\alpha)) > 0$, and $I(\alpha) > \tilde{I}(\alpha)$. By construction of $w^*(\alpha,\bar{r})$, $\bar{w}(\alpha)$, we have

$$\lambda(\tilde{I}(\alpha)) > \bar{w}(\alpha)\lambda'(\tilde{I}(\alpha)) \tag{25}$$

The additional contribution of $\alpha$ to total output does not exceed $F_E(\hat{T},\bar{E})\lambda'(\bar{I}(\alpha))(I(\alpha) - \bar{I}(\alpha))$ (since $E > \bar{E}$), but by (25) and the equilibrium condition

$$F_E(\hat{T},\bar{E}) = \bar{w}(\alpha)/\lambda(\tilde{I}(\alpha)) \tag{26}$$

for all $\alpha \in \bar{G}$, this cannot exceed $I(\alpha) - \tilde{I}(\alpha)$, the increase in his consumption. In fact, if $U(\alpha) > \tilde{U}(\alpha)$, so that $I(\alpha) - \bar{I}(\alpha) > 0$, the increase in consumption <u>strictly</u> exceeds his additional contribution to output.

Next, pick $\alpha \in D$. Then $\lambda(\bar{I}(\alpha)) = 0$ and so $\bar{I}(\alpha) = \tilde{R}(\alpha)$. Note that by the unemployability of these $\alpha$'s in equilibrium (Proposition 1),

$$F_E(\hat{T},\bar{E}) < w/\lambda(w + \bar{R}(\alpha)) \quad \text{for all} \quad w > \bar{w}(\tilde{R}(\alpha)) \tag{27}$$

Again, since $U(\alpha) > \tilde{U}(\alpha)$, and $\alpha \in D$, $I(\alpha) - \tilde{I}(\alpha) > \bar{w}(\tilde{R}(\alpha))$, and so (27) applies with $w = I(\alpha) - \bar{I}(\alpha)$. Hence the increase in output due to such an $\alpha$ is less than or equal to the L.H.S. of

$$F_E(\hat{T},\tilde{E})\lambda(I(\alpha)) = \frac{F_E(\hat{T},\tilde{E})\lambda(\bar{R}(\alpha) + I(\alpha) - \bar{I}(\alpha))}{I(\alpha) - \bar{I}(\alpha)} [I(\alpha) - \bar{I}(\alpha)] < I(\alpha) - \bar{I}(\alpha) \tag{28}$$

where the R.H.S. of (28) represents the increase in his consumption. Moreover, if $\mu(D) > 0$, it follows from the <u>strict</u> concavity of $F$ in $E$, that the increase in <u>total</u> output as a result of all this is <u>strictly</u> less than the increase in the total consumption of all $\alpha \in D$. Observe that to obtain this strict inequality, an argument of the kind used for $\alpha \in E$ will <u>not</u> work).

Now, given $\mu(B) > 0$ and $C = \Phi$, it follows that either $\mu(D) > 0$ or $\mu(B \cap E) > 0$. In either case, the total increase in output must fall short of the total increase in consumption, contradicting the feasibility of $U(\alpha)$.

<u>Case II:</u> $C \neq \Phi$.

Here, we first prove that $E > \bar{E}$. Suppose not, so that $E < \bar{E}$. Pick $\alpha \in C$. His removal from work caused an output loss of <u>at least</u> $\lambda(\bar{I}(\alpha))F_E(\hat{T},\bar{E})$ (by $E < \bar{E}$ and concavity of $F$). Since $U(\alpha) > \bar{U}(\alpha)$,

(29) $$R(\alpha) + \bar{w}(R(\alpha)) > \tilde{I}(\alpha) = \tilde{R}(\alpha) + \tilde{w}(\alpha) \quad .$$

where $R(\alpha)$ is (non work) income given to $\alpha$ under $\{U(\alpha)\}$.

This implies $R(\alpha) > \tilde{R}(\alpha)$. For if $R(\alpha) < \tilde{R}(\alpha)$, $\bar{w}(R(\alpha)) < \bar{w}(\tilde{R}(\alpha)) < \tilde{w}(\alpha)$, contradicting (29) (here we have used the fact that $\bar{w}(\cdot)$ is nondecreasing). This means that the extra output <u>freed</u> by $\alpha$ as a result of not working is <u>less</u> than or equal to $\tilde{w}(\alpha)$, and given (26) this means that no "net" output is added to the economy as a result of removing $\alpha$ from work.

So the "residual" output available to all $\alpha \in A\backslash C$ does not exceed the earlier "residual" equilibrium output (and it is <u>strictly</u> less, if $\mu(B \cap C) > 0$. Since $I(\alpha) > \tilde{I}(\alpha)$ for $\alpha \in A\backslash C$ (strictly more for $\alpha \in B \cap (A\backslash C)$), this is a contradiction to the feasibility of $\{U(\alpha)\}$ (because either $\mu(B \cap C) > 0$ or $\mu(B \cap (A\backslash C)) > 0$). Therefore $E > \tilde{E}$.

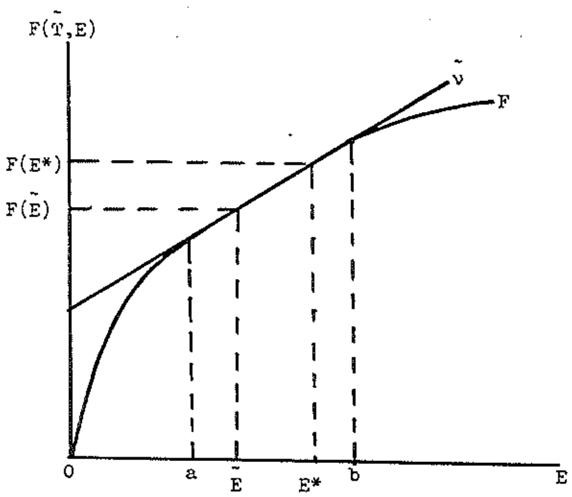
The remainder of the argument runs as follows. We pick a subset of $D \cup E$, and lower their levels of work-efficiency under the new allocation, and their utilities, to that prevailing under the original equilibrium. Denote this fresh allocation by primed variables (e.g. $E'$, $R'(\alpha)$, $U'(\alpha)$). We will choose this subset $F$ of $D \cup E$ so that in the latest allocation, $E' = \tilde{E}$ (clearly such a subset exists). Note that the reduction process may "absorb" extra output; we will prove that this is not possible and that, if anything, net output will be <u>released</u> by members of $F$ as we lower their work efficiencies. Transfer such output to members of $C$ in any way we please.

If $\alpha \in F \cap E$, then $\lambda(\tilde{I}(\alpha)) > 0$. By an argument similar to that for $\alpha \in E$ in Case I, we can show that $I(\alpha) - \tilde{I}(\alpha)$ [the necessary reduction to bring such $\alpha$'s back to the level $\bar{U}(\alpha)$] is not less than the reduction in total output as $I(\alpha)$ is lowered to $\tilde{I}(\alpha)$, and in fact is strictly more if $U(\alpha) > \tilde{U}(\alpha)$. If $\alpha \in F \cap D$, then use the argument for $\alpha \in D$ in Case I to conclude, again, the same inequality, with strict inequality applying if $\mu(F \cap D \cap B) > 0$.

After these adjustments, it is clear that we are left with an allocation $U'(\alpha)$ which is Pareto-superior to $\tilde{U}(\alpha)$. (If $\mu(F \cap B) > 0$, then net output is released to $\alpha$-types in $C$, so $U'(\alpha) > U(\alpha)$ on a set of positive measure. Otherwise, outside $F$, $U'(\alpha) = U(\alpha) > \tilde{U}(\alpha)$ on a set of positive measure, anyway). But this allocation has $E' = E$, which contradicts our demonstration in the first part of case II, that Pareto superior allocations with $C \neq \Phi$ have to exhibit a higher number of efficiency units than $\bar{E}$. Q.E.D.

3. Possibility of Pareto-inefficiency if $F$ is not strictly concave: Here, we sketch a brief example of how an equilibrium may fail to achieve Pareto optimality. Diagram 9 displays one profit maximizing choice of $\bar{E}$ in an equilibrium.

Think of such an equilibrium as a regime 2 equilibrium, with involuntary unemployment among the landless. Then there is another allocation, which creates, say, $E^* - \bar{E}$ new efficiency units by simply putting some of the landless to work. Moreover, the extra output so generated, $F(E^*) - F(\bar{E})$, exactly suffices to pay the new employed landless at the efficiency wage, which exceeds the reservation wage. So these people are better off and nobody is worse off, demonstrating the




Equilibrium $\tilde{\nu}$ coincides with a linear stretch of F (from a to b). $\tilde{E}$ is a profit maximizing choice.

Diagram 9

Pareto-inefficiency of the equilibrium.

Three remarks: (i) this example illustrates how a "blocking coalition" may arise from those in "compensated equilibrium" (see footnote 35) (ii) the example depends crucially on the fact that the production function is concave, but not strictly so, and (iii) the Pareto-improvement is supportable, at least in this case, by a competitive equilibrium, too (note that all values of E between a and b are profit-maximizing for the employers).

4. Proof of Proposition 5: As in the proof of Proposition 2,

construct a correspondence $E'(r)$ corresponding to the new land distribution $(t'(\alpha))$ after a partial land reform. Denote by primes the other relevant variables (functions) corresponding to the new equilibrium.

We first establish that $\min E'(r) > \min E(r)$, and $\max E'(r) > \max E(r)$. To see this, note that $\mu(B'(r)\setminus B(r)) > 0$, since none of the previously employed lose land, and because the gain in land among some of involuntarily unemployed and "near involuntary unemployed" pushes their $v(\alpha,\tilde{r})$ to $v'(\alpha,\tilde{r})$ and below $\bar{v}$. This immediately yields $\min E'(r) > \min E(r)$. A similar argument establishes that $\mu(G'(r)\setminus G(r)) > 0$, and hence that $\max E'(r) > \max E(r)$.

If $\tilde{E} \in E'(\tilde{r})$, then the old output-efficiency units configuration continues to be the (unique) equilibrium.

Otherwise,

$$E(\tilde{r}) = \tilde{E} < \min E'(\tilde{r}) \quad , \tag{30}$$

and since $E'(r)$ is bounded as $r \to \infty$, and satisfies the properties of Lemmas 1 and 2, and because $E(r) \to \infty$ as $r \to \infty$, there is $\tilde{r}' > \tilde{r}$ and $\tilde{E}' \in E'(\tilde{r}')$ such that

$$\tilde{E}' = E(\tilde{r}') \quad . \tag{31}$$

Because $E(\cdot)$ is increasing, we have

$$\tilde{E}' > \tilde{E} \tag{32}$$

From $\tilde{E}'$, construct the new (unique) equilibrium as in the proof of Proposition 2. This displays a higher output.

Finally, observe that if $\nu(\alpha,\tilde{r})$ has no flats in the region of the earlier equilibrium, $\tilde{G} = \bar{B}(r)$, and so $\tilde{G} \subseteq B'(r)$. But this proves that (30) must hold, and therefore that the new equilibrium has strictly higher output. Q.E.D.

5. Proof of Proposition 6: Let $(\hat{T},I(\hat{T}))$ be an equal division solution. Define $\hat{\nu} \equiv F_E(\hat{T},\lambda(I(\hat{T})))$, $\hat{r} \equiv F_T(\hat{T},\lambda(I(\hat{T})))$. Choose the reservation wage function "low" enough so that, in particular, $\bar{w}(\hat{r}\hat{T}) < \lambda(I(\hat{T}))\hat{\nu}$, and define $\tilde{G} = [0,1]$, with $\hat{w}(\alpha) \equiv \hat{w} \equiv \lambda(I(\hat{T}))\nu$ for all $\alpha \in [0,1]$. Recall that by the equal distribution postulate, $t(\alpha) = 1$ for all $\alpha \in [0,1]$. It is easy, now, to verify all the conditions of Proposition 2, except condition (i), (iii), and (iv) which we show explicitly.

We show that

$$\hat{\nu} > \min_{w > \bar{w}(\hat{r}\hat{T})} \frac{w}{\lambda(\hat{r}\hat{T} + w)} \tag{33}$$

and

$$\hat{w} > \arg \min_{w > \bar{w}(\hat{r}\hat{T})} w/\lambda(\hat{r}\hat{T} + w) \tag{34}$$

which, together with the other verifications, will complete the proof of the proposition.

Note, first, that if $w = \bar{w}(r\hat{T})$ in (33), then we're done, since

$$\hat{\nu} = \frac{\hat{w}}{\lambda(I(\hat{T}))} = \frac{\hat{w}}{\lambda(\hat{r}\hat{T} + \hat{w})} > \frac{\bar{w}(\hat{r}\hat{T})}{\lambda(\hat{r}\hat{T} + \bar{w}(\hat{r}\hat{T}))} \tag{35}$$

(to verify (35), note that $I(\hat{T}) = \hat{r}\hat{T} + \hat{w}$ by (7) and Euler's theorem, and $\hat{w} > \bar{w}(\hat{r}\hat{T})$ by assumption).

Finally, if the reservation wage constraint is not binding, the solution $w^*$ to (33) is characterized by

$$\lambda(\hat{r}\hat{T} + w^*) = w^*\lambda'(\hat{r}\hat{T} + w^*) \tag{36}$$

Now observe that by our construction, $I(\hat{T})$ is the larger of (at most) two roots to (7), and therefore

$$F_E(\hat{T},\lambda(I(\hat{T}))) \cdot \lambda'(I(\hat{T})) < 1 \tag{37}$$

But, recalling the definition of $\hat{w}$, this means that

$$\lambda(I(\hat{T})) > \hat{w}\lambda'(I(\hat{T}))$$

$$\text{or,} \quad \lambda(\hat{r}\hat{T} + \hat{w}) > \hat{w}\lambda'(\hat{r}\hat{T} + \hat{w}) \tag{38}$$

So comparing (36) and (38), $\hat{w} > w^*$, which, together with the definition of $\hat{w}$, completes verification of (iv). Finally, note that

$$\frac{\hat{w}}{\lambda(\hat{r}\hat{T} + \hat{w})} = \hat{\nu} > \min_{w > \bar{w}(\hat{r}\hat{T})} \frac{w}{\lambda(\hat{r}\hat{t} + w)} \tag{39}$$

which verifies conditions (i) and (iii). Q.E.D.

6. Proof of Proposition 7: Define $\hat{T}_o$ as the minimum value of $\hat{T}$ such that a solution to (7) exists. It is easy to check that at this minimum value,

$$F_E(\hat{T}_o,\lambda(I(\hat{T}_o)))\lambda'(I(\hat{T}_o)) = 1 \tag{40}$$

By (7) and Euler's Theorem,

$$(41) \qquad I(\hat{T}_o) = F(\hat{T}_o,\lambda(I(\hat{T}_o))) = F_T\hat{T}_o + F_E\lambda(I(\hat{T}_o))$$
$$> F_E\lambda(I(\hat{T}_o))$$
$$= \lambda(I(\hat{T}_o))/\lambda'(I(\hat{T}_o)) \quad .$$

From (41), it is easily seen that $I(\hat{T}_o) < \hat{I}$, and noting that $I(\hat{T})$ is strictly increasing and unbounded in $\hat{T}$, we have $\hat{T}_1$ well defined and greater than $\hat{T}_o$.

To proceed further, we establish that

$$(42) \qquad \nu(\hat{T}) \equiv F_E(\hat{T},\lambda(I(\hat{T})))$$

is an <u>increasing</u>, unbounded, function in $\hat{T}$, for all $\hat{T} > \hat{T}_1$. Verification of this is straightforward but tedious; we mention only that it depends on (a) $I(\hat{T}) > \hat{I}$ for $T > \hat{T}_1$, and (b) the fact that equation (37) holds with strict inequality for all $T > \hat{T}_1$.

Using this, and noting that at $\hat{T}_1$,

$$(43) \qquad \nu(\hat{T}_1) < \frac{\hat{I}}{\lambda(\hat{I})}$$

there exists $\hat{T}_2 > \hat{T}_1$ such that $\nu(\hat{T}_2) = \hat{I}/\lambda(\hat{I})$.

Finally, we will show that for $\hat{T} \in [\hat{T}_1,\hat{T}_2)$, equal distributions of $\hat{T}$ generate competitive equilibria involving full employment and no malnourishment, while there exist <u>unequal</u> distributions of $\hat{T}$ giving rise to involuntary unemployment and malnourishment.

The first part of the preceding statement follows directly from Proposition 6, and the observation that $I(\hat{T}) > \hat{I}$ for $\hat{T} > \hat{T}_1$. We establish the second part.

Pick any $\hat{T} \in [\hat{T}_1, \hat{T}_2)$. To make things easy, we note that in the equal-distribution equilibrium, (39) holds with strict inequality (since (37) does). In other words, if one constructs the correspondence $E(r)$ (as in the proof of Proposition 2), $E(r)$ is a singleton at $\hat{r}$, the equilibrium rental rate. It is obvious that for small changes in $r$, $E(r)$ continues to be a singleton (since $\nu(r)$ continues to strictly exceed $\nu(\alpha, r)$). Thus pick a small interval $[r_a, r_b]$ around $\hat{r}$, where $E(r)$ is a singleton, and with the additional property that $\nu(r_a) < \hat{I}/\lambda(\hat{I})$. This is possible because at equilibrium, $\hat{\nu} = \nu(\hat{r}) < \hat{I}/\lambda(\hat{I})$.

Now define a function $E(r, \delta)$ on $[r_a, r_b] \times [0, \delta_o]$ (where $\delta_o$ is chosen suitably small - see below) in the following way.

First, define $t(\alpha, \delta)$ (these will be land distributions "slightly" modified from the equal distribution) as follows:

$$
\begin{aligned}
(44) \qquad t(\alpha,\delta) &= 0 \quad , \quad \alpha \in [0,\delta] \\
&= \frac{2(\alpha - \delta)}{(2 - 3\delta)\delta} \quad , \quad \alpha \in [\delta, 2\delta) \\
&= \frac{2}{2 - 3\delta} \quad , \quad \alpha \in [2\delta, 1]
\end{aligned}
$$

(diagram 10 below summarizes (44))

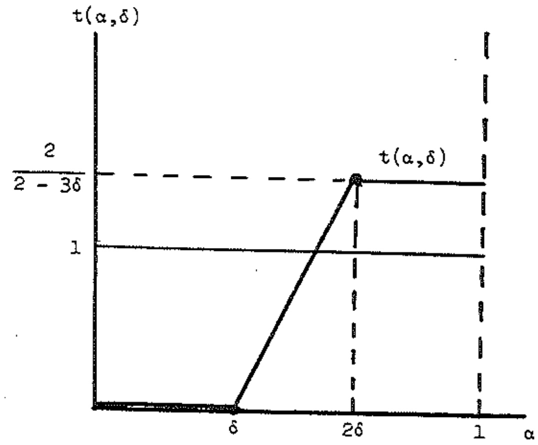



Diagram 10: A "t(α,δ)" land distribution

Of course, we chose $\delta_o$ small enough so that $t(\alpha,\delta)$ is well-defined for all $\delta \in [0,\delta_o]$. It can be checked that $\int_0^1 t(\alpha,\delta)d\alpha = 1$. Remembering that reservation wages were chosen low enough in the equal distribution problem, we also take care to choose $\delta_o$ small enough so that reservation wages still do not impose a binding constraint, for all $r \in [r_a,r_b]$.

Now, define $E(r,\delta)$ as the correspondence analogous to $E(r)$, for the land distribution $t(\alpha,\delta)$.

Of course, $E(r,0) = E(r)$. It is easily checked that if $r_a$, $r_b$, and $\delta_o$ are chosen suitably, $E(r,\delta)$ is a singleton, too, for each $\delta \in [0,\delta_o]$ and $r \in [r_a,r_b]$ (this simplifies our argument, so we emphasize it). So, with some abuse of notation, think of $E(r,\delta)$ as function on $[r_a,r_b]$, for each $\delta$.

Finally, observe that, by the construction of $t(\alpha,\delta)$, for each $r \in [r_a, r_b]$, $E(r,\delta)$ is continuous in $\delta$, at $\delta = 0$. To summarize, at $\hat{r}$,

$$E(\hat{r},0) = E(\hat{r}) \tag{45}$$

(where $E(r)$ is defined as in the proof of Proposition 2),

$$r_a < \hat{r} < r_b \tag{46}$$

and $E(r,\delta)$ is continuous in $\delta$, at $\delta = 0$, for $r \in [r_a, r_b]$. So for $\delta$ close to 0, but positive, there is $\hat{r}(\delta)$ in $[r_a, r_b]$, so that

$$E(\hat{r}(\delta),\delta) = E(\hat{r}(\delta)) \tag{47}$$

This is easily seen to generate an equilibrium, as we did in the proof of Proposition 2. But because $\hat{r}(\delta) > r_a$, and $\nu(r_a) < \hat{I}/\lambda(\hat{I})$, it must be true that $\nu(\hat{r}(\delta))$, the new equilibrium marginal product, is <u>also</u> less than $\hat{I}/\lambda(\hat{I})$.

By Proposition 1, condition (ii), it follows that the landless (all $\alpha \in [0,\delta]$) are unemployed and malnourished in this equilibrium, and some of the small landholders are also involuntarily unemployed and malnourished. Q.E.D.

7. <u>Proof of Proposition 8</u>: Suppose not. Then for some $\hat{T} > \hat{T}_2$, there exists a land distribution $t(\alpha)$ such that

$$\tilde{\nu} < \hat{I}/\lambda(\hat{I}) \quad . \tag{48}$$

(otherwise it is easy to check that malnourishment and involuntary unemployment is impossible).

By definition of $\hat{T}_2$ and the fact that $\nu(\hat{T})$ is increasing in $\hat{T}$ for $\hat{T} > \hat{T}_1$ (see proof of Proposition 7, equation (42)), we have

(49) $$\nu(\hat{T}) > \hat{I}/\lambda(\hat{I}) \quad , \quad \text{for} \quad T > \hat{T}_2$$

so that, combining (48) and (49) with the concavity of $F$ in $E$, the equilibrium under $t(\alpha)$ involves a larger number of efficiency units than that achievable under equal division; i.e.

(50) $$\int_{[0,1]} \lambda(\tilde{I}(\alpha))d\mu(\alpha) > \lambda(I(\hat{T}))$$

Now consider the following maximization problem:

(51) $$\max_{I(\alpha)} \int_{[0,1]} \lambda(I(\alpha))d\mu(\alpha)$$

subject to the feasibility constraint

(52) $$\int_{[0,1]} I(\alpha)d\mu(\alpha) = F(\hat{T}, \int_{[0,1]} \lambda(I(\alpha))d\mu(\alpha))$$

We prove that $I(\alpha) = I(\hat{T})$ for $\alpha$ a.e. in $[0,1]$ is the unique solution. This will contradict (50) (since $\tilde{I}(\alpha)$ is a feasible allocation), and establish the proposition.

When $\lambda(\cdot) > 0$, it is strictly concave. So, wherever the solution to (51) involves $\lambda(I(\alpha)) > 0$ on a set of positive measure, $I(\alpha)$ must be constant, say equal to $\tilde{I}$.

And whenever $\lambda(I(\alpha)) = 0$, it must be that $I(\alpha) = 0$, otherwise (51) is not solved. So the solution to (51) is of the form: on a set

A of $\mu$-measure $a > 0$, $I(\alpha) = \bar{I}$, and on $[0,1]\backslash A$, a.e., $I(\alpha) = 0$. We prove that $a = 1$, and therefore $\bar{I} = I(\hat{T})$.

Suppose, on the contrary, that $a < 1$. Then $\bar{I} > I(\hat{T})$. Now, by (52), we have

$$\text{(53)} \qquad a\bar{I} - I(\hat{T}) = F(T, a\lambda(\bar{I})) - F(\hat{T}, \lambda(I(\hat{T})))$$
$$< [a\lambda(\bar{I}) - \lambda(I(\hat{T}))] F_E(\hat{T}, \lambda(I(\hat{T})))$$
$$< [a\lambda(\bar{I}) - \lambda(I(\hat{T}))]/\lambda'(I(\hat{T}))$$

since for all $\hat{T} > \hat{T}_2$ (in fact $> \hat{T}_0$), (37) holds with strict inequality. Rearranging (53),

$$\text{(54)} \qquad \frac{\lambda(I(\hat{T}))}{\lambda'(I(\hat{T}))} - I(\hat{T}) < a \left[\frac{\lambda(I)}{\lambda'(I(\hat{T}))} - \bar{I}\right]$$

Now pick $x$ such that $\lambda(x) > 0$. Then it is easy to verify that $G(I) \equiv (\lambda(I)/\lambda'(x)) - I$ is strictly decreasing in $I$ for all $I > x$. But given $\bar{I} > I(\hat{T})$, the fact that the L.H.S. of (54) is positive, and $a < 1$, this contradicts (54). Q.E.D.

## 8. An Example

We assume

$$\text{(55)} \qquad \lambda(I) = \bar{\lambda} > 0 \quad \text{if} \quad I > \hat{I} > 0$$
$$= 0 \quad \text{if} \quad I < \hat{I}$$

$$\text{(56)} \qquad t(\alpha) = 1/(1 - \underline{\alpha}) \quad \text{for} \quad \alpha > \underline{\alpha} > 0$$
$$= 0 \quad \text{for} \quad 0 < \alpha < \underline{\alpha}$$

(57) $$\bar{w}(R) = 0 \quad \text{for all} \quad R > 0$$

(58) $$F(E,T) = E^a T^{1-a} \quad , \quad 0 < a < 1 \quad .$$

In words, (55) says that the consumption-ability relationship is a step-function, (56) says that it is a two-class economy, (57) says that the reservation wage is nil for all persons, and (58) postulates a Cobb-Douglas production function. (55) and (56) violate the conditions assumed in Section 3.1. (For example, both $\lambda(I)$ and $t(\alpha)$ have so far been assumed to be continuous.) Clearly, though we can approximate them by functions satisfying those conditions as closely as we like. The example is thus a valid one to use for illustrating the theory. It also indicates that assumptions of continuity, etc. are essentially simplifying devices for the model.

Using (55)-(57) in equation (1) we find

(59) $$w^*(\alpha,r) = \hat{I} \quad \text{for} \quad 0 < \alpha < \underline{\alpha}$$

$$= \max \{0, \hat{I} - r\hat{T}/(1 - \underline{\alpha})\} \quad \text{for} \quad 1 > \alpha > \underline{\alpha} \quad .$$

Likewise, using (55)-(57) in equation (2) yields

(60) $$\nu(\alpha,r) = \hat{I}/\bar{\lambda} \quad \text{for} \quad 0 < \alpha < \underline{\alpha}$$

$$= \max \{0, \hat{I} - r\hat{T}/(1 - \underline{\alpha})\}/\bar{\lambda} \quad \text{for} \quad 1 > \alpha > \underline{\alpha} \quad .$$

We will first vary $\hat{T}$ so as to illustrate Proposition 3 and the claims made in Sections 3.4-3.6. It is in fact simplest to write down the equilibrium conditions for regime 2 (Section 3.5 and Diagram 6b) because $\tilde{\nu}$ is anchored to $\hat{I}/\lambda(\hat{I})$. So, on using (55)-(60) in Proposition 3 we note that the equilibrium conditions in regime 2 are:

(61) $$\tilde{E} = \bar{\lambda}(1 - \alpha_1) \quad , \text{ where } \quad 0 < \alpha_1 < \underline{\alpha} < 1$$

(62) $$\tilde{r} = \bar{\lambda}^a(1 - \alpha_1)^a(1 - a)\hat{T}^{-a}$$

(63) $$\tilde{\nu} = a\bar{\lambda}^{(a-1)}(1 - \alpha_1)^{(a-1)}\hat{T}^{(1-a)}$$

and

(64) $$\tilde{\nu} = \hat{I}/\bar{\lambda} \quad . \text{ 48/}$$

Equations (61)-(64) are four in number, and there are four unknowns, $\tilde{E}$, $\tilde{r}$, $\tilde{\nu}$ and $\alpha_1$, to solve for. Using (63) and (64) we note that

(65) $$\alpha_1 = 1 - [a\bar{\lambda}^a\hat{T}^{(1-a)}/\hat{I}]^{1/(1-a)}$$

Now, in regime 2 one must have $0 < \alpha_1 < \underline{\alpha} < 1$. Using this in equation (65) we conclude that <u>given</u> $\underline{\alpha}$, for the economy to be in regime 2 $\hat{T}$ must satisfy the inequalities:

(66) $$(1 - \underline{\alpha})[\hat{I}/a\bar{\lambda}^a]^{1/(1-a)} < \hat{T} < [\hat{I}/a\bar{\lambda}^a]^{1/(1-a)} \quad ,$$

that is, if $\hat{T}$ is neither too large nor too small, (Section 3.5).

From (66) we conclude that the economy is in regime 3 if $\hat{T} > [\hat{I}/a\bar{\lambda}^a]^{1(1-a)}$, no matter what the distribution of land holdings is. Since $\alpha_1 = 0$ in regime 3 (Diagram 6c) competitive equilibrium can be explicitly computed to be

(67) $\bar{E} = \bar{\lambda}$ ; $\bar{r} = \bar{\lambda}^a(1 - a)\hat{T}^{-a}$ ; and $\bar{\nu} = a\bar{\lambda}^{(a-1)}\hat{T}^{(1-a)} > \hat{I}/\bar{\lambda}$ .

From (66) we also conclude that the economy is in regime 1 (Section 3.4 and Diagram 6a), if $\hat{T} < (1 - \underline{\alpha})[\hat{I}/a\bar{\lambda}^a]^{1/(1-a)}$.

The regime 1 equilibrium exhibits employment of all from $\underline{\alpha}$ to 1 as long as $\hat{T}$ is not too small. To calculate this bound, assume first that the equilibrium set $\tilde{G} = [\underline{\alpha},1]$; then

(68) $$\bar{E} = \bar{\lambda}(1 - \underline{\alpha}) \quad , \quad \bar{r} = \bar{\lambda}^a(1 - \underline{\alpha})^a(1 - a)\hat{T}^{-a}$$

and $$\bar{\nu} = a[\bar{\lambda}(1 - \underline{\alpha})]^{a-1}\hat{T}^{1-a} < \hat{I}/\bar{\lambda}$$

And this is an equilibrium as long as $\bar{\nu} > \nu(\alpha,\bar{r})$ for all $\alpha \in [\underline{\alpha},1]$, or if

(69) $$a[\bar{\lambda}(1 - \underline{\alpha})]^{a-1}\hat{T}^{1-a} > \{\hat{I} - \bar{r}\hat{T}/(1 - \alpha)\}/\bar{\lambda}$$

Substituting for $\bar{r}$ and rearranging, one obtains

(70) $$\hat{T} > a^{1/(1-a)}(1 - \underline{\alpha})(I/a\bar{\lambda}^a)^{1/(1-a)}$$

Note that, since $a < 1$, the R.H.S. of (70) is smaller than the L.H.S. of (66), which is the borderline for regime 1 equilibria.

If $\hat{T}$ doesn't satisfy (70), then we're in regime 1 equilibria where only a subset of $[\underline{\alpha},1]$ is employed. No generality is lost by choosing this subset to be of the form $[\alpha_1,1]$, where $\alpha_1 > \underline{\alpha}$.

For such equilibria, $\bar{\nu} = \nu(\alpha,\bar{r})$; in other words, defining

(71)
$$\tilde{E} = \bar{\lambda}(1 - \alpha) \quad , \quad \tilde{r} = \bar{\lambda}^{a}(1 - \alpha_1)^{a}(1 - a)\hat{T}^{-a} \quad ,$$
$$\text{and} \quad \tilde{v} = a[\bar{\lambda}(1 - \alpha_1)]^{a-1}\hat{T}^{1-a} < \hat{I}/\lambda$$

as the equilibrium magnitudes, we solve for $\alpha_1$ by noting that

(72)
$$a[\bar{\lambda}(1 - \alpha_1)]^{a-1}\hat{T}^{1-a} = \{\hat{I} - \tilde{r}\hat{T}/(1 - \underline{\alpha})\}/\bar{\lambda}$$

Rearranging, $\alpha_1$ is the solution to

(73)
$$a\bar{\lambda}^{-a}(1 - \alpha_1)^{a-1}\hat{T}^{1-a} + [(1 - a)\bar{\lambda}^{a}(1 - \alpha_1)^{a}\hat{T}^{1-a}]/(1 - \underline{\alpha}) = \hat{I}$$

This describes the regimes.

Using (61) and (65), we note that in regime 2, aggregate employment, and hence aggregate output, are independent of the distribution of land holdings. This is not so in regime 1. First consider the regime 1 subcase where $\hat{T}$ satisfies (70).

Here a more equal land distribution (lowering of $\underline{\alpha}$) results in greater equilibrium employment and output.

Next, consider the subcase of regime 1 where $\hat{T}$ is too small to satisfy (70). In this case, land reforms (lowering of $\underline{\alpha}$) has the perverse effect of lowering output and employment! This can be checked by verifying that $\alpha_1$ has to increase in (73) in response to a decrease in $\underline{\alpha}$. The intuition is that already employed workers become less productive, contracting output. After all, the landless (who now gain some land) are out of the picture to begin with. Observe this does not contradict Proposition 5, which deals with land transfers from pure rentiers to the employed. In this example, there is no landed gentry. We also will note that such types of behavior are impossible if the economy is endowed with enough land to support everyone, in principle, without malnourishment. To this we now turn.

Consider an economy only moderately endowed with land, but enough to make it technologically feasible to feed everyone adequately. In the present example, the borderline case $\hat{T}_1$ is given by the solution to $\bar{\lambda}^{a}\hat{T}^{(1-a)} = \hat{I}$, and so

$$\hat{T}_1 = [\hat{I}/\bar{\lambda}^{a}]^{1/(1-a)} \tag{74}$$

Note that this immediately rules out the possibility of the economy being in the subcase of regime 1 where (70) fails. So the possibly perverse effects of land reform cannot occur in this case.

Obviously, the only way the market can sustain the equal division allocation of (74) is if land is equally distributed to begin with. But if the allocation is anything else, some people must be malnourished and involuntarily unemployed.

To confirm this explicitly note first that (74) and the second inequality in (66) together imply that the economy can never be in regime 3, if $\underline{\alpha} > 0$. It is in regime 1 if

$$(1 - \underline{\alpha})[\hat{I}/a\bar{\lambda}^{a}]^{1/(1-a)} > [\hat{I}/\bar{\lambda}^{a}]^{1/(1-a)} \tag{75}$$

(see the first inequality in (66)), and is in regime 2 if

$$(1 - \underline{\alpha})[\hat{I}/a\bar{\lambda}^{a}]^{1/(1-a)} < [\hat{I}/\bar{\lambda}^{a}]^{1/(1-a)} , \tag{76}$$

or

$$\underline{\alpha} > 1 - a^{1/(1-a)} . \tag{77}$$

If (77) holds then (61)-(65) and (74) allow us to conclude that

(78) $$\alpha_1 = 1 - a^{1/(1-a)} ,$$

which is independent of $\underline{\alpha}$. It can also be verified for all $\alpha \in [\alpha_1, 1]$, $\bar{w}(\alpha) = \hat{I}$. Thus all who are employed receive the "efficiency wage" of the landless.[49/] In this regime there are, therefore, three income groups: The landless unemployed $(\alpha \in [0, \alpha_1))$ who earn nothing, the landless employed $(\alpha \in [\alpha_1, \underline{\alpha}))$ who earn $\hat{I}$ and the landowners, who work and receive rental income to the tune of

$$\hat{I} + \bar{\hat{r}}\bar{T}/(1 - \alpha) = \hat{I}[1 + a^{a/(1-a)}(1 - a)/(1 - \underline{\alpha})] .$$

Suppose next that the economy is in regime 1. The condition for this is seen from (77) to be

(79) $$\underline{\alpha} < 1 - a^{1/(1-a)} .$$

From the argument we have made above, we know that the only possible regime 1 equilibria consistent with (74) are those where all the landed work and all the landless are unemployed. From (68) and (74) we conclude the wage income of the landholders is $a(1 - \underline{\alpha})^{a-1}\hat{I}$ and their rental income (per landowner) is $(1 - \underline{\alpha})^{a-1}(1 - a)\hat{I}$, so that their total income is $\hat{I}/(1 - \underline{\alpha})$.

We have now confirmed the intuition that if $\underline{\alpha} = 0$ (i.e., land is equally distributed) the competitive equilibrium sustains full employment, and each person consumes $\hat{I}$.

Footnotes

1/ See for example FAO [1974]. Poleman [1977] and Srinivasan [1983] present balanced assessments of such estimates. By undernourishment we mean at this point protein-calorie deficiency, recognizing of course that a balanced diet contains many other nutrients, and that a person's state of health depends as well on the medical and sanitary facilities available to him and made use of by him. Pascual et.al. [1976] contains a succinct account of nutritional requirements.

2/ See FAO [1957, 1963, 1973, 1974] for systematic revisions of the energy needs of the "reference man." Their 1973 assessment is a daily need of 2600 kiloCalories for maintenance and 400 kiloCalories for moderate activity for an average male aged between 20-39, weighing 65 kilogams and living in a mean ambient temperature of 10° C.

3/ For obvious reasons, we are indentifying malnourishment with undernourishment here.

4/ A sustained investigation of nutrition needs in a developing country is the continuing series of reports by Dr. C. Gopalan and his associates, for India. A brief summary of his group's findings is in Gopalan [1983]. Bardhan and Srinivasan [1974] is an excellent collection of essays identifying the incidence of poverty, and by implication malnutrition, in India. (Indeed, a common methodology for arriving at a figure for the poverty line is to estimate the minimum income which, given the proportion of income the poor spend on food, enables one to meet basic nutrition needs.) Rao [1982], Bardhan [1984] and Dasgupta [1984] are more up-to-date reports on these matters. It should be noted that there is considerable evidence that a person's metabolic efficiency in the use of energy adjusts to alterations in his energy intake. In a recent essay Srinivasan [1983] uses this, among other points, to argue against the methodology underlying the FAO assessments of the nutrition needs of a person and thereby the extent of malnutrition in the world today.

5/ The term "protein-calorie malnutrition", or PCM, is used to label the effect, not the cause. the cause is inferred from the effect. Thus, in the nutrition literature it is a clinical term, and it refers to a variety of forms, the two most well known being kwashiorker and marasmus. See for example, Caliendo [1979].

6/ One may of course ask in what sense a person can be said to be rich and yet be hungry--unless it is by anorexic compulsion.

7/ Of course, one <u>can</u> provide an explanation for this; for example, via labour turnover models. (See Stiglitz [1974].) What we are saying is that the cause of rigid wages is often treated somewhat cavaliarly in the development literature.

8/ The idea therefore has to do with the possible discontinuity of the function relating the realized well-being of a person to his characteristics. In the Arrow-Debreu theory under convex structures such a function is continuous. See Section 3.4 below.

9/ To be sure this is a caricature. But only by a tiny amount. The most sustained welfare-theoretic exploration of the Lewis construct, namely Marglin [1976], could envisage capital accumulation in the industrial sector as the <u>only</u> means of alleviating rural poverty.

10/ The World Bank, for example, has been publishing them annually in their World Development Reports.

11/ The idea that asset (or consumption) redistribution should have priority over growth in one's thinking, because it is conducive to growth, has been emphasized greatly in a somewhat different framework by Irma Adelman.

12/ Durnin and Passmore [1967] provides an illuminating account of the various energy requirements for different sorts of activities. We should emphasize that by "current" we don't mean the current <u>moment</u>, only that the time-scale of the effect of malnutrition on ability to work is short compared to the period of analysis of concern to the development economist. We should also emphasize that by "consumption", we mean an aggregate concept, including nutrients, education, health, sanitary services and so forth. In a disaggregated model one would obviously wish to distinguish them. See, for example, Behrman and Wolfe [1984] who present empirical results on the relative importance of current income and mothers' educational attainments on families' nutritional status in Nicaragua.

13/ Bliss and Stern [1978b] and Strauss [1984] contain valuable discussions of the empirical evidence and present their own. An early assessment was FAO [1962]. David [1970] provides some evidence from the English experience.

14/ See, for example, the very instructive work of Sukhatme, e.g. Sukhatme [1978].

15/ Of course, this presumes that such laborers are not in short supply to the firm. All this is taken care of in the analysis that follows.

16/ In their formal analysis neither postulated a market for land. That is, their analysis pertained to a single decision unit, not to a set of interacting decision units. If the 'farm' is interpreted as an entire economy, as is done in Mirrlees [1975], then the problem that is solved by the the decision unit is a central planning problem in a land-poor economy, not the outcome in a decentralized market economy.

17/ To see this consider the related but more extreme version of the problem, which is discussed among philosophers under the heading "life-boat ethics". Three people, stranded in a life-boat together, have enough food for precisely two until they get ashore. Equal sharing guarantees starvation for all three. What should they do? The Utilitarian answer is, of course, to draw lots on who should throw himself overboard. The need for unequal division of consumption because of minimum survival requirements has been demonstrated in a most terrifying manner by relief workers in famine-stricken Ethiopia.

There is a great deal of anthropological evidence of unequal consumption allocation within poor families in various parts of the world. In most instances the brunt falls on female members—female infanticide being the extreme form. It is possible to find economic explanations for this sex-bias. For a careful documentation of this for North-West India, see Miller [1981]. Bardhan [1974, 1984] presents both evidence and an interesting range of economic explanations for India.

18/ Of course, assuming that $z$ was less than the efficiency wage to begin with.

19/ Land owning workers still retain an advantage in that no landless worker would be employed until all the former are.

20/ We are grateful to Gavin Wright of Stanford University for this reference.

21/ Since we will be thinking of a wage-based economy it would be more appropriate to think of the output as a cash crop which can be traded internationally at a fixed price for rice. It should be added that the one-good structure bars us from addressing a number of important related issues. But this is the subject of another paper.

22/ We also suppose that $F(E,T)$ satisfies the Inada conditions (see Appendix). These are technical conditions designed to streamline proofs. They are innocuous. The assumption of strictly diminishing marginal product is essential, though, for one of the results, and this we note below.

23/ 'Smoother' labour-leisure choices can easily be built in, but it would violate the spirit of the exercise so much that we shan't introduce it.

24/ For a thorough analysis of abnormal times, in the sense the term is being used here, see Sen [1981].

25/ Or more precisely, he compares his maximal income if he is working to the sum of his reservation wage and maximal nonwage income if he is not working.

26/ Each person's landholding is thus public knowledge. We want to avoid basing involuntary unemployment on asymmetric information in the market. For this see Weiss [1980].

27/ Labour is thus a differentiated factor since different labourers own in general different quantities of land.

28/ Clearly, a formally equivalent development of the model is possible by focussing on the (single) wage for efficiency units of labour, as is indeed done below. However, the particular approach we choose yields more insights.

29/ A formal identification of this particular equilibrium concept with a compensated Walrasian equilibrium can be made. Such an identification would necessarily involve an infinity of commodities, each different value of labour power (or labour quality) being identified as one such commodity. We are grateful to Peter Hammond for pointing out the connection. For a definition of compensated equilibrium, see, e.g. Arrow and Hahn [1971]. Debreu [1962] had earlier termed it a quasi-equilibrium.

30/ The theory that we are developing here can certainly accommodate Diagram 1a, but it requires additional, fairly complicated exposition. So we avoid it. The reader can extend the arguments that follows to this case. Indeed, we will indicate some of these extensions as we go along. In the text we shall continue to describe properties of various functions by the help of diagrams. In the Appendix these properties will be formally stated.

31/ Given that the $\lambda$ function is of the form depicted in Diagram 1b, the right hand side of equation (1) has a unique value. If the $\lambda$ function is of the s-shaped form of Diagram (1a) the right hand side of (1) isn't necessarily unique. When not we would choose the largest solution (which in fact exists) and define $w^*(\alpha,r)$ as the largest solution.

32/ In Diagrams 4a and 4b the consumption-ability curve of Diagram 1b has been translated to the left by the rental income, $rTt(\alpha)$, of $\alpha$-person. We have done this so as to be able to display $w^*(\alpha,r)$ diagramatically.

33/ We have not been able to find reasonable assumptions that will generate the U-shape, so we do not impose any such structure in the mathematical arguments of the appendix. This necessitates the use of some fairly complicated technical arguments.

34/ This is not, in general, true for the more complicated consumption-ability curve of Diagram 1a, but that doesn't affect the main arguments.

35/ All relevant functions such as $D(\alpha)$ are taken to be measurable. Lebesgue measure is denoted by $\mu(\cdot)$. Observe that the two stated conditions regarding $w(\alpha)$ define it uniquely for each employed $\alpha$-type.

36/ In general, our equilibrium corresponds to a compensated Walrasian equilibrium, and such an equilibrium need not be Pareto-efficient. It is, if a subset of those in compensated equilibrium cannot form a "blocking coalition", and in fact in this case the equilibrium would be in the "core" (see Hammond [1984]).

37/ Of course, it is conceivable that $\bar{v}$ is so low that the "poorest employable person" (i.e., $\alpha_1$) has his reservation wage equal to the market wage, but this is so unlikely that we exclude this case. Examples we have worked out suggest that the major portion of the declining part of $\nu(\cdot,r)$ is taken up by $\alpha$-types whose reservation wages aren't binding.

38/ Of course, the well-to-do may not accumulate, or accumulate slowly so that the time taken to reach the final regime will be great. For an illuminating discussion of this range of questions see DasGupta [1975].

39/ Diagram 7 looks at a land reform in regime 1; clearly, the case of regime 2 can be similarly analyzed.

40/ The assumption of a unique competitive equilibrium can be dropped, but then one would have to look at the stable equilibria. We avoid these to rule out unnecessary technical complications.

41/ In general, if $\nu(\alpha,r)$ has no "flats" at the original equilibrium, output will strictly increase.

42/ This, of course, destroys our ordering of $\alpha$'s so that $t(\alpha)$ is nondecresing, but it is trivial to make the necessary technical adjustments.

43/ This excludes the "tangency case" where there is exactly one solution. One can show, and this will be done in the proof of Proposition 7, that the smallest $\hat{T}$ for which a solution to (7) exists involves an $I(\hat{T}) \leq \hat{I}$. So $\hat{T}_1$, to be described below is uniquely defined.

44/ It helps to think of the reservation wage function as being identically zero in the relevant range, for this final section, as its presence adds nothing to the development of our basic point.

45/ This statement should not be taken to mean that there is no connection between inequality and unemployment in resource - rich economies, but only that the causal chain running through malnourishment and consequent inadequate ability to supply effort is not of the first importance for rich economies.

46/ It should be emphasized that because the increasing-returns in the consumption-ability relation is personalized in each individual, its effects haven't been softened despite an appeal to a large economy. That increasing-returns-to-scale in production can, in some circumstances, be softened in a large-economy is well known. See e.g., Novshek [1980].

47/ The Arrow-Debreu theory doesn't ever claim to do so. It is of course the great power of the Arrow-Debreu analysis to have found (sufficient) conditions under which involuntary unemployment won't occur.

48/ Note that $\tilde{G} \equiv \{\alpha \mid D(\alpha) = 1\} = [\alpha_1, 1]$.

49/ This doesn't contradict Proposition 4 because the consumption-ability relationship assumed in this example violates Diagram 6b.

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