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ABSTRACT

Equilibria in a Dynamic Global Game: The Role of Cohort Effects*

We introduce strategic waiting in a global game setting with irreversible investment. Players can wait in order to make a better informed decision. We allow for cohort effects and discuss when they arise endogenously in technology adoption problems with positive contemporaneous network effects. Formally, cohort effects lead to intra-period network effects being greater than inter-period network effects. Depending on the nature of the cohort effects, the dynamic game may or may not satisfy dynamic increasing differences. If it does, our model has a unique rationalizable outcome. Otherwise, there exist parameter values for which multiple equilibria arise because players have a strong incentive to invest at the same point in time others do.

JEL Classification: C72, C73, D82 and D83
Keywords: coordination, equilibrium selection, global game, period-speciifc network effects, strategic complementarities and strategic waiting

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1 Introduction

Often the optimal action of an economic agent is complementary to the actions undertaken by other agents. For example, a consumer's payoff from buying computer software is typically increasing in the number of other consumers who use that software. Or, think of a consumer who decides to buy a durable consumption good such as a car. As more consumers buy this brand of car, more repair shops will have the know-how and spare parts to repair the car quickly.[1] Models of situations in which the agents' optimal actions are complementary to each other are often plagued by multiple equilibria with self-fullfilling beliefs: If a player expects the other players to buy the software, then it is in her best interest to buy it as well. If a player expects the other players not to acquire the software, she wants to refrain from buying. This multiplicity is annoying from an economic policy point of view. Without an adequate theory of equilibrium selection, one cannot use these theories to predict the market outcome. How then does one judge, for example, whether policies to subsidize/tax the adoption of information technology should be implemented? How does one predict the market power of firms who sell their products in markets with network externalities?

For two-player coordination games, Carlsson and van Damme (1993), henceforth CvD, developed an equilibrium selection theory, which Morris and Shin (1998) extend to a continuum of players. CvD assume that the agents' payoffs depend on the action chosen by the other agent in the economy and some unknown state of the world θ. Agents receive different signals about θ, which generate beliefs about the state of the world and a hierarchy of higher order beliefs. CvD refer to their model as a global game and give conditions under which the equilibrium is unique.[2]

[1] Complementarity of optimal actions is also a key ingredient of many models of macroeconomic coordination failures such as currency crises, debt crises, bank runs, financial crashes, and Keynes-type underemployment (Obstfeld (1996), Cole and Kehoe (2000), Diamond and Dybvig (1983), Bryant (1983)). Milgrom and Roberts (1990) discuss other examples of games with strategic complementarities such as R&D competition, oligopoly, coordination in teams, arms races, and pretrial bargaining.

[2] We refer to any binary action game with strategic complementarity, incomplete information, and in which for some types it is a dominant strategy to adopt one action while for others it is a dominant strategy to adopt the other as a global game. That heterogeneity of agents can lead to a unique equilibrium in situations in which the agents' actions are complementary to each other was shown by Postlewaite and Vives (1987) in a bank-run model. For a comprehensive survey of the global game literature, see Morris and Shin (2003). For an extension of the global game approach to many action games, see Frankel, Morris and Pauzner (2003).

Thus, the global game framework enables researchers to predict behavior in coordination games. It has been applied to a wide variety of contexts within a static framework.[3] In reality, however, many economic coordination problems are essentially dynamic. Players can always postpone their investment decisions in order to make a better informed decision. This paper investigates conditions under which the global game approach can be extended to model dynamic technology adoption problems.

To address this question, we build a dynamic global game. We consider a continuum of investors, who have the opportunity to engage in a risky investment project in either of two periods. Investments are irreversible. Payoffs depend positively on the realization of a random variable, which we refer to as the fundamental, and on the mass of investors. All players receive some noisy private information concerning the realization of the fundamental. For very high signals, it is a dominant strategy to invest immediately and for very low signals it is a dominant strategy not to invest. For intermediate signals, a player's optimal behavior depends on how other investors act. If a player decides to wait, she gets a more informative signal concerning the realization of the fundamental at the cost of foregone profits. We work with a flexible dynamic payoff structure in which a player's gain of investing not only depends on the total mass of players who invest, but also on *when* the other players invest. We say that our payoffs exhibit an early (late) mover cohort effect if the early (late) adopters enjoy more network benefits from the other early (late) adopters than from the late (early) ones. Four main results emerge from our analysis.

First, we show that cohort effects can arise endogenously in a dynamic set-up with contemporaneous network effects. We discuss three archetypical technology adoption problems. In the first, which we call "Fixed Lifespan" (FL), players decide to adopt a technology that becomes obsolete after two periods. In the interim period in which late movers have not invested yet, early movers are subject to a contemporaneous network effect that only depends on the mass of early movers. Since the technology of the early movers becomes obsolete while late movers are still using the technology, late movers benefit from a contemporaneous network effect in period 3 that depends only on the mass of late movers. This interpretation

[3] It is used, for example, to model currency crises (Morris and Shin (1998), Corsetti et al. (2004)), bank runs (Goldstein and Pauzner (2003), Rochet and Vives (2002)) and car-dealer markets (Dönges and Heinemann (2000)).

is thus characterized by an early and a late mover cohort effect. In the second interpretation, which we call "Joining a Nascent Club" (NC), players must decide whether to become member of a club. The more players who join the club, the greater its appeal. At time one the club's member base may grow in the future. At time two the club's member base has reached a "mature" level (and will remain constant in the future). For the same reason as above, this interpretation exhibits an early mover cohort effect. For late members, however, the network benefit in any period depends only on the total mass of members (regardless of when the other players joined the club). Hence, this interpretation is void of any late mover cohort effect. In the last interpretation, which we refer to as "Pledging to Invest" (PI), players commit whether or not to invest before the technology becomes available. As the technology becomes available to all players at the same point in time, there is neither an early nor a late mover cohort effect.

We next introduce a condition on the ex-post payoff function called *dynamic increasing differences*. Call both a change from not investing at all to investing at time two and a change from investing at time two to investing at time one, a move to a higher action.[4] Dynamic increasing differences implies that as a higher percentage of the population takes a higher action, it becomes weakly more profitable to take a higher action. For example, it requires that as more players invest late rather than not at all, it becomes weakly more profitable to invest at time one. Our second result shows that a technology adoption problem that exhibits contemporaneous increasing differences does not necessarily exhibit dynamic increasing differences. In particular, dynamic increasing differences requires there to be no late mover cohort effect and it is thus violated by the FL interpretation. The other interpretations, however, satisfy dynamic increasing differences.

Our third result proves that dynamic increasing differences imply the existence of a unique rationalizable outcome. We start by observing that active players who have a "very high" second-period signal always want to invest,[5] since they believe that the fundamental is so good that investing is profitable no matter what actions the other players choose. Now consider a player who has an "extremely high"

[4]This definition is based on an ex-post perspective of the "action space" according to which a player either invested in period one, invested in period two, or did not invest at all. Clearly, the definition is not based on the normal form action space (i.e. the set of pure strategies) nor on the extensive form game as we, for simplicity, ignore the (infinite) set of histories. A rigorous definition of dynamic increasing differences is provided below.

[5]A player is said to be active if she has not invested yet.

first-period signal so that she foresees that her second-period signal will be very high even if she gets bad news. She strictly benefits from investing immediately and saving the waiting costs if she expects no other player to invest.

Now consider a player who has a "high" but not an "extremely high" first-period signal. If she expects no other player to invest in either period, then she would also prefer to refrain from investing. As she possesses a flat prior concerning the realization of the fundamental, it is equally likely that the other players received a higher or lower signal than herself. Therefore, in equilibrium, she cannot expect that no one invests. As her signal is "high," her knowledge that all players with an extremely high signal invest at time one and that all active players with a very high signal invest at time two, induces her to invest at time one as well. Similarly, the knowledge that all players with an extremely high signal invest at time one and that all active players with a very high signal invest at time two, gives active players with a high (but not a very high) signal a strict incentive to invest at time two. This will, in turn, convince players with slightly less favorable signals to also invest, etc... This process of iterative elimination of dominated strategies ends at some cutoff vector $(\overline{k}_1, \overline{k}_2)$. Mirroring the above argument, there is a critical cutoff vector $(\underline{k}_1, \underline{k}_2)$ such that a player refrains from investing in period t whenever she has a signal below $\underline{k}_t$. We next observe that these cutoff vectors give rise to symmetric switching equilibria. In the final step, we exploit the nature of symmetric switching equilibria to show that if the ex-post payoff function satisfies dynamic increasing differences, then the extremal switching equilibria coincide, i.e. $(\overline{k}_1, \overline{k}_2)$ $= (\underline{k}_1, \underline{k}_2)$.

Fourth, we characterize symmetric switching equilibria for a wide range of parameter values. This enables us to illustrate why multiple equilibria can arise if dynamic increasing differences are violated. In essence, if dynamic increasing differences are violated, then players have an incentive to invest at the same point in time at which other players do. If this incentive is strong enough, it gives rise to self-fullfilling expectations according to which some players invest at time two if and only if they anticipate other players to do the same.

This is not the only paper to introduce dynamic elements in a global game. Chamley (1999) studies a dynamic global game in which there is uncertainty about the distribution of investment costs. The distribution of investment costs evolves stochastically through time. Players use the observed previous behavior to update their beliefs about the state of the world. If there is sufficient heterogeneity in the population, each period can be analyzed as a static global game and the equilibrium is unique.[6] Contrary to our paper, there is a new population of players in every period. Thus, players cannot choose when to invest.

Morris and Shin (1999) study the onset of currency crises using a dynamic global game in which the fundamental follows a Markov process. As long as there has been no successful attack, all players choose whether or not to attack in every period. In each period, the past realizations of the fundamental are common knowledge and players observe a private signal regarding its current realization. If the private signal is sufficiently precise, each period can be analyzed as a static global game and the model has a unique equilibrium.[7] In contrast to our model, investments are not irreversible.

Dasgupta (2003) introduces strategic waiting in a global game with irreversible investment. Players can invest in two periods. If a player delays, she observes a noisy signal about the past economic activity at the cost of foregone profits. Dasgupta provides a condition under which his game is characterized by a unique equilibrium within the class of symmetric switching strategies. In his model players wait to benefit from social learning, while in our model players delay to obtain a more precise signal. Furthermore, we allow for cohort effects and do not restrict attention to symmetric switching strategies.

Burdzy, Frankel, and Pauzner (2001) investigate a complete-information dynamic model in which the state evolves stochastically through time. In each period, a continuum of players is randomly matched to play a 2x2 game with strategic complementarities. Under the assumptions that (i) in some states of the world playing

[6]In a static global game set-up, if players observe a public and a private signal and the public signal is sufficiently precise, then multiple equilibria prevail as players can use the public signal as a coordination device (see Hellwig (2002)). In Chamley's model, if the population is sufficiently heterogenous, the inferences players draw based on past observed behavior are less precise. Because the parameter of the distribution of investment costs changes only slowly over time, heterogeneity is needed to rule out equilibria in which players ignore their private signals and use their common *equilibrium knowledge* as a "public signal" to coordinate behavior. A related effect leads to multiple equilibria in Angeletos, Hellwig and Pavan (2003); see Section 6.

[7]Toxvaerd (2002) analyzes merger waves using a similar set-up as Morris and Shin (1999). Maintaining the assumption that all past realizations of the fundamental are common knowledge, Toxvaerd and Giannitsarou (2003) provide a general framework for finite horizon models in which each period can be analyzed as a static global game.

one action is dominant while in others the other is dominant and that (ii) in each period a player has only a small chance of revising her action, they characterize the unique equilibrium.[8] A similar set-up is used by Frankel and Pauzner (2000) to investigate a model of sectoral choice and by Oyama (2003) to analyze economic fluctuations in less developed countries. In contrast to our paper, these papers require that only a small set of players can revise their action at any given point in time and they do not allow for strategic waiting.

Echenique (2004) investigates the set of subgame perfect equilibria in extensive-form dynamic games with strategic complementarities. While his set-up differs from ours, he also observes that static strategic complementarities do not imply dynamic complementarities — although for a different reason.[9]

The remainder of this paper is organized as follows. In section 2, we introduce our formal model. Section 3 relates the parameters of our model to different economic environments. In section 4, we define dynamic increasing differences and show that they imply a unique rationalizable outcome. Section 5 characterizes some symmetric switching equilibria. We demonstrate why strong cohort effects can lead to multiple equilibria in Section 6. Final comments are summarized in section 7. All proofs can be found in the appendix.

2 The model

A continuum of risk-neutral players with mass one have the opportunity to undertake one risky investment project. Investments are irreversible. A player can invest at time one, at time two, or can decide not to invest at all. If player i decides

[8] Levin (2001) adopts this framework to considers a many action game in which players move according to an exogenous sequence. He shows that the equilibrium is unique under a no influence condition, while otherwise there may be multiple equilibria.

[9] Echenique says that an extensive-form game satisfies strategic complementarities, whenever a switch to a higher action following any given history, induces rivals to choose (weakly) higher actions following every history. He observes that extremal outcomes (e.g. cooperation in the infinitely repeated prisoners' dilemma) may rely on "punishments" or taking low actions following certain histories. If a player switches to taking high actions following such punishment histories also (e.g. always cooperates in the prisoners' dilemma independent of past play), then the best reply of his opponent is to sometimes take a lower action (i.e. defect). Thus, even if the static game is one of strategic complementarity, the dynamic game may not be. Note that this argument is based on the observation of past play, which we abstract from.

to invest at time one, she gets a utility U_1^i equal to:

$$U_1^i = \theta + n_1 + \alpha n_2 - 1,$$

where n_1 (n_2) denotes the mass of players who invest at time one (two). The state of the world θ is randomly drawn from a uniform distribution along the entire real line. A period-two investor enjoys a utility equal to:

$$U_2^i = \tau(\theta + \beta n_1 + n_2 - 1 - \Delta).$$

If player i decides not to invest in both periods, she gets zero. Throughout, we assume that $\tau, \alpha, \beta \in [0,1]$ and that $\Delta \geq 0$. We postpone the discussion of the economic motivation for our payoff structure until the next section.

All players possess a private and imperfect signal concerning the realized state of the world. Formally, player i's first-period signal $s_1^i = \theta + \epsilon_2^i + \epsilon_1^i$, and his second period signal $s_2^i = \theta + \epsilon_2^i$. The errors ϵ_2^i are uniformly distributed in the population over the interval $[-\epsilon, \epsilon]$. Half of the population receives an error $\epsilon_1^i = -\epsilon$, and half of the population receives an error $\epsilon_1^i = \epsilon$. Errors ϵ_1^i and ϵ_2^i are independently distributed in the population.

Note that our model possesses some "desirable" features that simplify the characterization of equilibria and enable a direct comparison with the static counterparts of our model. First, note that s_1^i is constructed by adding noise to s_2^i. In statistical terms, this means that s_2^i is a sufficient statistic for s_1^i. In particular, this implies that $E(\theta|s_2^i, s_1^i) = E(\theta|s_2^i)$. Second, we know that $\theta = s_2^i - \epsilon_2^i$. Hence, $\theta|s_2^i \sim U[s_2^i - \epsilon, s_2^i + \epsilon]$, and $E(\theta|s_2^i) = s_2^i$. Third, as illustrated in Figure 1, the first-period signals are also uniformly distributed around θ.

[Insert here Figure 1]

The lower (upper) part of Figure 1 represents the time-two (time-one) distribution of signals. Recall that at time two signals are uniformly distributed between $\theta - \epsilon$ and $\theta + \epsilon$. Suppose $s_2^i \in [\theta, \theta + \epsilon]$. If player i's first period noise equals $-\epsilon$, $s_1^i \in [\theta - \epsilon, \theta]$. If player i's first period noise equals $+\epsilon$, then her first period signal $s_1^i \in [\theta + \epsilon, \theta + 2\epsilon]$. The same logic applies to a player whose second-period signal lies between $\theta - \epsilon$ and θ: Depending on the realization of her first-period noise, she will either lie between $\theta - 2\epsilon$ and $\theta - \epsilon$ or between θ and $\theta + \epsilon$. As the prior probability that $\epsilon_1^i = \epsilon$ equals $\frac{1}{2}$, it follows that $s_1^i \sim U[\theta - 2\epsilon, \theta + 2\epsilon]$. Fourth, one can apply a similar argument to show that $\theta|s_1^i \sim U[s_1^i - 2\epsilon, s_1^i + 2\epsilon]$. Finally, note that $E(\theta|s_1^i) = s_1^i$.

The timing of the game is as follows:
0) Nature chooses θ. All players receive their first-period signals.
1) All players simultaneously decide whether to invest or wait.
2) Player i observes whether $\epsilon_1^i = \epsilon$ or $\epsilon_1^i = -\epsilon$ but not n_1. If she did not invest at time one, she decides whether or not to do so at time two.
3) All players receive their payoffs.

Each player's time-one action space, A^1, equals {invest, not invest}. Player i's time-two action space, A^2, equals {invest, not invest} if a_1^i = not invest, and equals {not invest} if a_1^i = invest. Player i's observable history at time one is $H_1^i = \{s_1^i | s_1^i \in \Re\}$ and at time two is $H_2^i = \{(s_1^i, s_2^i) | s_1^i \in \Re, s_2^i \in \{s_1^i - \epsilon, s_1^i + \epsilon\}\} \times A^1$. Let $\sigma^i = (\sigma_1^i, \sigma_2^i)$ denote player i's behavioral strategy, where $\sigma_1^i(s_1^i)$ represents the probability with which player i invests at time one given her first period signal and $\sigma_2^i(s_1^i, s_2^i)$ represents the probability with which player i invests at time two given (s_1^i, s_2^i) and given that she did not invest in the first period. (Trivially, a player cannot invest in the second period if a_1^i =invest, i.e. if she already invested in the first period.) We denote a strategy profile by σ.

Frequently, we will refer to symmetric switching strategies. A strategy profile is a symmetric switching strategy profile if it can be parameterized by a single vector $k \equiv (k_1, k_2)$ with the interpretation that: (i) $\sigma^i(s_1^i) = 1$ if and only if $s_1^i > k_1$, (ii) $\sigma^i(s_1^i, s_2^i) = 1$ if and only if $s_2^i > k_2$ for all i. An equilibrium in symmetric switching strategies is a k^* such that player i's strategy is a best response at every information set given (i) her beliefs about the state of the world, and given (ii) the equilibrium behavior of all other agents.

For future reference, let

$$h(s_2^i, \sigma) \equiv s_2^i + E(\beta n_1 + n_2 | s_2^i, \sigma) - 1 - \Delta. \tag{1}$$

$h(s_2^i, \sigma)$ is the expected payoff of a player who invests in the second period after getting signal s_2^i, given the strategy profile σ. Similarly, we define

$$W(s_1^i, \sigma) \equiv \frac{\tau}{2} \max\{0, h(s_1^i + \epsilon, \sigma)\} + \frac{\tau}{2} \max\{0, h(s_1^i - \epsilon, \sigma)\}. \tag{2}$$

$W(s_1^i, \sigma)$ denotes the gain of waiting for player i, given her first-period signal s_1^i and given that all other players play according to σ. If player i postpones her investment decision, then with probability 1/2 she will get "bad news," i.e. she will learn that at time one she was too optimistic because $\epsilon_1^i = +\epsilon$. With probability 1/2, however, she will receive "good news" in the sense that she will learn that $\epsilon_1^i = -\epsilon$. Equation (2) states that player i's gain of waiting equals her expected second-period payoff given that she will make an optimal second-period investment decision (i.e. not invest at time two if and only if her gain of investing is negative). For brevity, define

$$g(s_1^i, \sigma) \equiv s_1^i + E(n_1 + \alpha n_2 \mid s_1^i, \sigma) - 1 - W(s_1^i, \sigma). \tag{3}$$

Trivially, it is optimal to invest in the first period for a player with a signal s_1^i (who believes that all her rivals play according to σ) if and only if $g(s_1^i, \sigma) \geq 0$.

3 Economic interpretations

The general payoff structure of our model nests a wide variety of more specific models. We provide three detailed interpretations below.

1 Fixed Lifespan (FL). Suppose players can invest in a new technology of unknown quality. The technology can be used for T periods. For simplicity, players are only allowed to invest in period 1 or period 2 and have a common discount factor δ. Call a player who invests at time one (two) an (a) early (late) adopter. When investing, players need to pay a setup cost $s \geq 0$. The (net of any per-period cost) return of the investment in period t ($t = 1, ..., T$), is given by $v_t^i = \tilde{\theta} + m_t$, where m_t denotes the mass of players who are using the technology at time t. Assume, for the sake of simplicity, that $T = 2$. In this case the payoff of a player investing in period 1 is given by

$$V_1^i = (1+\delta)\tilde{\theta} + (1+\delta)n_1 + \delta n_2 - s,$$

and of a player investing in period 2 is given by

$$V_2^i = \delta(1+\delta)\tilde{\theta} + \delta n_1 + \delta(1+\delta)n_2 - \delta s.$$

Setting $\theta = \tilde{\theta} - \frac{s}{(1+\delta)} + 1$ and $U_t^i = \frac{V_t^i}{(1+\delta)}$ shows that this interpretation is a special case of our model in which $\tau = \delta$, $\alpha = \delta\beta < \beta = \frac{1}{1+\delta} < 1$, and $\Delta = 0$.

To illustrate this interpretation, suppose everyone has the opportunity to buy a video player. The more people who buy a video player, the more videos, video rental stores, etc... become available. A video player can only be used for two periods. At time three a DVD player will be introduced in our economy. As DVD technology is superior to video technology, from time three on, no one wants to buy a new video player anymore. People, however, only switch to the superior DVD technology once their video player becomes "too old" (i.e. early adopters switch to the superior technology at time three, while late adopters switch to the new technology at time four). Whenever $\alpha < 1$ ($\beta < 1$), we say that our model exhibits an early (late) mover *cohort effect*. Hence, our FL interpretation exhibits early and late mover cohort effects. This is intuitive: At time one the early movers do not enjoy any network benefits from the late movers. Therefore early movers care more about the mass of players who adopt the technology at time one than about the mass of players who adopt it at time two (which explains why in this case $\alpha < 1$). Late movers know that the installed base will become smaller at time three because early movers will switch to the new technology. Therefore, $\beta < 1$.

2 Joining a Nascent Club (NC). In this interpretation all players must decide whether or not to join a "secret" club like Freemasonry, the Rosicrucian movement, Opus Dei, etc... Call period one the "start-up" phase. Period two represents the "mature" phase. The larger the club's member base, the more utility it provides to all its members. In period one, the movement is still relatively new and its member base may grow in the future. In period two, the club's member base has reached its mature level and remains constant thereafter. To become a member of the club, players must bear a fixed cost equal to $F \geq 0$.[10] Once someone becomes a member, she must pay a per-period membership fee $s \geq 0$. If a player joins the club at time one, she gets

$$\begin{aligned} V_1^i &= (\tilde{\theta} + n_1 - F - s) + \delta(\tilde{\theta} + n_1 + n_2 - s) + \delta^2(\tilde{\theta} + n_1 + n_2 - s) + ..., \\ &= \frac{1}{1-\delta}(\tilde{\theta} + n_1 + \delta n_2 - s) - F. \end{aligned}$$

If she joins the club at time two, she gets

$$V_2^i = \frac{\delta}{1-\delta}(\tilde{\theta} + n_1 + n_2 - s) - \delta F.$$

[10] For example, people can only join the Rosicrucian movement after having shown to possess enough knowledge of the Bible. Similarly, to become a Freemason, one must also pass a series of tests.

Setting $\tilde{\theta} = \theta + s - 1 + (1-\delta)F$ and $U_t^i = V_t^i(1-\delta)$, the reader can check that this interpretation is a special case of our model in which $\alpha = \tau = \delta < 1$, $\beta = 1$ and $\Delta = 0$.[11] For the same reason as above, this interpretation possesses an early mover cohort effect. However, in this interpretation late members know that they will never suffer from the early members' switching to another club. Therefore, this interpretation is void of any late mover cohort effect.

3 Pledging to Invest (PI). Suppose there are two periods in which players can commit to invest in a future project. For example, firms may commit to buy some land in a soon-to-be developed industrial zone (or individuals may commit to become a member of some club or join a lobbying organization). In the first period, the land is sold at a lower price than in the second period (or there is a reduced membership rate). The more players who invest in either period, the better the infrastructure provided (or the more exciting it will be to visit the club or the more influential the lobbying organization will be). In period 3, all players that committed to invest pay the amount due and start getting the benefit from the planned activity. This can be captured by a model in which $\alpha = \beta = \tau = 1$ and $\Delta > 0$. In this interpretation an early investor (or member) knows that the network externality only depends on the future size of the network. Hence, this interpretation is void of any cohort effects.

While the PI interpretation may seem somewhat special, it is in line with the classical papers on network externalities (e.g. Farrell and Saloner (1985)), which assume that the network effect only depends on the total number of adopters (independent of when each player adopts the technology). The preceding analysis questions the validity of this modeling choice for many environments. In particular, it seems to us that for many technology adoption problems the FL interpretation is more natural. The importance of this modeling choice is shown below.

Our payoff structure also encompasses many other interpretations. For example, suppose consumers must decide whether or not to buy a software program.[12] At time two the producer releases a new version (say, version 2.0) that is partially incompatible with the one sold at time one (say, version 1.0). In this interpretation

[11] Note also that the FL interpretation coincides with the NC one if the number of periods T is infinite and per period profits are constant. In this interpretation, all players decide whether or not to construct a new plant in period one or two. Once build, each plant generates an infinite stream of constant per-period net profits.

[12] We are grateful to Larry Karp for suggesting this interpretation.

cohort effects are driven by the fact that the early and late adopters use different and only partially compatible technologies.

One special feature of our model is that it is void of social learning, i.e. players do not observe past investment decisions. While specific, this assumption is sometimes realistic. For example, the secret organizations we list in our NC interpretation do not divulge the size of their member base. Similarly, imagine our players must decide whether to invest in a foreign country. If FDI statistics are not released, completely unreliable, or only released with a big delay, investors cannot infer how many other firms have (recently) invested and it may be natural to abstract from social learning. The robustness of our insights with respect to the introduction of social learning is discussed below.

4 Dynamic increasing differences and uniqueness

In this section, we define dynamic increasing differences and relate it to our economic interpretations. Finally, we show that dynamic increasing differences imply a unique rationalizable outcome.

From an ex-post perspective, a player either did not invest (which we refer to as action 0), invested in the second period (which we refer to as action a_2), or invested in the first period (which we refer to as action a_1). Think of not investing as the lowest action and investing in the first period as the highest action. Denote the difference in ex-post payoffs between investing in the second period and not investing by

$$\Delta U^i(a_2, 0) \equiv \tau(\theta + \beta n_1 + n_2 - 1 - \Delta),$$

and denote the difference between investing in the first period and investing in the second period by

$$\Delta U^i(a_1, a_2) \equiv \theta + n_1 + \alpha n_2 - 1 - \tau(\theta + \beta n_1 + n_2 - 1 - \Delta).$$

We say that the ex post payoff function exhibits *dynamic increasing differences* if and only if:

$$(i) \quad \frac{\partial \Delta U^i(a_2, 0)}{\partial n_2} = \tau \geq 0,$$

$$(ii) \quad \frac{\partial \Delta U^i(a_1, a_2)}{\partial n_2} = \alpha - \tau \geq 0,$$

$$(iii) \quad \frac{\partial \Delta U^i(a_2, 0)}{\partial n_1} - \frac{\partial \Delta U^i(a_2, 0)}{\partial n_2} = \tau(\beta - 1) \geq 0,$$

$$(iv) \quad \frac{\partial \Delta U^i(a_1, a_2)}{\partial n_1} - \frac{\partial \Delta U^i(a_1, a_2)}{\partial n_2} = 1 + \tau - \tau\beta - \alpha \geq 0.$$

Condition (i) states that as more players invest in the second period, investing in the second period becomes more attractive relative to not investing. This condition is implied by the fact that the contemporaneous payoff function exhibits increasing differences. Condition (ii) requires that if more players invest in period 2, it becomes weakly more profitable to invest early. Intuitively, it implies that as more players invest in the second period, there is no additional gain from switching and investing late rather than early. It thus requires that the inter-period network effect α, which measures the increase in payoff for a player who invests immediately, is no less than the discount factor τ, which measures the increase in payoff for a player who invests late. Condition (iii) states that as more players move from investing late to investing early, it becomes weakly more profitable to invest late rather than not to invest at all. Observe that this condition can only be satisfied in the absence of late mover cohort effects, i.e. if $\beta = 1$. Finally, condition (iv) states that as more investors invest early rather than late, investing early becomes more profitable. This condition is always satisfied. We conclude that our ex-post payoff function exhibits dynamic increasing differences if and only if $\alpha \geq \tau$ and $\beta = 1$. This implies the following observation:

PROPOSITION 1 *The "Joining a Nascent Club" (NC) and the "Pledging to Invest" (PI) interpretations exhibit contemporaneous and dynamic increasing differences. The "Fixed Lifespan" (FL) interpretation exhibits contemporaneous but not dynamic increasing differences.*

We discuss the implications of dynamic increasing differences below.

PROPOSITION 2 *If there are positive waiting costs ($\tau < 1$ or $\Delta > 0$) and if the ex-post payoff function satisfies dynamic increasing differences, there exists an essentially unique rationalizable outcome.*[13]

[13] In static global games, one typically derives the existence of a unique rationalizable equilibrium. In our dynamic game, we only show the existence of a unique rationalizable outcome as one cannot iteratively delete strategies that prescribe different behavior following out-of-equilibrium information sets.

Intuitively, the argument proceeds as follows. Suppose player i did not invest at time one. Then, not investing (at time two) is dominated for her whenever $s_2^i > 1 + \Delta$. Now consider any type who has a signal $s_1^i > 1 + \Delta + \epsilon$. This player foresees that she would want to invest in the second period independent of whether she receives good or bad news. But under the assumption that no other player invests, it is obvious that waiting and investing for sure is dominated by investing immediately and saving the waiting cost. Indeed, understanding that she will invest in the second period if $s_2^i > 1 + \Delta$, a player at time one can calculate her benefit of investing under the assumption that no other player invests in either period, and determine a cutoff level $\overline{s}_1^1$ such that for all higher first-period signals, she prefers to invest immediately. Dynamic increasing differences ensure that investing dominates not investing for all players who receive a signal $s_1^i > \overline{s}_1^1$ or $s_2^i > 1 + \Delta$. The reason is that as other players invest (in either of the two periods), investing early becomes even more attractive relative to investing late, which in turn becomes even more attractive relative to not investing at all. Call $\overline{s}^1 = (\overline{s}_1^1, 1 + \Delta)$.

A rational player anticipates that all other players invest (in one of the two periods) if they receive a sufficiently high signal. Now, consider a player with a second-period signal slightly below $1 + \Delta$ and suppose she has not invested in period one. Because a player with signal $1 + \Delta$ expects *at least* half of the population to invest, a player with a signal close to $1 + \Delta$ strictly prefers to invest. Similarly, a player with a signal $s_1^i = \overline{s}_1^1$ must expect half of the population to invest at time one and perhaps some other players to invest at time two. But as the number of early (and late) investors increase, dynamic increasing differences imply that waiting becomes less desirable. Hence, we can determine a new cutoff vector $\overline{s}^2 = (\overline{s}_1^2, \overline{s}_2^2)$ where both $\overline{s}_t^2$'s are computed such that if $s_1^i = s_1^2$ ($s_2^i = s_2^2$), player i is indifferent between investing and waiting (investing and not investing) given that player i anticipates all other players to invest at time one (two) if their first-period signals are higher than s_1^1 (if they did not invest at time one and if their second-period signals are higher than $1 + \Delta$). Repeating this procedure, we get a decreasing sequence of cutoff vectors. This sequence must converge, as investing is dominated for sufficiently low signals. Furthermore, it must converge to a symmetric switching equilibrium. To see this, note that a player with a signal $\overline{s}_1^\infty$ must be indifferent between investing at time one and waiting knowing that the other players invest at time one (two) if their first-period signals lie above $\overline{s}_1^\infty$ (if they did not invest at time one and if their second-period signals lie above $\overline{s}_2^\infty$).

The reason is that it must be optimal for players with higher signals to invest, and it cannot be strictly optimal for players with slightly lower first-period signals to invest by construction of the sequence. Players with even lower first period signals prefer not to invest if their rivals play according to $(\overline{s}_1^\infty, \overline{s}_2^\infty)$ because they have a lower estimate of the fundamental and expect less players to invest, which makes waiting more desirable. A similar argument ensures that a player with a second-period signal $\overline{s}_2^\infty$ is indifferent between investing and not investing.

For players with sufficiently low signals it is a dominant strategy not to invest, even if they expect all other players to invest. Mirroring the above argument, one can construct an increasing sequence of cutoff vectors below which every player refrains from investing. This sequence also converges to a symmetric switching equilibrium, which we refer to as the lower equilibrium. To complete the proof, we suppose that the iterative elimination from above and below converge to different symmetric switching equilibria and show that this leads to a contradiction. To see the logic underlying the contradiction, observe that a player who receives a signal equal to the first- (second-) period cutoff level is indifferent between investing and waiting (and not investing). Thus, a player who receives a cutoff signal must expect less investment activity in the higher equilibrium as she is more optimistic about the fundamental. For a player who receives a first- or second-period cutoff signal, the expectation of the total number of investors is only a function of $k_1 - k_2$ because all realizations of the fundamental are equally likely. In the higher equilibrium, a player with a signal equal to the second-period cutoff level expects lower investment activity only if less players have already invested in the first period, that is if k_1 is higher relative to k_2. This requires that $k_1 - k_2$ must have a higher value in the higher equilibrium. But a player with a signal equal to k_1 expects a lower level of investment activity only if k_2 is relatively higher, which ensures that she expects less players to invest in the second period. This implies, however, that $k_1 - k_2$ must have a lower value in the higher equilibrium, which establishes the contradiction. Thus, there exists a unique symmetric switching equilibrium and hence a unique rationalizable outcome.

5 Characterization of symmetric switching equilibria

In this section, we characterize some symmetric switching equilibria,[14] which we will use to illustrate how the absence of dynamic increasing differences can lead to multiple equilibria in the following section.

A necessary condition for a strategy profile k^* in which $k^*_t < \infty$ (for $t = 1, 2$) to be an equilibrium (strategy profile) in symmetric switching strategies is that it satisfies the following two equations:

$$g(k^*_1, k^*) = 0, \tag{4}$$

$$h(k^*_2, k^*) = 0. \tag{5}$$

Equation (4), which can be rewritten as

$$k^*_1 + E(n \mid k^*, s^1_i = k^*_1) - 1 = W(k^*, s^1_i = k^*_1),$$

states that a player possessing a first-period signal $s^i_1 = k^*_1$ must be indifferent between investing and waiting. Equation (5) says that a player who receives a second-period signal $s^i_2 = k^*_2$ is indifferent between investing and not investing. In case $k^*_1 = \infty$, equation (4) must be replaced by the condition $g(s^i_1, k^*) \leq 0$, for all s^i_1. That is, it must be optimal to refrain from investing for all first period signals. Similarly, in case $k^*_2 = \infty$, condition (5) must be replaced by the condition $h(k^*_2, k^*) \leq 0$ for all s^i_2.

In general, $h(\cdot)$ and $g(\cdot)$ need not be monotonic. Hence, (4) and (5) are only necessary conditions. Equilibrium also requires that $g(\cdot)$ (respectively $h(\cdot)$) is nonnegative if $s^i_1 > k_1$ (respectively $s^i_2 > k_2$) and nonpositive otherwise. To economize on notation, we will from now on denote equilibrium strategy profiles (and candidate equilibria) by k rather than k^*.

We refer to an equilibrium k in which no player invests in the second period as an *immediate investment equilibrium*. Formally, k is an immediate investment equilibrium if and only if $k_2 \geq k_1 + \epsilon$.

[14] This characterization is complete for a wide range of parameter values including all interpretations given in Section 3. For a formal proof, see the working paper version of this paper.

Proposition 3 *There exists an immediate investment equilibrium if and only if $\Delta \geq -\frac{1}{2} + \epsilon + \frac{3}{4}\beta$. In an immediate investment equilibrium $k_1 = \frac{1}{2}$.*

The parameter condition under which an immediate investment equilibrium exists is intuitive. As the payoff reduction for late movers Δ increases, players have an incentive to move early and thus an immediate investment equilibrium is more likely to exist. As β decreases, a player who deviates in order to invest late enjoys a smaller (inter-period) network effect, which makes deviating less attractive. Hence, as β decreases, an immediate investment equilibrium is more likely to exist. To understand why an increase in ϵ makes it harder to sustain an immediate investment equilibrium, consider a player with a signal $s_1^i = 1/2$. This player is uncertain about whether the fundamental θ is high enough to make her investment profitable. As ϵ increases, more uncertainty about θ is resolved between period one and two, which makes it more desirable to wait in order to receive more information.

To further understand the role of ϵ, it is useful to note that the expected network benefit for a player with a signal $s_1^i = k_1$ equals 1/2 in an immediate investment equilibrium. Intuitively, player i knows that all players possessing a signal higher (lower) than herself invest (do not invest) at time one. Player i asks herself the question: What is the mass of players who received a first-period signal greater than k_1? Player i knows that θ lies in a 2ϵ neighborhood of s_1^i. If $\theta > s_1^i$ ($\theta < s_1^i$), she knows that more than half of the population possess a signal higher (lower) than herself. Given hat $\theta | s_1^i$ is symmetrically distributed around s_1^i, player i knows that the event $\theta > s_1^i$ is as likely to occur as the event $\theta < s_1^i$. Therefore $E(n_1 | s_1^i = k_1, (k_1, \infty)) = 1/2$.

Now, for the sake of argument, suppose there is no inter-period network effect for late movers ($\beta = 0$) and that $\Delta = 0$. Then, an immediate investment equilibrium does not exist whenever $\epsilon > 1/2$. Intuitively, in an immediate investment equilibrium a player with a signal $s_1^i = k_1$ is indifferent between investing and not investing, which is the action she will take if she decides to wait. So her expected payoff must be zero. Furthermore, as discussed above, she expects half of the population to possess a better signal than herself. So her expected gain from the network effect is 1/2. But if $\epsilon > 1/2$, this player could wait, forfeit the expected network effect and only invest if she learns that she was too pessimistic. In this case her expected payoff when getting good news changes by $\epsilon - 1/2$, while her expected payoff when getting bad news remains zero. So if $\epsilon > 1/2$ this is a profitable deviation and an immediate investment equilibrium cannot exist. In general, the more uncertainty about the fundamental is revealed before the second period, the more attractive it becomes to wait, and the less likely it is that an immediate investment equilibrium exists.

We will refer to an equilibrium in which players with high signals invest immediately and players with intermediate signals wait and invest later when receiving good news — but not when receiving bad news — as an *informative waiting equilibrium.* When solving for informative waiting equilibria, it is convenient to slightly relax the definition of symmetric switching equilibria and solve for all symmetric strategy profiles that can be characterized by a vector (k_1, k_2) with the interpretation that (i) $\sigma^i(s_1^i) = 1$ if and only if $s_1^i > k_1$, and (ii) for all $s_2^i < k_1+\epsilon$, $\sigma(s_1^i, s_2^i) = 1$ if and only if $s_2^i > k_2$. We refer to such equilibria as weak symmetric switching equilibria. The difference to our earlier definition is that we only require switching behavior on the equilibrium path. In the out-of-equilibrium event that a player with signal $s_1^i > k_1$ did not invest and gets a signal $s_2^i > k_1 + \epsilon$, we do not solve for this player's optimal behavior explicitly.[15] Formally, an informative waiting equilibrium is a (weak symmetric switching) equilibrium in which $k_1 - \epsilon < k_2 < k_1 + \epsilon$.

For brevity, let $x \equiv 4\epsilon + \beta$ and let

$$D \equiv -16\Delta + 16\epsilon - 8 + 12\beta + [(2-\alpha) - (2-\tau)x]^2,$$

$$\Delta^a \equiv \Delta^b + \frac{1}{16}\,[(2-\alpha) - (2-\tau)x]^2,$$

$$\Delta^b \equiv -\frac{1}{2} + \frac{3}{4}\beta + \epsilon,$$

$$\Delta^c \equiv \frac{\beta}{4}(1+\tau) - \frac{(1+\alpha) + 4\epsilon(1-\tau)}{4}.$$

We are ready to characterize when an informative waiting equilibrium exists.

PROPOSITION 4 *There exists an informative waiting equilibrium* (k_{11}, k_{21}) *if the following three conditions are satisfied: (a)* $\Delta \leq \Delta^a$*, (b) either* $(2-\alpha) > (2-\tau)x$ *or* $\Delta \leq \Delta^b$*, and (c)* $\Delta > \Delta^c$*. In this informative waiting equilibrium*

$$k_{11} = \frac{1}{8}\,\{\tau(\tau-2)x^2 + 2x[1-(1-\alpha)(1-\tau)] + (1-\alpha)^2 + 3 + (x\tau - \alpha)\sqrt{D}\},$$

[15] We, nevertheless, require player i's strategy to be sequentially rational, i.e. she must invest in the second period if and only if it is profitable for her to do so. This optimal behavior may, however, require a player with signal $s_2^i > k_2$ not to invest following an out-of-equilibrium history in which she did not invest when receiving a signal $s_1^i > k_1$.

$$k_{21} = \frac{1}{8}\{-\sqrt{D}(\alpha+8\epsilon-x\tau)+x^2\tau^2+2\tau x(1-\alpha-4\epsilon-x)+(2-\alpha)^2-8(1-\alpha)\epsilon+2\alpha(1+x)+16\epsilon x\}.$$

Furthermore, there exists an informative waiting equilibrium (k_{12}, k_{22}) *if the following three conditions are satisfied: (a)* $\Delta \leq \Delta^a$*, (d)* $(2-\alpha) > (2-\tau)x$*, and (e)* $\Delta > \Delta^b$*. In this informative waiting equilibrium*

$$k_{12} = \frac{1}{8}\ \{\tau(\tau-2)x^2 + 2x[1-(1-\alpha)(1-\tau)] + (1-\alpha)^2 + 3 - (x\tau - \alpha)\sqrt{D}\},$$

$$k_{22} = \frac{1}{8}\{\sqrt{D}(\alpha+8\epsilon-x\tau)+x^2\tau^2+2\tau x(1-\alpha-4\epsilon-x)+(2-\alpha)^2-8(1-\alpha)\epsilon+2\alpha(1+x)+16\epsilon x\}.$$

Conversely, there exists no other informative waiting equilibrium.

To understand under what conditions an informative investment equilibrium exists, suppose first that $(2-\alpha) < (2-\tau)x$, as is the case if the ex post payoff function satisfies dynamic increasing differences. Then, since condition (d) is violated, the (k_{12}, k_{22}) equilibrium does not exist. Next, observe that in this case conditions (a) and (b) are satisfied whenever Δ is too low to sustain an immediate investment equilibrium, i.e. when waiting to act on more information is profitable. The role of condition (c) is to ensure that the relevant decision for a player with signal $s_1^i = k_1$ is whether to wait for good news or whether to invest immediately. If it is violated, the player would prefer to invest in the second period also when getting bad news (which explains why condition (c) gives a lower bound on Δ). Condition (c) is always satisfied when the ex post payoff function exhibits dynamic increasing differences. In that case the only reason to wait is to collect information in order to make a better informed decision. So if a player would prefer to invest when getting bad news, she could invest immediately and save the waiting costs. If cohort effects are such that dynamic increasing differences are violated, however, one may want to wait and invest both when getting good and bad news. In this case a player waits in order to benefit from a higher network effect. Nevertheless, condition (c) is a weak condition, which is also satisfied in the FL interpretation.

We are left to consider the case in which $(2-\alpha) > (2-\tau)x$. Trivially, this implies that conditions (b) and (d) are satisfied. Thus, the equilibrium (k_{11}, k_{21}) exists for all $\Delta \in (\Delta^c, \Delta^a]$ and the equilibrium (k_{21}, k_{22}) exists for all Δ in the nonempty interval $(\Delta^b, \Delta^a]$. Since an immediate investment equilibrium exists for all $\Delta > \Delta^b$, this implies that *if* $(2-\alpha) > (2-\tau)x$*, there exist values of* Δ *for which our model has multiple equilibria* as long as $\Delta^a \geq 0$. A necessary — though not sufficient — condition for $(2-\alpha) > (2-\tau)x$ is that the ex post payoff function violates dynamic increasing differences. Observe first that as cohort effects increase (i.e. α or β decrease), the condition $(2-\alpha) > (2-\tau)x$ is more likely to be satisfied. One interpretation of this fact is that as cohort effects become more important, dynamic coordination becomes more important. A player then only wants to invest if she believes that the other players invest at the same point in time. Second, as τ decreases and players discount future benefits more, the condition is less likely to hold. Third, as the uncertainty ϵ increases, the condition is less likely to hold. Intuitively, as ϵ increases more uncertainty about the fundamental is revealed before the second period. For a player who is unsure about whether she should invest in the first period, it is thus more profitable to wait for additional news and relatively less important to invest when the other players invest. As the coordination aspect becomes relatively less important, multiple equilibria are less likely to exist.

Note that our game is characterized by multiple equilibria *despite* the fact that (i) we only focus on symmetric switching equilibria, (ii) we work with a uniform prior along the entire real line, and (iii) there is no social learning in our model. Hence, the absence of social learning allows us to identify cohort effects as the driving force behind our multiplicity result.

6 Cohort effects and multiplicity

In this section, we provide illustrative examples to discuss why the lack of dynamic increasing differences can lead to multiple equilibria.

Example 1. Consider the FL game with a discount factor of $\delta = 3/5$ and let $\epsilon = 1/64$. Using the normalization introduced in Section 3, one has $\tau = 3/5$, $\alpha = 3/8$, $\beta = 5/8$ and $\Delta = 0$. Since $\Delta > \Delta^b = -1/64$ in this case, Proposition 3 implies that there exists an immediate investment equilibrium. Furthermore, as in addition $\Delta < \Delta^a \approx 0.046$ and $(2-\alpha) > (2-\tau)x$, Proposition 4 implies that two informative waiting equilibria exist in this example.

Indeed, in the above example there exist multiple continuation equilibria for some first-period cutoff levels. To see this, consider the second-period continuation game of the immediate investment equilibrium; i.e. suppose it is common knowledge that all players invested in the first-period if and only if they received a signal $s_1^i > 1/2$. Obviously, there exists a continuation equilibrium in which no player invests as otherwise the immediate investment equilibrium would not exist. We now show that there exists another continuation equilibrium in which an active player invests if and only if she receives a signal $s_2^i > 1/2$. Consider first a player whose $s_2^i = \frac{1}{2}$. Using Lemma 2 (which can be found in the Appendix) it is easy to verify that $E_2(n_1 \mid 1/2, (1/2, 1/2)) = 1/2$ and that $E_2(n_2 \mid 1/2, (1/2, 1/2)) = 3/16$. Player i's gain of investing, given her anticipation that all active players with a $s_2^i \in [\frac{1}{2}, \frac{1}{2}+\epsilon]$ invest at time two, equals

$$h(\frac{1}{2}, (\frac{1}{2}, \frac{1}{2})) = \frac{1}{2} + \frac{5}{8}\frac{1}{2} + \frac{3}{16} - 1 = 0.$$

We next show that $\frac{\partial h(s_2^i, (\frac{1}{2}, \frac{1}{2}))}{\partial s_2^i} > 0$ for all $s_2^i \in [1/2, 1/2+\epsilon]$, thereby proving that it is indeed optimal to invest for all signals above 1/2 in the continuation game. One has

$$\frac{\partial h(s_2^i, (\frac{1}{2}, \frac{1}{2}))}{\partial s_2^i} = 1 + \frac{\beta}{2\epsilon}[n_1(s_2^i + \epsilon, (\frac{1}{2}, \frac{1}{2})) - n_1(s_2^i - \epsilon, (\frac{1}{2}, \frac{1}{2}))]$$

$$+\frac{1}{2\epsilon}[n_2(s_2^i + \epsilon, (\frac{1}{2}, \frac{1}{2})) - n_2(s_2^i - \epsilon, (\frac{1}{2}, \frac{1}{2}))].$$

Using Lemma 2 to substitute all relevant $n_1(\cdot)$'s and $n_2(\cdot)$'s in the above equation, the reader can check that

$$\frac{\partial h(s_2^i, (\frac{1}{2}, \frac{1}{2}))}{\partial s_2^i} \geq 1 + \frac{2\beta - 1}{8\epsilon} > 0 \qquad \forall s_2^i \in [1/2, 1/2+\epsilon].$$

Hence, there also exists a continuation equilibrium in which all active players with a $s_2^i \in [1/2, 1/2+\epsilon]$ invest at time two.

Thus, in Example 1 the continuation game is not a global game. This bears some resemblance with the models of Angeletos, Hellwig and Pavan (2003) and Chamley (1999). Nevertheless, the multiplicity is driven by another force. In the papers mentioned above, all players observe a public signal, which informs them about the state of the world. For example in Angeletos, Hellwig and Pavan players observe that a devaluation has not occurred. From this, they can deduce in equilibrium that devaluing is never a dominant strategy for the central bank. Hence, attacking is never a dominant strategy in the continuation game. In Chamley, players draw inferences about the state of the world based on the outcome induced by past aggregate behavior and their private signals. As the observation of past behavior is shared by all agents, it also acts as a public signal. In particular, if the observation becomes too informative about the state of the world, then Chamley's model has multiple equilibria.[16] Because aggregate past behavior is not observed in our model, the above does not drive the multiplicity result in our model. Moreover, if investments were perfectly reversible as in Angeletos, Hellwig and Pavan, our model would predict a unique rationalizable outcome.[17] Similarly, if there were no endogenous timing decision, as in Chamley's model, we would obtain a unique rationalizable outcome.

The multiplicity in our model comes from the fact that with irreversible investment and strong cohort effects, players have an incentive to invest when other players invest. This gives rise to a coordination problem regarding the timing of investment. Example 2 further illustrates this timing problem.

Example 2. $\beta = 1$; $\alpha = \frac{1}{2}$; $\epsilon = \frac{1}{16}$; $\tau = 0.99$ and $\Delta = 0.315$.

In this example

$$\Delta^b = \frac{1}{4} + \frac{1}{16} = 0.3125 < \Delta = 0.315 < \Delta^a = \Delta^b + \frac{1}{16}((2-\alpha) - (2-\tau)x)^2 = 0.316,$$

and

$$(2-\alpha) - (2-\tau)x \simeq 0.24 > 0.$$

Hence, from Propositions 3 and 4 we know that our example is characterized by multiple equilibria.

As $\beta = 1$, if players follow a cutoff strategy in the first period, then in Example 2 the continuation game is always a well defined global game. More precisely, fix any k_1 and construct an induced game in which nature invests on behalf of a player in the first period if and only if the player receives a signal $s_1^i > k_1$ and which is otherwise identical to the above example. We now argue that this induced game is always dominance solvable. Clearly it is not optimal to invest for very low signals. Iteratively eliminating strategies from below, there will be a unique level $\underline{s}_2^\infty$ below which it is optimal to refrain from investing. In case $\underline{s}_2^\infty \geq k_1 + \epsilon$, iterative elimination of dominated strategies implies that no player wants to invest in the

[16]See the discussion in the Introduction.

[17]To continue to abstract from social learning, one would need to assume that the first-period investment payoffs are only observed after the second-period investment decision.

second period. Hence, consider the case in which $\underline{s}_2^\infty < k_1 + \epsilon$. In this case, a player who receives a signal $s_2^i = \underline{s}_2^\infty$ is indifferent between investing and not investing if she anticipates all other active players with a higher second-period signal to invest. As $\beta = 1$, the profitability of investing late only depends on the state of the world and on the total number of investors in either period. Now consider a player with a higher second-period signal who anticipates all other active players with a higher signal than herself to invest. This player is thus more optimistic about the state of the world and must (weakly) expect more players to invest in either period. Hence, she strictly prefers to invest. In particular, for a player with signal $s_2^i = k_1 + \epsilon$, the condition that all active players with a higher signal than herself invest is vacuous and therefore she strictly prefers to invest. Iterative eliminating strategies from above, hence, implies that all players with a signal above $s_2^i = \underline{s}_2^\infty$ have a strict incentive to invest.

Why then does Example 2 have multiple equilibria? The reason is that as $\alpha < \tau$, dynamic increasing differences are violated, and a player who anticipates that more players wait and invest at time two has a higher incentive to do the same. On the other hand, if she thinks that the other investors are more likely to invest at time one, waiting becomes less attractive. Hence, the lack of dynamic increasing differences gives rise to a coordination problem regarding the timing of investment.

Observe that in both examples, for sufficiently high signals, all undominated strategies must prescribe player i to invest at time one. Similarly, for sufficiently low signals, all undominated strategies must prescribe player i to never invest. There are, however, no signals for which a player — regardless of her rivals' strategies — prefers to wait and invest at time two. This raises the following question: "Is the multiplicity result driven by an insufficiently rich signal structure?" The reasoning behind this question is as follows: In the presence of cohort effects, players face a two-dimensional coordination problem; first, they must decide whether to invest or not, and, in case they decide to invest, they must choose their investment period. Signals, however, are only one-dimensional and one cannot apply a process of iterative deletion of dominated strategies to the second dimension of our coordination problem as there is no player for whom it is dominant to invest at time 2. This reasoning, however, is incomplete.[18] The global game literature derives its results through iterative elimination of dominated strategies — exploiting the

[18]Recall also that if the game satisfies dynamic increasing differences, then there exists a unique equilibrium even though there exists no signal for which it is dominant to invest at time 2.

fact that players have increasing best replies.[19] Admittedly, increasing best replies are not a necessary condition for the introduction of noise to reduce the set of equilibria (see, for example, Guesnerie (1992) and Mason and Valentinyi (2003)), but without them few general results exist and there can be no presumption that equilibrium would be unique if one added another signal on the optimal timing of investment. To fully address this question, however, one would need to solve a dynamic global game with multiple signals. To the best of our knowledge, there exists no paper that investigates the impact of multiple signals in any global game setting, and such an extension is beyond the scope of the paper.

7 Conclusion

We investigated irreversible investment decisions with positive network effects using a dynamic global game approach. In contrast to most papers on global games, we did not focus on determining conditions on the prior distribution and signaling distribution that give rise to a unique equilibrium. Instead, we used a Laplacian prior, a simple signaling technology, and abstracted from social learning. This allowed us to focus on the interaction between positive network effects and irreversible investments. We showed that with irreversible investments positive contemporaneous network effects do not necessarily imply dynamic increasing differences.

Using this fact, we illustrated that in a dynamic setting the global game approach may not give rise to a unique prediction. If dynamic increasing differences are violated, a dynamic coordination aspect arises: Players have an incentive to invest at the same time others do. If this dynamic coordination aspect is strong enough, our global game has multiple equilibria.

If players observe a public signal about past investment activity, results by Hellwig (2002) and Dasgupta (2003) suggest that this reinforces the tendency towards multiplicity in our model. If the public signal is sufficiently imprecise and if payoffs satisfy dynamic increasing differences, however, Dasgupta's work also suggests a uniqueness result within the class of monotone strategies. More research is needed to fully characterize the set of equilibria in dynamic global games with social learning.

[19]See for example Frankel, Morris and Pauzner (2003).

Our results highlight the consequences of ignoring cohort effects if investments are irreversible. Even in the financial sector, reversing ones investment decision is typically costly. We believe an interesting question for future research is how big the impact of such transaction costs are in dynamic models of speculative attacks or other macroeconomic coordination failures. In our model, cohort effects rest on technological factors. We believe that cohort effects also arise in other contexts for different reasons: For instance, a successful speculative attack is more likely to occur when all speculators attack the currency at the same time than if they were to attack the currency peg at different moments in time. Future research may also shed some light on the nature and causes of cohort effects in different economic environments.

Appendix

We start by establishing a few Lemmas and by introducing some notation.

LEMMA 1 $E(n_j|s_1^i,k) = \frac{1}{2}E(n_j|s_2^i = s_1^i + \epsilon, k) + \frac{1}{2}E(n_j|s_2^i = s_1^i - \epsilon, k)\ \forall j = 1,2.$

The above holds trivially and its proof is omitted.

LEMMA 2 *One has*

$$n_1(\theta,k) = \begin{cases} 0 & \text{if } \theta < k_1 - 2\epsilon, \\ \frac{2\epsilon+\theta-k_1}{4\epsilon} & \text{if } k_1 - 2\epsilon \leq \theta < k_1 + 2\epsilon, \\ 1 & \text{if } k_1 + 2\epsilon \leq \theta, \end{cases}$$

and $\forall k_2 \in (k_1 - \epsilon, k_1 + \epsilon)$, *one has*

$$n_2(\theta,k) = \begin{cases} 0 & \text{if } \theta < k_2 - \epsilon, \\ \frac{\epsilon-k_2+\theta}{4\epsilon} & \text{if } k_2 - \epsilon \leq \theta < k_1, \\ \frac{k_1+\epsilon-k_2}{4\epsilon} & \text{if } k_1 \leq \theta < k_2 + \epsilon, \\ \frac{k_1+2\epsilon-\theta}{4\epsilon} & \text{if } k_2 + \epsilon \leq \theta < k_1 + 2\epsilon, \\ 0 & \text{if } k_1 + 2\epsilon \leq \theta. \end{cases}$$

[Insert Figure 2 here]

Proof: Consider Figure 2. The two thick lines represent all realizations of $(\epsilon_2^i, \epsilon_1^i)$. All players who received $\epsilon_1^i = \epsilon$ (respectively $\epsilon_1^i = -\epsilon$) are located on the upper (respectively lower) thick line. Recall that $\Pr(\epsilon_1^i = \epsilon) = \frac{1}{2}$, ϵ_2^i and ϵ_1^i are independently distributed, and ϵ_2^i is uniformly distributed in the population. Hence, the population is uniformly distributed on the two thick lines whose total lengths add up to 4ϵ. The diagonal represents the combination of all $(\epsilon_2^i, \epsilon_1^i)$ such that $\epsilon_1^i + \epsilon_2^i = k_1 - \theta$. Observe that all players who lie to its northeast possess a first-period signal $s_1^i > k_1$.

Denote the points $(-\epsilon,-\epsilon)$ by a, $(\epsilon,-\epsilon)$ by b, $(-\epsilon,\epsilon)$ by c, (ϵ,ϵ) by d, $(k_1-\theta-\epsilon,\epsilon)$ by e, $(k_1-\theta+\epsilon,-\epsilon)$ by e', and $(k_2-\theta,-\epsilon)$ by f. Note that e and e' lie on the diagonal. Furthermore, observe that this diagonal goes through the upper (respectively lower) thick line if and only if $k_1 - 2\epsilon \leq \theta \leq k_1$ (respectively $k_1 \leq \theta \leq k_1 + 2\epsilon$). To compute $n_1(\cdot)$, consider thus the following cases: (i) $\theta < k_1 - 2\epsilon$, (ii) $k_1 - 2\epsilon < \theta < k_1$, (iii) $k_1 < \theta < k_1 + 2\epsilon$, and (iv) $k_1 + 2\epsilon < \theta$. In case (i), e lies to the right of d and hence $n_1(\cdot) = 0$. Case (ii) is illustrated in Figure 2. Here the length of the line between d and e is $2\epsilon + \theta - k_1$ and hence $n_1(\cdot) = \frac{2\epsilon+\theta-k_1}{4\epsilon}$. In case (iii), e' lies between a and b and $n_1(\cdot) = \frac{1}{2} + \frac{\epsilon-k_1+\theta-\epsilon}{4\epsilon} = \frac{2\epsilon+\theta-k_1}{4\epsilon}$. In case (iv), e' lies to the left of a and $n_1(\cdot) = 1$.

We are left to compute $n_2(\cdot)$. A player invests at time two if and only if he lies to the east of the vertical "$k_2 - \theta$" and to the southwest of the diagonal. Note that the vertical "$k_2 - \theta$" cuts the lower thick line if and only if $k_2 - \epsilon \leq \theta \leq k_2 + \epsilon$. The Lemma considers the case in which $k_1 - \epsilon < k_2 < k_1 + \epsilon$. This implies that $k_2 - \epsilon < k_1 < k_2 + \epsilon < k_1 + 2\epsilon$. Consider thus the following five cases: (i) $\theta \leq k_2 - \epsilon$, (ii) $k_2 - \epsilon \leq \theta \leq k_1$, (iii) $k_1 \leq \theta \leq k_2 + \epsilon$, (iv) $k_2 + \epsilon \leq \theta \leq k_1 + 2\epsilon$, and (v) $k_1 + 2\epsilon \leq \theta$. In case (i), $b \leq f$ and thus $n_2(\cdot) = 0$. Case (ii) is illustrated in Figure 2. Since a player invests at time two if and only if he lies in $[f, b]$, one has $n_2(\cdot) = \frac{\epsilon-k_2+\theta}{4\epsilon}$. In case (iii), $a \leq f < e' \leq b$. Thus, $n_2(\cdot) = \frac{1}{4\epsilon}[(k_1 - \theta + \epsilon) - (k_2 - \theta)] = \frac{k_1+\epsilon-k_2}{4\epsilon}$. In case (iv) $f \leq a \leq e' \leq b$ and it is easy to verify that $n_2(\cdot) = \frac{k_1+2\epsilon-\theta}{4\epsilon}$. In case (v) $f < e' \leq a$ and thus $n_2(\cdot) = 0$. Q.E.D.

Lemma 3 *For any k that solves equations (4) and (5) and for which $k_2 \in (k_1 - \epsilon, k_1 + \epsilon)$, one has $h(s_2^i, k) < 0$ if $s_2^i < k_2$ and $h(s_2^i, k) > 0$ if $s_2^i \in (k_2, k_1 + \epsilon)$.*

Proof: Since

$$h(s_2^i, k) = s_2^i + \int_{s_2^i-\epsilon}^{s_2^i+\epsilon} [\beta n_1(\theta, k) + n_2(\theta, k)] d\theta - 1 - \Delta,$$

Leibnitz's rule implies that

$$\frac{\partial h(s_2^i, k)}{\partial s_2^i} = 1 + [\beta n_1(s_2^i + \epsilon, k) + n_2(s_2^i + \epsilon, k) - (\beta n_1(s_2^i - \epsilon, k) + n_2(s_2^i - \epsilon, k))].$$

We have shown in Lemma 2 that $n_1(\cdot)$ is weakly increasing in θ and therefore a sufficient condition for $h(\cdot)$ to be strictly increasing is that

$$n_2(s_2^i + \epsilon, k) \geq n_2(s_2^i - \epsilon, k).$$

By Lemma 2, $n_2(\cdot)$ is weakly increasing in θ for all $\theta \leq k_2 + \epsilon$ and hence $h(s_2^i, k)$ is a strictly increasing function in s_2^i for all $s_2^i \leq k_2$. Since k solves the equations

(4) and (5), $h(k_2, k) = 0$ and we conclude that $h(s_2^i, k) < 0$ if $s_2^i < k_2$.

Next, consider $s_2^i \in (k_2, k_1 + \epsilon)$. Since $h(k_2, k) = 0$, one can rewrite $h(s_2^i, k)$ as

$$h(s_2^i, k) = (s_2^i - k_2) + \beta[E(n_1 \mid s_2^i, k) - E(n_1 \mid s_2^i = k_2, k)] + [E(n_2 \mid s_2^i, k) - E(n_2 \mid s_2^i = k_2, k)].$$

As $s_2^i > k_2$, the first term is positive. Since, by Lemma 2, $n_1(\theta, k)$ is weakly increasing in θ, Leibnitz's rule implies that $E(n_1 \mid s_2^i, k)$ is weakly increasing in s_2^i. Hence $[E(n_1 \mid s_2^i, k) - E(n_1 \mid s_2^i = k_2, k)] \geq 0$. Thus a sufficient condition for $h(s_2^i, k) > 0$ is that

$$[E(n_2 \mid s_2^i, k) - E(n_2 \mid s_2^i = k_2, k)] \geq 0. \tag{6}$$

To prove that condition (6) is satisfied, we establish below that (i) $E(n_2 \mid s_2^i, k)$ is a concave function in s_2^i for all $s_2^i \in (k_2, k_1 + \epsilon)$, and that (ii) $E(n_2 \mid s_2^i = k_1 + \epsilon, k) = E(n_2 \mid s_2^i = k_2, k)$. By Leibnitz's rule,

$$\frac{\partial E(n_2 \mid s_2^i, k)}{\partial s_2^i} = \frac{1}{2\epsilon}[n_2(s_2^i + \epsilon, k) - n_2(s_2^i - \epsilon, k)],$$

and thus

$$\frac{\partial^2 E(n_2 \mid s_2^i, k)}{\partial (s_2^i)^2} = \frac{1}{2\epsilon}[\frac{\partial n_2(s_2^i + \epsilon, k)}{\partial s_2^i} - \frac{\partial n_2(s_2^i - \epsilon, k)}{\partial s_2^i}].$$

Using the facts that $k_2 + \epsilon < s_2^i + \epsilon < k_1 + 2\epsilon$, $k_2 - \epsilon < s_2^i - \epsilon < k_1$, and Lemma 2, it is easy to check that $\frac{\partial^2 E(n_2 | s_2^i, k)}{\partial (s_2^i)^2} = -\frac{1}{4\epsilon^2}$.

We are left to show that $E(n_2 \mid s_2^i = k_1 + \epsilon, k) = E(n_2 \mid s_2^i = k_2, k)$. Using Lemma 2, one has

$$E(n_2 \mid s_2^i = k_1 + \epsilon, k) = \frac{1}{2\epsilon}\int_{k_1}^{k_2+\epsilon} \frac{k_1 + \epsilon - k_2}{4\epsilon} d\theta + \frac{1}{2\epsilon}\int_{k_2+\epsilon}^{k_1+2\epsilon} \frac{2\epsilon + k_1 - \theta}{4\epsilon} d\theta,$$

and

$$E(n_2 \mid s_2^i = k_2, k) = \frac{1}{2\epsilon}\int_{k_2-\epsilon}^{k_1} \frac{\epsilon - k_2 + \theta}{4\epsilon} d\theta + \frac{1}{2\epsilon}\int_{k_1}^{k_2+\epsilon} \frac{k_1 + \epsilon - k_2}{4\epsilon} d\theta.$$

Thus

$$E(n_2 \mid s_2^i = k_1 + \epsilon, k) - E(n_2 \mid s_2^i = k_2, k) = \frac{1}{8\epsilon^2}[\int_{k_2+\epsilon}^{k_1+2\epsilon} (2\epsilon + k_1 - \theta) d\theta - \int_{k_2-\epsilon}^{k_1} (\epsilon - k_2 + \theta) d\theta].$$

Integrating this last expression shows that $E(n_2 \mid s_2^i = k_1 + \epsilon, k) - E(n_2 \mid s_2^i = k_2, k) = 0$. Q.E.D.

Let Σ^0 be the set of all strategies. Let Σ^n be the set of all strategies that are undominated after n rounds of iterative elimination of dominated strategies. Let $\sigma^n \in \Sigma^n$. Let $\underline{s}_t^n(\sigma^n)$ be the supremum below which σ^n prescribes all players to refrain from investing with positive probability at time t. Let $\underline{s}_t^n = \inf\{\underline{s}_t^n(\sigma^n) \mid \sigma^n \in \Sigma^n\}$. Call $\underline{s}^n = (\underline{s}_1^n, \underline{s}_2^n)$ the strategy in which all players invest at time one if and only if $s_1^i > \underline{s}_1^n$ and in which all active players invest at time two if and only if $s_2^i > \underline{s}_2^n$. Let

$$\hat{g}(s_1^i, \hat{\sigma}^n, \sigma^n) \equiv (s_1^i - 1)(1 - \frac{\tau}{2}(I_{\{h(s_1^i-\epsilon,\sigma^n)>0\}} + I_{\{h(s_1^i+\epsilon,\sigma^n)>0\}}))$$

$$+\frac{1}{2}E_2(n_1|\hat{\sigma}^n, s_1^i - \epsilon)(1 - \tau I_{\{h(s_1^i-\epsilon,\sigma^n)>0\}}) + \frac{1}{2}E_2(n_1|\hat{\sigma}^n, s_1^i + \epsilon)(1 - \tau I_{\{h(s_1^i+\epsilon,\sigma^n)>0\}})$$

$$+\frac{1}{2}E_2(n_2|\hat{\sigma}^n, s_1^i - \epsilon)(\alpha - \tau I_{\{h(s_1^i-\epsilon,\sigma^n)>0\}}) + \frac{1}{2}E_2(n_2|\hat{\sigma}^n, s_1^i + \epsilon)(\alpha - \tau I_{\{h(s_1^i+\epsilon,\sigma^n)>0\}})$$

$$-\frac{\tau}{2}(I_{\{h(s_1^i-\epsilon,\sigma^n)>0\}}(-\epsilon - \Delta) + I_{\{h(s_1^i+\epsilon,\sigma^n)>0\}}(\epsilon - \Delta)),$$

where $I_{\{.\}}$ denotes the indicator function, and $\hat{\sigma}^n \in \Sigma^n$. In words, if $\beta = 1$ then $\hat{g}(s_1^i, \hat{\sigma}^n, \sigma^n)$ denotes the difference between player i's gain of investing and her gain of waiting given that all the other players follow strategy $\hat{\sigma}^n$ and given that, at time two, player i decides to invest or not, *under the assumption that all players follow strategy* σ^n *instead of* $\hat{\sigma}^n$. If $\beta = 1$, trivially $\hat{g}(s_1^i, \sigma^n, \sigma^n) = g(s_1^i, \sigma^n)$. We first state and prove the following lemma.

LEMMA 4 *If* $\beta = 1$ *and* $\alpha \geq \tau$, $g(s_1^i, \underline{s}^n) \geq g(s_1^i, \sigma^n)$ $\forall s_1^i$ *and* $\forall \sigma^n \in \Sigma^n$.

Proof: Observe that

$$g(s_1^i, \underline{s}^n) - \hat{g}(s_1^i, \sigma^n, \underline{s}^n) = \frac{1}{2} \sum_{s_2^i \in \{s_1^i - \epsilon, s_1^i + \epsilon\}} \{[E_2(n_1|\underline{s}^n, s_2^i) \tag{7}$$

$$-E_2(n_1|\sigma^n, s_2^i)](1-\tau I_{\{h(s_2^i,\underline{s}^n)>0\}})+[E_2(n_2|\underline{s}^n, s_2^i)-E_2(n_2|\sigma^n, s_2^i)](\alpha-\tau I_{\{h(s_2^i,\underline{s}^n)>0\}})\}.$$

For each s_2^i, define the expression between {...} of the above equation as

$$f(s_2^i, \sigma^n) \equiv (1 - \tau I_{\{h(s_2^i,\underline{s}^n)>0\}})m(s_2^i, \sigma^n) + (\alpha - \tau I_{\{h(s_2^i,\underline{s}^n)>0\}})m'(s_2^i, \sigma^n),$$

where $m(s_2^i, \sigma^n)$ and $m'(s_2^i, \sigma^n)$ are defined as

$$m(s_2^i, \sigma^n) \equiv \frac{1}{2\epsilon} \int_{s_2^i - \epsilon}^{s_2^i + \epsilon} (n_1(\theta, \underline{s}^n) - n_1(\theta, \sigma^n)) d\theta,$$

$$m'(s_2^i, \sigma^n) \equiv \frac{1}{2\epsilon} \int_{s_2^i - \epsilon}^{s_2^i + \epsilon} (n_2(\theta, \underline{s}^n) - n_2(\theta, \sigma^n)) d\theta.$$

As all players with a $s_1^i > \underline{s}_1^n$ invest under $\underline{s}^n$, it follows that $m(s_2^i, \sigma^n) \geq 0$ for all s_2^i and for all σ^n. Hence, if $m'(s_2^i, \sigma^n) \geq 0$, then $f(s_2^i, \sigma^n)$ is positive. Thus suppose that $m'(s_2^i, \sigma^n) < 0$. Then $f(s_2^i, \sigma^n)$ is bounded below by $(1 - \tau I_{\{h(s_2^i, \underline{s}^n) > 0\}})(m(s_2^i, (\sigma_1^n, \underline{s}_2^n)) + m'(s_2^i, (\sigma_1^n, \underline{s}_2^n)))$. Note that

$$m(s_2^i, (\sigma_1^n, \underline{s}_2^n)) + m'(s_2^i, (\sigma_1^n, \underline{s}_2^n)) =$$

$$\frac{1}{2\epsilon} \int_{s_2^i - \epsilon}^{s_2^i + \epsilon} [(n_1(\theta, \underline{s}^n) + n_2(\theta, \underline{s}^n)) - (n_1(\theta, (\sigma_1^n, \underline{s}_2^n)) + n_2(\theta, (\sigma_1^n, \underline{s}_2^n)))] d\theta,$$

which is nonnegative and we conclude that $g(s_1^i, \underline{s}^n) \geq \hat{g}(s_1^i, \sigma^n, \underline{s}^n)$. As $g(s_1^i, \sigma^n)$ prescribes optimal time-two behavior, trivially $\hat{g}(s_1^i, \sigma^n, \underline{s}^n) \geq g(s_1^i, \sigma^n)$, and thus $g(s_1^i, \underline{s}^n) \geq g(s_1^i, \sigma^n)$ $\forall \sigma^n \in \Sigma^n$. Q.E.D.

Let $\overline{s}_t^n(\sigma^n)$ be the infimum above which σ^n prescribes all players to invest at time t with probability 1. Let $\overline{s}_t^n = \sup\{\overline{s}_t^n(\sigma^n) \mid \sigma^n \in \Sigma^n\}$. Call $\overline{s}^n = (\overline{s}_1^n, \overline{s}_2^n)$ the strategy in which all players invest at time one if and only if $s_1^i > \overline{s}_1^n$ and in which all active players invest at time two if and only if $s_2^i > \overline{s}_2^n$.

Proof of Proposition 2:

It follows from Lemma (4) that, conditional on s_1^i, investing in the first period is dominated after $n+1$ rounds of iterative elimination of dominated strategies if and only if $g(s_1^i, \underline{s}^n) < 0$.

Furthermore, $(n_1 + n_2)(\theta, \underline{s}^n) \geq (n_1 + n_2)(\theta, \sigma^n)$ for all $\sigma^n \in \Sigma^n$ because $\underline{s}^n$ prescribes an active player to invest whenever investing is not dominated after n rounds of iterative elimination of dominated strategies. Using this fact, it is easy to check that $h(s_2^i, \underline{s}^n) \geq h(s_2^i, \sigma^n)$ for all $\sigma^n \in \Sigma^n$. Hence, conditional on s_2^i, investing in the second period is dominated after $n+1$ rounds of iterative elimination if and only if $h(s_2^i, \underline{s}^n) < 0$. We conclude that when iteratively deleting dominated strategies from below, we can restrict attention to switching strategies $\underline{s}^n$.

We now show by induction that $\underline{s}_1^n$ and $\underline{s}_2^n$ are increasing sequences. Trivially, $(\underline{s}_1^0, \underline{s}_2^0) = (-\infty, -\infty)$. Because it is a dominant strategy not to invest for sufficiently low first- and second-period signals, $(\underline{s}_1^1, \underline{s}_2^1) >> (\underline{s}_1^0, \underline{s}_2^0)$. We are left to show that, $\underline{s}^{n-2} \leq \underline{s}^{n-1}$, implies that $\underline{s}^{n-1} \leq \underline{s}^n$.[20]

We first show that $\underline{s}_1^{n-1} \leq \underline{s}_1^n$. By definition of $\underline{s}_1^{n-1}$, we know that $g(\underline{s}_1^{n-1}, \underline{s}^{n-2}) = 0$. As $\underline{s}^{n-1} \in \Sigma^{n-2}$, from Lemma (4) we know that $g(\underline{s}_1^{n-1}, \underline{s}^{n-2}) \geq g(\underline{s}_1^{n-1}, \underline{s}^{n-1})$. $\underline{s}_1^n$ cannot be strictly lower than $\underline{s}_1^{n-1}$ because, by definition of $\underline{s}_1^{n-1}$, $\forall s_1^i < \underline{s}_1^{n-1}$ all strategies which prescribe player i to invest at time one with positive probability are dominated ones. As $g(\underline{s}_1^{n-1}, \underline{s}^{n-1}) \leq 0$ and as there exists some $\tilde{s}_1^i$ for which it is a dominant strategy to invest (i.e. for which $g(\tilde{s}_1^i, \underline{s}^{n-1}) > 0$), continuity of $g(\cdot)$ implies that there exists $\underline{s}_1^n$ such that $g(\underline{s}_1^n, \underline{s}^{n-1}) = 0$.

Next, we show that $\underline{s}_2^{n-1} \leq \underline{s}_2^n$. It is obvious that $E_2(n_1 + n_2 | s_2^i, \underline{s}^{n-2}) \geq E_2(n_1 + n_2 | s_2^i, \underline{s}^{n-1})$. Therefore,

$$\underline{s}_2^{n-1} + E_2(n_1 + n_2 | \underline{s}_2^{n-1}, \underline{s}^{n-1}) - 1 - \Delta \leq \underline{s}_2^{n-1} + E_2(n_1 + n_2 | \underline{s}_2^{n-1}, \underline{s}^{n-2}) - 1 - \Delta = 0$$

This implies that $\underline{s}_2^{n-1} \leq \underline{s}_2^n$ because $s_2^i + E_2(n_1 + n_2 | s_2^i, \underline{s}^{n-1}) - 1 - \Delta$ is strictly increasing in s_2^i and by definition

$$\underline{s}_2^n + E_2(n_1 + n_2 | \underline{s}_2^n, \underline{s}^{n-1}) - 1 - \Delta = 0.$$

Hence, as $n \to \infty$, $\underline{s}^n$ converges to some cutoff vector $\underline{s}$ that satisfies $g(\underline{s}_1, \underline{s}) = 0$ and $h(\underline{s}_2, \underline{s}) = 0$. Using reasoning that mirrors the one for $\underline{s}^n$, shows that $\overline{s}^n$ is a decreasing sequence that, as $n \to \infty$, converges to a cutoff vector $\overline{s}$ that satisfies $g(\overline{s}_1, \overline{s}) = 0$ and $h(\overline{s}_2, \overline{s}) = 0$. Observe that, by construction of $\overline{s}$ and $\underline{s}$, $\underline{s}_1 \leq \overline{s}_1$ and $\underline{s}_2 \leq \overline{s}_2$.

We are left to show that $\underline{s} = \overline{s}$. Suppose otherwise. Both $\underline{s}$ and $\overline{s}$ solve the following system of equations.

$$g(k_1, k) = 0, \tag{8}$$

$$h(k_2, k) = 0. \tag{9}$$

First, observe that if $\underline{s}_1 = \overline{s}_1$ then $\underline{s}_2 = \overline{s}_2$ because $h(k_2, (k_1, k_2))$ is strictly increasing in k_2. Thus, $\underline{s}_1 < \overline{s}_1$. $\underline{s}_2$ and $\overline{s}_2$ must be chosen such that $h(\underline{s}_2, \underline{s})$ and

[20]When comparing two vectors, we use $\leq$ to indicate that for all i, the ith component of the first vector is $\leq$ to the ith component of the second vector.

$h(\overline{s}_2, \overline{s}) = 0$, which implies that

$$(10) \qquad (\overline{s}_2 - \underline{s}_2) = E_2(n_1 + n_2|\underline{s}_2, \underline{s}) - E_2(n_1 + n_2|\overline{s}_2, \overline{s})$$

To gain some insight about $E_2(n_1+n_2|\cdot)$'s consider the following two pair of cutoffs (k_1', k_2) and (k_1'', k_2). Both strategies possess the same second-period cutoffs but suppose without loss of generality that $k_1' < k_1''$. Consider any arbitrary (s_1^i, s_2^i). Clearly, if player i invests in either period under strategy (k_1'', k_2), she also invests under strategy (k_1', k_2). Hence, $E_2(n_1 + n_2|k_2, (k_1, k_2))$ is weakly increasing in $k_2 - k_1$. Hence a necessary condition for for equation (10) to hold is that

$$(11) \qquad \overline{s}_2 - \overline{s}_1 < \underline{s}_2 - \underline{s}_1.$$

Now consider a player whose $s_2^i = k_1 + \epsilon$. Consider two different second-period cut-off levels k_2' and k_2'', and suppose without loss of generality that $k_2' < k_2''$. Consider any $s_1^i < k_1$. Clearly, if player i invests at time two under (k_1, k_2''), she will also do so under (k_1, k_2'). Hence, $E_2(n_2|k_1 + \epsilon, (k_1, k_2))$ is weakly decreasing in $k_2 - k_1$. The same logic can be applied to $E_2(n_2|k_1 - \epsilon, (k_1, k_2))$. Thus, it follows from (11) that

$$(12)\; E_2(n_2|\overline{s}_1 + \epsilon, \overline{s}) \geq E_2(n_2|\underline{s}_1 + \epsilon, \underline{s}) \text{ and } E_2(n_2|\overline{s}_1 - \epsilon, \overline{s}) \geq E_2(n_2|\underline{s}_1 - \epsilon, \underline{s}).$$

We also know that $\underline{s}_1$ and $\overline{s}_1$ must be chosen such that $g(\underline{s}_1, \underline{s}) = g(\overline{s}_1, \overline{s}) = 0$. Note that

$$g(\overline{s}_1, \overline{s}) - \hat{g}(\underline{s}_1, \underline{s}, \overline{s}) = (\overline{s}_1 - \underline{s}_1)(1 - \frac{\tau}{2} I_{\{h(\overline{s}_1+\epsilon,\overline{s})>0\}} - \frac{\tau}{2} I_{\{h(\overline{s}_1-\epsilon,\overline{s})>0\}})$$
$$+\frac{1}{2}(E_2(n_2|\overline{s}_1 + \epsilon, \overline{s}) - E_2(n_2|\underline{s}_1 + \epsilon, \underline{s}))(\alpha - \tau I_{\{h(\overline{s}_1+\epsilon,\overline{s})>0\}})$$
$$+\frac{1}{2}(E_2(n_2|\overline{s}_1 - \epsilon, \overline{s}) - E_2(n_2|\underline{s}_1 - \epsilon, \underline{s}))(\alpha - \tau I_{\{h(\overline{s}_1-\epsilon,\overline{s})>0\}}).$$

From (12) and from the fact that $\overline{s}_1 > \underline{s}_1$, it follows that $g(\overline{s}_1, \overline{s}) > \hat{g}(\underline{s}_1, \underline{s}, \overline{s})$. As $\hat{g}(\underline{s}_1, \underline{s}, \overline{s}) \geq g(\underline{s}_1, \underline{s})$ it follows that, under (11), $g(\overline{s}_1, \overline{s}) > g(\underline{s}_1, \underline{s})$, a contradiction. Q.E.D.

Proof of Proposition 3:

In an immediate investment equilibrium no player invests in the second period. Hence,

$$h(s_2^i, k) = s_2^i + \beta E(n_1|s_2^i, k) - 1 - \Delta.$$

It follows from the derivation of $n_1(\theta, k)$ in Lemma 2 that $n_1(\theta, k)$ is weakly increasing in the fundamental θ. Thus,

$$E(n_1|s_2^i, k) = \frac{1}{2\epsilon} \int_{s_2^i-\epsilon}^{s_2^i+\epsilon} n_1(\theta, k)\ d\theta$$

is weakly increasing in s_2^i, and hence $h(s_2^i, k)$ is strictly increasing in an immediate investment equilibrium. Therefore, there exists a unique k_2 such that $h(s_2^i, k) \leq 0$ if and only if $s_2^i \leq k_2$. By definition, we look for an equilibrium in which $k_2 \geq k_1+\epsilon$, which implies that $h(s_2^i = k_1+\epsilon, k) \leq 0$. Hence, the gain of waiting must be equal to zero. Therefore, k_1 must be set such that a player who possesses a signal $s_1^i = k_1$ is indifferent between investing and not investing. Thus k_1 solves the following equation

$$k_1 + E(n_1 \mid s_1^i = k_1, k) - 1 = 0.$$

Using the function $n_1(\theta, k)$, derived in Lemma 2, and the fact that

$$E(n_1 \mid s_1^i = k_1, k) = \frac{1}{4\epsilon} \int_{k_1-2\epsilon}^{k_1+2\epsilon} n_1(\theta, k)\ d\theta,$$

it is easy to verify that $E(n_1 \mid s_1^i = k_1, k) = \frac{1}{2}$. Thus, in an immediate investment equilibrium $k_1 = \frac{1}{2}$. Using this fact to rewrite the condition that no player has an incentive to invest in the second period, i.e. that $h(s_2^i = k_1 + \epsilon, k) \leq 0$, gives

$$\frac{1}{2} + \epsilon + \beta E(n_1 \mid s_2^i = k_1 + \epsilon, k) \leq 1 + \Delta. \tag{13}$$

Similarly, using $n_1(\theta, k)$ and the fact that

$$E(n_1 \mid s_2^i = k_1 + \epsilon, k) = \frac{1}{2\epsilon} \int_{k_1}^{k_1+2\epsilon} n_1(\theta, k)\ d\theta,$$

it is easy to verify that $E(n_1 \mid s_2^i = k_1 + \epsilon, k) = \frac{3}{4}$. Substituting this into equation (13) and rewriting yields $\Delta \geq -\frac{1}{2} + \epsilon + \frac{3}{4}\beta$, which is a necessary condition for an immediate investment equilibrium to exist. Because we already established that $h(s_2^i, k)$ is strictly increasing, it suffices to show that $g(\cdot)$ is (weakly) increasing to show that an immediate investment equilibrium exists whenever $\Delta \geq -\frac{1}{2} + \epsilon + \frac{3}{4}\beta$. First, observe that for all $s_1^i < k_2 - \epsilon$, one has

$$g(s_1^i, k) = s_1^i + E(n_1 \mid s_1^i, k) - 1,$$

which is strictly increasing in s_1^i because $E(n_1 \mid s_1^i, k)$ is weakly increasing in s_1^i. Second, for all $k_2 - \epsilon < s_1^i < k_2 + \epsilon$,

$$g(s_1^i, k) = s_1^i + E(n_1 \mid s_1^i, k) - 1 - \frac{\tau}{2} h(s_1^i + \epsilon, k).$$

Using Lemma (1) and equation (1), one can rewrite the above equation as

$$g(s_1^i, k) = (1-\frac{\tau}{2})s_1^i + \frac{1}{2}[E(n_1 \mid s_2^i = s_1^i - \epsilon, k) + (1-\tau\beta)E(n_1 \mid s_2^i = s_1^i + \epsilon, k) - \tau\epsilon - (2-\tau) + \tau\Delta].$$

Since $E(n_1 \mid s_2^i = s_1^i - \epsilon, k)$ and $E(n_1 \mid s_2^i = s_1^i + \epsilon, k)$ are weakly increasing in s_1^i, and $\tau, \beta \leq 1$, $g(s_1^i, k)$ is strictly increasing in s_1^i in this subcase. Third, for all $k_2 + \epsilon < s_1^i$, one has

$$g(s_1^i, k) = s_1^i + E(n_1 \mid s_1^i, k) - 1 - \frac{\tau}{2}[h(s_1^i - \epsilon, k) + h(s_1^i + \epsilon, k)].$$

Rewriting this equation using Lemma (1) and equation (1) yields

$$g(s_1^i, k) = (1-\tau)s_1^i + \frac{(1-\tau\beta)}{2}[E(n_1 \mid s_2^i = s_1^i - \epsilon) + E(n_1 \mid s_2^i = s_1^i + \epsilon)] - (1-\tau) + \tau\Delta.$$

Since $E(n_1 \mid s_2^i = s_1^i - \epsilon)$ and $E(n_1 \mid s_2^i = s_1^i + \epsilon)$ are weakly increasing in s_1^i, and $\tau, \beta \leq 1$, $g(s_1^i, k)$ is weakly increasing in s_1^i in this subcase. Q.E.D.

Proof of Proposition 4:

Rewriting (4) and (5) using the fact that $k_1 - \epsilon < k_2 < k_1 + \epsilon$ in an informative waiting equilibrium gives

$$k_1 + \frac{1}{2} + (\frac{\alpha}{8\epsilon})(k_1 + \epsilon - k_2) - 1 - \frac{\tau}{2}\{k_1 + \epsilon + \frac{3\beta}{4} + \frac{1}{16\epsilon^2}(k_1 + \epsilon - k_2)(k_2 + 3\epsilon - k_1) - 1 - \Delta\} = 0,$$

$$k_2 + \beta\{\frac{1}{4} + \frac{1}{4\epsilon}(k_2 + \epsilon - k_1) + \frac{1}{16\epsilon^2}(k_1 + \epsilon - k_2)(k_2 + 3\epsilon - k_1)\} - 1 - \Delta = 0.$$

Thus, (4) and (5) are a pair of quadratic equations, which is equivalent to a fourth order polynomial. Hence, there exists a routine procedure to solve this system of equations. Using mathematica to solve this system of equations shows that there are only two pair of roots (k_{11}, k_{21}) and (k_{21}, k_{22}). Rewriting, gives the expressions given in the proposition above. Because (4) and (5) are necessary conditions for an equilibrium, all informative waiting equilibria ar either of the form (k_{11}, k_{21}) or (k_{21}, k_{22}).

Observe that all roots are real if and only if $D \geq 0$. This requires that

$$16\epsilon - 8 + 12\beta + [(2-\alpha) - (2-\tau)x]^2 \geq 16\Delta.$$

Rewriting gives condition (a).

(k_{11}, k_{21}) is a valid solution only if $k_{11} - \epsilon < k_{21} < k_{11} + \epsilon$, because otherwise the functional form of (4) and (5) would differ from the one used above. That is, we require that (i) $-\epsilon < k_{11} - k_{21}$ and that (ii) $k_{11} - k_{21} < \epsilon$. Using the fact that

$$k_{11} - k_{21} = \epsilon[1 - \alpha - (2 - \tau)x + \sqrt{D}], \tag{14}$$

condition (i) holds if and only if

$$(2 - \tau)x - (2 - \alpha) < \sqrt{D}.$$

Note that this inequality is satisfied if either $(2 - \alpha) > (2 - \tau)x$ or if

$$[(2 - \tau)x - (2 - \alpha)]^2 < -16\Delta + 16\epsilon - 8 + 12\beta + [(2 - \alpha) - (2 - \tau)x]^2.$$

Rewriting gives condition (b).

Using $k_{11} - k_{21} = \epsilon[1 - \alpha - (2 - \tau)x + \sqrt{D}]$, to rewrite condition (ii) gives

$$\sqrt{D} < \alpha + (2 - \tau)x.$$

Squaring this inequality on both sides and rewriting yields

$$-16\Delta + 16\epsilon + 12\beta - 4(1 + \alpha) - 4x(2 - \tau) < 0,$$

which is equivalent to condition (c) in the proposition. Hence, conditions (a), (b), and (c) are necessary conditions for (k_{11}, k_{21}) to characterize an equilibrium.

Similarly, (k_{12}, k_{22}) is a valid solution only if both (i) $-\epsilon < k_{12} - k_{22}$ and (ii) $k_{12} - k_{22} < \epsilon$ hold. Using the fact that $k_{12} - k_{22} = \epsilon[1 - \alpha - (2 - \tau)x - \sqrt{D}]$, condition (i) holds if and only if $\sqrt{D} < (2 - \alpha) - (2 - \tau)x$. Hence, condition (i) requires that $(2-\alpha) > (2-\tau)x$, which is condition (d) in the proposition, and that $D < [(2-\alpha)-(2-\tau)x]^2$, which is equivalent to condition (e) in the proposition. We conclude that conditions (a), (d) and (e) are necessary conditions for (k_{12}, k_{22}) to characterize an equilibrium. (Note also that $k_{12}-k_{22} = \epsilon[1-\alpha-(2-\tau)x-\sqrt{D}] < \epsilon$.) Hence, we have established that no other informative waiting equilibrium than the ones characterized in the proposition exist. To show that (k_{11}, k_{21}) and (k_{12}, k_{22}) are indeed equilibria under the above conditions, we are left to verify that (i) $h(s_2^i, k) < 0$ for all $s_2^i < k_2$, (ii) $h(s_2^i, k) > 0$ $\forall s_2^i \in (k_2, k_1 + \epsilon]$, and that (iii) $g(s_1^i, k) < 0$ if and only if $s_1^i < k_1$. Conditions (i) and (ii) follow from Lemma (3). The proof of Condition (iii) is available upon request. Q.E.D.

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Figure One: Time-one distribution of signals

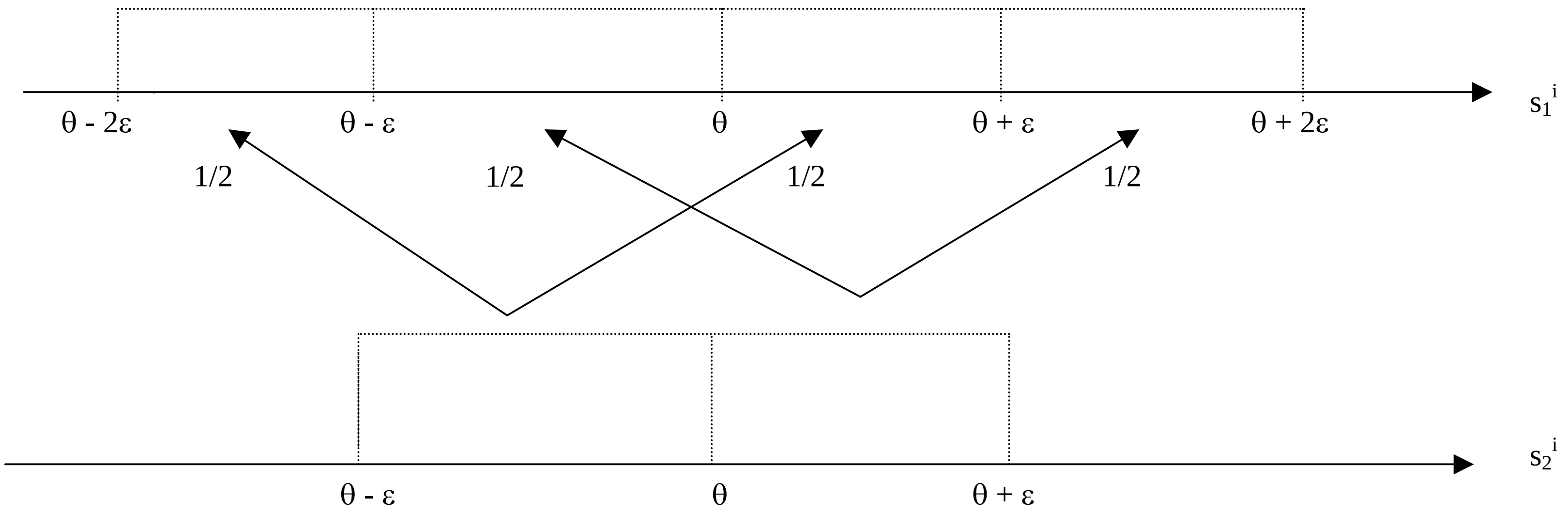

Figure 2: Graphical representation of the mass of players investing in the two periods as a function of (k_1,k_2).