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ABSTRACT

Income Distribution and Demand-Induced Innovations*

We utilize Schmookler's (1966) concept of demand-induced invention to study the role of income inequality in an endogenous growth model. As rich consumers can satisfy more wants than poor consumers, both prices and market sizes for new products, as well as their evolution over time, are determined by the income distribution. We show how a change in the distribution of income affects the incentive to innovate and hence long-run growth. In general, less inequality tends to discourage the incentive to innovate, but this depends on the nature of the redistribution.

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"Does man simply invent what he can, so that the inventions he makes in any period are essentially those which became possible in the previous period? Or is it to man's wants with their different and changing intensities, and to economic phenomena associated with their satisfaction, that one must primarily look for the explanation? In short, *are inventions mainly knowledge-induced or demand-induced?"*

Jacob Schmookler (1966), Invention and Economic Growth, p.12.

1 Introduction

In his seminal book "Invention and Economic Growth," Schmookler (1966) emphasizes the importance of "demand-induced inventions" – the fact that an invention requires not only pre-existing knowledge but also *a sufficiently urgent want that consumers seek to satisfy.* Demand-induced inventions have not received much attention in recent theories of innovation and growth.[1] In these theories, it is typically assumed that each potential innovation is equally useful and the demand-side plays a passive role.

In this paper, we present an endogenous growth model which puts emphasis on the demand-side. By elaborating Schmookler's concept of "demand-induced inventions" we study how the distribution of income affects the process of innovation and long-run growth. Our starting point is that (i) the various wants are *not equally urgent* and that (ii) it depends on *the level of a consumer's income* which want he or she is able to satisfy. In terms of Schmookler's introductory question, the intensity of a particular want *changes* over time as economic growth increases the consumers' willingness to pay for this want's satisfaction; and this intensity will *differ* across consumers as the rich have a higher willingness to pay than the poor.

To capture the idea that some wants are more urgent than others, we introduce the concept of *hierarchic preferences* which ranks the various wants by their priority. In this hierarchy, the most basic needs are ranked first and the more luxurious wants are ranked behind. As the number of currently satisfiable wants is limited by the number of existing goods, it requires innovations to satisfy additional wants. In other words, the scope for innovations consists of the list of those currently unsatisfiable wants which can conceivably be met by technical means.

[1] In line with Schmookler (and unlike Schumpeter), we use the terms "invention" and "innovation" synonymously.

The direction of actual innovation is determined by the relative urgency of these wants.

In such a framework of analysis, the distribution of income plays a crucial role for the evolution and profitability of new markets, and thus for long-run growth. To see how the mechanism works, consider the dynamics of demand and profits for a firm after a successful innovation. This market is initially small as only rich consumers purchase a new product. With growing incomes, the extent of the market expands and firms earn higher profits. The growth in profits is either due to the *intensive* margin (old consumers are willing to pay a higher price) or to the *extensive* margin (additional consumers are willing to purchase at the same price). Hence the income distribution affects the value of an innovation in a complex way by shaping an innovator's demand curve and shifting it in the growth process.

While our analysis holds for a general distribution of income, the intuition is most easy to grasp in the context of a two-class society. With two classes, the distribution of income is determined by two parameters: relative incomes and relative population sizes of rich and poor. How does inequality affect the incentive to innovate? Assume there is less income inequality due to a *lower relative income* of rich to poor. This has two opposing effects on innovation incentives. On the one hand, such a redistribution reduces the riches' willingness to pay and the innovator's profits – as long as the new product is sold exclusively to the rich. On the other hand, such a redistribution makes the poor better off and allows them to afford more goods. This has a favorable effect on innovators' profits as the market for a new product will develop more quickly into a mass market. We show that, on net, the former effect always dominates the latter. This is because profits become more *"backloaded":* The profit flow is lower early in the life cycle and higher later on. Due to discounting, the early fall in profits outweighs the later increase and the value of an innovation decreases. In sum, because of more backloaded profits, lower relative incomes have a negative effect on the incentive to innovate which reduces growth.

When a more even distribution comes from a *larger population size of the rich*, and each rich class member has a lower income (i.e. incomes are "less concentrated" in the hands of a few rich). Such a change in the distribution affects the incentive to innovate through a *"market size effect"* and a *"price effect".* The market size effect has a positive impact on the profit flow, because there are more individuals purchasing the new good right from the start-up of the business. The price effect goes in the opposite direction. As the willingness to pay for a new product decreases with a less wealthy rich class, innovators are forced to charge lower prices. The relative size of these two effects clearly depends on the scope of price setting by innovators. When no appropriate substitutes for the innovative products exist, the price effect dominates the market size effect. However, if there are such substitutes the scope for price setting by innovators is more limited and the market size effect becomes dominant. More equality discourages innovative activities in the former case, but fosters it in the latter.

The paper is organized as follows. Section 2 provides a brief review of related literature. In Section 3 we specify our crucial assumptions on consumers' preferences and discuss consumers' optimal choices in that context. Section 4 presents our assumptions on the income distribution and Section 5 discusses technology and the firms' price setting behavior. Section 6 analyzes a unique balanced growth equilibrium and analyzes the relationship between inequality and growth. Section 7 discusses other equilibria. Section 8 allows for a non-innovative sector and discusses the implications for the inequality-growth relation. In Section 9 we look at the robustness of our results with respect to the basic assumptions. Section 10 concludes.

2 Related literature

At a general level, our paper is related to models that emphasize the importance of market size and profit incentives for innovative activities which is central in R&D based growth models (Aghion and Howitt, 1992, Grossman and Helpman, 1991, Romer, 1990, Segerstrom et al, 1990). However, in these models unsatisfied wants are all alike, so the issue of income inequality and demand-induced innovations does not arise in any interesting way. In other words, these models typically assume homothetic preferences implying that income distribution does not play any role for the market demand functions of innovating firms. In our model, preferences are non-homothetic so changes in the distribution of income affect patterns of demand and, in particular, the dynamics of demand for new goods.

Our paper is also related to the recent literature on inequality and growth. However, the mechanism that gives rise to such a relationship is quite different from the ones that have been discussed. The literature has either focused on the role of capital market imperfections (see Galor and Zeira (1993), Banerjee and Newman (1993), Aghion and Bolton (1997), Galor and Tsiddon (1997), Galor and Moav (2003), and many others) or on political mechanisms (Bertola (1993), Persson and Tabellini (1994), Alesina and Rodrik (1994), and others) or on a combination of these (see papers by Bénabou (1996, 2004)). In contrast, our model does neither rely on capital market imperfections nor on politico-economic mechanisms but emphasizes the interaction of product market power and distribution-determined demand for innovative products.

Murphy, Shleifer, and Vishny (1989) have studied the role of distribution of income for the adoption of modern technologies when consumers have hierarchic preferences. While their formulation of hierarchic preferences corresponds to a special case of our model, they analyze the role of income distribution in a static (development) context. There is no scope for price setting of new firms in their model, so any effect of income distribution is transmitted via its effect on market size. In contrast, we focus on a dynamic (endogenous growth) context and study the evolution of prices and market size for innovators.[2]

In another recent paper, Matsuyama (2002) discusses the interdependence between growth and distribution under hierarchic preferences. In his model the distribution of income determines market demand in sectors where learning and technical progress is possible. Growth may initially benefit only the rich but, depending on the income distribution, may ultimately trickle down to the poor. In Matsuyama (2002), growth results from learning-by-doing, i.e. a by-product of production on otherwise perfect output markets. In contrast, in our model growth is driven by industrial R&D and income distribution affects growth due to its impact on incentives to engage in innovative activities.

Greenwood and Mukoyama (2001) do address the problem of how the size distribution of income affects innovation incentives. Their focus, however, is a partial equilibrium one in which a durable goods monopolist chooses the optimal timing of an innovation given that consumers with unequal incomes have different starting dates of purchasing the product. In contrast, our model is embedded into an general-equilibrium framework, in which aggregate factor prices and the growth rate are itself determined by the income distribution. This allows us to address the question of how income inequality affects long-run growth.

Saint-Paul (2004) investigates distribution and growth in the context of a dynamic mo-

[2]There is a small literature that emphasizes the role of demand for innovation incentives (Falkinger, 1990, 1994, Chou and Talmain, 1996, Li, 1996, Zweimüller, 2000, Glass, 2001, and Zweimüller and Brunner, 2004). However, none of these papers focuses on the role of inequality on the structure of prices and the exclusion of poor consumers from the innovator's markets – issues which are central to our model of income distribution and demand-induced inventions.

nopolistic competition model when consumers have non-homothetic preferences. His paper is based on the representative-agent assumption and studies the role of growth (productivity versus variety/creativity) on the distribution of income among factors of production. In contrast, we assume heterogenous consumers and address the opposite chain of causality, that is, how distribution affects growth. Bourguignon (1990) studies the role of demand for growth and distribution under rather general assumption on demand behavior. He shows conditions on price and income elasticities under which growth enhances (reduces) inequality. In contrast, our paper studies the opposite direction of causality and asks how inequality affects growth.

A further related literature is concerned with "directed technical change" (Kennedy, 1964, Acemoglu, 1998, 2002, 2003). These papers put emphasis on the incentives to adopt particular technologies and the consequences of these technology choices for the distribution of income among factors of production. In contrast, the heterogeneity in our model occurs on the preference side rather than on the supply side. This allows us to study the *effect of* an unequal distribution of income on the incentives to innovate.

The empirical literature on demand-induced technical change starts with Schmookler (1966) who finds a roughly proportional relationship between sales and successful patent applications. Most of the more recent empirical research has focused on the pharmaceutical industry. Kremer (2001a, 2001b) shows in various papers why research on vaccines for Malaria, tuberculosis, and the strains of HIV is so minimal - despite the fact that so many individuals in poor countries suffer from these diseases. His main explanation relies on the demand side: Potential vaccine developers fear that they would not be able to sell enough vaccine at a sufficient price to recoup their research expenses.[3] Finkelstein (2004) also provides evidence that investment in vaccines research responds strongly to policy-induced changes in expected revenues. Acemoglu and Linn (2004) investigate the effect of potential market size on innovation of new drugs by looking at demographic changes. They find substantial effects: an increase in the potential market by 1 % for a given drug category increases the number of drugs in that category by about 5 %.[4]

[3]An additional explanation for this result lies in the difficulty to enforce property rights on medicaments in developing countries.

[4]Further evidence supportive for the relevance of our analysis comes from studies that emphasize the importance of income distribution for demand patterns and market sizes. Looking at consumption patterns in Germany, Bonus (1973) finds that ownership of consumer durables, such as cars, cameras, televisions, and refrig-

3 The demand side

3.1 Preferences

Consider an economy with many potentially producible differentiated products indexed by $j \in [0, \infty)$.[5] Consumers' preferences are 'hierarchic' in the sense that the goods are ranked according to their priority in consumption. Goods with a low index have high priority, goods with a higher index have lower priority. We assume separability and model the hierarchy as follows. Consuming $c(j)$ units of good j yields utility $v(c(j)) \cdot \xi(j)$. In other words, utility is determined by a 'baseline' utility $v(\cdot)$, the same for all differentiated goods, and a 'weighting function $\xi(j)$ with $\xi'(j) < 0$. This formulation yields a ranking of the various products: low-j goods get a high weight (have high priority in consumption) and vice versa.

We need to restrict the functions $v(\cdot)$ and $\xi(\cdot)$. We first assume that purchasing a differentiated product is a 'take-it or leave-it' decision: either a product is consumed in quantity 1 or is not consumed, so that $c(j) \in \{0, 1\}$. This allows us to normalize the baseline utility to $v(0) = 0$ and $v(1) = 1$. This first assumption is primarily for tractability and analytical convenience. Second, and more importantly, we assume that the hierarchy function takes the form $\xi(j) = j^{-\gamma}$ with $\gamma \in [0, 1)$. The instantaneous utility function can then be written as $u(\{c(j)\}) = \int_0^\infty j^{-\gamma} c(j) dj$ and, when the first N goods in the hierarchy are consumed, takes the constant-elasticity form $u(\{c(j)\}) = N^{1-\gamma} / (1 - \gamma)$. This second assumption will be required below to generate a balanced growth path.[6]

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erators is strongly positively related to household income. Jackson (1984) provides evidence for both predictions using micro data from the Consumer Expenditure Survey of the BLS. He finds that the richest income class consumed twice as many different products as the poorest class. Falkinger and Zweimüller (1996) present similar evidence from aggregate cross-country data of the U.N. International Comparison Project. It turns out that the richest economy in the sample, the U.S., consumed five times as many product categories in "significant" quantity, than Tanzania, the poorest economy in the sample. Dalgin, Mitra and Trindade (2004) show that inequality is an important determinant of import demand in the standard gravity model. They divide the trade flows into necessary and luxury goods and show that the the relation between inequality and imports can be largely interpreted as a result of hierarchic preferences.

[5] In Section 8 below, we will consider also goods that imperfectly substitute for these differentiated products. To keep the analysis as simple as possible we first disregard such products.

[6] In the equilibria studied below households consume "along the hierarchy": a consumer who purchases a range of N goods will purchases the first N goods in the hierarchy. The restriction $\gamma \in [0, 1)$ makes sure that the integral $\int_0^N j^{-\gamma} di$ does not diverge.

Consumers have an infinite time horizon. Their objective function can be written as[7]

$$U(\tau) = \int_{\tau}^{\infty} \frac{1}{1-\sigma} \left[\int_{0}^{N(t)} j^{-\gamma} c(j,t) dj \right]^{1-\sigma} e^{-\rho(t-\tau)} dt. \tag{1}$$

where ρ and $1/\sigma$ denote the rate of time preference and the rate of intertemporal substitution.

3.2 Consumption choices

Consumers are unequally endowed with labor and wealth. The intertemporal budget constraint of a household can be written as

$$\int_{\tau}^{\infty} \int_{0}^{N(t)} p(j,t) c(j,t) dj \cdot e^{-R(t,\tau)} dt \leq \int_{\tau}^{\infty} w(t) l \cdot e^{-R(t,\tau)} dt + V(\tau), \tag{2}$$

where $N(t)$, $p(j,t)$, and $w(t)$ denote, respectively, the mass of available differentiated products, the price of variety j, and the wage rate at date t. $R(t,\tau) = \int_{\tau}^{t} r(\tau) d\tau$ is the cumulative discount factor between dates τ and t, l is the (time-invariant) labor endowment of household , and $V(\tau)$ is the initial wealth level owned by the household.

The household maximizes (1) subject to the budget constraint (2). Setting up the Lagrangian and defining $u(t) \equiv u(\{c(j,t)\})$, it is straightforward to obtain the first order conditions for $c(j,t)$

$$c(j,t) = \begin{cases} 1, & p(j,t) \leq z(j,t), \\ 0, & p(j,t) > z(j,t) \end{cases} \tag{3}$$

where the willingness to pay $z(j,t)$ is defined by

$$z(j,t) \equiv j^{-\gamma} \frac{e^{R(t,\tau)-\rho(t-\tau)}}{\mu} u(t)^{-\sigma}.$$

The parameter μ denotes the Lagrangian multiplier, the marginal utility of wealth at the initial date τ. (This can be translated into the more familiar 'time-t' marginal utility of wealth $\lambda(j,t) = \mu e^{-R(t,\tau)+\rho(t-\tau)}$). The first two equations in (3) state that, at date t, consumer will purchase the differentiated good j, if its price $p(j,t)$ does not exceed the willingness to pay $z(j,t)$; and will not purchase otherwise.

[7] Here it is assumed that the $N(t)$ goods supplied at date t coincide with the first $N(t)$ goods in the consumption hierarchy. In other words, there are no "holes" in the distribution of supplied products along the hierarchy. This will be the case in the equilibria studied below, where innovators introduce always the most urgently wanted goods.

The consumer's willingness to pay $z(j,t)$ is larger the lower the position of good j in the hierarchy (i.e. the higher the priority of good j). Furthermore, $z(j,t))$ is the higher the smaller is the consumer's marginal utility of wealth $\lambda(j,t) = \mu e^{-R(t,\tau)+\rho(t-\tau)}$. Obviously, rich consumers have a lower marginal utility of wealth and their willingness to pay is higher.

4 Distribution

It is assumed that all consumers have the same objective function (1) but differ in their endowments. To keep things as simple as possible, we assume there are two types of consumers, poor P and rich R, with population size β and $1-\beta$, respectively. (We discuss in Chapter 9 below that our analysis can be extended in a straightforward way to allow for more than two income classes). All households derive income from working and from shares in profits that accrue in the monopolistic firms. We assume further that each household has the same income composition (identical labor and profit shares). Hence the ratio of the income level of the poor relative to per capita income is $\theta_P < 1$, and the corresponding ratio of the rich is $\theta_R > 1$. Obviously the income shares of poor and rich must sum up to unity, so we have $(1-\beta)\theta_R+\beta\theta_P = 1$. Taking $\vartheta \equiv \theta_P$ as the exogenous parameter, we have $\theta_R = (1-\beta\vartheta)/(1-\beta)$. Hence the two parameters β and ϑ fully characterize the income distribution.[8]

We note that the assumption of identical income shares is restrictive and can be disputed both on empirical and theoretical grounds. It implies, for instance, that the distribution of income and the distribution of wealth are identical, whereas in reality the latter is typically more unequal than the former. Furthermore it implies that there is no feedback from the functional distribution to the personal distribution of income, whereas in reality changes in factor prices typically have an impact on the size distribution of income. There are basically two reasons why we adopt this assumption. The first is tractability. When both groups of consumers have the same relative endowments with labor and firm shares, the personal (or size) distribution of income is independent of the factor income distribution (which is endogenously determined in the model). Allowing for differences in the income composition would considerably complicate the analysis without adding much additional economic insight. [9] The second reason is that,

[8]The corresponding Lorenz-curve is piecewise linear with slope ϑ up to population share β; and slope $(1-\beta\vartheta)/(1-\beta)$ for population shares between β and 1.

[9]For an analysis of this point in the context of a static model see Zehnder (2004).

under our assumptions on preferences, an identical income composition implies that all households have the same optimal savings rate. This implies that, in fact, the initial distribution persists over time. Any impact of income inequality arises from demand-effects due to hierarchic preferences and none of the inequality effects is due to differences in the propensities to save between rich and poor consumers arising from changes in factor shares.

5 Technology and price setting

5.1 Production technology and technical progress

The supply side of the model is simple. Labor is the only production factor and the labor market is competitive. The market clearing wage at date t is denoted by $\tilde{w}(t)$. The goods are produced in monopolistic firms under increasing returns to scale. Before a good can be produced the firm has to make an 'innovation'. This gives the firm exclusive access to the blueprint of the new good and guarantees monopoly position.[10] The innovation cost are modelled by a set-up cost equal to $\tilde{F}(t)$ labor units. Once this set-up cost has been incurred, the firm has access to a linear technology that require $\tilde{b}(t)$ units of labor to produce one unit of output.

Innovations imply technical progress. We assume that the knowledge stock of this economy equals the number of known designs $N(t)$. The labor coefficients in the sector that produces differentiated goods are inversely related to the stock of knowledge. Hence we have $\tilde{F}(t) = F/N(t)$ and $\tilde{b}(t) = b/N(t)$ where $F > 0$ and $b > 0$ are exogenous parameters. Wages grow pari passu with productivity, $\tilde{w}(t) = wN(t)$, where $w > 0$ is a constant. Hence the cost of production of differentiated products stay constant over time as $\tilde{w}(t)\tilde{F}(t) = wF$ and $\tilde{w}(t)\tilde{b}(t) = wb$ are constant over time. We choose the marginal production cost in the differentiated sector as the numeraire $wb = 1$, then $w = 1/b$.

5.2 Prices of the differentiated goods

Producers of differentiated products are in a monopoly position and can set prices above marginal cost of production. In order to determine the monopoly price we need the monopolist's demand function. Consider the demand for good j. (For convenience, we omit time-indices

[10]By assumption, we rule out that there is duplication. So when a new good is 'invented' there is one and only one firm that incurs that fixed cost and captures the respective market.

in this Subsection). As consumption is a binary choice, market demand for good j depends on how many consumers are willing to purchase at a given price $p(j)$. With two groups of consumers the market demand function is a step function (Figure 1). At prices that exceed the willingness to pay for the rich, $p(j) > z_R(j)$, demand is zero and the demand curve in Figure 1 coincides with the vertical axes. For prices that do not exceed the willingness to pay for rich, but are strictly larger than the willingness to pay for the poor, $p(j) \in (z_P(j), z_R(j)]$, market demand equals the population size of the rich $1-\beta$. Finally, for prices lower than or equal to the willingness to pay of the poor $p(j) \leq z_P(j)$ market demand equal the size of the whole population in the economy which is unity.[11]

A monopolist will either charge $z_R(j)$ and sell only to the rich (point A in Figure 1) or charge $z_P(j)$ and sell to the whole population (point B in Figure 1), whichever yields the higher profits. The corresponding profit levels are, respectively, $[z_R(j)-1](1-\beta) \equiv \Pi_R(j)$ (point A) and $[z_P(j)-1] \equiv \Pi_{tot}(j)$ (point B).

Figure 1

Suppose N products are supplied by monopolistic firms and denote by N_i the range of goods purchased by consumer i. Which firms set high prices $z_R(j)$ and which ones set low prices $z_P(j)$? Note first that a situation where all N firms charge $z_R(j)$ and sell only to the rich cannot be an equilibrium. If the poor would not buy any differentiated products at all, their willingness to pay for goods $j \to 0$ would become infinitely large. Hence there must be some $j > 0$ such that $\Pi_{tot}(j) \geq \Pi_R(j)$ or equivalently, $z_P(j) - z_R(j)(1-\beta) \geq \beta$, which implies that $z_P(j)/z_R(j) > 1-\beta$. This leads us to the following

Proposition 1 *Firms set the prices such that for all goods* $j \in [0, N_P]$, $p(j) = z_P(j)$, *and for all* $j \in (N_P, N_R]$ *we have* $p(j) = z_R(j)$, *where* $0 < N_P < N_R \leq N$.

Proof. We know from equation (3) that $z_i(j) = (j^{-\gamma}/\lambda_i)u_i^{-\sigma}$. (Recall the definition $\lambda_i(j,t) = \mu_i e^{-R(t,\tau)+\rho(t-\tau)}$). Hence it is straightforward to calculate $\partial\Pi_R(j)/\partial j = -(\gamma/j)\, z_R(j)(1-$

[11]Obviously, if there are more types of consumers, there are more such kinks, and in the case of continuous distribution we have a smooth demand function. In any case, under the take-it or leave-it assumption the shape of the demand function reflects the distribution of the consumers' willingnesses to pay.

β) and $\partial \Pi_{tot}(j)/\partial j = -(\gamma/j)\, z_P(j)$. Since $z_P(j)/z_R(j) > 1-\beta$, we have $\partial \Pi_R(j)/\partial j > \partial \Pi_{tot}(j)/\partial j$ implying that $\Pi_{tot}(j) - \Pi_R(j)$ decreases with j. Since the poor consume always a positive subset of the differentiated goods, there exist a good $N_P > 0$ such that for any $j \leq N_P$, we have $\Pi_{tot}(j) \geq \Pi_R(j)$. Finally, $N_P = N_R$ cannot be an equilibrium. In that case, the poor and the rich spend the same amount, hence the rich do not exhaust their budget constraint and their marginal utility of wealth would be zero and $z_R(j)$ equals infinity. It is then profitable for a monopolist to deviate and sell only to the rich. ■

Hence Proposition 1 implies that the prices of the differentiated products are

$$p(j,t) = \begin{cases} z_P(j,t) & j \in [0, N_P(t)] \\ z_R(j,t) & j \in (N_P(t), N_R(t)]. \end{cases} \tag{4}$$

Proposition 1 implies that the poor consume all goods $j \in [0, N_P]$ and the rich consume all goods $j \in [0, N_R]$ where $0 < N_P < N_R \leq N$. This means 'consumption follows the hierarchy' in the sense that consumer i purchases only the first N_i products in the hierarchy and no products $j > N_i$. The poor purchase low-j goods, that is goods that satisfy the most urgent wants. The rich purchase not only those necessities, but can also afford more luxurious goods. These observations lead us to the following

Proposition 2 *The equilibrium is characterized by one of two regimes. In the first regime,* $N_P(t) < N_R(t) = N(t)$, *the rich purchase all products that firms can produce. In the second regime,* $N_P(t) < N_R(t) < N(t)$, *the rich purchase only a subset of all producible goods.*

Proposition 2 has important implications for the characteristics of the balanced growth path. We will see below that, along this path, the ratios $N_P(t)/N(t) < 1$ and $N_R(t)/N(t) \leq 1$ are constant over time. When $N_R(t)/N(t) = 1$, and rich consumers purchase all producible goods, the most recent innovator sells the new product to the rich right away and to the poor later one. We will call this situation the "regime IS" ("innovate and sell"). When $N_R(t)/N(t) < 1$, rich consumers never purchase all producible goods. The most recent innovator has to wait for a while until there is positive demand for new good. We will refer to this case the "regime IW" ("innovate and wait"). In the next Section we will discuss in some detail the IS-case, and will refer briefly to regime IW in Section 7.

Regime IW may appear like a strange outcome to some readers. After all, in the real world there are always very rich people able to pay a *very* high price for any new product. So such an equilibrium outcome is just an artefact of the two-class assumption (and which can arise only when the two classes are sufficiently similar). We will nevertheless consider regime IW, for two reasons. First, it helps us to understand the relationship between inequality and growth under a more general distributions with more than two groups. (We discuss this in Section 9). Second, a situation where innovators incur costs in order to capture a new market may in fact be more than a theoretical possibility.[12] Such an outcome may occur when the innovation costs are low and the prospective (future) market is sufficiently profitable.

6 Balanced growth: Regime IS

6.1 The allocation of resources across sectors

The economy's resources consist of the stock of knowledge $N(t)$ and homogeneous labor supplied by each household in the economy. At any date t, $N(t)$ is predetermined but affects current productivities $\tilde{b}(t)$ and $\tilde{F}(t)$. Total labor supply is normalized to unity. Since innovation is costly, a part of the economy's resources are employed in an R&D sector that develops blueprints for new products. The remaining labor force employed in the production of final output. The allocation of labor resources across sectors is endogenously determined. We denote by L_N the number of production workers and by L_I the number of research workers. The demand for production labor is given by $L_N(t) = \int_0^{N(t)} [b/N(t)] [\beta c_P(j,t) + (1-\beta) c_R(j,t)]\, dj$. As the rich consume all feasible products and the poor only a subset $n(t) \equiv N_P(t)/N(t)$, this simplifies to $L_N(t) = b [\beta n(t) + (1-\beta)]$. The demand for research workers depends on $\dot{N}(t)$, the level of innovation activities at date t. As introducing a new product requires $F/N(t)$ labor units, the demand for research workers is $L_I = F\dot{N}(t)/N(t) = Fg(t)$.

A perfect labor market ensures that the labor supply is fully employed at each date, so $1 = L_N + L_I$. Using the above expressions for L_I and L_N, the economy's resource constraint can be written as

$$1 = b [\beta n(t) + (1-\beta)] + Fg(t). \qquad (5)$$

The dynamic analysis below focuses on a balanced growth path, along which the allocation of labor across the two sectors stays constant over time. ¿From equation (5) it is obvious, that a

[12] As an example, The Economicst (April 6, 2000) notes in an article on "patent wars": "Biotech companies, which often have nothing to sell for years, find their value residing solely in their intellectual property."

balanced growth path is only possible if $n(t) = n$ and $g(t) = g$ do not change over time.

6.2 Prices and interest rate along the balanced growth path

How do the prices of some product j evolve along the balanced growth path? Denote the period when the good is introduced by τ. At that date, the innovating firm charges the price $p(j,\tau) = z_R(j,\tau)$ and the rich start to purchase. As their income grows, rich households are willing to pay more for any given product and the innovator can raise its price. It is straightforward to calculate the rate of change in the price from the definition of $z_R(j,t)$. Recall that $z_R(j,t) = j^{-\gamma} e^{R(t,\tau)-\rho(t-\tau)} u_R(t)^{-\sigma}/\mu_R$ and that – as households consume "along the hierarchy" – $u_R(t) = N(t)^{1-\gamma}/\left(1-\gamma\right)$. Using this in the expression for $z_R(j,t)$, taking logs and the derivative with respect to time t yields

$$\frac{\dot{p}(j,t)}{p(j,t)} = \frac{\dot{z}_R(j,t)}{z_R(j,t)} = r(t) - \rho - \sigma\left(1-\gamma\right)\frac{\dot{N}_i(t)}{N_i(t)}. \tag{6}$$

Prices grow at that rate until firms find it optimal to attract also the poor as customers. At that date, firms cut prices from what the rich are willing to pay $z_R(j,s)$ to what the poor can pay $z_P(j,s)$. After date s, price changes are determined by the rate of change in $z_P(j,s)$. The willingness to pay of the poor is $z_P(j,t) = j^{-\gamma} e^{R(t,\tau)-\rho(t-\tau)} u_P(t)^{-\sigma}/\mu_P$ where $u_P(t) = \left[n \cdot N(t)\right]^{1-\gamma}/\left(1-\gamma\right)$. Since n is constant along the balanced growth path, $\dot{z}_P(j,t)/z_P(j,t)$ yields exactly the same expression as equation (6).

We can make a slightly different thought experiment and look at the evolution of the price $p(N_i(t),t)$. This is the price for the good with least priority (i.e. the most luxurious good) that is purchased by consumer i. Setting $j = N_R(t) = N(t)$ in the above expression for $z_R(j,t)$ and $j = N_P(t) = n \cdot N(t)$ in the expression for $z_P(j,t)$, taking logs and the derivative with respect to time t yields (in each case)

$$\frac{\dot{p}(N_i(t),t)}{p(N_i(t),t)} = r(t) - \rho - \left(\gamma + \sigma\left(1-\gamma\right)\right)\frac{\dot{N}(t)}{N(t)}.$$

Along a balanced growth path, the menu of differentiated goods increases the at the same rate as $N(t)$ for both types of consumers. This rate is constant and given by g. Furthermore, the price of the most recently developed good $N(t)$ has to stay constant over time, otherwise the resources devoted to R&D L_I would change (see the next Subsection). Using $\dot{p}(N(t),t) =$

$\dot{p}(n \cdot N(t), t) = 0$ we can solve the above equation for the interest rate

$$r(t) = \rho + g\left(\gamma + \sigma\left(1 - \gamma\right)\right), \tag{7}$$

which, unsurprisingly, is also constant along the balanced growth path. Obviously, equation (7) is the equivalent in our model to the familiar Euler equation in the standard growth model, and is identical to it in the absence of a consumption hierarchy $\gamma = 0$. Note further that, in the special case where $\sigma = 0$, the hierarchy parameter γ tells us how an increase in the range of consumed goods affects the utility flow, just like the elasticity of marginal utility in the standard model.

Reinserting the interest rate (7) into equation (6), we see that $\dot{p}(j, t)/p(j, t) = g\gamma$, hence the price of a particular good increases at a constant rate. However, the price of a particular *hierarchical position in the consumption hierarchy,* $j/N(t)$, is independent of t. In other words, the *structure of prices* stays constant along the balanced growth path.

Clearly the price structure is determined by the endogenous variables g, $p(N(t), t)$, and $p(N_P(t), t)$. For further use, it will be convenient to focus on the price of the most recently innovated product and define $p \equiv p(N(t), t) = z_R(N(t), t)$. Furthermore, it will be convenient to express the price $p(N_P(t), t)$ in terms of the endogenous variables p and n. We know from Proposition 1 that the firm supplying good N_P is indifferent between selling to the whole customer base and selling only to the rich, as N_P satisfies the arbitrage condition $z_P(N_P) - 1 = [z_R(N_P) - 1](1 - \beta)$. (For simplicity, we omit time indices). We know further that, from equation (3), $z_R(N_P) = n^{-\gamma} z_R(N)$. Using this and the definition $z_R(N) = p$, we can solve the arbitrage condition for the price of good N_P

$$p(N_P(t), t) = z_P(N_P(t), t) = \beta + (1 - \beta)n^{-\gamma}p.$$

In sum, the price of a new good starts out with price p, increases at rate $g\gamma$ thereafter, drops to $\beta + (1 - \beta)n^{-\gamma}p$ once firms find it optimal to attract also the poor as customers, and increases at rate $g\gamma$ thereafter.

Four comments on the evolution of prices are in order. First, it is obvious that the discontinuous evolution of prices with a discrete jump when the poor start to purchase, is due to our assumption of two groups of consumers. Our analysis can be extended to many groups. In that case there would be many such changes in prices in order to attract additional groups or customers. Second, we have not allowed for learning-effects in the production of a particular variety. To the contrary, the evolution of prices (or markups) reflects the fact that a particular good experiences a higher willingness to pay because the consumers have already satisfied. The *relative* position of good j in the hierarchy, $j/N(t)$, decreases. In this sense, a good that was previously a luxury good, has become a necessity. This is reflected in increasing willingnesses to pay and rising prices for this product. Third, we did not allow for any changes in the market structure. This implies that once firms have made a successful innovation, they conquer a monopolistic position on the market for that good an keep these position forever (for instance, due to infinitely lived patents). Since prices grow at a constant rate $g\gamma$ this implies that prices grow without bound and the most necessary goods will approach infinity. An easy way to cope with this problem would be to introduce finite patent protection and allow for perfect competition once patents have expired. While this would considerably complicate the involved equations, - although the model could still be solved - allowing for finitely lived patents would not add any substantial insights. In particular, as long as patents expire *after* the poor have started to purchase the product, the relationship between inequality and growth will remain (qualitatively) unchanged. Finally, in our discussion in Section 9 below we will come to the issue of price dynamics and show that, by allowing for a hierarchical structure on the cost side, that our model can be easily adjusted to generate an equilibrium outcome where prices do *not* increase over time.

6.3 The innovation process

Up to now we have taken a continuous introduction of new products (and corresponding increases in productivity) for granted. We now look at the incentives to conduct R&D and introduce new products.

It is assumed that there is free entry into the R&D sector and the equilibrium is a situation of zero profits in which the cost and the value of an innovation are exactly balanced. The *cost* of an innovation is given by $wF = F/b$ whereas the *value* of an innovation remains to be determined. Note first that innovation efforts will be targeted towards those goods for which consumers are willing to pay most. In Schmookler's sense, firms will target those wants "that consumers want to satisfy badly enough". Hence the process of product innovations will follow the consumption hierarchy.

To calculate the value of an innovation we need to know the profit flow following the introduction of a new product. A successful firm has initial demand $1-\beta$ up until the date when prices are cut and also the poor are attracted as customers. From that date onwards, all consumers purchase and demand equals unity. Denote by Δ the time interval during which only the rich purchase a new product. Obviously, Δ must satisfy $N_P(t+\Delta) = N(t)$. Along a balanced growth path, where N_P grows at the constant rate g and we can write $N_P(t)e^{g\Delta} = N(t)$. Taking logs and solving for Δ yields

$$\Delta = -\ln[N_P(t)/N(t)]/g = -(\ln n)/g,$$

Note that $\Delta > 0$ as $n < 1$. Obviously, the duration Δ during which an innovator sells only to the rich is long if (i) the poor are very poor (so the fraction of goods the poor can afford n is small) and (ii) the growth rate g is low.

Recalling the evolution of prices and noting that we have normalized the marginal production cost to unity, the profit flow equals $(1-\beta)\left(pe^{g\gamma(s-t)}-1\right)$ at dates $s \in [t, t+\Delta)$ (when the firm sells only to the rich) and equals $\left[\beta + (1-\beta)n^{-\gamma}p\right] e^{g\gamma(s-t-\Delta)} - 1$ at dates $s \geq t+\Delta$ (when all households purchase the good). The value of an innovation equals the value of this profit flow, discounted at rate r. Calculating this value and setting it equal to the costs of an innovation yields the zero-profit condition of the innovation sector

$$\frac{F}{b} = (1-\beta)\left(\frac{p}{r-g\gamma} - \frac{1}{r}\right) + \beta n^{r/g}\left(\frac{1}{r-g\gamma} - \frac{1}{r}\right). \tag{8}$$

6.4 Solving for the equilibrium growth rate

We can now solve for the balanced growth equilibrium. We use Proposition 1 to rewrite the budget constraints (2) of poor and rich consumers, respectively, as

$$\begin{aligned} wl_P + (r-g)\frac{V_{tP}}{N_t} &= \left[\beta n + (1-\beta)pn^{1-\gamma}\right]\frac{1}{1-\gamma}, \text{ and} \\ wl_R + (r-g)\frac{V_{tR}}{N_t} &= \left[\beta n + (1-\beta)pn^{1-\gamma}\right]\frac{1}{1-\gamma} + p\frac{1-n^{1-\gamma}}{1-\gamma}. \end{aligned}$$

Dividing the former equation by the latter and making use of our assumption on the endowment distribution $l_P/l_R = V_{tP}/V_{tR} = \theta_P/\theta_R = \vartheta(1-\beta)/(1-\vartheta\beta)$ yields an equation that can be solved for p

$$p = \varphi(n) = \frac{(1-\vartheta)\beta}{1-\beta}\frac{n}{\vartheta - n^{1-\gamma}}, \text{ with } \varphi'(n) > 0. \tag{9}$$

The intuition for the positive relationship between p and n is straightforward: *For a given degree of inequality* (as represented by the exogenous parameters β and ϑ), a situation where the poor want to purchase a larger range of the differentiated products (n is higher) goes hand in hand with a situation where the rich are willing to pay a higher price for the product of the most recent innovator (p is higher).

We see from equation (9) how this restricts the relevant range of p and n. Note that $n < 1$ and $p > 1$. Equation (9) implies that p reaches infinity at $n = \vartheta^{1/(1-\gamma)} < 1$. Moreover, we see that there exists a critical value of n, call it m, such that $\varphi(m) = 1$. Hence the relevant ranges for the endogenous variables p and n are $p \in [1, \infty)$ and $n \in \left[m, \vartheta^{1/(1-\gamma)}\right]$.

To determine the growth rate we are now left with two equations in the two unknowns g and n. The first equation is (5). We obtain the second equation from rewriting the R&D equilibrium condition (8) using equations (7) to replace r and (9) to replace p

$$\frac{F}{b} = \frac{(1-\beta)\,\varphi(n) + \beta n^{\rho/g+(\gamma+\sigma(1-\gamma))}}{\rho + g\sigma\,(1-\gamma)} - \frac{1-\beta+\beta n^{\rho/g+(\gamma+\sigma(1-\gamma))}}{\rho + g\,(\gamma+\sigma\,(1-\gamma))}. \tag{10}$$

It is obvious that no closed-form solution for the equilibrium growth rate g and the consumption share of the poor n exists.

We know discuss the conditions under which a general equilibrium exists. In particular, we will show that, with a "flat" hierarchy (γ relatively small), this equilibrium is unique. (The implications of a "steep" hierarchy are discussed later). To characterize the general equilibrium it is convenient to draw the two conditions in the (g, n) space (Figure 2). We will refer to the former as the ZP-curve ("zero-profit") and the latter as the RC-curve ("resource-constraint"). We now discuss the properties of these tow curves in the following two Lemmas.

Lemma 1 *a) The ZP-curve crosses the n-axis at n^{ZP} where n^{ZP} satisfies $\varphi(n^{ZP}) = 1 + F\rho/\left[(1-\beta)\,b\right] > 1$ which implies that $n^{ZP} > m$. b) When $\gamma \leq \bar{\gamma}$, with $\bar{\gamma} \equiv \sigma F\rho/(\sigma F\rho + b)$, the ZP-curve is monotonically increasing in the (g, n)-space. c) The ZP-curve becomes a vertical line as $n \to \vartheta^{1/(1-\gamma)}$.*

Proof. See Appendix A1. ■

Part a) of Lemma 1 characterizes the slope of the ZP-curve. The ambiguity arises because the effect of g on the value of an innovation is a priori unclear (whereas the effect of n is always positive). A higher g raises the interest rate, but it also raises mark-ups and flow profits. The former effect is the familiar discounting effect. The latter effect arises due to hierarchic preferences and is stronger the larger is γ.[13] Provided the hierarchy is sufficiently flat, the former effect dominates the latter. In that case, the ZP-curve has positive slope: any increase in n has to be offset by a corresponding increase in g to make sure that the zero-profit condition (10) is satisfied. Part b) follows from equation (9). There we have seen that, as $n \to \vartheta^{1/(1-\gamma)}$, we have $\varphi(n) \to \infty$. To counteract large values of $\varphi(n)$, g must also grow large, otherwise condition (10) would be violated. Finally part c) of Lemma 1 says that, irrespective of particular parameter constellations, the ZP-curve always starts out to the right of m.

Lemma 2 *a) The RC-curve crosses the n-axis at* $n^{RC} \geq m$ *if* $1/b \geq \beta m + 1 - \beta$*. b) The RC-curve is continuous and monotonically decreasing in* (g, n)*-space.*

Proof. Follows from equation (5). ■

Lemma 2 discusses slope and position of the RC-curve. The intuition for the negative slope is straightforward. A higher growth rate g requires more labor resources. With fully employed labor resources this is only possible with less labor in production, hence a lower n. Part b) of Lemma 2 makes sure that the RC-curve lies inside the relevant range of the n so that a situation where the rich consume all products and the poor consume at least fraction m of the available products is feasible, with the available resources.

Figure 2

The above characterizations of the ZP- and the RC-curve allows us to state the following

Proposition 3 *Suppose the hierarchy is flat,* $\gamma \leq \bar{\gamma}$*. a) If* $n^{ZP} < n^{RC}$*, there exists a unique general equilibrium with a positive growth rate* $g > 0$*. b) If* $n^{ZP} \geq n^{RC}$*, the unique equilibrium is stagnation* $g = 0$*.*

Proof. Follows immediately from Lemmas 1 and 2. ■

Having established conditions for existence and uniqueness of a general equilibrium, we now turn to the question of our central interest: How does the extent of inequality affect long-run growth? The following proposition gives an answer to this question.

[13] Recall from our previous discussion that prices increase at rate $g\gamma$. Hence, with a higher g, mark-ups and profits increase at a larger rate.

Proposition 4 *Suppose the hierarchy is flat, $\gamma \leq \bar{\gamma}$, and a unique equilibrium with a positive growth rates exists. a) An increase in relative incomes of the poor ϑ decreases the growth rate g and decreases the consumption share of the poor n. b) An increase in the group share of the poor β – holding $\theta_P = \vartheta$ constant – increases g and has an ambiguous effect on n.*

Proof. See Appendix A3 ■

Recall that an increase in the distribution parameter ϑ implies less inequality, whereas an increase in the population share of the poor β – holding $\theta_P = \vartheta$ constant – implies higher inequality. Hence proposition states that more income inequality increases innovation and growth.

To see the mechanism that establishes the positive link between inequality and growth, let us first look what happens when the parameter ϑ increases. Such a change increases the relative income of the poor at the expense of the rich. From equation (5) it is obvious that the RC-curve is unaffected from that change. However, ϑ enters equation (10) via $\varphi(n)$, so the ZP-curve is affected. When ϑ increases, $\varphi(n)$ decreases, see equation (9). From inspection of (10) we see that, holding g constant, any reduction in $\varphi(n)$ must be offset by a corresponding increase in n, otherwise (10) would be violated. Hence the ZP-curve must shift to the right. Obviously, the new equilibrium has a lower growth rate and a higher consumption share of the poor.

The result is the outcome of two opposing effects. On the one hand, the rich suffer a loss in income which decreases their willingness to pay for new products. This is bad news for innovators' profits as the willingness to pay of the rich determines prices immediately after the innovation. On the other hand, higher incomes raise the willingnesses to pay of the poor. This affects the evolution of prices and market size. When the poor can pay more, firms will find it optimal (i) to cut prices and expand markets earlier than before; and (ii) to charge higher prices thereafter. However, these positive effects are always dominated by the above negative effects. The reason is discounting: negative effects occur immediately, whereas positive effects accrue later. In sum, an increase in ϑ decreases the value of an innovation. To establish equilibrium, growth must decrease.

Consider next the impact of an increase in the parameter β. Provided that ϑ remains constant, this increases relative incomes of the rich but leaves the poor unaffected. Such a change affects both the RC-curve and the ZP-curve. From (5), it is obvious that any increase in β must be offset by a corresponding increase in n. Hence the RC-curve shifts to the right. Moreover, the ZP-curve shifts to the left. An increase in β decreases the right-hand-side of (10) directly and increases it indirectly because $\varphi(n)$ increases in β. It is shown in Appendix A3 that the direct effect dominates. To offset this, n has to decrease.

Hence an increase in income inequality that results from a larger population share of the poor β increases the growth rate. To get the intuition, let us look at the zero-profit condition (10). Even though a higher β implies a lower number of rich people and hence thinner markets for the most innovative products, the fewer rich can pay higher prices for these products as their relative income has increased (recall that θ_R is increasing in β). On balance, the price effect outweighs the market size effect and implies higher profits for innovators early on. This raises the value of an innovation. To establish equilibrium, growth has to increase. We can look at the problem also from the perspective of the resource constraint (5). With a larger population size of the poor β saves resources for the economy as a whole, as the poor consume only a subset of the products that are supplied on the market. These idle resources can be employed in research, raising the economy's growth rate.

7 Other balanced growth equilibria

In this Section we explore the case where the hierarchy is steep, $\gamma > \bar{\gamma}$. In such a situation the nature of the general equilibrium may change. In particular, the ZP-curve may become non-monotonic, giving rise to multiple equilibria and the regime IW – where innovators have to wait until their new good is demanded – may become an equilibrium. (In such an equilibrium, innovators anticipate that there will be demand in the future and discount the resulting profit flow. By incurring research effort early, they preempt potential competitors and conquer a monopoly position on a new market.)

Let us consider the equilibrium in regime IW where new firms have a waiting time until the new product is demanded in positive amounts. In regime IW, conditions (5), (9), and(10) change to

$$1 = gF + bn[(1-\beta)/m + \beta], \tag{11}$$

$$\frac{1-\vartheta}{(1-\beta)\,\vartheta} = \frac{1-m^{1-\gamma}}{\beta m + (1-\beta)m^{1-\gamma}}, \tag{12}$$

and

$$\frac{F}{b} = \left(1 - \beta + \beta m^{r/g}\right)\left[\frac{1}{r - g\gamma} - \frac{1}{r}\right] \cdot \left(\frac{n}{m}\right)^{r/g}, \tag{13}$$

where $r = \rho + g(\gamma + \sigma(1-\gamma))$ by equation (7). These are three equations in the three unknowns g, m, and n. Here $m \equiv N_P(t)/N_R(t)$ denotes the relative consumption levels between poor and rich consumers. Note that m is determined directly from equation (12) and depends only on the hierarchy parameter γ and the distribution parameters β and ϑ. It does not depend on technological factors and is independent of the growth rate.[14] This allows to characterize the equilibrium graphically using equations (11) and (13).

In panel a) of Figure 3 it is shown that with steep hierarchy an equilibrium in the regime IW is possible. When the hierarchy is steep the ZP-curve is backward bending. The slope is negative when g is low and becomes positive for higher values of g such that the ZP-curve curve bends back in the IS regime. Again, once $n \to \vartheta^{1/(1-\gamma)} > m$ the ZP-curve becomes a vertical line. A positive growth equilibrium in the regime IW is more likely when F is very small so the ZP-curve is satisfied for low values of n. . In that case, the (low) costs of innovation activities pay off even when the willingness to pay of rich consumers is initially low and innovators earn positive profits only in the future. This is intuitive, in such a case innovators are willing to incur research cost even if they have to wait.

Panel b) of Figure 3 shows a situation where there are *multiple equilibria*. Point A is a stagnation equilibrium, and points B and C are growth equilibria, drawn in the IW regime.[15] The reason why multiple equilibria occur lies in a strong *demand effect* when hierarchy is steep. The demand of an innovating firm depends on the economy-wide growth rate. If innovators expect high growth they anticipate that their market will quickly develop into a mass market, with fast-growing prices and profits. Hence optimistic growth expectations support an equilibrium with a high incentive to innovate. If innovators expect low growth,

[14] Note that we have already calculated m in our discussion of equation (9) above. There we have defined m as the limit case in regime IS, such that $\varphi(m) = 1$. In other words, when the rich are willing to pay exactly the marginal production cost we are at the limit $n = m = N_P/N_R$. When the willingness to pay is even lower, we are in regime IW. In that regime m is constant, whereas n falls short of m. Hence there exists a smooth transition between regimes IS and IW.

[15] The way panel b) in Figure 4 is drawn is exemplary. By means of simulations it turns out that the bad equilibrium is always stagnation, whereas positive growth equilibria may be either both in regime IW, both in regime IS, or one (with the lower growth rate) in IS and the other (with the higher growth rate) in IW.

profit expectations and the resulting incentive to innovate is correspondingly low. Hence a low level of innovative activities is sustained by pessimistic expectations and vice versa.

The presence of steep hierarchy leaves the impact of income inequality on growth remains qualitatively unchanged for most cases. The situation is exactly the same as in the IS-regime. In the IW-situation, the situation is slightly more complicated. More equality not necessarily implies that more resources are needed (for a given growth rate). The reason is that the poor do consume more but the rich consume less goods. Graphically, an increase in the relative income of the poor ϑ shifts the ZP-curve to right but also the RC-curve is affected (in regime IW an increase in ϑ raises m, and shifts the RC-curve to the right). However, simulations render the result that the overall change is a reduced growth rate when ϑ increases or β decreases.

Proposition 5 *Assume the hierarchy is steep, $\gamma > \bar{\gamma}$, and a unique equilibrium with a positive growth rates exists. a) An increase in relative incomes of the poor ϑ decreases the growth rate g and decreases the consumption share of the poor n and increases the consumption share of the rich m. b) An increase in the group share of the poor β – holding $\theta_P = \vartheta$ constant – increases the consumption share of the rich m, decreases the growth rate g and has an ambiguous effect on n.*

Figure 3

8 A non-innovative sector

The above analysis has shown that, under most circumstances, higher inequality tends to increase growth. The driving force behind this result is the impact of inequality on price setting behavior of firms. When the rich are more wealthy, they pay higher prices for new products spurring the incentive to innovate. One could argue that the set-up of our model gives innovative firms too much market power. In the absence of any reasonable substitute for the innovative products, monopolistic producer will be able to fully exploit their monopoly position. In order to tackle this issue we extend our model for a non-innovative sector. We assume this sector produces a good that substitutes (albeit imperfectly) for the innovative products. Clearly, this should put an tighter limit on the price setting scope of innovators.

When prices of innovative products are too high, consumers will devote their expenditures towards to the relatively cheaper non-innovative products.

We denote the (homogenous and perfectly divisible) output of the non-innovative sector by x. (A possible different interpretation of x is leisure). For simplicity we assume (i) the non-innovative sector produces output with a linear technology that requires a labor units to produce one unit of output. (ii) There is no technical progress in this sector, hence the labor coefficient a is constant over time. (iii) The output market is competitive, prices equal marginal cost of production $p_x(t) = \tilde{w}(t)a = waN(t)$.

Assume further that, just like before, utility on the set of differentiated products. In addition, however, utility is positively affected by the quantity of the traditional good x. We assume a Cobb-Douglas relationship with parameter ν, where $0 \leq \nu < 1$.[16] Hence the utility function changes to

$$u(x, \{c(j)\}) = x^{\nu} \int_0^N j^{-\gamma} c(j) dj. \tag{14}$$

In the first part of Appendix A4, we show that in Regime IS the equilibrium conditions change as follows.[17] The full-employment condition (RC-curve) has to account for employment in the non-innovative sector, so equation (5) changes to

$$1 = gF + b(\beta n + 1 - \beta) + a \frac{\nu}{1-\gamma} \left[\beta^2 n + \beta(1-\beta) n^{1-\gamma} \varphi(n) + (1-\beta)\varphi(n)\right], \tag{15}$$

whereas the zero-profit condition (ZP-curve) remains the same as in (10). However, as mentioned above, the scope for price setting of innovative firms, as captured by the function $\varphi(n)$, is now different. Equation (9) changes to

$$\varphi(n) = \frac{\left[\nu\beta + (1+\nu)\frac{1-\vartheta}{\vartheta}\frac{\beta}{1-\beta}\right] n}{1 + \nu - \left[1 + \nu(1-\beta) + (1+\nu)\frac{1-\vartheta}{\vartheta}\right] n^{1-\gamma}}, \text{ with } \varphi'(n) > 0. \tag{16}$$

(It is straightforward to verify that, setting $\nu = 0$, equations (15) and (16) simplify to (5) and (9) above.) The additional (third) term in (15) equals labor demand in the non-innovative

[16]The Cobb-Douglas implies that expenditure shares are *per se* not systematically related to the income level. In equilibrium, however, rich and poor consumers have a different expenditure share of the homogenous product because the poor consume less differentiated products and face a different average price level for these products than the rich.

[17]Here we study only regime IS. For a treatment of regime IW, see Foellmi and Zweimüller (2004). However, the inequality growth-regime is qualitatively the same in both regimes.

sector; and the enriched expression for $\varphi(n)$ in (16) accounts for the fact that the scope for price setting for innovators is now more limited due to the presence of goods that can substitute (albeit imperfectly) the differentiated products. As can be checked by differentiating (16) the price $\varphi(n)$ indeed decreases in ν for a given n.

Note that RC-curve defined by equation (15) is downwards sloping in the (n, g) space. Hence, it can be shown (see Foellmi and Zweimüller, 2004) that, with a flat hierarchy, a unique general equilibrium of this extended model exists.[18] Moreover, it is straightforward to analyze the impact of income inequality on growth in this extended model. The leads us to the following

Proposition 6 *a) An increase in the relative income share of the poor ϑ leads to an unambiguous reduction in the growth rate. b) An increase in the population share of the poor β has now an ambiguous impact on growth.*

Part a) of Proposition 6 states that our previous result – a lower distance in incomes between rich and poor is harmful for long-run growth – survives even when we introduce a non-innovative sector. The reason is the same as before: lower incomes of the rich and higher incomes of the poor lead to more backloaded profits. This reduces the incentive to innovate. Hence in the extended model exactly the same mechanism is at work as in the basic model without a non-innovative sector. The backloading-effect of income inequality on the profit flow is robust to the presence of a non-innovative sector.

However, Part b) of Proposition 6 states that the effect of the population share β is different in the extended model as compared to the basic model without an non-innovative sector. As the increase in β – holding ϑ constant – implies the (fewer) rich are now more wealthy, they are willing to pay more for the most recent innovator's product. Hence the price of the most recent innovator's product increases, while the size of the market decreases. When ν is negligible (the non-innovative sector is absent), the price effect always dominates the market-size effect. However, when ν is sufficiently large, the price effect is weakened and the market-size effect dominates. In that case, *more income inequality* due to an increase in β *reduces* the growth rate. The reason for this result is that the presence of a substitute limits the market power

[18]The case of a steep hierarchy does not add particular substance to the analysis. Just like in the basic model without a non-innovative sector (i) the ZP-curve bends backward (the parameter ν does not enter the ZP-equation directly); and (ii) multiple equilibria are possible. Just like in the basic mode, the relationship between inequality and growth with a steep hierarchy is qualitatively identical to the one with a flat hierarchy.

of monopolistic firms. With $\nu = 0$, in the absence of adequate substitutes, consumers are "forced" to purchase the monopolistic products at very high prices. In contrast, with $\nu > 0$, such a substitute exists. When prices become too high, demand is directed towards the non-innovative sector. This limits the scope for the price setting of the monopolistic firms. Price effects become weaker and the market size effect becomes dominant.

The introduction of a non-innovative sector opens up the possibility for a further, and quite different, outcome. Unlike in the basic model, the balanced growth equilibrium could feature a situation where not only the rich but also the poor purchase *all* innovative goods, hence $N_P = N_R = N$. In the absence of a non-innovative sector, such a situation could never happen. If rich and poor consumed the same number of goods, their expenditures would be identical – the rich would not exhaust their budget constraint. However, if a non-innovative sector exists, the rich simply consume more traditional goods to exhaust their budget constraint. In other words, the presence of a non-innovative sector puts an upper bound on the willingness to pay of the rich for differentiated products. We shown in the second part of Appendix A4, that the resource-constraint and the zero-profit conditions change to

$$1 = gF + b + bp\frac{1 + v - \vartheta}{(1-\gamma)\,\vartheta}. \tag{17}$$

and

$$\frac{F}{b} = \frac{p}{\rho + g\sigma\,(1-\gamma)} - \frac{1}{\rho + g\,(\gamma + \sigma\,(1-\gamma))}. \tag{18}$$

with g and p being the unknowns. (Note that when $N_P(t) = N(t)$, n equals unity and can no longer serve as an endogenous variable. It is replaced by p, the price the poor are willing to pay for the most recent innovator's product). It is easy to show (see Foellmi and Zweimüller (2004)) that, under certain parameter constellations, there exists a unique balanced growth equilibrium in which both (17) and (18) are satisfied. In such an equilibrium, the inequality-growth relation changes fundamentally.

Proposition 7 *a) In a balanced growth regime, where both rich and poor consumer purchase all differentiated products, so that $N_P(t) = N_R(t) = N(t)$, an increase in the relative income of the poor ϑ unambiguously increases growth. b) A change in the population share of the poor β leaves the growth rate unaffected.*

Proof. see Proposition 4 of Foellmi and Zweimüller (2004). ■

The result in part a) of Proposition 7 is very intuitive. When the monopolistic firms sell to both groups, the willingness to pay by the poor is decisive for price setting. When ϑ increases, the poor are willing to pay more for new goods, hence innovators can raise prices and increase profits. This raises the innovation incentives and increases growth. On the other hand, a change in the groups size β has no effect. As long as not only the rich, but also the poor consume the most recent innovator's product, neither the size of the market nor the price that the innovator can charge are affected. Hence innovation incentives are unaffected.[19]

9 Discussion

Our model has emphasized the role of income distribution for the incentive to conduct industrial R&D when innovations are "demand-induced". We have started out from very simplifying assumptions both on the distribution of income and on the form of preferences. We now discuss the robustness of our results with respect to changes in these assumptions and mention some empirical implications.

Distribution Our discussion was based on the assumption that there are only two types of consumers. We show now that the analysis can be extended to a general distribution. We first discuss the characteristics of an equilibrium without a non-innovative sector and look at the effects of redistribution. We then show how results change when we consider the more general case when there is a non-innovative sector and discuss the impact of distributional changes in that context.

Suppose there are more than two groups and index groups by h, $h = 1, ..., \bar{h}$ with group $h = 1$ the richest and $h = \bar{h}$ the poorest group. With $\bar{h}$ groups of consumers, firms face an $\bar{h}$-step demand function and have the choice either to sell only to the richest group $h = 1$,

[19] To get a sense for the quantitative implications consider the following numerical example. Set $F = 5, b = 0.3, \sigma = 2,\ \gamma = 0.3,\ \rho = 0.02,$ and $\nu = 0.8$. As default values for inequality we set $\vartheta = 0.8$ and $\beta = 0.5$.

The growth rate monotonically decreases in β. For $\beta = 0.1$ the growth rate g equals 2.11% and for $\beta = 0.9$ we get $g = 1.61\%$.

On the other hand, an increase in ϑ affects growth negatively as long as $N_P(t) < N(t)$, i.e. the poor do not buy all goods. At $\vartheta = 0.7$ the growth rate $g = 2.25\%$ and at $\vartheta = 0.92$ we get $g = 1.40\%$.

However, for $\vartheta > 0.92$ the poor buy all goods $N_P(t) = N(t)$. As stated in Proposition 7 further increases of ϑ increase the growth rate and it reaches $g = 2.17\%$ when there is full equality, $\vartheta = 1$.

or the richest and second-richest $h \leq 2$, ..., or to all consumers $h \leq \bar{h}$. When there is *no* non-innovative sector, the equilibrium can be characterized as follows. The richest group will purchase all N goods, the second richest group will purchase $N_2 < N$ goods, the third richest group will purchase $N_3 < N_2 < N$ goods, and so on. In other words, there will be $\bar{h}$ different market sizes, with the most basic goods having the largest markets and the most luxurious goods having the smallest markets. During his life-cycle, the innovating firm will cut prices $\bar{h} - 1$ times, each price-cut attracting one additional group of consumers. In other words, each group becomes "critical", in the sense that an innovator's price equals to the groups' willingness to pay during *some* period of the life cycle.

What are the effects of a redistribution of income under a general distribution of income? We can apply our results from Propositions 4a and 5a in a straightforward way. As all groups are critical, any change in income of a particular group affects prices during a particular period in an innovator's life cycle. A redistribution of income from a richer class to a poorer class reduces the profits of more recent innovators and increases the profits of less recent innovators. This implies more backloaded profits and reduces the incentive to innovate. Hence, in the absence of a non-innovative sector, more inequality is beneficial for innovation and growth.

Consider next the more general situation when there is *a non-innovative sector*. In that case, not all groups are necessarily "critical" consumers. As we have seen in Section 8, this may be the case already in the two-class society. In an equilibrium studied there (see the discussion before Proposition 7) there is more than one group able to afford the product supplied by the most recent innovator. In that case the richest group is never "critical" and the level of their income does *not* play a role for the value of an innovation. In the presence of a non-innovative sector the rich can spend "left-over" income on the homogenous good. This logic extends to the general case. A new innovator may find it optimal to sell initially not only to the richest group but to the H_1 richest groups where $1 \leq H_1 \leq \bar{h}$. Groups $h = 1, ..., H_1 - 1$ are not critical and their income does not affect the value of an innovation. When an innovator cuts prices to attract additional consumers, the price cut may be such that not only the "next" group, but *more* additional groups are attracted as customers. In other words, the market after the first price cut encompasses the H_2 richest groups where $H_1 + 1 \leq H_2 \leq \bar{h}$. When $H_2 > H_1 + 1$ there is at least one group ($h = H_1 + 1$) that is not critical. Similar reasoning applies for further price cuts. The equilibrium is then characterized as follows. When innovators undertake $P - 1$

discrete price cuts, the life cycle can be divided into P periods (where a "period" is defined by the time between two successive price cuts). During the first period market size is H_1, during the second period market size is H_2, ..., and during the last period market size is $H_P = \bar{h}$. The consumers can be grouped into P classes such that $1 \leq H_1 \leq H_2 \leq ... \leq H_P = \bar{h}$ richest classes buy innovative products $[0, N_i]$ where $N = N_1 > N_2 > ... > N_P > 0$.

Now consider the effects of a redistribution of income. Suppose there is a transfer from a rich group h to a poorer group h' that involves only consumers purchasing the same set of innovative products, such that $H_i < h < h' < H_{i+1}$. This implies that neither the market size nor the prices are affected, so innovation incentives and long-run growth remain unchanged after such a redistribution. Similarly, when this redistribution involves non-critical consumers purchasing different sets of products, $H_i < h < H_{i+1} < h'$ there will not be any change in prices and market sizes either. Hence changes in income distribution involving non-critical consumers does not affect innovative activities and long-run growth.

Now assume that a transfer occurs from a richer group $h < H_i$ to a poorer group $h = H_i$ (a group of "critical" consumers). As some critical consumers have become richer, but no critical consumers have become poorer, the innovator can increase prices and profits during the corresponding period in the life cycle, without any change in market size, prices, or profits in other periods. This raises the value of an innovation and hence raise the growth rate. Obviously, the same increase in incentives to innovate could be generated from a redistribution from poor to rich where the latter are critical but the former are not.

Finally, let us next consider a transfer between critical groups, say from a richer group $h = H_1$ to a poorer group $h = H_i$ with $i > 1$. This implies lower profits during the first period and higher profits during the ith period, hence more backloaded profits (see Propositions 4a and 6a). However, such a transfer will result in a lower price distortion, which diminishes the share of the non-innovative sector. If the market size does not change in the two groups case (Proposition 6a: change in θ holding β constant), the outcome that profits are more backloaded dominates and growth is lower. On the other hand, if the progressive transfer is associated with an increase in the market size (lowering β) the effects from a lower price distortion can dominate (Proposition 6b). As these patterns can already be observed in the case with two groups they will be present with a more general distribution as well.

In sum, our analysis can be extended to a general distribution in a straightforward way.

However, the relationship between income distribution and growth is a complex one, and depends on the nature of the redistribution.

Preferences We have assumed a very simple form of preferences. Goods are indivisible, either consumed or not; and the consumption hierarchy can be represented by weighting the utilities from satisfying the various wants with a power function. The restriction of the weighting function to take the power form $i^{-\gamma}$ is essential. It implies that demand functions (and monopoly prices) only depend on the relative (rather than the absolute) position of the product in the hierarchy. As a result, the maximized static utility function can be expressed as a function of total (current) expenditure levels, the function taking the constant elasticity form with parameter γ. In other words, in intertemporal problems with a continuum of goods, assuming additive separability and weighting by a power function is the *equivalent* of assuming a CRRA-felicity function in the one-good growth model.[20] In either case, these functional forms guarantee a constant rate of consumption growth when rates of interest and time preference are constant over time.

The assumption that goods have to be indivisible and either consumed or not, however, is not essential. As mentioned in footnote 11, every subutility function $v(\cdot)$ would do. The model could still be solved with utility functions that allow consumers to choose not only whether or not to consume a certain item, but also how much to consume. To get the situation where poor consumers cannot afford to purchase certain items and non-negativity constraints become binding we need a subutility function with the additional property $v'(0) < \infty$. This implies that the subutility function must be non-homothetic: for instance, a quadratic form of $v(\cdot)$ has this property whereas the homothetic CES-utility function does not.[21]

Evolution of prices The model's prediction that the price of a particular product increases over time does not match well with the empirical facts. For instance, in a their classical study on the innovation-diffusion process, Gort and Klepper (1982) examined the evolution of real product prices for 46 product innovations. They also found that, in almost all cases, prices fell

[20]More precisely, Foellmi (2003) has shown that a felicity function of the form $u(\{c(j)\}) = \int_0^\infty j^{-\gamma} v(c(j))dj$ where $v' > 0 > v''$- is CRRA in expenditure levels if the price of good j can be expressed as a function of its relative position j/N only. This clearly holds true in the present model.

[21]Foellmi and Zweimüller (2003) solve a representative agent model with quadratic (hierarchic) preferences to study growth and structural change as a result of non-linear Engel-curves.

after the introduction of a new product.

As our concern is to understand the implications of "demand-induced" innovations for the inequality growth-relation we have kept our assumptions on technology and market structure very simple. Allowing for market entry – for instance, as a results of expired patents – would erode the innovating firm's market power and allow for falling product prices.

Note, however, that our model can be adjusted in a rather straightforward way for falling real prices such that a decrease in prices is *due to unequally distributed incomes.* Assume there is a 'hierarchical structure' on the cost- rather than the preference-side. More precisely, suppose that the marginal production costs are given by $mc(j) = (j/N(t))^{\alpha}$ with $\alpha > 0$, so that goods at a higher position in the hierarchy are relatively more costly to produce. This captures the idea that products which have been on the market for a longer period are cheaper to produce. (Still, there is a spillover effect working via the total number of developed goods in the economy that decreases required labor inputs for each firm). Obviously, the balanced growth properties of the model still hold as the utility-cost ratio is proportional to $(j/N(t))^{-\alpha}$ which is a (decreasing) power function. How do prices evolve under such circumstances? With symmetry on the demand side, the willingness to pay of a consumer with a given level of income is the same for all goods, with the rich (poor) having a high (low) willingness to pay for any given product. This implies that prices will be high initially, when only the rich consumers purchase the new good, and will gradually fall as firms with lower costs find it attractive to cut prices in order to attract also poorer customers.

Empirics of distribution and growth Our analysis suggests that the aggregate relationship between inequality and long-run growth may be quite complex. The model predicts, that depending on circumstances, the relationship between inequality and growth may be positive or negative. We note that this result is in line with the recent empirical literature on this issue. While early studies such as Alesina and Rodrik (1994), Persson and Tabellini (1994), and Perotti (1996) found that there is a negative relationship between inequality and growth, several recent studies cast doubt on the robustness of this result. For instance, Barro (2000) finds that this relationship is negative for poor countries but positive for rich countries. Forbes (2000) finds that, in panel-data, a positive relationship exists. Banerjee and Duflo (2003) find there exists a highly non-linear relationship between the two variables. While in these empirical studies the causality issue is not entirely clear, the available evidence suggests that there is no clear-cut monotonic relationship between inequality and long-run growth. Hence the predictions of our model may help to rationalize this seemingly contradictory empirical evidence. In the context of our model, changes in the distribution of income may affect the long-run growth rate in either direction, depending on the nature of the redistribution.

10 Conclusions

In this paper we have integrated Schmookler's concept of "demand-induced inventions" into an endogenous growth framework. A natural implication of this concept is that incentives to innovate depend on the distribution of income. We have identified various channels by which a redistribution may affect growth in such a context. First, a transfer from richer to poorer consumers leads to more "backloaded" profits, that is lower profits initially, and higher profits later on. This effect reduces the incentive to innovate and harms growth. Second, a higher population size of the rich (holding the income of the poor constant), has a price- and a market-size effect. The market-size effect is clearly positive as a larger number of household purchase new products. The price-effect is negative because the rich are less wealthy (roughly the same income has to be shared among a larger group of rich people) forcing firms to lower their prices. Whether the market size- or the price-effect dominate depends on the innovator's market power. This power is high, when there is no important substitute for the innovative products and vice versa.

While the intuition for our results is most easy to grasp in the context of a two-class society, we have also shown that our results can be extended to a general distribution. In particular, we have shown that the impact on growth depends on the particular way that income is redistributed and whether or not the redistribution involves "critical" consumers. A *progressive* income transfer from (non-critical) very rich towards critical poorer consumers increases the growth rate. As long as the price of new goods falls short of the willingness to pay of the very rich, lowering the income of the very rich does not harm the innovators' profits. Increasing the income of critical poor consumers, however, has a positive effect on future profits thus raising the incentive to innovate and increasing growth. At the same time, also a *regressive* income transfer may increase growth, if the transfer involves a redistribution from non-critical poorer to critical richer consumers.

Needless to say that our model can be extended in several directions. Here we want to mention two extensions that seem particularly interesting in the present context. First, our analysis has assumed that growth is driven by the increase in technological knowledge that arises with the introduction of new final goods. Other potentially important sources of technical progress were ruled out by assumption. For instance, no learning within sectors takes place. However, when the amount of learning depends on the size of the market, we have most learning when all demand is concentrated among a few number of sectors. Clearly this would establish a bias towards a situation where more inequality reduces growth.

A second extension refers to the fact that our model exhibits a scale effect. Removing the scale effect by introducing a quality (or cost saving) dimension (as in the models survey by Jones, 1999) will qualitatively change the mechanism by which inequality affects growth. Besides the effect that inequality affects the innovation process within a sector, higher inequality is associated with more product diversity and aggregate R&D expenditures will be spread out across a larger number of sectors. Hence via this channel, higher inequality will be associated with less technical progress in the aggregate.

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Appendix A1: Proof of Lemma 1

Part a. In integral notation, the value of innovation, henceforth denoted by B, is given by

$$B = \int_t^{t+\Delta} (1-\beta)\left(\varphi(n)e^{g\gamma(s-t)} - 1)e^{-r(s-t)}ds + \int_{t+\Delta}^{\infty} \left(\left[\beta + (1-\beta)n^{-\gamma}\varphi(n)\right] e^{g\gamma(s-t-\Delta)} - 1\right) e^{-r(s-t)}ds.$$

Evaluate this expression at $g \to 0$ (note that $\Delta \to \infty$).

$$B|_{g=0} = \int_t^{\infty} (1-\beta)\left(\varphi(n^{ZP}) - 1\right) e^{-\rho(s-t)}ds = \frac{1-\beta}{\rho}\left(\varphi(n^{ZP}) - 1\right) = \frac{F}{b}$$

We solve for $\varphi(n^{ZP}) = 1 + F\rho/\left[(1-\beta)\, b\right]$and Lemma 1a. follows.

Part b. To determine the slope of the ZP-curve, it suffices to check the signs of the partial derivatives with respect to the endogenous variables. First, it can be seen directly that the value of innovations increases in n. Second, take the derivative with respect to g and we get, note that $\Pi_R(N(t+\Delta)) = \Pi_{tot}(N(t+\Delta))$,

$$\begin{aligned}\frac{\partial B}{\partial g} &= \int_t^{t+\Delta} (1-\beta)\left[(\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\varphi(n)e^{g\gamma(s-t)}\right] e^{-r(s-t)}(s-t)ds \\ &\quad + \int_{t+\Delta}^{\infty} \left[(\sigma(1-\gamma)+\gamma) - \sigma(1-\gamma)\left(\beta + (1-\beta)n^{-\gamma}\varphi(n)\right) e^{g\gamma(s-t-\Delta)}\right] e^{-r(s-t)}(s-t)ds.\end{aligned}$$

We give first a sufficient condition for the second integral above to be negative. If $\gamma \leq \bar{\gamma} \equiv \sigma F\rho/\left(\sigma F\rho + b\right)$, the following holds $n^{-\gamma}\varphi(n) > \varphi(n) \geq \varphi(n^{ZP}) = 1 + F\rho/\left[(1-\beta)\, b\right] \geq 1 + \gamma/\left[\sigma(1-\gamma)(1-\beta)\right]$ where $\varphi(n) \geq \varphi(n^{ZP})$ is true when the curve is upward sloping. The condition on $\varphi(n)$ implies that the term in brackets of the second integral is nonpositive.

The same condition also implies the first integral to be negative: We directly see that $\sigma(1-\gamma)+\gamma-\sigma(1-\gamma)\varphi(n)e^{g\gamma(s-t)} \leq 0$ if $\varphi(n) \geq 1+\gamma/\left[\sigma(1-\gamma)\right]$, which is a weaker condition.

Part c. ¿From equation (9) we see that $\varphi(n) \to \infty$ as $n \to \vartheta^{1/(1-\gamma)}$. Hence, the keep the value of innovation B constant, the growth rate g must go to infinity as $n \to \vartheta^{1/(1-\gamma)}$.

Appendix A2: Proof of Proposition 2 It suffices to ask how the equilibrium curves defined by (15) and (10) are affected. From the static equilibrium condition (9) we see that $\frac{\partial\varphi}{\partial\theta} < 0$ and $\frac{\partial\varphi}{\partial\beta} > 0$. A rise in θ does not affect RC, since this parameter does not appear. A rise in β, however, implies that less resources are needed, RC shifts up. To discuss the shifts of ZP note that $\Pi_{tot}(j) = [\beta n^{\gamma} + (1-\beta)\varphi(n)] \left(\frac{j}{N}\right)^{-\gamma} - 1$ and $\Pi_R(j) = \left[\varphi(n)\left(\frac{j}{N}\right)^{-\gamma} - 1\right](1-\beta) = \varphi(n)(1-\beta)\left(\frac{j}{N}\right)^{-\gamma} + \beta$. Using the formula for $\varphi(n)$ from equation (9) we get the expression $\varphi(n)(1-\beta) = \frac{\beta(1-\theta)n}{\theta - n^{1-\gamma}}$. Hence, $\varphi(n)(1-\beta)$ falls in θ and increases in β. With n fixed, we directly get the result that $\frac{\partial\Pi_{tot}(j)}{\partial\theta} < 0$, $\frac{\partial\Pi_R(j)}{\partial\theta} < 0$ and $\frac{\partial\Pi_{tot}(j)}{\partial\beta} > 0$, $\frac{\partial\Pi_{tot}(j)}{\partial\beta} > 0$. Consequently, the ZP-curve shifts to the right when θ increases and it shifts to left when β increases.

Appendix A3: Equilibrium equations for regime IW When neither the poor nor the rich can afford all differentiated products available on the market we are in regime IW. Equation (11) obtains straightforward. When $N_P < N_R < N$ the demand for labor is $gF_N + b\left[(1-\beta)\, N_R/N + \beta N_P/N\right].$ Using $n \equiv N_P/N$ and $m \equiv N_R/N$ expression (11) immediately follows.

To get equation (12) note first that goods $j \in (N_R, N]$ have no demand. The structure of prices can be expressed by using (3), $p(N_R) = 1$, and (from monopolist N_P's arbitrage condition) $p(N_P) = (1-\beta)(N_P/N_R^{-\gamma}) + \beta$. Now use these expressions for prices and equation (7) to rewrite budget constraints (2) as

$$\begin{aligned} wl_P + (r-g)\,\frac{V_{tP}}{N_t} &= \left[\beta m + (1-\beta)m^{1-\gamma}\right]\frac{1}{1-\gamma}, \text{ and} \\ wl_R + (r-g)\,\frac{V_{tR}}{N_t} &= \left[\beta m + (1-\beta)m^{1-\gamma}\right]\frac{1}{1-\gamma} + \frac{1-m^{1-\gamma}}{1-\gamma}. \end{aligned}$$

Dividing the former equation by the latter, using our distributional assumption, yields equation (12).

Finally, to obtain equation (13), note first that innovators have waiting time. How long is the waiting time? On a balanced growth path with rate g, this waiting time δ is defined by the equation $N_R(t)e^{g\delta} = N(t)$, or equivalently, $\delta = -\ln(N_R/N)/g = -\ln(n/m)/g$. Hence demand is zero during the time interval $[t, t+\delta)$, $1-\beta$ during the interval $[t+\delta, t+\Delta)$, and 1 from $t+\Delta$ onwards. The price is 1 at date $t+\delta$, increases at rate $g\gamma$ until date $t+\Delta$ when it falls to $\beta + (1-\beta)m^{-\gamma}$, and increases at rate $g\gamma$ thereafter. The production cost remain constant over time. It is then straightforward to calculate the present value of this profit flow. Replacing r by equation (7), δ by $-\ln(n/m)/g$, and Δ by $-\ln n/g$ we get condition (13).

Appendix A4: Equilibria with a non-innovative sector With non-innovative goods the intertemporal problem of the household changes to

$$\max \int_{\tau}^{\infty} \frac{1}{1-\sigma} \left[x_i(t)^{\nu} \int_0^{N(t)} j^{-\gamma} c(j,t) dj \right]^{1-\sigma} e^{-\rho(t-\tau)} dt$$

$$\text{s.t.} \quad 0 \leq \int_t^{\infty} w(s) l_i e^{-R(s,t)} ds + V_i(t) - \int_t^{\infty} \left[p_x(s) x_i(s) + \int_0^{N(s)} p(j,s) c_i(j,s) dj \right] e^{-R(s,t)} ds.$$

The first order conditions, respectively, for $c_i(j,s)$ and $x_i(s)$ are

$$\begin{aligned} c_i(j,s) &= 1 \qquad p(j,s) \leq x_i(s)^{\nu} j^{-\gamma} \frac{e^{R(s,t)-\rho(s-t)}}{\mu_i} \equiv z_i(j,s), \\ c_i(j,s) &= 0 \qquad p(j,s) > z_i(j,s), \\ v x_i(s)^{\nu-1} \int_0^{N(s)} j^{-\gamma} c_i(j,s) dj &= \frac{\mu_i}{e^{R(s,t)-\rho(s-t)}} p_x(s). \end{aligned}$$

The case $N_P < N_R = N$

Recall that p is the price of the most recent innovator, and from the above first order conditon that $p(j,t) = p \cdot (j/N(t))^{-\gamma}$ for goods $j \in (N_P(t), N(t)]$ and $[\beta + (1-\beta)pn^{-\gamma}] \cdot (j/N(t))^{-\gamma}$ for goods $j \in [0, N_P(t)]$. We use this, and the above first order condition to express to calculate equilibrium consumption of non-innovative goods as

$$\begin{aligned} x_P &= \frac{\nu N}{1-\gamma} \frac{[\beta + (1-\beta)pn^{-\gamma}]}{wa} n, \text{ and} \\ x_R &= \frac{\nu N}{1-\gamma} \frac{p}{wa}. \end{aligned}$$

Furthermore, we use the above expressions for x_R and x_P to rewrite the budget constraints of rich and the poor consumers, respectively, as

$$\begin{aligned} wl_P + (\rho - (1-\gamma) g) \frac{V_{tP}}{N_t} &= \left[\beta n_P + (1-\beta) p n^{1-\gamma} \right] \frac{1+\nu}{1-\gamma}, \text{ and} \\ wl_R + (\rho - (1-\gamma) g) \frac{V_{tR}}{N_t} &= \left[\beta n_P + (1-\beta) p n^{1-\gamma} \right] \frac{1}{1-\gamma} + p \frac{1+\nu - n^{1-\gamma}}{1-\gamma}. \end{aligned}$$

Dividing the former equation by the latter and making use of our assumption on the endowment distribution $l_P/l_R = V_{tP}/V_{tR} = \theta_P/\theta_R = \vartheta(1-\beta)/(1-\vartheta\beta)$ yields equation (16) in the text.

With non-innovative goods, the resource constraint is given by $1 = b\left[\beta n + (1-\beta)\right] + a\left[\beta x_P + (1-\beta) x_R\right] + Fg$. We replace x_P and x_R by the above expression and p by $\varphi(n_P)$. This yields equation (15) in the text.

The case $N_P = N_R = N$

Now denote by $p = z_P(N)$ the willingness to pay of the *poor* for the most recent innovator's product. From the first order condition (third condition) we get

$$x_P = \frac{\nu N}{1-\gamma}\frac{p}{wa}.$$

We cannot calculate the corresponding value of x_R as the willingness to pay for the rich is strictly larger than p. Hence we cannot directly replace μ_R in the first order condition for x_R. To find an expression for x_R we use the budget constraints, respectively, for the rich and the poor consumers

$$\begin{aligned} wl_P + (r-g)\frac{V_{tP}}{N_t} &= p\frac{1+\nu}{1-\gamma}, \text{ and} \\ wl_R + (r-g)\frac{V_{tR}}{N_t} &= \frac{p}{1-\gamma} + wax_R. \end{aligned}$$

Dividing the former equation by the latter and using our assumption on distribution allows us express x_R as

$$x_R = \frac{p}{wa(1-\gamma)}\frac{\nu(1-\beta\vartheta)+1-\vartheta}{(1-\beta)\vartheta}. \tag{19}$$

The resource constraint in regime $N_P = N_R = N$ is given by $1 = b + a\left[x_P + (1-\beta)x_R\right] + gF$. Replacing x_R and x_P by the above equations yields equation (17) in the text.

To get the second equation in g and p, consider the R&D equilibrium condition. The value of an innovation in the regime $N_P = N_R = N$ can be easily calculated. The innovator of date t has demand 1 from date t onwards and charges a price p that increases at rate $g\gamma$. Straightforward calculation yields equation (18) in the text.