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ABSTRACT

Jump-and-Rest Effects of US Business Cycles*

One of the most extended empirical stylized facts about output dynamics in the United States is the positive autocorrelation of output growth. This paper shows that the positive autocorrelation can be better captured by shifts between business cycle states rather than by the standard view of autoregressive coefficients. This result is extremely robust to different non-linear alternative models and also applies not only to output, but also to the most relevant macroeconomic variables.

JEL Classification: C22, E27 and E32
Keywords: business cycles, output growth and time series

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1 Introduction

The positive and significant autocorrelation of output growth is one of the few empirical observations about business cycle dynamics that is widely accepted in the literature. Recently, Timothy Cogley and James Nason (1995) documented this stylized fact for the U.S. economy over short horizons corroborating the results already found by other authors as Charles Nelson and Charles Plosser (1982), Mark Watson (1986), or John Campbell and Gregory Mankiw (1987). All of these papers follow the standard view that the autocorrelation in output is well characterized by autoregressive processes.

This observed positive autocorrelation has been of crucial importance to evaluate the empirical relevance of real-business-cycle (RBC) models in characterizing the output dynamics. Along this line, Timothy Cogley and James Nason (1995) pointed out the difficulties of RBC models to reproduce this recognized pattern. On the one hand, standard RBC models, even when adding intertemporal substitution, capital accumulation, or cost of adjusting the capital stock, have weak internal propagation mechanisms and need to be complemented with exogenous sources of dynamics in order to match the autocorrelation found in the data. On the other hand, non-standard RBC models, as those that assume lags or costs of adjusting labor input, are only partially successful since they need to incorporate implausibly large transitory shocks. Consequently, these authors consider the difficulties to match the autocorrelation of the data as a failure of RBC models and suggest that RBC theorists ought to devote further attention to modeling internal sources of propagation in order to replicate the *right* pattern of output dynamics.

The purpose of this paper is to provide empirical evidence in favor of a novel alternative view of aggregate growth dynamics. We show that output growth is characterized by a recurrent sequence of shifts between two steady states of high and low growth means. This sequence explains the dynamics of output growth better than the standard autocorrelated time series alternative. For this attempt, we begin our analysis in a simple scenario in which we assume that the switches between the two states coincide with the widely accepted record of turning points identified by the National Bureau for Economic Research (NBER). Under this assumption, we obtain that, once the business cycle phases are accounted for, the autocorrelation in output growth is no longer significant, being the system dynamically complete.

However, we understand the limitations in terms of availability and endogeneity of using the NBER sequence to model the dynamic specification of output growth. In order to overcome those limitations, we propose nonlinear extensions to the baseline model that provide inference of the business cycle shifts without any of the inconveniences of exogeneously considering the location of the NBER turning points. To prevent our study against the dependence of our results to any particular nonlinear specification, we use a wide range of nonlinear proposals and we find that, since they all are able to identify sequences of business cycle states that are similar to the NBER chronology, the absence of autocorrelation is remarkably robust to any of these nonlinear models. According to these results, we can conclude that the expected U.S. output growth displays a dynamics as simple as a series that switches back and forth between the two fixed equilibria. For large enough shocks, output growth shows sharp transitions from one regime to the other regime. However, smaller shocks have no dynamic effect and output growth fluctuates around each of these states as a white noise exhibiting no conditional autocorrelation.

To be sure that we are appropriately addressing the actual data generating process for output growth, we carry out several robustness checks. First, we check that the absence of autocorrelation is an intrinsic characteristic of the output growth series and it is not a consequence of the particular sample period selected in the paper or the last output growth releases. Second, we obtain that the recurrence of declines and recoveries proposed by the NBER's dating committee is the sequence of business cycle dummies which reduces the autocorrelation in output growth the most. Third, while we have primarily focused on output growth, we detect that the absence of autocorrelation has been an important secular regularity affecting other key macroeconomic aggregates, such as real consumption, investment, and sales, typically assumed to be positively autocorrelated and estimated by using autoregressive parameters. Finally we empirically show that multiequilibria models in which the shifts among equilibria are governed by Markov chains may be good starting specifications in order to replicate the main U.S. business cycle characteristics.

This new characterization of output growth (and other economic aggregates) has several important implications. First, our findings can be interpreted as empirical evidence in favor of recent developments in theoretical macroeconomics that explain output dynamics as stochastic switches between periods of low and high growth with different sources of business cycle fluctuations. Examples of these papers are George Evans; Seppo Honkapohja and Paul Romer (1998) that rely on complementarities among different types of capital goods, and Costas Azariadis and Bruce Smith (1998) where adverse selection problems in financing capital goods create credit cycles associated with business cycles. Second, and coming back to the Cogley-Nason findings, our results may serve as a guideline to resuscitate theoretical models that were neglected because positive autoregressive parameters were accepted as roughly the true source of the output growth short-run persistence. Finally, from a technical point of view, predictions, impulse responses, and dynamic multipliers obtained in nonlinear contexts become much simpler and more intuitive since they solely rely on our believes about current and future states of the cycle and also, the absence of autocorrelation minimizes the mathematical complexity and the computational cost of simulation and calibration exercises.

The paper is organized as follows. Section 2 outlines the standard and new stylized facts about the U.S. economy, provides a simple scenario to take them into account, and introduces the main characteristics of the absence of autocorrelation. Section 3 examines the robustness of this new fact to the sample period, to the business cycle chronology, and to other real aggregates. Section 4 reveals how the results of the nonlinear specifications, that generate inferences about the business cycle timing, corroborate the previous findings. Section 5 evaluates the empirical reliability of our new characterization of output growth. Section 6 concludes.

2 New facts about output growth dynamics

2.1 Stylized facts

The time series literature reports three stylized facts about postwar output growth dynamics in the United States: Output growth is positively autocorrelated, exhibits a remarkable business cycle dependence, and experienced a decline in volatility in the mid eighties. Quotes to these facts are all over in the literature, but we can easily appreciate them just by looking at the series. Figure 1 presents those facts for the growth rate of U.S. real Gross Domestic Product (GDP) for the period 1953.1 − 2004.1. In this figure, Chart 1 reports the total and partial sample autocorrelation functions for output growth, along with the ninety-five confidence bands ($\pm 2/\sqrt{T}$, where T is the sample size). Chart 2 plots the output growth series, along with several shaded areas that correspond to the NBER recessions, and a vertical dashed line that refers to 1984.1. Finally, Chart 3 shows the kernel density estimate of output growth before and after the volatility break of 1984.1.

As shown in Chart 1, the pattern of the total sample autocorrelation function appears consistent with the simple geometric decay of autoregressive processes of order one, henceforth $AR(1)$. In addition, the partial autocorrelation function could be viewed as dying out after one lag, also consistent with the $AR(1)$ hypothesis with an autoregressive parameter of about 0.32. This standard result suggests that output growth presents positive autocorrelation and has been adopted by the literature as an empirical stylized fact. In fact, this is the motivation of Cogley and Nason (1995) to review the standard theoretical real-business-cycle (RBC) models and to incorporate exogenous sources of dynamics in order to replicate this impulse dynamics.

Chart 2 and the first column of Table 1 reveal that, while output growth fluctuates around its mean of 0.81, the broad changes of direction in the series seem to mark quite well the NBER-referenced business cycles. During expansions, output growth is usually higher (mean of 1.04) than its unconditional mean, but declines significantly within recessions (mean of -0.51). However, these business cycle differences do not seem to affect output volatility (standard deviations of 0.75 in expansions and 0.85 in recessions). Simple tests of the null of no different within-recessions and within-expansions means and variances are clearly rejected for the means and non rejected for the variances (p-values of 0.00 and 0.40, respectively).

Finally, Chang-Jim Kim and Charles Nelson (1999), and Margaret McConnell and Gabriel Perez-Quiros (2000) among other authors, have recently detected a substantial moderation in output growth volatility, with the suggestion that this moderation is well modeled as a single break in the mid eighties. We show empirical evidence in favor of this fact in the first column of Table 1. In particular, we update the supremum, exponential, and average tests used by Margaret McConnell and Gabriel Perez-Quiros (2000) to corroborate that 1984.1 is still the more appropriate break date to consider the structural change in volatility. This fact is also illustrated in Figure 1 (Chart 3), where it is clear that, after the break, the distribution of output growth is more tightly centered about its mean. The results of the Kolmogorov-Smirnov tests and the Wilconxon tests of equality of the quartiles are also displayed in Table 1, where the null of no change in the distribution of output growth is clearly rejected. However, contrary to the case of the business cycle, this break does not seem to affect the mean but the volatility. The former only moves from 0.81 to 0.80 while the latter dramatically falls from 1.14 to 0.54. This result is reinforced by the standard tests of no different means and variances that show p-values of 0.93 and 0.00, respectively.

2.2 A simple approach

In order to deal with the previous facts about the output growth dynamics, a good place to start is a simple linear autoregressive model. The evidence presented in the previous section supports a first order process as the best first candidate. First column of Table 2 presents the estimates of the model, labelled as $M1$,

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t, \tag{1}$$

where y_t represents output growth at time t, and $\varepsilon_t \sim N(0, \sigma)$ which is identically and independently distributed over time. As stated in textbooks, the estimated autoregressive coefficient is about 0.32 and generates an endogenous propagation of impulses that accounts for the positive autocorrelation stated above. That is, the k-period ahead impact of an unanticipated shock is estimated to be 0.32^k. Figure 2 (Chart 1) shows the in-sample fitting of this model by plotting both the actual and the estimated growth rates. As expected, after the negative shocks that characterize the peaks, output growth falls during recessions. However, it is interesting to realize that, in all recessions, due to the smooth dynamics implicit in this autoregressive model, estimates notably remain above the actual series.

The simple model in $M1$ can be easily extended to take into account the volatility break just by assuming that $\varepsilon_t \sim N(0, \sigma_t = d_0 + d_1 B_t)$, where B_t is a dummy that equals one in the period $1984.1 - 2004.1$. Second column of Table 2, labelled as $M2$, presents the estimates of this specification. The estimate of the coefficient d_1 is negative and statistically significant showing the reduction in volatility of output growth.

2.3 *Jump-and-rest* effect of business cycles

In this section we look at how business cycle fluctuations influence the positive autocorrelation of output growth documented in the previous section. To address this question, the simplest way of taking into account the whole set of stylized facts is by adding a dummy variable to the previous baseline model, $M2$, that equals one in the NBER recessionary periods. We understand the limiations in terms of availability and endogeneity of using the NBER sequence. Advanced reader can skip this section and go directly to the non-linear modelization. However, we consider that this section is a good ilustration of the nature of the results obtained with the more sophisticated modelizations.

We use N_t to denote the dummy variable that capture the NBER recession periods. The different ways in which the break in volatility dummy (B_t) and the NBER dummy (N_t) can modify the previous regressions are numerous. A general characterization of several of these modifications can be summarized by the following expression:

$$y_t = a_0 + a_1 y_{t-1} + b_0 N_t + c_0 B_t + c_1 B_t N_t + \varepsilon_t, \tag{2}$$

where $\varepsilon_t \sim N(0, \sigma_t = d_0 + d_1 B_t + d_2 N_t)$. Models $M3$ to $M5$ are generalizations of the standard linear autoregressive specification with heteroskedasticity $M2$.

In model $M3$ the NBER dates are allowed to interact with the intercept (b_0 different from 0).[1] This extension clearly improves the specification with respect to $M2$. Model $M3$ already reflects one of the main empirical findings of this paper: once the business cycle movements of output growth have been taken into account, the autoregressive parameter is no longer statistically significant. According to this result, the U.S. economy seems to be characterized by two different steady states. In the first one, the average growth rate of output is positive while in the second one is negative. In each of these states, output growth fluctuates around its mean value as a white noise exhibiting no autocorrelation.

Contrary to the autoregressive processes, the next period expected impact of an unanticipated one-unit increase in current output growth is no longer one third. Instead, the impact depends on the magnitude of the shock. To understand this point, let us take model $M4$ that, according to the result of the significativity test, imposes to $M3$ the excluding restriction that the autoregressive parameter is zero. Now assume the economy is in the negative

[1] We failed to obtain any statistically relevant finding from the obvious general proposal. For example, allowing the NBER dates to interact with the autoregressive coefficient produces an estimate of -0.09 with standard error of 0.18.

growth steady state. For low shocks, the expected impact on output growth is zero. Thus, output growth is expected to remain at its negative growth state mean of -0.42. However, for drastic innovations, large enough to change the state of the economy, the expected instantaneous impact on output growth is 1.37, and zero in subsequent periods, leading output growth to rise until its positive growth state mean of 0.95.[2] Figure 2 (Chart 2) illustrates this dynamics: output growth estimates switch sharply at turning points and remain constant at each steady state mean within states. This is why we call this particular effect of business cycles on output growth dynamics *jump-and-rest* effect of business cycles.

Before following with this analysis, it is worth to examine whether this simple model can be accepted as adequate as the first order autoregressive model usually does in the standard literature. In particular, model $M4$ residuals have zero mean (p-value of 0.21) and are normally distributed (the p-value of the Jarque-Bera normality test is 0.14). More importantly, we have to check whether this model is dynamically complete. If we had erroneously eliminated the first order autocorrelation of model $M3$, the unestimated model dynamic would have appeared in the residuals of the resulting model (that is, model $M4$), so they would have been serially correlated. However, if there was nothing to be gained by adding any lag of output growth to model $M4$, its residuals would be white noise. These residuals are plotted in Figure 3, Chart 1. In Chart2, we show the autocorrelation functions of the residuals, which support the white noise prior since the autocorrelations at the various lags are statistically insignificant. More formal tests of the null

[2] We return to this point in the next section in an attempt to provide an estimate of the threshold that marks the magnitude of shocks that are able to change the expected growth, and a description of the transition between states.

to detect possible serial correlation in the residuals are the Durbin-Watson, the Breusch-Godfrey, and the Brock-Dechert-Scheinkman tests. The former test shows a statistic of 1.78 that lies in the no autocorrelation zone (about $1.69 - 2.31$). The second test presents a p-value of 0.18, which does not allow us to reject the null hypothesis that residuals are white noise. The last test, for pairs of residuals that lie in hypercubes of size 1.75 times the standard deviation, shows a p-value of 0.28, which confirms that the residuals are independent.[3]

Before ending this section, we address in Table 2 two additional minor questions about output growth dynamics. The first one has to do with the potential business cycle dependence of output volatility. To examine this question, model $M5$ adds the NBER dummy to the specification of the standard deviation (d_2 different from 0). Following the $M5$ estimates, we conclude that, when the volatility break is accounted for, the recessionary dummy does not affect output volatility (p-value 0.10). The second issue deals with the analysis of whether the reduction in volatility induces a narrower gap in the business cycle means. In this respect, model $M6$ includes the volatility dummy in the mean specification (c_0 and $c1$ different from 0). The resulting estimates show that the break significantly affects the business cycle dynamics (p-value of joint significance of these dummies 0.007). According to Figure 3, Chart 3, this implies that the volatility reduction may be due to both a narrowing gap between growth rates during recessions and expansions as in Chang-Jim Kim and Charles Nelson (1999), and a decline in output volatility as in Margaret McConnell and Gabriel Perez-Quiros (2000).[4]

[3] According to the quarterly frequency of output growth, we conduct the Breuch-Godfrey test using four lags.

[4] Output growth mean falls from 1.17 to 0.87 in expansions and rises from -0.56 to -0.25 in recessions after the volatility break. In addition, its standard deviation is reduced

3 Robustness analysis

In this section we investigate the robustness of the *jump-and-rest* effect of business cycles in three different ways. First, we examine whether the absence of autocorrelation is a recent development or if it is robust to the sample period considered. Second, we check to what extent this effect is related to the particular sequence of business cycles proposed by the NBER. Finally, we study whether this effect is limited to output growth or shared by other U.S. major macroeconomic aggregates.

3.1 Is the *jump-and-rest* effect robust to the sample period?

We have detected that, accounting for the business cycle phases, additional autoregressive parameters are no longer statistically significant. However, an interesting question to analyze could be if this fact is just a consequence of the sample period studied or, instead, if it is an intrinsic characteristic of the output growth dynamics.

This question is addressed in Figure 4 (first row of charts) by using a recursive approach estimation of output growth. That is, we start by estimating the autoregressive parameters for a short sample covering from 1953.1 to 1963.1. Then, we iteratively expand the initial sample by one observation and re-estimate the parameters in two different scenarios. In the first one, we assume the process to be the simple first-order autoregressive specification stated in (1). Chart 1a shows the OLS estimates of the slope parameter and Chart 1b plots the p-value of the null of non-significativity. In these graphs, we observe a secular decrease in the magnitude of the slope parameter whilst

from 0.88 to 0.46.

it always remains very highly significant. The second scenario modifies the autoregressive process by the inclusion of the additive NBER-recessionary dummy variable N_t. Chart 1c shows that, once we allow for business cycle shifts around turning points, the autoregressive parameter becomes negligible, and Chart 1d reveals that it has never been statistically significant. These results confirm that, once accounted for the business cycle shifts, the absence of positive autocorrelated parameters in the output growth specification is robust to the sample period.

3.2 On the uniqueness of the NBER cycles

Up to this point, we have identified that the NBER business cycle fluctuations represented by a particular sequence of zeroes (expansions) and ones (recessions) has absorbed and continues to absorb the autocorrelation in the output growth dynamics. An obvious question that arises in the development of this property is to examine whether this is common to a few or to many other business cycle sequences, or whether the reduction in the autocorrelation of output growth achieved by the NBER chronology converts their sequence in "unique" in some sense.

In order to address this question, we propose different exercises. First, we want to examine to what extent the *jump-and-rest* effect remains significant under minor diferences in turning points identifications. For this attempt, we use leads and lags of the NBER additive dummy as regressors in the OLS regression of GDP growth rates on an intercept and on its lagged value. That is, we estimate

$$y_t = \alpha + \beta_i y_{t-1} + \gamma_i NBER_{t-i} + \varepsilon_t, \tag{3}$$

for $i = -4,..., 0,..., 4$, where the random error ε_t is iid normal with mean 0 and variance σ^2. In Figure 5, we present the estimated coefficients γ_i for each value of i, along with their 95% confidence intervals. As we can observe, only the coefficient γ_0 eliminates the correlation in the data. All the other values of i imply confident intervals that do not contain the value $\gamma_i = 0$. Therefore, minor differences in turning point identification imply the lost of the *jump-and-rest* effect of the business cycles.

In a second exercise, we consider by how much the autoregressive reduction achieved by the NBER chronology is shared by other business cycle sequences. This exercise is performed in two scenarios. In the first one, we create business cycle sequences that share the same business cycle properties as the NBER-dated phases. For this attempt, we generate $10,000$ blocks of recessions and expansions generated from a Markov process whose probabilities of staying in expansions, of staying in recessions, and of changing the state give an expected value of the blocks equal to the ones observed in the NBER data. With these $10,000$ series of zeroes and ones, we repeat the regressions outlined in (3), where, instead of using NBER leads and lags, we use each of the generated dummies. The result cannot accept the the null hypothesis of $\gamma_i = 0$ in any case. Actually, the minimum value of the t-statistic is 3.68. In the second scenario, we want to avoid the dependence of the analysis with respect to the NBER business cycle characteristics. In this case, we randomly generate $1,000$ threesomes of probabilities of staying in expansions, staying in recessions, and switching the regime.[5] For each of these threesomes, we generate $1,000$ business cycle dummies and repeat the previous regression exercise. Remarkably, our result is qualitatively the same: of the $1,000,000$ regressions (that is, $1,000$ threesomes times $1,000$ dummies)

[5] In order to obtain business cycle dummies with economic sense, we impose that the probabilities of staying in each state are above one half, and that the probability of staying in expansions is greater than the probability of staying in recessions.

the minimum t-statistic of the null of $\gamma_i = 0$ is 3.55. Thus, these results reinforce the idea that the autocorrelation reduction is consistent with some particular business cycle characteristics associated to the sequence proposed by the NBER.

Finally, we would like to go even further and try to compare its ability against all the possible combinations of zeroes and ones. However, due to the current capacity of our personal computers, the problem seems to be intractable (206 observations imply $2^{206} = 1.02 * 10^{62}$ possible combinations).[6] As an alternative, we propose an algorithm for seeking a global minimal value in the autocorrelation significativity over a huge amount of competing business cycle dummies, but trying to keep the problem computationally feasible. We start the algorithm by generating the $65,536$ different combinations of recessions and expansions for the first 16 observations.[7] We drop from this set of possible combinations those that do not have a minimum size block of two observations (this implies $19,856$ combinations left). As usual, we use the remaining combinations as additive business cycle dummies in the first order autoregressive regression and keep only those k combinations that provide a p-value of the null hypothesis of $\gamma_i = 0$ that is smaller than or equal to the one obtained by using the NBER sequence. Now, we consider that those k selected business cycle sequences could be followed by an expansion (add one more zero) or by a recession (add one more one), obtaining $2 * k$ business cycle combinations. With these $2 * k$ combinations, we repeat the exercise of regresing them as dummies in the first order autoregression.

[6] In fact, we were able to develop an algorithm that examines the jump-and-rest effect in any combination of zeroes and ones. However, according to our preliminary results, we would have required more than 1 year of iterations to finish up the calculations.

[7] We tried with different starting sample sizes but they yielded the same results.

We then continue with this process until we get to the last observation.[8] Out of this algorithm we obtain that there is only one sequence of zeroes and ones that reduces the autocorrelation in the GDP data more that the NBER recession dummy. This sequence is exactly the same that the NBER recession but adding as recession periods the quarters 1990.3, 1991.2 and 2001.1.[9] Therefore the 1991 recession may start one period before and end one period later, and the last recession may end one period later, as already pointed out by Camacho (2004) in an independent study.

We then continue with this process until we get to the last observation leading to the following results. First, up to 1981, we obtain just one combination out of the algorithm, the NBER combination. Second, recessions usually imply that the number of selected combinations increases but they always stabilize after we add a few more observations. In particular, the 1981 recession is the noisiest since it creates up to sixty selected blocks that stabilizes in the nineties. Third, only the recession in the nineties leaves one combination that dominates the NBER recession indicator: according to this algorithm, one period before and after the official 1991 recession should also be considered as recession periods (1990.3 and 1991.2). Something similar happens with the last NBER recession that, according to the algorithm, should last one more quarter (the algorithm locates the trough in 2001.1).[10]

Summing up all of these results, we find that the NBER recession periods

[8] We understand that the best analysis would come from examining all the possible combinations of zeroes and ones. However, given the impossibility of doing that, we think that the approach that we follow here is reasonable because it directly relates with the property of robustness across time that has been examined in the previous subsection.

[9] This 2001.1 correction is not necessary when repeating the algorithm taking into account the heteroskedasticity associated to the volatility break in 1984.1.

[10] The results are robust to the heteroskedasticity associated with the break in 1984.1. However, in this case, the last NBER recession does not need correction.

represent a succession of blocks of zeroes and ones with a property never previously found in the literature. Our results support the hypothesis that there is something "special" about the sequence of business cycles established by the NBER since it is very close to be the one that absorbs the autocorrelation of the GDP growth series the most.[11]

3.3 Does it affect other U.S. macroeconomic aggregates?

Table 1 (last six columns) analyzes whether the stylized facts that have been previously documented for output growth appear in other U.S. real macroeconomic variables. In particular, the analysis includes the rate of growth of Personal Consumption Expenditures (PCE), Gross Private Domestic Investment (GPDI), Government Consumption and Investment (GCI), Exports of Good and Services (EGS), and Final Sales of Domestic Product (FSDP). In all of these series but GCI, the business cycle phases seem to affect the first but not the second moment. The decline in volatility is significant in all the series, by using both informal tests of different standard deviations (p-values of 0.00) and formal structural break tests (vast majority of p-values below 0.05). The timing in this reduction is in either mid eighties (82.3 for EGS, 84.1 for GDP and GDPI, and 85.1 for IGS) or early nineties (92.1 for PCE and 92.4 for FSDP), with the exception of government expenditures whose break date occurs in the mid sixties. In addition, with the exception of consumption, the moderation in volatility is associated with reductions in the

[11] According to our results, we consider that this particular property of the NBER cycles may be used as an alternative way of identifying the business cycle phases in other countries. However, this is out of the scope of this paper and we think that it could be material for further research.

conditional variance after a break, not with different volatility in different business cycle phases. Specifically, in the case of consumption, the p-value of equal (within recessions and within expansions) standard deviations is 0.00. In the rest of macroeconomic variables, their respective p-values are always higher than the standard significance level of 0.05.

As in the case of output, the analysis of the autocorrelations is the main interest of this paper. The first four rows of Table 1 show that the autoregressive coefficients of consumption, investment and sales are positive and statistically significant. Their point estimates are 0.29, 0.16 and 0.18, and their p-values are 0.00, 0.02, and 0.00, respectively. However, they become negligible and statistically insignificant when the additive NBER dummy is introduced in their respective baseline first order autoregressive processes. Specifically, their point estimates become 0.08, -0.06, and 0.01, and their p-values increase to 0.23, 0.38, and 0.85, respectively.

Finally, as documented in Figure 4, this empirical fact seems to be very robust to the sample period considered. The secular reduction of the autoregressive parameters is shared by consumption and investment growths but they are always highly statistically significant. However, once the NBER business cycle phases are accounted for, the magnitudes of these parameters are dramatically reduced and never statistically significant. The case of sales is somehow special because, even though the *jump-and-rest* effect of business cycles affects its dynamics since the mid eighties, the slope parameter in a simple autoregressive regression is not significant for series that end prior to these years.

4 Nonlinear models of output growth

Up to this point, we have tried to confront two rather different views about the behavior of output growth dynamics. The first one is the standard description of output growth as a first order autoregressive process. This specification implies that an unanticipated shock gradually mitigates over time. The second one is the primary finding of this paper and depicts output growth as fluctuating around two steady states that coincide with the NBER expansions and recessions, probably with a narrower gap after the volatility break of the mid eighties. In this case, the expected impact of relatively high shocks (high enough to be able to shift the state of the economy) is the difference between these two steady state values. However, once the economy moves from one regime to the other one, output growth behaves like a white noise, so the impact dies out just in one period. Hence, subsequent relatively small shocks have no expected impact on output growth.

Even though we have found evidence in favor of the second one, the scenario proposed to develop the analysis was simple and had limited empirical application. In particular, we assumed to observe the discrete shifts between states directly since we used the dichotomous NBER variable as known at each time period. Additionally, this assumption implied a potential endogeneity problem of using the NBER dummy as an explanatory variable that has been constructed under the basis of knowing the actual value output growth. We overcome these two problems by using nonlinear extensions to the baseline model presented in the previous section. These specifications are useful because they provide inference about the probability of business cycle shifts in each period with information available up to that period. Furthermore, they allow us to correct the endogeneity problem that may affect the estimations of the previous section. Finally, we show that the main conclusions of this study are invariant to the wide range of nonlinear specifications that propose to account for the business cycle dynamics of output growth.

4.1 Self-exciting threshold autoregressive (SETAR)

In the autoregressive model enlarged with the business cycle dummy, the mean growth rate switches between business cycle states through the intercept term according to the NBER official classification. One possible way to endogenize the business cycles is the SETAR model, originally proposed by Howell Tong (1978).[12] In SETAR models, the regime is assumed to be determined by the value of an observed lagged dependent variable, y_{t-p}, relative to a threshold c. In particular, based on the previous analysis, we propose the following two-regime SETAR model

$$y_t = a_0 + b_0 I(y_{t-d}) + a_1 y_{t-1} + \varepsilon_t, \tag{4}$$

where $\varepsilon_t \sim N(0, \sigma_t = d_0 + d_1 B_t)$. In these models, $I(y_{t-d})$ is an indicator function taking the value of one when $y_{t-d} \geqslant c$, and zero otherwise. It is worth to note that the shifts between the two states is instantaneous by assumption and marked by the changes in the value of the indicator function from zero to one or viceversa.

Since the SETAR model is piecewise linear, all parameters can be easily estimated by maximum likelihood, provided we know the value of the threshold, c. However, since the threshold is unknown, we solve the maximization problem by searching the value of the threshold over the observed values of y_{t-d}. Finally, we choose the threshold and the lag of output growth that maximize the corresponding log-likelihood function.[13]

[12] For an overview of SETAR models, see Bruce Hansen (1999) and the references therein.

[13] Following Bruce Hansen (1999), we restrict the maximum value of d to be the maxi-

Table 3 shows the parameter estimates in the columns labeled as SETAR. The estimates of the baseline model, that appears in the first column as SETAR1, reveal that the maximum likelihood is achieved for a threshold of 0.16. Thus, the first regime is reached whenever last period's output growth is larger than 0.16 and is associated with a large conditional mean. The second regime appears when output growth is lower than 0.16 and is associated with a low mean. In order to put some additional light in the identification of the SETAR regimes, Figure 6 (Chart 1) plots the values of the indicator function, along with the NBER recessions. Typically, the indicator function is one, that is when past growth is roughly negative, at the official recessions. This confirms that, even though we have not imposed it a priori, the SETAR model clearly makes endogenous the dynamics of business cycles.

Something crucial for the interest of this paper is that the autoregressive parameter is statistically insignificant (the p-value for this test is about 0.09). This result leads to the model SETAR2 which directly excludes the autoregressive parameters.[14] This confirms our previous findings that, contrary to the standard analysis of output growth, provided we account for the business cycle asymmetries, output growth is not autocorrelated. This result corroborates that the absence of autocorrelation when accounting for business cycles was independent of the potential endogeneity induced by considering the business cycle phases as the ones identified by the NBER.

These findings have important implications for analyzing output growth reactions to shocks. Let us assume that output growth at time t is, say, equal to 0.20. Since this value is above the threshold level of 0.16 in SETAR2, the

mum lag length in the autoregressive specification, and the thresholds to contain at least 10% of observations in each regime.

[14]We obtain the p-value by comparing models SETAR1 and SETAR2 and testing, using a likelihood ratio test, the null hypothesis of autoregressive parameter equal to zero.

economy is in the expansive phase of the business cycle. If other factors are held fixed, the expected impact of any shock greater than -0.04 (threshold minus actual growth) is zero, and the expected growth for this and subsequent periods should be the rate of growth of expansions 0.90. However, negative shocks that lower the economy below the threshold, change the state of the economy, lowering the expected growth to 0.14 $(0.90-0.76)$ instantaneously.

4.2 Smooth transition autoregressive (STAR)

The hypothesis that the U.S. output growth can switch between two states according to the value of an observed lagged variable with respect to a threshold may be generalized by using the STAR models of Timo Teräsvirta (1994). The generalization comes from the fact that these models allow for more gradual transitions between the different regimes by replacing the indicator function in (4) with the logistic transition function:[15]

$$F(y_{t-1}) = \frac{1}{1+\exp\left[-g\left(y_{t-1}-c\right)\right]}. \tag{5}$$

The role of the transition function is then to allow the mean growth rate to change monotonically with the values of the transition variable, y_{t-1}, with respect to the threshold c. The parameter g, usually called smoothing parameter, determines the degree of smoothness of the transition from one regime to the other, in the sense that the higher the parameter the sharper the change (the steeper the slope of the transition function at the threshold).

As in the case of SETAR models, the STAR specification allows us to provide the statistical regimes with economic meaning. For this attempt, the

[15] We do not consider exponential transition functions since they are symmetric around the threshold. These specifications would imply that local dynamics would be the same for expansions and recessions.

last two columns of Table 3 contain the estimates of the different STAR models that we consider. Also, Figure 6 in Charts 2 and 3 show (one minus) the transition function. Let us associate the first regime to the values of lagged growth rate lower enough than the threshold to drive the transition function to zero. Hence, from an economic point of view, this regime may be considered as a recession and, according to the parameter estimates, coincides with periods of relatively low conditional expected growth estimates. As the value of lagged growth increases, the transition function changes monotonically from zero to one. In the limit, for very high lagged growth rates that are obviously associated to expansions, the transition function reaches one, and the parameter estimates lead to relatively higher values of the conditional growth rate. Hence, the closer to one the transition function is, the more probable the economy be in expansion. This is why Chart 3 plots the value of one minus the value of the transition function. This chart suggests that periods of low transition function values (high values of one minus the transition function) correspond to the official recessions fairly well, which confirms that the regimes may be interpreted as business cycle phases.

Again, the most important conclusion in the STAR specification is that the autoregressive parameter is insignificant (p-value of 0.13). Thus, our final conclusions should be based on the simpler model STAR2, that excludes the insignificant autoregressive parameter of model STAR1. Finally, we obtain a very high value of the smoothing parameter, which indicates that the transition from one business cycle phase to the other is very quickly. This means that the STAR model behaves very similar to the SETAR model. This results can be seen in Figure 6 (Chart 2), where the transition function changes from zero to one almost instantaneously when lagged growth reaches the threshold.

4.3 Markov-Switching autoregressive (MS)

Probably, this is the most popular and most successful specification for a nonlinear model of GDP growth in the U.S. Initially formulated by James Hamilton (1989), was modified by Margaret McConnell and Gabriel Perez-Quiros (2000) to capture the break in volatility. As in STAR models, the MS specification does not impose the change in regime to be sharp. However, in MS models, as opposite to STAR models, shifts are governed by an unobservable state variable that is assumed to follow a Markovian scheme with two regimes and fixed probabilities of transition from one to another.

According to the original specification of James Hamilton (1989), output growth may be decomposed into an state-dependent mean, that takes on value μ_1 in the first state and μ_0 in the second state, and a stationary process u_t,

$$y_t = \mu_{S_t} + u_t, \tag{6}$$

where u_t follows an $AR(1)$.[16] This specification implies that

$$y_t = \mu_{S_t} + \phi_1(y_{t-1} - \mu_{S_{t-1}}) + \varepsilon_t, \tag{7}$$

with $\varepsilon_t \sim N(0, \sigma)$. Therefore, the autocorrelation of output growth may be independently determined by both the shifts in the mean of the process and the autoregressive parameter.

Since the transition between states is assumed to follow a first order Markov chain, probabilities are determined by

$$P(S_t = i/\Omega_{t-1}) = P(S_t = i/S_{t-1} = j), \tag{8}$$

where Ω_t represents all the information set in period t. This specification is modified by Margaret McConnell and Gabriel Perez-Quiros (2000) by allow-

[16] In the original proposal, James Hamilton (1989) allows for four autoregressive lags. However lags of order two or higher are not statistically significant.

ing for two independent Markov processes that capture the two stylized facts, the change in mean (goberned by S_t) and the break in volatility (goberned by V_t). Therefore, they propose the model

$$y_t = \mu_{S_t,V_t} + \phi_1 (y_{t-1} - \mu_{S_{t-1},V_{t-1}}) + \varepsilon_t, \tag{9}$$

with $\varepsilon_t \sim N(0, \sigma_{V_t})$.

The results of this regression are displayed in Table 4. As shown in the table, the Hamilton's original specification, labelled as MS1, implies that the autoregressive parameter is 0.31 and statistically significant (standard error of 0.10). This would imply that, contrary to our previous findings, in the determination of the data generating process, autocorrelation matters. However, this result is not robust to including the second stylized fact, the change in volatility. Once we take into account both facts at the same time, as shown in MS2, the autoregressive parameter decays to 0.08, with a standard error of 0.09, and is clearly non significant. Thus, confirming our previous results, the serial correlation in logarithmic changes of real GDP seems to be better captured by shifts between states rather than by the autoregressive coefficients.

Figure 7 (Charts 1 and 2) gives a clear intuition of the nature of these results. As Chart 1 shown, the original Hamilton's model provides statistically significant autoregressive parameter because it does not provide reasonable inferences on the sequence of recessions and expansions identified by the NBER. One potential reason is that the model lacks a mechanism to account for the volatility reduction. In this respect, Chart 2 shows that, once we control for the volatility reduction, the model provides inferences about the business cycles that are in close agreement with the NBER reference cycle, and in this case, there is no room for more autocorrelation in the data.

Given that the autocorrelation is not significant in the data, we try a new MS specification of a model with no autoregressive parameter. The results are displayed in the third column of Table 4, model MS3, and the probabilities of recession and low variance in Chart 3 of Figure 7. Compared with the probabilities depicted in Chart 2, it is straightforward to conclude that lagged values of output growth do not help at all in forming inference of either the identification of the business cycle phases or in the determination of the timing of the volatility break.

Finally, as in the case of STAR models, the MS approach may also be used to infer the degree of abruptness in the transitions between business cycles. As Chart 7 shows, the filtered probability of low mean dramatically increases about the peaks and decreases about the troughs determined by the NBER dating committee.[17] This goes in line with our previous finding that the transitions from expansions to recessions and viceversa are sharp.

5 Model evaluation

In this section, we evaluate the different estimated models in terms of their forecast errors, by recursively comparing actual with one-period-ahead forecasts of output growths. In addition, we examine the extent to which the best of the non-linear models is able to generate cyclical behavior consistent with the actual data.

5.1 Forecast accuracy

To evaluate the forecast accuracy of one of these models, we consider Mean Squared Error (MSE), that is the average of the squared difference between

[17] For example, the probability of low mean rises about $1,250\%$ and falls about 62% in the first peak and trough, respectively.

actual and forecasts of output growth. However, to compare the forecast accuracy of competing models, we use two different kinds of statistical measures. The first type are usually called tests of equal forecast accuracy. Among them, we consider the Diebold-Mariano (DM), Modified Diebold-Mariano (MDM), Wilcoxon signed-rank (WILC), Morgan-Granger-Newbold (MGN), and Meese-Rogoff (MR) tests, all of them described in Francis Diebold and Roberto Mariano (1995) and David Harvey; Stephen Leybourne; and Paul Newbold (1997). The second type are the forecast encompassing tests (ENC). These tests are based on the fact that, if one model's forecasts encompass the other, then nothing can be gained by combining forecasts. Hence, additional competing forecasts should be statistically insingnificant in the regression of actual output growth on the models' forecasts.

Table 5 examines the ability of a simple linear AR model, and the non-linear specifications SETAR, STAR and MS. In addition, we compare our results with the well-know multivariate representation of the dynamics of the main US macroeconomic variables described in Robert King; Charles Plosser; James Stock; and Mark Watson (1991, henceforth KPSW). This consists on a vector error correction model of output, consumption and investment with two cointegration relationships. In the in-sample analysis, the MS model exhibits MSE reductions of about one-half, despite the competing model that we consider, and these reductions appear to be statistically significant using the whole set of tests of equal forecast accuracy. In addition, the encompassing tests show that forecasts from the MS model incorporate all the relevant information about output growth in competing forecasts, with the unique exception of the KPSW. Hence, everything points toward the MS model as the best model to fit the in-sample values of output growth.

The out-of-sample analysis, on the other hand, is based on recursive onestep-ahead forecasts. That is, the sample is successively enlarged with an additional observation and, to construct each of these forecasts, all the parameters are reestimated. However, prior to developing these forecasts, it may be determined at what time a forecaster would have recognized the volatility slowdown dated in the middle of the eighties. To address this question, Figure 8 uses the approximation suggested by Bruce Hansen (1997) to plot the p-values of the supremum test defined in Donald Andrews (1993) and the exponential and average tests developed in Donald Andrews and Werner Ploberger (1994) to test the structural break in the volatility of the series of GDP growth successively enlarged with one additional observation during the period $1997.1 - 2004.1$. This figure reveals that a clear signal of the structural break does not appear until the nineties. This result restricts the out-of-sample analysis to the relatively short forecast period $1991.1 - 2004.1$. For this period, the MS model again exhibits the lowest MSE. Although, probably due to the very short forecasting period, its forecast accuracy does not seem to be superior to its competitors since the forecast accuracy tests show p-values that are considerably large. However, the null that forecasts from this model still encompass the forecasts from other models can not be rejected at any standard significance level.

5.2 Adelman tests

The previous section suggests that the MS model is a reasonable starting point to forecast GDP growth. However, apart from describing first and second moments reasonably well, to be considered a good representation of the actual data generating process, we should ask whether this class of models are also able to generate cyclical behavior consistent with the data. We perform this exercise by comparing several business cycle characteristics of the data generated by this class of models with those generated by the actual data.

There is an extensive literature on business cycle characteristics which concentrates on the duration, amplitude and shape of the cycle. In this paper, we focus on the aspects of the cycle proposed by Don Harding and Adrian Pagan (2002) and Grant McQueen and Steven Thorley (1993) since they lead to a reasonable representation of the overall form of the typical cycle. In particular, for each of the two phases of the cycle, we consider the *duration* or average number of periods in the state of the cycle, the *amplitude* or percentage of gain in an expansion and loss in a recession, the *cumulative* movements between phases or percentage of wealth accumulated in expansions and lost in recessions, and the *excess* cumulated movements or difference between actual cumulative movements and the triangle approximation to cumulative movements.[18] In addition, we report measures of *sharpness* that compare growth rate changes two quarters around turning points.[19] Finally, one additional characteristic that should be generated by the MS process if it pretends to match the observed characteristics of the data is the sample *autocorrelation*.

The description of these business cycle characteristics must be accomplished first by isolating the turning points in the series. This is specially problematic when we try to report the cyclical behavior of thousands of generated time series. In this paper, we follow the well-known Bry-Boschan dating procedure to identify the countries' business cycle turning points because it

[18] In the definition of the cumulative movements between the phases of the cycle, wealth is defined as the accumulation of GDP production in each period of time.

[19] For a comprehensive overview of these measures, we refer the reader to the original papers.

is quick, easy to implement, and commonly accepted in the literature.[20]

First two columns of Table 4 provide an overview of the business cycle characteristics concerning the actual data. Expansions are about six times longer than recessions. The amplitude of expansions is also much larger than in recessions, basically because the latter are short-lived. This may also induce that, in expansions, the cumulated gains are much higher than the cumulated loses of recessions. The measures of excess show that contractions are similar and expansions are different from the triangle approximation of the cumulated loses and gains, respectively. The sign of the excess in expansion is consistent with the rapid recovery in the early part of the expansion that has been documented in the literature. Finally, according with the results of Grant McQueen and Steven Thorley (1993), the sharpness of troughs is roughly twice the sharpness of peaks, which support the view that peaks are relatively more rounded than troughs.

Now, it is turn to examine the ability of the MS model to match the characteristics found in the data. For this attempt, we collect the estimates of the model MS3 displayed in the third column of Table 3, generate $10,000$ Montecarlo time series simulations using these estimates, identify their turning points with the Bry-Boschan algorithm, and compute the set of business cycle characteristics generated by each of these simulations. Last two columns of Table 6 provides some summary statistics for the business cycle characteristics generated by the MS model: the mean, the standard deviation, and the percentile of the Montecarlo distribution in which the actual business cycle statistic is placed. Because the actual business cycle statistics are not in the extreme tails of the Montecarlo distributions, the MS model does a

[20] The Bry-Boschan algorithm isolates the local minima and maxima in a series, subject to reasonable constraints on both the length and amplitude of expansions and contractions.

reasonable job of producing recessions and expansions with business cycle characteristics consistent with those of the actual data. For the purpose of this paper, of noticeable interest is the ability of the MS model to generate time series with similar average correlation than the observed in the data, specially if we recall that the process that generates the simulations does not include any autoregressive parameter. This confirms the empirical reliability of the *jump-and-rest* effect of business cycles and the ability of the Markov switching representation to generate time series with business cycle characteristics similar to the ones of the observed data.

6 Conclusion

In this paper, we have found empirical evidence in favor of what we call the *jump-and-rest* effect of business cycles: Once we take into account the business cycle recessions and expansions sequence that is provided by the NBER, and the break in volatility at the mid eighties, there is no autocorrelation in the U.S. output growth rate. We have shown that this result is robust to the sample period, to many other alternative sequences of business cycle dates, to other macroeconomics aggregates such as consumption, investment, and sales, and to several alternative non-linear specifications determining endogenously the timing of the turning points. We believe that this result can be considered as "a new stylized fact of the U.S. economy".

The consequences of this new fact for both empirical and theoretical subsequent macroeconomic analysis are diverse and depend on the interest of the reader. From an empirical point of view, this simple dynamics facilitates the understanding and developing of forecasts, reduces to the minimum the complexity of impulse response functions and dynamic multipliers, specially those developed in nonlinear contexts, and simplifies the simulation and calibration analysis by overcoming unsolved computational problems. From a theoretical point of view, these finding provide empirical support to those theoretical models that describe the data generating process of output growth as a succession of equilibria between high and low growth. In addition, the *jump-and-rest* dynamics put additional lines to investigate the empirical reliability of theoretical simulations. Finally, it may serve as a guideline to resuscitate theoretical models that were neglected when autoregressive parameters were accepted as the source of the positive autocorrelation of output growth.

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Table 1. Summary statistics of U.S. macroeconomic series and analysis of the break in volatility

Statistics of US Macroeconomic series

		GDP	PCE	GPDI	GCI	EGS	IGS	FSDP
	Coefficient	0.32	0.29	0.16	0.17	-0.27	-0.05	0.28
AR(1)	test (1)	0.00	0.00	0.02	0.01	0.00	0.49	0.00
	Total	0.81	0.87	1.01	0.50	1.46	1.51	0.81
	Recessions	-0.51	0.12	-4.69	0.29	1.81	-1.34	-0.11
	Expansions	1.04	1.01	2.03	0.54	-0.51	2.02	0.97
Mean	test (2)	0.00	0.00	0.00	0.29	0.00	0.00	0.00
	Before 1984	0.81	0.86	0.99	0.61	1.47	1.43	0.82
	After 1984	0.80	0.93	1.05	0.46	1.45	1.65	0.80
	test (2)	0.40	0.57	0.93	0.42	0.96	0.68	0.87
	Total	0.94	0.71	4.65	1.20	4.00	3.60	0.76
	Recessions	0.85	0.90	4.80	1.42	4.24	3.28	0.84
Standard	Expansions	0.75	0.58	3.85	1.16	3.86	4.03	0.62
deviation	test (3)	0.93	0.00	0.15	0.19	0.55	0.18	0.06
	Before 1984	1.14	0.78	5.46	1.65	4.97	4.33	0.82
	After 1984	0.54	0.37	3.15	0.99	2.08	1.87	0.43
	test (3)	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Analysis of the break in volatility

Break tests:

		GDP	PCE	GPDI	GCI	EGS	IGS	FSDP
	Date	84.1	92.1	84.1	67.1	82.3	85.1	92.4
	Supremum (4)	0.00	0.00	0.05	0.19	0.04	0.01	0.04
	Exponential (4)	0.00	0.00	0.16	0.05	0.01	0.00	0.02
	Average (4)	0.00	0.00	0.04	0.01	0.00	0.00	0.02

Kolmogorov-Smirnov Test of equal distributions before and after the break:

		GDP	PCE	GPDI	GCI	EGS	IGS	FSDP
	Statistic	0.23	0.25	0.15	0.24	0.20	0.24	0.22
	Critical value	0.17	0.20	0.20	0.19	0.17	0.18	0.22

Wilconxon test of equal quartiles before and after the break:

		GDP	PCE	GPDI	GCI	EGS	IGS	FSDP
	Total	0.28	0.46	-1.49	-0.26	-0.41	-0.38	0.38
First	Before break	0.15	0.41	-1.87	-0.63	-1.65	-1.13	0.26
Quartile	After break	0.48	0.64	-1.15	-0.10	0.26	0.46	0.55
	test (5)	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Total	1.32	1.32	3.70	1.26	3.28	3.49	1.26
Third	Before break	1.68	1.36	4.54	1.95	4.05	4.17	1.30
Quartile	After break	1.14	1.09	2.96	1.02	2.64	2.86	1.03
	test (5)	0.00	0.00	0.00	0.00	0.00	0.00	0.01

Notes. Real and seasonally adjusted variables in columns are Gross Domestic Product (GDP), Personal Consumption Expenditures (PCE), Gross Private Domestic Investment (GPDI), Government Consumption and Investment (GCI), Exports of Goods and Services (EGS), Imports of Goods and Services (IGS), and Final Sales of Domestic Product (FSDP). Numbers are the *p*-values of the following nulls: (1) significativity of the slope parameter in an AR(1) specification for the growth rates, (2) no different means, (3) no different standard deviations, (4) no volatility break as described in Margaret McConnell and Gabriel Perez-Quiros (2000), (5) no different quartile.

Table 2. Simple linear time series models of U.S. output growth

	M1	M2	M3	M4	M5	M6
a_0	0.54	0.54	0.89	0.95	0.94	1.17
	(0.07)	(0.07)	(0.08)	(0.05)	(0.05)	(0.09)
a_1	0.32	0.32	0.05			
	(0.06)	(0.06)	(0.07)			
b_0			-1.30	-1.37	-1.37	-1.73
			(0.18)	(0.15)	(0.15)	(0.20)
c_0						-0.30
						(0.10)
c_1						0.61
						(0.30)
d_0	0.89	1.07	0.91	0.90	0.91	0.88
	(0.03)	(0.07	(0.06)	(0.06)	(0.07)	(0.06)
d_1		-0.57	-0.44	-0.44	-0.44	-0.42
		(0.08)	(0.08)	(0.08)	(0.08)	(0.07)
d_2					-0.01	
					(0.10)	
lnL	-265.73	-265.75	-215.54	-217.19	-217.18	-212.36

Notes. Entries refer to estimates and standard errors (in parenthesis) that correspond to an AR(1) for output growth extended with additive and multiplicative dummies that control for business cycles and volatility break. Last row refers to the log-likelihoods as stated. These models refer to the following expression:

$$\mathrm{y}_{\mathrm{t}} = a_0 + a_1 y_{t-1} + b_0 N_t + c_0 B_t + c_1 B_t N_t + \varepsilon_t,$$
$$\varepsilon_t \sim N(0, \sigma_t), \quad \sigma_t = d_0 + d_1 B_t + d_2 N_t.$$

The dummy B_t equals one in the period 1984.1-2004.1, and the dummy N_t equals one in the NBER periods of recession.

Table 3. SETAR and STAR models of U.S. output growth

	SETAR1	SETAR2	STAR1	STAR2
a_0	0.75	0.90	0.18	0.11
	(0.10)	(0.05)	(0.14)	(0.14)
b_0	-0.55	-0.76	0.58	0.81
	(0.18)	(0.13)	(0.20)	(0.15)
a_1	0.15		0.14	
	(0.09)		(0.09)	
g			27.26	15.57
			(29.70)	(17.11)
c	0.16	0.16	0.12	0.15
			(0.10)	(0.07)
d_0	1.07	1.09	1.08	1.09
	(0.07)	(0.07)	(0.07)	(0.07)
d_1	-0.61	-0.62	-0.61	-0.62
	(0.08)	(0.08)	(0.08)	(0.08)
lnL	-50.39	-51.87	-50.55	-51.73

Notes. Entries refer to estimates and standard errors (in parenthesis) that correspond to SETAR and STAR specifications for output growth for the following expressions:

$$y_t = a_0 + b_0\, I(y_{t-d}) + a_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim N(0, d_0) \quad if \;\; t < 1984.1, \text{and } \varepsilon_t \sim N(0, d_0 + d_1) \quad if \;\; t \geq 1984.1.$$

The term $I(y_{t-d})$ is an indicator function that takes the value 0 or 1 depending on $y_{t-d} \geq 0$ for the SETAR model and it is the transition function stated in the main text for the STAR model. Last row refers to the log-likelihoods.

Table 4. Markov-switching model of U.S. output growth

	MS1	MS2	MS3
μ_{11}	0.95 (0.07)	1.20 (0.16)	1.25 (0.13)
μ_{21}	-0.88 (0.21)	-0.34 (0.29)	-0.31 (0.24)
μ_{12}		0.91 (0.06)	0.91 (0.06)
μ_{22}		0.05 (0.23)	0.04 (0.18)
a_1	0.31 (0.10)	0.09 (0.10)	
σ^2_1	0.57 (0.08)	0.81 (0.14)	0.78 (0.13)
σ^2_2		0.17 (0.03)	0.17 (0.03)
p_{11}	0.95 (0.03)	0.93 (0.03)	0.92 (0.03)
p_{22}	0.45 (0.25)	0.73 (0.10)	0.74 (0.09)
q_{11}		0.99 (0.01)	0.99 (0.01)
q_{22}		0.99 (0.01)	0.99 (0.01)
lnL	-261.07	-243.46	-243.76

Notes. Entries refer to estimates and standard errors (in parenthesis) that correspond to the Markov-switching model stated as follows:

$$y_t = \mu_{S_t,V_t} + \phi_1(y_{t-1} - \mu_{S_{t-1},V_{t-1}}) + \varepsilon_t,\ \varepsilon_t \sim N(0,\sigma_{V_t})$$

Last row refers to the log-likelihoods.

Table 5. In-sample and out-of-sample accuracy

		RMSE	DM	MDM	WILC	MGN	MR	ENC
	AR	1.89	0.00	0.00	0.00	0.00	0.00	0.19
	SETAR	1.92	0.00	0.00	0.00	0.00	0.00	0.17
IN	STAR	1.92	0.00	0.00	0.00	0.00	0.00	0.20
	KPSW	1.96	0.00	0.00	0.00	0.00	0.00	0.00
	MS	1.00	---	---	---	---	---	---
	AR	1.11	0.22	0.23	0.33	0.24	0.21	0.73
	SETAR	1.09	0.42	0.42	0.49	0.51	0.49	0.56
OUT	STAR	1.09	0.46	0.46	0.34	0.44	0.43	0.92
	KPSW	1.10	0.55	0.55	0.89	0.59	0.60	0.11
	MS	1.00	---	---	---	---	---	---

Notes. First column is the relative mean squared error. Other columns refer to the *p*-values of Diebold-Mariano (DM), Modified Diebold-Mariano (MDM), Wilcoxon signed-rank (WILC), Morgan-Granger-Newbold (MGN), Meese-Rogoff (MR) and forecast encompassing tests (ENC). In-sample and out-of-sample refer to 1953.1-2004.1 and 1997.1-2004.1, respectively.

Table 6. Summary statistics for actual data and Markov-switching simulations

	Actual		Simulations	
	Expansions	Recessions	Expansions	Recessions
Duration	20.4	3.6	28.8 (11.77) [0.21]	2.8 (0.99) [0.17]
Amplitude	0.20	-0.022	0.29 (0.12) [0.21]	-0.020 (0.007) [0.30]
Excess	-0.227	-0.0028	-0.224 (0.65) [0.34]	-0.0004 (0.008) [0.28]
Cumulative	3.75	-0.04	8.07 (8.12) [0.23]	-0.06 (0.004) [0.64]
	Peaks	Troughs	Peaks	Troughs
Sharpness	0.024	0.039	0.030 (0.006) [0.17]	0.031 (0.006) [0.92]
Autocorrelation	0.32		0.23 (0.09) [0.87]	

Notes. Definitions for these business cycle characteristics are in the text. Standard deviations are in parenthesis. The percentile in the simulations that the actual business cycle characteristic occupies is in square brackets.

Figure 1. Stylized facts about output growth: 1953.1-2004.1

Chart 1. Total and partial autocorrelation functions

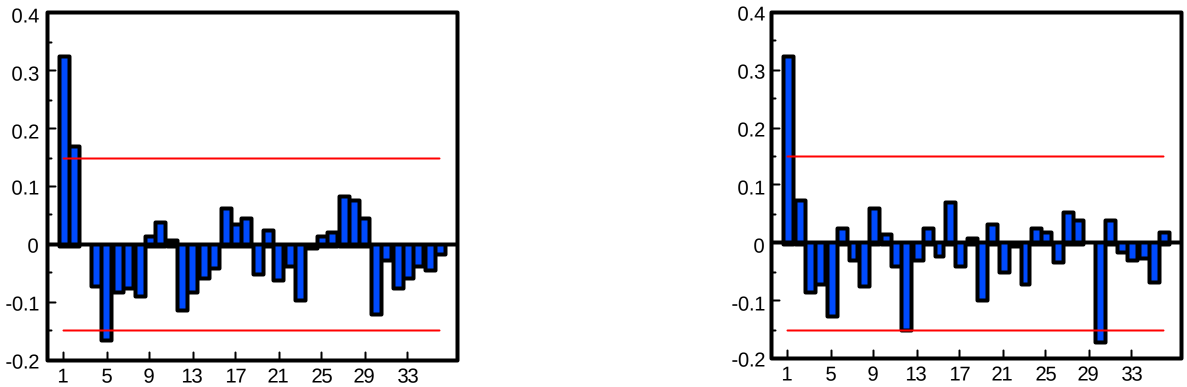
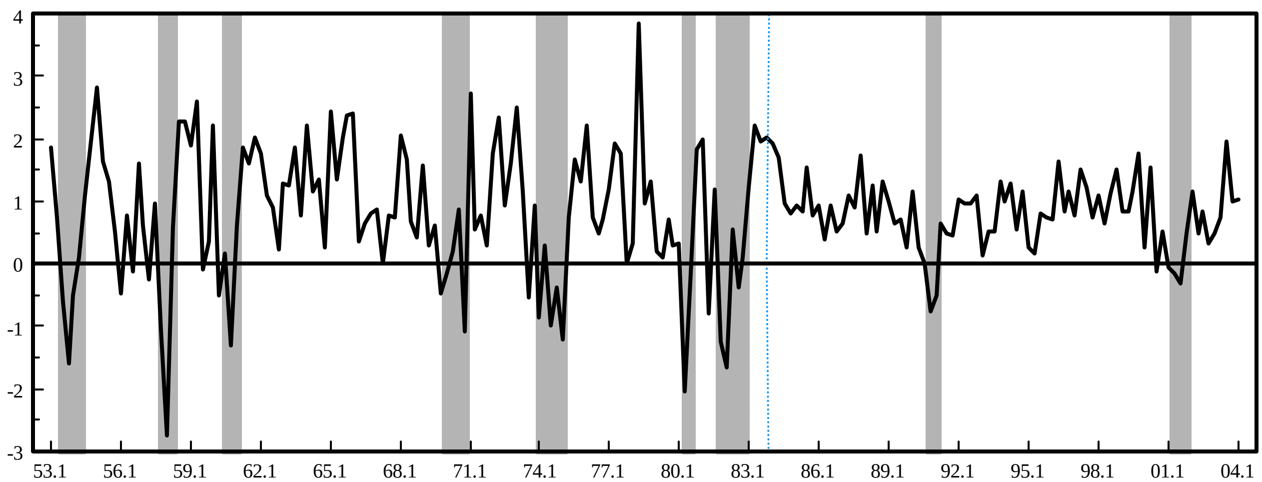


Chart 2. U.S. Real GDP Growth



Note: Shaded areas refer to the NBER recessions. Dashed line corresponds to the volatility break

Chart 3.Kernel density estimates before and after 1984.1

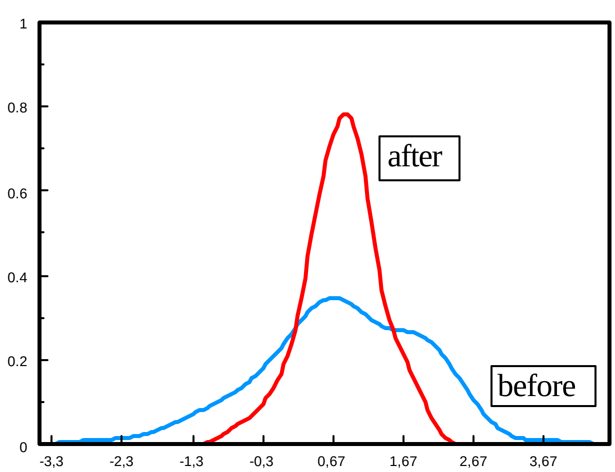


Figure 2. Output growth estimates: 1953.1-2004.1

Chart 1. Actual versus AR(1)

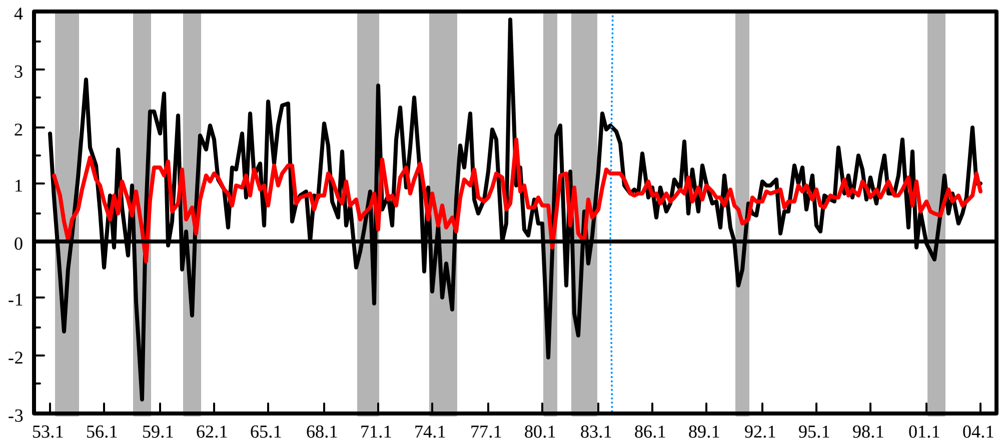


Chart 2. Actual versus two-states with volatility break estimates

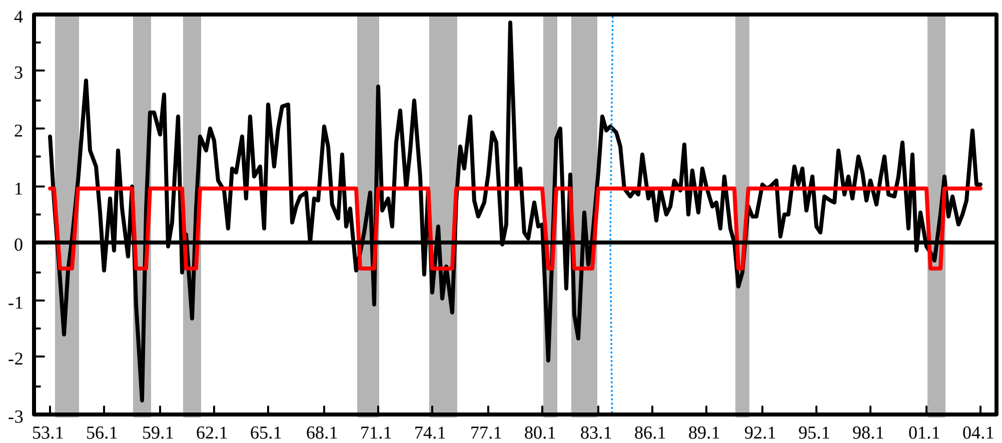
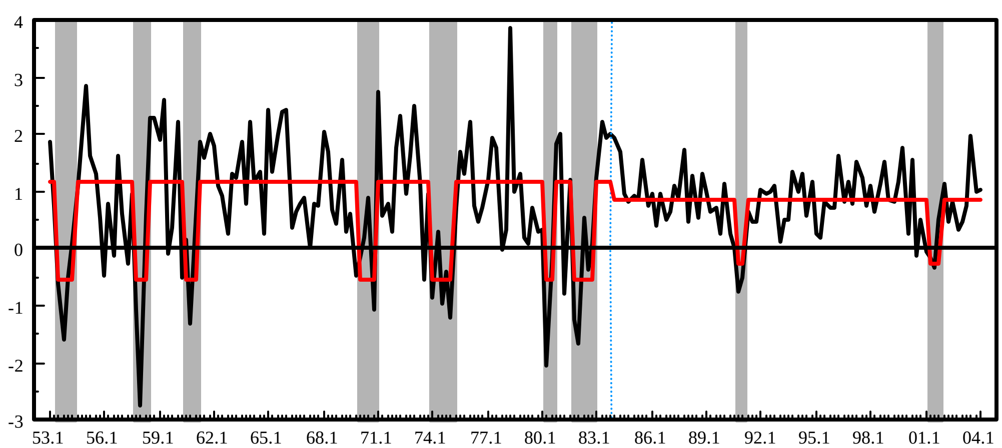


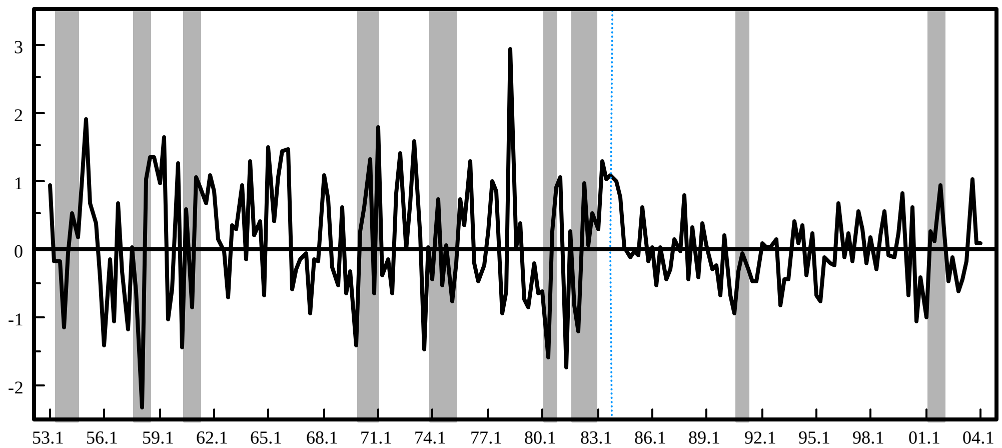
Chart 3. U.S. Actual versus four-states with volatility break estimates



Note: Shaded areas refer to the NBER recessions. Dashed line corresponds to the volatility break

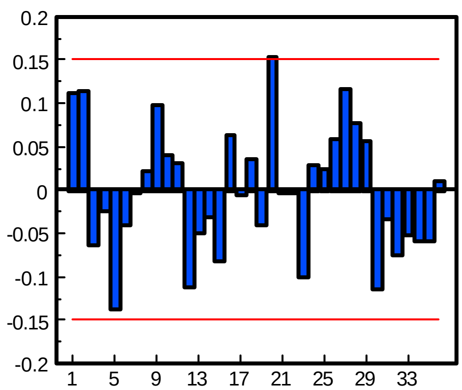
Figure 3. Residuals analysis

Chart 1. Residuals from linear model enlarged with dummies (2 means)



Note: Shaded areas refer to the NBER recessions. Dashed line corresponds to the volatility break

Chart 2. Total and partial autocorrelation functions



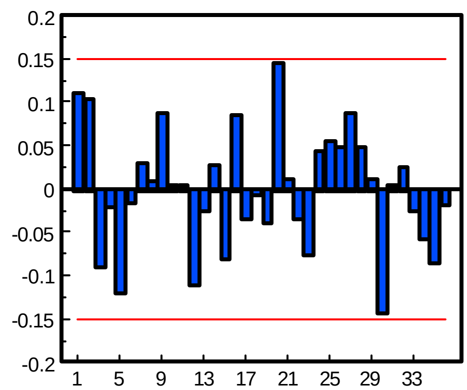
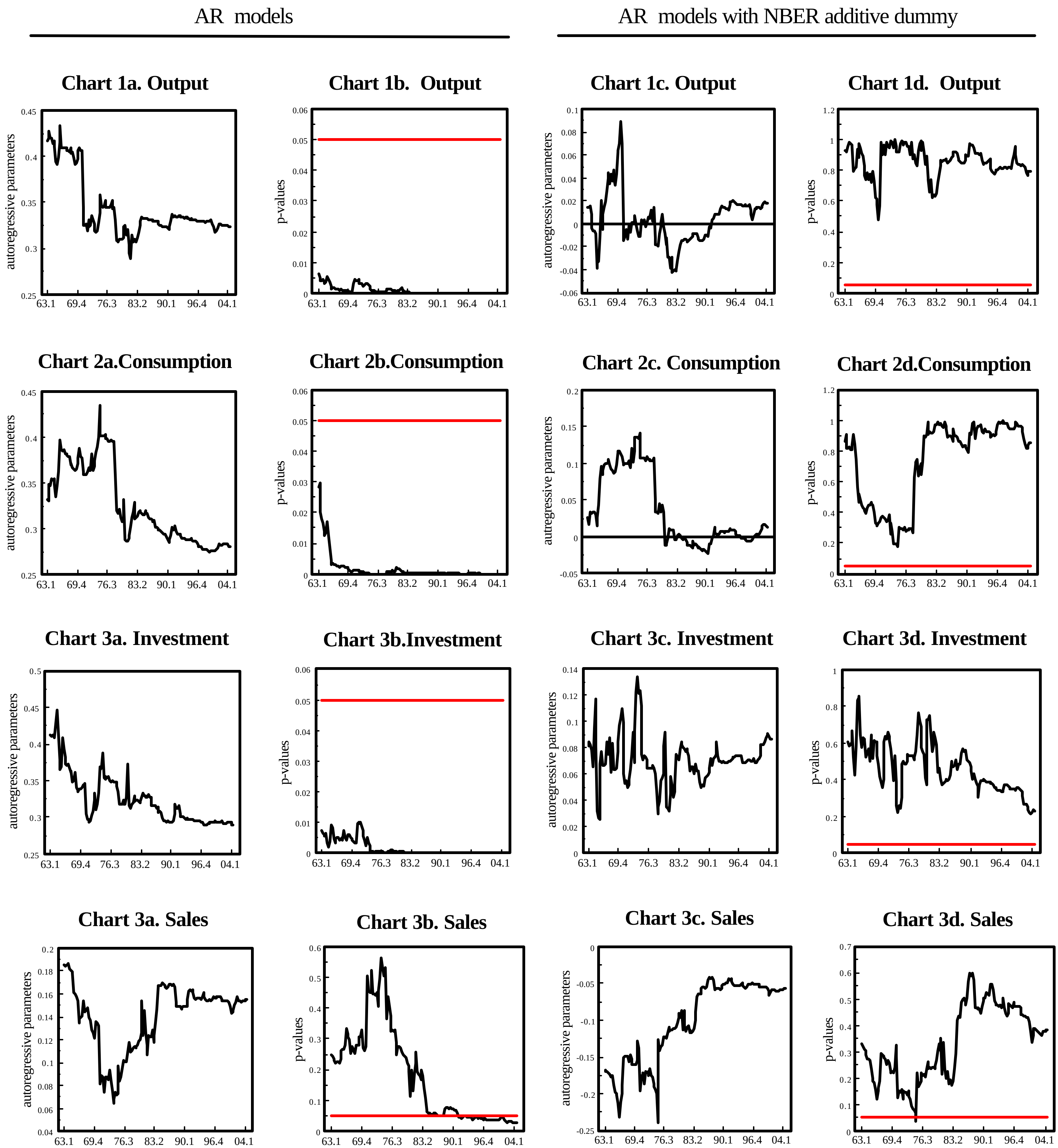
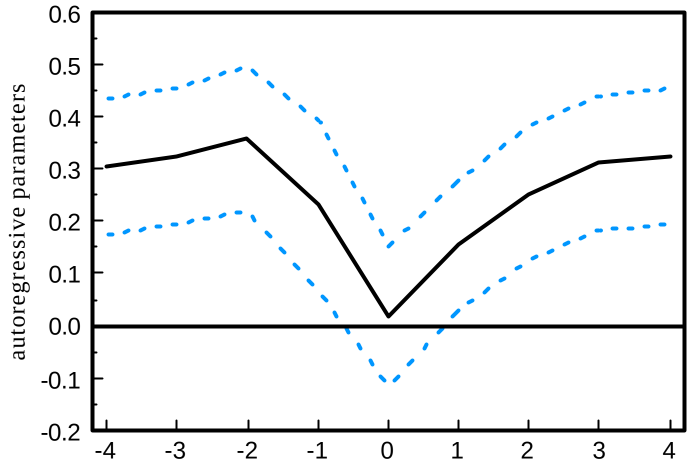


Figure 4. Recursive estimation



Note: Charts labeled with 1, 2, 3, and 4 refer to the rate of growth of U.S. real output, consumption, investment, and sales, respectively. Charts labeled with *a* refer to the recursive estimates of the AR(1) slope parameters while charts labeled with *d* refer to the same estimates but obtained by adding a NBER-recessionary dummy. Charts labeled with *b* and *d* refer to their respective *p*-values of the non-significativity null. Horizontal lines refer to the 0.05 significance value.

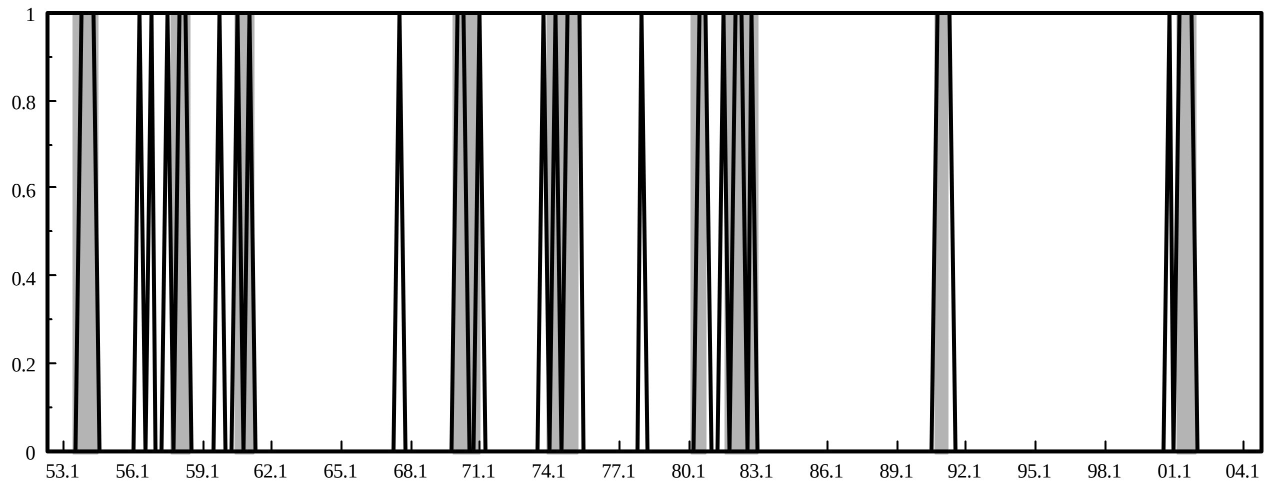
Figure 5. Regresion with leads and lags of the NBER sequence



Note: Dashed lines corrspond to 5% confidence intervals.

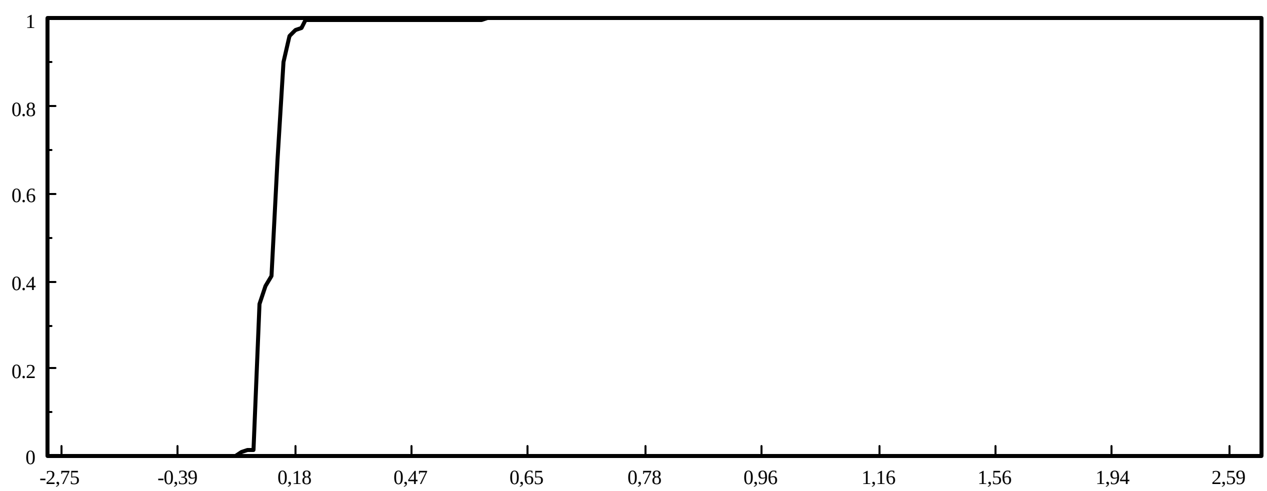
Figure 6. SETAR and STAR models of output growth

Chart 1. Probabilities of recession from the TAR model



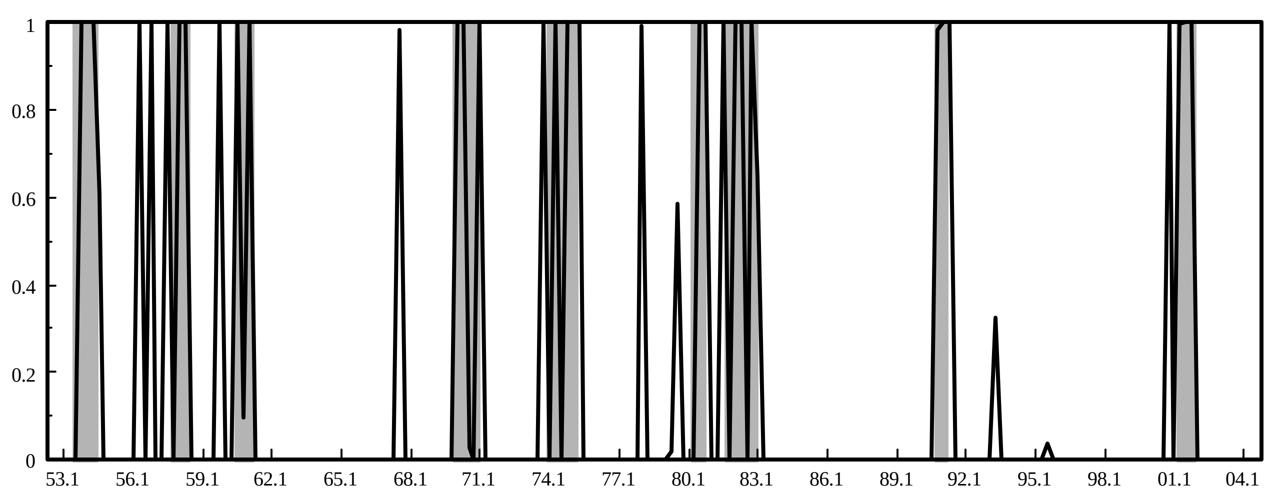
Note: Shaded areas refer to the NBER recessions.

Chart 2. Probability of recession at t versus GDP growth at t-1 from the STAR model



Note: This chart corresponds to the logistic transition function.

Chart 3. Probabilities of recession from the STAR model



Note: This chart plots one minus the transition function. Shaded areas refer to the NBER recessions.

Figure 7. Markov-switching model of output growth

Chart 1. Probability recession form Hamilton original model

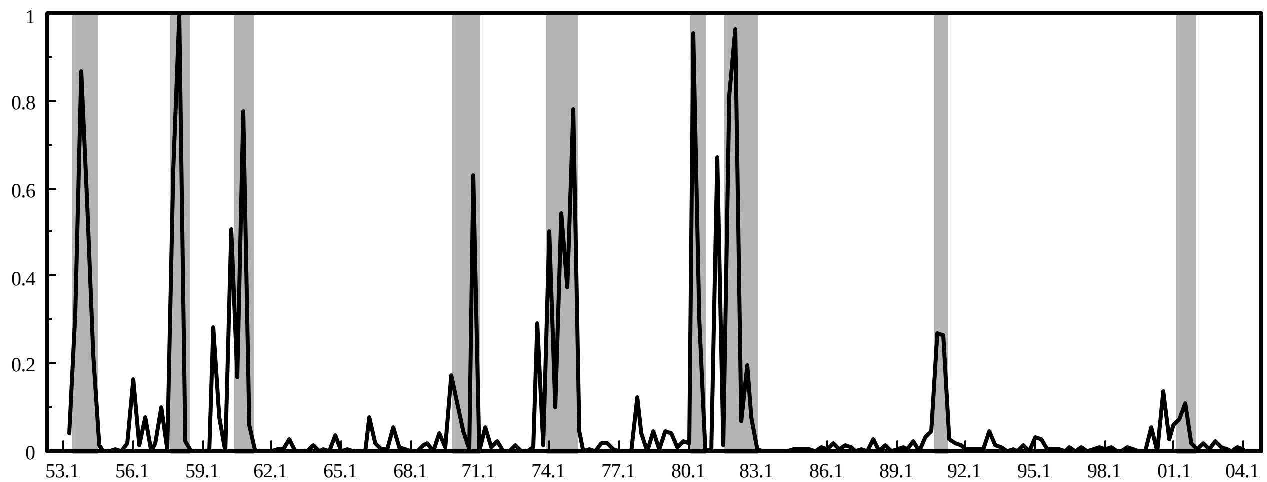


Chart 2. Probabilities from MS with four means, AR(1) parameter, and structural break

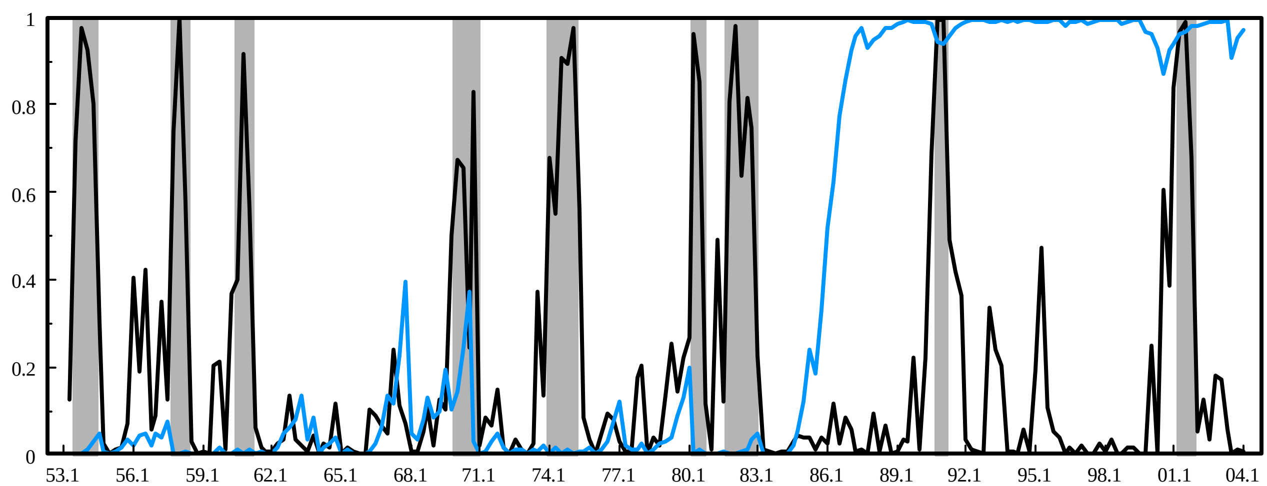
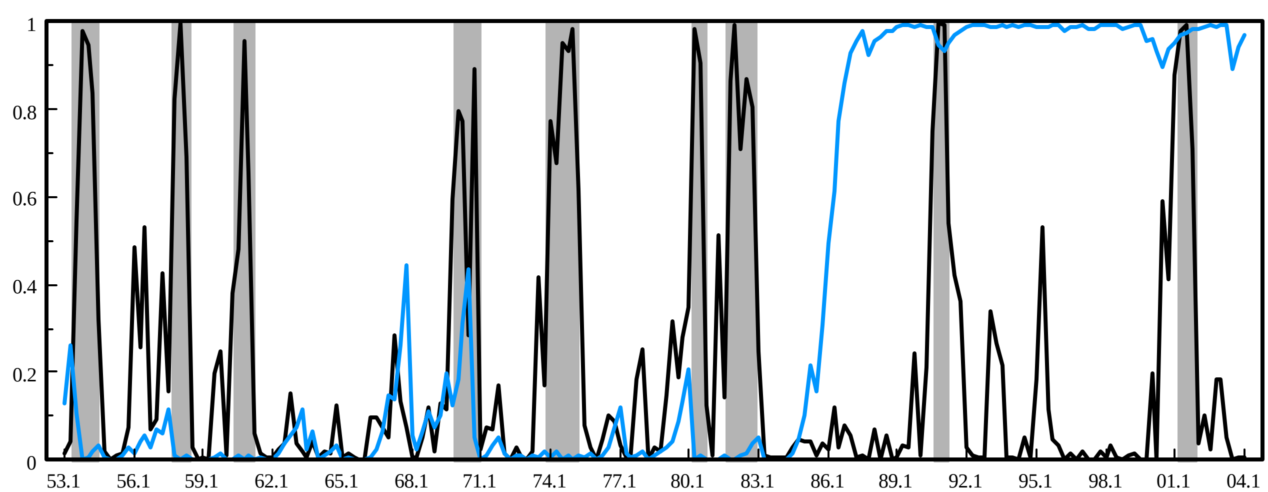
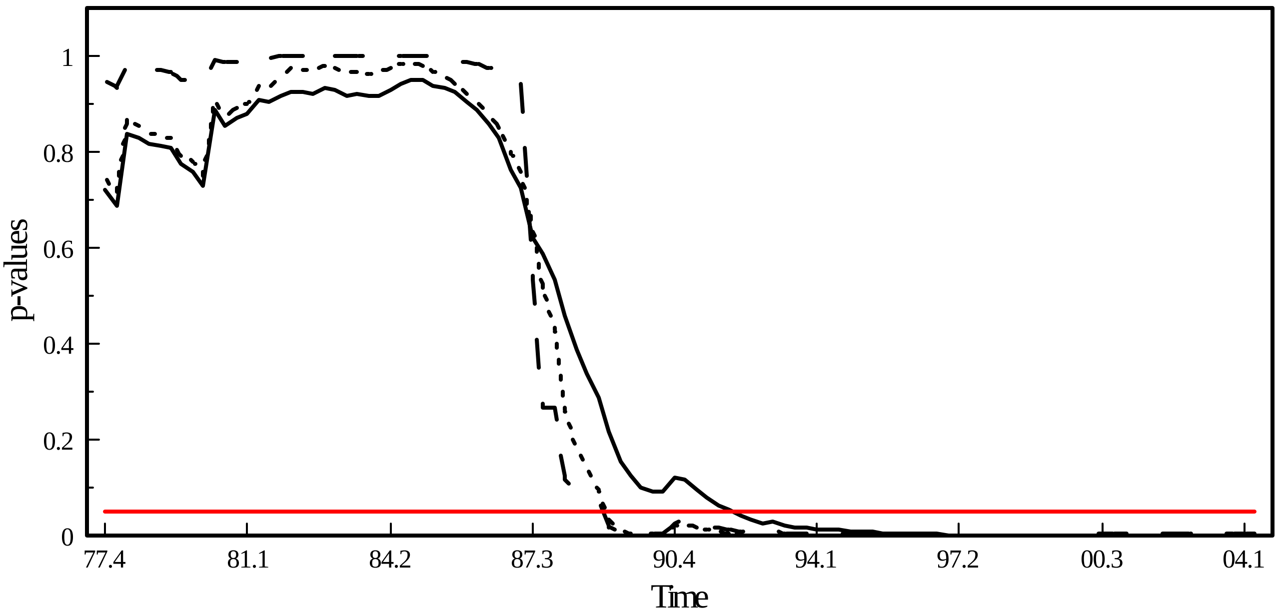


Chart 3. Probabilities from MS with four means and structural break



Note: These charts plot filtered probabilities. Black lines refers to the probability of low mean. Blue lines refers to the probability of low variance. Shaded areas are the NBER recessions.

Figure 8. Real-time structural break analysis



Notes: Using the approximation of Hansen (1997), this figure plots the *p*-values of the supremum (dashed line) test developed by Andrews (1993) and the exponential (dotted line) and average (straight line) tests suggested by Andrews and Ploberger (1994) applied to the GDP growth rate enlarged with one additional observation during the period 1997.4-2004.1. Horizontal line refers to the 0.05 *p*-value.