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ABSTRACT

Trade and Growth with Heterogeneous Firms*

This paper explores the impact of trade on growth when firms are heterogeneous. Our findings can be viewed as relevant to the trade and growth literature on one hand and the heterogeneous-firms trade theory on the other. Our main finding – that freer trade is both anti-growth and welfare worsening from a purely dynamic perspective – contrasts with most findings in the endogenous growth literature. We also show that market-entry costs are anti-growth, but heterogeneity *per se* is pro-growth. As concerns the heterogeneous-firms literature our main finding is a static-vs.-dynamic trade-off in terms of productivity gains. Freer trade raises measured productivity in a level sense but slows measured productivity growth.

JEL Classification: H32 and P16

Keywords: dynamic versus static efficiency, heterogeneous firms and trade and endogenous growth

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Trade and growth with heterogeneous firms

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ABSTRACT

This paper explores the impact of trade on growth when firms are heterogeneous. Our findings can be viewed as relevant to the trade and growth literature on one hand and the heterogeneous-firms trade theory on the other. Our main finding – that freer trade is both anti-growth and welfare worsening from a purely dynamic perspective – contrasts with most findings in the endogenous growth literature. We also show that market-entry costs are anti-growth, but heterogeneity per se is pro-growth. As concerns the heterogeneous-firms literature our main finding is a static versus dynamic trade-off in terms of productivity gains. Freer trade raises measured productivity in a level sense but slows measured productivity growth.

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1. INTRODUCTION

Until the 1980s, trade theory assumed away intra-industry trade for convenience, but empirical evidence revealed that much of world trade was exactly of the assumed-away kind (Grubel and Lloyd 1975). In response, the so-called new trade theory (Helpman and Krugman 1985) incorporated imperfect competition and increasing returns to account for intra-industry trade. The

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modelling choices made by new trade theorists assumed away, again for convenience, differences among firms. Recent empirical evidence, however, shows that differences among firms are crucial to understanding world trade. For example, firm differences within sectors may be more pronounced than differences between sector averages, and most firms – even in traded-goods sectors – do not export at all (Bernard and Jensen 1995, 1999a,b, 2001; Clerides, Lach and Tybout 1998, Aw, Chung, and Roberts 2000, Eaton, Kortum, and Kramarz 2004; see Tybout 2003 for a survey). In response, what might be called the ‘new new’ trade theory incorporated firm-level heterogeneity to account for the many of the new firm-level facts. The main theoretical papers in this rapidly expanding literature are Bernard, Eaton, Jensen and Kortum (2003), Melitz (2003), Helpman, Melitz and Yeaple (2004), Bernard, Redding and Schott (2004), Bernard, Eaton, Jensen, and Schott (2003), Melitz and Ottaviano (2003), and Yeaple (2005).

Our paper studies the growth effects of greater openness by embedding a heterogeneous-firms trade model in a product-innovation endogenous growth model (Grossman and Helpman 1989, 1991, Romer 1986, 1990, Rivera-Batiz and Romer 1991a, b, Dinopoulos and Segerstrom 1999a, b, Keller 2004). We show that openness slows growth even though it raises the level of productivity via selection and share-shifting effects. We also characterise the welfare impact of greater openness. We show that the market-determined number of varieties and marginal cost cut-off points (for exporting and domestic production) are socially optimal, but the market growth rate is sub-optimal. Moreover, we find that greater openness is welfare worsening in a dynamic sense, although its overall welfare impact is ambiguous since the static effect is positive. Additionally, we show that greater international knowledge spillovers are pro-growth and welfare improving, as is domestic deregulation that lowers the cost of introducing new varieties. Finally, we show that greater heterogeneity in firms’ marginal costs tends to be pro-growth. To make these points as simply as possible, we work with a new-new trade model related to Melitz (2003) and Helpman, Melitz and

Yeaple (2004) and a product-innovation growth model related to Grossman and Helpman's product innovation model (Grossman and Helpman 1991 chapter 4).

The paper is organised in six sections after the introduction. The next two present the model (section 2) and work out the long-run growth path (section 3). Section 4 studies the static and dynamic welfare aspects of the market-determined growth path. All of this is done without assuming an explicit distribution of firm-level marginal costs. Section 5 introduces a specific distribution (Pareto) and derives closed form solutions for all variables. Using these, Section 6 studies the growth effects of greater openness, domestic deregulation and a change in the degree of heterogeneity. Section 7 presents our concluding remarks.

2. A HETEROGENEOUS FIRMS TRADE MODEL WITH GROWTH

Following Melitz (2003), our model's foundation is the standard Helpman-Krugman monopolistic competition trade model (Helpman and Krugman 1985). We work with two identical nations, a single primary factor L , and a single consumption-good sector. Competition takes the form of Dixit-Stiglitz monopolistic competition among firms facing iceberg trade costs. Each firm's cost function is linear and involves a one-time variety-development fixed cost and constant marginal production costs. We think of this one-time cost as reflecting the sunk cost of developing a new variety.

To this standard new-trade model, market-entry costs are added. Thus selling a new variety in a particular market requires the firm to pay a sunk cost. We think of these market-entry costs as reflecting the cost of adapting the variety to market-specific standards, regulations and norms. The market-entry costs are assumed to differ systematically for locally produced varieties and imported varieties. The market-entry cost for locally made products is F_D (D for domestic) and F_X for imported varieties (X for exports).

Following the heterogeneous-firms trade literature (Bernard, Eaton, Jensen and Kortum 2003, Melitz 2003, Helpman, Melitz and Yeaple 2004, Bernard, Redding, Schott 2004, Bernard, Eaton, Jensen, and Schott 2003, Melitz and Ottaviano 2003, and Yeaple 2005) the model also allows for heterogeneity with respect to firms' marginal production costs; in particular, we follow Melitz (2003) and Helpman, Melitz and Yeaple (2004) in working with the Hopenhayn (1992a, 1992b) mechanism of development and selection of firms with heterogeneous marginal costs. In either nation, a typical firm j has a unit input coefficient – denoted as a_j – that is revealed once the firm has paid the variety-development cost; the 'a' is drawn from a density function $G[a]$ that has positive probability over the range $0 \leq a \leq a_0$.

The only really new element we add is “knowledge spillovers” in the sense of the standard product innovation endogenous growth model (Grossman and Helpman 1991). We interpret the three fixed costs – the variety-development cost and the two market-entry costs – as involving units of ‘knowledge’, where knowledge is created by an ‘innovation sector.’ The initial variety-development cost requires F_I units of knowledge (I for innovation), while adapting it to local and export market conditions requires F_D units and F_X units of knowledge, respectively. The production function for knowledge is:

$$(1) \quad Q_K = L_I / b_I ;$$

where Q_K is the flow of knowledge, L_I is the labour input and b_I is the unit labour input coefficient in the I sector.

In words, the knowledge-producing sector (innovation sector or I-sector for short) produces one unit of knowledge capital with b_I units of L , so the marginal cost of a unit of knowledge is $w b_I$, where w is the wage. We assume that b_I is a parameter to individual firms but it declines as the sector's cumulative learning rises as in the endogenous growth literature. Many justifications of this intertemporal externality are possible. Romer (1990), for instance, rationalizes it by referring to the

non-rival nature of knowledge. Grossman and Helpman (1991) assert that it reflects the impact of ‘public knowledge’ that is created automatically along with the private, patentable knowledge. Given the nature of knowledge, it is natural to assume that fixed costs are also sunk, although this plays no role in our analysis since we follow the heterogeneous-firms trade literature and focus exclusively on steady states. As in Melitz (2003), we assume that firms face a constant probability of ‘death’; the firm-death rate is a Poisson process with a hazard rate of δ (this enables transitions between steady states, although, as mentioned, the transitions are not explicitly characterised). Unlike Melitz (2003) and Helpman, Melitz and Yeaple (2004), we assume a positive discount rate so we can make intertemporal welfare comparisons.

3. THE EQUILIBRIUM

This section works out the steady state growth rate and characterises the equilibrium distribution of firm-level productivity.

3.1. Instantaneous equilibrium

Following standard practice in the endogenous growth literature, we first work out the instantaneous equilibrium, i.e. the economy’s equilibrium at a point on the long-run growth path, taking the growth rate as given. This boosts intuition since the analysis is almost identical to that of a no-growth heterogeneous-firms trade model.

3.1.1. Market-entry decisions: The cut-off conditions

Entry decisions are the fulcrum of this model and there are two types: 1) the market-entry decision facing firms that have decided to produce, and 2) the variety-introduction decision facing potential firms. Although these are linked, we treat them in sequence.

Firms enter a particular market if the benefit of doing so exceeds the cost. The benefit of entering a market will depend upon the firm's anticipated sales and these, in turn, will depend upon the firm's competitiveness in the market, i.e. its marginal selling cost relative to that of its competitors.

Relying on the standard logic of fixed costs, we can already anticipate that only firms with sufficiently low marginal costs will enjoy market shares that are high enough to justify the sunk market-entry costs. Since there are two market-entry costs (F_X and F_D) there will be two thresholds, one for domestic sales and one for export sales, denoted a_D and a_X . To flesh out this intuition, we characterise the operating profit that firms would earn if they decided to enter a given market.

Under Dixit-Stiglitz competition, firms find it optimal to charge a constant operating profit margin, $1/\sigma$, where $\sigma > 1$ is the constant elasticity of substitution among varieties.¹ Thus the operating profit that a firm earns in a particular market – denoted as π – equals the value of the firm's sales times $1/\sigma$. Since the mass of firms in each market will be rising steadily along the equilibrium growth path, it proves convenient to write sales as a market-specific market share times total expenditure in the market, E .

The market share function. The Dixit-Stiglitz market share is a simple function of the firm's price relative to the average price of all its competitors. By symmetry, wages are equalised internationally, so taking labour as numeraire and using the well-known constant mark-up pricing feature of Dixit-Stiglitz competition, firm j 's price in its local market is $a_j / (1 - 1/\sigma)$. If it exports, the firm's price in its export market is $\tau a_j / (1 - 1/\sigma)$, where $\tau \geq 1$ represents the usual iceberg trade costs (τ

¹ Specifically, denoting c as consumption of a typical variety and Θ is the set of consumed varieties utility of the

representative consumer is given by $U = \left(\int_{i \in \Theta} c_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}$. The constant operating profit mark-up follows directly from the constant cost-price mark-up; for example, rearranging the first order condition for local sales, $p(1-1/\sigma) = a$ we get that operating profit earned on local sales is $(p-a)c = pc/\sigma$, where c is local sales.

units must be shipped to sell one unit in the export market). Thus, the Dixit-Stiglitz market share function $s[m]$ is²:

$$(2) \quad s[m] = \left(\frac{1}{n}\right) \frac{m^{1-\sigma}}{\Delta}; \quad \Delta = \int_0^{a_D} a^{1-\sigma} dF[a] + \phi \int_0^{a_X} a^{1-\sigma} dF[a]; \quad 0 \leq \phi \equiv \tau^{1-\sigma} \leq 1$$

where m is the marginal selling cost (i.e. ‘ a ’ for local sales and τa for export sales), and Δ is the weighted average of firms’ marginal selling costs in a particular market; also Δ is related to the CES price index, viz. $P = \{(n\Delta)^{1/(1-\sigma)}\}/(1-1/\sigma)$. Moreover, $F[a]$ is the density function of the a ’s of all firms selling in this market ($F[a]$ is identical in the two nations). Observe that ϕ ranges from zero when trade is perfectly closed ($\tau=\infty$) to unity when trade is perfectly free ($\tau=1$); we refer to ϕ as the ‘free-ness’ of trade.

Equation (2) is pivotal, so we make five observations that facilitate intuition and subsequent analysis: i) The ratio, $m^{1-\sigma}/\Delta$, is a measure of the firm’s market-specific competitiveness, i.e. its marginal selling cost relative to an average of its competitors’ marginal selling costs; ii) Roughly speaking, a firm’s market share exceeds $1/n$ to the extent its competitiveness is above average; iii) A firm’s market share (and thus its operating profit) increase as its marginal cost falls, ceteris paribus; iv) The average of firms’ marginal selling costs in a market, Δ , depends on the two cut-off marginal costs, a_D and a_X , and the distribution function for active varieties, namely $F[a]$; v) As n grows, as it will along the steady state growth path, all existing firms’ market shares decline in tandem and at the rate at which n grows.

Cut-off conditions and 3 firm types. Given the sunk cost nature of market-entry costs, the relevant benefit of market entry is the present value of the market-specific operating profit. Using (2), a firm’s operating profit in its local and export market will be $s[a]E/\sigma$ and $\phi s[a]E/\sigma$,

² Firms are atomistic so firm shares are infinity small, namely $s[m]di$, but the density-weighted integral of these is unity.

respectively, where E is expenditure in the relevant market. Defining the discount rate as γ , a firm's benefit from entering its local and export market is, respectively, $s[a]E/\sigma\gamma$ and $\phi s[a]E/\sigma\gamma$. Of course, γ is an endogenous variable in a growth model, but it is time invariant in steady state, so we take it as given for the moment. The two market-entry cut-off conditions are:

$$(3) \quad s[a_D] \frac{E}{\sigma \gamma} = b_I F_D ; \quad s[a_X] \frac{E}{\sigma \gamma} = b_I \frac{F_X}{\phi}$$

where a_D and a_X are the cut-off marginal costs for entering the local market and the export market respectively (D stands for domestic and X for export); $b_I F_D$ and $b_I F_X$ are the fixed market-entry costs in units of the numeraire; the b_I converts the F 's, which are in units of knowledge, into the numeraire so they can be compared to the present values.

The two cut-off marginal costs define three types of firms: D-types (firms that sell only domestically), X-types (firms that export), and N-types (non-producers). Varieties with marginal production costs less than a_X export their variety (X-types), those with marginal costs below a_D sell domestically (D-types), and those with a 's above a_D produce nothing.

It is a well established empirical fact that only a fraction of firms export (Bernard and Jensen 1995, 1999a, Eaton, Kortum, and Kramarz 2003). If the exporting firms are to constitute only a fraction of all active firms then it must be that $a_X < a_D$ and from (3):

$$(4) \quad \frac{a_X}{a_D} = \left(\frac{F_X / \phi}{F_D} \right)^{\frac{1}{1-\sigma}} < 1 \quad \Leftrightarrow \quad \frac{F_X / \phi}{F_D} > 1$$

Thus, the empirical fact tells us that $(F_X/\phi)/F_D$ is greater than unity.

The distribution of active firms. Given that only varieties with $a < a_D$ are observed in the market, $dF[a]$ from (2) equals $g[a]/G[a_D]da$, where $g[a]$ the probability distribution function (pdf) of a .³

3.1.2. New variety introduction: the I-sector free entry condition

The Hopenhayn-Melitz approach folds variety development into manufacturing firms' activity. That is, the process of developing a new variety and the determination of its marginal cost are viewed as being undertaken by the firm that eventually manufactures the new variety. Since variety-introduction is the engine of endogenous growth in our model, it proves convenient to separate innovation and manufacturing by introducing an explicit innovation sector (I-sector for short).

To this end, we assume a perfectly competitive I-sector that makes and sells 'patents' for new varieties. Firms in this sector first develop the fundamental knowledge behind a new variety and then, if warranted, they also invent the knowledge necessary to meet the market-entry costs, i.e. the knowledge necessary to adapt the variety to market-specific standards and regulations. As mentioned above, creating a new variety requires F_I units of knowledge, while adapting it to local and export market conditions requires F_D units and F_X units of knowledge, respectively. Thus, after developing a new variety, the innovator checks the associated 'a' to determine the variety's type. N-types are abandoned, but the innovator sinks a further F_D units of knowledge for D-types and F_D+F_X for X-types. The patents are sold at competitive prices, namely the present value of the variety's operating profit. The value of the resulting variety is a function of its associated marginal cost. While calculating these values is easy, it is not necessary.⁴

The decision of whether to develop a new variety can be cast in terms of the ex ante expected present value of a 'winning' variety (winning in the sense that it is actually produced). Because we

³ See Melitz (2003) for a proof.

⁴ The values are: $v_N=0$ for N-types, $v_D=a^{1-\sigma}E/\sigma\gamma\Delta$ for D-types and $v_X=(1+\phi)a^{1-\sigma}E/\sigma\gamma\Delta$ for X-types.

focus on steady states, the ex ante likelihood of getting a winner with any particular ‘a’ is exactly the same as the actual distribution of a’s for winners already in the market. In other words, the ex ante expected operating profit of a winner must exactly match the average operating profit earned in the market. This average operating profit is $E/\sigma n$ because total operating profit worldwide equals $2E/\sigma$ (due to the constancy of the Dixit-Stiglitz operating profit margin) and the worldwide mass of varieties is $2n$. Thus:

$$\text{Ex ante expected value of a ‘winner’} = \frac{E}{\sigma n \gamma}$$

Of course, innovators do not come up with a winner every try; only new varieties with $a < a_D$ will be ‘winners’. It is straightforward to calculate the ex ante expected fixed cost of getting a winner (i.e. developing a D or X type patent). The answer is:

$$(5) \quad \bar{F} \equiv F_D + F_X \frac{G[a_X]}{G[a_D]} + F_I \frac{1}{G[a_D]}$$

The first right-hand term is the fixed cost for local sales – an expense that every winner will incur. The second term reflects the fact that some ‘winners’ will be X-types so their developer will find it profitable to incur F_X as well; $G[a_X]/G[a_D]$ is the probability of being an X-type conditional on being a winner. The third right-hand term reflects the ex ante expected variety development cost, i.e. F_I times the inverse probability of getting a winner on a random draw since, roughly speaking, $1/G[a_D]$ is the number of ‘tries’ needed to get a winner.⁵

⁵ Since we work with a continuum of goods, there are an uncountable number of varieties and a fraction δ of these must be replaced at all moments. Thus the I-sector must develop an uncountable infinity of new varieties (even ignoring the new varieties necessary to keep n growing). By the Law of Large numbers, this means that the average fixed cost per new varieties is exactly $b_I \bar{F}$ at all instants. Or, to put it differently, there is no aggregate uncertainty in this model.

Given the expected cost and benefit of a winner, the expected pure profit of devoting resources to the development of new varieties will be $(E/\sigma n\gamma - b_I \bar{F})$; the expected fixed costs \bar{F} is multiplied by b_I to convert the units of knowledge into units of numeraire. Free entry in the I-sector drives this to zero, so the free entry condition for variety introduction is⁶:

$$(6) \quad \frac{E/\sigma}{n\gamma} - b_I \bar{F} = 0$$

As we shall see, the discount factor γ depends on the growth rate of varieties since a faster introduction of new varieties means a faster decline of each existing firm's market share as per (2). Thus, it will be the discount factor γ that adjusts to ensure this equality along the long-run equilibrium growth path rather than the mass of firms as in heterogeneous-firms trade models without growth.

It is important to note that all active firms earn pure profits. Their revenue exceeds their variable costs by more than would be needed to amortize their sunk costs. The reason, of course, is that free entry into innovation stops at the point where the ex ante expected value of pure losses on N-types is balanced by the ex ante expected value of pure profits earned on D types and X types.

⁶ The direct approach to formulating the condition for zero-expected-profit-from-innovation is to calculate the ex ante expected benefit net of market entry costs, i.e. $\int_0^{a_D} \{a^{1-\sigma}(E/\sigma n\Delta\gamma) - F_D\} dG[a] + \phi \int_0^{a_X} \{a^{1-\sigma}(E/\sigma n\Delta\gamma) - F_X\} dG[a] + 0$, which simplifies to $(E/n\sigma\gamma\Delta)(A[a_D] + \phi A[a_X])$ minus $(G[a_D]F_D + G[a_X]F_X)$, where $A[x] \equiv \int_0^x a^{1-\sigma} dG[a]$. Notice, however, that $G[a_D]\Delta = (A[a_D] + \phi A[a_X])$ from (2) and the fact that $dF[a] = g[a]/G[a_D] da$. Thus, the benefit less market-entry costs is $G[a_D](E/n\sigma\gamma) - G[a_D]F_D - G[a_X]F_X$. This is set equal to F_I in the direct approach. Dividing this through by $G[a_D]$ validates our indirect approach in (6). We adopt the indirect approach since it allows us to deal more clearly with growth. First, it allows a direct comparison with standard endogenous growth models (which do not have market-entry costs) and it facilitates analysis of growth effects by concentrating the impact of openness and parameter changes in the ex ante

3.2. Saving, investment and growth

To finish our characterisation of the growth path, we need to address three further issues: 1) the equilibrium discount rate, 2) the utility maximising division of income between consumption and saving/investment, and 3) the time paths of the cut-offs, a_D and a_X .

3.2.1. Intertemporal utility maximisation

To deal with these issues, we work with simple intertemporal preferences, namely:

$$(7) \quad U = \int_0^{\infty} e^{-\rho t} \ln C_t dt; \quad C = \left(\int_{i \in \Theta} c_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}; \quad Y \equiv E + S; \quad Y \equiv L + \frac{E}{\sigma}$$

where ρ is the rate of pure time preference, and Θ is the (time-varying) set of consumed varieties.

Consumers divide their income, Y , between current expenditure, E , and saving/investment, S . The final identity is that income equals labour income, L , plus all operating profit E/σ (there is no profit in the I-sector). Utility optimisation yields the standard Dixit-Stiglitz demand functions and an optimal expenditure path that is characterised by a transversality condition and the standard Euler equation⁷:

$$(8) \quad \dot{E}/E = r - \rho$$

where ‘ r ’ is the rate of return on investment. Once we characterise the utility optimising evolution of consumption expenditure, this equation will yield r , the rate of return that consumers require as

expected cost of getting a winner, namely \bar{F} .

⁷ Formally, income is $L+rK$, where K is wealth, r is the rate of return, and $\dot{K} = L+rK-E$ describes wealth accumulation. The Hamiltonian is $e^{-\rho t} \ln(E/P) + \theta(L+rK-E)$ and r and the path of P are taken as exogenous by the consumer/saver. The necessary conditions are $e^{-\rho t}/E = \theta$ and $-\theta r = \dot{\theta}$ plus a transversality condition. Manipulation involving the time derivative of the first condition, $\dot{\theta}/\theta = -\rho - \dot{E}/E$, and substitution of the second condition yields the Euler equation.

compensation for postponing consumption. Note also that determining the utility-maximising path of expenditure also pins down the savings/investment path. Expenditure on goods E equals income less spending on new units of knowledge. Since there are zero pure profits in the I-sector, the value of spending on new knowledge equals the value of inputs, namely the amount of labour devoted to innovation, which we denote as L_I . Thus $E=L+E/\sigma-L_I$, so:

$$(9) \quad E = \frac{L - L_I}{1 - 1/\sigma} \quad \Leftrightarrow \quad L_I = L - E(1 - 1/\sigma)$$

3.2.2. The innovation sector learning curve

Given the knowledge spillovers in the innovation sector, we have that b_I declines as n rises. More specifically, we follow standard practice in the product innovation growth literature and assume:

$$b_I = \frac{1}{n + \lambda n^*}; \quad 0 \leq \lambda \leq 1$$

where the positive parameter λ measures the international dimension of spillovers (this is for the Home nation, but the Foreign nation's is identical since $n=n^*$ by symmetry). The seminal Grossman-Helpman product-innovation model considered the extremes of λ , namely no international knowledge spillovers, $\lambda=0$, and perfect spillovers, $\lambda=1$. Given this and the constant death-rate of firms, δ , constant employment in the I sector implies a constant output of new varieties⁸:

$$(10) \quad g = \frac{L_I}{\bar{F}/(1 + \lambda)} - \delta; \quad g \equiv \frac{\dot{n}}{n}$$

⁸ The flow of new varieties equals $L_I/\bar{F} b_I - \delta n$; dividing by n and using $b_I=1/n(1+\lambda)$ yields the expression.

where g is the growth rate of n and L_I is I-sector employment, which is related to E by (9).

Solving for the time-path of E . The economy can be described as a dynamic system consisting of two differential equations, the Euler equation (8) and the growth of varieties equation (10). To solve these for the steady state, we must specify the state variable. Although the steady state is unaffected by the choice of state variables, the ease of calculation depends greatly on this choice. Following Grossman and Helpman (1991), we choose E as the state variable. The fruit of this choice is immediately apparent. First, it pins down the steady-state rate of return as $r=\rho$ given (8) and the requirement that $\dot{E}=0$ in steady state. Moreover, if E stops evolving, (9) tells us that L_I also stops evolving, so from (10), we know that g is constant along the steady-state growth path. To summarise:

Result 1: **Expenditure E , investment L_I and the growth rate of varieties g are time-invariant along the steady state growth path; also $r=\rho$.**

3.2.3. Evolution of the cut-offs

The fact that the cut-off marginal costs, a_D and a_X , are time-invariant on the long-run growth path is easily shown. Inspection of (4) shows that the ratio is time invariant, however, we must rule out the possibility that both are evolving. To this end, consider the present value of a D-type firm. If the variety is introduced at time zero, the expected present value to its owner is:

$$\int_0^{\infty} \left\{ e^{-(\rho+g+\delta)t} \left(\frac{a^{1-\sigma}}{\Delta} \right) \frac{E}{\sigma n_0} \right\} dt \qquad a_X \leq a \leq a_D$$

Here $e^{-\rho t}$ reflects the fact that the consumer/owner requires a return of $r=\rho$ in order to forego the consumption necessary to finance the creation of a new variety. The term $e^{-\delta t}$ reflects the probability that the variety is still alive at t given the Poisson firm-death process, and the term e^{gt} reflects the rate at which the new firm's market share (and thus operating profit) declines due to the expansion

of competitors as per (2). Solving this integral we get the first expression in (3) with $\gamma=\delta+\rho+g$.

Intuitively, the gross discount rate γ , reflects pure time preference ρ , the constant hazard rate of firm-death δ , and the rate at which the typical variety's market share falls g .

Solving a similar integral for export sales gives use the second expression in (3) with the same value for γ . Likewise, solving for ex ante present value a 'winning' variety yields $\gamma=\delta+\rho+g$.

Consequently, the two cut-off conditions and the I-sector free entry condition are:

$$(11) \quad \frac{a_D^{1-\sigma} E / \sigma}{(\delta + \rho + g)n_0 \Delta} = b_I F_D, \quad \frac{a_X^{1-\sigma} E / \sigma}{(\delta + \rho + g)n_0 \Delta} = \frac{b_I F_X}{\phi}, \quad \frac{E / \sigma}{n_0(\delta + \rho + g)} = b_I \bar{F}$$

Here we are considering a typical point on the steady state growth path and normalise this to $t=0$.

Recalling the learning curve, $b_I=1/n_0(1+\lambda)$, we see that n_0 drops out of both the left-hand and right-hand sides of all three conditions. Or, in other words, the level of n cancels out of the conditions characterising the steady state growth path (as it must since n will grow forever). Using (4), we know that the ratio of the cut-offs, a_X/a_D , is time-invariant and this, combined with the definition of \bar{F} implies that \bar{F} will evolve over time, if and only if a_D does. However, the I-sector free entry condition (6) and Result 1 imply that \bar{F} is time-invariant, so a_D and a_X must be constant along the steady state growth path. To summarise:

Result 2: **Cut-offs a_D and a_X are time invariant along the long-run growth path.**

Given the time invariances mentioned in Results 1 and 2, the dynamic system can be reduced to three equations – the two cut-off conditions and the innovation-sector's free entry condition – involving three unknowns, g , a_D and a_X . The only thing left to find is the utility maximising level of E .

Using the definition of expenditure $E=L+E/\sigma-L_I$, the definition of b_I , and the value of γ together with (10) and (6) yields the useful and intuitive result that the utility-maximising E equals

permanent income, namely, the income from labour, L , plus the rental rate on the steady-state value of the nation's capital stock, which equals $\bar{F}/(1+\lambda)$ ⁹:

$$(12) \quad E = L + \rho\bar{F}/(1 + \lambda)$$

A typical assumption in no-growth heterogeneous-firms trade models is that $\rho=0$, so $E=L$; we take $\rho>0$ to facilitate intertemporal welfare calculations. Using (12) to eliminate E , we can write (11) in terms of g , a_D and a_X , namely:

$$(13) \quad \frac{a_D^{1-\sigma}(L(1+\lambda) + \rho\bar{F})}{(\delta + \rho + g)\sigma\Delta} = F_D, \quad \frac{a_X^{1-\sigma}(L(1+\lambda) + \rho\bar{F})}{(\delta + \rho + g)\sigma\Delta} = \frac{F_X}{\phi}, \quad g = \frac{L(1+\lambda)}{\sigma\bar{F}} - \delta - \rho\left(1 - \frac{1}{\sigma}\right)$$

Where the ex-ante expected fixed costs \bar{F} is defined in (5).

Observe that, as in most endogenous growth models, the long-run growth rate is higher when consumers are more patient, when the death rate is lower, and the world supply of primary factor is larger. Of course, g need not be positive but as a regularity condition for the model, we suppose that parameters are such that growth is positive (for example, this would hold if L were big enough).

3.3. Discussion of the growth path

The growth path in our model shares many similarities with the growth path of product-innovation growth models that assume homogenous varieties. The long-run growth path is characterised by a time-invariant division of labour between the production of consumption goods and capital goods (knowledge in our model), with L_1 units of labour devoted to the production of knowledge and $L-L_1$ units to consumer-goods production. Despite the constant labour input, the knowledge output rises

⁹ From (6), $b_{it}=1/n_i(1+\lambda)$ and the free entry condition, the value of a average produced variety is $\bar{F}/n_i(1+\lambda)$ so the value of all varieties is $\bar{F}/(1+\lambda)$, recalling that N -types have zero value.

steadily due to the learning curve and this allows the same amount of I-sector labour to produce an ever growth stock of varieties.

Observe that g can be thought of as the equilibrating variable that is analogous to the role of n in no-growth heterogeneous-firms trade models. In models without growth, n rises to the point where the ex ante expected benefit of expanding the stock of produced varieties is zero. Because knowledge spillovers remove n from the calculation, it is the speed of innovation that ensures zero expected pure profit from variety introduction. The economic logic is clear. A faster growth of n drives down the expected benefit of introducing a new variety since it boosts the rate at which innovators expect the market share (and thus profits) of a new variety to decline; see (2).

The constant flow of new varieties together with the time invariant employment of labour in the production of consumption goods means that consumption/production of each variety continuously declines at a rate of g . The representative consumer sees her real consumption, namely E/P , rise over time due to the love-of-variety feature of Dixit-Stiglitz preferences. In particular, the growth rate of real consumption is:

$$(14) \quad g_c = \frac{1}{\sigma - 1} \left\{ \frac{L(1 + \lambda)}{\sigma \bar{F}} - \delta - \rho \left(1 - \frac{1}{\sigma}\right) \right\}$$

since $P = \{(n\Delta)^{1/(1-\sigma)}\} / (1 - 1/\sigma)$.

4. WELFARE

We now turn to welfare issues with our primary focus on the question of whether the market produces growth that is too fast or too slow. We begin, however, with an evaluation of static welfare questions.

4.1. Static welfare issues

There are two static issues to consider: 1) does the market provide the right mass of varieties at any point in time? And, 2) does it provide the right cut-offs? As to the former question, we follow the Grossman and Helpman (1991 Appendix A.3.3) procedure of focusing on two externalities. The entering firm does not consider the extra consumer surplus it generates since it cares only about profit, but it also does not consider the impact it has on the profits of existing varieties.

4.1.1. Optimal mass of firms

Denoting the sum of all operating profit as Π , the indirect utility function is:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \{\ln E - \ln P\} dt; \quad P \equiv (n\Delta)^{1/(1-\sigma)} / (1 - \frac{1}{\sigma}); \quad E \equiv L - L_I + \Pi$$

We consider a small perturbation of the laissez-faire steady-state growth path of n and evaluate whether this raises utility. Differentiating with respect to n at time T :

$$(15) \quad \frac{dU_T}{dn_T} = \int e^{-\rho(t-T)} \left(\frac{1}{E_T} \frac{dE_T}{dn_T} - \frac{1}{P} \frac{d(P_T)}{dn_T} \right) dt$$

Noting that Δ is independent of n , differentiation of P yields the consumer surplus effect directly:

$$(16) \quad \frac{d \ln P}{dn} = \frac{-1}{n(\sigma - 1)}$$

The profit destruction effect is more subtle. We are considering a slight perturbation of the steady-state growth path of n . Having an extra variety at time T entails a slight increase in investment at T , but this creates a flow of operating profit as well. Since we are working from the steady-state path, where the cost and benefit of an extra variety are equal, the direct impact of the extra variety is nil.

However, this extra variety depresses the profit of all existing varieties, so $dE/dn = d\Pi/dn$.

Specifically, the average domestic operating profit falls from $E/\sigma n$ to $E/\sigma(n+dn)$. Multiplying this by the unperturbed n :

$$\frac{d\Pi}{dn} = -\frac{E}{\sigma n} + \frac{1}{\sigma} \frac{dE}{dn}$$

Using this and $dE/dn=d\Pi/dn$, we have $(1/E)dE/dn=-1/n(\sigma-1)$. Plugging this and (16) into (15) we see that $dU/dn=0$. In short, the consumer surplus effect and the profit destruction effect exactly cancel out, so the laissez-faire mass of varieties is also socially optimal (as usual in the static Dixit-Stiglitz model and its endogenous growth applications; this is true in a constrained optimum sense only, see Dixit and Stiglitz, 1977).

4.1.2. Optimal cut-offs

Evaluation of the socially optimal cut-offs follows a similar procedure. We consider the equilibrium growth path with the market-determined cut-offs and consider a small perturbation in these cut-offs. Consider first a small increase in a_D , i.e. each nation would see a small increase in the number of locally produced varieties. As before, introduction of these new varieties would have a direct effect – the cost of their development minus the present value of their operating profit – but this would be zero given the cut-off condition that defines a_D ; the same holds for a_X . Additionally, there would be an indirect effect on consumer surplus and profit destruction as before, however once again these will cancel out. The basic reason is that $1/P=(n\Delta)^{1/(\sigma-1)}/(1-1/\sigma)$ and the average profit of home firms falls from $\Delta(E/n\Delta\sigma+\phi E/n\Delta\sigma)$ to $\Delta(E/n(\Delta+d\Delta)\sigma+\phi E/n(\Delta+d\Delta)\sigma)$. Using the same procedure as above, it is easy to show that $dU/d\Delta=0$ when considering a slight perturbation of Δ around its steady-state growth path. Plainly this is true whether the perturbation of Δ comes from a perturbation of a_D or a_X . To summarise:

Result 3: **The laissez-faire mass of varieties and cut-off points along the steady-state growth path are socially optimal.**

4.2. Dynamic efficiency

The knowledge spillovers that drive endogenous growth in our model constitute an intertemporal externality and are thus a source of social inefficiency as in the homogenous-firms trade and growth literature. Each new variety lowers costs of all future innovations, but atomistic innovators ignore this when making their choices. As a consequence, the laissez-faire growth path will be too slow from the social perspective. To see this, we solve for the socially optimal, steady-state growth rate.

The representative consumer's utility can be written in terms of the steady state growth rate by employing $E=(L-L_1)/(1-1/\sigma)$ where $L_1=(g+\delta)\bar{F}/(1+\lambda)$; see (9) and (10), and $P=(n_0e^{gt}\Delta)^{1/(1-\sigma)}/(1-1/\sigma)$

so utility in terms of g is $U = \int_{t=0}^{\infty} e^{-\rho t} \{ \ln(L - (g + \delta)\bar{F}/(1 + \lambda)) - \ln(e^{gt} n_0 \Delta)/(1 - \sigma) \} dt$. Solving:

$$(17) \quad U = \frac{\ln\{L - (g + \delta)\bar{F}/(1 + \lambda)\}}{\rho} + \frac{g}{\rho^2(\sigma - 1)} + \frac{\ln \Delta + \ln n_0}{\rho(\sigma - 1)}$$

Solving the first order condition with respect to g , the socially optimal growth rate, g^{soc} , is¹⁰:

$$(18) \quad g^{soc} = \left(\frac{L}{\sigma\bar{F}/(1 + \lambda)} - \rho(1 - \frac{1}{\sigma}) \right) \sigma - \delta$$

Comparing this to the laissez-faire g in (13), we see that $g^{soc} + \delta = \sigma(g^{soc} + \delta)$ so the social growth rate is higher since $\sigma > 1$.

¹⁰More formal reasoning uses the Hamiltonian approach. Taking L_1 as the state variable, using $E=(L-L_1)/(1-1/\sigma)$, the current value Hamiltonian is $H = \text{constant} + \ln(L - L_1) + \ln(n\Delta)/(\sigma - 1) + \theta n(L_1 / \bar{F} - \delta)$ (we set $\lambda=0$ to lighten notation). The necessary and sufficient conditions are (10), the transversality condition, $\dot{\bar{F}} = \theta n(L - L_1)$, and $\dot{\theta}/\theta = (\rho + \delta) - 1/((\sigma - 1)\theta n) - L_1 / \bar{F}$. Taken together they imply $\rho\bar{F}/(L - L_1) - 1/(\sigma - 1)$ equals $\theta n(\dot{\theta}/\theta + \dot{n}/n)$ and this approaches zero as t approaches infinity. Thus, at steady state, we get the expression in the text with $\lambda=0$. This approach is more general in that it allows the possibility of a time-varying growth rate.

Intuition for this result is simple. The true social cost of an innovation is not $b_1 \bar{F}$ as perceived by the private firms; it is $b_1 \bar{F}$ minus the present value of the labour savings in the innovation sector implied by the knowledge spillovers. Thus the social free entry condition for the I-sector is:

$$(19) \quad \frac{(L + \rho \bar{F} / (1 + \lambda)) / \sigma}{n_0 (\delta + \rho + g^{soc})} = \frac{\bar{F}}{n_0 (1 + \lambda)} - \frac{(g^{soc} + \delta) \bar{F} / (1 + \lambda)}{n_0 (\delta + \rho + g^{soc})} \left(1 - \frac{1}{\sigma}\right)$$

Solving this yields (18). To interpret this, observe that the private benefit from innovation (shown on the left hand side) should equal the private cost $b_1 \bar{F}$ minus the present value of the spillovers-induced cost savings (the last term on the right side). The cost savings aspect of the last term can be seen by noting, from (10), that $\bar{F} (g^{soc} + \delta) / (1 + \lambda)$ equals the social L_I , and the $(1 - 1/\sigma)$ reflects the usual inter-sectoral imperfect competition distortion (the consumption good sector is imperfectly competitive with a price mark-up of $1/(1 - 1/\sigma)$ whereas the I-sector is perfectly competitive).

Result 4: **The laissez-faire growth rate is too low from the planner's perspective.**

This is a well known result in endogenous growth literature with homogenous firms (first shown by Grossman and Helpman); our value added is to show it also extends to the heterogeneous firms case.

5. ANALYTIC SOLUTIONS

All of the analysis up to this point has been conducted without resort to a functional form for G . Indeed much of the subsequent analysis can also be conducted in this manner, but the reasoning is clearer when we have explicit solutions for the cut-offs and growth rate. In particular, given the dependence of Δ and \bar{F} on the conditional and unconditional distributions of the a 's, we cannot solve (13) explicitly without assuming an explicit functional form for G . Following standard practice in the literature, we assume a Pareto distribution whose cumulative density function is:

$$(20) \quad G[a] = \left(\frac{a}{a_0}\right)^k \quad 0 \leq a \leq a_0$$

where k and a_0 are the ‘shape’ and ‘scale’ parameters, respectively. Given (20), and assuming the regularity condition at $\beta \equiv k/(\sigma-1) > 1$ so the integrals converge, we can explicitly solve for Δ and \bar{F} to get¹¹:

$$(21) \quad \Delta = \frac{a_D^{1-\sigma}(1+\Omega)}{1-1/\beta}, \quad \bar{F} = \frac{F_D(1+\Omega)}{1-1/\beta}; \quad 0 \leq \Omega \equiv \phi^\beta T^{1-\beta} \leq 1, \quad T \equiv \frac{F_X}{F_D}, \quad \beta \equiv \frac{k}{\sigma-1} > 1$$

The variable Ω (a mnemonic for ‘openness’) summarising the two types of goods-trade barriers our model, so it is worth pointing out four features of Ω that facilitate intuition and subsequent analysis:

(1) Ω measures the combined protective effects of higher fixed and variable trade costs; (2) $\Omega=0$ with infinite τ and/or infinite F_X/F_D , (3) $\Omega=1$ with zero iceberg costs and $F_X=F_D$; (4) we can also express Ω as $\phi(F_X/F_D\phi)^{1-\beta}$ and this tells us that as long as the inequality in (4) holds (this reflects the stylised fact that not all firms export), Ω is bound between zero and unity. Using (21), we can solve (13) to get explicit, closed form solutions for g , a_D and a_X ¹²:

$$(22) \quad g = \frac{(\beta-1)L(1+\lambda)}{\beta F_D(1+\Omega)\sigma} - \delta - \rho\left(1 - \frac{1}{\sigma}\right); \quad a_D = a_0 \left(\frac{F_I(\beta-1)}{F_D(1+\Omega)}\right)^{\frac{1}{k}}; \quad a_X = a_0 \left(\frac{\Omega(\beta-1)F_I}{(1+\Omega)F_X}\right)^{\frac{1}{k}}$$

We turn now to considering the impact of greater openness on growth.

¹¹ The expression for Δ follows directly from (2) and (20) noting that $F[a]$ in (2) is $G[a]/G[a_D]$, so $dF[a] = ka^{k-1}/a_D^k$. Using this in the cut-off condition for a_D implies $E/n\sigma\gamma = F_D(1+\Omega)/(1-1/\beta)/(1+\lambda)$ and this equals $b_I \bar{F}$ by (6).

¹² To find a_D , note that $\bar{F} = F_D(1+\Omega) + (a_0/a_D)^k F_I$ from (5) and (20) but $\bar{F} = F_D(1+\Omega)/(1-1/\beta)$ from (21). Equating these and solving for a_D yields the expression in (22). Using this in (4) implies the expression for a_X in (22) after some rearrangement. Finally, we get g from (6) using \bar{F} from (21), (12), and $\gamma = \rho + \delta + g$.

6. GROWTH EFFECTS OF MARKET OPENNING

In our model, there are three types of barriers to the free flow of goods and ideas: (1) iceberg trade costs ϕ , (2) technical barriers to trade that make it more expensive to enter the export market than it is to enter the domestic market T , and (3) natural or man-made barriers that reduce the knowledge spillovers in innovation $1-\lambda$.

Since $\beta > 1$ and ϕ rises towards unity as iceberg trade costs fall while T falls towards unity as technical barriers to trade fall, greater openness – i.e. $d\Omega > 0$ – measure the impact of lower fixed or variable trade barriers. Differentiating (22) using the definition of Ω in (21):

$$(23) \quad \frac{dg/g}{d\phi/\phi} = \left(\frac{L(1+\lambda)\beta}{g\sigma\bar{F}} \right) \frac{-\Omega}{1+\Omega} < 0, \quad \frac{dg/g}{dT/T} = \left(\frac{L(1+\lambda)}{g\sigma\bar{F}} \right) \frac{\Omega(\beta-1)}{1+\Omega} > 0, \quad \frac{dg}{d\lambda} = \frac{L}{\sigma\bar{F}} > 0$$

In other words, greater openness to trade in goods – i.e. $dT < 0$ and/or $d\phi > 0$ – slows growth. Notice that the growth effects of both forms of goods-trade liberalisation are amplified as overall openness rises, i.e. as Ω approaches unity. By contrast, greater openness in terms of the flow of ideas ($d\lambda > 0$) tends to speed growth. To summarise:

Result 5: **Greater goods market openness unambiguously slows the introduction of new varieties and thus slows real consumption growth. The impact is magnified as overall trade openness rises. Greater openness in terms of international knowledge spillovers, by contrast, is pro-growth.**

Intuition. These results are straightforward when one considers the fulcrum of the model, namely the decision to introduce a new variety. The counterintuitive aspect of it stems from general equilibrium effects. Consider the polar cases where no firms export, or all firms export. In our model, an individual X-type firm believes it maximises profit by exporting, however, in general equilibrium the average market share of all firms and thus the average benefit from producing

would be exactly the same whether all firms export, or no firms export. However, when all firms export, all would face a fixed cost of $F_I/G[a_D]+F_X+F_D$, whereas in the no export case, the fixed cost would be lower, namely $F_I/G[a_D]+F_D$. Naturally, the attractiveness of introducing varieties would, *ceteris paribus*, be less in the all-export case since the expected benefit of a winner would be the same as in the no-export case, but the expected fixed cost would be greater. The anti-growth effect therefore comes from the fact that greater openness raises the ex ante expected fixed cost of developing a variety without, in general equilibrium, raising the benefit of developing a variety. This naturally makes variety development less attractive so the rate of development must slow to rebalance the costs and benefits.

More directly, the entry decision balances the ex ante expected gain from introducing a new variety – this is always $E/\sigma(\rho+\delta+g)$ – against the ex ante expected cost of a new variety, namely $\bar{F}/(1+\lambda)$. Goods-market openness has no impact on the benefit of introducing a new variety, but it does affect the cost, as inspection of the definition of \bar{F} given in (5) reveals. In the special case of a Pareto distribution we can see the point more directly. As inspection of the second equation in (21) shows, greater goods-market openness (measured by Ω) raises the cost of a newly produced variety. Similarly, as international spillovers get better, i.e. as λ approaches unity, then the expected cost of a new variety falls.

Domestic deregulation. Plainly there is a pro-growth effect of domestic de-regulation without any change in openness (which corresponds to a proportional reduction in F_D and F_X), i.e. with no change in F_X/F_D or in ϕ (this follows from inspection of the definition of \bar{F}).

Finally, it is also obvious that a reduction in F_I would be pro-growth. Indeed a subsidy to innovation paid for by a lump-sum tax could lead the market to grow at the socially optimal rate, as in the homogenous-firms trade and growth models.

6.1. Comparison with the Grossman-Helpman product innovation model

The anti-growth impact of greater openness may seem at odds with the findings of the existing trade and growth literature. Grossman and Helpman (1991), for example, show that an autarky-to-free-trade opening is pro-growth. The original Grossman-Helpman model did not consider marginal changes in iceberg trade cost, but extensions of that model show that marginal reductions in iceberg trade costs have no impact on the endogenous growth rate. The question therefore arises: what causes the difference? Our model differs from the canonical product-innovation growth model along two important dimensions – the inclusion of heterogeneity and the inclusion of market-entry costs. We consider the impact of the two in turn.

In the rest of this subsection, we assume $\lambda=0$ as Grossman and Helpman do in one of their cases. The results are not qualitatively affected by this, but the reasoning is clearer.

6.1.1. Impact of market-entry costs

Consider the special case of our model where there are no market-entry costs, i.e. $F_X=F_D=0$. In this case, both cut-offs equal the upper bound of the marginal cost range, namely a_0 , so every firm becomes an X-type; there are no D-types and no N-types.¹³ Even without assuming $G[a]$ is Pareto, we get an analytic solution for the growth rate. By inspection of (5), noting that $G[a_X]=G[a_D]=1$ when $a_D=a_X=a_0$, $\bar{F}=F_1$, so the growth rate of n is:

$$g = \frac{L}{\sigma F_1} - \delta - \rho \left(1 - \frac{1}{\sigma}\right)$$

Observe that openness never enters this expression. As in the homogeneous-firms product-innovation growth model, marginal changes in iceberg trade costs have no growth effects. From this

¹³ As in the standard Helpman-Krugman trade model, every firm earns positive operating profit on every sale, so all firms would export.

we clearly see that the indirect impact of freer trade on the expected fixed cost of a new variety is the key to the anti-growth effects. As we shall see, heterogeneity is actually good for growth.

6.1.2. Impact of heterogeneity

There are two ways of varying the degree of heterogeneity of marginal costs when $G[a]$ is Pareto – a change in the shape parameter k and a change in scale parameter a_0 . Plainly, the increase in k affects both the mean and the variance of marginal costs. In particular, the first and second moments of (20) are, respectively:

$$(24) \quad E(a) = \frac{k}{1+k} a_0, \quad V(a) = \frac{k}{2+k} a_0^2 - E(a)^2 = \frac{E(a)^2}{k(2+k)}$$

Thus a rise in k with no change in a_0 raises the mean and lowers the variance. To focus purely on the heterogeneity aspects, we can change a_0 and k simultaneously to induce a mean-preserving spread (MPS) of G . As inspection of the second expression (24) shows, this involves a drop in k and a rise in a_0 . (NB: since $\beta > 1$ is a regularity condition for expectations to exist, the lower bound for k in these calculations is $\sigma - 1$, where $\sigma > 1$). As it turns out, a_0 does not affect the equilibrium growth rate; see (22), so the change in g with respect to a MPS of $G[a]$ works only through the fall in k . As usual, all growth effects operate through the impact on \bar{F} – even allowing for a general G as per (13) – so the growth effect of a MPS depends upon:

$$(25) \quad \left. \frac{dg}{d\bar{F}} \frac{d\bar{F}}{d\beta} \frac{d\beta}{dk} \right|_{MPS} = \frac{-L}{\sigma \bar{F}^2} \left\{ \frac{\bar{F}}{\beta(\beta-1)} + \frac{F_D}{1-1/\beta} \Omega \ln \frac{T}{\phi} \right\} \frac{1}{\sigma-1} < 0$$

where the inequality comes from the fact that $\ln(T/\phi) > 0$ by (4). Thus a MPS of G (i.e. a lower k) is pro-growth.

Intuition for this result flows from the interaction between heterogeneity and the market-entry costs. The heterogeneity means that some firms will be extraordinarily profitable and thus will find it

worth their while to pay two market-entry costs. As the heterogeneity narrows, most firms' competitiveness – in the sense of (2) – tends towards the average level of competitiveness, that is, the model gets closer to the representative firm model in which all firms export (Grossman and Helpman, 1991). Indeed a MPS of G can be shown to decrease the fraction of firms that are X -types. As a result, the average fixed cost \bar{F} is lower, but the ex ante average market share and thus average profitability is the same. By inspection of (13), a lower \bar{F} raises the long-run growth rate.

Implications for the trade and growth link. Finally, we turn to the question of whether marginal-cost heterogeneity per se is responsible for the anti-growth impact of greater openness. This is essentially a question of the cross derivative, $(d^2g/d\Omega dk)$. What we see from (25) is that $d^2g/d\Omega dk$ is negative. Because a MPS of G requires k to fall, we see that higher heterogeneity in the MPS sense tends to diminish the anti-growth effect. This is most clearly seen in the polar case of no heterogeneity, in the sense that all firms had a marginal cost of a_0 . In this case, every firm would have a market share exactly equal to the average, and either all firms would export (as would be the case of the representative firm model) or none would, so the ex post fixed cost paid by every firm would be identical. In either case, greater openness would have no impact on \bar{F} and so no growth effect. To summarise the two sets of findings:

Result 6: **The anti-growth effect of greater openness in our model is due the assumption of both fixed market-entry costs and heterogeneity. If market-entry costs are zero, or all heterogeneity is eliminated, greater trade freeness has no growth impact (as in the homogeneous-firms trade and growth literature). Note also that a mean-preserving increase in heterogeneity per se is pro-growth, but an increase in skewness (i.e. $dk > 0$) is anti-growth.**

6.2. Welfare effects of openness

Greater openness has a positive static welfare effect in a Melitz-style heterogeneous-firms trade model (Baldwin and Forslid 2004, Melitz and Ottaviano 2003). Here we show that when we allow growth to be endogenous, greater openness has an unambiguously negative welfare effect from a purely dynamic perspective under the regularity condition that growth would be positive even with perfect openness (defined as $\Omega=1$). By ‘dynamic perspective’ we mean the thought-experiment where the static welfare impact is zero.

To see this, we write the change in the present value of utility with respect to openness holding constant the static effects that operate via a_D and a_X as:

$$\frac{\partial U}{\partial g} \frac{dg}{d\Omega} < 0$$

From (17) U is concave in g and we know that the market g is below the optimal g^{SOC} , so dU/dg evaluated at the market rate is positive. We also know that $dg/d\Omega$ is negative, so the overall term is negative.

6.2.1. Including static effects

Including the static welfare effects investigated by Baldwin and Forslid (2004) and Melitz and Ottaviano (2003), we note that the full impact of greater openness is:¹⁴

$$\frac{dU}{d\Omega} = \frac{1}{\rho(1+\Omega)} \left(\frac{1}{1 + \frac{L(1-1/\beta)}{\rho(1+\Omega)F_D}} - \frac{(1-1/\beta)L/\sigma}{\rho(\sigma-1)F_D(1+\Omega)} + \frac{1+1/\beta}{\sigma-1} \right)$$

¹⁴ Melitz and Ottaviano (2003) have heterogeneous firms, but not market-entry costs, so their analysis does not apply direct to the model considered here.

where we used (22) and (17) to simplify. Plainly, we could find an L/F_D that was low enough for this to be positive, or high enough for this to be negative. Note also that this term is more likely to be positive, the higher is Ω to start with. This shows that the net welfare impact is ambiguous taking the dynamic and static effects together. To sum up:

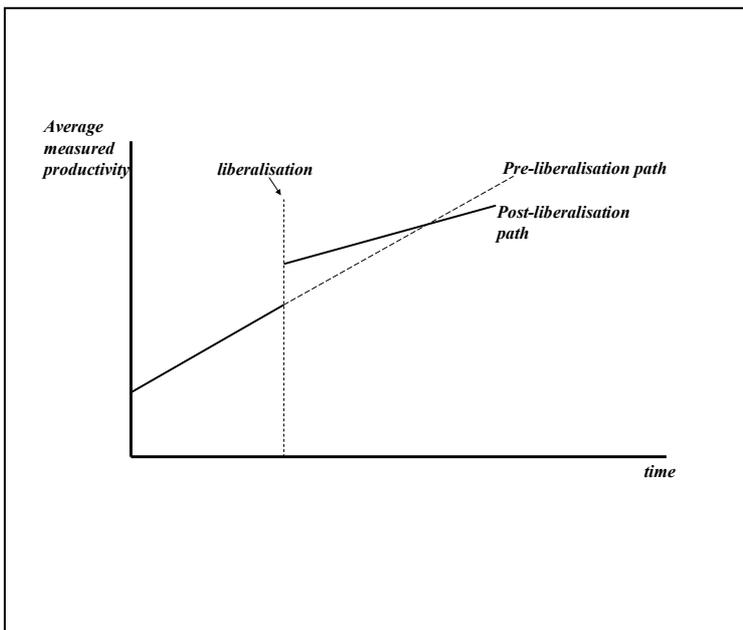


Figure 1: Static versus dynamic productivity effects of freer trade

Result 7: Greater openness is welfare worsening from a purely dynamic perspective, but it has ambiguous welfare effects overall since the purely static effects are positive. Moreover, the latter effects are more likely to dominate if trade is already largely free to start with

6.3. Static versus dynamic productivity effects

A major finding of heterogeneous-firms trade theory is that trade openness can raise productivity via selection and share-shifting effects (Melitz 2003). Defining ‘measured productivity’ as the ratio of real output to labour input, we note that openness also boosts productivity in our model, at least in a static sense. More specifically and considering only the manufacturing sector, total sales and labour input per nation are E and $L-L_1$ respectively, so the value of sales per production worker is

$E/(L-L_1)=1/(1-1/\sigma)$ from (9). The CES price index for national firms is $(n\Delta)^{1/(1-\sigma)}/(1-1/\sigma)$. Solving the integral, and using (10), (12) and (20) measured labour productivity is¹⁵:

$$(26) \quad \text{measured productivity} = \frac{1}{a_D} \left(\frac{n(1+\Omega)}{1-1/\beta} \right)^{1/(\sigma-1)}$$

Since greater openness lowers a_D and raises Ω , it raises industry's average productivity in a static sense, i.e. taking n as given.

However, freer trade also slows the growth rate of n so, freer trade would slow the growth rate of productivity. Thus our heterogeneous-firms trade and growth model displays a trade off between static and dynamic efficiency gains. The static versus dynamic productivity trade off is schematically illustrated in Figure 1.

6.3.1. An alternative productivity measure

The measured productivity in (26) is not the only conceivable measure of productivity. For example, Melitz (2003) focuses on the weighted average of the a 's using relative output shares as weights. His measure is basically the CES price index ignoring the mass of firms and the constant mark-up, i.e. $\Delta^{1/(\sigma-1)}$ in our notation, or $\tilde{\varphi}$ in his.¹⁶ This result also holds in our model as inspection of (21) shows. This measure of productivity, however, has some shortcomings in the context of our

¹⁵ The expression is $\{E/(L-L_1)\}/P^P$, where P^P is the producer price index. Substituting $P^P=\{a_D/(1-1/\sigma)\}\{n/(1-1/\beta)\}^{1/(1-\sigma)}$, which is the solution with the Pareto distribution, yields the expression in the text.

¹⁶ For the closed economy case, this definition of weighted productivity is justified as follows in the Melitz article (see

footnote 9). Using (20), write $\tilde{\varphi} = \int_0^{a_D} a \left(\frac{a^{-\sigma}}{\tilde{\varphi}^{-\sigma}} \right) \frac{ka^{k-1}}{a_D^k} da$ and we see that the idea is that the weight on firm j 's

marginal cost, a_j , is the ratio of j 's output (which is proportional to $a_j^{-\sigma}$) over the output of a firm with the weighted average marginal cost equal to $\tilde{\varphi}$ (which is proportional to $\tilde{\varphi}^{-\sigma}$).

model. Since this definition of productivity is time invariant in our model, this measure would tell us that there was zero productivity improvement despite the fact that real consumption and utility was rising steadily. In essence, the exclusion of love-of-variety effects from the productivity measure implies that it cannot pick up the growth mechanism in the Grossman- Helpman product innovation model. To summarise:

Result 8: Greater openness raises industrial productivity in a static sense but slows its growth rate when productivity is defined as real output per worker and the ideal price index is used to calculate real output. If, however, the measure of productivity is the weighted average of firm-level productivities, as in Melitz (2003), taking relative output shares as the weights, then openness has a positive static effect and no dynamic effect.

7. CONCLUDING REMARKS

This paper explores the impact of trade on growth when firms are heterogeneous and face sunk market-entry costs. Our findings can be viewed as speaking to the trade and growth literature on one hand, and the new-new trade theory on the other.

- As far as trade and growth is concerned, our main finding – that freer trade slows growth – contrasts with most findings in the homogeneous-firms endogenous growth literature. More specifically, we find that greater openness is both anti-growth and welfare worsening from a purely dynamic perspective; greater openness overall, however, has an ambiguous welfare effect since the static gains from trade are positive. We also show that market-entry costs are anti-growth, but heterogeneity per se is pro-growth.
- As far as the new-new trade theory is concerned, our main finding is that there is tension between static and dynamic productivity effects. Although freer trade improves industry

productivity in a level sense, it harms it in a growth sense, at least when ideal price indices are used to measure real output.

Since this is the first paper to introduce heterogeneous firms into an endogenous growth model, we worked with symmetric nations to keep the analysis as streamlined as possible. We have worked out the model with nations that differ only in terms of size (i.e. L 's) and find that the model and results go through unaltered in a qualitative sense, since, as the working paper version of Helpman, Meltiz and Yeaple (2004) shows, the cut-offs are identical despite the unequal market sizes. One line of research that would be interesting would be to explore the impact of unilateral trade liberalisation. However, this analysis becomes very involved since asymmetric iceberg costs produce asymmetric cut-offs and n 's. We therefore leave this task for future research.

Plainly, our growth model is not the only possible one and other growth mechanisms may reverse the anti-growth impact of openness that we find. Deep down, however, the basic logic that results in a lower mass of varieties in no-growth heterogeneous-firms trade models also tends to result in slower growth – the basic reason is that ' n ' is the margin of adjustment that eliminates expected pure profit in the no-growth model while g is the margin in our model. Given this, it is worth recalling the generality of the finding that freer trade reduces ' n ' in the no-growth models. Because only sufficiently efficient firms export, a marginal opening that induces extra firms to begin exporting brings relatively efficient firms into the market, thereby increasing the degree of competition. The extra competition forces out some of the highest marginal cost firms. On net, the result is tougher competition since the new competitors have systematically lower marginal costs than the exiting firms. The tougher competition, however, requires a drop in the mass of firms to restore expected profits to zero. Thus a growth model with heterogeneous firms where openness was pro-growth would have to work with a very different growth mechanism than the one traditionally used in the trade and endogenous growth literature.

REFERENCES

- Aw, B., Chung, S. and Roberts, M., 2000, 'Productivity and Turnover in the Export Market: Micro Evidence from Taiwan and South Korea', *World Bank Economic Review* 14, 65-90.
- Baldwin, Richard E. and Rikard Forslid (2004). "Trade liberalisation with heterogeneous firms," CEPR Discussion Paper 4635.
- Bernard A. and Jensen B, 1995, 'Exporters, Jobs and Wages in U.S. Manufacturing, 1976-1987' *Brookings Papers on Economic Activity, Microeconomics*, 67-119
- Bernard A. and Jensen B., 2001, 'Why Some Firms Export', NBER working paper 8349
- Bernard A. and Jensen, B., 1999b, 'Exporting and Productivity: Importance of Reallocation', NBER working paper 7135.
- Bernard, A. and Jensen, B., 1999a, 'Exceptional Exporter Performance: Cause, Effect, or Both?' *Journal of International Economics* 47, 1-26
- Bernard, A.B., J. Eaton, J.B. Jensen and S. Kortum (2003) "Plants and Productivity in International Trade", *American Economic Review*, Vol. 93, No. 4, September, 1268-1290.
- Bernard, Andrew B., Stephen Redding, Peter K. Schott (2004), "Comparative Advantage and Heterogeneous Firms," NBER Working Paper No. 10668.
- Bernard, Andrew, J. Bradford Jensen, and Samuel Kortum (2003). "Plants and productivity in international trade", *American Economic Review*, 93, 4, 1268-1290.
- Bernard, Andrew, Jonathan Eaton, J. Bradford Jensen, and Peter Schott (2003). "Falling Trade Costs, Heterogeneous Firms and Industry Dynamics", mimeo, Tuck School.

- Clerides S., Lach, S. and Tybout, J., 1998, 'Is Learning by Exporting Important? Micro-Dynamic Evidence from Colombia, Mexico, and Morocco', *Quarterly Journal of Economics*, 133, 903- 904
- Dinopoulos, E. and Segerstrom P, (1999b), "A Schumpeterian Model of Protection and Relative Wages", *American Economic Review*, 89, 450-473.
- Dinopoulos, E. and Segerstrom P., (1999a), "The Dynamic Effects of Contingent Tariffs", *Journal of International Economics*, 47, 191-222.
- Eaton, Jonathan and Samuel Kortum (2002). "Technology, Geography and Trade," *Econometrica*, 70, 5, pp 1741-1780.
- Eaton, Jonathan, Samuel Kortum, and Francis Kramarz (2004), "Dissecting Trade: Firms, Industries, and Export Destinations", *American Economic Review Papers and Proceedings*, 94, 2004, 150-154.
- Grossman, G.M. and Helpman, E., 1989. "Trade; Innovation; and Growth," Papers 154, Princeton, Woodrow Wilson School - Public and International Affairs.
- Grossman, Gene and Elhanan Helpman. (1991). *Innovation and Growth in the World Economy*. Cambridge, MA: MIT Press.
- Grubel, Herbert G., P.J. Lloyd (1975) *Intra-Industry Trade: The Theory and Measurement of International Trade in Differentiated Products* (London, Macmillan).
- Helpman, Elhanan and Paul Krugman (1985), *Market structure and Trade*, MIT Press.
- Helpman, Elhanan, Marc Melitz and Stephen Yeaple (2004). "Export versus FDI with heterogeneous firms," *American Economic Review*, 94, 1, pp. 300-317.

- Hopenhayn, Hugo (1992a). Entry, Exit, and Firm Dynamics in Long Run Equilibrium. *Econometrica* 60:1127-1150.
- Hopenhayn, Hugo (1992b). Exit, Selection, and the Value of Firms, *Journal of Economic Dynamics and Control* 16:621-653.
- Keller, W. (2004). "International technology diffusion,' forthcoming *Journal of Economic Literature*.
- Melitz, M. (2003). "The impact of trade on intraindustry reallocations and aggregate industry productivity," *Econometrica*, 71, pp 1695-1725.
- Melitz, Marc and Gianmarco Ottaviano (2003), "Market Size, Trade and Productivity", mimeo.
- Rivera-Batiz, L. and P. Romer, 1991a, Economic integration and endogenous growth, *Quarterly Journal of Economics* 106, 531-555.
- Rivera-Batiz, L. and P. Romer, 1991b, International trade with endogenous technological change", *European Economic Review* 35, 715-721.
- Romer, P., 1986, Increasing returns and long-run growth, *Journal of Political Economy* 94, 1002-1037.
- Romer, P., 1990, Endogenous technological change, *Journal of Political Economy* 98, 71-102.
- Tybout, J., 2003, Plant- and Firm-level Evidence on "New" Trade Theories', in E. K. Choi and J. Harrigan (eds) *Handbook of International Trade* Blackwell, Oxford.
- Yeaple, S. (2005), Firm Heterogeneity, International Trade, and Wages, *Journal of International Economics*, 65(1), pp 1-20.