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Dennis Bams, Thorsten Lehnert
and Christian C Wolff

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Dennis Bams, LIFE, Maastricht University
Thorsten Lehnert, LIFE, Maastricht University
Christian C Wolff, LIFE, Maastricht University and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Loss Functions in Option Valuation: A Framework for Model Selection

In this paper, we investigate the importance of different loss functions when estimating and evaluating option pricing models. Our analysis shows that it is important to take into account parameter uncertainty, since this leads to uncertainty in the predicted option price. We illustrate the effect on the out-of-sample pricing errors in an application of the *ad hoc* Black-Scholes model to DAX index options. Our empirical results suggest that different loss functions lead to uncertainty about the pricing error itself. At the same time, it provides a first yardstick to evaluate the adequacy of the loss function. This is accomplished through a data-driven method to deliver not just a point estimate of the pricing error, but a confidence interval.

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Dennis Bams
LIFE
Maastricht University
PO Box 616
6200 MD Maastricht
THE NETHERLANDS
Tel: (31 43) 388 3838
Fax: (31 43) 325 8530
Email: w.bams@berfin.unimaas.nl

Thorsten Lehnert
LIFE
Maastricht University
PO Box 616
6200 MD Maastricht
THE NETHERLANDS
Tel: (31 43) 388 3859
Fax: (31 43) 388 4875
Email: t.lehnert@berfin.unimaas.nl

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Christian C Wolff
LIFE
Maastricht University
PO Box 616
6200 MD Maastricht
THE NETHERLANDS
Tel: (31 43) 388 3838
Fax: (31 43) 388 4875
Email: c.wolff@berfin.unimaas.nl

For further Discussion Papers by this author see:
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1. Introduction

The adequacy of an option-pricing model is typically evaluated in an out-of-sample pricing exercise. We naturally prefer the method that minimizes the price differences to the observed market prices. However, the choice of the particular loss function for the in-sample estimation and the out-of-sample evaluation influences the result of that model selection process. Christoffersen and Jacobs (2002) show that the evaluation loss can be minimized by taking the same loss function for in-sample estimation and out-of-sample evaluation. In contrast, empirical researchers are inconsistent in their choice of the loss functions. They do not align the estimation and evaluation loss function and therefore the results of these studies may be misleading. For example the statistics literature already argued that the choice of the loss function is part of the specification of the statistical model under consideration (e.g. Engle (1993)). Therefore, it may happen that a misspecified model may outperform a ‘correctly specified’ model, if different loss functions in estimation and evaluation are used.

The majority of empirical option valuation studies use different loss functions at the estimation and evaluation stage; examples are among others Hutchinson et al. (1994), Bakshi et al. (1997), Chernov and Ghysels (2000), Heston and Nandi (2000) and Pan (2002). The results of these studies regarding model selection are therefore questionable. In contrast, Dumas et al. (1998) (DFW) and Lehnert (2003) tested the out-of-sample performance of their model using identical loss functions in the estimation and evaluation stage.

While Christoffersen and Jacobs (2002) show the importance of the loss function in option valuation, they do not recommend one particular loss function. However, the particular loss function used in the empirical analysis characterizes the model specification under consideration. Therefore, it is still possible that even if the loss functions are aligned, a misspecified model may outperform a ‘correctly specified’ model when the ‘inappropriate’ loss function is used. They correctly suggest that the alignment is more a rule-of-thumb than a general theorem and that the usefulness has to be evaluated in empirical work. The general problem with loss functions is that the choice of a particular one is heavily *subjective* and determined by the user of the option valuation model. Depending on the particular purpose of the model, like hedging, speculating or market making, one or the other loss function is preferred. Using different loss functions, the user puts more or less weight on the correct pricing of options

with different moneyness. The purpose of this paper is to provide an *objective* measure to evaluate different loss functions.

In option valuation, not only the pricing model plays an important role, but also the parameter values of these pricing models. Parameters are usually estimated based upon historical data. When a particular phenomenon is not present in the historical data, the parameters of the distribution function that are intended to account for the phenomenon are estimated with considerable uncertainty, as reflected by the standard errors of the parameter estimates. Uncertainty in the parameter estimates leads to uncertainty in the forecasted future price process and, hence, uncertainty in the out-of-sample pricing error. We will show that it is important to take into account estimation risk. Estimation risk refers to the fact that point estimates of parameters, resulting from an estimation procedure, do not necessarily correspond to the underlying true parameters. There is still uncertainty about these true values. The trade-off between the average pricing error and the precision of reported pricing errors provides a first yardstick to evaluate the adequacy of a particular loss function or option pricing model.

The aim of this paper is to provide an empirical selection approach to arrive at the most suitable loss function for a given data set. The method was proposed by Bams et al. (2002) in order to evaluate Value-at-Risk models and in this paper we apply it to the problem of model selection in an option valuation context. In our view, such an approach should deal with uncertainty in the reported out-of-sample pricing errors that stems from parameter uncertainty.

In the next section we set up the econometric framework.. We explain our testing procedure using a standard option pricing model, the so-called ad-hoc Black-Scholes model. In section 3 we describe the data and section 4 provides the empirical results for the standard model. In section 5 we demonstrate that the results are insensitive to the choice of the underlying option pricing model and replicate the analysis for a more sophisticated GARCH option pricing model. Finally, section 6 concludes.

2. Econometric framework

For the empirical analysis, we first use an alternative to the prominent ad-hoc Black-Scholes model of Dumas, Fleming and Whaley (1998) provided by Derman (1999)¹. We allow each

¹ In a later section, we generalize the analysis using a structural model.

option to have its own Black-Scholes implied volatility depending on the exercise price K and time to maturity T and use the following functional form for the options implied volatility:

$$IV_{ij} = \omega_0 + \omega_1 M_{ij} + \omega_2 M_{ij}^2 + \omega_3 T_j + \omega_4 T_j^2 + \omega_5 M_{ij} T_j, \quad (1)$$

where IV_{ij} denotes the implied volatility and M_{ij} the ‘moneyness’² of an option for the i -th exercise price and j -th maturity. T_j denotes the time to maturity of an option for the j -th maturity. For every exercise price and maturity we can compute the implied volatility and derive option prices using the Black-Scholes model.

We estimate the parameters of the ad-hoc Black-Scholes model by minimizing the particular loss function: (1) the implied volatility error $IVMSE = \frac{1}{N} \sum_{i=1}^n \sum_{j=1}^{l_i} (IV_{ij}^{\hat{}} - IV_{ij})^2$,

(2) the absolute pricing error $MSE = \frac{1}{N} \sum_{i=1}^n \sum_{j=1}^{l_i} (\hat{c}_{ij} - c_{ij})^2$ and (3) the relative pricing error

$\% MSE = \frac{1}{N} \sum_{i=1}^n \sum_{j=1}^{l_i} \left(\frac{\hat{c}_{ij} - c_{ij}}{c_{ij}} \right)^2$. \hat{c}_{ij} denotes the theoretical call price, c_{ij} is the observed

call price, $IV_{ij}^{\hat{}}$ is the implied volatility associated with the theoretical call price, IV_{ij} is the implied volatility associated with the observed call price, n is the number of exercise prices, l_i represents the number of prices available out of all maturities for the i -th exercise price and N is the total number of contracts included on the particular trading day.

Let p denote the vector of unknown parameters ω_i and let $L(p)$ denote the associated likelihood function. The covariance matrix of parameter estimates follows:

$$C = - \left(\frac{\partial^2 L(p)}{\partial p \partial p'} \right)^{-1} \Bigg|_{p=\hat{p}} \quad (2)$$

² We introduce the following notation: Moneyness is defines as $\frac{K_i}{F_j}$, where K_i is the exercise price and F_j is the forward price of an option for the i -th exercise price and j -th maturity, respectively.

where \hat{p} denotes the vector of maximum-likelihood point estimates for the unknown parameters. In the out-of-sample algorithm that we propose, this covariance matrix plays a crucial role since it reflects parameter uncertainty.

Parameter uncertainty may be incorporated by sampling from the parameter distribution. Asymptotic distribution theory leads to the following distribution for the parameter estimates:

$$\hat{p} \sim N(p, C) \quad (3)$$

where \hat{p} are the parameter values that maximize the likelihood function, and C denotes the associated covariance matrix of the parameter estimates. In a Bayesian framework we sample from:

$$p \sim N(\hat{p}, C). \quad (4)$$

Consider H samples³, which are denoted with $p^{(1)}, \dots, p^{(H)}$. For all these parameter values, we calculate theoretical call option prices for all traded options. This leads to H values for the pricing error, e.g. denoted with $MSE(p^{(1)}), \dots, MSE(p^{(H)})$ for the absolute pricing error. Therefore, instead of arriving at one MSE, we now have an entire *sample* of MSEs. The uncertainty in the MSE may be quantified by calculating confidence intervals of the MSE. The expression of the MSE shows that also the parameters may be treated as random variables that have impact on the size and the uncertainty in the estimated MSE. In order to test the adequacy of the assumed loss function, we propose an out-of-sample analysis.

3. Data and Methodology

We use daily closing DAX 30 index options and futures prices for a period from January 2000 until December 2000. The raw data set is directly obtained from the EUREX, European Futures and Options Exchange. The market for DAX index options and futures is the most active index options and futures market in Europe. Therefore it is an interesting market for testing option-pricing models.

³ We conduct the empirical analysis using $H=5000$ samples.

For index options the expiration months are the three nearest calendar months, the three following months within the cycle March, June, September and December, as well as the two following months of the cycle June, December. For index futures the expiration months are the three nearest calendar months within the cycle March, June, September and December. The last trading day is the third Friday of the expiration month, if that is an exchange-trading day; otherwise, the option expires on the exchange-trading day immediately prior to that Friday.

We exclude options with less than one week and more than 25 weeks until maturity and options with a price of less than 2 Euros to avoid liquidity-related biases and because of less useful information on volatilities. Instead of using a static rule and exclude options with absolute moneyness⁴ of more than 10% (see DFW), we exclude options with a daily turnover of less than 10,000 Euros. This rule was applied after carefully analyzing the particular data set (see Lehnert (2003)). Among others DFW argue that options with absolute moneyness of more than 10% are not actively traded and therefore contain no information on volatilities. Therefore an obvious solution is to filter the available option prices and include all options that are actively traded, inside or outside the 10% absolute moneyness interval. In particular, in volatile periods deep out-of-the-money options are highly informative if they are actively traded. As a result, each day we use a minimum of 3 and a maximum of 4 different maturities for the calibration.

The DAX index calculation is based on the assumption that the cash dividend payments are reinvested. Therefore, when we calculate option prices, theoretically we do not have to adjust the index level for the fact that the stock price drops on the ex-dividend date. But the cash dividend payments are taxed and the reinvestment does not fully compensate for the decrease in the stock price. Therefore, in the conversion from e.g. futures prices to the implied spot rate, we observe empirically a different implied underlying index level for each maturity⁵. For this reason, we always work with the underlying index level implied out from futures or option prices.

In particular we are using the following procedure for one particular day to price options on the following trading day:

⁴ In our notation, absolute moneyness is defined as $|K/F-1|$, where K is the exercise price and F is the forward price.

⁵ Since the stocks underlying the index portfolio pay dividends, the present value of expected future dividends is different for different lifetimes of the futures or options contracts.

First, we compute the implied interest rates and implied dividend adjusted index rates from the observed put and call option prices. We are using a modified put-call parity regression proposed by Shimko (1993). Put-call parity for European options reads:

$$c_{ij} - p_{ij} = [X_t - PV(D_j)] - K_i e^{-rf_j T_j} \quad (5)$$

where X_t is the underlying index level at time t , c_{ij} and p_{ij} are the observed call and put closing prices, respectively, with exercise prices K_i and maturity T_j , $PV(D_j)$ denotes the present value of dividends to be paid from time t of option valuation until the maturity of the options contract and rf_j is the continuously compounded interest rate that matches the maturity of the option contract. Therefore we can infer a value for the implied dividend adjusted index level for different maturities, $X_t - PV(D_j)$, and the continuously compounded interest rate for different maturities, rf_j . In order to ensure that the implied dividend adjusted index value is a non-increasing function of the maturity of the option, we occasionally adjust the standard put-call parity regression. Therefore we control and ensure that the value for $X_t - PV(D_j)$ is decreasing with time to maturity, T_j . Since we use closing prices for the estimation, one alternative is to use implied index levels from DAX index futures prices assuming that both markets are closely integrated.

Second, we estimate the parameters of ad-hoc Black-Scholes model by minimizing the particular loss function (e.g. the difference between the market implied volatilities (from daily closing prices) and the implied volatilities of theoretical option prices for calls and puts predicted by the model). Given reasonable starting values, we price European calls and puts with exercise price K_i and time to maturity T_j . We repeat this procedure with the usual optimization method (Newton-Raphson method) and obtain the parameter estimates that minimize the particular loss function. The goodness of fit measure for the optimization is the mean squared error criterion.

Third, having estimated the parameters in-sample, we turn to out-of-sample valuation performance and evaluate how well each day's estimated models value the traded options at the end of the following day. We filter the available option prices according to our criteria for the in-sample calibration. The futures market is the most liquid market, and the options and the futures markets are closely integrated. Therefore it can also be assumed that the futures price is more informative for option pricing than just using the value of the index. For every observed futures

closing price, we can derive the implied underlying index level and evaluate the option. Given a futures price F_j with time to maturity T_j , spot-futures parity is used to determine $X_t - PV(D_j)$ from

$$X_t - PV(D_j) = F_j e^{-rf_j T_j} \quad (6)$$

where $PV(D_j)$ denotes the present value of dividends to be paid over the lifetime of the futures contract, T_j , and rf_j is the continuously compounded interest rate (the interpolated EURIBOR rate) that matches the maturity of the futures contract (or time to expiration of the option). If a given option price observation corresponds to an option that expires at the time of delivery of a futures contract, then the price of the futures contract can be used to determine the quantity $X_t - PV(D_j)$ directly.

The maturities of DAX index options do not always correspond to the delivery dates of the futures contracts. In particular, for index options the two following months are always expiration months, but not necessarily a delivery month for the futures contract. When an option expires on a date other than the delivery date of the futures contract, then the quantity $X_t - PV(D_j)$ is computed from various futures contracts. Let F_1 be the futures price for a contract with the shortest maturity, T_1 and F_2 and F_3 are the futures prices for contracts with the second and third closest delivery months, T_2 and T_3 , respectively. Then the expected future rate of dividend payment d can be computed via spot-futures parity by:

$$d = \frac{r_3 T_3 - r_2 T_2 - \log(F_3 / F_2)}{(T_3 - T_2)} \quad (7)$$

Hence, the quantity $X_t - PV(D) = X_t e^{-dT}$ associated with the option that expires at time T in the future can be computed by⁶

$$X_t e^{-dT} = F_1 e^{-(rf_1 - d)T_1 - dT}. \quad (8)$$

⁶ See e.g. the appendix in Poteshman (2001) for details.

This method allows for a perfect match between the observed option price and the underlying dividend adjusted spot rate. Given the parameter estimates and the implied dividend-adjusted underlying we can calculate option prices and compare them to the observed option prices of traded index options. For the out-of-sample part the same loss functions for call options are used. The predictive performance of the various models are evaluated and compared by using the root mean squared valuation error criterion. We compare the predicted option values with the observed prices for every traded option. We repeat the whole procedure over the out-of-sample period and conclude which loss function minimizes the uncertainty in the out-of-sample pricing error.

4. Empirical results

For each trading day of the year 2000, we estimated the model using closing prices of traded options that fulfill our criteria. On average 84 option prices are used for the calibration and evaluation of the models, with a minimum of 62 and a maximum of 155. The model is estimated three times and each time with a different loss function. Therefore, we estimate the model by minimizing one loss function and determine the root mean squared error (RMSE) according to the loss function used. At the same time, we also determine the RMSE according to the other two loss functions. Table 1, Panel A reports the average RMSEs over the whole period (January 1st, 2000-December 29th, 2000). The diagonal of the Table corresponds to the ‘loss’ from using the same loss function at the estimation and evaluation stage. The off-diagonal entries report the ‘losses’ from using different loss function in- and out-of-sample. As expected, calibrating the model using one loss function also results in a minimum pricing error (in bold) for that particular loss function. The estimated parameters when calibrating the model using one loss function are only sub-optimal when looking at the pricing error regarding a different loss function.

[Table 1]

In a next step, we use the model calibrated on one trading day to price all traded options on the following day (closing prices). Again, we estimate the model using one loss function, but also evaluate the model using another loss function. Therefore, we change the specification at the evaluation stage. Table 1, Panel B reports the average RMSEs for the out-of-sample pricing

exercise. Still, as expected, using the same loss function at the estimation and evaluation stages minimizes the pricing error at the evaluation stage (in bold). We confirm the results of Christoffersen and Jacobs (2002)). Therefore, we cannot conclude which loss function is the preferred one, because the individual results are not directly comparable.

An objective measure in option valuation could be the precision of the particular pricing error in the out-of-sample analysis. Therefore, we use our simulation approach in order to derive a whole sample of out-of-sample pricing errors by incorporating the uncertainty in the parameter estimates from the calibration. Using the most appropriate loss function should result in the smallest confidence interval around the mean pricing error; actually for all loss functions. The results show that the distribution of pricing errors is heavily skewed to the right, meaning that significant pricing errors may occur, because, even when all parameter estimates are significant, there is a lot of uncertainty in the estimates. In order to make the uncertainty of different pricing errors comparable, we define a criterion that measures the actual uncertainty relative to the mean:

$$precision = \frac{UpperBound - LowerBound}{Mean}, \quad (9)$$

where upper/lower bound refers to the empirical upper/lower bounds of the 95% confidence interval around the mean, respectively and mean refers to the average pricing error. The choice of a particular criterion is of course arbitrary, but our precision measure is at least plausible, because of the following logical characteristics: the measure goes to zero if there is no uncertainty in the parameter estimates and hence, in the pricing error and we obtain positive values in case of uncertain parameter estimates. This measure is a standardized criterion and allows to objectively compare the different loss function under consideration. Table 2, Panels A and B report the average values of the measure for the in- and out-of-sample pricing exercise. The results show that there are obvious differences in the results for the in- and out-of-sample analyses.

[Table 2]

From the in-sample analysis, we see that the conclusions of Christoffersen and Jacobs (2002) are also valid for our type of analysis: aligning the loss function at the estimation and evaluation stage also reduces the relative uncertainty in the pricing error. However, the results do not allow us to recommend one or the other loss function. While we cannot draw a conclusion from the in-sample analysis, we can conclude from the out-of-sample analysis that using the absolute Euro pricing error at the estimation stage, minimizes the uncertainty in the pricing error at the evaluation stage for all loss functions (in bold). Therefore, it might be considered as the most appropriate, objective loss function in option valuation, because it minimizes the relative uncertainty in the out-of-sample pricing error regardless which loss function is used to evaluate the model.

5. Comparison with a GARCH Option Pricing Model

So far the empirical analysis has focused on testing the impact of different loss functions on the precision of the pricing error using an ad-hoc Black-Scholes model as the underlying option pricing model. We now consider the popular GARCH option pricing models and investigate whether the previous results can be generalized for more structural models. To document this, we replicate the analysis for a parsimonious GARCH specification, which only contains volatility clustering and a leverage effect.

In a Gaussian discrete-time economy the value of the index at time t , X_t , can be assumed to follow the following dynamics (see e.g. Duan (1995)):

$$r_t = \ln\left(\frac{X_t}{X_{t-1}}\right) + d_t = \mu_t + \sigma_t e_t \tag{10}$$

$$e_t | \Omega_{t-1} \sim N(0,1) \text{ under probability measure } P,$$

$$\ln(\sigma_t^2) = \omega + \alpha \ln(\sigma_{t-1}^2) + \beta(|e_{t-1}| - \gamma e_{t-1})$$

where the conditional mean is defined as $\mu_t = rf_t + \lambda \sigma_t$, d_t is the dividend yield of the index portfolio, rf_t is the risk-free rate, λ is the price of risk and Ω_{t-1} is the information set in period $t-1$ and the combination of β , γ , b and δ captures the leverage effect.

Duan (1995) shows that under the Local Risk Neutral Valuation Relationship (LRNVR) the conditional variance remains unchanged, but under the pricing measure Q the conditional expectation of r_t is equal to the risk free rate rf_t :

$$E^Q[\exp(r_t)|\Omega_{t-1}] = \exp(rf_t), \quad (11)$$

Therefore, the LRNVR transforms the physical return process to a risk-neutral dynamic. The risk-neutral Gaussian GARCH process reads⁷:

$$\begin{aligned} r_t &= rf_t - \frac{1}{2}\sigma_t^2 + \sigma_t \varepsilon_t \\ \varepsilon_t | \Omega_{t-1} &\sim N(0,1) \text{ under risk-neutralized probability measure } Q, \\ \ln(\sigma_t^2) &= \omega + \alpha \ln(\sigma_{t-1}^2) + \beta(|\varepsilon_{t-1} - \lambda| - \gamma(\varepsilon_{t-1} - \lambda)) \end{aligned} \quad (12)$$

where the term $-\frac{1}{2}\sigma_t^2$ gives additional control for the conditional mean. In Equation (12), ε_t is not necessarily normal, but to include the Black-Scholes model as a special case we typically assume that ε_t is a Gaussian random variable. The unconditional volatility level is equal

to $\sqrt{\exp\left(\frac{\omega + \beta E[(|\varepsilon - \lambda| - \gamma(\varepsilon - \lambda)]]}{1 - \alpha}\right)}$ and can be evaluated numerically. The parameter α

measures the persistence of the variance process.

The locally risk-neutral valuation relationship ensures that under the risk neutral measure Q , the volatility process satisfies

$$Var^Q[r_t|\Omega_{t-1}] = Var^P[r_t|\Omega_{t-1}] = \sigma_t^2. \quad (13)$$

A European call option with exercise price K_i and maturity T_j has at time t price equal to:

$$c_{ij} = \exp(-rf_t T_j) E_t^Q[\max(X_t - K_i, 0) | \Omega_{t-1}] \quad (14)$$

⁷ This type of GARCH specification is sufficient for the purpose at hand. Nevertheless, extensions of the model can improve the pricing performance (see e.g. Lehnert (2003)).

For this kind of derivative valuation models with a high degree of path dependency, computationally demanding Monte Carlo simulations are commonly used for valuing derivative securities. We use the recently proposed simulation adjustment method, the empirical martingale simulation (EMS) of Duan and Simonato (1998), which has been shown to substantially accelerate the convergence of Monte Carlo price estimates and to reduce the so called ‘simulation error’. As starting values for the calibration, we make use of the time-series estimates from the equivalent time-series GARCH model using approximately three years (752 trading days) of historical returns. In addition, we use two time-series parameter estimates for the option calibration: the long run volatility $\bar{\sigma}$ equal to the relatively stable 3-year historical standard deviation and the risk premium parameter λ . Using the time-series estimates for the price of risk is common practice, but our variance targeting approach is different to the one used in other studies (e.g. Heston and Nandi (2000)). They perform a constrained calibration in which the parameters λ and the local volatility are restricted to the time-series GARCH-estimates. In contrast, we estimate the local volatility together with the other parameters. Fixing the stationary volatility level stabilizes the estimation process dramatically without influencing the pricing performance of the model. In particular, in recent years there is some support for the hypothesis that the information provided by implied volatilities from daily option prices is more relevant in forecasting volatility than the volatility information provided by historical returns (e.g. Blair et al. (2001)). Therefore, an estimate of the local volatility from option prices directly might be more informative than the time-series estimate. It is also interesting to note that the stationary volatility level is known to be unstable over time when estimated from option prices; a fact that is typically not discussed in empirical option pricing studies.

In the following, we precisely replicate the empirical analysis described in the previous sections using the GARCH model as the underlying option pricing models. The in- and out-of-sample average pricing errors of the model are presented in Table 3, Panels A and B. In general, the results confirm the findings of the previous section. Therefore, using the same loss function at the estimation and evaluation stage minimizes the in- and out-of-sample pricing error at the evaluation stage (in bold). Additionally, when comparing the two pricing models under consideration, the results are consistent with previous findings (e.g. see Heston and Nandi (2000)). The ad-hoc Black-Scholes model typically overfits the data in-sample, but when

evaluated out-of-sample, it typically underperforms GARCH-type option pricing approaches. The results are consistent and do not depend on the particular loss function used at the estimation or evaluation stage.

[Tables 3 and 4]

Table 4 presents the in- and out-of-sample results for the relative uncertainty in the predicted pricing error. Again, the results suggest that it is important to consider estimation risk, defined as the uncertainty that point estimates of parameters, resulting from an estimation procedure, do not necessarily correspond to the underlying true parameters. Therefore, when looking at the precision of the pricing error, again the findings of the previous section can be confirmed. Aligning the estimation and evaluation loss functions minimizes the in-sample precision of the pricing error at the evaluation stage (Panel A), but using the absolute Euro pricing error at the estimation stage minimizes the uncertainty in the pricing error at the evaluation stage regardless which loss function is used to evaluate the model (Panel B). Therefore, the results from the previous section are robust and seem to be independent of the underlying option pricing model under consideration.

6. Conclusions

This paper investigates the important empirical issue concerning model selection in an option valuation context. So far, the empirical literature has mainly focused on the relative performance of various option valuation models. The role and the importance of the loss functions at the estimation and evaluation stages have been overlooked frequently. We propose a data-driven method that allows us to evaluate the relative performance of different loss function. The approach allows us to promote a particular loss function. Using the absolute pricing error criterion at the estimation stage minimizes the uncertainty in the parameter estimates and therefore maximizes the precision of the out-of-sample pricing error regardless which loss function is used at the evaluation stage. We confirm the empirical results of Christoffersen and Jacobs (2002) and find strong evidence for their conjecture that the absolute pricing error criterion “may serve as a general purpose loss function in option valuation applications”. The results are far-reaching for the option valuation literature, because researchers are typically

inconsistent in their choice of the loss functions and results are therefore incomparable. Of course, the choice of the loss function is subjective, but the framework proposed in this paper allows identification of the most appropriate loss function for the purpose at hand.

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Table 1: In- and Out-of-Sample Pricing Errors, Ad-hoc Black-Scholes Model

| Panel A: In-Sample Pricing Errors | | | |
|-----------------------------------|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.007 | 3.622 | 0.077 |
| € RMSE | 0.010 | 1.912 | 0.081 |
| % RMSE | 0.019 | 8.203 | 0.026 |

| Panel B: Out-of-Sample Pricing Errors | | | |
|---------------------------------------|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.012 | 7.818 | 0.139 |
| € RMSE | 0.013 | 6.756 | 0.114 |
| % RMSE | 0.021 | 11.600 | 0.076 |

Notes. The table presents the average in- and out-of-sample pricing errors from the daily estimation and evaluation of the model. Each day the model is estimated using one particular loss function and evaluated using traded options on the following day. At the evaluation stage, we also compute the results for the remaining loss functions. The table presents the differences in implied volatilities in percentages (IV RMSE), the absolute price differences in Euros (€ RMSE) and the relative pricing errors in percentages (% RMSE), respectively. The figures in bold refer to the best result given a particular loss function at the evaluation stage.

Table 2: Precision of Pricing Errors, Ad-hoc Black-Scholes Model

| Panel A: Precision of Pricing Errors (In-Sample) | | | |
|--|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.158 | 0.624 | 0.652 |
| € RMSE | 0.160 | 0.076 | 0.299 |
| % RMSE | 0.429 | 0.705 | 0.281 |

| Panel B: Precision of Pricing Errors (Out-of-Sample) | | | |
|--|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.286 | 0.406 | 0.589 |
| € RMSE | 0.127 | 0.142 | 0.230 |
| % RMSE | 0.389 | 0.537 | 0.401 |

Notes. The table presents the average precision of the in- and out-of-sample pricing errors from the daily estimation and evaluation of the model. Each day the model is estimated using one particular loss function and evaluated using traded options on the following day. At the evaluation stage, we also compute the results for the remaining loss functions. The table presents the figures for the precision criterion defined in Equation (9). The figures in bold refer to the best result given a particular loss function at the evaluation stage.

Table 3: In- and Out-of-Sample Pricing Errors, GARCH Option Pricing Model

| Panel A: In-Sample Pricing Errors | | | |
|-----------------------------------|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.008 | 3.258 | 0.072 |
| € RMSE | 0.009 | 2.338 | 0.069 |
| % RMSE | 0.017 | 7.592 | 0.039 |

| Panel B: Out-of-Sample Pricing Errors | | | |
|---------------------------------------|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.011 | 7.329 | 0.128 |
| € RMSE | 0.012 | 6.047 | 0.093 |
| % RMSE | 0.019 | 9.834 | 0.072 |

Notes. The table presents the average in- and out-of-sample pricing errors from the daily estimation and evaluation of the model. Each day the model is estimated using one particular loss function and evaluated using traded options on the following day. At the evaluation stage, we also compute the results for the remaining loss functions. The table presents the differences in implied volatilities in percentages (IV RMSE), the absolute price differences in Euros (€ RMSE) and the relative pricing errors in percentages (% RMSE), respectively. The figures in bold refer to the best result given a particular loss function at the evaluation stage.

Table 4: Precision of Pricing Errors, GARCH Option Pricing Model

| Panel A: Precision of Pricing Errors (In-Sample) | | | |
|--|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.185 | 0.714 | 0.752 |
| € RMSE | 0.191 | 0.126 | 0.358 |
| % RMSE | 0.502 | 0.778 | 0.324 |

| Panel B: Precision of Pricing Errors (Our-of-Sample) | | | |
|--|--------------|--------------|--------------|
| Estimation | IV RMSE | Evaluation | |
| | | € RMSE | % RMSE |
| IV RMSE | 0.385 | 0.513 | 0.689 |
| € RMSE | 0.215 | 0.190 | 0.390 |
| % RMSE | 0.502 | 0.678 | 0.523 |

Notes. The table presents the average precision of the in- and out-of-sample pricing errors from the daily estimation and evaluation of the model. Each day the model is estimated using one particular loss function and evaluated using traded options on the following day. At the evaluation stage, we also compute the results for the remaining loss functions. The table presents the figures for the precision criterion defined in Equation (9). The figures in bold refer to the best result given a particular loss function at the evaluation stage.