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ABSTRACT

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JEL Classification: E31, E40, E52, E58, E62 and E63
Keywords: interest rate rules, monetary policy and unique equilibrium

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Monetary Policy with Single Instrument Feedback Rules.*

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Abstract

We consider standard cash-in-advance monetary models and show that there are interest rate or money supply rules such that equilibria are unique. The existence of these single instrument rules depends on whether the economy has an infinite horizon or an arbitrarily large but finite horizon.

Key words: Monetary policy; interest rate rules; unique equilibrium.

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1. Introduction

In this paper we revisit the issue of multiplicity of equilibria when monetary policy is conducted with either the interest rate or the money supply as the instrument of

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policy. There has been an extensive literature on this topic starting with Sargent and Wallace (1975), including a recent literature on local and global determinacy in models with nominal rigidities. We show that it is possible to implement a unique equilibrium with an appropriately chosen interest rate feedback rule, and similarly with a money supply feedback rule of the same type. This is a surprising result because while it is well known that interest rate feedback rules can implement a locally unique equilibrium, it is no less known that they generate multiple equilibria globally.

We show that the reason for the results is the model assumption of an infinite horizon. In finite horizon economies, the number of degrees of freedom in conducting policy does not depend on the way policy is conducted. The number is the same independently of whether interest rates are set as functions of the state only, or as backward, current or forward functions of endogenous variables.

In analogous finite horizon economies, the number of degrees of freedom in conducting policy can be counted exactly. The equilibrium is described by a system of equations where the unknowns are the quantities, prices and policy variables. There are more unknowns than variables, and the difference is the number of degrees of freedom in conducting policy. It is a necessary condition for there to be a unique equilibrium that the same number of exogenous restrictions on the policy variables be added to the system of equations. Single instrument policies are not sufficient restrictions. They always generate multiple equilibria. This is no longer the case in the infinite horizon economy, as we show in this paper.

Whether the appropriate description of the world is an infinite horizon economy or the limit of finite horizon economies, thus, makes a big difference for this particular issue of policy interest, i. e. whether policy conducted with a single instrument, such as the nominal interest rate, is sufficient to determine a unique competitive equilibrium.

As already mentioned, after Sargent and Wallace (1975) and McCallum (1981), there is a large literature on multiplicity of equilibria when the government follows either an interest rate rule or a money supply rule. This includes the literature on local determinacy that identifies conditions on preferences, technology, timing of markets, and policy rules, under which there is a unique local equilibrium (see Bernanke and Woodford (1997), Clarida, Gali and Gertler (1999, 2000), Carlstrom and Fuerst (2001, 2002), Benhabib, Schmitt-Grohe and Uribe (2001a), Dupor (2001) among others). This literature has in turn been criticized by recent work on global stability that makes the point that the conditions for local determinacy can also be conditions for global indeterminacy (see Benhabib, Schmitt-Grohe and Uribe (2001b) and Christiano and Rostagno, 2002).

Our modelling approach is close to Adão, Correia and Teles (2003) for the case with sticky prices. In this paper it is shown that even at the optimal zero interest rate rule there is still room for policy to improve welfare since it is possible to use money supply to implement the optimal allocation in a large set of implementable allocations. This paper is also close to Adão, Correia and Teles (2004) where we show that it is possible to implement unique equilibria in environments with flexible prices and prices set in advance by pegging state contingent interest rates as well as the initial money supply. Bloise, Dreze and Polemarchakis (2004) and Nakajima and Polemarchakis (2005) are also related research.

We assume that fiscal policy is endogenous. Exogeneity of fiscal policy could be used, as in the fiscal theory of the price level to determine unique equilibria.

The paper proceeds as follows: In Section 1, we consider a simple cash in advance economy with flexible prices. In Section 2, we show the main result of the paper that there are single instrument feedback rules that implement a unique equilibrium. In Section 3 we show that in analogous finite horizon environments the single instrument rules would generate multiple equilibria. In Section 4, we show that the results generalize to the case where prices are set in advance. Section 5 contains concluding remarks.

2. A model with flexible prices

We first consider a simple cash in advance economy with flexible prices. The economy consists of a representative household, a representative firm behaving competitively, and a government. The uncertainty in period $t \geq 0$ is described by the random variable $s_t \in S_t$ and the history of its realizations up to period t (state or node at t), $(s_0, s_1, ..., s_t)$, is denoted by $s^t \in S^t$. The initial realization s_0 is given. We assume that the history of shocks has a discrete distribution. The number of shocks in period t is $\#S_t$ and the number of states in period t is Φ_t, so that $\Phi_t = (\Phi_{t-1})(\#S_t)$.

Production uses labor according to a linear technology. We impose a cash-in-advance constraint on the households' transactions with the timing structure described in Lucas and Stokey (1983). That is, each period is divided into two subperiods, with the assets market operational in the first subperiod and the goods market in the second.

2.1. Competitive equilibria

Households The households have preferences over consumption C_t, and leisure L_t, described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right) \right\} \tag{2.1}$$

where β is a discount factor. The households start period t with nominal wealth $\mathbb{W}_t$. They decide to hold money, M_t, and to buy B_t nominal bonds that pay $R_t B_t$ one period later, where R_t is the gross nominal interest rate at date t. They also buy $B_{t,t+1}$ units of state contingent nominal securities. Each security pays one unit of money at the beginning of period $t+1$ in a particular state. Let $Q_{t,t+1}$ be the beginning of period t price of these securities normalized by the probability of the occurrence of the state. Therefore, households spend $E_t Q_{t,t+1} B_{t,t+1}$ in state contingent nominal securities. Thus, in the assets market at the beginning of period t they face the constraint

$$M_t + B_t + E_t Q_{t,t+1} B_{t,t+1} \leq \mathbb{W}_t \tag{2.2}$$

Consumption must be purchased with money according to the cash in advance constraint

$$P_t C_t \leq M_t. \tag{2.3}$$

At the end of the period, the households receive the labor income $W_t N_t$, where $N_t = 1 - L_t$ is labor and W_t is the nominal wage rate and pay lump sum taxes, T_t. Thus, the nominal wealth households bring to period $t+1$ is

$$\mathbb{W}_{t+1} = M_t + R_t B_t + B_{t,t+1} - P_t C_t + W_t N_t - T_t \tag{2.4}$$

The households' problem is to maximize expected utility (2.1) subject to the restrictions (2.2), (2.3), (2.4), together with a no-Ponzi games condition on the holdings of assets.

The following are first order conditions of the households problem:

$$\frac{u_L(t)}{u_C\left(t\right)} = \frac{W_t}{P_t}\frac{1}{R_t} \tag{2.5}$$

$$\frac{u_C\left(t\right)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}}\right] \tag{2.6}$$

$$Q_{t,t+1} = \beta \frac{u_C(t+1)}{u_C(t)} \frac{P_t}{P_{t+1}},\ t \geq 0 \tag{2.7}$$

From these conditions we get $E_t Q_{t,t+1} = \frac{1}{R_t}$.Condition (2.5) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage adjusted for the cost of using money, R_t. Condition (2.6) is an intertemporal marginal condition necessary for the optimal choice of risk-free nominal bonds. Condition (2.7) determines the price of one unit of money at time $t+1$, for each state of nature s^{t+1}, normalized by the conditional probability of occurrence of state s^{t+1}, in units of money at time t.

Firms The firms are competitive and prices are flexible. The production function of the representative firm is linear

$$Y_t \leq A_t N_t$$

The firms maximize profits $\Pi_t = P_t Y_t - W_t N_t$ subject to the production function. The equilibrium real wage is

$$\frac{W_t}{P_t} = A_t. \tag{2.8}$$

Government The policy variables are taxes, T_t, interest rates, R_t, money supplies, M_t, state noncontingent public debt, B_t. We can define a policy as a mapping for the policy variables $\{T_t, R_t, M_t, B_t,\ t \geq 0, \text{ all } s^t\}$, that maps sequences of quantities, prices and policy variables into sets of sequences of the policy variables. Defining a policy as a correspondence allows for the case where the government is not explicit about some of the policy variables. Lucas and Stokey (1983) define policy as sequences of numbers for some of the variables. Adao, Correia and Teles (2003) define policy as sequences of numbers for all the policy variables. Here we allow for more generic functions (correspondences) for all the policy variables. We do not allow for targeting rules that can be defined as mappings from prices, quantities and policy variables to prices and quantities.

The period by period government budget constraints are

$$M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_{t-1}G_{t-1} - P_{t-1}T_{t-1},\ t \geq 0$$

Let $Q_{t+1} \equiv Q_{0,t+1}$, with $Q_0 = 1$. If $\lim_{T\to\infty} E_t Q_{T+1} \mathbb{W}_{T+1} = 0$, then the budget constraints of the government can be written as the sequence of budget constraints for $t \geq 0$,

$$\sum_{s=0}^{\infty} E_t Q_{t,t+s+1} M_{t+s} \left(R_{t+s} - 1\right) = \mathbb{W}_t + \sum_{s=0}^{\infty} E_t Q_{t,t+s+1} P_{t+s} \left[G_{t+s} - T_{t+s}\right] \quad (2.9)$$

Market clearing Market clearing in the goods and labor market requires

$$C_t + G_t = A_t N_t,$$

and

$$N_t = 1 - L_t.$$

We have already imposed market clearing in the money and debt markets.

Equilibrium A competitive equilibrium is a sequence of policy variables, quantities and prices such that the private agents solve their problems given the sequences of policy variables and prices, the budget constraints of the government are satisfied and the policy sequence is in the set defined by the policy.

The equilibrium conditions for the variables $\{C_t, L_t, R_t, M_t, B_t, T_t, Q_{t,t+1}\}$ are the resources constraint

$$C_t + G_t = A_t(1 - L_t),\ t \geq 0 \quad (2.10)$$

the intratemporal condition that is obtained from the households intratemporal condition (2.11) and the firms optimal condition (2.8)

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},\ t \geq 0 \quad (2.11)$$

as well as the cash in advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), and the budget constraints (2.9).

3. Single instrument feedback rules.

In this section we assume that policy is conducted with either interest rate or money supply feedback rules. We show that there are single instrument feedback rules that implement a unique equilibrium for the allocation and prices. The proposition for an interest rate feedback rule follows:

Proposition 3.1. *When the fiscal policy is endogenous and monetary policy is conducted with the interest rate feedback rule*

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

where ξ_t is an exogenous variable, there is a unique equilibrium.

Proof: Suppose policy is conducted with the interest rate feedback rule $R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}$. Then the intertemporal and intratemporal conditions, (2.6) and (2.11) can be written as

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0 \tag{3.1}$$

$$\frac{u_C(t)}{u_L(t)} = \frac{\frac{\xi_t}{\beta E_t \xi_{t+1}}}{A_t},\ t \geq 0 \tag{3.2}$$

These conditions together with the cash in advance conditions, (2.3), and the resource constraints, (2.10), determine uniquely the variables C_t, L_t, P_t and M_t.

The budget constraints (2.9) are satisfied for multiple paths of the taxes and state noncontingent debt levels■

The forward looking interest rate feedback rule that guarantees uniqueness of the equilibrium resemble the rules that appear to be followed by central banks. The nominal interest rate reacts positively both to the forecast of future consumption and to the forecast of the future price level. In this there is a difference to the feedback rules that are usually considered in that it depends on the future price level rather than inflation.

Depending on the exogenous process for ξ_t, it is possible to decentralize any feasible allocation distorted by the nominal interest rate. The first best allocation, at the Friedman rule of a zero nominal interest rate, can also be implemented. With $\xi_t = \frac{1}{\beta^t}$, $t \geq 0$, condition (3.2) becomes

$$\frac{u_C(t)}{u_L(t)} = \frac{1}{A_t},\ t \geq 0$$

which, together with the resource constraint (2.10) gives the first best allocation $C_t = C^*(A_t, G_t)$ and $L_t = L^*(A_t, G_t)$. The price level $P_t = P^*(A_t, G_t)$ can be obtained using (3.1), i.e.

$$\frac{u_C(C^*(.), L^*(.))}{P_t} = \frac{1}{\beta^t},\ t \geq 0,$$

and the money supply is obtained using the cash-in-advance constraint, $M_t = P^*(A_t, G_t)C^*(A_t, G_t)$.

Allocations where inflation is zero can also be implemented even if in this flexible price environment they are not optimal. There are multiple such allocations with nominal interest rates satisfying

$$R_t = \frac{u_C(C(A_t, G_t, R_t), L(A_t, G_t, R_t))}{\beta E_t u_C(C(A_{t+1}, G_{t+1}, R_{t+1}), L(A_{t+1}, G_{t+1}, R_{t+1}))}, \; t \geq 0$$

where the functions C and L are the solution for C_t and L_t of the system of equations given by (2.10) and (2.11).

For each path of the nominal interest rate, $\{R_t\}$, associated with zero inflation, there is a unique path for $\{\xi_t\}$ up to a constant term $\overline{P}$, which is the price level,

$$\frac{u_C(C(A_t, G_t, R_t), L(A_t, G_t, R_t))}{\overline{P}} = \xi_t, \; t \geq 0.$$

In an economy where the use of money becomes negligible which corresponds to the cash-in-advance condition

$$v_t P_t C_t \leq M_t. \tag{3.3}$$

where $v_t \to 0$, there is a single path for the nominal interest rate consistent with zero inflation,

$$R_t = \frac{u_C(C(A_t, G_t, 1), L(A_t, G_t, 1))}{\beta E_t u_C(C(A_{t+1}, G_{t+1}, 1), L(A_{t+1}, G_{t+1}, 1))}, \; t \geq 0.$$

where $C(A_t, G_t, 1) = C^*(A_t, G_t)$ and $L(A_t, G_t, 1) = L^*(A_t, G_t)$, since in the cashless economy the nominal interest rate is not distortionary.

An analogous proposition to Proposition 3.1 is obtained when policy is conducted with a particular money supply feedback rule.

Proposition 3.2. *When the fiscal policy is endogenous and the policy is conducted with the money supply feedback rule,*

$$M_t = \frac{\beta R_{t-1} C_t u_C(t)}{\xi_{t-1}}$$

there is a unique equilibrium.

Proof: Suppose policy is conducted according to the money supply rule $M_t = \frac{\beta R_{t-1} C_t u_C(t)}{\xi_{t-1}}$. Then, the equilibrium conditions

$$P_t C_t = \frac{\beta R_{t-1} C_t u_C(t)}{\xi_{t-1}}$$

obtained using the cash in advance conditions (2.3), and conditions

$$\frac{u_C(t)}{P_t} = \xi_t$$

obtained from the intertemporal conditions (2.6), in addition to the resource constraints, (2.10), and the intratemporal conditions (2.11), determine uniquely the four variables, C_t, L_t, P_t, R_t in each period $t \geq 0$ and state s^t.

The taxes and debt levels satisfy the budget constraint (2.9)■

The result that there are single instrument feedback rules that implement a unique equilibrium is a surprising one. In fact it is well known that interest rate rules may implement a determinate equilibrium, but not a unique global equilibrium. To illustrate this, consider the case where monetary policy is conducted with constant functions for the policy variables. We will show that in that case an interest rate policy generates multiple equilibria. That result is directly extended to the case where the interest rate is a function of contemporaneous or past variables.

3.0.1. Conducting policy with constant functions.

In this section, we show that in general when policy is conducted with constant functions for the policy instruments, it is necessary to determine exogenously both interest rates and money supplies.

The equilibrium conditions are the resources constraints, (2.10), the intratemporal conditions (2.11), the cash in advance constraints (2.3), the intertemporal conditions (2.6) and the budget constraints (2.9) that can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s u_C(t+s) C_{t+s} \left(\frac{R_{t+s} - 1}{R_{t+s}} \right) = u_C(t) \frac{\mathbb{W}_t}{P_t} + E_t \sum_{s=0}^{\infty} \beta^s u_C(t+s) \frac{[G_{t+s} - T_{t+s}]}{R_{t+s}} \tag{3.4}$$

using (2.7).

These conditions define a set of equilibrium allocations, prices and policy variables. There are many equilibria. We want to determine conditions on the exogeneity of the policy variables such that there is a unique equilibrium in the allocation and prices. We first consider the case in which a policy are sequences of numbers for money supplies and interest rates.

From the resources constraints,(2.10), the intratemporal conditions (2.11), and the cash in advance constraints, (2.3), we obtain the functions $C_t = C(R_t)$ and $L_t = L(R_t)$, which for simplicity in the notation we ignore the dependence on the shocks, and $P_t = \frac{M_t}{C(R_t)}$, $t \geq 0$. As long as $u_C(C_t, L_t)C_t$ depends on C_t or L_t, excluding therefore preferences that are additively separable and logarithmic in consumption, the system of equations can be summarized by the following dynamic equations:

$$\frac{u_C(C(R_t), L(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}} \right], t \geq 0 \tag{3.5}$$

together with the budget constraints, (3.4).

Suppose the path of money supply is set exogenously in every date and state. In addition, in period zero the interest rate, R_0, is set exogenously and, for each $t \geq 1$, for each state s^{t-1}, the interest rates are set exogenously in $\#S_t - 1$ states. The number of additional restrictions in period t is $\Phi_t - \Phi_{t-1}$, where Φ_t is the number of states in t. In this case there is a single solution for the allocations and prices. Similarly, there is also a unique equilibrium if the nominal interest rate is set exogenously in every date and state, and so is the money supply in period 0, M_0, as well as, for each $t \geq 1$, and for state s^{t-1}, the money supply in $\#S_t - 1$ states. The budget constraints restrict, not uniquely, the taxes and debt levels.

The proposition follows

Proposition 3.3. *Suppose policy are constant functions. In general, if money supply is determined exogenously in every date and state, and if interest rates are also determined exogenously in the initial period, as well as in* $\Phi_t - \Phi_{t-1}$ *states for each* $t \geq 1$, *then the allocations and prices can be determined uniquely. Similarly, if the exogenous policy instruments are the interest rates in every state, the initial money supply and the money supply, in* $\Phi_t - \Phi_{t-1}$ *states, for* $t \geq 1$, *then there is in general a unique equilibrium.*

The proposition states a general result. In the particular case where the preferences are additively separable and logarithmic in consumption, and money supply is set exogenously in every state, there is a unique equilibrium in the allocations and prices. There is no need to set exogenously the interest rates as well. This example is helpful in understanding the main point of the paper, that the degrees of freedom in conducting policy depend on how policy is conducted and on other characteristics of the environment.

3.0.2. Current or backward interest rate feedback rules .

We have shown Proposition 3.3. assuming that policy was conducted with constant functions for the policy variables. However, the use of interest rate rules that depend on current or past variables clearly preserves the same degrees of freedom in the determination of policy, as identified in that proposition. When fiscal policy is endogenous, it is still necessary to determine exogenously the levels of money supply in some but not all states. The corollary follows

Corollary 3.4. *When policy is conducted with current or backward interest rate feedback rules and fiscal policy is endogenous, there is a unique equilibrium if the money supply is set exogenously in* $\#S_t - 1$ *states, for each state* s^{t-1}, $t \geq 1$, *as well* M_0.

4. Understanding the Results: Finite vs Infinite horizon.

We have shown that there are interest rate rules that implement a unique equilibrium but that current or backward feedback rules do not. This means that even if the same number of instruments is set exogenously, the remaining degrees of freedom in determining policy depend on how those degrees of freedom are filled. This happens because the model economy has an infinite horizon.

If the economy had a finite horizon it would be characterized by a finite number of equations and unknowns. In that case the number of degrees of freedom in conducting policy is a finite number that does not depend on whether policy is conducted with constant functions, functions of future, current or past variables, as long as these functions are truly exogenous, i.e. independent from the remaining equilibrium conditions.

To determine the degrees of freedom in the case of a finite horizon economy amounts to simply counting the number of independent equations and unknowns. We proceed to considering the case where the economy lasts for a finite number of periods $T+1$, from period 0 to period T. After T, there is a subperiod for the clearing of debts, where money can be used to pay debts, so that

$$\mathbb{W}_{T+1} = M_T + R_T B_T + P_T G_T - P_T T_T = 0$$

The first order conditions in the finite horizon economy are the intratemporal conditions, (2.11) for $t = 0, ..., T$, the cash in advance constraints, (2.3) also for $t = 0, ..., T$, the intertemporal conditions

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right], \ t = 0, ..., T-1$$

$$Q_{t,t+1} = \beta \frac{u_C(t+1)}{u_C(t)} \frac{P_t}{P_{t+1}}, \ t = 0, ..., T-1 \tag{4.1}$$

and, for any $0 \leq t \leq T$, and state s^t, the budget constraints

$$\sum_{s=0}^{T-t} E_t Q_{t,t+s+1} M_{t+s} (R_{t+s} - 1) = \mathbb{W}_t + \sum_{s=0}^{T-t} E_t Q_{t,t+s+1} P_{t+s} [G_{t+s} - T_{t+s}]$$

where $E_0 Q_{T+1} \equiv \frac{E_0 Q_T}{R_T}$.

The budget constraints restrict, not uniquely, the levels of state noncontingent debts and taxes. Assuming these policy variables are not set exogenously we can ignore this restriction. The equilibrium can then be summarized by

$$\frac{u_C(C(R_t), L(R_t))}{\frac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{\frac{M_{t+1}}{C(R_{t+1})}} \right], \ t = 0, ..., T-1 \tag{4.2}$$

Note that the total number of money supplies and interest rates is the same. There are $\Phi_0 + \Phi_1 + ... + \Phi_T$ of each monetary policy variable. The number of equations is $\Phi_0 + \Phi_1 + ... + \Phi_{T-1}$. In order for there to be a unique equilibrium need to add to the system $\Phi_0 + \Phi_1 + ... + 2\Phi_T$ independent restrictions. One possibility is to set exogenously the interest rates in every state and in addition the money supply in every terminal node. Similarly there is a unique equilibrium if the money supply is set exogenously in every state and the interest rates are set in every terminal node. In this sense, the two monetary instruments are equivalent in this economy.

When policy is conducted with the forward looking feedback rule in Section 2, the policy for the interest rate in the terminal period R_T, cannot be a function of variables in period $T + 1$. If these rates are exogenous constants, it still remains to determine the money supply in every state at T.

In this finite horizon economy there is an exact measure for the degrees of freedom in conducting policy. In an economy that lasts from $t = 0$ to $t = T$, these are $\Phi_0 + \Phi_1 + ... + 2\Phi_T$. This measure does not depend on how policy is conducted, whether with constant functions or functions of endogenous variables.

While in economies with a finite horizon the number of degrees of freedom is not affected by the way policy is conducted, that is not the case when the economy has an infinite horizon. The issue of whether the actual economy is best represented by a model with an infinite horizon or by a model with a finite but arbitrarily large horizon thus becomes a relevant issue in this policy question, whether it is possible to implement a unique equilibrium with single instrument feedback rules.

5. Price setting restrictions

In economies with a finite horizon price setting restrictions do not affect the degrees of freedom in conducting policy. They add the same number of equations as unknowns, so that single instrument feedback rules do not insure uniqueness. In infinite horizon economies, it could be the case that price setting restrictions interact with the policy rules and affect the degrees of freedom in conducting policy. In this section we show that when prices are set in advance the single feedback rules in propositions 3.1 and 3.2 still insure uniqueness, so that the results derived above are robust to this form of price stickiness.

We modify the environment to consider price setting restrictions. There is a continuum of goods, indexed by $i \in [0, 1]$. Each good i is produced by a different firm. The firms are monopolistic competitive and set prices in advance with different lags.

The households have preferences described by (2.5) where C_t is now the composite consumption

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1,$$

where $c_t(i)$ is consumption of good i. Households have a demand function for each good given by

$$c_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} C_t.$$

where $p_t\left(i\right)$ is the price of each good i and P_t is the price level,

$$P_t = \left[\int p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{5.1}$$

The households' intratemporal and intertemporal conditions are as before, (2.5), (2.6) and (2.7).

The government must finance an exogenous path of government purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 0 \tag{5.2}$$

where $g_t(i)$ is public consumption of good i. Given the prices on each good i in units of money, $P_t(i)$, the government minimizes expenditure on government purchases by deciding according to

$$\frac{g_t(i)}{G_t} = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \tag{5.3}$$

The resource constraints can be written as

$$(C_t + G_t) \int_0^1 \left(\frac{p_t\left(i\right)}{P_t} \right)^{-\theta} di = A_t N_t. \tag{5.4}$$

We consider now that firms set prices in advance. A fraction α_j firms set prices j periods in advance with $j = 0, ...J-1$. Firms decide the price for period t with the information up to period $t-j$ to maximize:

$$E_{t-j} \left[Q_{t-j,t+1} \left(p_t(i) y_t(i) - W_t n_t(i) \right) \right]$$

subject to the production function

$$y_t(i) \leq A_t n_t(i)$$

and the demand function

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{5.5}$$

where $y_t(i) = c_t(i) + g_t(i)$, and $y_t(i)$ and $n_t(i)$ are respectively the output of good i and labor used on that good.

The optimal price is

$$p_t(i) \equiv p_{t,j} = \frac{\theta}{(\theta-1)} E_{t-j}\left[\eta_{t,j}\frac{W_t}{A_t}\right]$$

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1}P_t^{\theta}Y_t}{E_{t-j}\left[Q_{t-j,t+1}P_t^{\theta}Y_t\right]}.$$

The price level at date t can be written as

$$P_t = \left[\sum_{j=0}^{J-1}\alpha_j\left(p_{t,j}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{5.6}$$

The equilibrium conditions when prices are set in advance contain more variables and equations than the flexible price counterpart. The number of variables added, the prices of the restricted firms, is the same as the number of restrictions added. Since these added equations are functions of current and past variables the degrees of freedom in conducting policy are the same as in the flexible price economy. It is straightforward to show that the single instrument rules proposed implement a unique equilibrium also in this case.

6. Concluding Remarks

The problem of multiplicity of equilibria under an interest rate policy has been addressed, after Sargent and Wallace (1975) and McCallum (1981), by an extensive literature on determinacy under interest rate rules. Interest rate feedback rules on endogenous variables such as the inflation rate can, with appropriately chosen coefficients, deliver locally determinate equilibria. There are still multiple equilibria but only one of those equilibria stays in the proximity of a steady state.

In this paper we show that in a simple monetary model with flexible prices or prices set in advance there are interest rate feedback rules, and also money supply feedback rules, that implement unique equilibria. The interest rate feedback rules are forward rules that resemble the policy rules that central banks follow.

The results are not robust to a change in the theoretical environment on which it is hard to take a stand. The model economy has an infinite horizon. Suppose that we considered instead the analogous finite horizon economy. In that economy, for an arbitrarily large horizon, single instrument feedback rules would not implement unique equilibria.

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