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AUCTION: ON THE INCIDENCE
OF COMMISSIONS IN AUCTION
MARKETS**

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ABSTRACT

How to Win Twice at an Auction: On the Incidence of Commissions in Auction Markets*

We analyse the welfare consequences of an increase in the commissions charged by intermediaries in auction markets. We argue that while commissions are similar to taxes imposed on buyers and sellers the question of incidence deserves a new treatment in auction markets. We show that an increase in commissions makes sellers worse off, but buyers may strictly gain. The results are therefore strikingly different from the standard result that all consumers weakly lose after a tax or a commission increase. Our results are useful for evaluating compensation in price fixing conspiracies; in particular they suggest that the method used to distribute compensations in the class action against auction houses Christie's and Sotheby's was misguided.

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1 Introduction

This paper is about the welfare effects of commissions in auction markets. Because commissions are like taxes on transactions, one could appeal to the literature on tax incidence: increasing taxes make both buyers and sellers weakly worse off, and the relative losses of buyers and sellers are a function of demand and supply elasticities. For instance, if supply is inelastic, buyers are not hurt by an increase in taxes; sellers will bear the full burden. If supply is elastic, both buyers and sellers will be worse off.

However, goods that are exchanged in auction markets are of a different nature than those in markets where tax incidence is usually analyzed: in an auction, the good is in effect a lottery. When the degree of competition (that is the number of bidders) changes, both the probability of winning and the expected price change, often in opposite directions. For this reason, the effect of commissions in auction markets deserves an explicit treatment. As we show here, welfare effects are quite different from those in usual markets: in particular, a winner may strictly gain – which can not happen in standard markets since *all* consumers lose (weakly) after a tax or a commission increase.

Intermediation in auctions is the rule rather than the exception. In the non digital world, auction houses like Christie's and Sotheby's have for many years gotten the lion's share of auction sales. New technologies have facilitated the creation of large market places like auction portals targeted to consumers or to part-suppliers and have magnified the importance of intermediation. While some may have hoped that these digital markets would replicate perfect competition, it has been quickly recognized that they are likely to generate switching costs for both buyers and sellers and to be characterized by the dominance of a few intermediaries.¹ Hence, abuse of dominant position and collusive agreements by some intermediaries are likely to appear in digital market places, as much as they do in other markets. Because commissions are integral to the revenues of the intermediaries, such abuses are likely to express themselves in a change in commission. The price fixing agreement between Christie's and Sotheby's from 1993 to 2000, is an illustration as is the recent concern of the European and U.S. antitrust authorities about B2B platforms (see for instance the remarks of the FTC

¹On recent economic issues in two-sided markets, see Caillaud and Jullien (2003), Rochet and Tirole (2004), Ellison and Fudenberg (2003), Baye and Morgan (2001).

Commissioner Swindle 2000, or the survey by Wiseman 2000).

Once an abuse like price fixing is identified, it belongs to the parties to claim damage and to be compensated by the courts. Parties usually get three times the monetary equivalent of their welfare loss. Understanding the welfare effects of a change in commissions is therefore not only of theoretical but also of practical interest.

We develop a model of intermediation with a large set of potential bidders and sellers. Sellers offer the same type of object and have the same valuation. Bidders are ex-ante symmetric but learn their valuation after having decided to participate, which they do when their expected utility is larger than their cost of participation. Buyers can participate in only one auction and the average number of bidders per auction is a measure of competitiveness of the auction. An intermediary sets and announces commissions that must be paid by buyers and sellers. We show first that the analysis can be reduced to the effect of a change in one variable, an “aggregate commission”, that is a nonlinear function of the buyer and seller commissions.

When the level of commissions change, sellers adjust their reserve price and therefore the minimum type of a bidder that could possibly win the auction, called the “reserve type”. A higher commission leads to an increase in the reserve type and therefore to a decrease in the expected utility of bidders. It follows that a higher commission induces a lower number of bidders participating in the auction, as well as the possibility of a higher price paid by the winner. It is therefore possible that bidders will be worse off. But it is also possible that bidders will be better off if they win with a lower price, which typically arises when there is less competition (that is a lower number of bidders per auction) and when the second highest price is larger than the reserve price. Thus, among the winners in the high-commission auction, those paying a price strictly larger than the reserve price are strictly better off than if they had been winners in the low-commission auction. Winners paying the reserve price are weakly worse off. This result is quite general and does not rely on particular choices of the distributions of valuations.

From an *ex-ante perspective*, as long as the cost of participation is the same for all agents, the ex-ante average welfare of bidders is the same before and after the increase in commissions. This result is similar to the standard result that when supply is inelastic consumers are not

affected by the commission.² However, if the cost of participation is not the same for all bidders, bidders are worse off with an increase in commissions: the welfare loss includes the welfare of those bidders who could have won with the low commission but cannot win with the high commission because sellers have modified their reserve price.

In the *ex-post stage*, the question of compensating the winner for an increase in commissions is complicated by the fact that while we observe the price he pays, we do not observe the probability he had of winning the object. Following established practice in consumer economics, we make the thought experiment of finding the monetary transfer that would make the winner indifferent between paying the current price or facing more competition but the initial commission. More competition would imply that he wins the object less often, but potentially at a lower price. For valuations that are uniformly distributed, we show that the winner would, on average, not be willing to pay to face a low commission and therefore more competition, that is, to be faced with a situation similar to the low-commission auction. Compensating the winner for an increase in commission would therefore make him win twice.

These results suggest that in auction markets, sellers are the main losers and that there can be reasons not to compensate buyers. While there may be a welfare loss, it may essentially be due to bidders who are excluded, either at the *ex-ante* stage or when bidders learn their valuation (the interim stage), because their valuation was lower than the reserve type of the seller in the high-commission auction. These phantom participants may deserve compensation but they are obviously impossible to identify.

The Christie's-Sotheby's collusion case is of particular relevance. The two auction houses were found guilty of colluding between 1993 and 2000. In 2001, a class-action suit ended by each auction houses agreeing to pay 256 millions dollars to the plaintiffs. Buyers and sellers who had bought or sold through either auction houses between 1993 (1995 for sellers) and 2000 in the United States were compensated . Buyers received the largest share in the settlement. With this decision, compensated bidders may have won twice, first because hammer prices may have been lower than without collusion, and secondly, because they were unrighteously compensated, while sellers may not have been properly compensated. The settlement was obviously based on poor understanding of how auctions work.

²This is also the flavor of the observation in Ashenfelter and Graddy (2003).

In section 2, we show that in an auction, potential buyers take into account an increase in their commission rate by shading their bid and are therefore not directly affected. This shading decreases the price, thus affecting the seller who can react by strategically changing the reserve type. To the extent that the reserve price affects the outcome of the auction (by increasing the price or by a failure to sell), buyers can be indirectly hurt. We introduce, in section 3, a model of an auctioneer as an intermediary who charges commissions on both sellers and buyers. The model takes into account all the decisions on buyers and sellers (participation decision, setting of the reserve price, bidding) as a function of the commissions rates. In sections 4 and 5, we show that the ex ante total welfare decreases but that effective bidders (those who have a chance of winning) are strictly better off if they pay a price higher than the reserve price of the seller and may be also on average better off with higher commissions.

2 When Consumers are Indifferent to Commissions

Consider a market for a homogenous good with a decreasing demand function $D(p)$ and an inelastic supply $S(p) = s$; let p^0 solve $D(p^0) = s$. A linear commission c on consumers leads to a new demand function $\hat{D}(p) = D(p/(1+c))$ since consumers anticipate that when the sale price is p the final price they pay is $(1+c)p$. Hence, consumers shave their willingness to pay proportionally to the commission rate. Since supply is inelastic, the equilibrium price is $p^c = p^0/(1+c)$ and consumer welfare is unchanged.

In auction markets with private values, efficient auctions are strategically equivalent to second-price auctions in which a bidder has a dominant strategy to bid his valuation. The price paid by the winner corresponds thus to the valuation of the second highest bidder. When the buyer's commission is raised, buyers reduce their bids by the same amount, and this results in a reduction of the hammer price, very much like in our simple demand-supply analysis. Thus, the entire burden of the commission ends up being borne by the seller since buyers fully endogenize the commission in their bidding. Hence, as noted in Ashenfelter and Graddy (2003) and Ashenfelter et al. (2003), the theory of private value auctions implies also that buyers are indifferent to the level of the commission when supply is inelastic.

Suppose now that the supply is elastic. Let p^0 solve $D(p^0) = S(p^0)$. In a usual market, the introduction of a commission leads to a new demand function $\hat{D}(p) = D(p/(1+c))$. The

supply function remains unchanged. In equilibrium, the price is $p^c < p^0$, and consumer welfare decreases since $p^c(1 + c) > p^0$. The relative elasticities of supply and demand determine the burden of the commission passed to the sellers. The welfare of an individual buyer depends only on the market price; this means that all buyers suffer from the increase in commissions: those who cannot buy because of the increase in price as well as those who buy at a higher price.

What could be different in auction markets? Supply, there, is elastic because a seller can increase his expected revenue by strategically setting a reserve price under which he refuses to sell the object.

Suppose there is one seller with valuation v_s and buyers with valuations v_i distributed on $[0, 1]$ according to a cumulative density function $F(\cdot)$. The commission rates are c_S and c_B for sellers and buyers, respectively. Hence if p is the hammer price, the buyer pays $p(1 + c_B)$, the seller receives $p(1 - c_S)$, and the intermediary (the auction house) collects $p(c_S + c_B)$. The following ratio, $\alpha = \frac{c_S + c_B}{1 + c_B}$, that we call the commission index, will play an important role. Note that α is (weakly) increasing in c_S and c_B . It takes its smallest value when $c_S = c_B = 0$.

In Appendix 6.1, we show that the optimal reserve price r for the seller is such that

$$\rho - \frac{1 - F(\rho)}{f(\rho)} = \frac{v_s}{1 - \alpha}, \quad (1)$$

where $\rho = r(1 + c_B)$ denotes the marginal type of bidder that is excluded by the reserve price r , that we dub “reserve type”. In an auction with no commission on buyers, $\rho = r$ and therefore the reserve price corresponds to the marginal type that is excluded from the auction. With a commission on buyers, this is no longer true. Note that under the usual assumption of monotonic hazard rate, the reserve type ρ increases with the commission rates (see (1)). However, the observed reserve price r can increase or decrease as a function of commissions.

The ex-ante surplus of a buyer in an auction with n bidders is

$$B^A(\rho, n) = \int_{\rho}^1 \left(\int_{\rho}^v F(x)^{n-1} dx f(v) \right) dv. \quad (2)$$

Note that the buyer’s surplus does not depend directly on the commissions. But it does

indirectly since commissions affect the reserve type ρ and potentially n the number of bidders in the auction. The ex-ante surplus of buyers decreases with ρ (see (2)) and the commissions.

If, after the increase in commissions, the winning bidder (observed ex-post) pays the reserve price³ he may lose. However, if he pays more than the reserve price, he does not lose. It is therefore easy to tell whether buyers should or should not be compensated, but it is clear that not all of them should be.

So far we have only analyzed how commissions influence the bidding decision of buyers and sellers (a reserve price can be interpreted as a bid). But since the ex-ante welfare of buyers and sellers decreases with an increase in commissions, fewer of them may participate in the auction if there are costs of participating, This will have important consequences for the welfare analysis since if fewer bidders participate in the auction (smaller n), successful buyers (those actually getting an object) can in fact be better off.

Participation decisions can be interpreted in terms of elasticity of supply and demand. The case we are mostly interested in is the one in which buyers' participation is more elastic than sellers' participation since it leads to welfare considerations that are very different from usual ones. Whether buyers or sellers have a more elastic participation is an empirical question, but it seems reasonable that buyers who are on the long side of the market would be more responsive in their participation decision. In particular that would be so if all sellers make positive profits and thus all participate. A decrease in the participation cost would not change their participation.

The following example gives a simple intuition of our results.

Assume that there are two auctions, one with low commission (L), the other with high commission (H). We assume for simplicity that commissions are paid only by sellers. In the L auction, the seller sets his reserve price at 4 and there are three bidders A, B and C whose valuations are $v_A = 6$, $v_B = 7$ and v_C (which will be set at different values, to illustrate what may happen). In the H auction, the reserve price is 5, and only bidders A and B participate. The winner in the H auction is B. He bids 7 and pays 6, the second highest price. We now compare this outcome with the one that would have prevailed in the L auction, in which the

³Obviously, the effective price paid is $\rho = r(1 + c_B)$, r being the transfer to the seller and rc_B the transfer to the auction house.

reserve price of 4 is lower, and bidder C is active. Here, B loses the auction to C, who pays 7 if his value is $v_C > 7$. Obviously, the welfare of bidder B is lower than in the H auction. If the valuation of bidder C had been $6 < v_C < 7$, bidder B would have won the auction, but would have paid $v_C > 6$, making him also worse off than in the H auction. Finally, if C had a valuation $v_C < 6$, B would have won the auction, paying 6 as in the H auction. Since the reserve price of 5 was not binding in the H auction in which he paid 6, he could not benefit from the lower reserve price in the L auction. Therefore, in no case would buyer B have been better off in the L auction. Table 1 summarizes the discussion.

Table 1

Auction	Reserve Price	v_A	v_B	v_C	Result of auction
H auction	5	6	7	-	B wins and pays 6
L auction	4	6	7	$v_C > 7$	C wins
	4	6	7	$6 < v_C < 7$	B wins but pays $v_C > 6$
	4	6	7	$v_C < 6$	B wins and pays 6

Suppose now that bidder A has a value of 3 instead of 6. This changes the analysis of what would have happened in the L auction. The effect is ambiguous since the competitive effect of the additional bidder C and the effect of the lower reserve price in the L auction go in opposite directions, and B may be better off or worse off in the L auction than in the H auction. The situation is summarized in Table 2.

Table 2

Auction	Reserve Price	v_A	v_B	v_C	Result of auction
H auction	5	3	7	-	B wins and pays 5
L auction	4	3	7	$v_C > 7$	C wins
	4	3	7	$4 < v_C < 7$	B wins but pays $v_C > 4$
	4	3	7	$v_C < 4$	B wins and pays 4

A more complete analysis is provided in the following three sections. We show that when the degree of competition decreases with higher commissions, successful bidders can be better off. In particular, as in this example, a buyer who does not pay the reserve price is better off, and it is possible that, on average, successful buyers will be better off.

3 The auction house as an intermediary

3.1 The model

Consider the following two-sided market. On the long side of the market, there is a mass N of potential buyers, each with a participation cost t , with distribution $G(\cdot)$. Assume that each buyer wants to purchase one unit of the good, and can participate in one auction only. Participating buyers have valuations v , that are identically and independently distributed according to $F(\cdot)$. On the other side of the market, there is a mass M of potential sellers with valuation v_s and an ex-ante participation cost t , with distribution $H(\cdot)$.

There is a unique auction house which acts as an intermediary between buyers and sellers. It organizes auctions for the objects that sellers are willing to sell and sets commission rates (c_S, c_B) , each a fixed proportion of the hammer price. Buyers are allocated among auctions with the same number of buyers in each auction.⁴ Let n denote the ratio of participating buyers over participating sellers. This ratio represents the degree of competition among buyers in the market. If all buyers and sellers participate, then $n = N/M$.

The auction format used is the second price (Vickrey) auction that is strategically equivalent to the English auction used by most auction houses.⁵ Sellers decide on a reserve price r ,⁶ and buyers make their bids.

3.2 Equilibrium

To solve for equilibrium, we need the expressions of the optimal reserve price r , the hammer price p , the surplus of sellers, $\Pi(\cdot)$ and of buyers, $B^A(\cdot)$. As we have seen in equation (2), it turns out that it is convenient to express the surplus as a function of the reserve type ρ . In fact, we can solve the model “independently” of commissions. The important variables for participation are the ex-ante welfare for buyers and the expected price for sellers. The welfare of buyers depends only on ρ and n . It is independent of commissions insofar as the effect of commissions on ρ and n are taken into account. Similarly the price expected by the

⁴This leads to the potential problem that the number of buyers in an auction is not an integer. We disregard this problem.

⁵Our results would not change if we used any other usual auction format, since the revenue equivalence principle can be applied in our framework.

⁶The fact that the reserve price is often secret, coincides with reality, but does not matter here.

seller is independent of commissions in the sense that it is obtained by simply multiplying a commission-independent price by the commission index.

Let $b_{(1)}$ and $b_{(2)}$ be the highest and second highest bids among the n bidders. With a reserve price r , the object is sold to the highest bidder only if $b_{(1)} \geq r$ at a hammer price equal to $\max\{r, b_{(2)}\}$. Let $F_{(1,n)}$ be the distribution of the first order statistic when there are n bidders and let $F_{(2,n)}(x|y)$ be the distribution of the second order statistic when the first order statistic is equal to y .

In this case, it is immediate to see that the dominant strategy of a bidder with valuation v is to bid $b = v/(1 + c_B)$. Hence, if there are n bidders, the i -th order bid is $b_{(i)} = v_{(i)}/(1 + c_B)$, where $v_{(i)}$ is the i -th order valuation among the n bidders.

The expected hammer price is then

$$\begin{aligned} p &= \int_{r(1+c_B)}^1 \int_0^x \max\left\{r, \frac{v}{1+c_B}\right\} dF_{(2,n)}(v|x) dF_{(1,n)}(x) \\ &= \int_{r(1+c_B)}^1 \left(\int_0^{r(1+c_B)} r dF_{(2,n)}(v|x) + \int_{r(1+c_B)}^x \frac{v}{1+c_B} dF_{(2,n)}(v|x) \right) dF_{(1,n)}(x) \end{aligned}$$

Making the change of variable,

$$\rho \equiv r(1 + c_B),$$

we can write the hammer price as,

$$p = \frac{1}{1 + c_B} I(\rho, n),$$

where

$$I(\rho, n) \equiv \int_{\rho}^1 \left(\int_0^{\rho} \rho dF_{(2,n)}(v|x) + \int_{\rho}^x v dF_{(2,n)}(v|x) \right) dF_{(1,n)}(x).$$

$I(\rho, n)$ corresponds to the hammer price in an auction with no commission when the reserve price is ρ (the reserve price and the reserve type are equal in that case.)

The seller's profit

The expected profit of the seller is now

$$\begin{aligned} \Pi(\alpha, \rho, n) &= \frac{1 - c_S}{1 + c_B} I(\rho, n) + F^n(\rho) v_s \\ &= (1 - \alpha) I(\rho, n) + F^n(\rho) v_s \end{aligned} \tag{3}$$

and a strategic seller chooses ρ to maximize Π .⁷

The buyer's surplus

Standard arguments imply that the marginal surplus of a buyer is the expected probability of winning. If the reserve price is r (a function of α), all buyers with valuation $v < \rho = r(1 + c_B)$ have a zero probability of winning. Hence, the interim expected surplus of a buyer with valuation v is

$$B^I(v, \rho, n) = \int_{\rho}^v F^{n-1}(x) dx.$$

Ex-ante, the surplus of a buyer is

$$B^A(\rho, n) = \int_{\rho}^1 B^I(v, \rho, n) f(v) dv. \quad (4)$$

This surplus is clearly decreasing in n (for fixed ρ) and decreasing in ρ (for fixed n).

The auctioneer's profit

The auction house sets the commission rates. Its revenue in each auction is $(c_S + c_B)I(\rho, n)/(1 + c_B) = \alpha I(\rho, n)$. Thus, its total profit equals $R = \alpha I(\rho, n) \cdot H(\Pi(\alpha, \rho, n) - v_s) \cdot M$. There exists a trade-off between the number of transactions and the revenue that commissions generate on each transaction. The focus of the paper is not to analyze how commissions are chosen but on the impact of a change in commissions on the welfare of buyers and sellers.

3.3 Participation decisions and equilibrium

Since commissions decrease the revenue of sellers, these are likely to change their behavior; in particular, they may decide not to participate or to participate but modify their optimal reserve price. This has a feed-back effect on the surplus of buyers and hence on their own participation. Therefore, a change in commissions modifies the number of participants and therefore the ratio of buyers to sellers.

Sellers' participation

⁷We assume that the seller chooses an optimal reserve price, but our qualitative results would still obtain if we simply assumed that ρ is increasing in α . That would be so if the seller sets $\rho = v_s/(1 - c_s)$ in order to guarantee to himself a price net of commission larger than his valuation v_s .

A potential seller takes $c = (c_S, c_B)$ as given, anticipates n and sets his reserve type ρ (which, given (1) depends only on c) to maximize $\Pi(\alpha, \rho, n) = (1 - c_S)p(\alpha, \rho, n) + F^n(\rho)v_s$, where $p(\alpha, \rho, n)$ is the expected hammer price in the auction, $F^n(\rho)$ represents the probability that the object goes unsold and, is kept by the seller who values it at v_s . He participates if $\Pi(\alpha, \rho, n) - t \geq v_s$. The mass of participating sellers is $H(\Pi(\alpha, \rho, n) - v_s) \cdot M$.

Buyers' participation

A potential buyer takes $c = (c_S, c_B)$ as given, anticipates n and r . He participates if his cost of participating, t is smaller than his expected surplus $B^A(\rho, n)$. The mass of participating buyers is $G(B^A(\rho, n)) \cdot N$.

Equilibrium

Proposition 1 *There exists a unique equilibrium with positive participation.*

Proof: There always exists an equilibrium in which no buyer and no seller participate. If there is positive participation, the degree of competition is characterized by the rational expectations equation:

$$n \cdot H(\Pi(\alpha, \rho, n) - v_s)M = G(B^A(\rho, n)) \cdot N.$$

As we show in the Appendix, ρ does not vary with n and only depends on α . Since α is a parameter, the previous equation depends only on n . The right-hand side is decreasing in n since G is an increasing function and the surplus of buyers B is decreasing with competition. The left-hand side is increasing in n since H is an increasing function and the profit of sellers Π is increasing with n . ■

The degree of competition n varies with commissions. The sign of this change depends on the relative elasticity of participation of buyers and sellers.

3.4 Aggregate commission index

We now show that the outcome of the auction depends on the commissions to the extent that these modify the commission index $\alpha = (c_B + c_S)/(1 + c_B)$. Any change in the commission structure that leaves α unchanged, leaves unaffected the payoffs of sellers, buyers and auction houses. Indeed, by inspection of (3), we see that the seller's profit and thus his choice of ρ

depends only on α . His optimal reserve price is equal to $r = \rho/(1 + c_B)$. The reserve price thus depends on the commission structure, but since buyers shade their bids by the same factor ($b = v/(1 + c_B)$), the marginal type of buyer who is excluded from the auction has a valuation $v = \rho$, which depends on α only.

For buyers, (4) shows that the surplus is equal to

$$\int_{\rho}^1 \int_{\rho}^v F^{n-1}(x) dx f(v) dv,$$

and depends on ρ , and thus on α only. Since participation decisions depend directly on the surplus, the equilibrium ratio of buyers to sellers n also depends only on α . It is then clear that the auctioneer's profit depends on α .

Proposition 2 *All commission rates (c_S, c_B) keeping $\alpha = (c_S + c_B)/(1 + c_B)$ constant generate identical surpluses and profits for all agents in the model (buyers, sellers and auction house).*

4 Welfare

The result of Proposition 2 allows us to restrict attention to the analysis of an increase in the commission index from α to $\hat{\alpha}$. In traditional markets, the increase of a commission shifts the supply and demand curves, which results in a higher net price for buyers and a lower net price for sellers. The welfare consequences are simple: all buyers and sellers are worse off.

In our model, it is less straightforward to evaluate the welfare consequences of an increase in the commission index, which leads to a higher reserve type and to a lower participation of buyers and sellers. The original auction market is thus characterized by a low commission index α , a reserve type ρ and a ratio of buyers to sellers equal to n . In the new market, the commission index increases to $\hat{\alpha} > \alpha$; it leads to a higher reserve type $\hat{\rho}$ and to a new ratio \hat{n} . The welfare considerations depend mainly on the ratio of buyers to sellers, that is how competitive the auction market is. This ratio has a direct influence on the welfare of buyers since it determines their probability of winning, and the expected price if they win.

Since in an auction market, only some of the bidders are successful, it is important to distinguish between ex-ante and ex-post welfare.

4.1 Ex ante welfare

At the ex-ante stage, buyers and sellers are necessarily worse off when the commission index increases.

Proposition 3 *Suppose that $v_s > 0$ and that sellers set their reserve price strategically. If $\hat{\alpha} > \alpha$, then the ex-ante welfares of buyers and sellers decrease.*

Proof: Suppose by way of contradiction that $B^A(\hat{\rho}, \hat{n}) > B^A(\rho, n)$. Since ρ increases when α increases, and since $B^A(\rho, n)$ is decreasing in ρ and n , it must be that $\hat{n} < n$. But if the ex-ante surplus of buyers increases, the number of buyers who participate is also larger. To have $\hat{n} < n$, it is then necessary that more sellers participate as well, but this is not possible since the ex-ante profit of sellers decreases with α and n . So $B^A(\hat{\rho}, \hat{n}) \leq B^A(\rho, n)$. It is then easy to see that sellers can not be better off. We have that $\Pi(\hat{\alpha}, \hat{\rho}, n) < \Pi(\alpha, \rho, n)$. Hence, the only way to make sellers better off would be to have more bidders at each auction, $\hat{n} > n$. But this would be impossible since a lower welfare for buyers decreases their participation while higher profits for sellers would increase their participation leading to less bidders per auction. ■

Part of the ex-ante welfare loss is due to the lower participation of potential bidders. The elasticity of participation of buyers and sellers (due to the distribution of participation costs) is one of the important factors that explains the impact of an increase in commissions. If buyers' participation is very elastic (this is the case when all buyers have the same positive cost of participation), the buyer's ex-ante surplus is left unchanged by the increase in commissions. The participation decision decreases competition in the market to the point that it compensates exactly the direct decrease in welfare due to higher commissions and reserve prices.

However, the welfare comparisons made in practice are for effective buyers and sellers; ex-ante welfare is not the right measure on which their compensation should be based since it includes the welfare of buyers who will never win in the new auction. In auction markets, the definition of an effective buyer is not as clear as in traditional markets. We can distinguish between *effective bidders* and *successful buyers*: the former are those with a valuation higher than the reserve type and who therefore have a chance – at the interim stage – to win the

object; the latter are the bidders who actually won the object, that is the observed winners. We consider first the welfare of effective bidders.⁸

Proposition 4 *When all bidders have the same cost of participation, effective bidders are ex-ante better off in the market with high commissions.*

Proof: When each bidder has the same cost of participation t , the ex-ante welfare does not change with commissions: We have $B^A(\rho, n) = t = B^A(\hat{\rho}, \hat{n})$. The effective bidders are those with a valuation larger than the reserve type $\hat{\rho}$. Their ex-ante welfare in the new auction is $\int_{\hat{\rho}}^1 B^I(v, \hat{\rho}, \hat{n})f(v)dv = \int_{\rho}^1 B^I(v, \hat{\rho}, \hat{n})f(v)dv$ since buyers with type in $[\rho, \hat{\rho}]$ have no chance to win the auction. But now we have that

$$\begin{aligned} \int_{\hat{\rho}}^1 B^I(v, \hat{\rho}, \hat{n})f(v)dv &= \int_{\rho}^1 B^I(v, \hat{\rho}, \hat{n})f(v)dv \\ &= \int_{\rho}^1 B^I(v, \rho, n)f(v)dv \\ &> \int_{\hat{\rho}}^1 B^I(v, \rho, n)f(v)dv. \end{aligned}$$

The last inequality comes from the fact that in the original market, buyers with type in $[\rho, \hat{\rho}]$ have a positive expected surplus. As long as participation is elastic, that is the number of buyers participating in the auction strongly react to a change of ex-ante welfare, implying thus that $B^A(\hat{\rho}, \hat{n})$ is not much smaller than $B^A(\rho, n)$, the previous argument would still hold.

■

By continuity the result holds as long as the participation decision of bidders is “sufficiently” elastic. The change of average *ex-post* welfare of successful buyers and sellers is usually what economists have in mind when they analyze the effect of an increase in commissions in order to compensate losers from such an increase. If, ex ante, buyers are worse-off, it is not clear whether this will also be the case ex post. Since sellers change their reserve

⁸Our definition of effective bidders broadly corresponds to the observed bidders in an English (ascending) auction, used in most salesrooms.

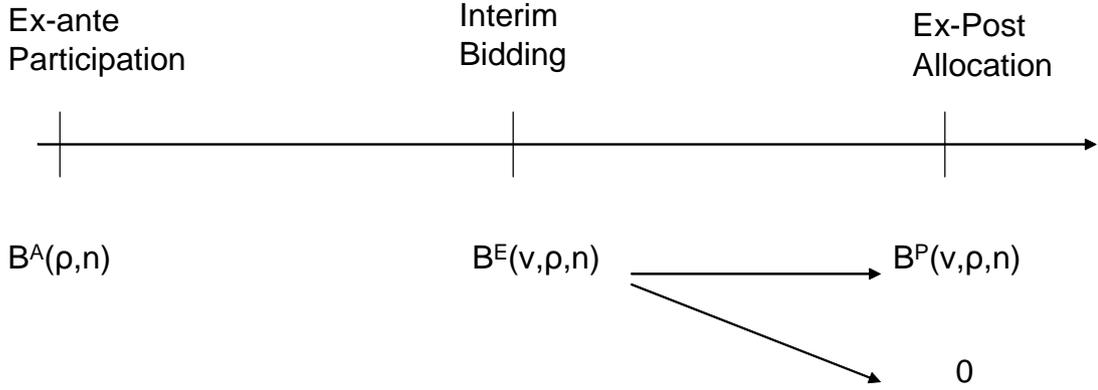


Figure 1: Welfare Considerations

types and since higher reserve types decrease ex-ante welfare of buyers and thus their participation, ex-post, buyers may face less competition (a lower n). Buyers who actually win end up paying less than they would have otherwise. Thus compensating the winning buyers might not be the best idea. We elaborate on this point in the next section.

We summarize this discussion in Figure 1, which shows the timing of the model and the various welfare definitions.

4.2 Ex-post welfare: Winning twice

The welfare of a participant in an auction is a function of the probability of winning and the expected price conditional on winning. To assess the welfare changes of participants, it is therefore necessary to take into account the joint variation of these two variables. This suggests that the welfare consequences of a change in commissions are different when assessed ex-ante (before the winner is known) and ex-post (after he is known).

The winner of an auction who has a valuation $v \geq \rho$ has on average a surplus of

$$B^P(v, \rho, n) = \int_{\rho}^v (v - \max(\rho, v_{(2)})) dF_{(2,n)}(v_{(2)} | v; n).$$

He pays the second highest bid if the second highest bid is larger than the reserve price, or pays the reserve price if there is no other larger bid. Indeed, if $v_{(2)} \geq \rho$ the hammer price is $v_{(2)}/(1 + c_B)$; otherwise, the hammer price is r .

When $v \geq \rho$, we can rewrite $B^P(v, \rho, n)$ as

$$\begin{aligned} & \int_0^\rho (v - \rho)(n - 1)f(z) \frac{F(z)^{n-2}}{F(v)^{n-1}} dz + \int_\rho^v (v - z)(n - 1)f(z) \frac{F(z)^{n-2}}{F(v)^{n-1}} dz, \\ &= (v - \rho) \left(\frac{F(r(1 + c_B))}{F(v)} \right)^{n-1} + \int_\rho^v (v - z)(n - 1)f(z) \frac{F(z)^{n-2}}{F(v)^{n-1}} dz, \\ &= v - \rho \left(\frac{F(\rho)}{F(v)} \right)^{n-1} - \int_\rho^v (n - 1)zf(z) \frac{F(z)^{n-2}}{F(v)^{n-1}} dz. \end{aligned}$$

Suppose that α increases to $\hat{\alpha}$, that is suppose that the reserve type increases from ρ to $\hat{\rho}$ and let n and \hat{n} be the corresponding equilibrium values of number of bidders per auction. We want to compare the expected surplus of winners when the commissions are $\hat{\alpha}$ to what they would have been with α . More competition implies that the winner wins the object less often, but potentially at a lower price.

The first dimension to look at is the price. Had he won in the other auction, a successful buyer would have paid a price determined in an environment with a lower reserve price but with more competition. The second dimension is the probability of winning. By definition, an observed winner has a probability of winning equal to one in the auction under consideration. In the market with lower commissions, he would have faced more competition. To make a meaningful comparison, we consider, as we did in the example of section 2, the following thought experiment. We fix the valuations of all the bidders present in the new auction, and add new bidders ($n - \hat{n}$, to be precise) drawing valuations for them. This decreases the probability of winning from 1 to $F^{(n-\hat{n})}(v)$ for an observed winner with a valuation of v .

The previous observations show that successful buyers may have gained when the intermediary increases the commission index but that they may have lost if the reserve price binds. One way to simplify the welfare analysis is therefore to verify whether the reserve price set by the seller was binding. An increase in commissions leads to a higher price only if the price paid is equal to the reserve price, otherwise the decrease in the degree of competition n would have driven the price down. Successful buyers who pay more than the reserve price are better off in a regime of high commissions and low competition than in a regime of low commissions and high competition.

Proposition 5 *When the number of bidders per auction decreases with an increase in commissions ($\hat{n} < n$), and when the observed winner of the auction does not pay the reserve price,*

then his welfare would be lower in the auction with lower commissions but more participation.

Table 3 summarizes the effects of increased commissions on the various welfare measures that we analyze.

Table 3

Sellers	Buyers (ex ante)	Effective bidders (interim $v > \hat{\rho}$)	Winners (ex post)
Worse off	Weakly worse off	Better off if participation is elastic	High types: Better off Low types: Worse off but better off if reserve price not binding

For valuations that are uniformly distributed, we also show that successful buyers with high valuation are better off in the scenario with high commissions, while lower types are worse off. We even show for a specialized example that, in fact, *on average*, these successful buyers would not be willing to pay to face more competition and a low commission, that is to be faced with a situation similar to the low-commission auction.

4.3 The case of uniformly distributed valuations

We first show that if type v wins in the $(\hat{\rho}, \hat{n})$ auction, then the expected surpluses $B^P(v, \hat{\rho}, \hat{n})$ and $B^P(v, \rho, n)$ satisfy a single crossing property.

Proposition 6 Consider $\hat{\rho} > \rho$ and $\hat{n} < n$.

- (i) For $v \in (\rho, \hat{\rho}]$, $B^P(v, \hat{\rho}, \hat{n}) - B^P(v, \rho, n) < 0$,
- (ii) If $n/\hat{n} \leq (1 - \rho^n)/(1 - \hat{\rho}^{\hat{n}})$, then $B^P(v, \hat{\rho}, \hat{n}) - B^P(v, \rho, n) < 0$ for all v ,
- (iii) If $n/\hat{n} > (1 - \rho^n)/(1 - \hat{\rho}^{\hat{n}})$, then there is single crossing: There exists a unique v_0 such that $B^P(v_0, \hat{\rho}, \hat{n}) - B^P(v_0, \rho, n) = 0$ and $(v - v_0)(B^P(v, \hat{\rho}, \hat{n}) - B^P(v, \rho, n)) > 0$ for all $v \neq v_0$.

Proof: See appendix 6.2. ■

Ex-post, we can divide buyers into two groups: those whose welfare increase after the increase in the commission index and those whose welfare decreased. Highest types are better off with higher commissions since they benefit from reduced competition and rarely suffer from the increased reserve price while lowest types are worse off since there is a high probability that the reserve price is binding when they win.

We now consider an example which shows that, on average, winners are better off in the auction with high commissions.

Example *An example in which successful buyers are better off after an increase in the commission index*

Assume that (a) all sellers have a valuation $v_s = \frac{1}{4}$, (b) sellers have no cost of participating, (c) buyers have valuations uniformly distributed on $[0, 1]$, (d) all buyers have the same participation cost $t = 0.025$,⁹ and (e) the commission ratio α goes from 0 to 50%.¹⁰

The optimal reserve type is

$$\rho(\alpha) = \frac{1}{2} + \frac{1}{8(1-\alpha)}$$

$$B^I(v, \rho, n) = \int_{\rho}^v (x)^{n-1} dx = \frac{(v)^n - (\rho)^n}{n}.$$

Thus the ex-ante surplus B^A of a buyer who expects n buyers in each auction is:

$$\begin{aligned} B^A(\rho, n) &= \int_{\rho}^1 B^I(v, \rho, n) f(v) dv \\ &= \int_{\rho}^1 \frac{(v)^n - (\rho)^n}{n} dv \\ &= \frac{1}{n(n+1)} - \frac{\rho^n}{n} + \frac{\rho^{n+1}}{n+1}. \end{aligned}$$

⁹Since we want to make the case that successful buyers can be better off on average, we make the assumption that all buyers have the same participation cost. This makes buyers' participation very elastic, and leads to the largest possible decrease in participation and the lowest possible degree of competition.

¹⁰We suppose for clarity of exposition that the commission is on sellers only. As a result of Proposition 2, this is without loss of generality. The increase from no commission to commissions of 50% is very large but this is meant to illustrate that even with such a steep increase, successful buyers are better off. Of course, since the degree of competition decreases continuously with α , there exists α and $\hat{\alpha}$ with $\alpha - \hat{\alpha}$ small for which the same results obtain.

For $\alpha = 0$, $\rho(0) = 5/8$, and $B^A(5/8, n) = (1 - (5/8)^n(1 + 3n/8))/n(n + 1)$.

For $\hat{\alpha} = 0.5$, $\rho(0.5) = 3/4$, and $B^A(3/4, n) = (1 - (3/4)^n(1 + n/4))/n(n + 1)$.

In Figure 2, we represent the welfare locus for both levels of the commission rates (and the corresponding reserve types). The upper (lower) curve is the one for the low (high) commission rate. The equilibrium ratio is determined when the ex-ante welfare curve crosses the (thick) straight line that represents the (constant) participation cost. The graph illustrates the fact that a constant participation cost leads to the highest possible decrease in the degree of competition.

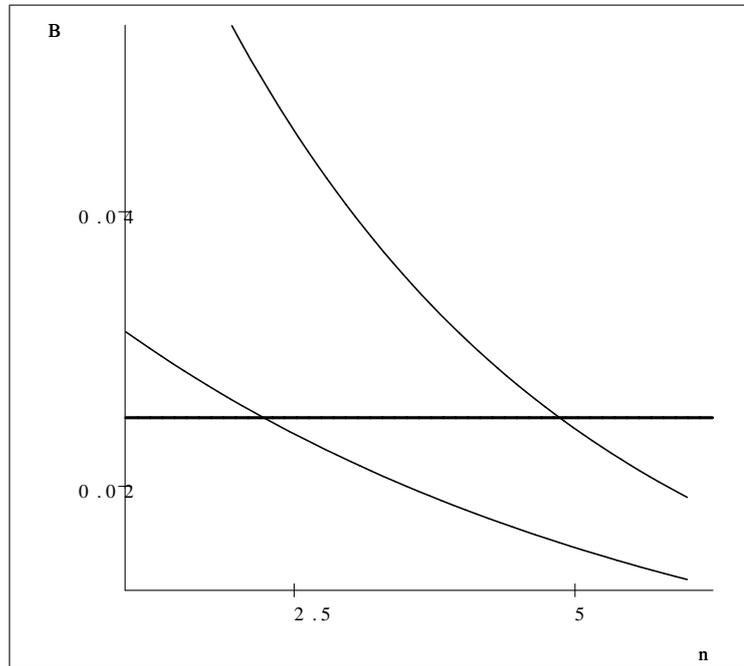


Figure 2: Ex-ante Welfare and Participation

The equilibrium number of participants is obtained when B^A is equal to the participation cost $t = 0.025$. This leads to $n = 4.8631$ for $\alpha = 0$ and $\hat{n} = 2.2277$ for $\hat{\alpha} = 0.50$.

The increase in the commission rate has two main effects on the price paid. First, the second order bid decreases, since the increase in commission leads to a decrease in the degree of competition: $\hat{n} < n$; this will decrease the hammer price. Second, the reserve type increases, so that the price paid by the winner is more likely to be equal to the reserve price. Whether

or not the (higher) reserve type is actually larger than the (lower) second order bid in the initial auction depends on how fast n decreases when ρ increases. The expected price paid by the winner of the auction if his valuation equals 1 is:

$$\begin{aligned} E(p|1) &= \rho F(r)^{n-1} + \int_{\rho}^1 (n-1)xf(x)F(x)^{n-2}dx \\ &= \rho^n + \frac{n-1}{n}(1-\rho^n). \end{aligned}$$

We can then compute the expected price paid by the highest type in both situations. These are: $p(\alpha = 0) = 0.81528$, and $p(\hat{\alpha} = 0.50) = 0.7876$. The expected price paid by the highest type is lower when commissions are high. He is better off with high commissions. We now compute the expected price paid by a winner who has a valuation v :

$$\begin{aligned} E[p|v] &= \rho \frac{F(\rho)^{n-1}}{F(v)^{n-1}} + \int_{\rho}^v (n-1)xf(x) \frac{F(x)^{n-2}}{F(v)^{n-1}} dx \\ &= \frac{1}{v^{n-1}} \left(\rho^n + \frac{n-1}{n}(v^n - \rho^n) \right). \end{aligned}$$

Figure 3 displays the expected price of a winner of type v in both auctions. We see the single-crossing property at work. High types ($v > 0.93985$) would pay a lower price in the auction with no commission. The downward effect on the price due to lower competition more than compensates the upwards pressure due to a higher reserve price. The effects are reversed for the low types.

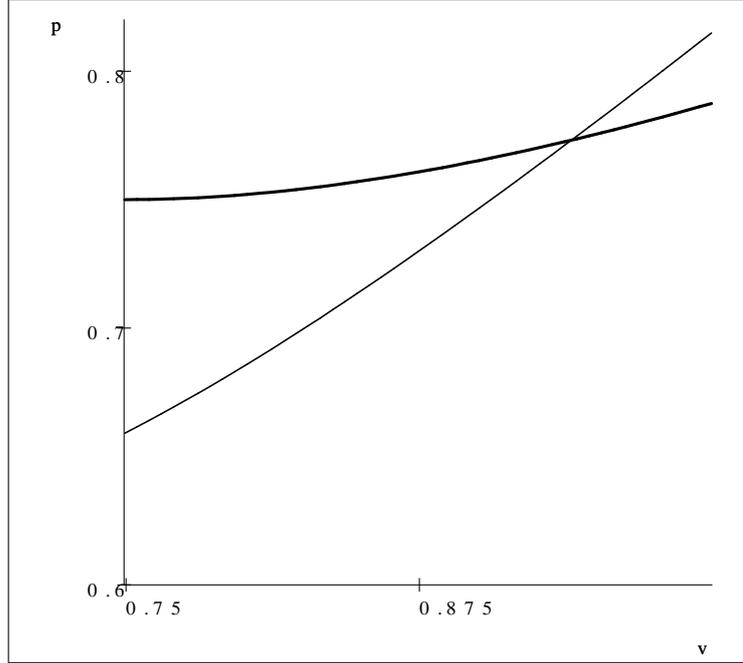


Figure 3: Expected price paid by the winner

Comparing prices is only comparing markets along one dimension. The second dimension is the probability of winning. And on that dimension, observed winners have clearly benefitted from the increase in commissions that has led to lower participation and lower competition. As suggested earlier, we make the thought experiment of finding the monetary transfer that would make the winner indifferent between paying the current price or face more competition but a lower commission (compensating variation).

On average, the welfare of a successful buyer of type v is :

$$B^P(v, \hat{\rho}, \hat{n}) = (v - E[p|v, \hat{n}]) = v - \frac{1}{v^{\hat{n}-1}} \left(\hat{\rho}^{\hat{n}} + \frac{\hat{n}-1}{\hat{n}} (v^{\hat{n}} - \rho^{\hat{n}}) \right).$$

We want to compare this to the welfare he would have had in the auction with low commissions, discounted with the probability that one of the additional bidders would have had a higher valuation, that is

$$(v - E[p|v, n]) F^{n-\hat{n}}(v).$$

Returning to our numerical example, we graph these two functions in Figure 4.

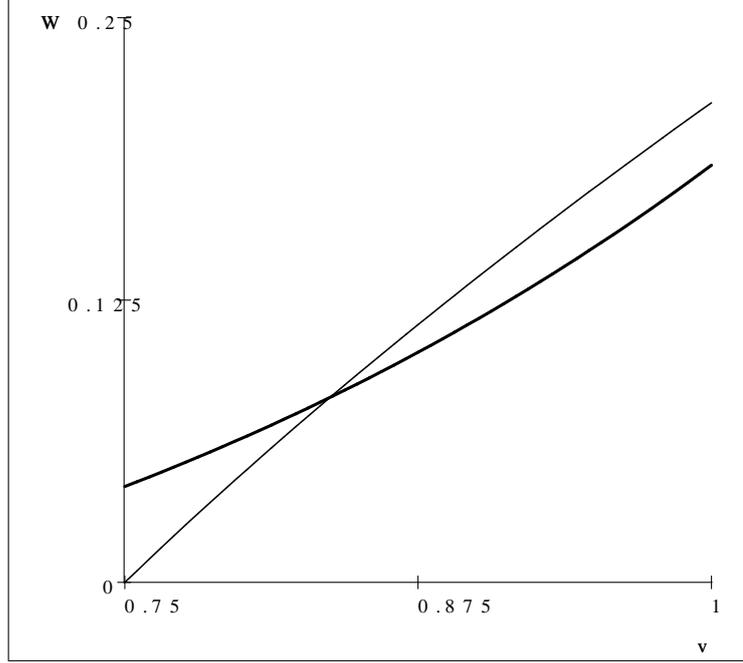


Figure 4 : Comparison of ex-post welfares

We now have to average these values with the density of winners to recover a meaningful comparison between the average welfare of successful buyers

$$\begin{aligned}
B^P(\hat{\rho}, \hat{n}) &= \int_{\hat{\rho}}^1 B^P(v, \hat{\rho}, \hat{n}) dF^{(1, \hat{n})}(v) \\
&= \int_{\hat{\rho}}^1 \left(v - \frac{1}{v^{\hat{n}-1}} \left(\hat{\rho}^{\hat{n}} + \frac{\hat{n}-1}{\hat{n}} (v^{\hat{n}} - \hat{\rho}^{\hat{n}}) \right) \right) (\hat{n} v^{\hat{n}-1}) dv \\
&= \frac{1 - \hat{n} \hat{\rho}^{\hat{n}} - \hat{\rho}^{\hat{n}} + \hat{\rho}^{\hat{n}+1} \hat{n}}{\hat{n} + 1},
\end{aligned}$$

and the welfare of those same buyers if they were participating in an auction with a low commission index but more buyers:

$$\begin{aligned}
&\int_{\hat{\rho}}^1 B^P(v, \rho, n) F^{n-\hat{n}}(v) dF^{(1, \hat{n})}(v) \\
&= \int_{\hat{\rho}}^1 \left(v - \frac{1}{v^{n-1}} \left(\rho^n + \frac{n-1}{n} (v^n - \rho^n) \right) \right) v^{n-\hat{n}} (\hat{n} v^{\hat{n}-1}) dv \\
&= \hat{n} \frac{1 - n \rho^n - \rho^n - \hat{\rho}^{n+1} + \hat{\rho} \rho^n n + \hat{\rho} \rho^n}{n(n+1)}.
\end{aligned}$$

Numerical calculations yield that the average welfare $B^P(\hat{\rho}, \hat{n}) = 0.253$ is larger than the corresponding welfare in an auction with low commission but more bidders showing that the winners are, on average, better off in the high commission market and would be ready to pay *not to be* in a market with lower commissions.

5 Conclusion

The welfare analysis of commissions is well understood in traditional markets and leads to the simple conclusion that buyers and sellers are worse off. In this paper, we have argued that a separate analysis is needed in auction markets with intermediaries. A key element of auction markets is that the goods that are exchanged are lotteries; as we show this implies that ex-ante and ex-post welfare comparisons are not necessarily identical. Generally the difference between ex-ante and ex-post welfare hinges on the observation that effective buyers can be better off with higher commissions whose negative impact is balanced by lower participation and a lower degree of competition. This intuition should also be operative in markets where there is uncertainty on the price that will be realized, for instance in search models. While rather intuitive, this observation has drastic consequences for evaluating the welfare effects of non competitive pricing and of collusion in markets with intermediation, including e-commerce, e-Bay like platforms and treasury bond auctions.

6 Appendix

6.1 Optimal Reserve Prices

The expected payment of a buyer with valuation $v \geq r$ can be written:

$$p(v, r) = rF^{n-1}(r) + \int_r^x y(n-1)f(y)F^{n-2}(y)dy.$$

The ex-ante expected payment of a bidder is:

$$\begin{aligned} E[p(x, r)] &= \int_r^1 p(v, r) f(x) dx \\ &= \int_r^1 \left(rF^{n-1}(r) + \int_r^x y(n-1)f(y)F^{n-2}(y)dy \right) f(x) dx \\ &= r(1-F(r))F^{n-1}(r) + \int_r^1 y(1-F(y))(n-1)f(y)F^{n-2}(y)dy. \end{aligned}$$

The expected revenue of the seller is thus $\Pi = (1 - \alpha) n E[p(x, r)] + F^n(r) v_s$. Differentiating with respect to r , we obtain:

$$\begin{aligned} \frac{\partial \Pi}{\partial r} &= (1 - \alpha) n (1 - F(r) - r f(r)) F^{n-1}(r) + n F^{n-1}(r) f(r) v_s \\ &= (1 - \alpha) n \left(1 - \left(r - \frac{v_s}{1 - \alpha} \right) h(r) \right) (1 - F(r)) F^{n-1}(r), \end{aligned}$$

where $h(x) = f(x)/(1 - F(x))$ is the hazard rate associated with distribution F .

Since $\partial \Pi / \partial r > 0$ at $r = v_s$, it is always optimal to choose a reserve price larger than the seller's valuation.

The optimal reserve price has to satisfy $\partial \Pi / \partial r = 0$. This will be true if

$$1 - \left(r - \frac{v_s}{1 - \alpha} \right) h(r) = 0,$$

so that

$$r - \frac{1 - F(r)}{f(r)} = \frac{v_s}{1 - \alpha}.$$

6.2 Proof of Proposition 4

We have,

$$\begin{aligned} \Delta(v) &\equiv B(v, \hat{\rho}, \hat{n}) - B^P(v, \rho, n) \\ &= \frac{v}{n \hat{n}} \left[n \left(1 - \left(\frac{\hat{\rho}}{v} \right)^{\hat{n}} \right) - \hat{n} \left(1 - \left(\frac{\rho}{v} \right)^n \right) \right]. \end{aligned}$$

Note that $\Delta(1) = [n(1 - \hat{\rho}^{\hat{n}}) - \hat{n}(1 - \rho^n)] / n \hat{n}$; this is negative only if $n / \hat{n} < (1 - \rho^n) / (1 - \hat{\rho}^{\hat{n}})$.

The sign of $\Delta(v)$ is equal to the sign of the term in brackets. The derivative of this term with respect to v is

$$\frac{d}{dv} \left(\hat{n} \left(\frac{\rho}{v} \right)^n - n \left(\frac{\hat{\rho}}{v} \right)^{\hat{n}} \right) = \frac{n\hat{n}}{v^{n+1}} \left(\hat{\rho}^{\hat{n}} v^{n-\hat{n}} - \rho^n \right).$$

Since $n - \hat{n} > 0$, the expression is increasing in v . As $v \leq \hat{\rho}$, $B^P(v, \hat{\rho}, \hat{n}) = 0 < B^P(v, \rho, n)$ and $\Delta(v) < 0$, which proves (i). The issue is whether there exist values of b for which $\Delta(v) > 0$.

Suppose that there exists v_0 such that $\Delta(v_0) = 0$; this requires that

$$\begin{aligned} \frac{n}{\hat{n}} &= \frac{1 - \left(\frac{\rho}{v_0} \right)^n}{1 - \left(\frac{\hat{\rho}}{v_0} \right)^{\hat{n}}} \\ \Leftrightarrow v_0^{n-\hat{n}} \hat{\rho}^{\hat{n}} - \rho^n &= \frac{n - \hat{n}}{\hat{n}} v_0^n \left(1 - \left(\frac{\hat{\rho}}{v_0} \right)^{\hat{n}} \right) \\ \Rightarrow v_0^{n-\hat{n}} \hat{\rho}^{\hat{n}} - \rho^n &> 0 \end{aligned}$$

where the last implication follows from $v_0 \geq \hat{\rho}$. Hence, for $v > v_0$, $\Delta(v) > 0$ and for $v < v_0$, $\Delta(v) < 0$. The existence of such a v_0 therefore requires that $\Delta(1) \geq 0$, in which case we have the single crossing property (iii). When $\Delta(1) < 0$, we must have $\Delta(v) < 0$ for all values of v , which proves (ii).

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