DISCUSSION PAPER SERIES

No. 4811

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INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

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ISSN 0265-8003

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Discussion Paper No. 4811
December 2004

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CEPR Discussion Paper No. 4811

December 2004

ABSTRACT

What We Don't Know About the Monetary Transmission Mechanism and Why We Don't Know It*

We study identification in a class of linear rational expectations models. For any given exactly identified model, we provide an algorithm that generates a class of equivalent models that have the same reduced form. We use our algorithm to show that a model proposed by Benhabib and Farmer [1] is observationally equivalent to the standard new-Keynesian model when observed over a single policy regime. However, the two models have different implications for the design of an optimal policy rule.

JEL Classification: C39, C62, D51, E52 and E58
Keywords: Benhabib-Farmer model, identification, indeterminacy and new-Keynesian model

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*This Paper was written in the summer of 2004 while Farmer was visiting the Directorate General Research as part of the European Central Bank’s Research Visitor Programme. He wishes to thank members of DG-Research for their kind hospitality. Farmer acknowledges the support of NSF award # SES 0418074. We thank participants at the University of Pennsylvania, Princeton University, New York University, the University of Kansas, the University of Indiana, the Dynamic Macroeconomics Conference in Copenhagen, the Society of Economic Dynamics Conference in Florence, the CEPR summer institute in Tarragona, the ESSIM Conference in Madrid, the Banque de France Workshop in Paris and the European Central Bank. Thanks especially to Fabio Canova who provided detailed comments on our working paper [4] which is an earlier an more extensive version of this Paper. The views expressed in this paper are those of the authors and do not necessarily represent those of the ECB.

Submitted 03 December 2004

NON-TECHNICAL SUMMARY

This Paper is about the general lack of identification in linear rational expectations models. We study identification in a class of three-equation monetary models and show that, using data from a single policy regime, it is not possible to tell whether a given period was associated with a policy that was driven purely by fundamental shocks; or whether sunspots also played a role. For any given exactly identified model, we provide an algorithm that generates a class of equivalent models that have the same reduced form. We use our algorithm to provide an example of the consequences of lack of identification. We establish an equivalence between a class of models proposed by Benhabib and Farmer and the standard new-Keynesian model. This result is disturbing since equilibria in the Benhabib-Farmer model are typically indeterminate for a class of policy rules that generate determinate outcomes in the new-Keynesian model.

1 Introduction

> It is my view, however, that rational expectations is more deeply subversive of identification than has yet been recognized: Christopher A. Sims, "Macroeconomics and Reality", [24], page 7.

This quote from Chris Sims is now twenty four years old but it has weathered well. It appeared in a paper that introduced vector autoregressions as an alternative to structural models at a time when the rational expectations agenda was in its infancy. A quarter of a century later, applied macroeconomists continue to estimate structural equations without paying careful attention to the identifying assumptions that one requires for a particular equation to make sense.

One popular approach to estimation of an equation that includes expectations of future variables is to replace the expectations by their realized values and to estimate the model using instrumental variables. This method, first discussed by McCallum [19], has been widely used in recent work on applied monetary economics to estimate the parameters of one or more equations in a New-Keynesian model of the monetary transmission mechanism.[1] Although it is possible to estimate a single equation using instruments, the assumptions that are necessary to make any particular identification valid in the context of a complete structural model are rarely spelled out.[2] In this paper we show that the New-Keynesian identifying assumptions are at best, untestable, and we provide a credible alternative identification scheme that provides a different answer to an important policy question: Should monetary policy be active or passive?

In an influential paper Clarida-Galí-Gertler [8] estimated central bank reaction functions before and after 1979 and embedded their estimated policy rule in a calibrated dynamic general equilibrium model. They found that, in their calibrated model, equilibrium was indeterminate when policy followed the estimated pre-1979 rule and that it was determinate when the post-1979 rule was substituted into the same calibrated environment. Their results have subsequently been substantiated by Boivin and Giannoni [7] and Lubik and Schorfheide [17] who independently estimated complete structural models.

Lubik and Schorfheide go beyond the estimation of a structural model. They provide a procedure that can be used to find the posterior odds ratio for the likelihood that a given set of data was driven by a determinate or

[1] Examples include Clarida et. al. [8], Gali and Gertler [12] and Fuhrer and Rudebusch [11].

[2] Examples of recent papers that make this, or related points, are those of Lindé [14], Lubik and Schorfheide [17], Nason and Smith [20] and Mavroedis [18].

an indeterminate policy regime. In related work, (Beyer and Farmer [3]), we provided two one equation models that have the same likelihood; one model was determinate and the other indeterminate. In this paper we go beyond the one equation example and we provide an algorithm that can generate equivalence classes of models with the same reduced form. We show, by means of an example, that the results that we report in our former work can be generalized to complete structural models. We provide examples of two alternative micro-based theories of the monetary transmission mechanism that have the same likelihood function and we show that, under a given policy rule, one of these theories leads to a determinate equilibrium and the other to an indeterminate equilibrium.

2 A Class of Linear Models

The purpose of this section is to introduce a class of structural linear rational expectations models (SLREs). This class is large enough to include most of the models used recently in work on the monetary transmission mechanism and real business cycle theory. The restriction to linearity is not as restrictive as it might at first appear since, linear models are often a good approximation to non-linear models around a steady state or around a balanced growth path.

We introduce a representation of an SLRE that we refer to as the *companion form.* A structural model can be written in companion form by adding a set of equations that define expectational errors. Once written in this way, it is relatively easy to compute the reduced form of the structure by applying a QZ decomposition to a pair of coefficient matrices that multiply vectors of current and lagged state variables.[3] Unlike alternative solution algorithms, these matrices do not need to be non-singular.

2.1 The Structural Form

Consider the following class of models,

$$AY_t + FE_t\left[Y_{t+1}\right] = B_1 Y_{t-1} + B_2 E_{t-1}\left[Y_t\right] + C + \Psi_v V_t, \tag{1}$$

$$E_t\left[V_t V_s'\right] = \begin{cases} \Omega_{vv}, & t = s, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

A, F, Ψ_v B_1 and B_2 are $l \times l$ matrices of coefficients, C is an $l \times 1$ matrix of constants, E_t is a conditional expectations operator and $\{V_t\}$ is a weakly

[3] The details of this procedure are described in Sims [25]. We use the term companion form for what Sims calls the canonical form of the model.

stationary i.i.d. stochastic process with covariance matrix Ω_{vv} and mean zero.[4] Lowercase letters are scalars, and uppercase letters represent vectors or matrices. We maintain the convention that endogenous variables appear on the left side of each equation and explanatory variables appear on the right.

2.2 The Companion Form

The structural form, Equation (1), is a system of l equations in $2l$ endogenous variables $\{Y_t, E_t[Y_{t+1}]\}$. To close the model one requires additional equations. Under the rational expectations assumption these are provided by the following definition

$$W_t = Y_t - E_{t-1}[Y_t]. \tag{3}$$

Combining Equations (1) and (3) we arrive at the following representation of a structural linear rational expectations model that we refer to as the *companion form*;

$$\overset{\tilde{A}_0}{\begin{bmatrix} A & F \\ I & 0 \end{bmatrix}} \overset{X_t}{\begin{bmatrix} Y_t \\ E_t[Y_{t+1}] \end{bmatrix}} = \overset{\tilde{A}_1}{\begin{bmatrix} B_1 & B_2 \\ 0 & I \end{bmatrix}} \overset{X_{t-1}}{\begin{bmatrix} Y_{t-1} \\ E_{t-1}[Y_t] \end{bmatrix}} + \overset{\tilde{C}}{\begin{bmatrix} C \\ 0 \end{bmatrix}} + \overset{\tilde{\Psi}_v}{\begin{bmatrix} \Psi_v \\ 0 \end{bmatrix}} V_t + \overset{\tilde{\Psi}_w}{\begin{bmatrix} 0 \\ I \end{bmatrix}} W_t. \tag{4}$$

We can write Equation (4) more compactly as follows:

$$\tilde{A}_0 X_t = \tilde{A}_1 X_{t-1} + \tilde{C} + \tilde{\Psi}_v V_t + \tilde{\Psi}_w W_t. \tag{5}$$

The variables W_t are referred to as non-fundamental errors and some or all of these variables are determined endogenously by making the additional assumption that Equation (5) defines a stationary time-series process for the complete set of endogenous variables X_t. In some cases, the assumption of stationarity is sufficient to pin down the values of W_t as functions of the fundamental shocks V_t. In other cases one must add additional structure to the problem by specifying an exogenous joint probability distribution for the shock vector $[V_t, W_t]'$.

[4] We will focus on the case of one lag, but our method can easily be expanded to include additional lags or additional leads of expected future variables.

2.3 The Reduced Form

The reduced form of an econometric model is a set of equations that explains each endogenous variable as a function of exogenous and predetermined variables. The reduced form of Equation (1) is given by the following equation,

$$X_t = \Gamma^* X_{t-1} + C^* + e_t, \tag{6}$$

where the reduced form residuals e_t are functions of the fundamental and non-fundamental shocks

$$e_t = \Psi_v^* V_t + \Psi_w^* W_t. \tag{7}$$

In the case of a unique equilibrium, Ψ_w^* is identically zero and, in this case, only the fundamental shocks influence the behavior of the system.

2.4 The Dynamics of the Reduced Form

The reduced form governs the behavior of the state variables Y_t and their expectations $E_t[Y_{t+1}]$. In computing the reduced form, there are three possible cases to consider: (1) there is a unique equilibrium (2) there are multiple stationary indeterminate equilibria or (3) no stationary equilibrium exists. In the following paragraphs we discuss cases (1) and (2).

In almost all cases, the reduced form parameter matrix Γ^* has reduced rank and it is possible to partition X_t into two disjoint subsets $X_t = (X_{1t}, X_{2t})$ such that X_{1t} is described by a VAR(1),

$$X_{1t} = \Gamma_{11}^* X_{1t-1} + C_1^* + e_{1t}, \tag{8}$$

$$e_{1t} = \Psi_{1v}^* V_t + \Psi_{1w}^* W_{1t}, \tag{9}$$

and X_{2t} is an affine function of X_{1t}

$$X_{2t} = C_2^* + M^* X_{1t}. \tag{10}$$

The one exception to this rule is when the equilibrium is indeterminate and the degree of indeterminacy is equal to l. In this case the matrix Γ_{11}^* has full rank and X_{2t} is empty.

In the familiar case of a unique equilibrium the number of unstable generalized eigenvalues of $\left\{\tilde{A}_0, \tilde{A}_1\right\}$ is equal to l.[5] In this case one can choose

[5] Our computational algorithm, `SysSolve`, uses a QZ decomposition and is based on Sims' code, `Gensys` [25]. The QZ decomposition for square matrices A and B is a pair of upper triangular matrices S and T and a pair of orthonormal matrices Q and Z such that $QTZ = A$, $QSZ = B$ and $QQ' = ZZ' = I$. The ratios $[S_{ii}]/[T_{ii}]$ of the diagonal elements of S and T are referred to as generalized eigenvalues or roots.

$X_{1t} = Y_t$ and Equation (8) has the form

$$\begin{aligned} Y_t &= \Gamma_{11}^* Y_{t-1} + C_1^* + e_{1t}, \\ e_{1t} &= \Psi_{1v}^* V_t. \end{aligned} \tag{11}$$

When the equilibrium is unique, the shocks W_t do not enter the reduced form and therefore, in that case X_{2t} is equal to $E_t[Y_{t+1}]$;

$$E_t[Y_{t+1}] = C_2^* + M^* Y_t, \tag{12}$$

and M^* and Γ_{11}^* are $l \times l$ matrices of full rank.

If the number of unstable generalized eigenvalues is less than l, the solution is said to be indeterminate. The degree of indeterminacy, r, is equal to $l - n$, where l is the dimension of Y_t and n is the number of unstable roots; r can vary between 1 and l. Although, in this case, it will still be possible to partition X_t and write the reduced form as a VAR(1) it may not be possible to choose this partition in a way that excludes $E_t[Y_{t+1}]$ from X_{1t}.

3 An Algorithm to Find Classes of Equivalent Models

In this section we provide an algorithm (implemented in Matlab as `FindEquiv`) to construct equivalence classes of structural models that have the same reduced form. For computational reasons we begin with a determinate model. This assumption is unrestrictive since our purpose is to establish, by means of an example, that there may exist determinate and indeterminate models that are observationally equivalent.

3.1 Structural and Reduced Form Parameters Defined

Consider a structural model given by the expression

$$\begin{bmatrix} A & F \end{bmatrix} \begin{bmatrix} Y_t \\ E_t[Y_{t+1}] \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ E_{t-1}[Y_t] \end{bmatrix} + C + \Psi_v V_t, \tag{13}$$

$$E_t[V_t V_t'] = \Omega_{vv} = I,$$

and define the vector of structural parameters

$$\theta = vec\left[(A, F, B_1, B_2, C, \Psi_v)'\right].$$

We refer to θ as the *true parameters* and to Equation (13) as the *true* model. The assumption that the covariance matrix of V_t is the identity matrix is unrestrictive since we allow for correlated shocks to the structural equations through the impact matrix Ψ_v.

The reduced form of Equation (13) is represented by the equation

$$X_t = \Gamma^* X_{t-1} + C^* + \Psi_v^* V_t, \tag{14}$$

and is parameterized by the vector

$$\phi(\theta) = vec\left[(\Gamma^*, C^*, \Psi_v^*)'\right].$$

This notation reflects the functional dependence of ϕ on θ. We refer to ϕ as the *reduced form* parameters.

3.2 Recovering an Equivalent Model Using Linear restrictions

Our next step is to forget that we know the true model and to trace the steps that would be followed by an econometrician who has access to an infinite sequence of data generated by the model and who uses this data to recover the reduced form parameters ϕ. The econometrician combines his estimated reduced form with an economic theory and recovers some possibly different model that we call $\bar{\theta}$.

Following Fisher [10], our econometrician establishes a set of linear equations linking ϕ to the structural parameters in his model, $\bar{\theta}$. He adds a set of linear restrictions of the form $R\bar{\theta} = r$ and solves the resulting linear equation system for $\bar{\theta}$ as a function of ϕ, r and R.

Let the structural model of the econometrician be denoted

$$\begin{bmatrix} \bar{A} & \bar{F} \end{bmatrix} \begin{bmatrix} Y_t \\ E_t[Y_{t+1}] \end{bmatrix} = \begin{bmatrix} \bar{B}_1 & \bar{B}_2 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ E_{t-1}[Y_t] \end{bmatrix} + \bar{C} + \bar{\Psi}_v \bar{V}_t. \tag{15}$$

We refer to $\bar{\theta}$, as the *equivalent parameters* and to Equation (15) as the *equivalent model.* Premultiplying (14) by $\begin{bmatrix} \bar{A} & \bar{F} \end{bmatrix}$ and equating coefficients leads to the following matrix equation

$$\underset{l\times 2l}{\begin{bmatrix} \underset{l\times l}{\bar{A}} & \underset{l\times l}{\bar{F}} \end{bmatrix}} \underset{2l\times(3l+1)}{\begin{bmatrix} \underset{2l\times 2l}{\Gamma^*} & \underset{2l\times 1}{C^*} & \underset{2l\times l}{\Psi_v^*} \end{bmatrix}} = \underset{l\times(3l+1)}{\begin{bmatrix} \underset{l\times l}{\bar{B}_1} & \underset{l\times l}{\bar{B}_2} & \underset{l\times 1}{\bar{C}} & \underset{l\times l}{\bar{\Psi}_v} \end{bmatrix}}. \tag{16}$$

After re-arranging Equation (16) and exploiting the properties of the Kronecker product, this system can be written as the following set of $l(3l+1)$

equations in the $l\,(5l+1)$ parameter vector $\bar{\theta}$:

$$\underset{l(3l+1)\times l(5l+1)}{H\,(\phi)}\;\underset{l(5l+1)\times 1}{\bar{\theta}} = \underset{l(3l+1)\times 1}{h}. \tag{17}$$

The details of this construction are given in Appendix A.

To recover a unique vector $\bar{\theta}$ that satisfies these equations we require an additional $2l^2$ independent linear restrictions which we assume are given by economic theory in the form of exclusion restrictions or as linear constraints. We parameterize these restrictions with a matrix R and a vector r such that

$$\underset{l(2l)\times l(5l+1)}{R}\;\underset{l(5l+1)\times 1}{\bar{\theta}} = \underset{l(2l)\times 1}{r}. \tag{18}$$

Stacking equations (17) and (18) leads to the system,

$$\underset{l(5l+1)\times l(5l+1)}{J}\;\underset{l(5l+1)}{\bar{\theta}} = \underset{l(5l+1)\times 1}{j}, \tag{19}$$

where

$$J = \begin{bmatrix} \underset{(3l+1)\times(5l+1)}{H} \\ \underset{l(2l)\times l(5l+1)}{R} \end{bmatrix} \text{ and } j = \begin{bmatrix} \underset{l(3l+1)\times 1}{h} \\ \underset{l(2l)\times 1}{r} \end{bmatrix}.$$

In order for the structural model to be identified, the matrix J must have full rank and the rows of Equation (18) must identify *different* structural equations. This requires that the rank and the order conditions (Fisher [10]) must be checked for each equation of the system. When identification is satisfied, the econometrician can recover the equivalent model $\bar{\theta}$ from the estimates of the reduced form (contained in ϕ) and the restrictions, contained in (18). By construction, $\bar{\theta}$ is observationally equivalent to the true model θ and both models lead to the same reduced form; that is,

$$\phi\,(\theta) = \phi\left(\bar{\theta}\right).$$

3.3 Remarks on the Variance Parameters

When equilibrium is indeterminate, the error vector $\bar{W}_t$ is only partly determined by $\bar{V}_t$ and some or all of the non-fundamental shocks must be determined by specifying a probability model for the joint distribution of $\left[\bar{V}_t, \bar{W}_t\right]'$. Equilibria may be indeterminate of differing degrees where the degree of indeterminacy, r, is equal to the number of elements of $\bar{W}_t$ that are not determined by fundamentals.

To close an indeterminate model we propose to partition $\bar{W}_t$ into two disjoint subsets $\left[\bar{W}_{1t}, \bar{W}_{2t}\right]$ where $\bar{W}_{1t} \in R^r$. We assume that

$$E\left[\begin{array}{c}\bar{V}_t \\ \bar{W}_{1t}\end{array}\right]=0,$$

and that $\left[\bar{V}_t, \bar{W}_{1t}\right]$ has variance-covariance matrix $\bar{\Omega}_1$, where

$$\bar{\Omega}_1=\left[\begin{array}{cc}\bar{\Omega}_{vv} & \bar{\Omega}_{vw_1} \\ \bar{\Omega}_{w_1v} & \bar{\Omega}_{w_1w_1}\end{array}\right].$$

Our approach treats W_{1t} as an additional set of fundamental shocks.[6]

4 Equivalent representations of the solution

When the original model is determinate and the equivalent model indeterminate, our solution algorithm generates different but equivalent VAR(1) representations of the same data generating process. Typically, the matrix Γ^* has zero columns that correspond to elements of X_{t-1} that are redundant in determining X_t.

Let Equation (20) represent the reduced form of a model that has a unique equilibrium,

$$\begin{aligned} Y_t &= \Gamma_{11}^* X_{t-1}+C_1^*+\Psi_v^* V_t, \\ E_t\left[Y_{t+1}\right] &= C_2^*+M^* Y_t. \end{aligned} \tag{20}$$

We assume that the econometrician identifies an equivalent model that has an indeterminate equilibrium and we write the reduced form of this model

[6] Our algorithm `FindEquiv` solves for the reduced form impact matrix Ψ_v^* for the true model and a pair of reduced form impact matrices $\bar{\Psi}_v^*, \bar{\Psi}_w^*$ for the equivalent model. When the model is indeterminate of degree r the algorithm chooses a probability model for W_t in which the first r elements of W_t are treated as fundamentals. We achieve this by setting

$$\underset{2l \times l}{\bar{\Psi}_w^*}=\left[\begin{array}{cc}\underset{2l \times r}{\bar{\Psi}_{w1}^*} & \underset{2l \times(l-r)}{0}\end{array}\right].$$

The algorithm solves the equations

$$\Psi_v^* I \Psi_v^{*\prime}=\left[\begin{array}{cc}\bar{\Psi}_v^* & \bar{\Psi}_w^*\end{array}\right] \bar{\Omega}\left[\begin{array}{c}\bar{\Psi}_v^{*\prime} \\ \bar{\Psi}_w^{*\prime}\end{array}\right]=\left[\begin{array}{cc}\bar{\Psi}_v^* & \bar{\Psi}_{w1}^*\end{array}\right] \bar{\Omega}_1\left[\begin{array}{c}\bar{\Psi}_v^{*\prime} \\ \bar{\Psi}_{w1}^{*\prime}\end{array}\right],$$

to find a matrix $\bar{\Omega}_1$ for which errors to the reduced form of the true model and the equivalent model have the same variance-covariance structure.

as follows;

$$\begin{aligned} X_{1t} &= \bar{\Gamma}_{11}^{*} X_{1t-1} + \bar{C}_1^{*} + \bar{\Psi}_v^{*} \bar{V}_t + \bar{\Psi}_w^{*} \bar{W}_t, \\ X_{2t} &= \bar{C}_2^{*} + \bar{M}^{*} X_{1t}. \end{aligned} \tag{21}$$

The algorithm we use to generate an equivalent model does not always choose a representation of the reduced form for which $X_{1t} = Y_t$. To establish observational equivalence we use a second algorithm, implemented in Matlab as `convert`, to rewrite the equivalent model using Y_t as the state variables. This leads to the representation

$$\begin{aligned} Y_t &= \bar{\Gamma}_{11}^{*} X_{1t-1} + \bar{C}_1^{*} + \bar{\Psi}_v^{*} \bar{V}_t + \bar{\Psi}_w^{*} \bar{W}_t, \\ E_t \left[Y_{t+1} \right] &= \bar{C}_2^{*} + \bar{M}^{*} X_{1t}. \end{aligned} \tag{22}$$

To check observational equivalence of the true model and the equivalent model one must make sure that in any given example,

$$\begin{aligned} \Gamma_{11}^{*} &= \bar{\Gamma}_{11}^{*}, \;\; C_1^{*} = \bar{C}_1^{*}, \\ C_2^{*} &= \bar{C}_2^{*}, \;\; M^{*} = \bar{M}^{*}. \end{aligned}$$

The solution algorithm `FindEquiv` generates a matrix $\bar{\Omega}$ such that

$$\Psi_v^{*} I_l \Psi_v^{*\prime} = \left[\begin{array}{cc} \bar{\Psi}_v^{*} & \bar{\Psi}_w^{*} \end{array} \right] \bar{\Omega} \left[\begin{array}{cc} \bar{\Psi}_v^{*} & \bar{\Psi}_w^{*} \end{array} \right]'.$$

This equality implies that the reduced forms of the two systems are observationally equivalent when the DGP is driven by shocks V_t with covariance matrix I_l and the equivalent system is driven by shocks $\left[\bar{V}_t, \bar{W}_t\right]$ with covariance matrix $\bar{\Omega}$.

5 Identification and the Monetary Transmission Mechanism

In this section we provide examples of two competing theories of the monetary transmission mechanism and we show that the two theories are observationally equivalent. Unlike previous examples of observational equivalence of the kind discussed by Sargent ([23]) the models we present in this section have different determinacy properties.[7]

[7] The code used to generate the example in this section is available at: http://farmer.sscnet.ucla.edu/

5.1 Two Alternative Models

Our first model is based on a New-Keynesian theory of aggregate supply. In this theory money has real effects because some agents are unable to adjust prices in every period. Our second model is based on the theory of aggregate supply outlined in Benhabib-Farmer [1]. In this theory money has real effects either because it is useful in production or because real balances influence labor supply.

The following equations represent a parameterized version of a three-equation version of the New-Keynesian model.

$$y_t + a_{13}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + f_{11}E_t\left[y_{t+1}\right] = b_{11}y_{t-1} + c_1 + v_{1t}, \tag{23}$$

$$a_{21}y_t + \pi_t + f_{22}E_t\left[\pi_{t+1}\right] = c_2 + b_{22}\pi_{t-1} + v_{2t}, \tag{24}$$

$$i_t + f_{32}E_t\left[\pi_{t+1}\right] = b_{33}i_{t-1} + c_3 + v_{3t}. \tag{25}$$

In our notation $[a_{ij}], [f_{ij}]$, and $[b_{ij}]$ represent the coefficient of variable j in equation i on contemporaneous endogenous variables, expected future variables and lagged endogenous variables. y_t is the output gap, i_t is the Fed. funds rate, π_t is inflation and v_{1t}, v_{2t} and v_{3t} are fundamental shocks to the equations of the model. c_i is the constant in Equation i.

Equation (23) is an "optimizing IS curve", Equation (24) is a New-Keynesian Phillips curve and (25) is a central bank reaction function. A model of this kind has been widely used to model the inflation process in a closed economy (Clarida et. al. [8], Galí-Gertler [12], Lindé [14], [15], Rotemberg and Woodford [22]) and a modified version of the model has been used to study inflation dynamics in open economies (Clarida et. al [9]).

To parameterize the 'true model' we chose parameters similar to those that have been estimated by Lubik and Schorfheide [17], Ireland [13] and Beyer et. al. [6]. Beyer et. al. provide a detailed discussion of the properties of this model under alternative estimation schemes and Beyer and Farmer [5] derive the implications of the restricted estimates for impulse responses to alternative shocks. Table 1 contains our specification of the New Keynesian DGP.

Table 1: Parameters of the NK Data Generation Process

Euler equation normalized for y_t

Var. Name	$i_t - E_t[\pi_{t+1}]$ a_{13}	$E_t[y_{t+1}]$ f_{11}	y_{t-1} b_{11}	constant c_1
	0.05	-0.5	0.50	0.0015

Phillips curve normalized for π_t

Var. Name	y_t a_{21}	$E_t[\pi_{t+1}]$ f_{22}	π_{t-1} b_{22}	constant c_2
	-0.5	-0.8	0.25	-0.0010

Policy rule normalized for i_t

Var. Name	y_t a_{31}	$E_t[\pi_{t+1}]$ f_{32}	i_{t-1} b_{33}	constant c_3
	-0.5	-1.1	0.8	-0.012

Our alternative model, based on Benhabib and Farmer ([1]), is represented in Equations (26)–(28). Equation (26) is identical to the optimizing IS curve in the New-Keynesian model; Equations (27) and (28) are different from their New-Keynesian counterparts.

$$y_t + a_{13}(i_t - E_t[\pi_{t+1}]) + f_{11}E_t[y_{t+1}] = b_{11}y_{t-1} + c_1 + \bar{v}_{1t}, \tag{26}$$

$$y_t + \bar{a}_{23}i_t = \bar{b}_{21}y_{t-1} + \bar{b}_{23}i_{t-1} + \bar{c}_2 + \bar{v}_{2t}, \tag{27}$$

$$\bar{a}_{31}y_t + \bar{a}_{32}\pi_t + i_t = \bar{b}_{33}i_{t-1} + \bar{c}_3 + \bar{v}_{3t}. \tag{28}$$

Equation (27) is the Benhabib-Farmer theory of aggregate supply by which a higher value of the nominal interest rate causes firms and households to economize on real balances. Since real balances are productive inputs to the real economy a reduction in real balances causes a loss of output. Benhabib and Farmer provide a theory that explains how this effect can be large even when the share of resources attributed to money as a productive asset is small. We have allowed for a propagation mechanism in this equation by including the lagged output gap and lagged nominal interest rate as additional variables.

Equation (28) is the policy rule. This differs from our New-Keynesian representation of policy in one respect; we have assumed that the Fed. responds to current inflation instead of to expected future inflation. This variation is important since we are searching for a version of the Benhabib-Farmer model that is observationally equivalent to the new-Keynesian model. The

Benhabib-Farmer aggregate supply curve does not depend on inflation and, since inflation appears contemporaneously in the New-Keynesian model, the Benhabib-Farmer model must introduce this variable elsewhere in the system if the two structural models are to have the same reduced form.

To identify the alternative model we computed the reduced form parameters $\phi(\theta)$ implied by our structural estimates of the New-Keynesian model and we imposed a set of 18 linear restrictions, six for each equation of the Benhabib-Farmer model.

Table 2: Equivalent Parameters of the Benhabib-Farmer Model

Euler equation, normalized for y_t

Var.	$i_t - E_t[\pi_{t+1}]$	$E_t[y_{t+1}]$	y_{t-1}	constant
Name	a_{13}	f_{11}	b_{11}	c_1
	0.05	-0.5	0.50	0.0015

Supply curve normalized for y_t

Var.	i_t	y_{t-1}	i_{t-1}	constant
Name	$\bar{a}_{23}$	$\bar{b}_{21}$	$\bar{b}_{23}$	$\bar{c}_2$
	0.09	0.74	-0.04	0.0064

Policy rule normalized for i_t

Var.	y_t	π_t	i_{t-1}	constant
Name	$\bar{a}_{31}$	$\bar{a}_{32}$	$\bar{b}_{33}$	$\bar{c}_3$
	-1.02	-0.26	0.63	0.0132

Table 2 reports the values of the structural parameters of the equivalent model that lead to the same reduced form as the New-Keynesian model. The most important finding of this analysis is that the equivalent model is indeterminate and may be driven, in part, by sunspot shocks.

The true New-Keynesian model has the reduced form

$$X_t = \Gamma^* X_{t-1} + C^* + \Psi_v^* V_t$$

where $X_t = (Y_t, E_{t-1}[Y_t])$ whereas the equivalent Benhabib-Farmer model has a reduced form

$$X_t = \Gamma^* X_{t-1} + C^* + \bar{\Psi}_v^* \bar{V}_t + \bar{\Psi}_{w1}^* \bar{W}_{1t}.$$

We checked that reduced form parameters $\{\Gamma^*(\theta), C^*(\theta)\}$ are indeed equal to those of the equivalent model, $\{\Gamma^*(\bar{\theta}), C^*(\bar{\theta})\}$ and we computed a variance-covariance matrix $\bar{\Omega}_1$ such that

$$\Psi_v^* I_l \Psi_v^{*\prime} = \begin{bmatrix} \bar{\Psi}_v^* & \bar{\Psi}_{w1}^* \end{bmatrix} \bar{\Omega}_1 \begin{bmatrix} \bar{\Psi}_v^* & \bar{\Psi}_{w1}^* \end{bmatrix}'$$

the shocks $(\bar{V}_t, \bar{W}_{1t})$ that drive the two models are observationally equivalent.

5.2 Comparative Dynamics of the Two Models

Table 3 presents a comparison of the generalized eigenvalues of the true model and the equivalent model arranged in descending order of absolute value. Stable roots are in boldface. The true model has three unstable roots leading to a unique determinate equilibrium. The equivalent model has the same three stable roots as the true model but one of the unstable roots is replaced by a generalized eigenvalue of zero.

Table 3: A Comparison of the Roots of the Two Models

Roots in Descending Order by Absolute Value	1st	2nd	3rd	4th	5th	6th
True Model	∞	1.39	1.39	**0.33**	**0.62**	**0.62**
Equivalent Model	∞	∞	**0**	**0.33**	**0.62**	**0.62**

The occurrence of an extra zero eigenvalue in the equivalent model implies that there is one degree of indeterminacy in the way the system responds to fundamental shocks. In any given period, contemporaneous fluctuations in output, the interest rate and inflation might in part be due to self-fulfilling beliefs.

5.3 Policy Implications of Observational Equivalence

A number of authors have taken up the issue of optimal policy in the new-Keynesian model. Michael Woodford [26] has argued that the central bank should strive to implement a policy that leads to a unique determinate rational expectations equilibrium since, if policy admits the possibility of indeterminacy, non-fundamental shocks may contribute to the variance of inflation and unemployment. This consideration suggests that a policy maker that dislikes variance should pick a policy rule that leads to a determinate equilibrium.

In a simple version of the new-Keynesian model equilibrium is determinate if the central bank responds to expected inflation by increasing the real interest rate and it is indeterminate if it responds by lowering it. In the former case, the central bank increases the nominal interest rate by more than one-for-one if it expects additional future inflation; a policy with this property is said to be *active*. In the latter case the central bank increases the interest rate by less than one-for-one if it expects additional inflation and in this case the policy is said to be *passive*.

In contrast, in simple a version of the Benhabib-Farmer model, equilibrium is determinate when the Fed follows a passive monetary policy. Our work suggests that an econometrician, by observing data from a period in which policy followed a stable rule, cannot tell whether the policy followed by the Fed led to a determinate or an indeterminate equilibrium.

Lubik and Schorfheide [17] provide an algorithm that, they claim, *can* discriminate between data that is generated by determinate and indeterminate models. It would seem that their work offers the policy maker the possibility of deciding what kind of rule to pick, based on observing past data. Our example sheds doubt on this claim. We have provided examples of two microfounded models of the data that have the same likelihood. One of these models is represented by a determinate equilibrium; the other is indeterminate. Lubik and Schorfheide's procedure will work once one has identified a given regime but, as Pesaran pointed out in his 1987 book [21], identification requires a priori restrictions on the lag length may be difficult to justify.

6 Conclusions

To summarize, this paper is about identification in linear rational expectations models. We provide an algorithm, implemented in Matlab, that generates equivalence classes of exactly identified models. This algorithm operates in three steps. First, the user specifies a "true" structural model, or Data Generating Process. Second, the algorithm is used to calculate the parameters of a reduced form: these parameters are functions of the parameters of the structural model. Third, the user specifies an alternative economic theory in the form of a set of linear restrictions. The linear restrictions, in combination with the reduced form parameters, allow the user to generate an equivalent structural model which is observationally equivalent to the true DGP.

Observational equivalence is not a new concept in the rational expectations literature. However, we provided an example based on the new-Keynesian theory of the monetary transmission mechanism in which the true model and the equivalent model have different determinacy properties. In our example we establish an equivalence between a class of models proposed by Benhabib and Farmer [1] and the standard new-Keynesian model. This we believe is a new and disturbing result since equilibria in the Benhabib-Farmer model are typically indeterminate for a class of policy rules that generate determinate outcomes in the new-Keynesian model.

References

[1] Benhabib, Jess and Roger E. A. Farmer, (2000). "The Monetary Transmission Mechanism" *Review of Economic Dynamics* Vol. 3,523–550.

[2] Beyer, Andreas and Roger E. A. Farmer (2003a). "Identifying the Monetary Transmission Mechanism Using Structural Breaks", European Central Bank working paper #275.

[3] ____(2003b). "On the Indeterminacy of Determinacy and Indeterminacy", European Central Bank working paper #277.

[4] ____(2004a). "On the Indeterminacy of New-Keynesian Economics", European Central Bank working paper #323.

[5] ____ (2004b). "Measuring the Effects of Real and Monetary Shocks in a Structural New-Keynesian Model". UCLA mimeo.

[6] Beyer, Andreas, Roger E. A. Farmer, Jérôme Henry and Massimiliano Marcellino (2004). "Factor Analysis in a New Keynesian Model" UCLA mimeo.

[7] Boivin, J. and M. Giannoni (2003), "Has monetary policy become more effective", *NBER Working Paper #9459.*

[8] Clarida, R., Galí, J. and M. Gertler, (2000). "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", *Quarterly Journal of Economics*, 115 (1), pp.147-180.

[9] ________________________________, (2002). "A Simple Framework for International Monetary Policy Analysis", *Journal of Monetary Economics*, 49(5), pp. 879–904.

[10] Fisher, F. (1966), *The Identification Problem in Econometrics.* New York: McGraw-Hill.

[11] Fuhrer, Jeffrey C., and Glenn D. Rudebusch (2003). "Estimating the Euler Equation for Output"", *Journal of Monetary Economics*, Volume 51, Issue 6, September 2004, Pages 1133-1153 .

[12] Galí, Jordi and Mark Gertler, (1999) "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics.*

[13] Ireland, Peter, (2002). "Money's Role in the Monetary Business Cycle", mimeo Boston College.

[14] Lindé, Jesper, (2001). "The Empirical Relevance of Simple Forward- and Backward-looking Models: A View from a Dynamic General Equilibrium Model", Sveriges Riksbank (Central Bank of Sweden) Working paper #130.

[15] ___________________"Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach", Sveriges Riksbank (Central Bank of Sweden) Working paper #129.

[16] Lubik, Thomas A. and Frank Schorfheide. (2002a). "Computing Sunspot Equilibria in Linear Rational Expectations Models," mimeo, University of Pennsylvania.

[17] __________________________________ (2002b). "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review* , forthcoming.

[18] Mavroeidis, Sophocles, (2002). "Identification and Mis-Specification Issues in Forward Looking Models", Department of Quantitative Economics, University of Amsterdam, Discussion paper 2002/21.

[19] McCallum, Bennett, (1976). "Rational Expectations and the Estimation of Econometric Models: An Alternative Procedure" *International Economic Review*, 1976, 17(2) , pp 484-90.

[20] Nason, James M. and Gregor W. Smith, (2003). "Identifying the New Keynesian Phillips Curve". Mimeo, Federal Reserve Bank of Atlanta.

[21] Pesaran, Hashem, (1987). *The Limits to Rational Expectations* Basil Blackwell, Oxford.

[22] Rotemberg, J.J and M. Woodford (1998), "An optimization-based econometric framework for the evaluation of monetary policy: expanded version", NBER Technical Working Paper 233.

[23] Sargent, Thomas, (1976). "The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics", *Journal of Political Economy*, Vol 84 (3) pp 631-640.

[24] Sims, Christopher A., (1980) "Macroeconomics and Reality," Econometrica, 48, 1, pp. 1-48.

[25] Sims, Christopher, A. (1996). "Solving Linear Rational Expectations Models". Princeton University mimeo.

[26] Woodford, Michael, (2003). *Interest and Prices* Princeton University Press.

Appendix A

This section defines the matrices H, h, J and j from Equations (17) and (19), Section 3.

$$\underset{l(3l+1)\times l(5l+1)}{\overset{H}{\left(\underset{l\times l}{[I_l]} \otimes \underset{(3l+1)\times(5l+1)}{\begin{bmatrix} \underset{((3l+1)\times 2l)}{\begin{matrix} \underset{2l\times 2l}{\Gamma^{*\,\prime}} \\ \underset{1\times 2l}{C^{*\,\prime}} \\ \underset{l\times 2l}{\Psi_v^{*\,\prime}} \end{matrix}} & \vdots & \underset{(3l+1)\times(3l+1)}{-I_{(3l+1)}} \end{bmatrix}} \right)}} \underset{l(5l+1)\times 1}{\overset{\bar{\theta}}{\left(vec \underset{(5l+1)\times l}{\begin{bmatrix} \underset{l\times l}{A^{\prime}} \\ \underset{l\times l}{F^{\prime}} \\ \underset{l\times l}{B_1^{\prime}} \\ \underset{l\times l}{B_2^{\prime}} \\ \underset{1\times l}{C^{\prime}} \\ \underset{l\times l}{\Psi_V^{\prime}} \end{bmatrix}} \right)}} = \underset{l(3l+1)\times 1}{\overset{h}{[0]}}, \qquad \text{(A1)}$$

$$\left(\underset{l(5l+1)\times l(5l+1)}{\begin{bmatrix} \underset{l(3l+1)\times l(5l+1)}{\left(\underset{l\times l}{\begin{bmatrix} I_l \end{bmatrix}} \otimes \underset{(3l+1)\times(5l+1)}{\left(\begin{bmatrix} \underset{((3l+1)\times 2l)}{\begin{matrix} \underset{2l\times 2l}{\Gamma^{*\,\prime}} \\ \underset{1\times 2l}{C^{*\,\prime}} \\ \underset{l\times 2l}{\Psi_v^{*\,\prime}} \end{matrix}} & \vdots - I_{(3l+1)} \end{bmatrix} \right)} \right)} \\ \underset{l(2l)\times l(5l+1)}{R} \end{bmatrix}} \right) \left(vec \underset{(5l+1)\times l}{\begin{bmatrix} \underset{l\times l}{\bar{A}^{\prime}} \\ \underset{l\times l}{\bar{F}^{\prime}} \\ \underset{l\times l}{\bar{B}_1^{\prime}} \\ \underset{l\times l}{\bar{B}_2^{\prime}} \\ \underset{1\times l}{\bar{C}^{\prime}} \\ \underset{l\times l}{\bar{\Psi}_v^{\prime}} \end{bmatrix}} \right) = \underset{l(5l+1)\times 1}{\begin{pmatrix} \underset{l(3l+1)\times 1}{[0]} \\ \underset{l(2l)\times 1}{[r]} \end{pmatrix}} \qquad \text{(A2)}$$