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MARKET SOLUTIONS TO THE PROBLEM OF STABILISING COMMODITY EARNINGS

Andrew Hughes Hallet and Prathap Ramanujam

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Centre for Economic Policy Research
6 Duke of York Street
London SW1Y 6LA
Tel: (44 71) 930 2963

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ABSTRACT

Market Solutions to the Problem of Stabilising Commodity Earnings

This paper examines and compares the effectiveness of hedging and buffer-stock strategies for stabilizing the revenues of individual producers who face different supply conditions in a market with uncertainty about prices and output. The results are obtained in a model-free framework, with arbitrary probability distributions and potentially inefficient markets. This approach avoids any dependence on particular parameter estimates, specific distributional assumptions, or restrictive assumptions about the market structure and information patterns, The analysis examines how far storage and transaction costs limit our ability to stabilize commodity earnings, to what extent the stabilization strategy followed by one producer would conflict with the strategies chosen by other producers in the same market, and whether those conflicts could destabilize the revenues of some producers. The results are illustrated with data from five primary commodity markets.

JEL classification: 130, 700
Keywords: commodity price, stabilization, hedging, bufferstocks

Andrew Hughes Hallett
Department of Economics
University of Strathclyde
Curran Building
100 Cathedral Street
Glasgow
G4 0NL
Tel: (44 41) 552 4400

Prathap Ramanujam
Ministry of Policy,
Planning and Implementation
Sethsiripaya
Battaramulla
Sri Lanka

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NON-TECHNICAL SUMMARY

Both hedging and stockpiling provide producers with an opportunity to reduce the risks associated with price and production uncertainties. Most of the literature on this topic has been concerned with the welfare justification for, and the consequences of, market stabilization. But there is the equally important problem of making the welfare analysis operational by determining *how* to stabilize earnings most effectively in a market where prices are volatile and production uncertain, once the welfare case for doing so has been established. The obvious application is to the stabilization of export earnings of heavily indebted developing countries that are crucially dependent on earnings from one or two commodity markets. Our results are very general, however, and can equally be applied to any market with price and output uncertainties.

Optimal hedging rules for a range of different objective functions, including that of minimizing the variance of earnings, have been derived by McKinnon (1967), Anderson and Danthine (1983), Newbery (1988) and Karp (1988). Stockpiling rules have been analysed by Nguyen (1980), Newbery and Stiglitz (1981), and Ghosh et al. (1987). All these studies, however, incorporate some crucial, but implausible, assumptions: typically, that prices and production are jointly normally distributed, that there is no private stockholding, that markets are unbiased and risks diversifiable, and that unlimited resources are available for stabilizing activities. Many authors have pointed out that stockholding does occur in practice. Together with financial constraints, stockpiling would generate the asymmetries and kurtosis in the price and quantity distributions that we observe in practice. Similarly, commodity producers do face significant risks but they operate predominantly in economies where there are few opportunities to diversify those risks by investing in other assets.

To deal with these criticisms of earlier analyses, we have recently developed hedging and stockpiling rules designed to minimize the variance of producers' earnings in a framework that is free of strong assumptions concerning probability distributions and model structures (Hughes Hallett and Ramanujam, 1990). In principle, those rules could be applied in any market, whatever the price distribution, and without relying on econometric estimates of the main supply and demand elasticities. They are, however, still subject to two important restrictions. First the stockpiling rules have been developed and tested only for producers taken as a group, whereas under hedging each producer can operate his own rule. The problem is that buffer-stock interventions designed to minimize the variance of earnings for the market as a whole depend on a single intervention parameter. Unless every producer faces exactly the same supply distribution, the rule that is optimal for the market as a whole will not be optimal for individual producers (a point made by Gemmill, 1985). The first purpose of this paper is to extend the analysis to the case of individual producers who face different supply conditions.

In addition, the hedging and stockpiling rules we derived in earlier work do not take account of operating costs and so are designed to minimize the variability of gross revenues. This may be a reasonable assumption if the stabilization activities are being funded by international agencies, such as UNCTAD, on behalf of the poorer and indebted LDC commodity producers, but it is clearly unreasonable at the level of individual producers. The second purpose of this paper is therefore to examine and compare hedging and stockpiling strategies' revenues *net* of storage and transactions costs.

Three restrictions remain in our analysis, however. We continue to assume market invariance; i.e., that the act of stabilizing revenues does not change the behaviour of market agents and traders. This assumption only matters to the extent that these changes in behaviour affect the first four joint or marginal moments of the bivariate price-quantity distributions. Second, we continue to take the long-run equilibrium price as the price level that stockpiling rules attempt to maintain. Third, all our conclusions depend on the assumption that earnings stabilization is the policy objective.

Applying our techniques to the five most volatile markets in the United Nation's list of 29 'core commodities', which might be candidates for stabilization agreements, we find that the optimal stabilization rules do vary between markets, but that they vary more between producers within one of those markets. In two out of five cases these differences result in disagreements over the type of strategy; the rest of the markets show disagreements over the degree of intervention that would be desirable. Fortunately, however, the losses from choosing the 'wrong' strategy do not appear to be very large; so it is better to do something rather than nothing, even if the distribution of the benefits remains a problem. Accounting for storage and transactions costs can, on the other hand, make a big difference. The degree of earnings stability achieved is likely to be severely reduced in markets with significant price volatility and with high unit values and/or strong price elasticities. In markets with those characteristics, producers will not be impressed by the scope for stabilization, and given the costs of running out of stock altogether they tend to switch to a hedging strategy if they have to pay their own costs. Elsewhere, costs make little difference.

One consequence of dealing with the stabilization problem faced by individual producers is that the strategy which each producer would choose for himself will not be optimal for the other producers (a problem of non-cooperation) or for the market as a whole (a problem of centralized planning) unless, of course, all producers face identical supply conditions. Conflicts may therefore arise about which strategy, hedging or buffer-stock interventions should be adopted, and about how vigorous the stabilization operations should be. If those disagreements among producers or between producers and a market authority are strong enough, it will be hard to operate any earnings-stabilization scheme. There is the additional problem that one producer's stabilization rule, or a market stabilization rule, may have the effect of destabilizing the earnings of another producer.

It turns out that the potential for disagreement is less severe in the hedging case since the use of a privately-optimal rule by one producer does not preclude the use of a different rule by another. But the associated earnings streams are correlated, so although conflicts can still arise they are not found to be serious. Under buffer-stock schemes, the potential losses are larger and may lead to one producer destabilizing the earnings of another; or, more likely, to a market-wide stabilization scheme destabilizing the earnings of one or more participants. In that sense centralized planning appears to be less helpful, explicit cooperation between private producers more helpful. Nevertheless, conflicts can be damaging when there is great diversity in the supply conditions facing producers and when there are large differences between the gains that can be realized from hedging rather than from the use of buffer stocks.

1. INTRODUCTION

Both hedging and stockpiling provide producers with an opportunity to reduce the risks associated with price and production uncertainties. Most of the literature on this topic has been concerned with the welfare justification for, and the consequences of, market stabilization. But there is the equally important problem of how to stabilize earnings most effectively in a market where prices are volatile and production uncertain, once the welfare case for doing so has been established. That makes the welfare analysis operational.

Optimal hedging rules for a range of different objective functions, including minimizing the variance of earnings, have been derived by McKinnon (1967), Anderson and Danthine (1983), Newbery (1988) and Karp (1988). Stockpiling rules have been analyzed by Nguyen (1980), Newbery and Stiglitz (1981), and Ghosh et al. (1987). All these studies incorporate some crucial but implausible assumptions; typically that prices and production are jointly normally distributed, that there is no private stockholding, that markets are unbiased and risks diversifiable, and that resources for stabilizing activities are not constrained. Many authors[1] have pointed out that stockholding does in fact occur, and that that, coupled with financial constraints, would imply the kind of asymmetries and kurtosis in the price and quantity distributions which we observe in practice. Similarly commodity producers do face significant risks but they operate predominantly in economies where there are few opportunities to diversify those

[1] Wright and Williams (1982), Hughes Hallett (1986).

risks onto other assets.

To deal with these criticisms we have recently developed hedging and stockpiling rules which minimize the variance of producers' earnings in a *distribution free* and *model free* set up (Hughes Hallett and Ramanujam, 1990). In principle those rules could be applied in any market, whatever the price distribution and without relying on econometric estimates of the associated supply and demand elasticities. They are, however, still subject to two important restrictions. First the stockpiling rules have been developed and tested only for producers taken as a group, whereas under hedging each producer can operate his own rule. The problem is that buffer stock interventions designed to minimize the variance of earnings for the market as a whole will depend on a single intervention parameter so, unless every producer faces exactly the same supply distribution, what is optimal for the market as a whole will not be optimal for individual producers (a point made by Gemmill, 1985). Our analysis therefore needs to be extended to the case where individual producers who face different supply conditions. That is the first purpose of this paper.

The second restriction is that these hedging and stockpiling rules do not account for their operating costs. They are therefore designed to minimize the variability of gross revenues. That may be reasonable if the stabilization activities are being funded by international agencies, such as UNCTAD for example, on behalf of the poorer and indebted LDC commodity producers. But it is clearly unreasonable at the level of individual producers. The second purpose of this paper is therefore to examine and compare hedging and stockpiling strategies for stabilizing revenues *net* of storage and transaction costs.

Three restrictions remain in our analysis, however. We continue to assume market invariance; i.e. that the act of stabilizing revenues does not change the behaviour of market agents and traders. However, this assumption only matters in as much as those changes in behaviour might affect the first four moments of the bivariate price-quantity distributions. It does not matter if the parameters of the underlying demand, supply or stockholding rules are changed (as happens in Turnovsky's (1983) model where elasticities depend on the variance of prices) so long as the *net* impact on those moments is negligible. Secondly, we continue to take the mean pre-stabilization price as the price level which stockpiling rules attempt to maintain. Formally that requires linear supply and demand functions but, as Gilbert (1986) points out, a relaxation of that specification would only lead to a redistribution of revenues between producers and consumers rather than to alterations in the "stabilizability" of the market. Since distributional questions are not our concern here, this restriction is not important to our results. Thirdly, all our conclusions are dependent on earnings stabilization as an objective; they might not survive in a fuller welfare analysis. However earnings are quadratic in prices and quantities, and Levy and Markowitz (1979) have demonstrated that the mean and variance of a quadratic objective is a very robust approximation to the general expected utility criterion of risk averse decision makers. So that restriction is also unlikely to be important. That said, we make no attempt to establish the welfare case for stabilization in this paper. It is simply assumed that the necessary welfare evaluations have been made and that producers now wish to determine the best way to achieve the earnings stability that they need.

Finally, earnings stability is defined in terms of the variance of random fluctuations about mean or trend values. Hence our hedging or stabilization rules are designed to "buy" predictability, rather than to induce stability in the sense of modifying the dynamics of the earnings stream. Similarly the question of whether the underlying prices are statistically stationary, stationary with discrete jumps, or trending, plays no role here since we are concerned with random fluctuations about some known trend or cycle.

2. NONPARAMETRIC STABILIZATION RULES

Markets may be stabilized using financial instruments or with buffer stock interventions. The welfare objection to using buffer stock interventions is that agents can achieve the same results more easily by taking appropriate positions in competitive futures markets. The cost of maintaining the buffer stock itself would also be saved. The difficulty with that argument is that a complete set of contingency markets does not exist for most commodity and financial markets and, in some cases, no such markets exist. In addition, many producers or producing countries are either too small or too indebted to secure the credit needed to adopt the necessary positions.

For these reasons it is important to determine which strategy is most effective for reducing earnings risk. If it is hedging, then policy needs to be directed at providing access to contingency markets. If that is not possible, compensatory finance, income support or buffer stock schemes aimed at replicating the hedging outcomes might be introduced – although that will be successful only if output fluctuations are fairly small (Hughes Hallett and Ramanujam, 1990). If however price stabilization is the better strategy we must evaluate (and cost) the buffer stocks needed. Producers therefore need to develop and compare optimal hedging and buffer stock strategies in order to determine their best revenue stabilization strategies.[2]

2.1 Operating on the Futures Markets: Optimal Hedging Strategies

Consider the j-th producer, j=1...n, who can produce a quantity q_j of some commodity which he can sell at a spot price p. However it takes some time to complete production and the final supply is subject to various shocks. In period 0 he plans to produce $\bar{q}_j$, but the actual output q_j is a random variable with mean $\bar{q}_j$ and variance σ_j^2. Similarly the, as yet unknown, spot price p is a random variable with mean $\bar{p}$ and variance σ_p^2. Nothing else is assumed about the joint probability distribution of (p,q_j).

As an alternative to receiving an uncertain revenue of $y_j=pq_j$, the producer can hedge on the futures market by selling (in period 0) a fixed quantity h_j for delivery in period 1 at a known price p_f. If he does that he will be obliged to sell his unhedged output q_j-h_j (including the supply shock $q_j-\bar{q}_j$) at the spot price (including the price surprise $p-\bar{p}$). His hedged revenue is therefore

$$y_j^h = p(q_j-h_j) + p_f h_j = pq_j + (p_f-p)h_j \tag{1}$$

If, in addition, the market is efficient so that forward prices are an unbiased predictor of future spot prices (i.e. $E(p_f)=\bar{p}$), then

$$E(y_j^h) = E(pq_j) = E(y_j^o) \tag{2}$$

where y_j^o denotes producer j's unhedged income. Hence hedging on an unbiased market leaves average revenues unchanged. Direct calculation now yields

$$V(y_j^h) = E(p^2q_j^2) - \left[E(pq_j)\right]^2 - 2h_j(\bar{p}\mu_{11}^j+\bar{q}_j\sigma_p^2+\mu_{21}^j) + h_j^2\sigma_p^2 \tag{3}$$

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[2] Some comparisons have been attempted before but only for aggregated producers where interventions are costless, where prices and quantities are jointly normally distributed (so there is no private stockholding), and where risks are diversifiable and prices unbiased. On this basis McKinnon (1967), Newbery and Stiglitz (1981), Gemmill (1985) and Gilbert (1985) prefer a market approach although the empirical results are mixed and show no clear superiority for hedging. Later work (Gilbert, 1988) is more favourable to buffer stocks interventions. It is not at all clear that any of these results would extend to our more general framework, or to the level of individual producers.

where $\mu^j_{ik}=E[(p-\bar{p})^i(q_j-\bar{q}_j)^k]$, for i,k≥1, are the bivariate moments from the joint density functions of (p,q_j) for each j. We can write $V(y^o_j)$ for $E(p^2q^2_j)-[E(pq_j)]^2$, and for convenience we shall also write $\rho_j\sigma_p\sigma_j$ for μ^j_{11} where ρ_j is the correlation coefficient between p and q_j. It is usually assumed that $\rho_j<0$ on the argument that a production shock which affects one producer will affect them all. A negative shock would then lead to a fall in aggregate supply and higher prices (and vice versa if the supply shock is positive). But there is no guarantee of this since a producer who is differentiated either geographically or in production structure may not suffer the same shocks as others and, depending on the demand elasticities, may even be able to profit from a larger market share during high price periods.

From (3) the optimal hedging rule is

$$h^*_j = \rho_j\bar{p}\sigma_j/\sigma_p + \bar{q}_j + \mu^j_{21}/\sigma^2_p \tag{4}$$

which is the result obtained by McKinnon (1967). If $\rho_j<0$, the greater is the output uncertainty relative to price uncertainty, the less should be hedged. Similarly the stronger the negative correlation, the smaller is the quantity that should be hedged, but the greater the (positive) asymmetry the larger the hedge. Notice also that if output variability is large enough, h^*_j may be negative and the producer should *buy* forward. Conversely if $\rho_j>0$ and μ^j_{21} is small, $h^*_j>\bar{q}_j$. So short or long positions can result from (4). Finally, if p and q_j are symmetrically and normally distributed, then $\mu^j_{21}=\mu^j_{12}=0$. If they are independent then $\rho_j=0$ as well. Those conditions would imply

$$h^*_j = \rho_j\bar{p}\sigma_j/\sigma_p + \bar{q}_j \quad \text{and} \quad h^*_j = \bar{q}_j \text{ respectively} \tag{5}$$

Under any hedging rule, the variance of revenue can be written as

$$V(y^h_j) = V(y^o_j) + [h^2_j - 2h_h h^*_j]\sigma^2_p$$

using (3) and (4) again. Hence under the optimal hedging rule we get

$$V(y^h_j) = V(y^o_j) - h^{*2}_j\sigma^2_p \tag{6}$$

That shows an unambiguous gain, in terms of risk reduction, over doing nothing. This risk reduction increases with increasing uncertainty in prices and quantities (i.e. σ_p and σ_j), with greater skewness($\mu^j_{21}>0$), but lower association between prices and quantity (assuming $\rho_j<0$). Thus, for a given average market size $\bar{q}_j$, "smaller" or more differentiated producers may benefit more from trading on the futures markets than would large producers who can influence spot prices.

2.2 Extensions: Private Stockholding and Speculation

Almost all commodity markets display positively skewed price distributions, and this is likely to be the result of stockholding activities. If the market exhibits excess demand, that can only be met by running down stocks. Supply is inelastic in the short term and once stocks are exhausted prices will rise very rapidly. If, on the other hand, there is excess supply, agents can accumulate stocks so long as they have sufficient finance. They may not wish to accumulate stocks indefinitely, but they can certainly accumulate more easily than decumulate and they can often hold stocks "in the ground" or fail to harvest. Hence prices fall by considerably less than they rise in the excess demand case (Hughes Hallett, 1986).

The same arguments imply that stockholding will generate large bivariate third moments. For example, large negative supply shocks are likely to push market prices high above their mean value because stocks and short run supply responses are inadequate to cope with the excess demand. But a positive supply shock would push market prices below their mean by a smaller amount since stocks can absorb the excess supply. Hence stockholding will imply

$$\mu_{12} = E\left[(p-\bar{p})(q-\bar{q})^2\right] > 0 \quad \text{and} \quad \mu_{21} = E\left[(p-\bar{p})^2(q-\bar{q})\right] < 0$$

What if the market is biased or inefficient: $E(p_f)=\bar{p}+k$ where $k\neq0$? Gilbert (1988) argues that a large number of speculators in a market where price risks are diversifiable will ensure that the futures price is an unbiased predictor of the corresponding spot price. On the other hand, in a world where spot and futures price are jointly determined and where there is little or no speculative activity and risks are not easily diversified, the futures price will generally be biased (Turnovsky, 1983; Kawai, 1983). However retracing steps (3) to (6) for the biased market case yields exactly the same hedging rule and variance reduction as in the unbiased case; equations (4) and (6) can be applied just as before (Hughes Hallett and Ramanujam, 1990). Average earnings, on the other hand, are affected since

$$E(y^h_j) = E(pq_j) + kh_j \tag{7}$$

follows from (1). This deals with the case where there are few speculators in the market and price risks are not easily diversifiable. Hedging will produce a systematic gain (loss) in expected earnings if $k>0$ ($k<0$). But the same hedge and gains in stability apply whether or not markets are inefficient or biased.

2.3 Non-Market Interventions: Optimal Price Stabilization Rules

Direct price stabilization requires producers to set up a buffer stock which will buy some or all of the market's excess supply at some target price level (to prevent prices falling further) and sell when there is excess demand at that price (to prevent them rising further). Naturally the amounts to be bought or sold in that operation will depend on the elasticities of demand and supply. But, given a homogenous commodity, all producers face the same demand schedule so that the interventions which each producer needs to stabilize his revenue will vary according to his own supply conditions. Everyone will therefore wish to run his own buffer stock operations and, unless everyone faces the same supply function, private buffer stocks will not stabilize market revenue and interventions which stabilize the market as a whole will be suboptimal for individual producers.

Let producer j's net buffer stock purchase be

$$BS_j = \lambda_j(S-D) \tag{8}$$

where $S = \Sigma q_j^S$ is aggregate supply, D is total demand, and λ_j is the proportion of excess demand/supply which producer j sells or buys at the target price level in order to stabilize his revenues. Producer j will pick λ_j to minimize the variance of his own earnings.[3] Suppose that he faces demand and supply schedules given by

$$D = a - bp + u \qquad a, b > 0 \tag{9}$$

$$q_j^S = \alpha_j + \beta_j p + v_j \qquad \alpha_j, \beta_j > 0$$

Suppose also that the target price level is the expected equilibrium price, $\bar{p}$. This assumption allows us to abstract from any transfers between market participants which would follow if $\bar{p}$ (or linear supply and demand functions) were not chosen and thus ensures comparability with the hedging strategy.

Post-stabilization prices, $\tilde{p}$, satisfy $BS+D=S$ where $BS=\Sigma BS_j$ is evaluated at $\tilde{p}$. In other words

[3] $\lambda_j=0$ implies no stabilization; $\lambda_j=1$ implies perfect stabilization (although the individual producer would probably find that hard to achieve). We are at present ignoring the costs and financial commitment involved in maintaining the buffer stock.

$$\tilde{p} = \bar{p} + (1-\lambda_j)(u-v)/(\beta+b) \tag{10}$$

where $\bar{p} = (a=\alpha)/(\beta+b)$, $v = \Sigma v_j$, $\beta = \Sigma\beta_j$ and $\alpha = \Sigma\alpha_j$. Hence producer j's stabilized revenue will be

$$y_j^S = \tilde{p}q_j = pq_j - \lambda_j(p-\bar{p})q_j \tag{11}$$

Thus

$$E(y_j^S) - E(y_j^O) = -\lambda_j\rho_j\sigma_p\sigma_j \tag{12}$$

so that price stabilization will raise expected earnings above the pre-stabilization or hedging level if $\rho_j<0$ (and it will reduce average earnings if $\rho_j>0$). This is because producers can sell all their expected output at stabilized prices, whereas they can hedge only their expected output at forward prices – the remaining supply shock will just fetch the spot price. Hence, in terms of average earnings, price stabilization is the better strategy the more negatively correlated are prices with supply shocks. That will depend on demand elasticities and the market (production) structure.

Notice that there will generally be a conflict of objectives under price stabilization since $V(y_j^S)$ will be minimized at a finite value $\lambda_j\neq 0$, while (12) shows $E(y_j^S)$ increases with λ_j if $\rho_j<0$ (and vice versa if $\rho_j>0$). In fact (11) and (12) imply

$$V(y_j^S) = (1-\lambda_j)^2 V(y_j^O) + \lambda^2\bar{p}^2\sigma_j^2 + 2\lambda_j(1-\lambda_j)\bar{p}\ \mathrm{Cov}(y_j^O,q_j) \tag{13}$$

and hence that the optimal buffer stock intervention is defined by (8) with[4]

$$\lambda_j^* = \frac{V(y_j^O) - \bar{p}\ \mathrm{Cov}(y_j^O,q_j)}{V(y_j^O) + \bar{p}^2\sigma_j^2 - 2\bar{p}\ \mathrm{Cov}(y_j^O,q_j)} = \frac{A_j}{B_j} \tag{14}$$

where $B_j = \mu_{22}^j - \rho_j^2\sigma_p^2\sigma_j^2 + 2\bar{q}_j\mu_{21}^j + \bar{q}_j^2\sigma_p^2$ and $A_j = B_j + \bar{p}\mu_{12}^j + \bar{p}\bar{q}_j\rho_j\sigma_p\sigma_j$.

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[4] The second equality in (14) is obtained by rewriting (13) as

$$V(y_j^S)=V(y_j^O)+ \lambda_j^2[(V(y_j^O)+ \bar{p}^2\sigma_j^2- 2\bar{p}\mathrm{Cov}(y_j^O,q_j)] - 2\lambda_j[V(y_j^O)- \bar{p}\mathrm{Cov}(y_j^O,q_j)]$$

and then substituting in the direct evaluations of $V(y_j^O)$ and $\mathrm{Cov}(y_j^O,q_j)$; i.e.

$$V(y_j^O) = \mu_{22}^j - \rho_j^2\sigma_p^2\sigma_j^2 + 2\bar{p}\mu_{12}^j + 2\bar{q}\mu_{21}^j + \bar{p}^2\sigma_j^2 + \bar{q}_j^2\sigma_p^2 + 2\bar{p}\bar{q}_j\rho_j\sigma_p\sigma_j$$

and $\mathrm{Cov}(y_j^O,q_j) = \mu_{12}^j + \bar{p}^2\sigma_j^2 - 2\bar{p}\bar{q}_j\rho_j\sigma_p\sigma_j$, to yield

$V(y_j^S) = V(y_j^O) + \lambda_j^2B_j - 2\lambda_jA_j$ and hence (14) as stated.

Finally, reorganizing (13) as indicated in footnote 4, and inserting λ_j^* from (14) gives

$$V(y_j^s) = V(y_j^o) - A_j^2/B_j \tag{15}$$

which shows an unambiguous gain in earnings stability since we can rewrite B_j as $E[y_j^o - E(y_j^o) - \bar{p}(q_j - \bar{q}_j)]^2 > 0$. However the conventional result that partial stabilization is optimal, $0<\lambda_j^*<1$, follows only if $0<A_j<B_j$. That will happen if μ_{12}^j and/or ρ_j are weakly negative so that $\mu_{12}^j + \bar{q}_j\rho_j\sigma_p\sigma_j < 0$. A simple sufficient condition for partial stabilization to be appropriate is weakly negative second and third *joint* moments. But if those moments are positive $\lambda_j^*>1$, and if they are strongly negative $\lambda_j^*<0$. Thus, as in hedging, positive p and q_j correlations will require "over-stabilization", while very strong negative correlations call for pro-cyclical "stabilization". Only in the special case where p and q_j are symmetrically distributed *and* $\rho_j<0$, is the conventional result $0<\lambda_j^*<1$ generally valid. If, in addition, p and q_j are independent then perfect stabilization ($\lambda_j^*=1$) is best.

2.4 When is Price Stabilization Preferable to Hedging?

When do buffer stock stabilization programmes produce greater earnings stability than hedging on the futures market? That turns on whether $A_j^2/B_j > h_j^{*2}\sigma_p^2$ or not. This may be tested numerically. But these expressions take exactly the same algebraic form, in terms of the (p,q_j) distribution faced by producer j, as they did for stabilizing aggregate earnings given the market's (p,q) distribution; compare Hughes Hallett and Ramanujam (1990). Therefore we can use the same decision tree test, applied to each (p,q_j) distribution in turn. This test is given in Figure 1.

3. CONFLICTS BETWEEN MARKET AND PRIVATE STABILIZATION SCHEMES

3.1 Hedging

Hedging changes the price distribution which an individual producer faces, but not the price distribution faced by other market participants, since matching demand and supply on the futures markets will leave the spot market's excess

Compute $\lambda_h = q_f^*/\bar{q}$ using (1); $\lambda_s = \lambda^*$ using (2); plus $d = \partial[V(y^h) - V(y^s)]/\partial\lambda = \mu_{22} - \rho^2\sigma_p^2\sigma_q^2 + \bar{q}\mu_{21} + \bar{p}\mu_{12}$ and $\lambda^+ = (d + \bar{q}\mu_{21})/(d - \bar{p}\mu_{12})$. Then test as follows:

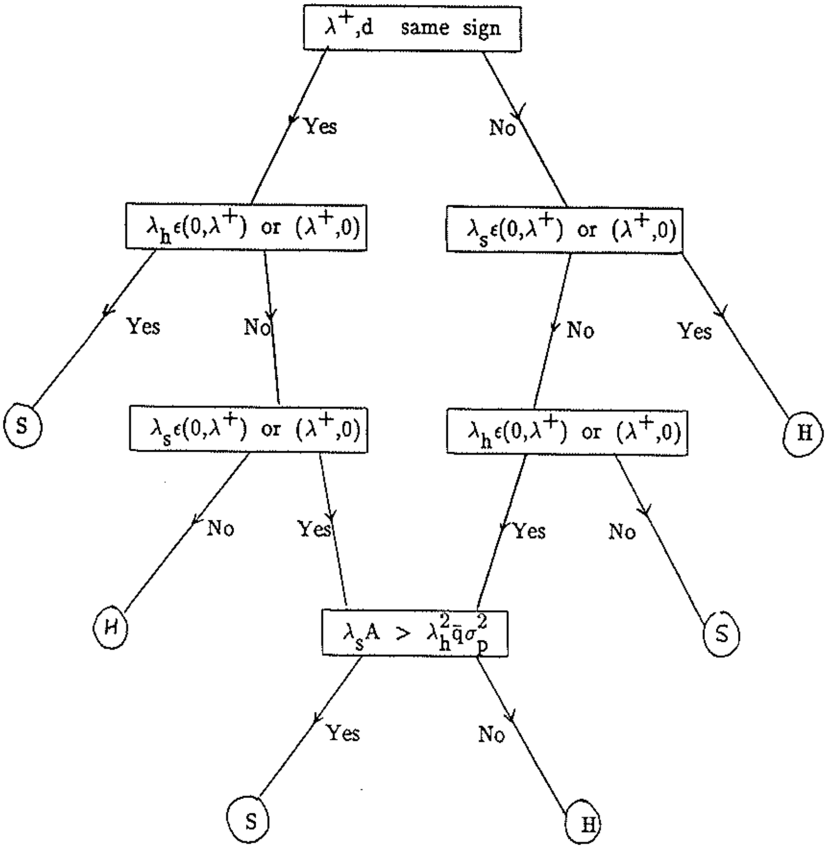

Figure 1. Decision tree

demand or supply position unchanged.[5] As a result, producer j will find no conflict between the rule which minimizes his own earnings variance and the rules which other producers may be using to minimize their earnings variance; the rules are independent in the sense that changes in producer k's rule will not affect producer j's optimal rule. Moreover, because each producer can determine his own received price distribution without influencing the price distribution of others, each rule is controlled by its own intervention parameter (h_j^* or $\lambda_j^h = h_j^*/\bar{q}_j$). Nevertheless it is easy to check that the earnings of different producers remain correlated. Hence producer j's hedging will affect producer k's earnings distribution, and $V(y_k)$ in particular. Consequently privately optimal hedging rules will not minimize the variance of market revenues as a whole unless all the supply distributions are the same. Similarly, a rule which is optimal for the market as a whole will not minimize earnings variances for individual producers. Inspection of (4) shows $\Sigma h_j^* \neq h^*$, where h^* is the optimal hedge for the market as a whole and is defined by (4) with joint moments μ_{11} and μ_{21} taken from the aggregate (p,q) distribution[6] unless $\mu_{11} = \Sigma_j \mu_{11}^j$ and $\mu_{21} = \Sigma_j \mu_{21}^j$. A simple sufficient condition for that is $\mu_{11}^j = \mu_{11}/n$ and $\mu_{21}^j = \mu_{21}/n$ for all j.

3.2 Price Stabilization

Any buffer stock intervention will change the price distribution for every producer – including those who choose not to participate in the stabilization programme. As a result the only thing that matters in a market of homogenous goods is the impact of the aggregate buffer stock; producers cannot determine their own price distributions even if they are operating private buffer stocks, and the degree of stabilization is actually a function of the aggregated interventions and a single intervention parameter.

The boot is on the other foot here. Under hedging every producer was free to minimize his own earnings variance and will not care that market revenues are not stabilized at the same time since there are no Pareto improvements to be had by doing so (either in terms of trading between variances, or in terms of trading

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[5] This follows from the market invariance assumption. The spot market's turnover will have fallen, however.

[6] See Hughes Hallett and Ramanujam (1990) for this result.

smaller variances for higher average earnings). That remains the case even if official policy is directed at opening up access to the futures markets. But under price stabilization, producers will care that interventions which minimize the earnings variance of one producer do not so so for others or for the market as a whole. They will also be concerned that the interventions which stabilize the market revenues do not do so for individual producers unless all the joint (p,q_j) distributions are identical; (14) shows $\lambda_j^* \neq \lambda_k^*$ and $\lambda_j^* \neq \lambda^*$, where $k \neq j$ and λ^* minimizes $V(y)$ for[7] $y=\Sigma y_j$. In this case the conflicts between private and aggregate stabilization rules do matter and it is not clear how policy should attempt to resolve those conflicts. For this reason we focus on analyzing the differences between buffer stock schemes and their policy implications in what follows.

3.3 Strategic Errors

The possibility of conflicts between aggregate and private stabilization schemes raises three important questions:

a) When does $\lambda_j^*=\lambda_k^*$ or λ^*? Those are the circumstances in which producer j's private actions will have the effect of stabilizing producer k's earnings at the same time, and also the circumstances in which individual producers will find that a buffer stock used to stabilize market revenues will maximize their own earnings stability.

   Sufficient conditions[8] for $\lambda_j^*=\lambda_k^*$ are $\mu_{i\ell}^j=\mu_{i\ell}^k$ for $i,\ell=0,1,2$; or $\mu_{12}^j=\mu_{12}^k=0$ and $\rho_j=\rho_k=0$. Similarly sufficient conditions for $\lambda_j^*=\lambda^*$ are either $\mu_{i\ell}^j=\mu_{i\ell}$ for $i,\ell=0,1,2$; or $\mu_{12}^j=\mu_{12}=0$ and $\rho_j=\rho=0$. The former condition is nearly the case where producers have identical supply distributions (but not quite since it refers only to joint moments up to fourth order). The latter condition

[7] λ^* is given by (14) with joint/marginal moments from the aggregate (p,q) distribution; Hughes Hallett and Ramanujam (1990). The non-linearity of (14) means that no simple conditions on the moments of (p,q_j) and (p,q) will ensure the aggregate and private intervention rules all coincide.

[8] The necessary and sufficient conditions, $\mu_{12}^j + \bar{q}_j\rho_j\sigma_p\sigma_j = \mu_{12}^k + \bar{q}_k\rho_k\sigma_p\sigma_k$, are too particular to be of any interest here. The corresponding conditions for hedging are $\mu_{11}^j=\mu_{11}^k$ and $\mu_{21}^j=\mu_{21}^k$.

requires symmetric distributions with uncorrelated prices and quantities. Both are rather more stringent than requiring all producers to have the same price elasticity of supply, as suggested by Gemmill (1985). Beyond these two special cases, interventions which are optimal for one producer will be suboptimal for another. Thus, unless those special cases hold, stabilizing market revenues will not minimize the earnings variability of market participants.

b) When would producer j choose the same strategy as an agency charged with stabilizing the market as a whole? Alternatively, how much would be lost because such an agency would choose either the wrong strategy from the individual producer's point of view, or suboptimal interventions within a given strategy? Given that public/private conflicts do not arise under hedging, this amounts to testing when the authority would select price stabilization when producer j would do better with a hedging strategy that he can control himself. Using the test in Figure 1, this happens if

$$\lambda^s A > h^{*2}\sigma^2 \qquad \text{in aggregate} \qquad (16)$$

$$\text{but} \qquad \lambda^s_j A_j < h^{*2}_j \sigma^2_p \qquad \text{for producer j.}$$

Now if $\theta = A^2/(Bh^{*2})$ but $\theta_j = A^2/(B_j h^{*2}_j)$, (16) can only hold when $\theta_j < \sigma^2_p < \theta$. The corresponding losses can be evaluated by computing $V(y^s_j)$ at λ^* or λ^*_k and comparing that to the optimized values of $V(y^h_j)$ or $V(y^s_j)$.

c) Under what circumstances could a market stabilization programme actually end up destabilizing producer j's income, or when would producer k's privately optimal interventions have the effect of destabilizing producer j's earnings? Taking the former case, that could only happen if $V(y^s_j) > V(y^o_j)$ which follows from the definition of $V(y^s_j)$ in footnote 4 if

$$A/B\{AB_j/B - 2A_j\} > 0.$$

Such an inequality can hold only when $\lambda^* > 2\lambda^*_j$ if $A>0$, or $\lambda^* < 2\lambda^*_j$ if $A<0$. Similarly producer k's actions would destabilize producer j's earnings if $\lambda^*_k > 2\lambda^*_j$ when $A_k>0$, or if $\lambda^*_k < 2\lambda^*_j$ when $A_k<0$. These inequalities[9] are in

[9] The corresponding condition for aggregate hedging to destabilize producer j's earnings is $h^* > 2h^*_j$ if $h \geq 0$ (but $h^* < 2h^*_j$ if $h<0$).

fact quite likely to hold when supply conditions vary significantly across different producers. They are more likely to hold for certain highly differentiated producers than between any producer and a market authority whose λ^* represents some kind of average of the λ_j^* values.

4. INTERVENTION COSTS: STABILIZING NET REVENUES

Stabilization operations, whether undertaken on the futures markets or through buffer stock interventions, have to be financed. Producers will need to take these operating costs – principally storage, transactions and financing costs – into account when deciding on their strategy and level of stabilizing activity. With the exception of the case where an external agency finances these operations as part of its aid or debt relief programmes, producers should aim to stabilize their earnings *net* of operating costs.

To a first approximation, operating costs will vary with either the physical volume or the value of the hedge on the buffer stock. The former represents any warehousing or storage costs. The latter represents the transaction costs consisting of the financing (or opportunity) costs of maintaining a buffer stock or of making the required margin payments on the forward contracts. These costs will be roughly proportional to the value of the buffer stock or forward contract. Note that the value of a forward contract is known in advance, but the actual buffer stock costs will depend on the realized (post-stabilization) market price which cannot be known beforehand. However the intended buffer stock is that which returns prices to their target level, $\bar{p}$. So producers will have to base all their decisions on a buffer stock evaluated at the same (planned) price.

4.1 Hedging Strategies with Costs

If there are storage costs, producer j's revenue will be reduced by rh_j, where r is the cost per unit hedged. We replace (1) with an expression for net revenue:

$$y_j^h = pq_j + (p_f - p - r)h_j \tag{17}$$

which, in contrast to (2), implies $E(y_j^h) < E(y_j^o)$ by an amount which depends on r and h_j. On the other hand, (17) shows that all our previous results carry over if the forward prices, p_f, are interpreted as *net* of costs. In particular, since h_j^* is independent of p_f, (4) and (6) continue to apply without modification.

It is hard to think of plausible reasons why hedging costs should depend on the volume rather than the value of the futures contract. The transactions costs (margin payments) case implies net revenues are given by

$$y_j^h = pq_j + (p_f - p)h_j - rp_f h_j \tag{18}$$

where r can be interpreted as the real market rate of interest. Once again (18) implies $E(y_j^h) < E(y_j^o)$. At the same time, direct calculation shows that $V(y_j^h)$ is unchanged:

$$V(y_j^h) = E\left[pq_j + (p_f - p)h_j - rp_f h_j\right]^2 - \left[E(pq_j) - rp_f h_j\right]^2$$

$$= V(y_j^o) + h_j^2\sigma_p^2 - 2h_j(\mu_{21}^j + \bar{q}_j\sigma_p^2 + \bar{p}\rho_j\sigma_p\sigma_j) \tag{19}$$

which is identical to (3). So, in this case as well, transactions costs do not affect the optimal hedging rule or the reduction in earnings variances which can be achieved; (4) and (6) continue to apply. In fact, the only consequence of including operating costs in the analysis is to introduce a new trade-off, controlled by choice of h_j, between maintaining average earnings and reducing their variance.

4.2 Buffer Stock Strategies with Costs

Warehousing costs are clearly important in this case. Those costs will be $r\lambda_j(S-D)$ where r is the unit storage cost. Buffer stock managers are likely to have a lot of difficulty in measuring excess demand and supply accurately, so it is convenient to reformulate the buffer stock rule (8) in terms of deviations of actual prices from their target level. That is easy to measure (indeed it requires no special model or analysis of the market to do so) and it puts the intervention rule into the form in which it would be used in practice.

Evaluating (8) at $\bar{p}$, using the supply and demand functions at (9), and then substituting again for v and u, yields

$$BS_j = \gamma_j(\bar{p} - p) \quad \text{where} \quad \gamma_j = \lambda_j(\beta + b) \tag{20}$$

where $v = \Sigma v_j$ and $\beta = \Sigma\beta_j$. Hence, if the storage costs are $r\lambda_j(\beta+b)(\bar{p}-p)$, producer j's net revenue will be

$$y_j^s = pq_j - \lambda_j(p - \bar{p})q_j - r\lambda_j(\beta + b)(\bar{p} - p) \tag{21}$$

which implies $E(y_j^s) = E(y_j^o) - \lambda_j\rho_j\sigma_p\sigma_j$, just as in (12). But

$$V(y_j^s) = (1-\lambda_j)^2 V(y_j^o) + \lambda_j^2 \bar{p}^2 \sigma_j^2 + r^2 \lambda_j^2 (\beta+b) \sigma_p^2 \qquad (22)$$

$$+ 2\lambda_j(1-\lambda_j)\bar{p}\ \mathrm{Cov}(y_j^o,q_j) + 2r\lambda_j^2(\beta+b)\mathrm{Cov}(p,q_j)$$

and

$$\lambda_j^* = \frac{V(y_j^o) - \bar{p}\ \mathrm{Cov}(y_j^o, q_j)}{V(y_j^o) + \bar{p}^2\sigma_j^2 - 2\bar{p}\ \mathrm{Cov}(y_j^o,q_j) + 2r(\beta+b)\mathrm{Cov}(p,q_j)} \qquad (23)$$

In contrast to the hedging case, it is now the earnings variance and optimal buffer stock interventions – and not the average earnings – which have been changed by allowing for operating costs. For calculation purposes we can rewrite (23) as $\lambda_j^* = A_j/\bar{B}_j$, and the variance reduction as $A_j^2/\bar{B}_j$, where A_j was given after (14) and where $\bar{B}_j = B_j + r^2(\beta+b)\sigma_p^2 + 2r(\beta+b)\rho_j\sigma_p\sigma_j$. The buffer stock interventions are now clearly smaller if either $\rho_j > 0$, or if $\rho_j < 0$ and the uncertainty is more in prices and demand than on the supply side (i.e. $\sigma_p \geq \sigma_j$). But large supply shocks and $\rho_j < 0$ could lead to larger interventions (and gains) than before.

The transactions costs case is very similar. The buffer stock needed to hold prices at their target level would cost $r\bar{p}\lambda_j(S-D) = r\bar{p}\lambda_j(\beta+b)(\bar{p}-p)$ at those prices. If r is once again the market rate of interest, this is the financing or opportunity cost of maintaining the buffer stock. Repeating steps (21) to (23) we find average earnings are still the same as in (12), but $(\beta+b)$ is replaced by $\bar{p}(\beta+b)$ throughout (22) and (23). Hence

$$\lambda_j^* = A_j/\bar{\bar{B}}_j \quad \text{and} \quad V(y_j^s) = V(y_j^o) - A_j^2/\bar{\bar{B}}_j \qquad (24)$$

where A_j is given below (14) and $\bar{\bar{B}}_j = B_j + r^2\bar{p}^2(\beta+b)^2\sigma_p^2 + 2r\bar{p}(\beta+b)\rho_j\sigma_p\sigma_j$. The same remarks about reduced interventions apply, but with added force since the presence of $\bar{p}$ will tend to increase $\bar{\bar{B}}_j$ above $\bar{B}_j$.

The upshot of all this is that operating costs will make little difference to producers' ability to use the futures markets to stabilize their earnings, although their average net revenue will be reduced. But those costs will, in a market with volatile prices, reduce their ability to use buffer stocks to stabilize prices/earnings although their average net revenue would be no lower. This is consistent with the empirical evidence that the main difficulty with buffer stock stabilization schemes is their financing and operating costs. Those costs have the effect of sharpening the inherent conflict between maintaining average earnings and increasing their stability.[10]

5. EMPIRICAL ANALYSIS

We have examined the price and earnings distributions for the major producers in five international commodity markets. From the 29 most important commodities reported in the World Bank's "Commodity Trade and Price Trends", which covers UNCTAD's list of 15 "core" commodities,[11] we selected Wheat, Coffee, Rubber, Copper and Tin to illustrate the scope which producers have for stabilizing their earnings. Three of these commodities (Coffee, Rubber, Tin) have or have had stabilization agreements; four are predominantly produced in developing economies (Coffee 90%, Rubber 99%, Tin 86%, Copper 60%, while Wheat is 97% produced in industrialized countries).[12] Our list contains two metals, two perennials, and one cash crop, to provide a roughly representative "sample" of commodity types.

We use estimates of the first four bivariate moments (i.e. $\bar{p}$, $\bar{q}_j$, σ_p^2, σ_j^2, and μ_{11}^j, μ_{12}^j, μ_{21}^j and μ_{22}^j) to construct optimal hedging and stabilization strategies for the sample period 1973–87.[13] We can then determine which strategy would be

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[10] See Hughes Hallett (1986).

[11] These "core" commodities were those picked out by the policy makers as prime candidates for stabilization. They were Wheat, Rice, Bananas, Sugar, Coffee, Cocoa, Tea, Rubber, Cotton, Jute, Wool, Iron Ore, Copper, Bauxite and Tin (see "Yearbook of International Trade Statistics", United Nations, New York, 1982, vol. 2).

[12] The coefficients of price variation over 1973–87 were 0.26 for Wheat, 0.34 for Coffee, 0.38 for Rubber, 0.44 for Copper, and 0.27 for Tin, with a maximum of 0.45 across all 29 commodities. For comparison, electrical machinery had a coefficient of price variation of 0.06 and clothing 0.08 for the same period. Price variability is measured here by the coefficient of variation of the US dollar price per unit deflated by the UN's index of export unit values for manufactured goods.

[13] The data used for the calculations reported here consists of monthly time series for January 1973–December 1987. It was obtained from: *Prices:* the UN's 'Monthly Commodity Price Bulletins' and the IMF's 'International Financial Statistics'. *Quantities:* For Coffee and Rubber: FAO's 'Monthly Bulletins on Agricultural Statistics'. For Copper and Tin: 'Metal Statistics' (Metallgesellschaft A.G., Frankfurt am Main). The quantity data was available only as quarterly data. By considering the monthly export volumes (UN Commodity Year Book, 1986) of countries which among them hold more than 60–70% of the export markets of the respective commodities, the quarterly output data was

best for stabilizing each producer's earnings, both net and gross earnings, in each market. These results allow us to make a rough assessment of the actual scope for earnings stabilization using the futures markets or buffer stocks, and of the potential conflicts between aggregate and private stabilization rules. Our results apply to individual producers who account for the majority of the supply in their markets: for Wheat it is 85% of the volume traded, for Coffee it is 44%, for Rubber 92%, for Copper 54% and for Tin 51%.

5.1 Tests of Non-Normality

The appendix contains a summary of the estimated moments of the spot price, production and earnings distributions for each producer in each market. In every case, the third moment shows strongly positive skews. This reflects the stockholding behaviour mentioned in section 2.2.

The appendix also reports the estimated joint moments from the bivariate (p,q_j) distribution for each commodity. One item of interest here is the signs of the correlation coefficients: only 8 out of 17 producers face the negative (p,q_j) correlation which is usually assumed in this sort of analysis. The others show weakly positive correlations. Nevertheless, this is sufficient to show that price stabilization would produce lower *average* earnings for the US in the wheat market; for the Ivory Coast in the Coffee market; for Malaysia, Indonesia and Thailand in the Rubber market; for Zambia and Zaire in the Copper market; and for Thailand in the Tin market. A second point is that 19 out of 34 bivariate third moments have the signs predicted for them as a result of stockholding behaviour. On that evidence it would be hard to argue that any of these distributions are symmetric and normal.

The usual test of whether a variable's distribution departs significantly from a normal distribution is either the Kolmogorov-Smirnov test (or one of its variants) or the χ^2 goodness of fit test. But these tests are not very powerful and do not reveal whether the normality in the distribution is due to "skewness" or due to "kurtosis" (Miller, 1986). As a result Pearson (1963, 1965) recommends the following test statistics be used to detect the non-normality:

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converted into monthly data. For Wheat: the Australian Bureau of Agricultural Economics. *Futures Prices*: 'The Financial Times' – Commodities Reviews of the Week on the first Saturday of the Month, January 1973 to December 1987.

Skewness: $\delta_1 = 1/n\Sigma(y_i-\bar{y})^3/[1/n\Sigma(y_i-\bar{y})^2]^{3/2} = \mu_3/\sigma^3$

Kurtosis: $\delta_2 = [1/n\Sigma(y_i-\bar{y})^4/\{1/n\Sigma(y_i-\bar{y}^2]^2\} - 3 = [\mu_4/\sigma^4] - 3$

The calculated values of δ_1 and δ_2 for the price and quantity distributions in our five markets are given in Table 1. Under a null hypothesis of normality, the critical values at a 1% significance level for δ_1, with a sample size of 180 is 0.424 and for δ_2 it is 2.49 (Pearson and Hartley, 1966). Three out of the five price distributions and 12 out of the 17 quantity distributions show asymmetries which are statistically significant. There is less evidence of significant kurtosis – none of the price distributions and only 3 out of 17 quantity distributions show it. But, overall, all five markets and hence all producers face significant departures from normality in their price-quantity distributions.

Table 1. Tests of Non-Normality by Commodity Market

Market	Spot Prices δ_1	Spot Prices δ_2	Producer	Quantity Traded δ_1	Quantity Traded δ_2
Wheat	.024	−.730	USA	.802*	.291
			Canada	1.071*	.794
			France	1.143*	2.349
			Australia	1.067*	1.736
			Argentina	1.534*	1.838
Coffee	.613*	1.249	Ivory Coast	.344**	−.765
			Columbia	.110	.294
			Brazil	.275	1.261
Rubber	.724*	.899	Malaysia	.292	.617
			Indonesia	1.149*	3.049*
			Sri Lanka	1.615*	3.498*
			Thailand	1.197*	2.999*
Copper	1.419*	2.055	Zaire	.513*	.139
			Chile	.052	.242
			Zambia	.609*	.642
Tin	.011	−.506	Malaysia	.020	.181
			Thailand	.781*	.488

* denotes significance at the 1% level; ** at the 5% level

5.2 Hedging versus Price Stabilization: No Intervention Costs

Table 2, parts (a)–(e), contains the result of applying optimal hedging and buffer stock stabilization strategies to minimize the variability of each producer's *gross* revenue. That is revenue stabilization without counting the storage or transactions costs involved, as might be appropriate if the exercise were to be part of some development project or aid programme. *Net* revenue stabilization, where producers must count the cost of their interventions, is dealt with in Table 3 and section 5.3.

a) Hedging: columns 2 and 3 of Table 2 show that the optimal hedge varies a certain amount between markets, but more so between producers within any given market. In the Wheat market, the largest producer (the US) should hedge significantly more than its expected output while Canada, France, Australia and Argentina should only sell between a half and three-quarters of their expected production forward. This difference is the result of a positive p, q_j correlation for the US, coupled with strong asymmetries in the form of $\mu_{21}^j<0$ and $\mu_{12}^j>0$, which makes $p<\bar{p}=E(p_f)$ the more likely outcome.[14] In that case hedging more than $\bar{q}_j$, and then buying any shortfall on the spot market, will yield extra profits on average. (This effect would be offset to some extent if $\mu_{12}^j>0$ since that would reduce the expected fall in p for a given output disturbance. But, by the same argument, $\mu_{21}^j<0$ would increase the optimal hedge.) That explains the US result; the negative correlations faced by the other producers produce the opposite effect, but modified now by both $\mu_{21}^j<0$ and $\mu_{12}^j<0$ (Argentina excepted).

Essentially the same things happen in the Coffee market, with $\rho_j>0$ leading the Ivory Coast to go short and $\rho_j<0$ leading Columbia and Brazil to hedge 80% of their expected output. These positions are less extreme because the μ_{21}^j and μ_{12}^j asymmetries are stronger and serve to modify the hedging rule towards full hedging. That happens again in Rubber (Malaysia and Indonesia go slightly short, Sri Lanka is three-quarters hedged); in Copper (Zaire and Zambia go

[14] See again the Appendix for these figures.

Table 2. Optimal Hedging and Price Stabilization Strategies in 5 Markets – With No Intervention Costs

Producer	$V(y_j^0)$	Hedging h_j^*	Hedging λ_j^h	Hedging % Reduction in Earnings Variance	Price Stabilization $\Delta Ey_j^s(\%)$	Price Stabilization λ_j^s	Price Stabilization % Reduction in Earnings Variance	Dominant Strategy
(a) Wheat								
USA	5.1×10^{10}	4161.6	1.522	26.0	–2.1	1.550	33.2	S
Canada	1.2×10^{10}	902.3	.694	5.1	+0.6	.702	6.2	S
France	5.0×10^{9}	749.7	.884	8.7	–0.0	.737	6.3	H
Australia	6.0×10^{9}	442.4	.481	2.5	+0.6	.343	1.5	H
Argentina	4.1×10^{10}	281.3	.717	1.5	+0.8	1.707	12.0	S
(b) Coffee								
Ivory Coast	1.7×10^{9}	25.6	1.247	43.9	–2.3	1.131	51.6	S
Columbia	4.5×10^{9}	33.8	.759	29.7	+2.0	1.455	90.9	S
Brazil	1.2×10^{10}	54.1	.828	29.1	+2.5	.982	47.7	S
(c) Rubber								
Malaysia	1.4×10^{9}	138.2	1.065	69.1	–0.4	1.064	71.7	S
Indonesia	5.9×10^{8}	86.9	1.187	63.9	–1.3	1.231	72.9	S
Sri Lanka	2.7×10^{7}	8.3	.760	12.7	+1.3	.818	19.9	S
Thailand	3.9×10^{8}	43.0	.997	23.9	–0.7	1.077	25.0	S
(d) Copper								
Zaire	1.5×10^{8}	16.5	1.125	24.9	–0.4	1.024	26.0	S
Chile	9.4×10^{8}	49.7	.860	36.9	+0.4	.967	43.8	S
Zambia	6.7×10^{8}	55.7	1.115	65.3	–0.1	1.078	67.1	S
(e) Tin								
Malaysia	6.3×10^{8}	5.2	.964	56.4	+1.0	.855	53.5	H
Thailand	2.1×10^{8}	3.1	1.618	59.7	–9.5	1.617	78.9	S

Units: y_j in 1000 \$; h_j^* in 1000 MTonnes; % or ratios elsewhere slightly short, Chile is 86% hedged); and in the Tin market (Thailand goes short, Malaysia is fully hedged). None of these results are changed if the forward markets are biased or inefficient - and in fact the biases would lead to changes of 1% or less in average earnings of 1% or less.

b) Price Stabilization: Under this strategy, the US and Argentinean market authorities would want to intervene very strongly in the Wheat market, while Canada, France and especially Australia would prefer relatively weak buffer stock interventions: see columns 5 and 6 in Table 2. Indeed both the US and Argentina need "over-stabilizing" interventions if they wish to minimize earnings instability. Canada and France, in contrast, require a 70% stabilization rule and Australia a 35% rule. There is a clear conflict of interest here between those who prefer vigorous interventions and those who would prefer only partial stabilization.

In fact the conflict may be worse than that because although the reductions in earnings variances are not very different from those achievable under hedging, it is clear that the US, Canada and Argentina would want to choose buffer stock stabilization schemes, while France and Australia would prefer to hedge on the futures market. It is possible that the relative gains are small enough for these differences on the choice of strategy to be overcome,[15] but agreement on how best to operate the chosen strategy might be more difficult to achieve. On the other hand, with the exception of the US's 33% reduction in income stability, none of these stabilization gains are large enough to cause a great deal of enthusiasm for either strategy.

The picture for the Tin market is somewhat similar. Thailand would want "over-stabilizing" interventions while Malaysia would only want partial stabilization. Thailand would prefer a hedging strategy, Malaysia price stabilization. This time the potential gains are larger: a 56% reduction in earnings variance in Malaysia and a 79% reduction in Thailand. Once again disagreement on strategies might not cause serious problems for Malaysia who would not lose much stability but would gain 1% in average revenue with a switch to price stabilization. Nevertheless Malaysia and Thailand are quite likely to disagree on the strength of the intervention rules.

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[15] Especially as France and Australia stand to make marginal gains in average earnings, to offset the smaller variance reductions, if they switch to price stabilization (col. 7).

The Coffee, Rubber and Copper market results suggest no disagreements between producers; they would all go for price stabilization schemes, by a wide margin in the Coffee case and by rather narrower margins in the Rubber and Copper markets. However there would be some differences of opinion about the form of the decision rules. One producer in each market would want only partial stabilization (albeit with intervention parameters of .82 to .98) while the majority would prefer mild "over-stabilization".

c) The Dominant Strategies: The gains in earnings stability are, with the exception of the Wheat market, quite large. For most producers they are in the range of reductions of 50% to 80% in earnings variability, although two producers in the Rubber market and one in the Copper market could only achieve 25% reductions. By and large, the bigger gains appear where the worst instability problems have arisen (Wheat again excepted). So these are strong results even if subject to a margin of error.

Second, assuming some flexibility on behalf of the two Wheat producers and one Tin producer who would marginally prefer a hedging strategy, price stabilization would be chosen in each of the five markets. However, except in the Coffee market and for the Thai Tin producers, the margins are pretty small – very seldom more than 5% points - and they are compensated by a rise in average earnings in 8 out of 17 cases. So the losses from choosing the "wrong" strategy are not large; it is significantly better for all producers to do something than to do nothing. This is a comforting conclusion in a world where estimated parameters inevitably introduce elements of approximation and error. Moreover, given some flexibility by those who would marginally prefer to hedge, producers individually always choose the same strategy as would minimize market revenues, even if they disagree on the precise form of decision rule. But the costs of those disagreements could be significant. Argentina's buffer stock activities in the Wheat market would actually destabilize Canadian, French and Australian earnings ($\lambda_j^s > 2\lambda_k^s$) and Thailand's activities in the Tin market come close to being destabilizing.

More important perhaps is the *distribution* of the gains between producers. That varies quite a bit in all five markets, and the average gain for the large producers individually can be quite different from the potential market gain (e.g.

for Copper the average in 45% against a potential market gain[16] of 80%) which suggests the private-market rule conflict may be serious and may affect the fringe producers disproportionately.

5.3 Hedging versus Price Stabilization Allowing for Intervention Costs

So far our results have shown which strategy is most effective, but not most cost-effective, for stabilizing earnings. How much difference would it make if producers had to take the costs of their interventions into account? Would they want to choose different stabilization strategies, and is the degree of revenue stability seriously reduced?

Table 3 reports the results of stabilizing net earnings after allowing for transactions and storage costs. For illustrative purposes we have assumed an interest rate of 10% p.a. and, since our data refers to monthly production figures, the costs have been assessed on a monthly basis (r=0.008).

As we saw in section 4, the hedging strategy and its earnings variance reduction are unaffected by the intervention costs. Average earnings are however reduced. It turns out that those reductions are small, typically about .75% and rising to 1.2% in two cases. Hence, except for the Wheat market, the earnings variance reductions by hedging are obtained relatively cheaply and producers are unlikely to change their views on the scope for stabilization by hedging.

The results of price stabilization are much less clear cut. Nothing much changes in the Wheat and Rubber markets (compare Table 2). Producers would still prefer price stabilization for net revenues – except for French and Australian Wheat farmers who still prefer hedging. The earnings variance reductions and intervention strategies are not noticeably changed. The degree of earnings stability achieved in the Coffee market is reduced a certain amount – but not by enough to change the dominance of buffer stocks as a stabilization instrument. So once again producers are unlikely to revise their views in the Wheat, Coffee or Rubber markets.

However, things are quite different in the Copper and Tin markets. In the Copper case, two producers (Zambia and Zaire) would want to switch to a hedging strategy if they have to pay their own costs. The third (Chile) would wish to stick with a price stabilization strategy. That produces yet another

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[16] See Hughes Hallett and Ramanujam (1990).

Table 3. Optimal Hedging and Price Stabilization Strategies Including Transaction and Storage Costs (r=.008)

Producer	Hedging			Price Stabilization			Dominant Strategy
	h_j^*	ΔEy_j^h (%)	% Reduction in Earnings Variance	ΔEy_j^s (%)	λ_j^s	% Reduction in Earnings Variance	
(a) Wheat							
USA	4161.6	–1.2	26.0	–2.1	1.550	33.2	S
Canada	902.3	–0.6	5.1	+0.6	.702	6.2	S
France	749.7	–0.7	8.7	–0.6	.737	6.3	H
Australia	442.4	–0.4	2.5	+0.6	.343	1.5	H
Argentina	281.3	–0.6	1.5	+0.8	1.707	12.0	S
(b) Coffee							
Ivory Coast	25.6	–0.9	43.9	–2.2	1.039	47.4	S
Columbia	33.8	–0.5	29.7	+1.9	1.409	88.1	S
Brazil	54.1	–0.6	29.1	+2.4	.972	47.2	S
(c) Rubber							
Malaysia	138.2	–0.8	69.1	–0.4	1.064	71.7	S
Indonesia	86.9	–0.9	63.9	–1.3	1.230	72.9	S
Sri Lanka	8.3	–0.6	12.7	+1.3	.816	19.9	S
Thailand	43.0	–0.8	23.9	–0.7	1.076	25.0	S
(d) Copper							
Zaire	16.5	–0.9	24.9	–0.3	.673	17.1	H (Change)
Chile	49.7	–0.7	36.9	+0.3	.925	41.9	S
Zambia	55.7	–0.1	65.3	–0.0	1.025	63.8	H (Change)
(e) Tin							
Malaysia	5.2	–0.8	56.4	+0.0	.007	0.41	H
Thailand	3.1	–1.2	59.7	–0.0	.002	0.08	H (Change)

Units: as Table 2.
Aggregate market price coefficients (β,b) are (0.167, 0.096); (0.25, 0.07); (0.065, 0.023); (0.71, 0.22); and (0.727, 0.064) for Wheat, Coffee, Rubber, Copper and Tin respectively.

market where producers would find it hard to agree on a stabilization strategy. Although Zambia would not lose much by joining a price stabilization scheme, it would cost Zaire and Chile quite a lot to change to price stabilization or hedging respectively.

By contrast, any previous disagreements in the Tin market are now removed; Thailand joins Malaysia in preferring a hedging strategy. Another major change appears to be that the stabilizing power of buffer stock interventions is almost completely wiped out by their cost.[17] No variance reductions in net earnings are possible. That is the result of a high mean price in a market which has reasonably strong price responses and a high price volatility compared to the other markets. It therefore appears that the net versus gross earnings distinction is only crucial for commodities which have a high unit value and price sensitive demand/supply responses.

6. PRIVATE VERSUS AGGREGATE MARKET STRATEGIES

We have emphasized that every buffer stock transaction will affect every producer's price distribution and that the degree of stability achieved depends on the aggregate buffer stock movements. Hence the price stabilization results reported in Table 2 are unlikely to emerge in a market of a homogenous good, since no producer will succeed in imposing his privately optimal rule on all the other producers in the market.[18] However, Table 2's calculations are useful because they allow us to assess how much the privately optimal intervention rules differ from one another and how much they differ from the aggregate or publically optimal rule. We can see where the public/private conflicts are most likely to occur, and how much imposing public or compromise rules would cost individual producers in terms of lost stability. The latter gives us an idea of each producer's incentive to participate in a price stabilization scheme.

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[17] It is interesting that Tin is the only market to show this characteristic because it was the Tin buffer stock that became insolvent in 1985 and caused the collapse of that market. According to these results, the Tin producers would have done better to instruct their buffer stock manager to follow a hedging strategy.

[18] The case of non-homogenous products is a good deal more complicated, but allows for non-cooperative behaviour and for producers to try to manipulate their own price distributions in order to stabilize their own earnings stream. That case is ruled out in our analysis by the assumption of a single price and the fact that our commodity markets all have a single reference price (any price differentials can be fully accounted for by quality/grade differences). Production from different sources is therefore taken to be perfectly substitutable.

The same conflict does not arise under hedging because each producer can affect the price distribution which he receives without changing that for other producers. Nevertheless, as we pointed out in section 3, the earnings distributions are still correlated, so it is not clear what impact a market hedging authority might have on individual earnings streams.

Table 4 considers three cases. The first two columns indicate the maximum gain which each producer could make via price stabilization rather than hedging. That merely establishes the relative advantage of price stabilization, ignoring intervention costs, were the market to adopt each producer's privately optimal decision rule in turn. These relative gains are variable but mostly quite small; less than 10% (Columbian coffee, Indonesian rubber, Thai tin, and the three small relative losses excepted). On that basis producers might not be too concerned about choosing the wrong type of strategy.

But these comparisons are rather artificial since no producer will in fact succeed in imposing his own optimal price stabilization rule on the whole market. These are two other possibilities: producers agree on a compromise intervention rule based on a weighted average of the privately optimal rules, or producers agree to use a rule which is optimal for the market as a whole. Columns 3 to 6 of Table 4 show the results of using compromise rules, where interventions are controlled by a single price stabilization parameter which is an average of the private parameters weighted by each producer's share in total production. For comparison, results are also included for the case where a compromise hedging rule is used in each market – i.e. where some market authority undertakes all hedging operations on behalf of the producers. These compromise rules imply rather small private losses; less than 7% for all but one producer who would lose 13% of his maximum stability. The gains over doing nothing are still substantial, except for the four smaller wheat producers who could have gained rather little anyway.

Those hedging results are also rather artificial because there is no compelling reason why policy makers should not just aim to provide access to the futures markets. The more interesting case is therefore the compromise price stabilization rule where the results depend on aggregate buffer stock movements. Nevertheless the story is remarkably similar in that case; small private losses (less than 7%) emerge for all but two producers. However those two producers (Columbian coffee, Thai tin) now lose about 40% of their very substantial privately optimal gains. Consequently the overall picture is that compromise

Table 4. The Losses in Earnings Stability When Market Rather Than Privately Optimal Stabilization Rules are Adopted (in %) – No Intervention Costs

Producer	Private Rules		Compromise Rules				Aggregate Market Rules			
			Hedging		Stabilization		Hedging		Stabilization	
	λ	Gain (a)	Gain (b)	Gain (c)	Gain (b)	Gain (c)	Gain (b)	Gain (c)	Gain (b)	Gain (c)
(a) Wheat										
USA	1.55	+9.7	–3.4	+22.6	–4.5	+28.7	–7.1	+18.9	–55.1	–21.9*
Canada	.70	+1.2	–1.4	+3.7	–2.0	+4.2	–26.4	–21.3*	–8.0	–1.8*
France	.88	–2.6	–0.4	+8.3	–1.5	+4.8	–4.0	+4.7	–17.6	–11.3*
Australia	.48	–1.0	–3.6	–0.9*	–7.2	–5.7*	–3.5	–1.0*	–6.4	–4.9*
Argentina	1.71	+10.7	–0.3	+1.2	–1.8	+10.2	–22.1	–20.6*	–15.0	–3.0*
			$\lambda^h = 1.052$		$\lambda^s = 1.086$		$\lambda^h = 1.052$		$\lambda^s = 1.048$	
(b) Coffee										
Ivory Coast	1.13	+13.7	–7.0	+36.9	–0.1	+51.5	–4.7	+39.2	–0.5	+51.1
Columbia	1.46	+87.1	–1.0	+28.7	–38.6	+52.3	–2.4	+27.3	–78.4	+12.5
Brazil	.98	+26.2	–0.1	+29.0	–3.1	+44.6	–0.7	+28.4	–0.3	+47.4
			$\lambda^h = 1.167$		$\lambda^s = 1.167$		$\lambda^h = 0.94$		$\lambda^s = 1.04$	
(c) Rubber										
Malaysia	1.06	+8.4	–0.2	+68.9	–0.5	+71.2	–0.1	+69.0	–0.1	+71.6
Indonesia	1.23	+24.9	–1.9	+61.0	–2.6	+70.3	–1.4	+62.5	–1.4	+71.5
Sri Lanka	.82	+8.2	–2.4	+10.3	–3.0	+16.9	–14.6	–1.9*	–10.2	+9.7
Thailand	1.08	+1.4	–0.2	+23.7	–0.0	+25.0	–0.3	+23.6	–0.0	+25.0
			$\lambda^h = 1.077$		$\lambda^s = 1.105$		$\lambda^h = 1.053$		$\lambda^s = 1.082$	
(d) Copper										
Zaire	1.02	+1.5	–0.4	+24.5	–0.0	+26.0	–0.2	+24.7	–0.0	+26.0
Chile	.97	+10.9	–1.5	+35.4	–0.3	+43.5	–2.0	+34.9	–0.3	+43.5
Zambia	1.08	+5.2	–2.1	+63.2	–0.5	+66.6	–1.3	+64.0	–0.4	+66.8
			$\lambda^h = 0.996$		$\lambda^s = 1.019$		$\lambda^h = 0.89$		$\lambda^s = 1.03$	
(e) Tin										
Malaysia	.96	–6.7	–4.0	+52.4	–6.0	+47.5	–3.0	+53.4	–8.6	+44.9
Thailand	1.62	+47.6	–13.3	+46.2	–45.3	+33.6	–9.1	+50.6	–10.0	+68.9
			$\lambda^h = 1.134$		$\lambda^s = 1.053$		$\lambda^h = 1.132$		$\lambda^s = 1.107$	

Key: Gain (a) = % gain of price stabilization over hedging in terms of reduction of the variance of earnings.
Gain (b) = % loss when compromise or aggregate market rule is adopted in place of the privately optimal rule.
Gain (c) = % gain/loss when compromise or aggregate market rule is adopted rather than doing nothing.
* denotes producers earnings are actually destabilized by this rule.

rules would not cost individual producers very much in terms of lost earnings stability – except for one or two particular cases. The private versus market rule conflict is therefore not very strong and the incentives to join a collective stabilization scheme are quite large unless: (a) one's private gains are very much larger than those of other producers in the same market (e.g. Columbia in the Coffee market, Thailand in the Tin market), or (b) the private gains are so small that the compromise rule becomes net destabilizing (e.g. Australia in the Wheat market).

Things are not so favourable for rules which are optimal for the market as a whole (columns 7 to 10 of Table 4). Again this includes the results of an aggregate hedging rule although there is no compelling reason why policy makers should want to impose it. The private losses are (mostly) larger than in the compromise rule case, confirming our earlier conjecture that average private rules will perform better, given diversity between producers, than rules which stabilize the market as a whole. Price stabilization seems to imply losses which are slightly smaller than hedging but with a few dramatic exceptions. The US in the Wheat market and Columbia in the Coffee market both stand to lose more than 50% of their potential gains, while there are no such "outliers" under aggregate hedging rules. Moreover these larger losses imply that a number of producers will find aggregate market rules net destabilizing – all the wheat producers and Sri Lankan rubber producers (i.e. all those whose maximum gains were 25% or less) would be destabilized. Consequently the conflicts between private and aggregate intervention rules are likely to prove a much greater obstacle when policy makers attempt to introduce market-wide stabilization policies. Those problems will emerge in markets where (a) the maximum potential stabilization gains are anyway small, or (b) there is great diversity in the potential private gains within the market, or (c) there is a considerable gain in using hedging over price stabilization (or vice versa).

7. CONCLUSIONS

This paper has derived and compared optimal hedging and price stabilization strategies for individual producers in markets with price and production uncertainties. These strategies are distribution free and model free, although the price stabilization case implicitly assumes linear supply and demand functions. We have also assumed the moments of the joint price-output distributions, but not necessarily the price elasticities or other model parameters, to be invariant to any new interventions.

The second contribution was to extend those strategies for stabilizing *net* revenues; that is to allow for transactions and storage costs. We have then applied our decision rules to the problem of stabilizing the revenues of the major producers in five important commodity markets. While two markets contained producers who would disagree on the type of strategy which should be adopted, the main differences were in the different strength of interventions which different producers would prefer. That would of course be no problem had they all decided to hedge on the futures markets. But, in fact, all but 3 out of 17 producers would select price stabilization. That makes agreement on the interactions a necessity.

Producers could alternatively agree to a compromise rule or to stabilize market revenues as a whole. We therefore examined the losses suffered by each producer when either a compromise intervention rule or one which stabilizes market revenues is imposed, taking that producer's privately optimal outcome as the point of comparison. It is clear that compromise rules lead only to small losses, and those losses are never large enough to disturb the superiority of price stabilization as a strategy (Table 4). The aggregate market rules perform rather worse because they would actually destabilize the earnings of producers whose potential gains from stabilization are small and because the stabilization gains are rather unevenly distributed.

Allowing for operating costs did make considerable changes in two of the five markets. There are small losses in the amount of earnings stability achieved (less than 5% in all but 3 out of 34 cases) and small changes in the average earnings levels (again less than 2.5% in all cases). However that is enough to make hedging superior for three more producers. The main change is that a buffer stock's ability to stabilize earnings is wiped out for high unit value commodities because the cost of financing the necessary stocks seriously reduces net earnings. Among our examples, this happens in the Tin market and, to a smaller extent, in the Copper market. Transactions and financing costs on the futures markets, on the other hand, imply relatively small falls in average earnings and no losses in stabilizing power. Hence, as a rule of thumb, hedging is likely to be more effective for stabilizing the earnings from high value commodities and price stabilization is likely to be superior for commodities with low unit values.

One final consideration is that, in practice, many buffer stock stabilization schemes have run into difficulties because they have inadequate stocks to restrain price rises in boom periods and inadequate finance to purchase stocks when prices are low. Naturally that has caused those schemes to collapse or become ineffective. One might therefore suspect that our results are too favourable to price stabilization since our buffer stock interventions appear to be imperious to the threat of stock-out or bankruptcy. However, as soon as the transactions cost terms are included, the dangers of stock-out or bankruptcy do influence the amount of stabilization undertaken since changes in the size and costs of holding stock contribute directly to the level and variability of *net* earnings. The larger the stock, the greater its financing costs. That would penalize any further increases and hence reduce the chances of financial collapse. On the other hand, the smaller the stock, the higher are net earnings and the more will interventions move away from possible stock-out. As a result we find *smaller* interventions when transactions costs are introduced, plus a tendency for producers to switch to hedging in order to reduce the chances of stock-out or bankruptcy. Tables 2 and 3, for example, show the intervention parameters are uniformly smaller when costs are incorporated, and a lot smaller in Copper and Tin where the danger of collapse is large. Only in Wheat, where the scale of intervention is so small as to rule out any significant danger of collapse, do we observe no appreciable changes.

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APPENDIX: Estimated Bivariate Moments of the Price-Quantity Distributions in 5 Markets 1973–1987

Wheat

	Mean	Variance	3rd Moment	4th Moment
Price: (US$/MT)	146.6	770.8	507.6	1.34×10^6
Quantity: ('000MT)				
USA	2735.1	1.699×10^6	1.777×10^9	9.499×10^{12}
Canada	1301.0	5.426×10^5	4.282×10^8	1.117×10^{12}
France	848.5	2.201×10^5	1.180×10^8	2.589×10^{11}
Australia	919.4	2.912×10^5	1.677×10^8	4.016×10^{11}
Argentina	392.2	1.709×10^5	1.084×10^8	1.413×10^{11}

	USA	Canada	France	Australia	Argentina
μ_{21}	–144560.3	–61457.7	–76309.8	–54980.2	–44776.5
μ_{12}	5.910×10^6	-9.591×10^5	-1.057×10^6	-1.472×10^6	9.186×10^5
μ_{22}	1.473×10^9	4.043×10^8	1.569×10^8	2.329×10^8	8.460×10^7
ρ	0.210	–0.082	0.0001	–0.147	–0.076

Coffee

	Mean	Variance	3rd Moment	4th Moment
Price: (US$/MT)	2852.3	1.158×10^6	7.639×10^8	5.698×10^{12}
Quantity: ('000MT)				
Ivory Coast	20.54	104.23	366.09	2.428×10^4
Columbia	44.51	223.27	366.00	1.642×10^5
Brazil	65.42	740.82	5551.9	2.339×10^6

	Ivory Coast	Columbia	Brazil
μ_{21}	2.336×10^6	-7.414×10^6	-3.482×10^4
μ_{12}	6481.6	3.833×10^5	2.632×10^5
μ_{22}	1.152×10^8	2.818×10^8	8.362×10^8
ρ	0.113	–0.1094	–0.1589

Rubber

	Mean	Variance	3rd Moment	4th Moment
Price: (US$/MT)	919.16	50002.3	8.106×10^6	9.748×10^9
Quantity: ('000MT)				
Malaysia	129.72	307.88	1577.53	3.429×10^5
Indonesia	73.26	156.90	2257.18	1.489×10^5
Sri Lanka	10.89	24.65	197.67	3948.3
Thailand	43.18	326.92	7072.87	6.412×10^5

	Malaysia	Indonesia	Sri Lanka	Thailand
μ_{21}	27310.7	27094.1	11117.37	-2.561×10^{11}
μ_{12}	5255.08	19241.8	96.62	–4652.14
μ_{22}	2.654×10^7	1.341×10^7	1.859×10^6	1.259×10^7
ρ	0.110	0.256	–0.139	0.067

Copper

	Mean	Variance	3rd Moment	4th Moment
Price: (US$/MT)	1600.1	1.409×10^5	7.514×10^7	1.005×10^{11}
Quantity: ('000MT)				
Zaire	14.69	44.56	152.99	6250.6
Chile	57.74	206.97	152.40	1.150×10^5
Zambia	49.98	92.87	545.15	31447.06

	Zaire	Chile	Zambia
μ_{21}	9.595	-5.96×10^5	2.00×10^5
μ_{12}	−932.46	1.039×10^4	−51.63
μ_{22}	5.132×10^6	4.0435×10^7	1.455×10^7
ρ	0.0407	−0.0624	0.105

Tin

	Mean	Variance	3rd Moment	4th Moment
Price (US$/MT)	10694.4	1.319×10^7	5.398×10^8	3.290×10^{14}
Quantity ('000MT)				
Malaysia	5.392	2.645	0.0867	22.255
Thailand	1.907	0.5982	0.3613	1.2482

	Malaysia	Thailand
μ_{21}	4.671×10^6	1.883×10^6
μ_{12}	−2620.61	1229.87
μ_{22}	2.935×10^7	1.002×10^7
ρ	−0.114	0.455