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**CONTENT AND ADVERTISING  
IN THE MEDIA: PAY-TV VERSUS  
FREE-TO-AIR**

Martin Peitz and Tommaso Valletti

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**Martin Peitz**, International University in Germany  
**Tommaso Valletti**, Imperial College, London and CEPR

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Centre for Economic Policy Research  
90–98 Goswell Rd, London EC1V 7RR, UK  
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

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## **ABSTRACT**

### **Content and Advertising in the Media: Pay-TV versus Free-To-Air\***

We compare the advertising intensity and content of programming in a market with competing media platforms. With pay-tv media platforms have two sources of revenues, advertising revenues and revenues from viewers. With free-to-air media platforms receive all revenues from advertising. We show that if viewers strongly dislike advertising, the advertising intensity is greater under free-to-air television. We also show that free-to-air television tends to provide more similar content whereas pay-tv stations differentiate their content. In addition, we compare the welfare properties of the two different schemes.

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Martin Peitz  
School of Business Administration  
International University in Germany  
76646 Bruchsal  
GERMANY  
Email: martin.peitz@i-u.de

Tommaso Valletti  
Imperial College London  
Tanaka Business School  
South Kensington Campus  
London  
SW7 2AZ  
Tel: (44 20) 7594 9215  
Fax: (44 20) 7823 7685  
Email: t.valletti@ic.ac.uk

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# 1 Introduction

A broadcasting media platform can only succeed if it has viewers. Otherwise, its revenues from advertising as well as its revenues from charging viewers would be zero. In the traditional world of free-to-air television, there is no direct mechanism for charging viewers, rather financing is obtained through advertising bought by firms that wish to promote their products. This environment typically produces a market failure since it is not possible for those viewers who really value a program and would be willing to pay a higher price to do so. The willingness-to-pay of viewers is not internalized by tv channels, thus resulting in sub-optimal allocations. With the appearance of encryption techniques and digital decoders viewers can be charged for their consumption of certain programming. Hence, each channel has two sources of income: subscription revenues and revenues from advertising.<sup>1</sup> New technologies have the potential to transform the TV market into an on-demand service where consumers are able to choose what they want to watch and when they want to watch it. One may thus conclude that the relevance of market failures should be diminished in the new digital world. The purpose of this paper is to investigate under which conditions such conclusion can be supported by a formal analysis.

Clearly, the coexistence of different forms of finance is not unique to broadcasting, as many markets combine revenue streams of different kinds. What is special here is the so-called two-sided nature of the market. Advertising is typically a nuisance for viewers. Therefore, the amount of advertising constitutes an indirect charge to consumers. Viewers are interested in programming with little advertising; hence advertisers exert a *negative* external effect on viewers. Conversely, advertisers are interested in a large number of viewers; hence viewers exert a *positive* external effect on advertisers. Platforms compete for viewers and advertisers and the question in relation to sources of finance revolves around whether the outcomes generated by the market match preferences and promote welfare.

This problem has attracted considerable interest in the academic literature that we review below. The move towards digital pay-tv has also initiated an important debate in many countries, challenging the rationale for Public Service Broadcasting (PSB). If, under pay-tv, digital media platforms generate the programmes that viewers want, then the justification for PSB interventions is reduced if not eliminated. An example of this discussion is taking place in the UK where the sector regulator, Ofcom, is currently reviewing the nature of PSB. To give an idea of the stakes involved, the total financial cost of PSB is estimated around £3bn a year in the UK, with the BBC accounting for more than 85% of this subsidy. Most of the total cost is made by the licence fee, but it

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<sup>1</sup>Recent numbers for European pay-tv channels show that advertising revenues are only a small part of total revenues (Observatoire Européen de l'Audiovisuel, Strasbourg, 2003). For example, Canal+ in Spain receives only 7 percent of revenues from advertising. Similarly, in the UK revenues from advertising are less than 10 percent of revenues from subscription.

also includes an imputed value of the analogue spectrum which is given free of charge to PSB providers.<sup>2</sup>

Our paper contributes to this debate in a stylized way. We miss one source of revenues, the licence fee which is typically levied on all viewers. We concentrate only on charges to advertisers and on subscription fees to viewers. We consider a situation where there is no BBC (the public sector broadcaster) and no PSB requirements. Instead we ask what the market would do naturally, in the absence of any obligation. This seems to us a natural first step to investigate if significant interventions (such as PSB provision) are needed to start with and, if so, under which regime they can play a bigger role. We propose a stylized model which allows us to compare content and advertising decisions in the conventional world of free-to-air television as opposed to the new world of pay-tv. We show how the welfare comparison between advertiser and price funded media is complicated by the viewers' attitudes towards advertising and by the intensity of competition. In particular, we show that the move from free-to-air to pay-tv is welfare-improving when competition in the market is sufficiently intense.

**Related Literature.** A series of papers has analyzed the provision of content and advertising in media markets. Closest to our paper is the work by Anderson and Coate (2003), Gal-Or and Dukes (2003) and Gabszewicz et al. (2001, 2002, 2004).<sup>3</sup>

Anderson and Coate (2003) consider a model of two competing media platforms with given content.<sup>4</sup> Advertising generates rents because it informs consumers about products but is a nuisance for viewers.<sup>5</sup> The natural question therefore is to ask whether the market over- or underprovides advertising. Under free-to-air, Anderson and Coate show that the equilibrium advertising level is below the optimal one if the nuisance of advertising is small and above it if the nuisance is large.

Content choice is the main point of interest in the works of Gabszewicz et al. (2001, 2002, 2004) and Gal-Or and Dukes (2003). In Gabszewicz et al. (2001, 2002) viewers are assumed to be indifferent about the level of advertising. This implies that the inter-group externalities between advertisers and viewers exist only in one direction, namely that advertisers like a media platform with many viewers. Gabszewicz et al. then show that, when viewers are not charged, both platforms provide the same content;

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<sup>2</sup>See Ofcom, "Review of public service television broadcasting - Phase 2", September 2004. In this document the regulator proposes to continue the current system of funding and to introduce a new provider in charge of introducing new technologies. Existing broadcasters, but not the BBC, could bid to operate this new provider. Final decisions are expected in 2005. A broad discussion of the economic rationale for PSB can be found in Armstrong and Weeds (2004).

<sup>3</sup>There are earlier contributions in the literature, but they typically do not consider the two-sided nature of the market. An influential example is Spence and Owen (1977) who consider program selection under pay-tv and under free-to-air.

<sup>4</sup>To be precise, the content of media platforms is either maximally differentiated or identical. It is then found that platforms do not choose identical content to avoid Bertrand-like competition.

<sup>5</sup>For an excellent overview on the economics of advertising see Bagwell (2003). He elaborates on the welfare properties of informative advertising compared to other forms of advertising.

that is, there is minimal differentiation in the content space. This is reminiscent of the Hotelling model in which prices are fixed so that providing the same content at the center of the content space is the only equilibrium. However, when viewers are charged there is maximal differentiation. In this case, the logic is the same as in d'Aspremont et al. (1979) because more differentiation reduces competition for viewers.

In Gabszewicz et al. (2004) viewers dislike advertising. Media platforms set content and advertising levels; advertisers are homogeneous. Because of the latter assumption advertising rates per viewer are constant. Instead of setting prices to viewers as in the standard Hotelling model, platforms set disutilities from advertising. In equilibrium, platforms choose maximal differentiation in the content space if the disutility from advertising is linear in the amount of advertising.

Gal-Or and Dukes (2003) present a different model in which two advertisers compete for viewers who are also the consumers of the products. More advertising increases the probability that a consumer becomes informed (as in Grossman and Shapiro, 1984). This implies that less advertising by the two advertisers leads to higher prices because each advertiser has a larger captive segment. At the first stage of the game, platforms decide on the content they offer. At the second stage, platforms and advertisers reach agreements on the amount of advertising and advertisers set prices for their products and choose advertising levels. Different from Gabszewicz et al. (2004) they find that both platforms offer the same content. The intuition for their results is the following; first, when a platform locates closer to the center it increases the number of viewers that will exclusively consume content from this platform (this effect is also present in Gabszewicz et al., 2004). Moving closer, however, also gives rise to a competition effect. In Gal-Or and Dukes (2003) less differentiation increases the platforms' profits because under program duplication advertisers choose a low level of advertising together with high product prices leading to high revenues. Since in their model any surplus is shared between advertiser and platform, platforms benefit from relaxed competition among advertisers in the product market.

More generally, our paper is related to the growing literature on two-sided markets (see e.g. Armstrong, 2004; Evans, 2003; Rochet and Tirole, 2003, 2004). Most closely to our work, Armstrong (2004, section 5) analyzes advertising competition among media platforms for given content, where advertisers can advertise on both platforms and platforms set prices on both sides of the market. He shows that there is always underprovision of advertising compared to the social optimum. In addition, if platforms set per-consumer advertising charges, a platform's revenues from advertising are passed onto consumers in the form of lower prices, since consumers are a "competitive bottleneck".

**Our contribution and plan of the paper.** As with the above papers we present a model of competing media platforms. Our model is similar to Armstrong (2004) and Anderson and Coate (2003), with the important difference that a major role in our model is played by endogenous content (as in Gabszewicz et al., 2001, 2002, 2004).

We distinguish between pay-tv, where advertisers and viewers have to pay, and free-to-air tv, where only advertisers have to pay. Our model is presented in section 2. Section 3 contains our analysis of pay-tv. Section 4 contains the analysis of free-to-air tv. In both cases, pay-tv and free-to-air tv, we endogenize the choice of content as in Gabszewicz et al. (2001, 2002). We show that their minimum differentiation result under free-to-air relies on their assumption that consumers are indifferent about advertising. Depending on the nuisance of advertising content is between minimum and maximum differentiation. In section 5 we compare equilibrium content and advertising under pay tv and free-to-air tv, depending on the potential differentiation between platforms and the nuisance from advertising. We also compare welfare under pay-tv and free-to-air when content is given and when content is chosen by platforms. Section 6 concludes.

## 2 The Model and Social Optimum

**Viewers.** Viewers consume advertising and content from either one of two programs. They constitute the buyer side in the market. The buyer side is of mass  $N$ . A buyer of type  $\beta$  has a particular taste for programming. Programming can be in the  $[0, 1]$ -interval. A buyer has preference parameter  $\beta \in [0, 1]$ , which reflects her favorite type of programming. If a program is located at  $d_i$  a buyer incurs a disutility  $\tau(\beta - d_i)^2$ , where  $\tau > 0$  is the disutility parameter from consuming programming content that does not satisfy a buyer  $\beta$ 's tastes.<sup>6</sup> A program also contains advertising. We assume that advertising and content are additively separable in the utility function.<sup>7</sup> Viewers are assumed to dislike advertising. The corresponding utility loss is  $\delta a_i$ , where  $\delta$  is the disutility parameter for advertising and  $a_i$  is the amount of advertising. All consumers are assumed to have the same parameter  $\delta$ . A viewer has to pay a fee  $s_i$  under pay-per-view. The indirect utility of a viewer of type  $\beta$  from consuming program 1 is

$$v - \delta a_1 - \tau(\beta - d_1)^2 - s_1$$

where  $v$  is the willingness-to-pay for perfect programming and zero advertising. We implicitly assume that  $v$  is sufficiently large such that all potential viewers view one program, that is, we have full market coverage. Similarly, for program 2. We also assume that viewers cannot mix the two programs – this means we are considering competition for a particular time slot rather than competition between two channels.<sup>8</sup>

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<sup>6</sup>The quadratic specification of transport costs is adopted to ensure existence in the location game.

<sup>7</sup>This assumption is also made in the cited literature above. It clearly is a restrictive assumption but avoids additional computational problems.

<sup>8</sup>Gal-Or and Dukes (2003) and Gabszewicz et al. (2004) consider channel competition so that viewers can mix. As shown by Anderson and Neven (1989), the resulting demand is the same one as the demand derived in the present model. See also Remark 2 and Section 6 below.

Then under pay-tv viewers pay for a particular show, that is, we are considering pay-per-view. If the two programs are located at  $d_1$  and  $1 - d_2$  on the line, there is a viewer  $b_1$  who is indifferent between the two programs,

$$-\delta a_1 - s_1 - \tau(b_1 - d_1)^2 = -\delta a_2 - s_2 - \tau((1 - b_1) - d_2)^2.$$

Solving for  $b_1$  one obtains

$$b_1 = \frac{d_1 + 1 - d_2}{2} - \frac{\delta(a_1 - a_2) + (s_1 - s_2)}{2(1 - d_1 - d_2)\tau}. \quad (1)$$

All viewers to the left of  $b_1$  view program 1 and all viewers to the right of  $b_1$  view program 2. Hence, the total number of viewers of program 1 is  $Nb_1$  and the total number of viewers of program 2 is  $Nb_2$  where  $b_2 \equiv 1 - b_1$ .

**Remark 1** *The discrete choice between platforms is a typical feature for newspapers. We thus can interpret our model also as a model of competing newspapers. In one segment of the market, newspapers charge for subscription, in another segment they are entirely financed through advertising.*

**Advertisers.** Advertisers of mass 1 sell products to viewers who are also the consumers of the products. Products are produced at constant marginal costs, which without loss of generality are set equal to zero. A product is produced at quality  $\alpha$  and consumers have willingness to pay  $\alpha$  for a good of quality  $\alpha$ . Each producer has monopoly power and can therefore extract the full surplus from consumers, that is, a product of quality  $\alpha$  is sold at price  $\alpha$ . Producers differ with respect to the quality of the good they offer. Quality is distributed on some interval  $[0, \alpha^{\max}]$  according to a p.d.f.  $F$  with  $F(0) = 0$  and a continuously differentiable density. As our lead example, we consider the case where  $F$  is uniform on  $[0, 1]$ .

Advertisers can only sell to those consumers which have seen the ad. If an advertiser advertises in a particular program it is assumed that all viewers of this program are aware of the corresponding product.<sup>9</sup> Advertisers can advertise in none, one, or both programs. Advertisers have to pay the advertising charge  $r_i$ . The profit for advertiser  $\alpha$  from advertising in program  $i$  is

$$N\alpha b_i - r_i.$$

The marginal advertiser for program  $i$ ,  $\underline{\alpha}_i = r_i/(Nb_i)$ , makes zero profit. Hence the amount of advertising in program  $i$  is

$$a_i = 1 - F\left(\frac{r_i}{Nb_i}\right). \quad (2)$$

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<sup>9</sup>In the terminology of the literature on two-sided markets, the viewers' market is a competitive bottleneck, see Armstrong (2004).



The advertising space  $a_i$  determines the advertising charge per viewer  $r_i/(Nb_i)$ ; it is not affected by the decisions of the competing platform.

**Remark 2** *If consumers mix between channels, aggregate demand is the same as above and therefore our positive analysis remains valid.<sup>10</sup> To establish the formal equivalence of the advertisers' profit functions in the mixing and the discrete choice setting it is required that, in the mixing case, the likelihood of watching a channel maps linearly into the likelihood/frequency of buying a product as a response to viewing the channel. This is compatible with advertising being directly informative.<sup>11</sup> However, in case of pay-tv, consumers often pay a flat subscription fee. In this case consumers have four options: no subscription, subscription to channel 1 or 2, and subscription to both channels. In order to mix channels, consumers have to pay both subscription fees. Our discrete choice analysis is then still appropriate provided that the gain from mixing is less than paying the subscription fee. This is clearly the case if channel content is sufficiently similar.*

**Media Platforms.** Media platform  $i$  invites advertisers to its platform. For this it provides advertising space  $a_i$  and attracts a number  $b_i$  of customers. We distinguish between two different technologies,

- free-to-air and
- pay-tv.

Under free-to-air a program is provided for free to customers. With pay-tv customers are charged a pay-per-view price  $s_i$ . Here, we allow for the subsidization of customers, that is,  $s_i$  can take negative values. Under free-to-air the price  $s_i$  is fixed equal to zero. Profit of program  $i$  is

$$\pi_i = Nb_i s_i + a_i r_i.$$

**The Content, Pricing and Advertising Game.** The two media platforms are the only players which are not atomless. They play a three-stage game. In the first stage, the platforms determine their content by choosing a location in the unit interval. In the second stage, they determine the space for advertising and set the subscription or pay-per-view price. In the third stage, advertisers decide where to advertise and

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<sup>10</sup>The model of mixing in a product differentiation context has first been analyzed by Anderson and Neven (1989). For an application to media markets see e.g. Gal-Or and Dukes (2003).

<sup>11</sup>Suppose that an advertiser considers placing an ad on one of the channels. In case it already advertises on the other channel it advertises at the same time so that consumers see the ad at most once and the decision to advertise in one is not affected by the decision to advertise in the other channel. A consumer buys the product if she sees the ad and if her willingness to pay is above the price. The value from placing the ad is then  $Nab_i$  as in the discrete choice setting.

$i = 1, 2$	platform $i$
$\alpha$	type of advertiser
$F$	distribution of advertisers' profits per customer
$a_i$	amount of advertising on platform $i$
$r_i$	advertising charge
$d_i$	location of platform
$\beta$	type of buyer
$b_i$	number of buyers at platform $i$
$s_i$	pay-per-view price
$N$	mass of customers
$\tau$	disutility parameter for content misspecification
$\delta$	disutility parameter for advertising

Table 1: Notation

viewers decide which channel to view. At this stage, advertisers on a platform exert a negative external effect on viewers of this platform and viewers of a platform exert a positive external effect on advertisers on this platform. Advertisers and viewers play an anonymous game. We can summarize the game as follows.

- Stage 1: Media platforms simultaneously decide on content  $d_i$ .
- Stage 2: Media platforms simultaneously decide on advertising space  $a_i$  and pay-per-view prices  $s_i$ , where  $s_i = 0$ , under free-to-air.
- Stage 3: Advertisers and viewers simultaneously take their decisions: advertisers place their ads in none, one, or both programs and viewers view program 1 or program 2.

We characterize subgame perfect Nash equilibria of this game. Note that in stage 2, media platforms are quantity setters in the advertising market.<sup>12</sup> Advertising charges  $r_i$  and  $r_j$  clear the market. Since  $F$  is invertible on the support of  $\alpha$  equation (2) can be rewritten. This gives expressions for the advertising charges

$$r_i = Nb_i F^{-1}(1 - a_i).$$

Our notation is summarized in Table 1.

**Welfare.** In our model with full viewer participation only the amount of advertising and the content of programs affect welfare. Hence welfare is the sum of a constant

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<sup>12</sup>Since advertising space fully determines the advertising rate per viewer it does not matter whether platforms set advertising space or rate per viewer. However, setting a rate per viewer requires that platforms are able to monitor the number of viewers.

$K = Nv$ , welfare with respect to content, and welfare with respect to advertising,  $W = K + W^{co} + W^{ad}$ . Welfare with respect to content is

$$W^{co} = -N\tau \int_0^{b_1} (\beta - d_1)^2 d\beta - N\tau \int_0^{b_2} (\beta - d_2)^2 d\beta.$$

Welfare with respect to advertising is

$$W^{ad} = Nb_1 \int_{\min\{\underline{\alpha}_1, \alpha^{\max}\}}^{\alpha^{\max}} (\alpha - \delta) dF(\alpha) + Nb_2 \int_{\min\{\underline{\alpha}_2, \alpha^{\max}\}}^{\alpha^{\max}} (\alpha - \delta) dF(\alpha).$$

**Social Optimum.** Welfare is maximized with respect to content when  $d_1^W = d_2^W = 1/4$  – these are the locations which minimize transportation costs if transportation costs are strictly convex as it is the case for our quadratic specification. Consider now welfare with respect to advertising. The derivative of  $W^{ad}$  with respect to  $\underline{\alpha}_i$  is

$$\frac{\partial W^{ad}}{\partial \underline{\alpha}_i} = -Nb_i(\underline{\alpha}_i - \delta)f(\underline{\alpha}_i).$$

If  $\delta < \alpha^{\max}$  the welfare maximizing number of advertisers is determined by  $\underline{\alpha}_i = \delta$  so that  $a_i^W = 1 - F(\delta)$ . Otherwise,  $a_i^W = 0$ . In the special case that  $F$  is uniform on  $[0, 1]$ ,  $a_i^W = \max\{1 - \delta, 0\}$ .

### 3 Pay-tv

In this section we analyze competition between pay-tv stations. Hence, each media platform has two instruments to make profits: it can sell advertising time to advertisers and it can charge buyers for watching its program. For given advertising levels and prices, advertisers and viewers take their decisions at stage 3: advertisers place their ads in none, one, or both programs and viewers view program 1 or program 2. The solution to the system of equations (1), (2) for  $i = 1, 2$  and  $b_2 = 1 - b_1$  is the equilibrium in stage 3. This determines how advertising charges react to pay-per-view prices  $s_i$  and to advertising levels  $a_i$ , which are set in stage 2.

**Equilibrium for given program content.**<sup>13</sup> The (subgame perfect) equilibrium at stage 2 is characterized by the system of four first-order conditions

$$\frac{\partial \pi_i}{\partial s_i} = N \left( b_i + \frac{\partial b_i}{\partial s_i} s_i \right) + a_i \frac{\partial r_i}{\partial s_i} = 0, \quad i = 1, 2 \quad (3)$$

$$\frac{\partial \pi_i}{\partial a_i} = N \frac{\partial b_i}{\partial a_i} s_i + r_i + a_i \frac{\partial r_i}{\partial a_i} \leq 0, \quad i = 1, 2 \quad (4)$$

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<sup>13</sup>This part of the analysis with exogenous content is related to section 6.1 in Anderson and Coate (2003) and section 5.2 in Armstrong (2004).

First-order conditions (3) must hold with equality if we allow for subsidies to viewers. In contrast, the amount of advertising cannot be negative. Therefore, first-order conditions (4) hold with equality only for interior solutions.

The advertising revenue per viewer is  $\rho(a_i) \equiv a_i F^{-1}(1 - a_i)$ . We need that this function is single-peaked, which is implied by the following assumption.

**Assumption.** *The advertising revenue per viewer  $\rho$  is concave in  $a_i$ .*

Note that in the uniform case  $\rho(a_i) = a_i(1 - a_i)$ , which is concave. Rewriting the first-order conditions for platform  $i$ , one obtains

$$b_i + [s_i + \rho(a_i)] \frac{\partial b_i}{\partial s_i} = 0, \quad (5)$$

$$\rho'(a_i) b_i + [s_i + \rho(a_i)] \frac{\partial b_i}{\partial a_i} \leq 0. \quad (6)$$

These two first-order conditions determine platform  $i$ 's amount of advertising. This is independent of content, own pay-per-view price and the competing platform's decisions, as stated in the following lemma.

**Lemma 1** *Media platform  $i$ 's profit maximizing amount of advertising is a constant. It is determined by*

$$\rho'(a_i) = \delta \quad (7)$$

if  $\delta < \alpha^{\max}$ . Otherwise it is 0.

**Proof.** Since  $\partial b_i / \partial a_i = \delta \partial b_i / \partial s_i$  inequality (6) can be rewritten as

$$\frac{\rho'(a_i)}{\delta} b_i + [s_i + \rho(a_i)] \frac{\partial b_i}{\partial s_i} \leq 0$$

If this condition is satisfied with equality so that  $a_i$  is positive, this condition together with (5) implies (7). Since  $\rho$  is concave, the best-response  $a_i$  is uniquely determined by (7). When  $\delta \geq \alpha^{\max}$  the condition holds with strict inequality, the platform is constrained by the fact that advertising space cannot be negative. Hence,  $a_i = 0$  and in this case the solution coincides with the social optimum. ■

Providing a fixed advertising space, which depends on the disutility parameter for advertising, is a best-response property of each media platform. As the disutility parameter becomes smaller, the profit maximizing advertising space  $a_i$  increases. In the absence of the viewers' reaction to advertising, i.e.  $\delta = 0$ , each media platform provides the monopoly advertising space  $\rho'(a_i) = 0$ .

In the uniform case advertising is  $a_i = \max\{(1 - \delta)/2, 0\}$ . For a sufficiently high disutility from advertising, in the example  $\delta > 1$ , the social optimum is implemented,

which is zero advertising. For a sufficiently low disutility from advertising, there is an underprovision of advertising. The reason is that platforms cannot absorb all rents from advertisers. In general, we have that in pay-tv markets there is always underprovision of advertising until the market is shut down by the media platforms.

To determine the pay-per-view price we solve the two first-order conditions (3) for given advertising space  $a_i$ ,  $i = 1, 2$

$$s_i = \frac{(1 - d_i - d_j)(3 + d_i - d_j)}{3}\tau + \frac{\delta(a_j - a_i)}{3} - \frac{2\rho(a_i) + \rho(a_j)}{3}.$$

The right-hand side of the expression which determines  $s_i$  consists of three terms: the standard Hotelling term and two terms that depend on advertising. The first of those two latter terms captures the viewers' disutility from ads. If platform  $i$  admits more advertising than the competing platform it has to compensate its own subscribers via a lower pay-per-view price. In other words, if advertising was given exogenously, more advertising on platform  $i$  leads to an asymmetry between platforms and the equilibrium price reaction is for platform  $i$  to price more aggressively and for platform  $j$  to price less aggressively. The second of those two latter terms reflects the role of advertising to generate revenues. The higher the advertising revenues per viewer  $\rho$  the more attractive viewers are. Hence, prices are lowered to subsidize viewers. This illustrates that the platform can adjust the pay-per-view price to the advertising space it provides.

As implied by Lemma 1, both firms provide the same advertising space,  $a_1 = a_2$ . Hence, in the equilibrium of stage 2

$$s_i = \frac{(1 - d_i - d_j)(3 + d_i - d_j)}{3}\tau - \rho(a_i).$$

There is a full pass-through of advertising revenues into lower pay-per-view prices. An immediate consequence is that the advertising revenues do not affect equilibrium profits of the two platforms (profit neutrality). Notice that the disutility parameter for ads,  $\delta$ , does not enter directly into the expression for equilibrium prices. However, it affects prices indirectly through the advertising space  $a_i$ .

In the uniform case, we can write the equilibrium pay-per-view prices as

$$s_i = \frac{(1 - d_i - d_j)(3 + d_i - d_j)}{3}\tau - \frac{(1 + \delta)}{2} \max \left\{ \frac{1 - \delta}{2}, 0 \right\}$$

so that  $s_i$  is decreasing in  $\delta$  for  $\delta < 1$  (this reflects the pass-through result as advertising revenues are maximized for  $\delta = 0$  and decrease for higher values of  $\delta$ ), and constant in  $\delta$  for  $\delta > 1$ . Equilibrium profits are

$$\pi_i = (1 - d_i - d_j) \frac{(3 + d_i - d_j)^2}{18} \tau N.$$

**Remark 3** *The result on equilibrium advertising space and viewers' prices has an analogy with two-part pricing. In our model, advertisers are the "sticky" part of the market since viewers are competitive bottlenecks. Platforms have two instruments at their disposal to attract viewers, advertising space and subscription prices. As with two-part prices, each platform sets the advertising space that maximizes the joint surplus of the platform and its viewers. It then extracts part of this joint surplus using the fixed subscription fee according to the intensity of competition with the rival platform. This explains why the platform decides to shut down the advertising market when  $\delta$  is higher than  $\alpha^{\max}$ : the disutility for viewers always exceeds any profit that could be made from advertisers. It also explains why, for lower values of  $\delta$ , the solution is given by (7) and there is always under-provision of advertising: the platform takes into account only the surplus of its viewers but not the advertisers' and sets the advertising space that equates at the margin the revenue per viewer to its disutility.*

**The Provision of Program Content.** Because of profit neutrality the analysis at the first stage reduces to the standard Hotelling model with quadratic transportation costs (d'Aspremont et al., 1979). Hence, media platforms locate at the extremes,  $d_1 = d_2 = 0$ . This shows that the result by Gabszewicz et al. (2001, 2002) still holds when consumers dislike ads, as long as media platforms can use unrestricted pay-per-view prices.

**Proposition 1** *In the unique subgame perfect equilibrium of the pay-tv market media platforms maximally differentiate media content. For  $\delta < \alpha^{\max}$ , both platforms provide advertising space according to  $\rho'(a_i) = \delta$  and pass all advertising revenues  $\rho$  onto viewers. For  $\delta > \alpha^{\max}$ , both platforms provide advertising-free programs. Equilibrium prices are  $s_i = \tau - \rho(a_i)$  and profits are neutral in  $\delta$ .*

Consider the uniform case and suppose that  $\delta < 1$ . As mentioned above,  $a_i = (1 - \delta)/2$ . The equilibrium prices are

$$s_i = \frac{1}{4} (4\tau - (1 - \delta^2)).$$

If  $4\tau > 1 - \delta^2$ , that is, there is enough potential content diversity, viewers have to pay a positive price. If the reverse holds, viewers receive a subsidy. In the remainder we will consider both cases when the subsidy is feasible or not. If the subsidy is feasible in practice (e.g. viewers can receive free decoders or other benefits), then there is no restriction on parameters. If on the contrary we restrict prices to be non-negative, then differences between pay-tv and free-to-air can arise only when  $\tau > (1 - \delta^2)/4$  since when  $\tau < (1 - \delta^2)/4$  the two systems would be identical (price is zero in both cases). We show below that this distinction between negative and non-negative prices does not affect our welfare results in a meaningful way.

Suppose, on the contrary, that  $\delta \geq 1$ . In this case  $p_i = \tau$ , as in the standard Hotelling model — in fact, this holds outside the uniform case if  $\delta \geq \alpha^{\max}$ .

## 4 Free-to-Air

In this section we analyze competition between free-to-air stations. Hence, each media platform can only make profits from advertising. For given advertising levels, advertisers and viewers take their decisions at stage 3.

**Equilibrium for given program content.**<sup>14</sup> The profit of media platform  $i$  is  $\pi_i = a_i r_i = N\rho(a_i)b_i$ . The first-order condition of profit maximization then becomes

$$\rho'(a_i)b_i + \rho(a_i)\frac{\partial b_i}{\partial a_i} = 0.$$

Rewriting equation (1) for free-to-air, when  $s_1 = s_2 = 0$ , we have

$$b_1 = \frac{d_1 + 1 - d_2}{2} - \frac{\delta(a_1 - a_2)}{2(1 - d_1 - d_2)\tau}$$

and  $b_2 = 1 - b_1$ . The system of first-order conditions can be written as

$$\rho'(a_i)b_i - \frac{\rho(a_i)\delta}{2(1 - d_1 - d_2)\tau} = 0.$$

Note that the second-order condition

$$\rho''(a_i)b_i + 2\rho'(a_i)\frac{\partial b_i}{\partial a_i} < 0$$

is satisfied because  $\rho$  has been assumed to be concave.

At a symmetric equilibrium given symmetry in the content space, i.e.  $d_1 = d_2 \leq 1/2$ , the market for viewers is evenly split, i.e.  $b_i = 1/2$ , and advertising levels satisfy

$$\rho'(a_i) = \frac{\rho(a_i)\delta}{(1 - 2d_i)\tau}. \quad (8)$$

This means that for any parameter constellation  $\delta > 0$ ,  $\tau > 0$  and  $d_1 = d_2 < 1/2$  there is a strictly positive amount of advertising that solves (8) because  $\rho(0) = 0$ . Note that the above equation uniquely determines  $a_i$  if  $\rho$  is log-concave and log-concavity is implied by concavity; the concavity of  $\rho$  has been assumed above. It is clear that  $a_i$  cannot be zero in equilibrium so that for  $\delta$  sufficiently large there is overprovision of advertising. To see this, imagine platforms offer zero advertising and do not duplicate content. They make zero profits and market shares are determined according to the positions in the content space. If a platform places a small amount of advertising it will lose some but not all viewers. Hence, it can make positive profits. Only if programs are perfectly duplicated profits are equal to zero, provided that  $\delta > 0$ . In fact, if platforms

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<sup>14</sup>Section 4.2 of Anderson and Coate (2003) provides a related analysis for fixed extreme locations, i.e.  $d_1 = d_2 = 0$ .

share the very same location on the Hotelling line there is an escalation to provide less advertising than the rival to attract viewers. This continues until the market for advertisers is shut down and equilibrium profits are equal to zero. Hence, we should not expect content duplication if  $\delta > 0$ . Different from free-to-air we also would not expect profit neutrality with respect to  $\delta$ .

In the special case that  $\delta = 0$ , advertising space is provided according to  $\rho'(a_i) = 0$  as in the monopoly case and the amount of advertising would be same as under pay-tv. In both cases platforms extract full monopoly profits from advertisers as viewers do not react to them. For any  $\delta > 0$  the amount of advertising is strictly lower than the monopoly.

How does the equilibrium advertising depend on the nuisance parameter  $\delta$ ? As stated above, when  $\delta = 0$  advertising would be set at the monopoly level. As  $\delta$  increases, viewers react negatively to ads and we should expect advertising levels to decrease with  $\delta$  as long as platforms do not share the same location. Furthermore, we should expect advertising space to be rather insensitive to  $\delta$  when content is very similar. This is confirmed by the following proposition.

**Proposition 2** *The equilibrium advertising space decreases as advertising becomes more of a nuisance,  $da_i/d\delta < 0$ . However, if programs are of almost identical content, equilibrium advertising space is insensitive to the nuisance parameter, i.e.*

$$\lim_{d_i \rightarrow 1/2} (da_i/d\delta) = 0.$$

**Proof.** Totally differentiating equation (8) we obtain

$$\begin{aligned} \rho''(a_i)da_i &= \frac{\rho'(a_i)\delta}{(1-2d_i)\tau}da_i + \frac{\rho(a_i)}{(1-2d_i)\tau}d\delta \\ \frac{da_i}{d\delta} &= \frac{\frac{\rho(a_i)}{(1-2d_i)\tau}}{\rho''(a_i) - \frac{\rho'(a_i)\delta}{(1-2d_i)\tau}} = \frac{\rho(a_i)}{(1-2d_i)\tau\rho''(a_i) - \rho'(a_i)\delta} \end{aligned}$$

Since  $\rho$  is concave and, in equilibrium,  $\rho' > 0$ , the amount of advertising necessarily decreases as viewers have a stronger preference against advertising, i.e.,  $da_i/d\delta < 0$ . Taking the limit as  $d_i \rightarrow 1/2$  we have

$$\lim_{d_i \rightarrow 1/2} \left( \frac{da_i}{d\delta} \right) = - \lim_{d_i \rightarrow 1/2} \frac{\rho(a_i)}{\rho'(a_i)\delta},$$

that is, if programs are almost duplicated, the marginal reduction in advertising is  $\rho(a_i)/\delta\rho'(a_i)$ , where the equilibrium advertising space depends on  $d_i$ . From (8) it follows that we can substitute  $\rho(a_i)/\rho'(a_i)$  by  $(1-2d_i)\tau/\delta$ . Hence,

$$\lim_{d_i \rightarrow 1/2} \left( \frac{da_i}{d\delta} \right) = - \lim_{d_i \rightarrow 1/2} \frac{(1-2d_i)\tau}{\delta^2} = 0. \blacksquare$$



In the uniform case, we obtain explicit expressions for the amount of advertising at symmetric locations  $d_1 = d_2$ ,

$$a_i = \frac{1}{2} + (1 - 2d_1)\frac{\tau}{\delta} - \sqrt{\frac{1}{4} + (1 - 2d_1)^2\frac{\tau^2}{\delta^2}}. \quad (9)$$

We can also answer how advertising reacts to the nuisance parameter, when this parameter is small, that is, when consumers do not mind much being exposed to advertising. Using equation (9), we can write

$$\frac{da_i}{d\delta} = -\frac{(1 - 2d_1)\tau \left[ \sqrt{\frac{\delta^2}{\tau^2} + 4(1 - 2d_1)^2} - 2(1 - 2d_1) \right]}{\delta^2 \sqrt{\frac{\delta^2}{\tau^2} + 4(1 - 2d_1)^2}}$$

Note that for  $\delta$  turning to zero, numerator and denominator both turn to zero. Using the L'Hospital rule, we have

$$\lim_{\delta \rightarrow 0} \frac{da_i}{d\delta} = -\frac{1}{8(1 - 2d_1)\tau}. \quad (10)$$

This shows that equilibrium advertising reacts strongly to the nuisance parameter  $\delta$  for  $\delta$  small, provided platforms offer similar content.

We now turn to the equilibrium analysis at the stage where platforms choose content.

**The Provision of Program Content.** The relocation tendency is expressed by

$$\begin{aligned} & \frac{\partial \pi_i}{\partial d_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} \\ &= N\rho(a_i) \left( \frac{\partial b_i}{\partial d_i} + \frac{\partial b_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} \right). \end{aligned} \quad (11)$$

There is no relocation tendency for interior solutions if the first-order condition at stage 1 holds, which can be written as

$$\frac{\partial b_i}{\partial d_i} + \frac{\partial b_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} = 0. \quad (12)$$

This equation can be expressed as

$$\frac{1}{2} + \frac{\delta(a_i - a_j)}{2(1 - d_i - d_j)^2\tau} + \left( \frac{\delta}{2(1 - d_i - d_j)\tau} \right) \frac{\partial a_j}{\partial d_i} = 0.$$

For symmetric locations, this simplifies to

$$\frac{1}{2} + \frac{\delta}{2\tau(1-2d_i)} \frac{\partial a_j}{\partial d_i} \Big|_{d_i=d_j} = 0. \quad (13)$$

At symmetric locations, more similar content leads to less advertising in equilibrium. This is stated in the following lemma.

**Lemma 2** *If  $\delta > 0$  then, at a symmetric equilibrium,  $(\partial a_j / \partial d_i) < 0$ .*

**Proof.** To determine the effect of a change in content on advertising space, we have to determine

$$\frac{\partial a_j}{\partial d_i} = - \frac{\frac{\partial^2 \pi_j}{\partial a_j \partial d_i} \frac{\partial^2 \pi_i}{(\partial a_i)^2} - \frac{\partial^2 \pi_i}{\partial a_i \partial d_i} \frac{\partial^2 \pi_j}{\partial a_j \partial a_i}}{\frac{\partial^2 \pi_j}{(\partial a_j)^2} \frac{\partial^2 \pi_i}{(\partial a_i)^2} - \frac{\partial^2 \pi_j}{\partial a_j \partial a_i} \frac{\partial^2 \pi_i}{\partial a_i \partial a_j}}.$$

In symmetric equilibrium, i.e.  $d_1 = d_2$ , this can be evaluated as

$$\frac{\partial a_j}{\partial d_i} \Big|_{d_i=d_j} = - \frac{2\rho\delta[\frac{\delta^2}{\tau^2}(2-d_1)\rho - \rho''(1-d_1)(1-2d_1)^2]}{\tau[-\frac{\delta^4}{\tau^4}\rho^2 + (2\frac{\delta^2}{\tau^2}\rho - \rho''(1-2d_1)^2)^2]} < 0. \quad \blacksquare$$

As a reference point consider the case that viewers do not care about advertising, i.e.  $\delta = 0$ . We find that

$$\frac{\partial \pi_i}{\partial d_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} > 0$$

for all  $d_i$ , evaluated at  $d_1 = d_2$ . This means that platforms have a tendency to provide more similar content so that program duplication results. This generalizes the result obtained by Gabszewicz et al. (2001, 2002). However, this result is not robust to introducing advertising as a nuisance. Consider the case  $\delta > 0$ . At  $d_1 = d_2 \rightarrow 1/2$ ,  $\partial a_j / \partial d_i \Big|_{d_i=d_j} = -\tau/\delta$ . Hence, expression (13) simplifies to  $1/2 - 1/[2(1-2d_i)]$  that is negative as  $d_i \rightarrow 1/2$ . Therefore, for any  $\delta > 0$  program duplication cannot occur.

**Lemma 3** *If  $\delta > 0$ , program duplication does not occur.*

This shows that the minimum differentiation result is not robust to introducing advertising as a nuisance in the viewers's utility function. The reason goes as follows: if  $\delta > 0$ , the platform with the smaller amount of advertising attracts all viewers under program duplication. Platforms would compete in a Bertrand-fashion by reducing the advertising space. If a platform differentiates its program by moving away from the center it can make positive revenues.

It is difficult to provide a full characterization of the equilibrium for any parameter constellation in the general case. Still, we can discuss some limiting cases. What happens if the nuisance parameter is large? In this case, maximal differentiation holds, as stated in the following lemma.

**Lemma 4** *If  $\delta$  is sufficiently large, media platforms maximally differentiate content.*

**Proof.** In the limit we obtain that

$$\lim_{\delta \rightarrow \infty} \left. \frac{\partial a_j}{\partial d_i} \right|_{d_i=d_j} = - \lim_{\delta \rightarrow \infty} \frac{2(2-d_i)\tau}{3\delta} = 0.$$

For large  $\delta$ , the derivative  $\partial a_j / \partial d_i|_{d_i=d_j}$  can be approximated by  $-2(2-d_i)\tau/(3\delta)$ . To obtain an interior solution, the first-order condition at stage 1 has to hold. Expression (13) simplifies to

$$\frac{1}{2} - \frac{2-d_i}{3(1-2d_i)} = 0.$$

However, since the left-hand side is always negative, there is a tendency to offer more differentiated content for symmetric locations. This shows that, for any symmetric equilibrium candidate,  $d_1 = d_2 = 0$ .

This result extends to finite values of  $\delta$  as long as  $\delta$  is high enough. In fact,

$$\begin{aligned} & \left. \frac{\partial \pi_i}{\partial d_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} \right|_{d_i=d_j=0} \\ &= \frac{1}{2} - \frac{\delta^2}{\tau^2} \frac{\rho(2\frac{\delta^2}{\tau^2}\rho - \rho'')}{-\frac{\delta^4}{\tau^4}\rho^2 + (2\frac{\delta^2}{\tau^2}\rho - \rho'')^2}. \end{aligned}$$

We have to show that the right-hand side of the equation is negative for  $\delta$  sufficiently large. This is equivalent to

$$\frac{1}{2} < \frac{\delta^2}{\tau^2} \frac{\rho(2\frac{\delta^2}{\tau^2}\rho - \rho'')}{-\frac{\delta^4}{\tau^4}\rho^2 + (2\frac{\delta^2}{\tau^2}\rho - \rho'')^2}.$$

For large  $\delta$  the right-hand side is approximately  $2/3$ . Equivalently, the previous inequality holds if  $\delta > \tau\sqrt{(1+\sqrt{2})(-\rho'')/\rho'}$ , which is a finite value since at equilibrium  $\rho'$  is positive. ■

For lower values of the disutility parameter we can show that the location of content is monotonically decreasing in  $\delta$ . From expression (13) we conduct comparative statics obtaining:

$$\begin{aligned} & \text{sign} \left( \left. \frac{\partial d_i}{\partial \delta} \right|_{d_i=d_j} \right) \\ &= \text{sign} \left( 2\rho''(1-2d_i)^3\tau^2[\delta^4\rho^2(5-d_i) \right. \\ & \quad \left. - 2(1-2d_i)^2(2-d_i)\delta^2\tau^2\rho\rho'' + (1-2d_i)^4(1-d_i)\tau^4\rho''^2] \right) < 0. \end{aligned}$$

We summarize our findings in the following proposition.

**Proposition 3** *In subgame perfect equilibrium of the free-to-air market, media platforms do not minimally differentiate media content for  $\delta > 0$ . Content differentiation is increasing in  $\delta$  and reaches maximal differentiation for  $\delta$  sufficiently large.*

We can also ask what happens as viewers view programs of different content as hardly (respectively very) substitutable, i.e., as  $\tau$  becomes large (respectively small). We do not have to run a formal analysis since it suffices to notice that advertising levels from (8) and content choice from (13) only depend on the ratio  $\delta/\tau$ . Hence the results we obtained for small (respectively large)  $\delta$  can be rephrased in terms of a large (respectively small) value of  $\tau$ . We thus state without proof the following result.

**Proposition 4** *In subgame perfect equilibrium of the free-to-air market, for given  $\delta > 0$ , media platforms do not maximally differentiate media content for sufficiently large  $\tau$ , while they choose maximal differentiation for  $\tau$  sufficiently small. Content duplication is increasing in  $\tau$ .*

Notice that equilibrium profits decline with  $\delta$ . This is because profits arise only from advertising and in a symmetric equilibrium each platform has 50% of the viewers. Thus profits are  $N\rho(a_i)/2$ . Function  $\rho(\cdot)$  is maximized when  $\rho'(a_i) = 0$  which from (8) occurs only when  $\delta = 0$ . Recalling that  $\partial a_i/\partial\delta < 0$ , it follows immediately that equilibrium profits are monotonically decreasing in  $\delta$ ,  $\partial\pi_i/\partial\delta = N(\rho'(a_i)/2)(\partial a_i/\partial\delta) < 0$ .

To explicitly determine equilibrium content, we return to the uniform case. Expression (13) in the linear case is:

$$\sqrt{\frac{1}{4} + (1 - 2d_1)^2 \frac{\tau^2}{\delta^2}} = \frac{(1 + 4d_1) + 4(1 - 2d_1)^2(3 + 4d_1) \frac{\tau^2}{\delta^2}}{2(2 - 3d_1 - 2d_1^2) \frac{\tau}{\delta}}$$

The previous equation implicitly defines equilibrium content as a function of the nuisance parameter and of the transportation cost when an interior equilibrium exists. Studying the LHS and the RHS of the previous equation, it can be shown that there is a unique root  $0 < d_1 < 1/2$  that satisfies the equation as long as  $\delta/\tau$  is low enough. When  $\delta/\tau$  is high enough, the LHS is always smaller than the RHS, thus the relocation tendency is always negative and extreme content differentiation is chosen. To be precise, maximal content differentiation arises when  $\delta/\tau$  is above the critical value  $\delta/\tau = 2\sqrt{5 + 4\sqrt{2}} \approx 6.53$ .<sup>15</sup>

Figure 1 gives the equilibrium programming content for  $\tau = 1$ . In line with the general case, for  $\delta = 0$  there is program duplication and for  $\delta$  sufficiently large ( $\delta > 6.53$ ) there is maximal content differentiation. For lower  $\delta$  the differentiation between programs,  $1 - 2d_i$ , is a decreasing, concave function in  $\delta$ .

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<sup>15</sup>In this analysis we did not check whether the solution to the first-order condition is indeed a maximizer so that the existence of equilibrium is not necessarily assured. In the appendix we address this issue.

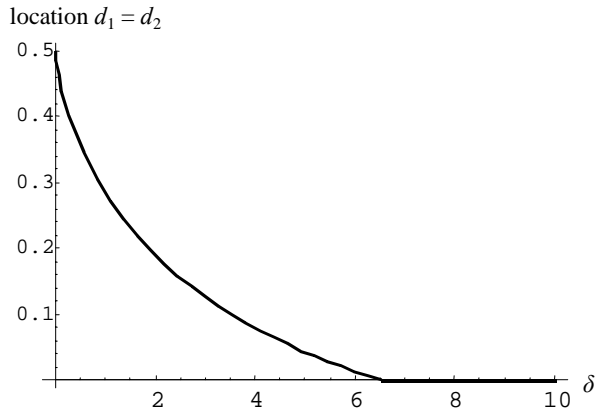


Figure 1: Content provision under free-to-air

## 5 Pay-Tv Markets versus Free-to-Air Markets

### 5.1 Advertising and Content Comparison

In this section we compare the market outcomes under the two different schemes, pay-tv and free-to-air. We start by comparing pay-tv and free-to-air for exogenous content provision.

**Exogenous content provision.** In pay-tv advertising is given by (7) while in free-to-air market it is given by (8). Since a monopolist would set  $\rho'(a_i) = 0$ , it follows that competing media platforms provide less advertising than a monopolist. Only at  $\delta = 0$ , the amount of advertising space is the same under pay-tv and free-to-air and coincides with the advertising provision of a monopolist. When  $\delta > \alpha^{\max}$ , then pay-tv implements the first-best advertising allocation, which in this case is 0 advertising, whereas free-to-air leads to a strictly positive amount of advertising, provided that content is not perfectly duplicated. In other words, if viewers strongly dislike ads, then pay-tv has necessarily less advertising. For lower values of  $\delta$ , if  $\rho(a_i) > (1 - 2d_i)\tau$  then free-to-air has again more advertising than pay-tv. Thus the general comparison between the two systems depends on content and the level of transportation costs. While content does not matter for advertising space in pay-tv, it does affect advertising in free-to-air. Similarly for transportation costs. Under perfect content duplication  $d_1 = d_2 = 1/2$ , advertising is zero for any value of  $\delta$  in free-to-air: in this case pay-tv either provides the same zero amount when  $\delta > \alpha^{\max}$ , or it strictly provides more advertising.

In the uniform case we make a more detailed comparison of equilibrium advertising depending on  $\delta$ ,  $\tau$ , and  $d_1(= d_2)$ . Advertising with free-to-air is always greater than advertising with pay-tv if  $\delta > 1$  and  $d_i < 1/2$ . If  $\delta < 1$  it is still greater if the following inequality holds:

$$\frac{1}{2} + (1 - 2d_1)\frac{\tau}{\delta} - \sqrt{\frac{1}{4} + (1 - 2d_1)^2\frac{\tau^2}{\delta^2}} > \frac{1 - \delta}{2}$$

which can be re-written as:

$$d_1 < \frac{1}{2} - \frac{1 - \delta^2}{8\tau}.$$

This inequality is likely to hold if programs are not easily substitutable ( $d_1$  small and  $\tau$  large). In particular, if one imagines extreme differentiation ( $d_1 = 0$ ), and recalling the non-negativity restriction on subscription prices derived in Section 3,  $\tau > (1 - \delta^2)/4$ , then the previous inequality is always satisfied. However this result does not hold in general if either subsidies are allowed or content is less differentiated. In fact, if programs are substitutable, competition for viewers under free-to-air makes platforms choose a restrictive advertising policy. This leads to less advertising than under pay-tv.

**Endogenous content provision.** Pay-tv always ends up providing maximal content differentiation, while free-to-air provides less diversity of content. Only when  $\delta$  is sufficiently high both systems provide the same (maximal) content diversity. Pay-tv content diversity is always excessive in terms of social welfare. On the other hand, there may be socially too little content diversity (if viewers do not strongly dislike ad) or excessive diversity (if viewers strongly dislike ads) under free-to-air. In the previous section we showed that location under free-to-air is monotonically decreasing between 0.5 and 0 as  $\delta$  increases. Hence there is always one particular value of  $\delta$  such that content provision under free-to-air is socially optimal ( $d_i = 1/4$ ).

The comparison of advertising space provided under the two schemes relates to the previous analysis, but now the content of the media platforms is endogenized. We illustrate the provision of advertising space in the uniform case (see Figure 2 for  $\tau = 1$ ; thus the non-negativity constraint on price,  $\tau > (1 - \delta^2)/4$ , is always satisfied).

For  $\delta = 0$ , both systems offer the same (monopoly) advertising level. As  $\delta$  is slightly increased, the chosen programs are close substitutes under free-to-air whereas there is maximal program differentiation under pay-tv. The slope of the function  $a_i(\delta)$  is  $-1/2$  under pay-tv whereas it is  $-\infty$  under free-to-air, when evaluated at  $\delta = 0$ . This extreme sensitivity of advertising arises from the endogenous location of free-to-air: at  $\delta = 0$  both platforms perfectly duplicate content, but as  $\delta$  increases they differentiate a bit, otherwise their advertising level would drop immediately from the monopoly level (when  $\delta = 0$ ) to zero (when  $\delta > 0$ ). When  $\delta$  is very small, programs are “almost” duplicated and advertising reacts very sharply to  $\delta$ . Hence, for the nuisance parameter  $\delta$  sufficiently small, there is less advertising under free-to-air than under pay-tv. Only when viewers strongly dislike advertising does pay-tv lead to less advertising. Since there is always social underprovision of advertising under pay-tv on the range of values for  $\delta$  such that the socially optimal amount of advertising is strictly positive, there is an

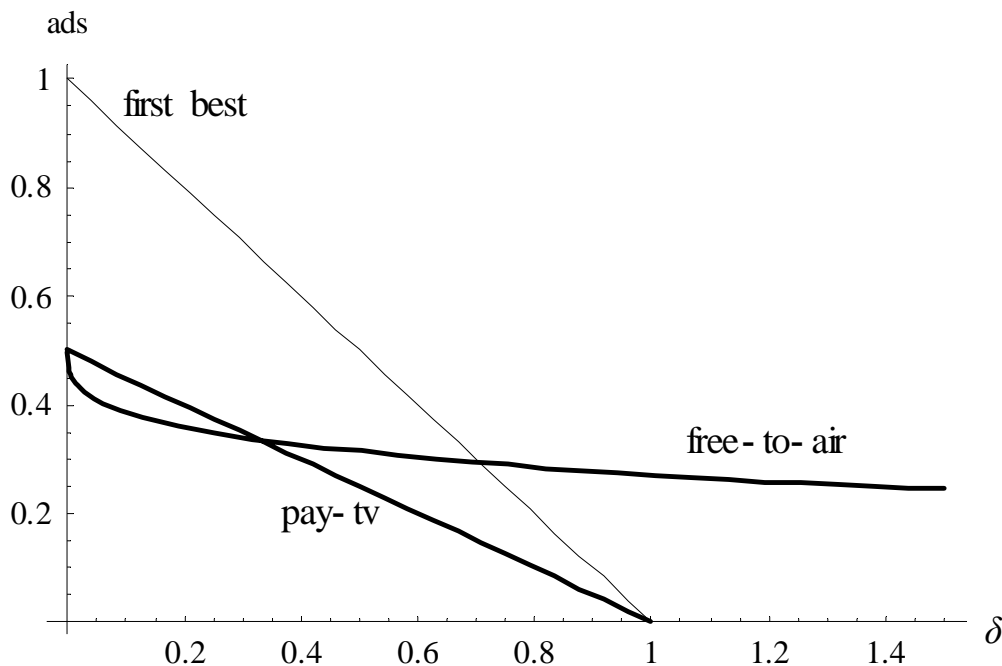


Figure 2: Advertising intensity with endogenous content

intermediate range of values for  $\delta$  such that the amount of advertising under free-to-air is socially better than under pay-tv. For larger values of  $\delta$  advertising under free-to-air is socially worse than under pay-tv. This is in particular the case for  $\delta > 1$ . These findings with respect to equilibrium advertising with endogenous content provision hold in the more general case provided that the indirect effect of the nuisance parameter on advertising is not too strong, as we summarize in the following proposition.

**Proposition 5** *Advertising with endogenous content provision is less under free-to-air than under pay-tv if  $\delta$  is sufficiently small and provided the direct effect of the nuisance parameter on advertising dominates the indirect in the limit as  $\delta$  turns to zero, and greater if  $\delta$  is sufficiently large. In the former case, social underprovision of advertising is more pronounced under free-to-air; in the latter case, free-to-air can lead to under- or overprovision of advertising.*

**Proof.** Lemma 1 describes the choice of advertising under pay-tv, while equation (8) describes advertising under free-to-air. If  $\delta > \alpha^{\max}$  then it follows immediately that free-to-air provides more advertising than pay-tv. If  $\delta = 0$  then the same level of advertising is chosen under both systems. Also, under pay-tv  $da_i/d\delta = 1/\rho''$  which always takes a finite value. On the other hand, under free-to-air for  $\delta$  close to zero we know that the endogenous content is “almost” the centre of the line. We can write

$$\lim_{\delta \rightarrow 0} \left( \frac{da_i(d_i(\delta), \delta)}{d\delta} \right) = \lim_{\delta \rightarrow 0} \left( \frac{\partial a_i(d_i(\delta), \delta)}{\partial \delta} \right) + \lim_{\delta \rightarrow 0} \left( \frac{\partial a_i(d_i, \delta)}{\partial d_i} \frac{\partial d_i}{\partial \delta} \right)$$

Consider the first expression on the right-hand side,

$$\lim_{\delta \rightarrow 0} \left( \frac{\partial a_i(d_i(\delta), \delta)}{\partial \delta} \right) = \lim_{\delta \rightarrow 0} \left( \frac{\rho(a_i)}{(1 - 2d_i(\delta))\tau\rho''(a_i) - \rho'(a_i)\delta} \right).$$

Since  $\rho'' \leq 0$  and  $\rho'$  is in equilibrium positive, the denominator is negative. Clearly the numerator is positive and, in equilibrium for  $\delta$  small, bounded away from zero. Hence the fraction is negative. Note furthermore that since  $\lim_{\delta \rightarrow 0} d_i = 1/2$  and  $\rho', |\rho''|$  have positive upper bounds, the denominator converges to zero. Hence, the whole expression tends to  $-\infty$ . The indirect effect

$$\lim_{\delta \rightarrow 0} \left( \frac{\partial a_i(d_i, \delta)}{\partial d_i} \frac{\partial d_i}{\partial \delta} \right)$$

has to be dominated by this direct effect. ■

## 5.2 Welfare Comparison

Our analysis in the previous subsection with respect to the social under- or overprovision of advertising leads naturally to a welfare analysis.

**Welfare with exogenous content provision.** For given content, the only difference in welfare under the two pricing schemes comes from different amounts of advertising. Our earlier results in Proposition 5 with respect to advertising levels directly translate into welfare results. In particular, for large  $\delta$  welfare is higher under pay-tv than under free-to-air because there is overprovision of advertising under free-to-air whereas the advertising level is socially optimal under pay-tv. For  $\delta$  sufficiently low, there is underprovision of advertising under the two pricing schemes. This underprovision is more pronounced under free-to-air when exogenous content of the two platforms is quite similar: in this case pay-tv is again socially desirable. However, there is an intermediate range of values for  $\delta$  and enough content differentiation such that there is severe underprovision of advertising under pay-tv whereas the advertising level is close to first-best levels under free-to-air. On this range, free-to-air is socially desirable.

In the uniform case, welfare under a pay-tv system and free-to-air are respectively:

$$\begin{aligned} W_{pay-tv}^{ad} &= \frac{3N}{8}(1 - \delta)^2 \text{ if } \delta < 1 \\ W_{free-to-air}^{ad} &= N(a_i(1 - \delta) - a_i^2/2) \text{ where } a_i \text{ is given by (9)} \end{aligned}$$

Figure 3 plots the results of the welfare comparison for  $\tau = 1$  (once again the non-negativity constraint on price,  $\tau > (1 - \delta^2)/4$ , is always satisfied). In line with



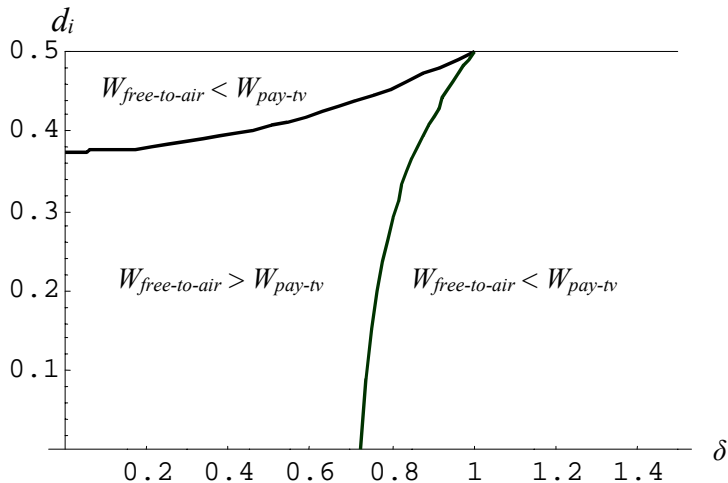


Figure 3: Welfare comparison for given differentiation of content

the general case, three regions arise. Free-to-air is preferred to pay-tv if the nuisance parameter is not too high and there is sufficient content diversity, i.e.  $d_i$  sufficiently small. Pay-tv is preferred if the nuisance parameter is sufficiently high or if content is sufficiently substitutable.

**Welfare with endogenous content provision.** The welfare comparison is somewhat more involved if platforms choose content. We start by making a number of observations.

First for  $\delta$  large, namely

$$\delta > \max\{\alpha^{\max}, \tau\sqrt{(1 + \sqrt{2})\rho''/\rho'}\},$$

platforms maximally differentiate content under both pricing schemes. In both cases there is the same (excessive) content diversity. Hence, only advertising matters for the welfare comparison: pay-tv is socially desirable for  $\delta$  large since it provides zero advertising while free-to-air overprovides it.

Second for  $\delta$  sufficiently small, free-to-air platforms do not maximally differentiate content so that free-to-air is socially preferred to pay-tv as far as content is concerned. Since on an intermediate range of  $\delta$  it is socially preferred also with respect to advertising, free-to-air leads to higher welfare than pay-tv for intermediate values of  $\delta$ .

The remaining question is which of the two schemes leads to higher welfare for  $\delta$  very small. Clearly, at  $\delta = 0$  both lead to the same welfare because in our model program duplication and maximal program differentiation involve the same welfare loss compared to the first-best and advertising levels under both schemes are equal to the monopoly advertising level. For very small  $\delta$ , the advertising level under pay-tv is

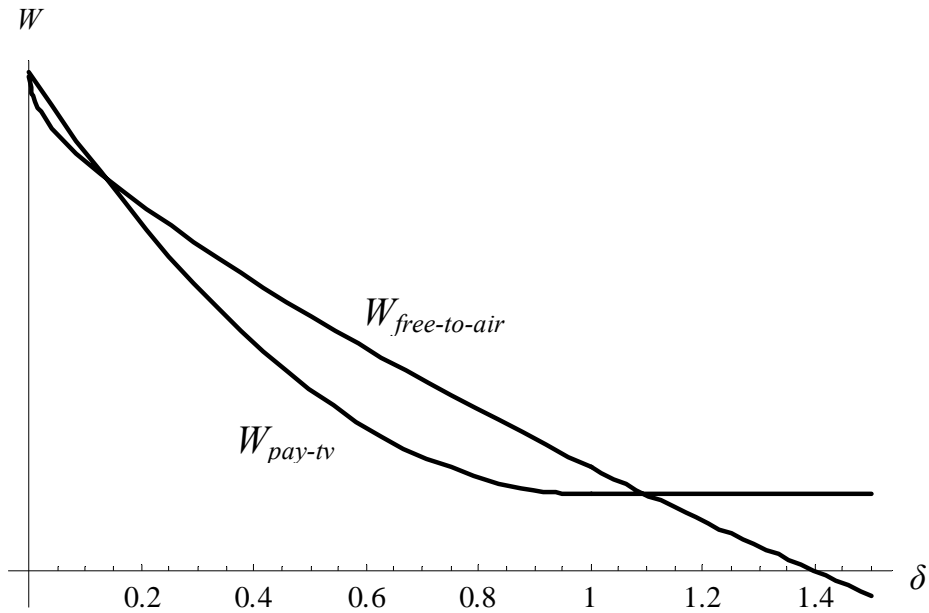


Figure 4: Welfare under pay-tv versus free-to-air for endogenous content

socially preferred but content provision under free-to-air is socially preferred. It has been argued above that, for given content, there is a more pronounced underprovision of advertising under free-to-air when content is almost perfectly duplicated. This still holds true under endogenous content provision: as  $\delta$  is increased slightly above 0, a free-to-air platform changes its content by a very small amount while it decreases sharply the number of ads it shows. Hence the welfare gain from better content is limited, while the under-provision of ads is exacerbated: pay-tv dominates over free-to-air for small nuisance if transportation costs  $\tau$  are not too high. This is illustrated for the uniform case in Figure 4 for  $\tau = 1$ . In this case, pay-tv has better welfare properties for  $\delta < 0.127$  and for  $\delta > 1.101$ .

If transportation costs  $\tau$  are small, pay-tv is preferred for a wide range of nuisance parameters: for highly competitive markets ( $\tau$  small) pay-tv dominates free-to-air for almost all  $\delta$  provided that viewers can be subsidized by platforms. Intuitively, as programs of different content are easily substitutable, under a free-to-air system platforms tend to advertise very little, exacerbating the underprovision result that would arise under pay-tv for most  $\delta$  such that the first-best provision of advertising is positive. However, for any  $\tau > 0$  there is an intermediate range of values for  $\delta$  such that free-to-air is the socially preferred scheme. For  $\tau$  small this range is narrow because the value for  $\delta$  such that free-to-air implements the first-best is close to 1. At  $\delta$  close to 1 the welfare loss under pay-tv due to the social underprovision of advertising is small compared to the first-best. In addition, for  $\delta$  close to 1 and  $\tau$  small, free-to-air also

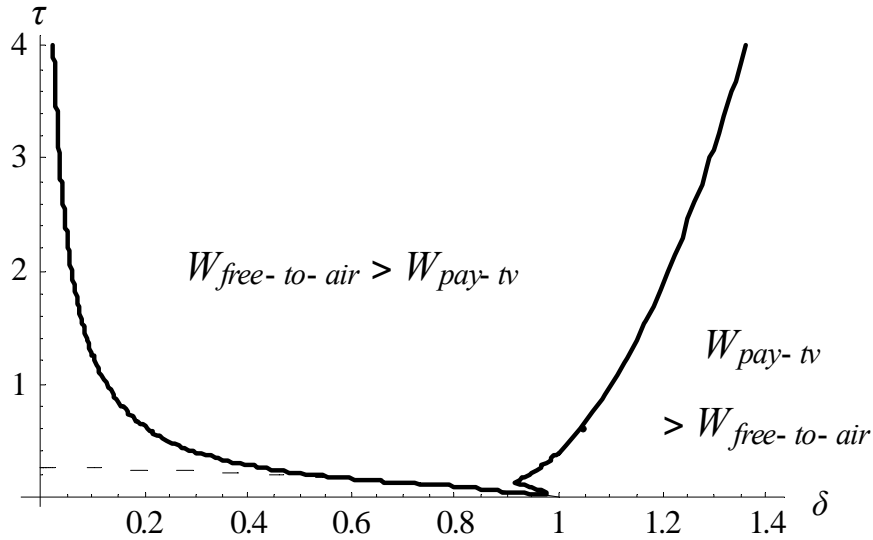


Figure 5: Welfare comparison in the  $\delta$ - $\tau$  space

leads to maximal content differentiation so that there is no social benefit from adopting free-to-air rather than pay-tv with respect to content.

As can be seen from Figure 5, the line in the  $\delta$ - $\tau$  space such that welfare under free-to-air is the same as welfare under pay-tv is essentially U-shaped. Notice that in the figure we have also reported the non-negativity constraint on price,  $\tau > (1 - \delta^2)/4$ . This is the dotted line in the bottom-left corner of the diagram. If pay-tv subscription prices cannot be negative, then the welfare analysis is only valid above the line (below the line the two systems lead to the same allocation and have the same welfare properties). If subsidies are allowed, then also the area below the curve can be included in the comparison. As it can be seen, the non-negativity constraint is immaterial for our main qualitative results.

If transportation costs are large, the nuisance parameter must be very small or very large for pay-tv to give higher welfare than free-to-air. For instance, if  $\tau = 100$ , then free-to-air is preferred for any  $\delta$  with  $0.000084 < \delta < 4.149$ . These findings generalize to the more general case.

**Proposition 6** *Welfare with endogenous content provision is greater under pay-tv, if, for given  $\tau$ , the nuisance parameter  $\delta$  is sufficiently small or sufficiently large or if, for given  $\delta$ , the potential differentiation between programs  $\tau$  is sufficiently small.*

**Sketch of Proof.** 1) Imagine that  $\delta$  is positive but close to zero. Then under free-to-air the location chosen is “almost” the centre of the line. Thus the two systems

reach approximately the same level of welfare over content. Free-to-air is marginally better than pay-tv with respect to content, but this is second-order compared to the underprovision of advertising which is much worse under free-to-air (provided Proposition 5 holds). Pay-tv results in higher welfare overall. 2) Take the case of high  $\delta$ , namely

$$\delta > \max\{\alpha^{\max}, \tau\sqrt{(1 + \sqrt{2})\rho''/\rho'}\}.$$

Both systems lead to maximally differentiated content, thus welfare differences arise only from advertising: free-to-air must result in lower welfare than pay-tv since the latter is efficient for this range of  $\delta$  while the former overprovides advertising. 3) Take the case of very small  $\tau$ . Content choice under free-to-air is characterized by Proposition 4. Welfare with respect to content is then the same for both systems. Differences in welfare arise only with respect to advertising. As  $\tau \rightarrow 0$  with  $d_i = 0$ , from (8) it turns out that free-to-air platforms tend to shut down the advertising market completely, while pay-tv simply underprovides advertising. The range for values of  $\delta$  such that free-to-air is socially better than pay-tv shrinks to zero as  $\tau \rightarrow 0$ . ■

When  $\tau$  is increased free-to-air is more likely to be socially preferred to pay-tv. As discussed in Section 4, under free-to-air platforms do not maximally differentiate over content for  $\delta$  not too large, thus free-to-air has better welfare properties with respect to content than pay-tv. However, from (8) free-to-air platforms tend to offer the monopoly level of advertising for  $\tau$  large, provided  $d_i$  is bounded away from  $1/2$ . Monopoly advertising according to  $\rho'(a_i) = 0$  becomes severely inefficient as the nuisance parameter  $\delta$  grows bigger. For sufficiently large  $\delta$  the welfare effect due to advertising dominates the welfare effect due to content and pay-tv is the preferred system.

## 6 Discussion and Conclusion

This paper has provided a positive and a normative analysis of competition between media platforms. Our main interest was the comparison between free-to-air and pay-tv. We asked under what conditions the ability to charge viewers directly makes commercial broadcasting like any other normal market, where competition is presumed to deliver efficiently without government intervention. We have shown how results depend quite crucially on the endogenous choice of content that is always extreme under a pay-tv system, while less so under free-to-air. We have found that, while market failures would still exist, pay-tv has better welfare properties than free-to-air when competition is very intense, or when the disutility from viewing ads is either very small or very large. In other situations trade-offs arise as in our model free-to-air typically provides better content diversification while it under- or overprovides advertising.

In the remainder we first review the role played by some of our modelling assumptions and discuss possible extensions and modifications. Secondly, we briefly discuss some implications of our model with respect to public policy, namely whether content

and advertising regulation is desirable. Finally, we relate our findings to the current debate on PSB obligations.

**Viewing only one channel or mixing between them.** We have assumed that viewers decide to view only one media program. This assumption may seem at first sight more appropriate for the newspaper market than a tv market, since there is persistence in a reader's choice of a newspaper over time. Clearly, our model can be applied to this market. In our view the application to media is also appropriate if one thinks that at any given point in time a viewer must choose one, and only one, channel. For instance, if two movies start at about the same time, a viewer will watch only one, not a mix of the two. Similarly for news programs, since by mixing one may either lose information or receive duplicates. Thus our approach has a literal interpretation as a single-product choice in some media markets. On the contrary, there are some media markets where mixing is more appropriate. For instance a listener may want to spend some time listening to classical music and some time listening to jazz. In this case, an alternative approach would be to follow Anderson and Neven (1989) and Gal-Or and Dukes (2003), where viewers are able to diversify their viewing/listening experience, obtaining a mix of programs to match their preferences. It can be shown that this alternative specification of individual preferences produces the same aggregate demand for both platforms if viewers, under pay-tv, pay only for the proportion of time they spend with a broadcaster (pay-per-view). Thus the same positive analysis conducted here would apply under this alternative approach. In addition, the same welfare analysis of advertising would hold. However, the welfare analysis with respect to content would differ since the first best of this alternative specification implies that the two platforms should be located at the extreme points of the line (so that every viewer would mix between the two programs), rather than the quartiles that are first best in our model. Thus, the welfare analysis under this alternative specification would tend to favor the pay-tv framework more often, as pay-tv always results in (efficient) maximal content differentiation.

**The role of expectations.** Some works have used the concept of "fulfilled" expectations, instead of solving the system of equations (1), (2) in the last stage of the game (see e.g. Ferrando et al., 2003). This alternative approach would simplify calculations but also lead to very peculiar results where the nuisance parameter does not play any role. Fulfilled expectations, in fact, cut any direct link between the two sides of the market. Hence the advertising space would always be set at the pure monopoly level, independently of  $\delta$ , both under free-to-air and under pay-tv. In the first stage, the location game would also be very simple: maximal differentiation would arise in pay-tv and minimal differentiation under free-to-air. As a consequence, total welfare would always be identical under both systems! As one departs from fulfilled expectations, all these results would not be robust.

**Advertising space and advertising prices.** A similar observation applies when the advertising rate is set exogenously, as assumed, for instance, by Gabszewicz et al.

(2004). Once again, there would be no direct link between the two sides of the market. When advertising rates are set by platforms instead, as we have already mentioned, our results would go unaltered if the choice variable is the advertising rate per viewer. On the other hand, the analysis would be considerably more complicated if platforms set lump sum advertising rates. This is addressed by Crampes et al. (2004) in a model with free entry along the Salop's circle, where the distance between firms is assumed to be equally spaced. They show that the viewers' subscription price under a pay-tv system is typically higher when platforms set advertising prices rather than quantities. This impacts on the number of firms that enter in equilibrium. The characterization of the equilibrium with endogenous content choice and advertising prices is left for further research.

**Competing for advertisers.** We have used a model of a "competitive bottleneck": if an advertiser wants to reach a particular viewer, he is obliged to contact him/her via a particular platform. Reisinger (2004) considers a model in which content is given but postulates that platforms compete for advertisers; more specifically, he assumes single-homing so that advertisers choose on which platform to advertise. He shows that if media platforms cannot charge viewers, closer substitutability of platforms for viewers can lead to higher profits because competition on the advertisers side is reduced. It seems interesting to study the provision of content in this context.

**Quality of programs.** In our model we have considered only one dimension of programming, and we have neglected that programs may be of different qualities. A broadcaster is able to obtain higher-quality programs by spending more money on production. A simple monopoly model with endogenous quality is likely to result in higher incentives to invest in high-quality programs with pay-tv than with free-to-air, since the firm can directly charge the viewers. Under competition, if quality is decided simultaneously with pricing (under pay-tv), then unambiguously pay-tv provides the socially optimal quality. This is again a result of what we called the "two-part pricing" analogy in the main text (Remark 3). Pay-tv broadcasters maximize joint surplus with their viewers, and extract part of such surplus using the subscription fee. Thus if quality has no commitment value, the welfare analysis under this extension would tend to favor the pay-tv framework more often than in our model. However, this conclusion is not so obvious when quality is decided prior to pricing decision. To see why, imagine our duopoly model with a linear distribution is simplified over one dimension (there is no nuisance from ads), but it is extended to account for quality: this is denoted as  $k_i$  and it is supplied by firm  $i$  at a cost  $C(k_i)$ . Quality and locations of programming are decided first, then platforms compete.

With pay-tv, the platforms charge a monopoly price to advertisers and price to viewers that reflects the quality differential. Equilibrium profits in the last stage are:

$$\pi_i = \frac{[k_i - k_j + (1 - d_i - d_j)(3 + d_i - d_j)]^2}{18(1 - d_i - d_j)} \tau N - C(k_i).$$

Restricting the attention to symmetric equilibria, in the first stage, the platforms choose a quality level defined by  $C'(k_i) = N/3$  *independently* from the location. If location is endogenized as well, the location game is the same one as before and platforms choose maximal differentiation.

With free-to-air, platforms still charge the monopoly price to advertisers and their equilibrium profits in the last stage are:

$$\pi_i = \frac{1 + d_i - d_j}{8}N + \frac{k_i - k_j}{8\tau(1 - d_i - d_j)} - C(k_i).$$

In the first stage, the platforms choose a quality level defined by  $C'(k_i) = N/[8\tau(1 - 2d_i)]$  in a symmetric equilibrium. Thus, if location is given exogenously, the quality level may be higher or lower than under pay-tv according to whether  $1/[8\tau(1 - 2d_i)]$  is greater or smaller than  $1/3$ . Since there is no nuisance from ads, once location is endogenized we again get the result of minimal differentiation. However, if  $d_i = 1/2$  there is no equilibrium in pure strategies over the quality level. Notice that the introduction of a small (but non zero) nuisance parameter should get rid of this extreme result. Platforms would locate close to each other but not in the mid-point. This implies that with a small nuisance parameter quality is likely to be higher under free-to-air since almost identical platforms tend to exhaust most of their advertising resources on program quality to attract bigger audiences. While this is only a conjecture, as we have not conducted a full analysis, it suggests that the monopoly and duopoly outcomes may differ considerably with endogenous location and quality choice.

**Content provision and content regulation.** In our analysis we have assumed that the content space is  $[0, 1]$ . However, it is conceivable that media platforms can offer more extreme content. Indeed, under pay-tv media platforms always choose content specification outside the  $[0, 1]$ -interval in the equilibrium of the model with an unconstrained content space. This in turn implies that pay-tv leads to a socially undesirable polarization of content that is not matched by the heterogeneity of viewers' tastes; this polarization is more pronounced than in the constrained version of the model. This shows that in the unconstrained version of our model pay-tv appears in a less favorable light than free-to-air. However, content regulation may effectively be of the sort that extreme content specifications, which apparently do not meet the tastes of viewers, are not allowed. Also note that under both systems, free-to-air and pay-tv, content regulation is welfare improving if one is prepared to believe that a social planner can identify and enforce the socially optimal content provision.

**Advertising bans or restrictions.** Our model provides some support for imposing advertising bans in the conventional world of free-to-air since if the nuisance parameter is high enough, this results in overprovision of advertising. In contrast, there is no reason for adopting restrictions under pay-tv. Note that in the case of an advertising ban under pay-tv all revenues must come from viewers. Advertising-free pay-tv no longer has the feature of a two-sided market and the standard Hotelling

analysis applies (see d'Aspremont et al., 1979). As a consequence, pay-tv with zero advertising leads to the same content as pay-tv with advertising but there is a more pronounced social underprovision of advertising in the presence of an advertising ban. This generalizes to binding advertising restrictions. Hence, an outcome of our model is that an advertising ban or restrictions under pay-tv are socially undesirable. While advertising restrictions can be introduced in an otherwise unregulated environment with private broadcasters, they have been imposed primarily in the context of public service broadcasting. In this context, we discuss the merits of advertising restrictions below.

**PSB obligations.** Our analysis contributes to the current debate on the merits of PSB obligations in the digital age. Public intervention in television is commonplace around the world. Almost every Western government supports PSB and funds it to some degree. The US stands out as a very important exception where funding comes almost exclusively from private sources. We have analyzed one particular aspect, namely the developments in payment mechanisms which create a range of different ways in which viewers pay for the programs they watch. Our approach has been rather extreme, in that we have assumed a “clean start” and compared the welfare properties of the two unregulated systems. Our results can be seen as a way of judging the implications of retaining PSB against a measurable alternative without PSB. Ofcom has recently concluded that in the UK the market alone would not provide a desirable range and diversity of TV content without the public subsidies now available, even in the digital age. Indeed, we have shown that the market does not provide an optimal diversity of content also in the digital age, which may call for some continuation of PSB obligations.<sup>16</sup> However, the second part of Ofcom’s statement is much more controversial as it is questionable if public subsidies could achieve the optimal choice.

Apart from requirements with respect to the diversity of TV content, PSB obligations often involve restrictions on advertising. In the extreme, such as for the BBC in the UK, advertising is banned. In other cases, such as Channel 4 in the UK which is not directly subsidized but gets cheap spectrum, and many public broadcasters in continental Europe, there is no advertising ban but advertising is subject to restrictions such as how much advertising they can air. Since restrictions on advertising reduce available revenues, subsidies can be seen as a compensation which allows broadcasters to break-even. PSB obligations with respect to advertising can be defended in our model under free-to-air because, for high nuisance parameters, there is social overprovision of advertising. In contrast, our model predicts that pay-tv cannot lead to an overprovision of advertising. Hence, PSB obligations which contain advertising

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<sup>16</sup>The move to digital has implications beyond those relating to the ability to charge subscribers. For instance, content can be personalized and stored much more easily than in the past. Digital compression techniques also provide capacity that should relax the spectrum constraint that has characterized the analogue era. The number of tv channels is thus likely to increase.



restrictions cannot be defended in our model under pay-tv. Then subsidies are not needed provided that unregulated pay-tv stations are profitable. This suggests that with respect to advertising PSB obligations are more questionable under pay-tv than under free-to-air.

### Appendix.

In this appendix we address the issue about the existence of subgame perfect equilibrium of the free-to-air market in the uniform case. The second-order condition at a symmetric interior equilibrium at stage 1 is:

$$\begin{aligned}
SOC|_{d_i=d_j} &= \frac{\delta}{2(1-2d_i)^2\tau} \left[ \frac{\partial a_i}{\partial d_i} - (1-2d_i) \frac{\partial^2 a_j}{\partial d_i^2} \right] \Big|_{d_i=d_j} \\
&= - \frac{32(2+d_1)(1-2d_1)}{[1+4(1-2d_1)^2\frac{\tau^2}{\delta^2}]^{3/2} [3+12d_1+4(1-2d_1)^2(1+8d_1)\frac{\tau^2}{\delta^2}]^2 \delta^3} \frac{\tau^3}{\delta^3} \\
&\quad \times \left( 4-5d_1+14d_1^2+14d_1^3+4(1-2d_1)^2d_1(49d_1^2+19d_1-3) \frac{\tau^2}{\delta^2} \right. \\
&\quad \quad \left. +8(1-2d_1)^4(106d_1^3+4d_1^2+6d_1-11) \frac{\tau^4}{\delta^4} \right. \\
&\quad \quad \left. -32(1-2d_1)^7(18d_1^2+6d_1+5) \frac{\tau^6}{\delta^6} \right)
\end{aligned}$$

where  $d_1$  takes the “equilibrium” value as derived from the first-order conditions. This expression is very cumbersome. However, notice that

$$SOC|_{d_1=0} = 256 \frac{\tau^2}{\delta^2} \frac{256 \frac{\tau^3}{\delta^3} (40 \frac{\tau^6}{\delta^6} + 22 \frac{\tau^4}{\delta^4} - 1)}{(1+4\frac{\tau^2}{\delta^2})^{3/2} (3+4\frac{\tau^2}{\delta^2})^2}$$

which is always negative as long as  $\delta/\tau > 2.33$ . Thus the local SOC is satisfied when indeed  $d_1 = 0$  is chosen (recall from Section 4 that this happens when  $\delta/\tau > 6.53$ ). Also notice that

$$sign(SOC_{d_1 \rightarrow 1/2}) = -sign\left(\frac{\tau^3}{\delta^3}/12\right) < 0.$$

Thus when  $\delta/\tau$  is very small, then  $d_1 \rightarrow 1/2$  and the local SOC is satisfied. We also checked for various intermediate values that the local SOC is satisfied and never found a violation of it. We thus conclude that the candidate equilibrium locations in the content space are local maximizers for each platform given the other platform’s “equilibrium” location so that there is zero relocation tendency at stage 1.

We also checked numerically that the contents which satisfy the first-order conditions constitute a (subgame perfect) equilibrium. We performed extensive numerical checks in the uniform case: for small, intermediate, as well as large  $\delta/\tau$  we always found that first-stage profit functions (given the “equilibrium” content of the competing platform) are well-behaved, in particular, platform  $i$ ’s solution to the first-order condition is a global maximizer given platform  $j$ ’s solution to the first-order condition.

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