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CAPITAL STRUCTURE UNDER IMPERFECT ENFORCEMENT

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ABSTRACT

Capital Structure under Imperfect Enforcement*

Building on a costly state verification framework, we propose a theory of capital structure with imperfect enforcement. In addition to being consistent with stylized facts on the choice of capital structure, it accommodates a range of empirical regularities on the repayment behaviour, such as strategic defaults of debt obligations, costly bankruptcy, investor intervention, and violations of absolute priority rules.

Keywords: cash diversion, costly state verification, financial contracts and outside equity

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Capital Structure under Imperfect Enforcement*

Hans K. Hvide and Tore Leite†

October 4, 2004

Abstract

Building on a costly state verification framework, we propose a theory of capital structure with imperfect enforcement. In addition to being consistent with stylized facts on the choice of capital structure, it accommodates a range of empirical regularities on repayment behavior, such as strategic defaults of debt obligations, costly bankruptcy, investor intervention, and violations of absolute priority rules.

Keywords: Cash Diversion, Costly State Verification, Outside Equity, Financial Contracts.

1 Introduction

Financial contracts typically do not specify repayments to investors as a detailed function of all payoff relevant variables. For example, debt contracts put some easily describable liability on the firm's cash flow through a fixed repayment. One tradition that attempts to model this phenomenon is based on the idea that upon repayment, insiders have superior information to the outside investors about the profitability of the firm and may therefore

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have an incentive to divert cash from outside investors. Outside investors, in turn, can mitigate the cash diversion incentive by partially or fully verify the true profitability of the firm through costly intervention. Seminal papers by Townsend (1979) and Gale and Hellwig (1985) show that under certain conditions simple debt contract can thus be optimal, i.e., a contract which promises a fixed repayment and where the creditor intervenes whenever the offered repayment falls short of this treshold.

Despite its initial successes, the Costly State Verification (CSV) tradition has fallen short of offering a complete framework to understand financial structure and repayment behavior.[1] First, the standard CSV setup does not accommodate strategic defaults of debt obligations by the borrower, features of repayment behavior that have received increasing attention from empirical researchers (Brown et al. 2003, Esty and Megginson, 2003). Moreover, the use of strategic defaults is a common explanation for why risk premia on corporate debt significantly exceed those implied by the pricing model of Merton (1974). Second, while in practice outside equity typically coexists with debt in the firm's financial structure, the standard CSV model is silent on the use this funding tool.[2]

The objective of the paper is to suggest a framework for understanding financial structure and repayment behavior that is based on the CSV tradition's informational and contractibility assumptions. An important feature of our approach is that we take standard debt and equity contracts as primitives in the model. This is standard in the capital structure literature (see Harris and Raviv, 1991), but should be contrasted to the traditional mechanism design approach as in e.g., Krasa and Villamil (2000), who consider optimal contracting under modified circumstances to those in Townsend (1979).[3]

[1]See e.g. Hart (1995) for a critique of the CSV approach.

[2]Indeed, as noted by Townsend (1979), "the [CSV] model as it stands may contribute to our understanding of closely held firms, but cannot explain the coexistence of publicly held shares and debt."

[3]We see pros and cons with both our more institutional approach and the more normative approach *of the mechanism design literature*, and cannot see that there are compelling arguments to discard any of two alternative approaches given the current state of knowledge.The mechanism design approach has the advantage of endogenously determining contractual forms. However, it also typically violate some important institutional rules; for example, there are legal limitations to which contracts can count as debt (to give a tax break), and legal rules that are designed to protect minority investors; such rules are hard to model very precisely in a mathematical fashion but still clearly calls for a limitation of the available contracts. In addition, it is not clear to us that our "sin" on this point is any worse than assuming a cash flow state space with three or fewer states, which is commonly done in the financial contracting literature.

With this as starting point, we derive a number of results that are consistent with stylized facts. Let us illustrate with some examples. For debt, the manager makes the lender a repayment offer that depends on the true cash flow of the firm, and the lender monitors with a probability that increases in the size of the default. Such lenience on part of the lender implies that there can be strategic defaults of debt repayments in equilibrium, in that the borrower defaults on his debt obligation even though he has sufficient cash on hand to repay in full.

While debt involves a fixed payment being promised to the outside investor, equity is issued with a promise to the investor of a fixed fraction of firm's cash flow. This fractional cash flow right is in turn supported by an unconditional right for the investor to intervene and verify. Combining debt and equity in the model allows us to consider the choice of the optimal capital structure. While pure debt financing will be shown to be optimal for a low funding requirement, the firm will have a mixed financing structure if the funding requirement is sufficiently high. This result is reminiscent of the pecking order theory of capital structure, where debt is the preferred financing instrument for high-NPV firms, and a combination of debt and (outside) equity is used by low-NPV firms.

Under a mixed financial structure, costly bankruptcy, as well as costly intervention by equity, both obtain with positive probabilities in equilibrium. Our model thus addresses Asquith, Gertner, and Sharfstein (1994) who, with reference to the Coase theorem, puzzle as to why costly bankruptcies are used so frequently, or at all, when less costly alternatives exist. Indeed, in the symmetric information approach to financial contracting originating from Hart & Moore (1989), which currently is the leading paradigm of financial structure under incomplete contracts, bankruptcy never obtains, and shareholders never intervene. This is also the case in the literature that considers the incentives to strategic default on risk premia on corporate debt, such as Mella-Barral and Perraudin (1997).

Equilibria with a mixed capital structure also involves a delegation of monitoring efforts, with creditors monitoring in low states and outside equity monitoring in high states. This prediction is similar to that of Dewatripont and Tirole (1994), where capital structure serves as combined disciplining tool for the manager and an incentive scheme for the firm's outside investors. An important difference between their approach and ours is that while they take contract enforcement as given, enforcement in our setting is imperfect due to verification costs.[4]

Some of the results that we derive have been accommodated in more partial theories before. Our more unified approach enables us to explore links and joint occurrences. For example, we find that a financing mix can be optimal even if it implies that violations of absolute priority rules can occur in equilibrium, in that outside equity receives a positive repayment even if creditors are not repaid in full. The literature on absolute priority violations (e.g., Bergman and Callen, 1991, Mella-Barral and Perraudin 1997, Bebchuk, 2002) deals with violations in favor of an inside owner-entrepreneur. In our setting, there are absolute priority violations favoring both inside and outside equity, which enables us to link intervention probability with the distribution of equity.

Other findings make us able to shed light on empirical findings that are not easily explained by existing theories. For example, Betker (1995) shows that absolute priority violations tend to occur for intermediate cash flow realizations, rather than for low cash flow realizations. This will be shown to be consistent with our setup, where the probability of an absolute priority violation first increases in the cash-flow realization and then decreases.

Our paper builds on Gale and Hellwig (1989), who consider a signaling game where the cash flow is fully revealed through the repayment offer from the insider to the outside investor. In their setting contracting plays no explicit role, nor does different classes of investors. In contrast, we allow for contracts to be written on verification state split of the cash flow. This enables us to endogenously determine capital structure and repayment behavior. Reinganum and Wilde (1986) consider a tax-evasion game where a tax payer submits an income statement to the IRS, and the IRS makes a sequentially rational, random audit. The main difference to our setting is that the "contract" between a tax-payer and the IRS (proportional taxation with a penalty for misreporting) is exogenously imposed by a third party (the "policy makers") rather than being determined by competitive forces. As we do, Reinganum & Wilde (1986) focus on separating equilibria

[4] Winton (1995) derives the optimal contract under CSV using the standard assumption that creditors are commited to verify whenever this is called for in the contract. He shows that optimal funding contract will consist of two classes of debt, with verification efforts delegated between the junior and the senior creditor. In our setting, verification is delegated betwen two outside investors, although unlike Winton (1995) we do not a priori assume the need for two investors to fund the firm.

of the audit game. Povel and Raith (2004) consider a setting with unobservable cash flows and moral hazard, where the manager being acquitted involves a loss in future private benefits. They find that debt is optimal under these conditions. While they do derive costly bankruptcies in equilibrium, they do not model outside equity and, as with Gale and Hellwig (1989) and Reinganum and Wilde (1986), their setting differ from ours in that verification state payoffs are not contracted upon. Persons (1997) imposes sequential rationality and stochastic monitoring in a CSV setting, but restricts attention to a two-state case, where it is difficult to make a meaningful distinction between debt and equity contracts. Krasa and Villamil (2000) and Krasa et al. (2003) derive optimal contracts under sequential rationality in a setting with limited commitment by the investor. They focus on equilibria without renegotiation (by fixing beliefs such that offers are either accepted with probability one or zero), in contrast to our approach that has the explicit purpose of making renegotiation a part of the equilibrium description.[5]

The rest of the paper is organized as follows. In Section 2, we present the basics of model and analyze pure debt and pure equity financing. In Section 3 we examine a mixed financing, and Section 4 concludes. All proofs, unless otherwise stated, appear in Appendices A and B.

2 Basic Setup

A risk-neutral entrepreneur-manager with an investment opportunity needs I units of funding. In this section, we assume that the manager must choose between (pure) debt and (pure) equity financing. Mixed financing is considered in Section 3. Markets for funding are competitive and financiers are risk-neutral, which implies that an investor will fund the entrepreneur if the expected repayment equals the funded amount I.

[5]Others who consider outside equity and debt financing under incomplete contracting include Fluck (1998), Fan and Sundaresan (2000), Myers (2000), and Anderson and Nyborg (2001), who operate in a symmetric-unverifiable information setup à la Grossman and Hart (1986) and Hart and Moore (1989). These papers focus on dynamic issues of repayment and do not derive an optimal mix of debt and outside equity. Boyd and Smith (1999) show that the optimal contract in a CSV type of setting can involve a mix of debt and equity. The payoff to outside equity in their model is supported by the observable part of the firm's cash flow, and hence their model does not explain the use of equity financing to projects that generate unobservable cash-flows.

After being financed, the manager invests and generates a cash flow x, a stochastic variable with density $f(.)$ and strictly positive support $[x_L, x_H]$. Upon realization, x is freely observed by the entrepreneur-manager, but observed (perfectly) by the outside investor only at a positive cost of verification, denoted by c_D for a debt claimant, and c_E for an equity claimant.[6] One interpretation is that c_D is a bankruptcy cost, and that c_E is the cost of partially or fully taking control of the firm's cash flow for the equity holder. Or, c_D and c_E could reflect the creditors' and the outside equity holder's respective cost for performing a thorough audit. We make two convenience assumptions. Firstly, $c_E, c_D < x_L$, i.e., that there is sufficient liquidity in the firm ex post to cover the verification cost. In addition, we assume that the cost of verification is borne by the firm's cash flow (as shown in Appendix C, this assumption is not crucial). After the manager observes the cash flow, he proposes a repayment offer to the investor.

Conditional upon the offer, the investor decides whether to accept (no verification) or reject (verification). Upon rejection, the cash flow is distributed according to the repayment contract. This contract can only specify a payout to the investor in the verification state, in the form of a debt or an equity contract. Debt is issued with a face value $D \in \Re_{++}$ along with a right on the part of the lender to verify (intervene) if D is not paid in full. If the creditor verifies, his payoff becomes $\min[D, x - c_D]$.[7] Outside equity we model as a linear contract that gives the investor a fractional right of $\beta \in (0, 1]$ to the firm's cash flow; if the outside shareholder verifies he gets $\beta(x - c_E)$. Linearity is consistent with laws protecting minority shareholders, in that a smaller ownership share should give proportionally the same cash flow rights (interpreted broadly as dividends, liquidation proceeds, or a takeover premium) as a larger ownership share.[8] The cash flow

[6] The model focuses on frictions in the repayment stage. Among the plausible effects that are not considered are moral hazard in the investment stage and the manager having private benefits from control.

[7] One can enrich the contractual space by allowing verification state payoffs to depend both on x (resources available) as before, and a cash flow report made by the manager. Such contracts would specify that the manager gets 0 if caught lying (an idea explored by Mookherjee and Png, 1989, and Persons, 1997). In the working paper version of the paper, we consider such contracts and show that they yield qualitatively the same results as the current contracts.

[8] Since debt and equity contracts in real life can be of a somewhat richer variety than captured by our model, we have taken care to check our results for robustness against alternative formulations. All our attempts in this direction indicate that the basic results of the paper hold through with such modified contracts.

right associated with equity is supported by an *unconditional* right to intervene.[9]

The model produces a class of signaling games, with the manager as a sender of a costly signal (the repayment offer) and the investor(s) as receiver, making a choice (verification or not) based on the repayment offer and his beliefs about the underlying cash flow. We investigate three versions of the model: pure debt financing, pure equity financing, and mixed financing, and employ the notion of sequential equilibrium.[10] We note that signaling games are known to have a multiplicity of sequential equilibria due to the flexibility that this equilibrium concept gives in specifying out-of-equilibrium path beliefs. In terms of equilibrium selection, we shall focus on separating equilibria, where the manager's repayment offers are strictly increasing in the true cash flow. We discuss equilibrium selection in Section 2.2.

2.1 Debt financing

Here we consider the case when the firm is purely financed by debt. The face value of debt D will be determined by the participation constraint of the creditor, but can be taken as exogenous at this point.

Formally, the entrepreneur makes a (deterministic) repayment offer $\tilde{D} : [x_L, x_H] \rightarrow [0, x_H]$ after observing x, where $\tilde{D} \leq x$ due to the zero initial funds of the entrepreneur. Note that the entrepreneur making a repayment offer $\tilde{D} < D$ is equivalent to proposing for the creditor to make a concession $D - \tilde{D}$ on the debt claim. Given an offer $\tilde{D} < D$, the creditor either accepts or rejects the concession pledge. If the creditor accepts, he receives $\tilde{D}$, and the manager gets the residual $x - \tilde{D}$. If the creditor rejects/verifies, he receives a payoff according to the written contract.[11] A strategy for the creditor is an

[9] The combination fractional cash flow right and unconditional right to intervene is consistent with equity as observed in practice, and is the same type of approach as e.g. Myers (2000) and Anderson and Nyborg (2001).

[10] Recall that in a *sequential equilibrium*, then at all nodes agents pick an optimal decision given their beliefs, and these beliefs should be updated in a Bayesian manner.

[11] Our approach here is similar to that in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997). Potentially, there is a third action open to the creditor, namely to put a counter-offer on the table. If the costs of making counter-offers are large relative to the cost for the manager to make counter-counter-offers, the solution of a Rubinstein (1982) type of bargaining game between the manager and the creditor would give the creditor less than accepting the offer $\tilde{D}$, and hence this third action would not be relevant. Fan and Sundaresan (2000) consider a setting which allows for varying relative

accept probability $Q(\tilde{D})$, where $Q(.)$ is a mapping from the set of possible repayments $[0, x_H]$ to a probability on $[0,1]$. For $\tilde{D} \geq D$, the contract dictates that $Q(\tilde{D}) = 1$, since the creditor may only reject offers less than D. For $\tilde{D} < D$, then $Q(\tilde{D})$ is the probability that the creditor accepts the concession on the debt claim proposed by the manager.

The basic trade-offs faced by the players are the following. The entrepreneur makes a repayment offer that trades off the probability of being verified (and hence reducing the net payoff) with his payoff if not being verified. The creditor, on the other hand, follows a verification strategy based on his conjectures about the underlying cash flow. On the one hand, verification is costly (by reducing the net cash flow) but on the other hand verification will call a diversion attempt by the manager. Stepwise, we now construct the equilibrium strategies of the (unique) separating equilibrium. In the next subsection we specify the underlying belief structure and discuss equilibrium selection issues.

In a separating equilibrium, the verification strategy of the creditor must be stochastic. To see why, note that in a separating equilibrium the repayment schedule $\tilde{D}(x)$ must be strictly increasing (up to the point where $\tilde{D} = D$) and therefore the offer $\tilde{D}$ reveals x. If the x implied through the offer gives a verification state payoff that exceeds $\tilde{D}$, the manager is "burning money", which cannot be consistent with equilibrium. On the other hand, if the implied x gives a verification state payoff that falls short of $\tilde{D}$, the creditor will verify with probability 1, which neither can be optimal play by the manager. Hence the only candidate separating equilibrium involves the creditor being indifferent as to whether to verify or not,

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{1}$$

Note that under $\tilde{D}(x)$, defaults that are purely strategic occur in equilibrium for $x \in [D, D + c_D]$, by which the manager defaults even though the firm has sufficient cash on hand to pay D. In the region $x \in [x_L, D]$ the default is partly liquidity-based and partly strategic.[12] For the manager to have incentives to follow $\tilde{D}(x)$, we must specify

bargaining strength of the inside equity holders and the creditors.

[12] The most plausible interpretation of the mixed strategy played by the creditor is that the entrepreneur

a verification strategy $Q(.)$ that makes $\tilde{D}(x)$ optimal play by the manager. It turns out that here exists a unique, closed-form, solution to this problem that is independent of the cash flow distribution $f(x)$. Intuitively, the investor's accept/reject strategy serves to control the manager's reporting behavior, and the manager *knows* the realization of x, so the prior distribution of x is irrelevant.

Since $\tilde{D}(x)$ is strictly increasing on $[x_L, D + c_D]$, an offer $\tilde{D}$ implies a "report" about the cash flow given by the inverse function $\tilde{D}^{-1}(x)$. We denote the implied report by $\tilde{x}$ and denote the manager's payoff conditional upon the implied report $\tilde{x}$ and the true state x by $U(\tilde{x}; x)$, written just $U(\tilde{x})$. For the manager's incentive-compatibility constraint to hold, $Q(.)$ must be constructed so that $U(\tilde{x})$ is maximized for $\tilde{x} = x$. The manager has no interest in offering the lender a payment that exceeds D, and the lender's right to demand verification is contingent on offers less than D. Consider therefore values of $\tilde{x}$ on the interval $[x_L, D + c_D]$, where $x - \tilde{x}$ is the magnitude of cash flow diversion compared to the proposed equilibrium strategy. First consider the case $x \in [x_L, D + c_D]$. We then have that,

$$\begin{aligned} U(\widehat{x}) &= Q(\tilde{x})[c_D + x - \tilde{x}] + [1 - Q(\tilde{x})]0 \\ &= Q(\tilde{x})[c_D + x - \tilde{x}] \end{aligned} \tag{2}$$

Since the manager gets nothing if the creditor rejects the concession pledge, the expected payoff of the manager just equals the equals the concession proposal $(c_D+x-\tilde{x})$ multiplied by its acceptance probability. We now maximize the manager's payoff with respect to $\tilde{x}$, where it is assumed that $Q(\tilde{x})$ is differentiable.

$$U'(\tilde{x}) = Q'(\tilde{x})[c_D + x - \tilde{x}] - Q(\tilde{x}) = 0 \tag{3}$$

faces a market of possible financiers, and where each financier may play a pure strategy on when to verify (e.g., to verify for any default larger than z, where z is some positive constant), so that the mixed strategy reflects the average behavior played by potential creditors, not the strategy played by each possible creditor. Under this interpretation, the offer function $\tilde{D}(x)$ is a best response to the average or expected play by creditors, not necessarily the best response to the particular creditor played (this is a standard interpretation of mixed strategy equilibria in the game-theoretic literature, see e.g., Rubinstein, 1991). The same interpretation is applicable to the equilibrium we derive under pure equity and under mixed financing.

For $\tilde{D}(x)$ to be optimal play by the manager, this function must be maximized for $x = \tilde{x}$, and hence,

$$Q(\tilde{x}) - Q'(\tilde{x})c_D = 0 \tag{4}$$

Solving this differential equation yields,

$$Q(\tilde{x}) = Me^{\frac{\tilde{x}}{c_D}} \tag{5}$$

where M is an integration constant. Now note that the equilibrium $Q(.)$ function must be continuous. Were it not for some $\tilde{D}$, the manager could ensure a jump in acceptance probability by offering slightly more than specified by $\tilde{D}(x)$. But in that case $\tilde{D}(x)$ cannot be optimal play. Using the corner condition $Q(D) = 1$ implied by this condition, we can get rid of the integration constant to get,

$$Q(\tilde{x}) = \begin{cases} e^{-\frac{D + c_D - \tilde{x}}{c_D}}, \ \tilde{x} \in [x_L, D + c_D] \\ 1, \text{ for } \tilde{x} \in [D + c_D, x_H] \end{cases} \tag{6}$$

Since the creditor is indifferent between accepting and not accepting the offer on the equilibrium path, he cannot gain from changing his strategy. That the entrepreneur will not gain from deviating is ensured by the construction of $Q(\tilde{D})$. We then have the following result.

Proposition 1 *(Debt) In equilibrium, the manager offers $\tilde{D} = D$ if $x \geq D + c_D$. If $x < D + c_D$, the manager makes a concession pledge by offering $\tilde{D} = x - c_D$. The lender accepts this concession pledge with probability $Q(\tilde{D}) = e^{\frac{\tilde{D}-D}{c_D}}$. The creditor will be less lenient the higher face value of debt D.*

It follows directly from the proposition that $Q(.)$ is increasing and convex in $\tilde{D}$. $Q(.)$ must be increasing in $\tilde{D}$ since a higher concession pledge must have a lower probability of being accepted than a smaller concession pledge.[13] If we think of the lender accepting

[13] To see that the (local) second order condition for maximum is satisfied, differentiate $U(\widehat{x})$ twice with respect to $\hat{x}$. Evaluated at $x = \hat{x}$, this yields $\frac{\partial^2 Q}{\partial \hat{x}^2} - 2\frac{\partial Q}{\partial \hat{x}} = -\frac{Q}{c_D} < 0$.

the entrepreneur's offer as the firm successfully restructuring its debt out of court and the lender rejecting the offer as the firm going to formal bankruptcy (under e.g., Chapter 7), this implies that firms are more likely to enter formal bankruptcy the larger their default.

The intuition for convexity of $Q(.)$ is as follows. Recall that,

$$-U'(\tilde{x}) = Q(\tilde{x}) - Q'(\tilde{x})[c_D + x - \tilde{x}] \tag{7}$$

The first term is the marginal gain from "cheating" by offering less than specified by $\tilde{D}(x)$ and the second term is the marginal loss from such behavior. The first term reflects that the manager gets more with probability $Q(.)$, i.e., if verification does not occur, and the second term reflects the higher verification probability that follows from a lesser offer. In a separating equilibrium, these two effects must counteract at $\tilde{x} = x$, or in other words,

$$-U'(\tilde{x} = x) = Q(x) - Q'(x)c_D = 0 \tag{8}$$

Now, when $\tilde{D}$ is low then $Q(.)$ is low and the gains from cheating is small simply because the probability of getting away with it is low. On the other hand the loss from cheating is proportional in $Q'(.)$. The only way to induce adherence to $\tilde{D}(x)$ is therefore for the cheating deterrence to be stronger the higher $\tilde{D}$, or in other words for $Q'(.)$ to be higher for higher $\tilde{D}$. Therefore, $Q(.)$ must be convex in $\tilde{D}$.

2.2 Beliefs and Equilibrium Selection

Let us first construct on- and off-equilibrium path beliefs that supports the separating equilibrium and then discuss equilibrium selection.

Begin by considering the beliefs of the creditor. The prior of the creditor is that x is distributed according to $f(x)$. For an offer $\tilde{D}$, we assume that the creditor assigns the following posterior beliefs $\pi(x)$ to the manager's type,

$$\pi(x) = \begin{cases} \text{unrestricted} & \tilde{D} < x_L - c_D \\ \text{degenerate at } \tilde{D} + c_D \quad \text{for} & x_L - c_D < \tilde{D} < D \\ \text{unrestricted} & \tilde{D} \geq D \end{cases} \tag{9}$$

In the case where $x_L - c_D > \tilde{D} > D$, the posterior beliefs are consistent with the manager's equilibrium strategy, i.e., that $x = \tilde{D} + c_D$ with probability one. For $\tilde{D} < x_L - c_D$, it will be strictly optimal for the creditor to verify independently his updated beliefs about x. We therefore put no restrictions on beliefs in that case (except, of course, that they are on the support of x). In the case where $\tilde{D} \geq D$, the beliefs of the creditor are also immaterial since he only has the option to accept the offer.

The model shares the property of many other signaling games that there are more than one equilibrium. In particular, as pointed out in a related setting by Krasa and Villamil (2000) and Krasa et al. (2003), there exists sequential equilibria of the Townsend (1979) type, where the creditor accepts an offer D with probability one and rejects all lesser offers with probability one. On the equilibrium path, only the offer $\tilde{D} = D$ and the offer $\tilde{D} = 0$ will therefore be observed. This sequential equilibrium is supported by optimistic beliefs about the identity of a deviating type. For example, an offer $\tilde{D} - \epsilon$ will be rejected due to beliefs assigning this offer to a type of entrepreneur with sufficient funds to pay D in full.

Gale and Hellwig (1989), in a closely related setting, discuss which type of equilibrium is the more plausible, the separating one (as in our setting) or the pooling equilibrium (as in Krasa and Villamil, 2000). Their argument is that pooling equilibria will not be strategically stable in the sense of Kohlberg and Mertens (1986). We can make the same argument in the current setting, by applying a version of the "Never Weak Best Response" criterion. Consider a pooling equilibrium C where any offer less than D is rejected with probability one. Suppose that the offer $D - \epsilon$ is made. Under the beliefs about the creditor's behavior specified by C, such an offer would never be optimal behavior by any type $x' > D$. To see why, observe that such a type will get payoff equal to $x' - D > 0$ by repaying D and payoff equal to $\max(x' - D - c_E, 0) < x' - D$ by offering the repayment $D - \epsilon$. On the other hand, types with $x < D - \epsilon$ are cash constrained and cannot have made the offer $D - \epsilon$. Thus one can argue that the offer $D - \epsilon$ could only have been made by a manager with $x \in [D - \epsilon, D)$. But if the creditor reasons in this manner, it must be optimal for him to accept the concession pledge $\tilde{D} = D - \epsilon$. This argument, then, undermines the pooling equilibrium C.

The specific structure of the repayment game makes us able to apply a more direct argument in support of the separating equilibrium. Suppose that the repayment stage of the game is modified as follows. With probability r there occurs a signal R which (for free) reveals the true cash flow to the creditor.[14] The signal is "soft" or only observed by the two involved parties (so that they cannot contract upon it), and comes after the entrepreneur makes his offer, but before the creditor makes his verification decision.

First observe that the separating equilibrium will not be affected by this additional structure, the reason being that in this equilibrium, the manager makes the same offer as he would have made in the case where the cash flow was freely observable.

Let us show that for any $r > 0$, the pooling equilibrium is eliminated. We consider $x \in [x_L, D + c_D]$, i.e., the part of the cash flow interval where a separating and a pooling equilibrium differ. The verification state payoffs are 0 to the manager and $x - c_D$ to the creditor. Start out by assuming that $x \in [D, D + c_D]$. In a pooling equilibrium, the entrepreneur offers D and gets the payoff $x - D$. Let us now consider the payoff for manager if he instead of repaying D makes the concession pledge $\tilde{D} = x - c_D < D$. If the signal does not occur, the creditor verifies, and the manager's payoff will be zero. If the signal occurs, the creditor learns the true cash flow, and will (optimally) accept the concession pledge.[15] The manager's payoff will in that case be c_D. Hence there can exist a pooling equilibrium only if,

$$x - D \geq rc_D \tag{10}$$

But since $c_D > 0$ then for any $r > 0$ there must exist $x \in [D, D + c_D]$ such that the manager has incentives to deviate from pooling. In other words, a pooling equilibrium cannot exist for any $r > 0$. Although strictly speaking redundant, one can see that the same argument can be applied for $x \in [x_L, D]$. In a pooling equilibrium, the manager offers 0, is rejected with probability 1, and gets zero. If the manager had instead made the concession pledge $\tilde{D} = x - c_D$ he gets zero if the signal does not occur and c_D if not. Since $rc_D > 0$, the manager will deviate from the pooling strategy.[16]

[14] The signal could for example be extracted from media coverage or from communication with the firm's auditor.

[15] To have the creditor optimally accepting the offer should be $x - c_D + \epsilon$, where ϵ is positive but small.

[16] A technical issue: The off equilibrium path beliefs of the creditor can be specified such that $\tilde{D} \neq \{0, D\}$ will be inconsistent with R. In that case, the manager may reject a concession proposal even if it according to R would be optimal to accept. [An example of such beliefs would be that the creditor after

2.3 Equity financing

Recall that in order to provide I units of funding, the shareholder requires an ownership fraction β that makes him willing to participate. The "dilution factor" β will be determined by the participation constraint of the outside investor, but can be viewed as exogenous at this point.

A strategy by the entrepreneur is an repayment function $\tilde{E}(x)$, where $\tilde{E}:[x_L, x_H] \rightarrow [0, x_H]$, and $\tilde{E} \leq x$. A strategy for the equity holder is an accept function $P(\tilde{E})$, where $P(.)$ is a mapping from the set of possible repayments $[0, x_H]$ to a probability on $[0, 1]$. As with debt, we focus on separating equilibria of the signaling game, i.e., equilibria where the shareholders cannot precommit to a monitoring strategy (see e.g., Admati et al, 1994, for a similar type of assumption) and where the entrepreneur reveals the cash flow x through his offer $\tilde{E}$.[17] Given a cash flow right β, the equity investor receives $\beta(x - c_E)$ in payoff if he decides to intervene.

Analogous to the case of pure debt financing, a separating equilibrium must have the property that the offer made by the manager makes the investor indifferent whether to verify or not,[18]

$$\tilde{E}(x) = \beta(x - c_E) \tag{11}$$

Note that $\tilde{E}(x)$ is invertible. As under pure debt financing, a separating equilibrium can only exist if there exists an accept probability function $P(\tilde{E})$ such that $\tilde{E}(x)$ is optimal play by the manager. *As in the case of debt*, this problem conveniently turns out to have a unique solution, which can be given a closed-form characterization.

Proposition 2 *(Outside equity) For a given β, in equilibrium the manager offers the investor $\beta(x - c_E)$, and the investor accepts the manager's offer with probability $P(\tilde{E}) =$*

an offer $\tilde{D} \neq \{0, D\}$ puts probability 1 to the manager's type being x_H].

One way to avoid this problem is to assume that the off equilibrium path beliefs of the creditor have full support (with the restriction $\tilde{D} \leq x$). This would be plausible if the manager may tremble when making the repayment offer. In that case, the manager would update his beliefs about x after observing R, thus eliminating the pooling equilibrium.

[17] The same equilibrium refinement arguments can be made for the equity equilibrium as in the debt case, and will not be repeated here.

[18] Appendix B is more elaborate on this point.

$$e^{\frac{\tilde{E}+c_E-\beta x_H}{c_E}}.$$

The probability of the outside equity holder intervening is decreasing in the size of the payment that the entrepreneur offers. This is intuitive, the higher the earnings and the higher the proposed dividend the less is the chance that shareholders will find it necessary to intervene. Note also that there is a positive probability of intervention for all x, in contrast to what the case is with debt financing.

As can readily be seen, $P(.)$ decreases in the ownership stake β. Intuitively, higher outside ownership increases the potential for the insider to divert cash away from the outsider, which forces the outsider to intervene with a greater probability. The straightforward implication is that higher outside ownership is associated with outsiders intervening more frequently.

Finally, let us specify the beliefs that underlie the separating equilibrium. The prior of the investor is that x is distributed according to $f(x)$. For an offer $\tilde{E}$, the investor assigns the following posterior beliefs $\pi(x)$ to the manager's type,

$$\pi(x)=\begin{cases}\text{unrestricted} & \tilde{E}<\beta(x_L-c_E)\\ \text{degenerate at } \dfrac{\tilde{E}+\beta c_E}{\beta} \quad \text{for} & \beta(x_L-c_E)\leq\tilde{E}\leq\beta(x_H-c_E)\\ \text{unrestricted} & \tilde{E}\geq\beta(x_H-c_E)\end{cases} \tag{12}$$

In the case where $\beta(x_L-c_E)\leq\tilde{E}\leq\beta(x_H-c_E)$, the posterior beliefs are consistent with the manager's equilibrium strategy, so that that $x=\dfrac{\tilde{E}+\beta c_E}{\beta}$ with probability one. For $\tilde{E}\leq\beta(x_L-c_E)$ it will be strictly optimal for the creditor to verify independently of his updated beliefs about x, and for $\tilde{E}\geq\beta(x_H-c_E)$ it will be strictly optimal for the creditor to accept independently of his updated beliefs about x, so we put no restrictions on beliefs in those cases. The manager believes that after a deviation, the offer will be accepted with probability $P(.)$. That the manager will not gain from deviating is ensured by the construction of $P(.)$.

2.4 Comparing Pure Debt and Pure Equity Financing

We now compare the performance of pure debt and pure equity financing. To raise I, the face value of debt must equal,

$$U_C = \int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx = I \tag{13}$$

The first integral is the payoff to the creditor in the default region, and the second integral is the payoff to the creditor in the region where the debt is fully repaid. The lowest D consistent with this condition will be the equilibrium face value of debt. The expected verification cost under debt financing equals,

$$V_C(D; .) = c_D \int_{x_L}^{D+c_D} [1 - Q(x; .)] f(x) dx \tag{14}$$

These two equations implicitly define the verification cost as a function of the exogenous parameters I and c_D. For equity financing, the dilution factor β required to raise the amount I equals,

$$U_E = \int_{x_L}^{x_H} \beta(x - c_E) f(x) dx = I \tag{15}$$

The expected verification cost under equity financing equals,

$$V_E(\beta; .) = c_E \int_{x_L}^{x_H} [1 - P(x; .)] f(x) dx \tag{16}$$

Let us now compare pure debt and pure equity financing.

Proposition 3 *(i)If $c_E \geq c_D$ then for any fundable I the expected verification cost under pure debt financing is lower than under pure equity financing. (ii) If $c_E = c_D$ the same range of I is fundable under debt and equity financing. If $c_E > (<) c_D$ the range of fundable I is smaller (greater) under pure equity financing than under pure debt financing.*

Proof. Let us start out by sketching the proof of (ii). Start out by assuming that $c_E = c_D = c$. The maximum fundable amount for pure debt financing is obtained for $D = x_H - c$ (a higher D would be redundant) and the maximum fundable amount for equity financing is obtained for $\beta = 1$. But in this case the pure debt and the pure equity contract are identical (i.e., both the expected payoff to investors and the expected verification costs are the same), and the set of fundable projects under the two contracts must also be the same. Given this observation it is easy to see that $c_E > (<) c_D$ implies that a smaller (greater) set of projects is fundable under pure equity financing than under pure debt financing.

Let us now prove (i). We show that the expected verification cost under debt is less than the expected verification cost under outside equity for $c_E = c_D = c$ and a fortiori for $c_E \geq c_D$. Recall that for $I = \bar{I}$, where $\bar{I}$ is the maximum fundable amount, the pure debt and the pure equity contract coincide and therefore $V_D(\bar{I}) = V_E(\bar{I})$. Consider therefore the case for which $I < \bar{I}$. To show that $V_D(.) < V_E(.)$ in this case, it is sufficient to show that $Q(x;.) > P(x;.)$ for $x \leq D + c$. From the expressions for $Q(x;.)$ and $P(x;.)$ it is easy to see that $Q(x;.) > P(x;.)$ if $D + c - x < \beta(x_H - x)$. Since $x > c$, this condition is implied by,

$$D < \beta(x_H - c) \tag{17}$$

Let us now show that this inequality is indeed satisfied. If debt and equity raises the same amount I then (β, D) must be such that,

$$\int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx = \int_{x_L}^{x_H} \beta(x - c_E) f(x) dx \tag{18}$$

Rearranging,

$$(1 - \beta) \int_{x_L}^{D+c} (x - c) f(x) dx = \int_{D+c}^{x_H} [\beta(x - c) - D] f(x) dx \tag{19}$$

Since $(1 - \beta) \int_{x_L}^{D+c} (x - c) f(x) dx > 0$, it must be the case that,

$$\int_{D+c}^{x_H} [\beta(x - c) - D] f(x) dx > 0 \tag{20}$$

This expression increases in x. Therefore $\beta(x_H - c) - D > 0$. It follows directly that $D < \beta(x_H - c)$. We have thus shown that for $c_E = c_D = c$ then pure debt dominates pure equity. ■

In other words, debt dominates equity when $c_E \geq c_D$. If $c_E < c_D$, then equity dominates debt for a high funding requirement, and debt dominates debt for a low funding requirement.[19]

3 Capital Structure

In the next subsection, we describe the extensive form of the financing game. Then we describe some of the properties of a separating equilibrium with a mixed capital structure, before we (partially) characterize the optimal capital structure. The last subsection discusses the main implications of our findings and relate them to theoretical and empirical work on capital structure.

Throughout the section, we assume that $c_E \leq c_D$.[20] We are not aware of systematic empirical work that compares the intervention cost of debt and the intervention cost of equity, but note that bankruptcy is a costly and lengthy process where all the claimants of the firm are involved, while monitoring from shareholders is a less extensive process with fewer claimants directly involved. In addition, since creditors obtain none of the upside potential of the firm, creditors may be too eager to liquidate the firm compared to first best, which incurs a social cost.[21]

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[19] It can be shown that pure debt financing dominates pure equity financing for a sufficiently small funding requirement I, where the intuition is similar to that in Proposition 7.

[20] We do not have a formal proof but have been unable to generate examples with an optimal mixed financing for $c_D < c_E$. An interesting conjecture is therefore that pure debt is the optimal contract under equal verification cost, i.e., $c_D = c_E$. To solve for the optimal contract in this setting is – at least on the face of it – a very complex variational calculus problem where the solution may depend on the underlying cash flow distribution.

[21] Another justification for $c_E \leq c_D$ is as follows. In an extended model, we can imagine the verification costs as being determined by ex-ante investments by the claimants (Boot and Thakor, 1993). The idea is that if the investors can make investments in monitoring technology before x is realized, then creditors will have less incentives than equity holders to invest, and we get $c_E < c_D$ as part of the equilibrium description. Cantillo (2004) contains a formalization of a similar idea applied to bank debt and publicly held debt.

3.1 Extensive Form

First, the manager finances the investment I with a fraction α in the form of debt and $(1-\alpha)$ in the form of equity, where $\alpha \in [0,1]$, and β and D are agreed upon.[22] The cash flow is then realized and observed only by the manager. We take the creditor to be the senior claimant in the repayment phase, meaning that the entrepreneur settles his accounts with the creditor before proposing a payout to the outside equity holder.[23]

Upon observing the true cash flow, the manager offers a debt repayment $\tilde{D}$ to the creditor, which the creditor accepts with probability $q(\tilde{D})$. If the creditor rejects the offer, he gets $\min[D, x - c_D]$, while the equity holder gets $\max[0, \beta(x - D - c_D)]$; in other words, absolute priority is upheld in case of bankruptcy. The entrepreneur's payoff is the residual. If the creditor accepts the manager's offer $\tilde{D}$, the manager proceeds to the shareholder with a repayment offer $\tilde{E}$, which the shareholder accepts with probability $p(\tilde{E}, \tilde{D})$. Note that this formulation requires that $\tilde{D}$ is observable to the equity holder.[24] If the shareholder accepts the offer $\tilde{E}$, the manager retains $x - \tilde{D} - \tilde{E}$. If the shareholder rejects the offer, and verifies, the shareholder receives $\beta(x - \tilde{D} - c_E)$, and the manager gets the residual.[25]

3.2 Properties of a Mixed Capital Structure Equilibrium

Here we analyze the subgame that ensues conditional on a mixed financing (i.e., $\beta, D > 0$). A sequential equilibrium specifies offer functions $\tilde{D}(.)$ and $\tilde{E}(.)$ by the manager, accept

[22] β and D are assumed to be agreed upon in a manner that excludes opportunistic behavior by a subset of the three agents at the contracting stage. This assumption means that the manager cannot issue debt after issuing equity without the consent of the shareholder (the shareholder's payoff depends upon D, but the creditor's payoff does not depend on β as will become apparent below). Stylistically, we can think of the investors as offering break-even schedules $D(\alpha I; .)$, $\beta(D, \alpha I; .)$, and the manager picking the prefered α.

[23] Contracts between the creditor and the equity holder involving the creditor (equity holder) subcontracting the intervention action to the equity holder (creditor) are prohibitively costly to enforce.

[24] The case where $\tilde{D}$ is unobservable to the equity holder (i.e., debt renegotiations are secret) so that p is a function of $\tilde{E}$ only, has qualitatively similar properties and was considered in the working paper version of the paper.

[25] We assume that the creditor by accepting waives any future rights to the cash flow. This is consistent with bankruptcy law as practiced in e.g., the U.S. where repudiation is limited to situations under which the creditor can show that he was coerced to accept the firm's offer (see Berglöf, Roland, and von Thadden, 2000, for a discussion).

functions $q(\tilde{D})$ and $p(\tilde{E})$ by the creditor and the shareholder, respectively, and a consistent set of beliefs that deem these strategies optimal. We focus on separating equilibria where both $\tilde{D}(.)$ and $\tilde{E}(.)$ are strictly increasing in x. Equilibrium selection will be discussed at the end of the section.

Note that the creditor's payoff does not depend upon β. Upon a repayment offer $\tilde{D}$, the creditor therefore faces the same trade-off as in the pure debt financing version of the model: Verification is costly since it reduces the net cash flow but has the advantage of calling a bluff from the manager. Moreover, the debt repayment strategy $\tilde{D}(x)$ of the entrepreneur in a separating equilibrium must be the same as under pure debt financing, i.e., the offer must make the creditor indifferent as whether to verify or not,

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{21}$$

The posterior beliefs of the creditor after observing an offer $\tilde{D}$ are the same as under pure debt financing, i.e., degenerate at $\tilde{D} + c_D$ for $x_L - c_D < \tilde{D} < D$, and unrestricted otherwise. An open question is whether there exists an accept probability function $q(\tilde{D})$ that makes $\tilde{D}(x)$ optimal play by the manager. To answer this question, it is necessary to first consider the equity subgame, since the manager's incentive to stick to $\tilde{D}(x)$ or not depends on what outcome he expects from the equity subgame.

Moving to the equity subgame, there are two cases of interest, the subgame reached after $\tilde{D} = D$ and the subgame reached after $\tilde{D} < D$. Let us consider these two cases in turn. When $\tilde{D} = D$, the true cash flow is not fully revealed, and the only interim beliefs of the shareholder (i.e., the posterior beliefs after $\tilde{D}$ is realized but before an offer $\tilde{E}$) consistent with the manager's strategy is that the cash flow is on the interval $[D + c_D, x_H]$ with a density equal to the prior of x.[26] It is straightforward to show that the separating solution to the equity subgame is identical to that in the pure equity financing game, with

[26] Of course, the prior on this interval should be multiplied by a factor $[\int_{D+c_D}^{x_H} f(x)dx]^{-1}$ to obtain the posterior beliefs.

x replaced by $x - D$ in the manager's offer function, i.e.,

$$\begin{aligned} \tilde{E}(x) &= \beta(x - D - c_E) \\ p(x) &= P(x) = e^{-\beta \frac{x_H - x}{c_E}} \end{aligned} \tag{22}$$

The second case occurs when $\tilde{D} < D$. If the creditor has rejected the concession pledge, the cash flow is distributed according to the contract, as specified above, and the beliefs of the shareholder are immaterial. In the more interesting eventuality that the creditor has accepted the concession pledge, the shareholder's beliefs over the remaining cash flow must be degenerate at c_D since any other beliefs would be inconsistent with the proposed equilibrium play $\tilde{D}(x)$. Therefore, the shareholder believes (with probability 1) that his payoff equals $\beta(c_D - c_E)$ if he rejects an offer by the manager. Optimal play by the manager must therefore be to ensure *acceptance with probability one* by offering the shareholder $\beta(c_D - c_E) + \varepsilon$, where ε is positive but small.[27] Let us summarize these arguments in a proposition.

Proposition 4 *In a separating equilibrium, the division of monitoring is completely specialized between the creditor and the shareholder; the creditor monitors (with a positive probability) when the cash flow is low, i.e., for $x < D + c_D$, and the shareholder monitors (with a positive probability) when the cash flow is high, i.e., $x \geq D + c_D$.*

We have now specified the functions $\tilde{D}(.)$ and $\tilde{E}(.)$ by the manager and the accept function of the shareholder $p(\tilde{E})$ in a separating equilibrium. Let us now proceed to derive the accept probability function $q(\tilde{D})$ of the creditor. As under pure debt financing, we find a unique, closed-form, solution.

It is specified by the contract that $q(D) = 1$, so we need to derive $q(.)$ for any concession pledge $\tilde{D} < D$. First note the close resemblance with the pure debt financing game; the payoff for the creditor is identical (holding D constant), and the payoff to the manager

[27] After a deviation $\tilde{E} < (>) \beta(c_D - c_E)$, Bayesian updating implies that the shareholder believes (with probability 1) that the remaining cash flow is c_D, and it is strictly optimal to reject (accept) the offer. We leave out the technical difficulties in specifying beliefs after $\tilde{E} > c_D$ (i.e., a contradiction of the shareholder's posterior beliefs) and simply assume that the equityholder will accept such an offer.

is the same, subtracted the proceeds to the shareholder, which is a constant over this interval if the creditor accepts. The $q(.)$ we solve for will therefore be similar to the solution for pure debt financing game, $Q(.)$, in terms of the curvature. Second, recall that in a separating equilibrium, the payoff of the manager must be continuous in $\tilde{D}$. If not, the manager would have incentives to deviate from $\tilde{D}(x)$ for certain points, since this would ensure a discontinuous increase in acceptance probability, lower verification costs, and hence a higher payoff than he would get from sticking to $\tilde{D}(x)$. Recall that for $\tilde{D} < D$, the shareholder's acceptance probability is zero, while for $\tilde{D} = D$, it is strictly positive. The creditor's accept probability must therefore increase discontinuously in the point $\tilde{D} = D$ to ensure continuity of the manager's payoff in this point. Denote $\lim_{\tilde{D} \to D^-} q(\tilde{D})$ by $q(D)^-$ and let the total intervention probability (in equilibrium) for the cash flow x equal the sum of the creditor's and the shareholder's intervention We then have,

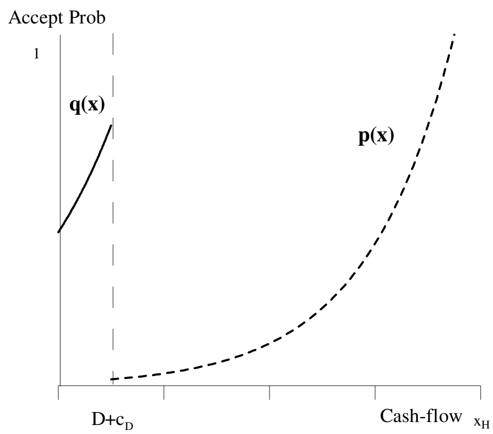
Proposition 5 *The equilibrium creditor accept probability function $q(.)$ has the following properties:*

(i) $q(D)^- < 1$ for $c_D > 0$

(ii) $q(.)$ is strictly decreasing in β

(iii) As x goes through the point $D + c_D$, the total accept probability drops discontinuously if $c_E < c_D$.

The intuition for the first part is that since the monitoring probability of the equity holder in equilibrium drops discontinuously as we move below the point $x = D + c_D$, a corresponding discontinuous increase in the monitoring probability of the creditor is needed to ensure continuity of the manager's payoff in the point $\tilde{D} = D$ and hence that the manager has incentives to stick to $\tilde{D}(x)$. Part (ii) implies that the accept probability of the creditor will be decreasing in the outside equity ownership ratio β, and hence it predicts that creditors will be less lenient with firms with greater fraction of equity held by outside shareholders. This obtains since an increase in β gives a decrease in the accept probability $p(x)$ of the shareholder, which in turn forces the creditor's accept probability to decrease to ensure continuity. The intuition for (iii) will become clearer from the following figure.



This figure depicts the accept probabilities in equilibrium. For a low x, there is a positive probability of the creditor monitoring, while the probability of the investor monitoring (conditional on the creditor not monitoring) is zero. For a higher x, there is a zero probability of the creditor verifying, and a positive (and decreasing) probability of the investor verifying. Hence monitoring is completely specialized in equilibrium; the creditor has the role of disciplining the entrepreneur in bad states, and the outside investor has the role of disciplining the manager in good states. The probability of the creditor verifying is discontinuous in the point $\tilde{D} = D$ (i.e., $x = D + c_D$) as in the original setup of Townsend (1979) but now without any assumed commitment power by the creditor. The total intervention probability decreases up to $\tilde{D} = D$, then increases discontinuously in that point, and from then onwards it decreases. The intuition is that since it is cheaper for the manager that the shareholder monitors, the total accept probability must drop in the point $x = D + c_D$ for the manager to have incentives to stick to $\tilde{D}(x)$.

Let us now discuss strategic defaults and priority violations in terms of the model. We have the following.

Proposition 6 *For high cash flows, i.e., $x \geq D + c_D$, the manager pays back the debt in full. For intermediate cash flows, i.e., $D \leq x < D + c_D$, the manager defaults on the debt obligation purely for strategic reasons. For low cash flows, i.e., $x < D$, the manager's default is partly due to liquidity shortfall and partly for strategic reasons. Priority violations occur with a positive probability for low cash flows, i.e., $x < D + c_D$. The probability of a priority violation increases in x on this interval.*

Defaults purely due to strategic reasons occur in equilibrium for $x \in [D, D + c_D]$ since the manager defaults even though the firm has sufficient cash on hand to pay out D. In the region $x \in [x_L, D]$, the default is partly liquidity-based and partly strategic. The creditor will accept a concession pledge even if he knows that it is (partly) strategic, and at the same time the expected repayment to the shareholder is strictly positive. This repayment pattern constitutes an absolute priority violation, since it implies that the equity holder may receive a positive payoff even though the lender is not paid the full value of his debt contract. We can note that without strategic defaults, priority violations would not be possible.[28]

We have focused on separating equilibria of the financing game. As in the pure debt financing case, there exist pooling equilibria where the creditor rejects any offer short of D and where this behavior disciplines the manager to default only when $x < D$. Analogous to in the pure debt financing case, such pooling equilibria does not satisfy strategic stability type of arguments. To see why, suppose that the manager deviated from the pooling equilibrium, by offering $D - \epsilon$. By the liquidity constraint, such an offer can only have been made by types $x \geq D - \epsilon$. Under the beliefs specified by a pooling equilibrium (the creditor has optimistic beliefs about the manager's type after observing a deviation), such a deviation would not be optimal behavior by a type $x > D + c_D$; observe that such a type would get the payoff (1-β)($x - D - c_D$) by defaulting, since the creditor rejects with probability 1. By not defaulting, such a type would get a higher payoff, at least (1-β)($x - D - c_E$), since this is what he gets if the shareholder rejects the repayment offer. From this argument, the creditor will after a deviation plausibly believe that the offer $D - \epsilon$ was made by a type on the interval $[D - \epsilon, D + c_D]$. But in that case it is optimal for the creditor to accept the concession pledge, which undermines the pooling equilibrium. Also analogous to the pure financing cases, pooling equilibria will not survive refinements where the investors with a small probability observes the true

[28] Bebchuck (2002) studies the effects of absolute priority violations on the ex-ante risk shifting incentives of borrowers, finding that debt that permits absolute priority violations induces stronger risk shifting incentives than debt that does not. The effect identified by Bebchuck can be generated in the present setting as well. While absolute priority violations in his setup are imposed exogenously by giving the borrower a fixed fraction of the firm's assets in any default state, the absolute priority violations in the present setting arise endogenously, due to the frictions created by the verification costs.

cash flow after the offer made by the manager.

3.3 Optimal Capital Structure

Let us now analyze the optimal capital structure, where we can obtain some insights although closed-form solutions are not feasible. For a given D and β, the expected verification cost is given by,

$$V(D,\beta) = c_D \int_{x_L}^{D+c_D} [1 - q(x;.)] f(x) dx + c_E \int_{D+c_D}^{x_H} [1 - p(x;.)] f(x) dx \tag{23}$$

The first term is the expected verification costs for low cash flows ($x \in [x_L, D+c_D]$), that arise due to creditor intervention, and the second term is the expected verification cost for high cash flows ($x \in [D+c_D, x_H]$), due to shareholder intervention. The objective of the entrepreneur is to pick the α that minimizes this expression, subject to the participation constraint of each investor (described in Appendix B). Note that for $\alpha = 0$, i.e., pure equity financing, the first term in $V(.)$ drops, and $p(x;.) = P(x;.)$. For $\alpha = 1$, i.e., pure debt financing, the second term drops, and $q(x;.) = Q(x;.)$. Trivially, for $c_D = c_E = 0$, any choice of capital structure will be optimal.

The first observation we can make about the optimal capital structure is the following.

Proposition 7 *The firm will never be 100% equity financed.*

Note that this result does not hinge on debt being risk free, as shown in the proof. The intuition behind the result is that debt issued in small quantities behaves much like risk-free debt; the verification region for debt (i.e., the cash-flow region where bankruptcy occurs with a positive probability) is small, and the acceptance probability in that region is large. That debt only will incur verification costs in low cash flow states suggests that debt financing will tend to be more preferred the less risky cash flows (i.e., more probability mass in the intermediate cash flow region), an argument that has been verified to hold in numerical analysis but which we are unable to support with a tight formal statement. That the debt ratio decreases in riskiness of the firm is a firmly corroborated empirical finding (see the survey by Harris and Raviv, 1991, and for more recent evidence, Fama and French, 2002).

Proposition 8 *(i)High-NPV firms (i.e., a low I) will be externally financed by debt only. (ii) Low-NPV firms (i.e., a high I) will issue some external equity.*

The intuition for (i) is similar to the intuition for the previous result. For a low funding requirement, under debt financing the verification region is small and with a high acceptance probability within this region. For equity, on the other hand, the verification region spans the whole interval for any $\beta > 0$, which by the convexity of $P(x; .)$ implies that verification costs for equity will be non-negligible even for low β. The intuition for (ii) follows directly from the fundability arguments in Proposition 3, part (i): for a high fundability requirement, pure debt financing is not feasible, and the firm must issue at least some external equity in optimum.[29]

Given that debt will always be issued, it is straightforward to show that if c_E is sufficiently low, the firm must optimally issue both debt and equity. Since the funding requirement of a project is more immediately observable than c_E and c_D, the following result seems more useful.

Proposition 9 *The optimal capital structure must be mixed for a sufficiently high funding requirement.*

The result follows directly from Proposition 7 and Proposition 8; for a high funding requirement, it must be the case that the firm issues equity, since the project would not be fundable with pure debt. On the other hand, debt will be part of the optimal financing for any funding requirement, and it follows that a mixture must occur for a high funding requirement.

3.4 Discussion

An important feature of our setup is that costly bankruptcy, strategic defaults, priority violations, and costly intervention by equity all obtain with positive probabilities. In

[29] An alternative intuition is that as the financing requirement is increased, the cost of pure debt financing increases in a convex manner, due to the combined effects of a larger verification region and an increased verification probability inside this region. At some point, therefore, equity will be cheaper on the margin than debt, and thus firms with a large funding requirements will have a mixed financing.

the symmetric information approach to financial structure as represented by Hart and Moore (1989), Fluck (1998), and Myers (2000), in equilibrium, bankruptcy never obtains and shareholders never intervene, even though contract enforcement in this literature is incomplete. Similarly, in the literature on absolute priority violations (e.g., Bergman and Callen, 1991, and Mella-Barral and Perraudin, 1997), bankruptcy never obtains, although the presence of bankruptcy costs creates a wedge between what can be paid and what will be paid, in equilibrium.

The standard approach in the literature on priority violations is to focus on the relation between debt and inside equity (e.g., Berkovitch, Israel, and Zender, 1998).[30] Our paper is richer in the sense that it includes both debt and different equity classes. To our knowledge, our paper is alone in relating bankruptcy probability and absolute priority violations to inside versus outside equity. In this respect, our model predicts that larger inside relative to outside equity ownership gives a more lenient creditor and hence a lower probability that the manager's offer will be rejected, which implies that higher inside ownership will be associated with a higher probability that debt restructurings will succeed. This prediction is consistent with empirical evidence of Betker (1995) who finds a positive correlation between CEO ownership and absolute priority violations. Betker (1995) does not test for the more direct prediction that is implied by our analysis of a positive correlation between insider ownership, successful restructuring and priority violations.

Another implication of the model is that the rejection probability of the creditor increases in the size of the default. This is the case whether the firm has outside equity in its capital structure or not. This prediction is consistent with Betker (1995) who finds that priority violations are larger the closer the firm is to solvency. Betker (1995), however, does not test the more direct empirical prediction of our model that firms closer to solvency are less likely to fail in their attempts to restructure their debt. The present paper may be useful in this respect in suggesting a structural relation to impose on the data.[31]

[30] Or on different classes of debt (e.g., Brown, 1989, Asquith et al., 1994, Franks and Nyborg, 1998).

[31] A puzzling finding from the influential Asquith et al. (1994) is that more profitable firms are not less likely to go bankrupt. This is in fact consistent with model in that more profitable firms (lower I) have a higher D and are therefore may be more likely to go bankrupt. Our results suggest that controling for capital structure composition, then this result should go away.

Let us finally relate our results on optimal capital structure to the two main established theories. Our results on optimal capital structure relates to the trade-off theory of capital structure, where the optimal mix of debt and equity obtains at the point where the tax benefit of debt, on the margin, equals the expected bankruptcy cost. In our setting, the optimal mix of debt and (outside) equity obtains when the expected intervention cost associated with debt, on the margin, equals the expected intervention cost of equity. A distinction to the trade-off theory is that the latter does not distinguish between inside and outside equity, since equity in this theory serves just as a buffer against bankruptcy. In the present theory, however, outside equity has an active role in disciplining the manager and there is therefore a clear distinction between inside and outside equity. This allows us to pin down priority violations in our framework, in contrast to in the trade-off theory. In the choice between internal and external funds the entrepreneur will prefer internal funds since the use of internal funds will reduce expected verification costs regardless of the mix of external funds. In the choice between debt and outside equity as external funds the entrepreneur has an initial preference for debt and issues outside equity only for sufficiently high funding requirement. As such, our theory has predictions reminiscent of pecking-order: inside funds are preferred, then debt, and if the funding need is large, issue outside equity along with debt.[32] Hence, one interpretation of our main contribution is to arrive at a theory of capital structure with some of the same basic predictions as trade-off and pecking-order theory, but with a much richer set of predictions about repayment behavior.

4 Conclusion

We have constructed a theory of capital structure that accommodates strategic defaults and priority violations, phenomena that are well-known from the empirical and theoretical literature, but has not been integrated into a theory of capital structure before. The model produces implications that are consistent with several other stylized facts, such

[32] For example, Leary and Roberts (2004) find that more profitable firms, firms with larger internal funds, and firms with smaller capital expenditures are less likely to seek external financing. This evidence is consistent with the pecking-order theory of finance, and it is consistent with our theory in which external funds are costly due to costly enforcement.

as bankruptcies occurring in equilibrium, a division of labor between different security holders in disciplining the entrepreneur, and the debt ratio of the firm decreasing in its funding requirement. Also, we find that firms with low funding requirements will issue only debt, while firms with sufficiently high funding requirements will issue a mix of debt and equity. In other words, firms may be all debt financed, but will never be all equity financed, even if the intervention cost of equity is much lower than that of debt.

The basis of the theory is the cash diversion problem. Under the simplest interpretation this says that the manager steals the money. Although this phenomenon may be important in less developed economies (Shleifer and Vishny, 1997, provide some examples), the cash diversion in the model may more reasonably be seen as a short form of a situation where the manager may divert the cash into unprofitable pet projects if given the opportunity, which creates a need for investors to discipline. In contrast to some earlier models of costly state verification, our added features are to require sequential rationality and to allow for equity contracts in addition to debt.

For future work, we see a range of possible extensions of the present framework. First, it may be of interest to introduce dynamics into the model, to discuss such issues as dividend policy and delays in debt repayments. A second extension would be to discuss commitment debt (where the creditors verify whenever the offered repayment falls short of some treshold, as in the equilibrium proposed by Krasa and Villamil, 2000) vis-a-vis non-commitment debt (as considered in the paper), and to allow for different seniority in debt claims. Dispersed investors in the securities market may have commitment through their free-rider status, while banks do not. A preliminary result from our analysis of this question indicates that non-commitment (bank) debt dominates commitment (security) debt for projects with a cash flow distribution which is skewed to the left, which is intuitively appealing, as the non-commitment debt would rely on verifying less often in low cash-flow states. Third, it would be of interest to extend the current setup to accommodate investments in monitoring technology. Equity holders may have stronger incentives than debt holders to invest in a monitoring technology, but on the other hand, it is not obvious how the private costs of such investments correspond to the social costs. If claimants may have private costs that differ from social costs when investing in monitoring technology this may have interesting implications for security design that lies outside the scope of the present paper.

5 References

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6 Appendix A: Proofs

Proof of Proposition 2. Let us first prove that a separating equilibrium must involve stochastic monitoring. This is a straightforward exercise where we assume that there exists a separating equilibrium with a deterministic verification strategy, and derive a contradiction. Under a deterministic monitoring rule, there must exist a constant $\bar{E}$ such that all repayment offers below $\bar{E}$ are rejected with probability 1, and the offer $\tilde{E} = \bar{E}$ is accepted with probability 1. But in that case, the manager's optimal strategy must be to offer $\bar{E}$ whenever $\beta(x - c_E) > \bar{E}$, i.e., whenever the verification payoff of the shareholder exceeds $\bar{E}$, because this saves the verification costs. Such a strategy is not separating, however, since an interval of manager types will pool at the offer $\bar{E}$.[33] Therefore, a separating equilibrium must involve stochastic monitoring for all x. For a stochastic verification strategy to be optimal play by the investor, he must be indifferent between accepting and rejecting an offer. It follows immediately that if there exists a separating equilibrium, the manager strategy $\tilde{E}(x)$ must be,

$$\tilde{E}(x) = \beta(x - c_E) \tag{24}$$

Let $\tilde{x}$ be the implied report by the manager given an offer $\tilde{E}$, i.e., $\tilde{x} = \tilde{E}^{-1}(x) = \tilde{E}/\beta + c_D$. For notational convenience we will work with $\tilde{x}$ instead of $\tilde{E}$ in the following. In a separating equilibrium, it must be the case that the manager's payoff is maximized for an implied report equal to the actual cash flow, i.e., that

$$U(\tilde{x}) = P(\tilde{x})[x - \beta(\tilde{x} - c_E)] + [1 - P(\tilde{x})](1 - \beta)(x - c_E) \tag{25}$$

[33] For $\bar{E}$ sufficiently large, this does not necessarily hold true. But, as can easily be shown such large values of $\bar{E}$ cannot be optimal play by the investor.

is maximized for $\tilde{x} = x$. Differentiating with respect to $\tilde{x}$ and setting $\tilde{x} = x$ we obtain the differential equation

$$P(\tilde{x})\beta - \frac{dP(\tilde{x})}{d\tilde{x}} c_E = 0 \tag{26}$$

Solving this differential equation yields

$$P(\tilde{x}) = K\mathbf{e}^{\frac{\beta\tilde{x}}{c_E}} \tag{27}$$

By using the corner condition $P(x_H) = 1$, we obtain

$$P(\tilde{x}) = \mathbf{e}^{-\frac{\beta(x_H - \tilde{x})}{c_E}} \tag{28}$$

We can note that the accept probability decreases in β and increases in c_E. Expressed in terms of the repayment offer $\tilde{E}$,

$$P(\tilde{E}) = \mathbf{e}^{-\frac{\beta x_H - c_E - \tilde{E}}{c_E}} \tag{29}$$

The (local) second order condition can be easily checked to hold: $\frac{d^2U(\hat{x})}{d\hat{x}^2} = \frac{\beta^2}{c_E}[P'(x - \tilde{x}) - P] = \frac{\beta^2}{c_E}[\frac{\beta}{c_E}P(x - \tilde{x}) - P] = -P\frac{\beta^2}{c_E} < 0$ for $\tilde{x} = x$. ■

Proof of Proposition 4. Here we first derive $q(.)$ and then prove some of its properties. Again it is notationally simpler to work with the implied report $\tilde{x}$ rather than $\tilde{D}$, where $\tilde{x} = \tilde{D}^{-1}(x) = \tilde{D} + c_D$. The strategy of the proof is to first derive the slope of $q(.)$ necessary to make $\tilde{D}(x)$ optimal play in the lower cash flow region, and then pin down the level of $q(.)$ through the continuity requirement. Conditional on $x, \tilde{x} \in [x_L, D + c_D)$, the manager's payoff equals,

$$U(\tilde{x}) = q(\tilde{x})[(1 - \beta)c_D + \beta c_E + x - \tilde{x}] \tag{30}$$

The expression reflects that the manager only gets a positive payoff if the creditor accepts the concession pledge. In a separating equilibrium, $\tilde{x} = x$ must be optimal. Differentiating

$U(\tilde{x})$ with respect to $\tilde{x}$ and setting $\tilde{x} = x$, we obtain the differential equation

$$q(x) - \frac{dq(x)}{dx}((1-\beta)c_D + \beta c_E) = 0 \qquad (31)$$

with solution

$$q(x) = Ke^{\overline{(1-\beta)c_D + \beta c_E}^{\;x}} \qquad (32)$$

By substitution,

$$q(\tilde{D}) = Ke^{\overline{(1-\beta)c_D + \beta c_E}^{\;\tilde{D}+c_D}} \qquad (33)$$

This function is increasing and convex in $\tilde{D}$. The constant K will be determined by the requirement that the manager's payoff is continuous in the point $\tilde{D} = D$, a task will be returned to below.

Let us now consider the manager's payoff when $x \in [D + c_D, x_H)$. In the proposed equilibrium, the manager clears his debts by repaying D to the creditor and then enters the equity subgame. As argued in the text, the manager's offer in the equity subgame must equal $\beta(x - D - c_E)$, i.e., the verification state payoff for the investor, and $p(\tilde{x}) = P(\tilde{x})$. The latter requirement ensures that $\tilde{x} = x$ is the optimal implied report by the manager. We therefore have that when $x \in [D + c_D, x_H)$, the equilibrium payoff to the manager equals

$$U(x) = p(x)((x-D)(1-\beta) + c_E\beta) + (1-p(x))(x-D-c_E)(1-\beta) \qquad (34)$$

The first term is the payoff for the manager if the shareholder accepts the offer $\tilde{E} = \beta(x - D - c_E)$, and the second term is the payoff for the manager if the shareholder rejects. Simplifying, we obtain,

$$\begin{aligned} U(x) &= (1-\beta)(x-D) + p(x)c_E\beta - (1-p(x))c_E(1-\beta) \\ &= (1-\beta)(x-D-c_E) + p(x)c_E \end{aligned} \qquad (35)$$

We now need to ensure that the manager's payoff function is continuous in the point $\tilde{D} = D$, a requirement will pin down K. It is straightforward to check that this construction makes the manager's payoff (locally) concave. Continuity implies that

$$\lim_{x\to(D+c_D)^-} U(x) = \lim_{x\to(D+c_D)^+} U(x), \text{ or} \tag{36}$$

$$\lim_{x\to(D+c_D)^-} \{q(x)[(1-\beta)c_D + \beta c_E)]\} = (1-\beta) \lim_{x\to(D+c_D)^+} \{x - D - c_E\} + \lim_{x\to(D+c_D)^+} \{p(x)c_E\}$$

Denote $\lim_{x\to(D+c_D)^-}\{q(x)\}$ by $q(D+c_D)^-$ and $\lim_{x\to(D+c_D)^+}\{p(x)\}$ by $p(D+c_D)$. Simplifying, we obtain,

$$\begin{aligned} q(D+c_D)^-[(1-\beta)c_D + \beta c_E)] &= (1-\beta)(c_D - c_E) + p(D+c_D)c_E \\ q(D+c_D)^- &= \frac{(1-\beta)(c_D - c_E) + p(D+c_D)c_E}{(1-\beta)c_D + \beta c_E} \end{aligned} \tag{37}$$

K is now determined by the equation

$$Ke^{\overline{(1-\beta)c_D + \beta c_E}^{\,D+c_D}} = \frac{(1-\beta)(c_D - c_E) + p(D+c_D)c_E}{(1-\beta)c_D + \beta c_E} \tag{38}$$

Solving for K and substituting in, we get,

$$q(x) = \frac{(1-\beta)(c_D - c_E) + p(D+c_D)c_E}{(1-\beta)c_D + \beta c_E} e^{-\frac{D+c_D-x}{(1-\beta)c_D+\beta c_E}} \tag{39}$$

To prove (i), subtract 1 from $q(D+c_D)^-$,

$$\begin{aligned} & q(D+c_D) - 1 \\ = & \frac{(1-\beta)(c_D - c_E) + p(D+c_D)c_E}{(1-\beta)c_D + \beta c_E} - 1 \\ = & -\frac{1 - p(D+c_D)(D+c_D)}{(1-\beta)c_D + \beta c_E} c_E < 0 \end{aligned} \tag{40}$$

To prove (ii), observe that

$$\begin{aligned} & q(D+c_D)^- - p(D+c_D) \\ = & \frac{(1-p)(1-\beta)(c_D - c_E)}{(1-\beta)c_D + \beta c_E} \geq 0, \end{aligned} \tag{41}$$

where it is clear that the inequality holds strictly for $c_D > c_E$. To prove (iii), it is sufficient to show that $\frac{\partial q}{\partial \beta} < 0$. This is straightforward but tedious and therefore omitted. ■

7 Appendix B: Optimal Capital Structure

We start out with some preliminaries. Let α be the fraction of the funding requirement financed by debt, and hence 1-α be the fraction financed by outside equity. The objective of the manager is to pick the financial structure $\alpha \in [0, 1]$ that minimizes expected verification costs, subject to the constraint that the outside investors are willing to participate and the conjectured repayment/verification behavior of a separating equilibrium. Recall that for given D and β, the expected verification cost under a mix of debt and outside equity equals,

$$V(D, \beta) = c_D \int_{x_L}^{D+c_D} [1 - q(x; .)] f(x)dx + c_E \int_{D+c_D}^{x_H} [1 - p(x; .)] f(x)dx \tag{42}$$

The participation constraints are,

$$\begin{aligned} U_C &= \int_{x_L}^{D+c_D} (x - c_D) f(x)dx + \int_{D+c_D}^{x_H} D f(x)dx = \alpha I \qquad (43) \\ U_E &= \int_{x_L}^{D+c_D} q(x; .)\beta(c_D - c_E) f(x)dx + \int_{D+c_D}^{x_H} \beta(x - D - c_E) f(x)dx = (1 - \alpha) I \end{aligned}$$

The first integral in U_C is the payoff to the creditor given that the cash flow is low and the manager defaults, and the second integral in U_C is the payoff to the creditor if the cash flow is high, and debt is repaid in full. The first integral in U_E is the payoff to the equity holder in low cash flow states. The shareholder here only gets a non-zero payoff if the creditor accepts a concession pledge, and absolute priority violations occurs. The second integral is the payoff in the high cash flow states. Combining the participation constraints, we can get rid of α,

$$\begin{aligned} I &= U_C + U_E = \int_{x_L}^{D+c_D} (x - c_D) f(x)dx + \int_{D+c_D}^{x_H} D f(x)dx \\ &\quad + \int_{x_L}^{D+c_D} q(x; .)\beta(c_D - c_E) f(x)dx + \int_{D+c_D}^{x_H} \beta(x - D - c_E) f(x)dx \qquad (44) \end{aligned}$$

This expression gives the (D, β) combinations that are sufficient to raise the capital requirement I, and the optimal capital structure must lie somewhere on the boundary defined by it.

It is convenient to express the first order condition for minimization of verification costs as

$$\frac{dV(.)}{dD} = \frac{\partial V(D,\beta)}{\partial D} + \frac{\partial V(D,\beta)}{\partial \beta}\frac{d\beta(D)}{dD} = 0. \tag{45}$$

Intuitively, an increased D gives a positive direct effect on the verification cost (through the first term) but a negative indirect effect in that β decreases (the second term). The optimal value of D defined by this equation can be substituted back into the expression for the (D, β) boundary to find the optimal β. Equipped thus with the optimal (β, D) one can subsequently find the optimal α through one of the participation constraints. Having described the system of equations that determines the three endogenous variables D^*, β^* and α^* (conditional on an interior solution) let us move to proving Proposition 7 and Proposition 8.

We first collect some derivatives. First,

$$\begin{aligned} \frac{\partial V}{\partial D} &= -c_D \int_{x_L}^{D+c_D} \frac{\partial q\,(x;.)}{\partial D} \cdot f(x)dx + c_D[1-q(D+c_D)] - c_E[1-p(D+c_D)] \quad (46) \\ \frac{\partial V}{\partial \beta} &= -c_D \int_{x_L}^{D+c_D} \frac{\partial q\,(x;.)}{\partial \beta} f(x)dx - c_E \int_{D+c_D}^{x_H} \frac{\partial p\,(x;.)}{\partial \beta} f(x)dx > 0 \end{aligned}$$

To derive $\dfrac{d\beta(D)}{dD}$, define

$$\Phi := U_C + U_E - I \tag{47}$$

By the implicit function rule,

$$\frac{d\beta}{dD} = -\frac{\Phi_D}{\Phi_\beta} \tag{48}$$

where subscript denotes partials. Writing out, we get,

$$\begin{aligned} \Phi_D = & \int_{D+c_D}^{x_H} f(x)dx + \int_{x_L}^{D+c_D} \frac{\partial q\,(x;.)}{\partial D}\beta(c_D - c_E)f(x)dx + \quad (49) \\ & q(D+c_D)\beta(c_D-c_E)f(D+c_D) - \beta(c_D-c_E)f(D+c_D) - \beta\int_{D+c_D}^{x_H} f(x)dx \end{aligned}$$

and

$$\Phi_\beta = \int_{x_L}^{D+c_D} q(x;.)(c_D - c_E)f(x)dx + \int_{x_L}^{D+c_D} \frac{\partial q(x;.)}{\partial \beta}\beta(c_D - c_E)dx + \int_{D+c_D}^{x_H} (x - D - c_E)f(x)dx \tag{50}$$

Simplifying and putting together,

$$\frac{d\beta}{dD} = -\frac{\Phi_D}{\Phi_\beta} = \tag{51}$$

$$-\frac{(1-\beta)\int_{D+c_D}^{x_H} f(x)dx + \beta(c_D - c_E)[\int_{x_L}^{D+c_D} \frac{\partial q(.)}{\partial D}dx - [1 - q(D+c_D)]f(D+c_D)]}{(c_D - c_E)[\int_{x_L}^{D+c_D} q(.)f(x)dx + \beta\int_{x_L}^{D+c_D} \frac{\partial q(.)}{\partial \beta}dx] + \int_{D+c_D}^{x_H}(x - D - c_E)f(x)dx}$$

Proof of Proposition 7. Here we show that the firm will never finance itself with pure equity financing. Let us first consider the case when risk-free debt can be issued., i.e., $c_D < x_L$. Suppose that the firm is fully equity financed. In that case the first term of $V(.)$ is zero and the second term is positive. This can clearly not minimize verification cost, as substituting some external equity for debt will reduce β, and hence reduce the second term of $V(.)$, without increasing the first term.

Let us now consider the more interesting case where debt is always risky, i.e., $c_D = x_L$. To show that the firm will always issue debt also in this case, it is sufficient to show that $\frac{dV(.)}{dD}_{D=0} < 0$, where

$$\frac{dV(.)}{dD}_{D=0} = \frac{\partial V(D,\beta(D))}{\partial D}_{D=0} + \{\frac{\partial V(D,\beta(D))}{\partial \beta}\frac{d\beta(D)}{dD}\}_{D=0} \tag{52}$$

Let us now collect terms. When D goes to zero, $\frac{\partial q(.)}{\partial D} = q(D+c_D) = 1$, and $\int_{x_L}^{D+c_D} f(x)dx \equiv$

0. Therefore,

$$\begin{aligned}
\frac{d\beta}{dD}_{D=0} &= -\frac{(1-\beta)}{\int_{x_L}^{x_H}(x-c_E)f(x)dx} < 0 \\
\frac{\partial V}{\partial D}_{D=0} &= -c_E(1-p(c_D)) < 0 \\
\frac{\partial V}{\partial \beta}_{D=0} &= -c_E\int_{x_L}^{x_H}\frac{\partial p\,(x;.)}{\partial\beta}f(x)dx
\end{aligned} \tag{53}$$

To end the proof, note that $\frac{\partial p\,(x;.)}{\partial\beta} = -\frac{(x_H-x)p(x;.)}{c_E} < 0$. Consequently,

$$\frac{dV(.)}{dD}_{D\to 0} = -c_E[1-p(c_D)] - \int_{x_L}^{x_H}(x_H-x)p(x)f(x)dx\frac{(1-\beta)}{\int_{x_L}^{x_H}(x-c_E)f(x)dx} < 0 \tag{54}$$

■

Proof of Proposition 8. Let us begin by proving (ii). Suppose that $I \in (\bar{I}_D, \bar{I}_E]$. By the definition of $\bar{I}_D$ and $\bar{I}_E$, the project is not fundable with pure debt financing, while it is fundable with pure equity financing. It follows immediately that at least some external equity must be used to fund the project. (i)Let us now prove that for sufficiently low I, it will be optimal to finance the firm through pure debt financing. We consider the non-trivial case $c_D = x_L$. Recall that,

$$\frac{dV(.)}{dD} = \frac{\partial V(D,\beta(D))}{\partial D} + \frac{\partial V(D,\beta(D))}{\partial\beta}\frac{d\beta(D)}{dD} \tag{55}$$

It is sufficient to show that when I goes to zero, this expression is negative. Note that as I goes to zero, both β and D must go to zero. As I becomes small, the first term of $\frac{\partial V(.)}{\partial D}$ must converge to zero since the interval of integration goes to zero. The second and third term of $\frac{\partial V(.)}{\partial D}$ goes to zero since both both $q(.)$ and $p(.)$ goes to unity when I goes to zero. Now consider $\frac{\partial V}{\partial\beta}$. When I becomes small, the first term goes to zero since the interval of integration goes to zero. Recall that $\frac{\partial p\,(x;.)}{\partial\beta} = \frac{x_H - x}{c_E}p(x;.)$. Therefore, the second term of $\frac{\partial V}{\partial \beta}$ converges to $x_H - x$ as I tends to zero. Now inspect $\frac{d\beta}{dD}$.

$$\frac{d\beta}{dD} = -\frac{\Phi_D}{\Phi_\beta} = \tag{56}$$

$$-\frac{(1-\beta)\int_{D+c_D}^{x_H} f(x)dx + \beta(c_D - c_E)[\int_{x_L}^{D+c_D} \frac{\partial q\,(.)}{\partial D}dx - [1 - q(D+c_D)]f(D+c_D)]}{(c_D - c_E)[\int_{x_L}^{D+c_D} q(.)f(x)dx + \beta\int_{x_L}^{D+c_D} \frac{\partial q\,(.)}{\partial \beta}dx] + \int_{D+c_D}^{x_H}(x - D - c_E)f(x)dx}$$

When I tends to zero, β goes to zero and the numerator goes to 1 (recall that $c_D = x_L$). The first term of the denominator goes to zero, and the second term goes to $Ex - c_E > 0$, where Ex is the expected cash flow, $\int_{x_L}^{x_H} xf(x)dx$. Therefore $\frac{d\beta}{dD}$ converges to $-\frac{1}{Ex - c_E}$ as I tends to zero. Putting terms together, we therefore have that,

$$\left.\frac{dV(.)}{dD}\right|_{I\to 0} = \frac{Ex - x_H}{Ex - c_E} < 0 \tag{57}$$

■

8 Appendix C: Who pays the verification cost

Let us here compare the case when the manager pays the verification cost, as considered in the main analysis, with the case when the investors pay the verification cost. We show that under debt financing, this question is immaterial in terms of net payoffs, while under equity financing the analysis becomes similar. The implications for mix are the same.

8.1 Debt financing

Recall that in the main analysis the equilibrium payoffs are determined by,

$$\begin{aligned} U_C &= \int_{x_L}^{D+c_D}(x - c_D)f(x)dx + \int_{D+c_D}^{x_H} Df(x)dx = I \qquad (58) \\ Q(x) &= e^{-\frac{D+c_D-x}{c_D}} \end{aligned}$$

The first equation determines the D necessary to fund I. For a given D, the expression for $Q(.)$ determines the expected verification cost (for brevity this expression is skipped).

Suppose now that the creditor pays the verification costs. We distinguish this case by using lower case letters. Suppose that a debt contract is written with the verification state payoff for the creditor $d(.)$ equal to,

$$\begin{aligned} &x \text{ for } x \in [x_L, d] \\ &d, \text{ for } x \in [d, x_H] \end{aligned} \tag{59}$$

Moreover, the contract assigns the creditor with a right to verify whenever $\tilde{d} < d$. In a separating equilibrium, the repayment offer then must equal,

$$\tilde{d}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, d] \\ d - c_D, \text{ for } x \in [d, x_H] \end{cases} \tag{60}$$

Since the creditor pays the verification costs, this repayment schedule makes the creditor indifferent as to whether verify or not. Note that the manager *always* defaults on his debt obligation.[34]

Given this repayment schedule, the expected payoff for the creditor becomes,

$$u_C = \int_{x_L}^{d} (x - c_D) f(x) dx + \int_{d}^{x_H} (d - c_D) f(x) dx \tag{61}$$

Turning to the equilibrium verification probability function, let us consider the interesting case where $(\tilde{x}, x) \in [x_L, d]$. For a given $q(.)$, the payoff for the manager equals,

$$\begin{aligned} u(\tilde{x}, x) &= q(\tilde{x})(c_D + x - \tilde{x}) + [1 - q(\tilde{x})]0 \\ &= q(\tilde{x})(c_D + x - \tilde{x}) \end{aligned} \tag{62}$$

Differentiating with respect to $\tilde{x}$, setting $\tilde{x} = x$, and solving the resulting differential

[34] Suppose that the manager offers $d - c_D + \epsilon$ when $x \in (d, x_H)$. In that case the creditor *strictly* prefers to accept, which justifies the corner condition $Q(d - c_D) = 1$ imposed later.

equation (with boundary condition $q(d-c_D)=1$) gives,

$$q(x) = e^{-\dfrac{d-x}{c_D}} \tag{63}$$

But now substitute in for $d = D + c_D$ in $u_C(.)$ and $q(.)$ to get,

$$\begin{aligned} u_C &= \int_{x_L}^{D+c_D} (x-c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx \\ q(x) &= e^{-\dfrac{D+c_D-x}{c_D}} \end{aligned} \tag{64}$$

But note that $u_C(.) = U_C(.)$ and $q(.) = Q(.)$. Hence the contract $\{D$, the manager pays$\}$ and the contract $\{d = D + c_D$, the creditor pays$\}$ are equivalent in terms of net payoffs. Finally, we have to check that the range of feasible net payoffs is the same. When the manager pays, the relevant range of contracts is $D \in [0, x_H - c_D]$. When the creditor pays, the relevant range of contracts is $d \in [c_D, x_H]$. But since the transformation is $d = D + c_D$, the range of feasible net payoffs must be the same in the two cases.

8.2 Equity financing

Recall that when the firm pays the verification costs the equilibrium payoffs are determined by,

$$\begin{aligned} U_E &= \int_{x_L}^{x_H} \beta(x-c_E) f(x) dx = \beta(Ex - c_E) = I \\ P(x) &= e^{-\beta\dfrac{x_H - x}{c_E}} \end{aligned} \tag{65}$$

The first equation determines β as a function of I, and the second equation determines the expected verification costs. Note that in this formulation, the shareholder essentially pays a fraction β of the verification costs, which can be interpreted as the shareholder being partially reimbursed for costs of engaging in a proxy contest.

Let us now consider the case where the equity holder pays the full verification cost.

We distinguish this case by using lower case letters. Suppose that an equity contract is written with the verification state payoff for the shareholder equal to βx. Assuming that β is sufficiently high to make $\beta x_L - c_E \geq 0$,[35] in a separating equilibrium the repayment offer must then equal,

$$\tilde{e}(x) = \beta x - c_E \tag{66}$$

Since the equity holder pays the verification costs, this repayment schedule makes the equity holder indifferent between verifying or not. The expected payoff for the equity holder then equals,

$$u_E = \int_{x_L}^{x_H} (\beta x - c_E) f(x) dx = \beta E x - c_E \tag{67}$$

It follows that for a given β, the payoff for the equity holder is higher when the firm pays the verification cost than when he pays it himself. Let us turn to the equilibrium verification probability function. For a given $p(.)$ function the payoff for the manager equals,

$$U(\tilde{x}, x) = p(\tilde{x})((1-\beta)\tilde{x} + c_E + x - \tilde{x}) + [1 - p(\tilde{x})][(1-\beta)x \tag{68}$$

Differentiating with respect to $\tilde{x}$ and setting $\tilde{x} = x$,

$$\begin{aligned} u'(\tilde{x}, x) &= p'(\tilde{x})(c_E + x - \tilde{x}) - p(\tilde{x})\beta \qquad (69) \\ &= p'(\tilde{x})c_E - p(\tilde{x})\beta \end{aligned}$$

Setting this expression equal to zero and solving the differential equation using the corner condition $P(x_H) = 1$ gives,

$$p(x) = e^{-\beta \frac{x_H - x}{c_E}} \tag{70}$$

But note that $p(.) = P(.)$. However, the required ownership fraction β in the alternative formulation will be higher, since the investor receives $\beta x - c_E$ in equilibrium rather than

[35] If this condition does not hold, then exactly the same analysis goes through if we assume that $\tilde{E} < 0$ is permissable. Allowing for $\tilde{E} < 0$ would reflect a venture financing type of situation where the manager can force the outside equity holder (venture capitalist) to choose between adding funds to the company or to take control, i.e., a type of staged financing contract. If $\tilde{E}$ is constrained to be non-negative and β is not sufficiently high to make $\beta x_L - c_E \geq 0$ then there will exist partially separating equilibria that are hard to characterize (see e.g., Bester and Strausz, 2001).

$\beta(x - c_E)$. Consequently, the accept probability function in the alternative formulation will have the same qualitative properties as the formulation in the text, but for any I the expected verification costs from pure equity financing will be higher.

8.3 Mixed financing

Although equity will be more costly when the investor pays (which is a reason for imposing in the contract that the manager pays verification cost, bringing us back to the main analysis), the results on capital structure will be qualitatively speaking identical to in the case where the manager pays, in that all Propositions 4-9 will continue to hold. To see why, observe that whenever $c_E < c_D$ then the set of fundable projects is still greater under pure equity financing than under pure debt financing (for $\beta = 1$, it is immaterial who pays the verification cost, as under pure debt financing), so that equity must be used under a high funding requirement (Proposition 8). It is also straightforward to show that debt will always be used in this alternative formulation (Proposition 7). The qualitative properties of a mixed equilibrium (Propositions 4-6) will also be the same.[36]

[36] With the caveat that strategic defaults and priority violations occur for all x, not only in the lower region as in the main analysis.