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ABSTRACT

A Portfolio Choice Model with Utility from Anticipation of Future Consumption and Stock Markets' Mean Reversion*

This Paper studies a consumption and portfolio choice problem of a long-lived investor who derives pleasure not only from current consumption, but also from contemplation of future consumption. These preferences are formalized by Kuznitz (2003a, 2003b, 2003c), in a model where all effects of future consumption on current well being are assumed to enter through a single variable – that is, the 'stock of future consumption' – analogously to habit-formation models. The main implications of the model concern the incentives for savings, and the fundamental sources of risk in financial markets. In this Paper it is shown that, when the stock market exhibits mean reversion, deriving utility from anticipation of future consumption has a tremendous effect on portfolio choice. In particular, mean allocation to stocks is much lower under the proposed preferences relative to the standard preferences, especially for high-risk averse investors.

Keywords: consumption, habit-formation, mean reversion and portfolio

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1 Introduction

The standard time-separable von Neumann-Morgenstern preferences suggest that a person's felicity is a function only of her current consumption level. Yet, it requires little effort to think of daily life examples where one's anticipation of future consumption affects her current well being. Actually, this idea has long been emphasized by Jevons (1871), and more recently was formally developed by Loewenstein (1987).[1] A main feature of Loewenstein's model is that it gives rise to time inconsistency.

In this paper we assume that investors derive utility from anticipation of future consumption, and that all effects of future consumption on current well being enter through a single variable — that is, the "stock of future consumption" — analogously to habit-formation models. Consequently, consumers are dynamically consistent. Most importantly, we show that these preferences, the main implications of which concern the incentives for savings and the fundamental sources of risk in financial markets, have a tremendous effect on portfolio choice when stock returns are mean reverting.

We solve numerically the consumption and portfolio choice problem of a long-lived consumer who faces a time-varying equity premium; the problem is solved for both the proposed preferences and the standard power utility. Three lines of empirical research have emphasized the importance of this problem. First, many empirical studies have found evidence that the excess return on stocks over treasury bills is predictable (see Keim and Stambaugh [1986], Campbell [1987], Campbell and Shiller [1988], Fama and French [1988, 1989], and Hodrick [1992]). Second, in articles on the equity premium puzzle it is usually claimed that average excess stock returns are too high to be consistent with the standard preferences with low relative risk aversion (see e.g. Mehra and Prescott [1985], Hansen and Jagannathan [1991], Kandel and Stambaugh [1991] Cochrane and Hansen [1992], and Campbell [1999]). Third, recently Campbell and Viceira (1999) have shown that when the expected return on stocks follows a mean reverting AR(1) process, calibrated to US stock market data, then the average demand for stocks substantially increases due to intertemporal hedging motives, implying that the equity premium puzzle

[1]Elster and Loewenstein (1992) extensively discuss the importance of utility from anticipation, and survey related literature.

becomes more severe.[2] These results suggest that it is important to study the problem of optimal consumption and portfolio choice when stock returns are mean reverting and investors have the general preferences proposed in this paper.

We consider the same discrete-time model for the return process as in Campbell and Viceira (1999); that is, we assume a constant riskless interest rate and a mean-reverting AR(1) process for the risky asset return, calibrated to US stock market data. Differently from Campbell and Viceira (1999) and similarly to Campbell et al. (2001), we consider a constrained version of the problem, in which the investors are not allowed to borrow at the riskless interest rate or to sell short the risky asset. Likewise, we allow the investor to have the general preferences proposed in this paper, in which the consumer's felicity is positively affected not only by current consumption, but also by anticipation of future consumption.

The proposed preferences imply a new dimension of risk in financial markets, as investors wish to smooth out fluctuations not only in their consumption, but in their wealth to consumption ratio as well. We find that these preferences have a tremendous effect on portfolio choice. In particular, mean allocation to stocks is much lower under the proposed preferences relative to the standard preferences, especially for high risk averse investors. The underlying mechanism is that under mean-reverting stock returns, the wealth to consumption ratio is positively correlated with the return on stocks. This increases the risk of investing in stocks for investors who care about their wealth to consumption ratio. Likewise, we find that saving rates are much higher in the current model relative to the standard model, as consumers save also for the sake of anticipating future consumption.

The paper proceeds as follows. Section 2 explains the motivation for the proposed preferences and surveys related literature. Section 3 presents the theoretical model. Section 4 describes the model's calibration and discusses its results. Section 5 concludes.

[2] See also Kim and Omberg (1996) and Wachter (2002) for analytical solutions in continuous time.

2 Utility from anticipation

2.1 Background

Few of us would doubt that much of the pleasure and pain we experience in daily life arises not from direct experience — that is, "consumption" — but from contemplation of our own past or future (Elster and Loewenstein, 1992). However, the traditional economic literature assumes, at least implicitly, that a person's felicity is a function only of her current consumption level.

Habit-formation models emphasize the effect of past consumption on the utility from current consumption through the "contrast effect".[3] Generally, these models assume that all effects of past consumption on current well being enter through a single state variable — that is, the "stock of past consumption". In recent years, habit-formation models have been applied to analyze many economic phenomena, and, in particular, to explain some asset pricing anomalies. (see Sundaresan [1989], Constantinides [1990], Abel [1990], Ferson and Constantinides [1991], Heaton [1995], Campbell and Cochrane [1999]).

Yet, the effect of future consumption on current well being has not received enough attention. Loewenstein (1987) was the first to formally develop the idea that individuals derive pleasure from anticipation of future consumption. Essentially, Loewenstein's model assumes that a person's instantaneous utility is equal to the utility from current consumption, plus some function of the discounted utility of consumption in future periods. The lifetime utility function implied by Loewenstein's model differs from the standard utility in that it relaxes the exponential discounting assumption — and thus, its main implication is time inconsistency.

Kuznitz (2003a, 2003b, 2003c) assumes that all effects of future consumption on the instantaneous utility function enter through a single variable — that is, the "stock of future consumption" — analogously to habit-formation models. The idea, thus, is that individuals enjoy future pleasures holistically — that is, without breaking the future into its parts. To put things differently, it is as though individuals live on two planes at each time point: The here and now (current consumption), and the future as a whole. Further,

[3]Frederick, Loewenstein, and O'Donoghue (2002) highlight the standing of habit-formation models within the strand that enriches the instantaneous utility function.

we assume that future well being is exponentially discounted, as in the standard model. Essentially, exponential discounting, together with the clear distinction which is made here between "future consumption" (a good of which only anticipatory utility is derived) and "actual consumption", allow Kuznitz's model to be dynamically consistent.

There is no single way to construct a single variable that captures all effects of future consumption on current well being, and one could form the "stock of future consumption" in fairly arbitrary ways. In Kuznitz's model existing market prices of (stochastic) future consumption are used to add up the pieces to the "stock of future consumption", and it is assumed that individuals evaluate random future well being within the usual expected utility framework.[4]

2.2 Related literature: Direct utility from wealth

Kuznitz's model may be linked to a model where direct utility is derived from wealth, an idea which has been traditionally associated with Pigou (1941), and more recently was explored by Bakshi and Chen (1996). Just to give the flavor, in discussing the factors which affect savings behavior, Pigou emphasizes "the present value of the direct amenity utility, in the form of power, sense of security and so on, if any, which a typically constituted man expects to derive from *having* his marginal unit of present savings, as distinct from the utility which he expects the future incomes due to that unit to yield" (1941, p.103).[5]

The link of Kuznitz's model to the aforementioned literature is that, in equilibrium, the "stock of future consumption" equals the amount of wealth available in the next period. Thus, as was also noted by Loewenstein (1987), direct wealth preference may actually be a "reduced form" of anticipatory utility, as through accumulation of wealth people imagine and thus derive immediate pleasure from the consumption which wealth could finance.[6]

[4]Using the existing market prices of (stochastic) future consumption to add up the pieces to the "stock of future consumption" has the virtue of keeping the model parsimonious.
[5]Emphasis in the original.

3 The model

3.1 Technology, markets, and preferences

Technology

We consider a discrete-time, infinite-horizon economy, with a continuum of consumers, who may differ in preferences and asset holdings. Time is denoted by $t = 0, 1, 2, \ldots$. Uncertainty is represented by a probability space (Ω, F, P) on which all stochastic processes are defined, where the information structure $\{F_t\}_{t=0}^{\infty}$ satisfies the usual assumptions. Throughout, random variables indexed by t (denoted by a tilde) are assumed to be F_t-measurable.

There exists a single consumption good, which may be consumed or invested in two assets. Asset f pays a sure dividend in every future period, and its price is constant through time; let r_f denote the one-period log return on f. Asset r pays a random dividend in every future period, and its price is state-dependent; let $\tilde{r}_{r,t+1}$ denote the one-period log return on r.

The approach is one of partial equilibrium — that is, both the riskless interest rate, and the stochastic process for returns are external to the model. We follow Campbell and Viceira (1999) in assuming that the expected excess log return on the risky asset, denoted by $S_t \equiv E_t \tilde{r}_{r,t+1} - r_f$, evolves as a stationary AR(1) process:

$$S_{t+1} = (1-\rho)\bar{S} + \rho S_t + \sigma_s \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \; i.i.d. \; N \; (0, 1-\rho^2), \tag{1}$$

where $\bar{S}$, σ_s, and ρ are the mean, standard deviation, and serial correlation of S_t, respectively. The unexpected excess log return on the risky asset, ν_{t+1}, is a normally distributed white noise process, correlated with innovations in S_t:

$$\tilde{r}_{r,t+1} = r_f + S_t + \nu_{t+1}, \quad \nu_{t+1} \sim \; i.i.d. \; N \; (0, \sigma_\nu^{\,2}), \quad \mathrm{corr}(\varepsilon_{t+1}, \nu_{t+1}) = \omega . \tag{2}$$

[6]It should be noted, though, that Pigou (1941) assumes that utility is derived only from the stock of capital,

Markets

We assume that the given price processes for the securities are arbitrage-free. This implies the existence for each date *t* of strictly positive prices of the consumption good at all future dates and in all future states; let $\{\tilde{M}_{t,t+n}\}_{n=1}^{\infty}$ denote prices of date and state contingent claims divided by states' probabilities. The spot price of one unit of consumption is normalized to one.

Preferences

Following Kuznitz (2003a, 2003b, 2003c) we assume that the consumer's instantaneous utility — her well being in period *t*, is a linear function of her utility from current consumption, and her expected utility from the "stock of future consumption" — defined as the next period value of total future consumption. Thus, the consumer's expected lifetime utility at date *t* is given by:

$$U\left(\{\tilde{c}_{t+n}\}_{n=0}^{\infty},\{\tilde{x}_{t+n+1}\}_{n=0}^{\infty}\right)=E_t\sum_{n=0}^{\infty}\beta^n[u(\tilde{c}_{t+n})+\delta u(\tilde{x}_{t+n+1})],\quad 0<\beta<1,\quad \delta>0, \qquad (3)$$

where β is the subjective time discount factor, δ is a scaling parameter that determines the relative weight the consumer attaches to anticipation of future consumption in the current well being (to be defined later), E_t is the conditional expectation operator with respect to F_t, c_t is consumption, and x_t is defined to be:

$$x_t \equiv c_t + E_t\sum_{n=1}^{\infty}\tilde{M}_{t,t+n}\tilde{c}_{t+n} \qquad (4)$$

for all $t \geq 0$. Finally, the function $u(\cdot)$ is given by:

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that is, not including the value of human capital, which weakens its link to the current model.

$$u(x) = \frac{x^{1-\gamma} - 1}{1-\gamma}, \qquad \gamma > 0, \tag{5}$$

where γ is the constant relative risk aversion coefficient.[7] Thus, the proposed preferences have the property that consumers' decision rules are scale-invariant.[8]

3.2 Optimization

At each date *t*, the consumer allocates her wealth, w_t, defined as the market value of total assets, between consumption, c_t, investment in the risky asset, k_t, and riskless savings, b_t. The period-*t* budget constraint is:

$$c_t + k_t + b_t \leq w_t, \tag{6}$$

$$c_t, k_t, b_t \geq 0 \tag{7}$$

for all $t \geq 0$, where w_0 is given. Likewise, $\tilde{w}_{t+1}$ satisfies:

$$\tilde{w}_{t+1} = k_t \tilde{R}_{r,t+1} + b_t R_f, \tag{8}$$

where $\tilde{R}_{r,t+1} = \exp(\tilde{r}_{r,t+1})$, $R_f = \exp(r_f)$. It should be noted that restriction (7) does not allow the consumer to borrow at the riskless rate (that is, to hold a leveraged portfolio) or to sell short the risky asset.

Let us define the value function for an individual in this economy, as the maximized value of the utility function (3). The value function depends on individual wealth, w_t, and on the single state variable for the economy, S_t:

[7] In appendix A.2 we show that the elasticity of intertemporal substitution in consumption is the reciprocal of the coefficient of relative risk aversion, γ, as is the case with the standard power utility function.

[8] That is, with a stationary investment opportunity set, consumers' decision rules do not change over time as the scale of the economy increases.

$$V(w_t, S_t) \equiv \max_{\{k_t,\dots,b_t,\dots\}} U\left(\{\tilde{c}_{t+n}\}_{n=0}^{\infty}, \{\tilde{x}_{t+n+1}\}_{n=0}^{\infty}\right), \tag{9}$$

where the maximization is subject to (6), (7) and (8). The value function (9) is the solution to the Bellman functional equation:

$$V(w_t, S_t) = \max_{\{k_t, b_t\}} \left\{u(c_t) + \delta E_t u(\tilde{x}_{t+1}) + \beta E_t V\left(\tilde{w}_{t+1}, \tilde{S}_{t+1}\right)\right\}. \tag{10}$$

The weak inequality in (6) will bind in an optimum; thus, solving for c_t, substituting into (10), and differentiating with respect to k_t and b_t (using [8]), yield the first-order necessary conditions, indexed by $j, t+1$:

$$-u'(c_t) + E_t\left\{\left[\beta V_1\left(\tilde{w}_{t+1}, \tilde{S}_{t+1}\right) + \delta u'(\tilde{x}_{t+1})\right]\tilde{R}_{j,\,t+1}\right\} = 0, \quad j = f, r, \tag{11}$$

where $\tilde{R}_{j,\,t+1}$, is the gross rate of return on asset *j* held from time *t* to *t*+1.
The envelope condition for this maximization is:[9]

$$V_1(w_t, S_t) = u'(c_t). \tag{12}$$

Substituting the envelope condition at date *t*+1 in the first-order necessary conditions yields the Euler equations:

$$u'(c_t) = E_t\left\{\left[\beta u'(\tilde{c}_{t+1}) + \delta u'(\tilde{x}_{t+1})\right]\tilde{R}_{j,\,t+1}\right\}, \quad j = f, r. \tag{13}$$

A solution of the agent's optimization problem must also satisfy the boundary condition:

$$\lim_{N\to\infty} E_t[\tilde{M}_{t,t+N}\,\tilde{w}_{t+N}] = 0. \tag{14}$$

[9]See Sargent (1987).

For a purpose, which will be clear immediately, it will be useful to derive the lifetime budget constraint too. To that end, let us note that given that there are no arbitrage opportunities, the date *t* portfolio decision is equivalent to a choice of the amount of date $t+1$ state-contingent wealth. Thus, one can write the period-*t* budget constraints as:

$$c_t + E_t\left[\tilde{M}_{t,t+1}\tilde{w}_{t+1}\right] \leq w_t\,, \tag{15}$$

$$c_t\,,\ \tilde{w}_{t+1} \ \geq\ 0 \tag{16}$$

for all $t \geq 0$. By iterating on the period-*t* budget constraints ([15] and [16]), and using the boundary condition (14), we obtain:

$$w_t = c_t + E_t \sum_{n=1}^{\infty} \tilde{M}_{t,t+n}\tilde{c}_{t+n}\,. \tag{17}$$

implying that $x_t = w_t$ for all $t \geq 0$. That is, in an optimum, the "stock of future consumption", x_{t+1}, equals the amount of wealth available in the next period, w_{t+1}. Thus, one can write the Euler equations (13), using more "familiar" terms:

$$u'(c_t) = E_t\left\{\left[\beta u'(\tilde{c}_{t+1}) + \delta u'(\tilde{w}_{t+1})\right]\tilde{R}_{j,\,t+1}\right\}, \quad j = f, r\,. \tag{18}$$

Intuition about the Euler equation

The Euler equations (13) state that along an optimal path the consumer cannot raise her expected utility by forgoing one unit of consumption in period *t*, investing it in any asset, and consuming the proceeds in period $t+1$, while benefiting from *anticipating* the marginal increase in "future consumption". Note that if $\delta = 0$, that is, when consumers' current well being is not affected by future consumption, the Euler equations collapse to those implied by the standard model.

The relative weight attached to anticipatory utility in current well being

Let us define the (state-contingent) relative weight the consumer attaches to anticipatory utility in current well being, as the expected marginal utility from the "stock of future consumption" scaled by the marginal utility from current consumption — in an optimum:

$$Z_t(S_t) \equiv \frac{\delta E_t u'(\tilde{x}_{t+1})}{u'(c_t)} = \delta E_t \left(\frac{\tilde{w}_{t+1}}{c_t} \right)^{-\gamma} . \tag{19}$$

Intuitively, the larger is future consumption relative to current consumption, the less preoccupied consumers will be, at the margin, with the future relative to the immediate experience (that is, current consumption).[10]

Practically, one can interpret $Z_t(S_t)$ as the consumer's (relative) valuation of the marginal unit of savings — due to the anticipatory utility it delivers. To put things differently, $Z_t(S_t)$ reflects the extra amount the consumer would be willing to pay for the marginal unit of savings, due to its anticipatory value.[11]

The unconditional mean of $Z_t(S_t)$, denoted by $\overline{Z}$, will be treated as a parameter in the model.[12]

3.3 Implications

Incentives for savings

The intertemporal marginal rate of substitution (IMRS) between consumption at time *t* and $t+1$ is given by:

[10]It should be noted that the fact that the "stock of future consumption", and "current consumption" are of different order of magnitude is innocuous, as we care only about the ratio of marginal utilities between the two.

[11]Note that Z_t is independent of the consumer's wealth.

[12]One should naturally expect $\overline{Z}$ — the mean ratio of marginal utilities between future consumption and current consumption to be frequency-dependent, as at high frequencies (for example, days), the future constitutes a larger portion of a person's lifetime than at low frequencies (for example, years).

$$IMRS_{t,t+1} = \frac{\beta u'(\tilde{c}_{t+1}) + \delta u'(\tilde{x}_{t+1})}{u'(c_t)}. \tag{20}$$

The intuition behind (20) is that current consumption delivers only consumption utility, while future consumption delivers both consumption utility and anticipatory utility — providing an additional incentive for saving. Thus, other things being equal, consumers in this economy would like to exhaust their resources at a slower rate than that implied by the neoclassical model.

Sources of risk

Substituting (5) into (20) and rearranging gives:

$$IMRS_{t,t+1} = \left(\frac{\tilde{c}_{t+1}}{c_t}\right)^{-\gamma} \left[\beta + \delta \left(\frac{\tilde{w}_{t+1}}{\tilde{c}_{t+1}}\right)^{-\gamma}\right], \tag{21}$$

implying a new dimension of risk in financial markets — that is, the risk to the "consumption multiplier", or the wealth to consumption ratio. Thus, in an optimum, consumers face not only the risk that next period's consumption will be hit (as is the case in the standard model), but also the risk that consumption in all subsequent future periods will be even more badly hit — embodied by a negative shock to the "consumption multiplier". Intuitively, low "consumption multiplier" is a bad predicament, since it implies that one's future standard of living is going to deteriorate, as the proportion of resources consumed at the present time is higher than the average.

To sum up, in the current model economy, individuals wish to smooth out fluctuations not only in their consumption, but in their wealth to consumption ratio as well. Hence, if the wealth to consumption ratio is positively correlated with the return on the risky asset, then investment in the latter will expose consumers to an additional risk, beyond the standard consumption risk.

3.4 Solving for consumption and portfolio decisions

Exploiting the fact that the preferences are scale-invariant, we can normalize the value function (9) — that is, define it for a consumer who has wealth equal to one. Thus, using (3), (5), and the fact that $x_t = w_t$ for all $t \geq 0$, the Bellman functional equation (10) can be written as:

$$V(S_t) = \max_{\{\alpha c_t,\ \alpha k_t\}} \left\{ \frac{1}{1-\gamma}[(\alpha c_t)^{1-\gamma} + \delta E_t (d\tilde{w}_{t+1})^{1-\gamma}] + \beta E_t V\left(\tilde{S}_{t+1}\right) \right\}, \tag{22}$$

where:

$$\alpha c_t \equiv c_t / w_t ,$$

$$\alpha k_t \equiv k_t / (w_t - c_t),$$

$$d\tilde{w}_{t+1} \equiv \tilde{w}_{t+1} / w_t$$

for all $t \geq 0$. Similarly, the Euler equations (18) can be written as:

$$(\alpha c_t)^{-\gamma} = E_t \left\{ (d\tilde{w}_{t+1})^{-\gamma} \left[\beta (\alpha \tilde{c}_{t+1})^{-\gamma} + \delta \right] \tilde{R}_{r,\,t+1} \right\}, \tag{23}$$

$$(\alpha c_t)^{-\gamma} = R_f E_t \left\{ (d\tilde{w}_{t+1})^{-\gamma} \left[\beta (\alpha \tilde{c}_{t+1})^{-\gamma} + \delta \right] \right\}. \tag{24}$$

Finally, the dynamic budget constraint (6)-(8) can be written as:

$$d\tilde{w}_{t+1} = (1 - \alpha c_t) \left[R_f + \alpha k_t (\tilde{R}_{r,\,t+1} - R_f) \right], \tag{25}$$

where:

$$0 \leq \alpha c_t \leq 1, \tag{26}$$

$$0 \leq \alpha k_t \leq 1 \tag{27}$$

for all $t \geq 0$. Thus, exploiting the scale-invariance of the model, we need to solve only for the proportion of wealth consumed and the proportion of savings invested in the risky asset for each state of nature, S_t. Given the solution we will be able to calculate the mean allocation to stocks, the first two moments of consumption growth and the wealth to consumption ratio, and other variables of interest.

3.5 Practical solution

In practice, using equations (1), (2), and (23)-(27), we solve the investor's problem backwards numerically from the last period to the first, given the parameters set (where in the last period the investor consumes all his resources). The solution is carried out for investors with long horizons (400 quarters) to approximate the solution to the infinite-horizon problem. Likewise, the problem is solved sequentially, that is, in each period the consumer first chooses the proportion of wealth consumed, and at the second stage she chooses the portfolio weight for the risky asset.

A number of recent papers have also presented numerical solutions to intertemporal consumption and portfolio choice problems. These include Kandel and Stambaugh (1996), Brennan et al. (1997), Balduzzi and Lynch (1999), Brandt (1999), Barberis (2000), Campbell et al. (2001), and Lynch (2001). These papers, except Campbell et al. (2001) study problems with a finite horizon and power utility defined over terminal wealth. Campbell et al. (2001) consider an infinite horizon and Epstein-Zin-Weil utility (Epstein and Zin, 1989; Weil, 1989) over consumption.

4 Model calibration

In this section we present the major results of the model's calibration. More specifically, we show the effect of the proposed preferences on portfolio choice, consumption growth, and the wealth to consumption ratio when stock returns are mean reverting. Mainly, we illustrate that investors' desire to have a smooth "consumption multiplier" across states of nature has a tremendous effect on portfolio choice, especially for high risk averse investors.

4.1 Parameter values

Return process parameters

For the return process parameters we follow Campbell and Viceira (2000) who estimate them from a time series of quarterly US financial data for the period 1947:1-1995:4. In their estimation, the risky asset is taken to be the CRSP value-weighted market portfolio, and the risk-free asset is the three-month Treasury bill. Table 1 summarizes the parameter choices. Mainly, the parameters set implies that the expected return on stocks is highly persistent, and that the correlation between unexpected stock returns and revisions in expected future stock returns is almost perfectly negative.

Preferences

We set the subjective time discount factor, β, to 0.94 in annual terms, as in Campbell and Viceira (1999).

We consider relative risk aversion coefficients, γ, ranging from 5 to 30, and mean relative weights attached to anticipatory utility in the current well being, $\overline{Z}$, ranging from 0.062 to 0.080. Likewise, it should be noted that we "fine tune" $\overline{Z}$ by changing δ, the scaling parameter.[13] We report, however, only the value of the former, which has a clear economic meaning.

Practically, given β, δ, and the return process parameters, we solve the investor's problem for different values of relative risk aversion, while focusing on the differences in behavior between the current model ($\overline{Z} > 0$) and the standard one ($\overline{Z} = 0$).

4.2 Results

Tables 2 and 3 report the mean allocation to stocks, and the first two moments of consumption growth and the wealth to consumption ratio for both the current model and the standard model. The results are reported for particular values of γ (relative risk aversion) and $\overline{Z}$ (the relative weight attached to anticipatory utility in the current well being).

The standard model

The calibration results for the standard model correspond with those reported by Campbell and Viceira (1999, 2000), and Campbell et al. (2001). More specifically, the results show that the demand for stocks is much higher when returns are mean reverting, compared with an i.i.d. environment (with the same mean and variance of returns). For example, with *RRA* of 10, the proportion invested in stocks rises from 21% to 81% when we move from an i.i.d. environment to a mean reverting one. This is explained by a hedge demand for stocks, as the correlation between unexpected stock returns and revisions in expected future stock returns is almost perfectly negative. Thus, mean reversion makes stocks a hedge against deteriorating investment opportunity set, as they tend to have high returns at times when expected returns are low. Alternatively, one can say that under mean reversion in returns the consumption risk of stocks is lower, since when returns are low, expected future returns are high, thus protecting consumption from falling too much. Actually, the existence of mean reversion in returns makes the equity premium puzzle much more severe, as even at high levels of relative risk aversion the proportion invested in stocks is very high (more than eighty percent), and correspondingly, consumption growth is highly volatile – see Table 3.

The current model: main result

Tables 2 and 3 clearly exemplify the effect of the proposed preferences on portfolio choice, and also imply the underlying mechanism for this effect. Specifically, the current model investors give a smaller weight in the portfolio to the risky asset in comparison to the "standard investors". The underlying mechanism is that under mean-reverting stock returns, the wealth to consumption ratio is positively correlated with the return on the risky asset (this is true for both the current model and the standard model).[14] This increases the risk of investing in stocks for the current model investors, as they wish to have a smooth wealth to consumption ratio across states of nature, and moreover, for them a low wealth to consumption is a bad predicament. Correspondingly, the scaled

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[13]It can be shown (by numerical illustrations) that, other things being equal, $\overline{Z}$ monotonically increases in δ — as long as a solution exists.

[14]See Campbell and Viceira (1999).

standard deviation of the wealth to consumption ratio is lower in the current model, compared with the standard model.[15] Likewise, we find that the effect of the preferences increases in the relative risk aversion of investors. For example, with *RRA* of 15, the difference in the proportion invested in stocks between the current model and the standard model is 49%, while with *RRA* of 5 the difference between the models is only 12.9%. Thus, investors' desire to have a smooth "consumption multiplier" across states of nature, interacting with high risk aversion, leads them to invest substantially less in stocks in comparison with the "standard investors". Finally, it should be noted that in an i.i.d. environment the proportion invested in stocks is the same for both the current model and the standard model. The reason is that in an i.i.d. environment the wealth to consumption ratio is constant across states of nature, and thus the implications, for attitudes toward risk, of the proposed preferences, do not differ from those of the standard preferences (see equation [21]).[16]

To sum up, the current model generates lower investment in stocks than the standard model when stock returns are mean reverting, due to investors' desire to have a smooth "consumption multiplier" across states of nature, where this effect increases in the relative risk aversion of investors.

Saving rates

The results show that the mean wealth to consumption ratio is higher for the current model compared with the standard model, implying that saving rates are higher in the former relative to the latter. This result is explained by the additional motive for saving implied by the current model. In particular, the model suggests that individuals save also due to the anticipatory utility savings deliver; that is, they benefit from *anticipating* the increase in future consumption.

[15]We scale the standard deviation of the wealth to consumption ratio by dividing it by the average wealth to consumption ratio.

Risk aversion and consumption volatility

Interestingly, the effect of the proposed preferences on the volatility of consumption growth is mixed. Essentially, for relatively low levels of relative risk aversion (below 15) consumption growth is more volatile in the current model compared with the standard model. However, considering high values of relative risk aversion the effect of the proposed preferences on the volatility of consumption growth works in the opposite direction. For example, with *RRA* of 30, consumption volatility goes down by 75% when $\overline{Z}$ increases from zero (the standard model) to 0.079. In fact, for the current model to be consistent with the smooth consumption we see in the postwar US data, we still have to consider high values of relative risk aversion (more than 30). The current model, however, fares much better than the standard model with this respect, since in order for the latter to generate the smooth consumption as in the data we have to consider much higher values of relative risk aversion.

4.3 Model intuition

Basically, under mean reverting stock returns, the consumption risk of stocks is lower relative to an i.i.d. environment, since the correlation between unexpected stock returns and revisions in expected future stock returns is negative. Thus, during times of bad returns, expected future returns are high, which protects consumption from falling too much (given that consumers wish to smooth out consumption over time $[\gamma > 1]$), and this is expressed by a higher than the average consumption to wealth ratio.[17] However, the positive correlation between the wealth to consumption ratio and the return on the risky asset increases the risk of stocks for the current model investors, since for them low wealth to consumption ratio is a bad predicament. Intuitively, low wealth to consumption ratio implies that one's future standard of living is going to deteriorate, as the proportion of resources consumed at the present time is higher than the average.

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[16]In an i.i.d. environment, the value of the subjective time discount factor does not affect the proportion of savings invested in the risky asset.
[17]See Lettau and Ludvigson (2001).

5 Concluding Remarks

Although in recent years it became "acceptable" to assume that the instantaneous utility function is affected by *memories* of past consumption, the idea of utility from *anticipation* of future consumption has not received enough attention. This paper formalizes this idea, and explores its theoretical and empirical implications for the consumption and portfolio decisions of a long-lived investor under stock market mean reversion.

The proposed preferences do not violate any of the conventional properties of rationality. Rather, they imply a new dimension of risk in financial markets, as investors wish to smooth out fluctuations not only in their consumption, but in their wealth to consumption ratio as well. Likewise, the proposed preferences suggest a new motive for saving, as consumers save also for the sake of anticipating future consumption.

The consumption and portfolio choice problem of the long-lived investor is solved numerically for both the proposed preferences and the standard power utility, where the risky asset return follows a mean-reverting AR(1) process, calibrated to postwar US data. The investor is not allowed to borrow at the riskless interest rate or to sell short the risky asset. The main result is that the desire to smooth out fluctuations in the wealth to consumption ratio leads consumers to invest substantially less in stocks relative to the prediction of the standard model, where this effect increases in the relative risk aversion of consumers. Moreover, the effect of the proposed preferences on portfolio choice exists only when stock returns are mean reverting. In an i.i.d. environment the proportion of the portfolio invested in stocks is the same for both the current model and the standard model.

Appendix A

A.1 Risk aversion

Relative risk aversion is usually defined as the elasticity of marginal value with respect to individual wealth, with any other state variables held constant:

$$RRA_t \equiv \frac{-w_t V_{11}(w_t, S_t)}{V_1(w_t, S_t)}. \tag{A.1}$$

Using the envelope condition:

$$V_1(w_t, S_t) = c_t^{-\gamma},$$

and some algebraic manipulation, we obtain:

$$RRA_t = \gamma \frac{\partial \log c_t}{\partial \log w_t}.$$

However, the model's scale-invariance entails:

$$\frac{\partial \log c_t}{\partial \log w_t} = 1,$$

implying that:

$$RRA_t = \gamma\ . \tag{A.2}$$

A.2 Intertemporal substitution

The elasticity of intertemporal substitution in consumption (which I denote by ψ) is conventionally defined as the partial derivative of the expected growth rate in consumption with respect to the risk-free rate:

$$\psi_t \equiv \frac{\partial\left[E_t(\Delta \log \tilde{c}_{t+1})\right]}{\partial \log R_{f,t+1}}. \tag{A.3}$$

In order to solve for ψ_t, let us rewrite equation (25) as:

$$E_t\left\{\left(\frac{\tilde{c}_{t+1}}{c_t}\right)^{-\gamma}\left[\beta+\delta\left(\frac{\tilde{w}_{t+1}}{\tilde{c}_{t+1}}\right)^{-\gamma}\right]\right\}=\frac{1}{R_{f,t+1}}. \tag{A.4}$$

Figuring out that a change in the short-term risk-free rate, ceteris paribus, has no effect on the (contingent) consumption-wealth ratio in the next period, it is straightforward that ψ_t equals the reciprocal of the coefficient of relative risk aversion, as with the standard power utility:

$$\psi_t \equiv \frac{1}{\gamma}. \tag{A.5}$$

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Table 1

Return Process Parameters

Parameter	Symbol	Value
Log risk-free return	r_f	0.00082
Unconditional expected excess log return	$\bar{S}$	0.01005
Autocorrelation of expected excess log return	ρ	0.957
Standard deviation of expected excess log return	σ_s	0.01895
Standard deviation of unexpected excess log return	σ_v	0.07626
Correlation between expected and unexpected returns	ω	-0.960

Note: The model is calibrated at a quarterly frequency — All parameters are at quarterly rates.

The values of the parameters are the estimates for the return process reported by Campbell and Viceira (2000) (sample period 1947:1-1995:4).

Table 2

Consumption and Portfolio Decisions (*RRA*=5, 10, 15)

Parameters*						
Relative risk aversion	5		10		15	
	Current Model	Standard Model	Current Model	Standard Model	Current Model	Standard Model
Mean weight attached to anticipation in current well being	0.062	—	0.080	—	0.063	—
Statistics						
Mean allocation to stocks	**0.639**	**0.768**	**0.442**	**0.812**	**0.341**	**0.831**
Mean allocation to stocks in an i.i.d. environment**	0.426	0.426	0.213	0.213	0.142	0.142
Mean consumption growth rate	1.015	1.001	1.011	1.002	1.006	1.003
Standard deviation of consumption growth rate	0.037	0.023	0.030	0.021	0.021	0.021
Mean wealth to consumption ratio	6121.9	75.7	1218.2	86.1	359.9	96.6
Scaled standard deviation of wealth-consumption ratio***	0.089	0.182	0.058	0.211	0.052	0.221

Note: The model is calibrated at a quarterly frequency — All relevant variables are at quarterly rates. The statistics are all based on the parameters for the return process reported in Table 1.

*We set the subjective time discount factor to 0.94 in annual terms.

**Relates to an i.i.d. environment with the same mean and variance of excess log returns.

***Calculated as the ratio of the standard deviation to the mean wealth to consumption ratio.

Table 3

Consumption and Portfolio Decisions (*RRA*=20, 25, 30)

Parameters*						
Relative risk aversion	20		25		30	
	Current Model	Standard Model	Current Model	Standard Model	Current Model	Standard Model
Mean weight attached to anticipation in current well being	0.071	—	0.075	—	0.079	—
Statistics						
Mean allocation to stocks	**0.244**	**0.837**	**0.192**	**0.835**	**0.157**	**0.828**
Mean allocation to stocks in an i.i.d. environment**	0.106	0.106	0.085	0.085	0.071	0.071
Mean consumption growth rate	1.005	1.004	1.004	1.005	1.004	1.006
Standard deviation of consumption growth rate	0.016	0.021	0.013	0.021	0.010	0.021
Mean wealth to consumption ratio	559.3	107.3	705.8	118.0	850.2	128.4
Scaled standard deviation of wealth-consumption ratio***	0.036	0.225	0.028	0.224	0.022	0.221

Note: The model is calibrated at a quarterly frequency — All relevant variables are at quarterly rates. The statistics are all based on the parameters for the return process reported in Table 1.
*We set the subjective time discount factor to 0.94 in annual terms.
**Relates to an i.i.d. environment with the same mean and variance of excess log returns.
***Calculated as the ratio of the standard deviation to the mean wealth to consumption ratio.