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ABSTRACT

Creating Competition Out of Thin Air: Market Thickening and Right-to-Choose Auctions*

We study a procedure for selling multiple heterogenous goods, which is commonly used in practice but rarely studied in the literature. The novel feature of this procedure is that instead of selling the goods themselves, the seller offers buyers the right to choose among the available goods. Thus, buyers who are after completely different goods are forced to compete for the same good, the 'right to choose'. Competition can be further enhanced by restricting the number of rights that are sold. This is shown both theoretically and experimentally. Our main experimental finding is that by auctioning 'rights-to-choose' rather than the goods themselves, the seller induces an aggressive bidding behaviour that generates more revenue than the theoretical optimal mechanism.

JEL Classification: C91 and D44
Keywords: behavioural mechanism-design, experimental auctions and right-to-choose auctions

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1 Introduction

This paper presents a theoretical and an experimental study of a procedure for selling multiple heterogenous goods. The novel feature of this procedure is that it induces a competition between buyers who are interested in completely different goods. While this procedure is commonly used in practice, it is rarely studied in the literature.

For expositional purposes we illustrate the subject of our study with the following simple example. Consider a record collector who has been collecting records over his lifetime. Suppose the collector decides to sell three albums of three distinct music styles. He chooses to sell one opera album (*Stravinsky's "Oedipus-Rex"*), one punk-rock album (*The Sex Pistols*) and one pop album from the 80's (*Duran Duran*). On the day of the sale six people show up to bid. However, because the three albums are so different the people who show up have very specific tastes: two value only the opera album and have no use for the others, another pair of buyers only value the punk album and the remaining two are only interested in the 80's album.

The record collector has to decide how to auction the records. One option is to hold three separate auctions and run them either sequentially (offering one record at a time as they would do in Sotheby's) or simultaneously (allowing bidders to bid simultaneously on any record they want until the auction ends). We call such auctions Good-by-Good auctions, or GBG for short.

An alternative is to combine the 'thin' markets for each of the goods into one 'thick' market by transforming the three distinct goods into three units of a new homogeneous good called a "right-to-choose". This can be achieved by adopting a procedure often used in real-estate auctions, which we call a *right-to-choose auction*, or RTC for short . Such an auction would consist of three phases. In the first phase, all six bidders submit sealed bids. The highest bidder wins the right to choose one of the three goods, and he pays the second-highest bid. The other five bidders are then told which good was taken, and the bidder, who wanted the same good as the winner, exits the auction. The remaining four bidders enter the second phase of the auction, where again bidders submit sealed bids, the highest bidder wins the right to choose one of the two remaining goods, and then both he and another bidder who wanted the same good as the winner, exit the auction. At the third and final phase, two bidders compete in what is essentially a standard second-price auction.

The novel feature of this auction is that it forces buyers of completely different goods (who would otherwise not compete against each other) to compete for the same good, namely the 'right-to-choose'. This can lead the highest bidder for one album to pay the bid of the highest bidder for another album.

While this feature seems appealing, a few moments of thought would lead us to realize that in the context of our example, the RTC auction is actually revenue equivalent to the sequential or simultaneous GBG auctions described above. In the symmetric equilibrium, both types of auctions lead to the same efficient allocation of albums. This is true because, when all is said and done, in this RTC auction, all a buyer needs do to win an object is to outbid the only other bidder who wants his record, the other bidders are actually irrelevant and hence not in direct competition with him. In this sense the RTC format does not thicken the market at all.

Interestingly, one can break the revenue equivalence by introducing inefficiency into the RTC auction via quantity restriction. In our example this means that the collector only sells two rights-to-choose, i.e., he conducts a RTC auction with only two phases. Since by running a RTC auction the record collector effectively becomes a monopolist with a fixed supply of three rights-to-choose, intuition suggests that he may be able to raise more revenue by not selling all of his inventory. Indeed, as we show in Section 3, the collector may raise more revenue by restricting output and selling only two rights to choose than by selling all three goods using any standard sequential or simultaneous good-by-good auction.[1] The quantity restriction works because it does force bidders, who are after completely different goods, to compete for the rights to choose. In this sense, the (restricted) RTC auction creates "competition out of thin air".

Further introspection would suggest, however, that even this quantity restricted RTC auction will not raise the most revenue since the seller can still do better, in theory, by using the "optimal auction": a GBG auction with an optimally set reserve price. The fact that RTC auctions are used in the real world suggests that despite the fact that they are not optimal *in theory* they still have a number of merits.

While the optimal auction in the above example was relatively simple, in slightly more enriched environments the optimal auction can be quite complicated, in contrast to the simple structure of the RTC format. Moreover, in some environments with heterogeneous goods, the optimal auction is not yet known (see Jehiel, Meyer-ter-Vehn, and Moldovanu

[1] It is immediate that quantity reduction in good-by-good auctions only reduce revenues further so such a trick can only work in conjunction with a right-to-choose auction or some type of auction that connects these markest or thickens them.

(2003)). Unlike optimal auctions, RTC auctions with quantity restrictions guarantee that a certain number of goods (two in the example used above) will be sold. Hence, the seller does not bear the risk of making zero revenue as he would with a reserve price. To carry out the optimal auction - and in particular, to set the optimal reserve price - the seller needs to know the distribution of buyers' valuations. In contrast, RTC auctions are "detail-free" in the sense that their execution is independent of any information that the seller or the buyers may have. But most importantly, our experimental evidence suggests that RTC auctions may actually outperform the theoretical optimal auction.

We focus on a simple model, which captures the basic ingredients of our simple example. A monopolist owns K unrelated goods. For each type of good, there are n risk-neutral buyers who value only that good. Buyers' valuations are purely private and are drawn independently from the same uniform distribution. The theoretical predictions for this model are the same as in our example: (1) a *RTC* auction in which all goods are sold generates the same expected revenue as conducting a separate auction for each good with no reserve prices, (2) more revenue can be generated by restricting quantity in the *RTC* auction (i.e., selling less than K goods), and (3) expected revenue is highest when the goods are sold via a GBG auction with optimal reserve prices.

In contrast to these theoretical predictions, our experimental findings show that a RTC auction in which *all goods are sold* raises more revenue on average than the GBG auction with optimal reserve prices. Compared to the inefficient *theoretically-optimal* auction, more revenue is raised by our (approximately) efficient RTC auction. Thus, our results suggest that sellers who are interested in pursuing a combined goal of revenue and efficiency should seriously consider a RTC auction where all goods are sold. Our findings also demonstrate that quantity restriction leads to more aggressive bidding behavior: when quantity is restricted by one unit, more revenue is raised in each phase of the restricted RTC auction than in the corresponding phase of the unrestricted auction. However, given the high revenue of the RTC auctions when all goods are sold, there is only little room left for quantity restriction to further intensify competition. Consequently, revenues are roughly equal in both cases.

The variance in the revenue is strictly smallest when quantity is reduced in the RTC auction. Thus, a risk-averse seller may be best off with a RTC auction where quantity is restricted. In particular, when a seller faces unexpected liquidity problems and has an urgent need to raise cash, the RTC auction with quantity reduction has an important practical advantage compared to the theoretical optimal auction. In the optimal auction there is always a danger that none of the goods is sold and zero revenue is raised. In contrast, the RTC auction gives the seller explicit control of the number of goods that he wants to sell so that a positive revenue is guaranteed.

The fact that our RTC auctions outperform the optimal auction in our experimental environment means that the behavior observed differs from that predicted by the theory. Since the observed behavior in the optimal auction closely followed the theoretical prescription (i.e., most subjects bid close to their value), our results are driven by the fact that the RTC design induced a bidding behavior that departs from the theory. We explain this departure by positing that individuals use a simple rule-of-thumb to resolve the trade-off between maximizing the probability of winning and minimizing the expected price. This rule-of-thumb takes the form of a cutoff strategy whereby an individual bids close to his value when that value is at or below some intermediate threshold; however, when his value exceeds that threshold, an individual shades his bid but at a decreasing rate. We discuss the intuition for using this rule of thumb and report the results of a series of econometric estimations that provide evidence in support of our explanation.

To summarize, this paper contributes to the literature on mechanism-design by proposing a detail-free procedure for selling multiple goods. We focus on a simple environment, which allows us to derive the theoretical optimal auction and compare its performance in the laboratory with that of the RTC auction. We find that in our simple set-up, the RTC auction outperforms the theoretical optimal auction. This suggests to us that in more complicated environments in which the optimal auction may be difficult to derive, sellers may benefit from using the RTC auction.

We proceed as follows. We begin with a discussion of related literature in Section 2. In Section 3 we present the theory of RTC auctions and derive a set of theoretical results. Section 4 describes our experimental design while Section 5 summarizes our results. We present our conclusions in Section 6.

2 Related literature

As mentioned in the Introduction, auction formats where instead of winning a specific object, bidders win the right-to-choose any of the yet unsold objects are often used to sell real-estate such as condominiums and land parcels. Most of these auctions either follow our sequential RTC design (see Ashenfelter and Genesove (1992)), or use the simultaneous "pooled auction" format. In a pooled auction each bidder submits a single sealed-bid, the highest bidder gets to choose his most preferred item, then the second highest bidder chooses among the remaining items, and so on until all items are sold. Menezes and Montiero (1998) derive the optimal bidding strategies for risk-neutral bidders in a pooled auction and show that in the homogeneous private-values case, this format is revenue-equivalent to a sequential auction of multiple items. In an experimental study, Salmon and Iachini (2003) compare the pooled auction to the simultaneous ascending auction and find that pooled auctions raise higher revenues than the simultaneous ascending auction. The pooled auction has the disadvantage that bidders frequently experience losses when they are forced to buy their less preferred goods at high prices. Salmon and Iachini report that the overbidding in their pooled auctions can neither be explained by loss aversion nor by risk aversion.

Burguet (2002) studies a model with two substitute goods and shows that the RTC auction in which all goods are sold is efficient but not optimal. However, the optimal mechanism turns out to be quite complex requiring information, which is not readily available to the seller. Burguet proposes a detail-free way of introducing inefficiency into the RTC auction, which improves the performance of the original auction: the seller should not reveal to the bidders which item was chosen by the winner of the previous phase. This is similar in spirit to running what is known as a "pooled auction" in which each buyer simultaneously submits a single sealed-bid, the highest bidder gets to choose his most preferred item, then the second highest bidder chooses among the remaining items, and so on until all items are sold. Clearly, such an auction is inefficient since a winning bidder may wind up paying a price higher than his value for the good he wins. Consequently, winning bidders may wish to opt out when their willingness to pay for each of the available goods is smaller than the price they need to pay.[2] We propose quantity restriction as an alternative means for introducing inefficiency into the RTC auction in a detail-free manner.

Goeree, Plott and Wooders (2003) introduce risk-averse bidders into Burguet's (2002) model. They show that in this case, RTC auctions in which *all goods are sold* raise more revenue than standard simultaneous or sequential ascending auctions. These authors also test their model in the laboratory, and provide evidence in support of their theoretical result. We interpret their findings as complementary to the present study. These findings suggest that the performance of the RTC auction may be robust to the buyers' degree of risk aversion, and to the degree of substitutability between the different goods. This

[2] In fact, under California law, winning bidders in pooled auctions have the right to opt out.

contrasts with the theoretical optimal auction, which is sensitive to both these properties. Goeree *et al.* (2003) do not consider the possibility of quantity restriction, and they do not compare their results with the optimal auction in their set-up.

Our paper follows a small literature that has focused on the design of robust, detail-free mechanisms. This literature was inspired by what became to be known as the "Wilson Critique", which called for the design of mechanisms that are independent of the details of the environment (see Wilson (1987)). Our approach to this critique has been to examine a detail-free procedure for stirring up competition in thin markets. An alternative approach is offered by Baliga and Vohra (2003) and Segal (2003) who study the design of detail-free auctions for thick markets with many buyers where the seller is free to discriminate between buyers by using bidder specific reserve prices. Under this approach, the seller uses the bids of buyers as sample points to estimate the true distribution of valuations: each buyer faces a distinct reserve price, which is based on the bids of the buyers. When buyers' valuations are conditionally independent, the revenue raised by this auction converges to the revenue generated by the optimal auction, given the true distribution.[3]

3 Theory

Consider a seller with K heterogeneous goods who faces the following demand structure. For each of the K goods there are exactly n risk neutral buyers. A randomly chosen buyer is equally likely to demand any of the K goods. Each buyer has a private value for only one of the K goods (the 'preferred' good), and has zero value for all other goods. All buyers independently draw the value for their preferred good from a uniform distribution on $[0, 1]$.[4]

There are several selling procedures that accomodate the above demand structure. The benchmark procedure consists of holding K separate second-price auctions, one for each good. We call this procedure a *good-by-good* (GBG) auction and denote it by $GBG(K, n)$,

[3] Two alternative approaches to the question of robustness of mechanisms are taken by Bergmann and Morris (2004) and Bose, Ozdenoren and Pape (2004). The first set of authors study the robustness of solution concepts to higher order beliefs. The second set of authors study optimal mechanism-design in the presence of ambiguity aversion. In particular, they show that the classical mechanisms of first and second price auctions with appropriate reserve prices are not optimal in this environment.

[4] All of our results can easily be generalized for a uniform distribution on $[\underline{v}, \overline{v}]$. We assume that for each good the seller has a value equal $\underline{v}$. The possibility to reduce quantity in a right-to-choose auction becomes even more attractive in situations where the seller is forced to sell the goods as a result of financial trouble while she has higher values for the goods than the lower end of the support.

where K stands for the number of goods and n for the number of buyers per good. In the remainder of this section we introduce an alternative selling procedure and present a set of results concerning the buyers' strategies and the seller's expected revenue at the symmetric subgame perfect Bayesian Nash equilibrium (SPBE). All proofs are relegated to the appendix.

A *right-to-choose* (RTC) auction in our framework proceeds as follows. There are $K - q \leq K$ phases, where q (which stands for "quantity-restriction") is a nonnegative integer, which is smaller than K. In the first phase, all nK bidders bid for the right to choose among the K available goods. The highest bidder in phase 1 wins the right to choose one of the K goods and pays the bid of the second highest bidder. At the end of this phase bidders are told which good was selected by the winner (bidders are not informed of the price paid by the winner). Bidders are then given the option to either exit the auction or stay and move to the next phase. Clearly, all of the $n - 1$ buyers who value the same good as the winner will at this point drop out of the auction. The remaining bidders participate in phase two, which is essentially the same as phase 1: bidders simultaneously submit bids, the highest bidder wins the right to choose one of the remaining goods, the winner pays the second highest bid and chooses a good, and all other bidders are informed of the good that was chosen. This continues until $K - q$ rights offered for sale are sold. A sequential right-to choose auction with K goods, $K - q$ phases and n bidders per good is denoted $RTC(K, K - q, n)$.[5] We choose the second-price rule to help make the analysis more tractable since this pricing rule simplifies the derivation of the equilibrium strategies.

A strategy in a $RTC(K, K-q, n)$ is a collection of $K-q$ functions, one for each phase, where each function maps a bidder's value into a bid for the corresponding phase of the auction. The second-price auction in each phase can be thought of as being an ascending-clock English Auction in which the price rises until the pen-ultimate bidder drops out and the remaining bidder wins at the last drop-out price (to maintain strategic equivalence with the second-price format, we assume that the auctioneer does not reveal drop-out prices of bidders who leave the auction). Hence, each bidder in the auction must determine a drop-out price for each phase in which he or she is still active. For a bidder with value v we denote this phase-k drop-out price by $b_k(v)$, where $k = 1$ is the initial phase and $k = K - q$

[5] In principle, an RTC auction could also be run simultaneously where bidders submit $K - q$ bids (one bid for each phase of the auction) all at once in addition to declaring which good they are interested in buying. We chose a sequential design because it highlights the trade-off between having a higher chance of obtaining a good now and paying a lower price in the future.

is the final phase.

There are several reasons for choosing such a simple model. First, the model can be easily tested in the lab. Second, the simplicity of the model allows us to derive an explicit closed form solution of the equilibrium bid functions.[6] Third, the model allows us to derive theoretical predictions for the sale of any number of goods. We emphasize this point because all previous works on sequential and bidders' choice auctions have focused on the case of two goods. Finally, our demand assumptions are not unrealistic especially since right-to-choose auctions can (and should) be used to sell very different types of objects, each with a thin market.

We focus on the SPBE in which all bidders use the same monotonic bid function in each phase. This means that each bidder knows that the winner in any given phase has a value at least as high as the winner in a subsequent phase. Therefore, each bidder has an incentive to wait and try to win in late phases of the auction. This incentive is offset by the probability that the bidder's good will be bought by another bidder.

In equilibrium each bidder balances the trade-off between lowering the price (by trying to win in late phases) and raising the probability of winning his good (by trying to win in early phases). To understand how this is done, consider a simple RTC auction with two goods, two phases and two bidders per good. In the second and final phase there are two bidders competing in a standard second-price auction. Each remaining bidder would therefore bid his value. Hence, $b_2(v) = v$.

Consider next the initial phase of the auction. The initial bid represents the highest amount a bidder is willing to pay for his good in phase one. In order to determine this amount, each bidder i considers the event in which he wins in the first phase and pays his bid (note that in this event the second highest bidder has the same value as bidder i). Thus, conditional on this event, bidder i with value v expects a net payoff of $v - b_1(v)$.

Bidder i can also delay his win to the second phase, in which case bidder j wins the first phase. Bidder i would enter the second phase only if the first phase winner (bidder j) does not take his good. The probability of this event is $\frac{2}{3}$ (this follows from the assumption that a randomly chosen bidder is equally likely to demand each of the goods). If player i makes it to the second phase, then in that phase he faces one other bidder with a value of v or less. Hence, bidder i expects to win the second phase and pay $\frac{1}{2}v$ (recall that bidders bid their values in the last phase). It follows, that the expected value for player i from

[6] For example, both Menezes and Monteiro (1998) and Iachini and Salmon (2003) do not solve explicitely for the equilibrium bid functions.

not winning in the first phase of the auction is $\frac{2}{3}\left(v - \frac{1}{2}v\right)$. This means that $b_1(v)$, the highest amount bidder i is willing to pay in order to win in the first phase, must make him indifferent between winning in that phase and winning in the next phase. I.e.,

$$v - b_1(v) = \frac{2}{3}\left(v - \frac{1}{2}v\right)$$

and so

$$b_1(v) = \frac{2}{3}v$$

The above reasoning allows us to derive the equilibrium bids when there are more than two goods, more than two phases and more than two bidders per good. These equilibrium bids are presented in the next proposition. For this proposition, we require the following notation. Denote by N the total number of bidders (i.e., $N = nK$) and let N_k denote the number of active bidders in phase k of the auction (i.e., $N_k = n(K - k + 1)$).

Proposition 1 *The $RTC(K, K, n)$ has a SPBE with the property that in each phase $1 \leq k \leq K$ every bidder uses the linear bid function $b_k(v) = \frac{(N_k-1)-(K-k)}{N_k-1}v$.*

The equilibrium described in Proposition 1 has the property that individuals bid a higher proportion of their value as the auction progresses, until eventually they bid their value in the final phase. Although the slope of the bid function increases with k, the expected price decreases with k. This follows from the fact that in equilibrium, bidders are indifferent as to when (i.e., in what phase) they win their good. Hence if bid functions are symmetric, high value bidders will win in early rounds and pay higher prices than low value bidders who win later on at lower prices.

The intuition underlying the SPBE of the $RTC(K, K, n)$ can also be applied to a RTC with quantity restriction. In particular, we apply this intuition to a RTC auction in which one of the goods is not sold. Here, too, we derive the equilibrium bid of phase k by making each bidder indifferent between winning and losing the current phase conditional on having the highest value, which is also the value of the second highest bidder. The only difference between the two auctions is that in $RTC(K, K - 1, n)$ bidders bid their value in round $K - 1$ because this is the final phase of the auction. Hence, using the same reasoning as in Proposition 1, we obtain the following result.

Proposition 2 *The $RTC(K, K - 1, n)$ has a SPBE with the property that in each phase*

$1 \leq k \leq K$ *every bidder uses the linear bid function*

$$b_k(v) = \frac{N_k - (K - k)}{N_k - 1} v \tag{1}$$

By comparing the equilibrium bids in Proposition 1 and 2, one sees that quantity restriction enhances the competition between bidders. This is evident from the fact that the bidding coefficient in phase k of $RTC(K, K-1, n)$ is higher by $\frac{1}{N_k - 1}$ than the corresponding bid in the $RTC(K, K, n)$. Although quantity restriction raises bids in every phase, there is one less phase and, hence, one less good that the seller sells. Therefore, it is not immediately clear whether quantity restriction in fact raises the seller's expected revenue. Questions concerning the expected revenue of the RTC auction are explored next.

Proposition 3 *The expected revenue in the SPBE of a $RTC(K, K, n)$ is equal to the expected revenue obtained in a $GBG(K, n)$ where bidders use weakly dominating strategies.*

The above result means that by simply combining K markets together and letting all bidders compete against each other for "rights-to-choose", the seller cannot extract more surplus (in equilibrium) from the buyers[7]. However, by introducing some inefficiency into the auction, one could break the revenue equivalence of RTC and GBG. This can be achieved by taking advantage of the fact that the seller in a RTC auction is simply a monopolist with a fixed supply of K rights-to-choose. When facing an inelastic known demand, a monopolist with a fixed supply would maximize revenue by selling less units than what he actually has. The question is, could a monopolist in our setting also increase his revenue by restricting quantity?

Quantity restriction clearly raises more revenue than a GBG auction when a seller offers two units that attract only two buyers, each of whom wants a different good. By auctioning each good separately via a $GBG(2, 1)$, the seller would essentially be giving the goods for free. In contrast, by auctioning only a single right-to-choose (i.e., executing a $RTC(2, 1, 1)$), the seller expects to earn the second order statistic. Note that in this case it is a weakly dominant strategy for buyers to bid their value. Hence, if neither the seller nor the buyers know the distribution of valuations, auctioning only a single right-to-choose guarantees the seller the second order statistic (whose value is unknown to the seller).

[7] This does not mean, as we will see later when we discuss our experimental results, that the mechanisms are equivalent behaviorally.

This example raises the question of whether quantity restriction leads to higher revenues in our set-up when $K > 2$ and $n > 1$. Our next result establishes that for any n there is K large enough such that a *RTC* auction with quantity restriction - more specifically, selling all but one good - raises more revenue than selling all goods. In particular, for the setting we use in our experiments, where $n = 2$, quantity restriction raises revenue for any $K > 2$.

Proposition 4 *For every n there exists a finite K^n such that for all $K \geq K^n$, restricting quantity by one unit raises the expected revenue in the RTC auction. In particular, for $n = 2$ we can set $K^2 = 3$.*

Note that from Propositions 3 and 4 it immediately follows that quantity restriction raises more revenue (in expectation) than auctioning *all* K goods in K separate second-price auctions. By restricting the number of rights sold, one creates a scarcity that does not otherwise exist. What's more, one is creating competition among buyers who have no real interest in competing with each other, and do not do so in GBG auctions since they value completely disjoint sets of goods.[8]

Clearly, quantity-restriction will not raise more revenue in a GBG auction. In this auction, each market is separate from the next and hence reducing the number of such markets simply reduces the expected revenue proportionately. Quantity restriction introduces an inefficiency in a similar way in which reserve prices do. In essence, quantity restriction is analogous to an endogenous reserve price determined by one of the bidders. This raises the question of whether a GBG auction with appropriately chosen reserve prices would raise more revenue than a RTC auction with quantity restriction. In particular, it is instructive to derive the revenue maximizing auction in our set-up.

Proposition 5 *Expected revenue is maximized by a $GBG(n, K)$ auction with a reserve price of $\frac{1}{2}$.*

This result follows from our assumptions on the demand for each of the K goods. Our assumptions imply that the payoff type of each bidder is uni-dimensional: it can be

[8] The principle of quantity reduction may also be profitable in settings where buyers have multi-unit demand. Noussair (1995) shows theoretically that bidders have incentives to strategically reduce their demand when there is multi-unit demand. The idea is that by asking for less in the auction, bidders get more as they manage to keep auction prices low. List and Lucking-Reiley (2000) and Kagel and Levin (2001) show in a series of experiments that bidders actually use the possibility to reduce demand in uniform-price auctions. In such situations, sellers may find it profitable to reduce the quantity offered for sale in the first place, thereby counterbalancing bidders' incentives for strategic demand reduction.

summarized by the value he assigns to his preferred good. Because bidders draw this value independently from the same distribution, there is no loss of generality from finding the optimal mechanism for a single market and conducting K separate replicas of this mechanism. A straightforward application of Riley and Samuelson (1981) yields that an optimal auction in the market for good k is a second-price auction with a reserve price of $\frac{1}{2}$.

The question we ask in our experiment is whether Proposition 5 also holds behaviorally.

4 Experimental Design and Procedures

4.1 Design

The experiment we ran consisted of six treatments. The first four treatments correspond to the four auction formats investigated in the previous section: a standard RTC auction in which all goods are sold ($RTC(K, K, n)$), a RTC auction in which one good was not sold ($RTC(K, K-1, n)$), a GBG auction where each good is sold using a second-price rule with no reserve prices ($GBG(K, n)$) and a GBG auction with an optimal reserve price of 50 ($OGBG(K, n)$). The remaining two treatments tested a variant of the RTC auction called a RTC auction with no information (or NIRTC for short), which we discuss in Section 5.4. Except for that particular section, we shall focus only on the first four treatments. All six treatments adhered to the design described in this section.

Since the tools to intensify competition are most relevant in thin markets, we set the number of bidders for each good equal to two ($n = 2$) in all treatments. We chose $K = 4$, so that in total eight subjects were competing for four goods. In some auctions we restricted the number of rights to be sold to $K - 1 = 3$, so our auctions consisted of $RTC(4, 4, 2)$, $RTC(4, 3, 2), GBG(4, 2)$ and $OGBG(4, 2)$. The value for each subject was independently drawn from a uniform distribution over the support $[0, 100]$. The experiments were performed at CREED, the experimental economic laboratory of the University of Amsterdam, as well as the experimental laboratory at the Center or Experimental Social Science at New York University.

For each experiment subjects were recruited from the general undergraduate population of these respective schools. The experiment lasted about one hour and twenty minutes, except for the good-by-good auction which typically lasted about one hour. Subjects were paid a show up fee in each location and earned the remainder of their money according to how they did during the experiment. Motivation and understanding of the instructions were good and average earnings were $18.2 in Amsterdam and $15.8 in New York. No significant behavioral differences were found across locations so we pool all observations from both subjects populations.[9]

Each group of subjects performed one and only one type of auction and repeated the auction 16 times after participating in a practice round. There were eight groups performing each treatment (four in Amsterdam and four in New York) so the total number of subjects recruited was 384 (eight groups of eight subjects in six treatments). Four different sets of values were generated for the first four groups of subjects in each treatment. The same exact sets of values were also used for the second four groups. Hence, each set of randomly generated values was used twice in the experiment in each treatment (once in New York and once in Amsterdam). We did this to ensure that any revenue differences were attributable to differences in behavior rather than differences in the vectors of random variables generated. This also allowed us to make some controlled comparisons of behavior.

Our design is summarized in Table 1.

Table 1

4.2 Procedures

In all six treatments subjects were seated in a computer lab in groups of 16 and separated into two sub-groups of eight subjects each. Subjects read the computerized instructions at their own pace. The instructions of the $RTC(4,3,2)$ auction are presented in Appendix B. Each group performed the same experiment, but once a subject was assigned to a group of eight, he or she remained in that group for the entire experiment. In all treatments each of the 16 periods began by each subject being shown the good he or she valued (either good A, B, C, or D) and the value of that good for the period. After this was presented on the screen the program asked them to bid. In all of the RTC auctions subjects were asked to bid in phases.

The RTC auctions proceeded as follows (the key differences between this procedure and that of the NIRTC is discussed in Appendix B). In phase 1 subjects submitted their bid and, using second price rules, the good was allocated to the highest bidder at the

[9] There was a small set of subjects who went bankrupt in New York. All observations occurring after these bankruptcies happened were dropped. This was the only difference in the behavior of subject pools noticed.

second highest price. If the winner selected the good of another subject, that subject was informed that his or her good was selected and was not allowed to bid in further phases of this period. The vector of submitted bids was not revealed to the bidders. The winner was the only one who knew the price at which he or she had bought the good. The bidders whose good was not won in the initial phase proceeded to phase 2. This phase, as well as those that followed it, proceeded in the same manner as phase 1.

When the next period began, subjects were allocated to different goods at random. Hence, each subject was randomly paired with one other subject who valued the same good as he. A new independent value was presented to them and bidding proceeded in phases as before. In the $RTC(4,4,2)$ there were four phases per period while in the $RTC(4,3,2)$ there were three phases per period. Finally, in the $GBG(4,2)$ and the $OGBG(4,2)$ auctions each period consisted of only one phase in which all subjects bid for the good they valued and faced one other subject who also valued that good.

Total earnings in the experiment consisted of the per-period earnings of subjects summed over all 16 periods. Subjects played for points which were converted into euros and dollars at the rate of 15 points per one dollar. Finally, to protect subjects from bankruptcy we gave each subject a 150 points at the beginning of the experiment and all losses during any period were subtracted from this amount.[10]

5 Experimental results

The theoretical results of Section 3 provide a set of testable hypotheses concerning the bidding behavior of subjects and expected revenues. In the SPBE of the auction, subjects are expected to bid in each phase according to a linear bid function in which bids are proportional to values. In particular, in a RTC auction, with or without quantity restriction, the equilibrium bid coefficients increase from phase to phase. The equilibrium bid coefficients obtained for the parameters of our experimental design, are presented in Table 2.

Table 2

[10] Of the 384 subjects, nine went bankrupt and this occurred in six groups (i.e. in some groups several people went bankrupt). All bankruptcies except one occurred in the NIRTC (3,4,2) and NIRTC(4,4,2) experiments. The one other bankruptcy occured in the RTC(4,4,2). Six of the nine bankruptcies occurred after period 11 (four in period 14). To purge the impact of bankruptcy on the data we drop all observations for subjects in any group after a subject had gone bankrupt.

Table 2 also compares two notions of expected revenue. The first notion, displayed in the fifth column, computes the expectation with respect to *all possible realizations of values*. This is the expected revenue predicted by the theory. The second notion, displayed in the last column, computes the expectation with respect to the *values drawn in our experiment*. This is the revenue that is expected to be generated when subjects with the values generated in our experiment bid according to the equilibrium. While the first notion assumes that the law of large numbers is at work in the generation of values, the second notion accounts for the fact that set of values generated in the experiment is finite. Note that there is little qualitative or quantitative difference between the numbers in these two columns.

5.1 Revenues

Before we analyze the bidding behavior of our subjects, let us compare the performance of the auctions in terms of revenue. Table 3 presents the mean revenues generated by our subjects in each phase and treatment along with their standard deviations. It also presents the revenues expected to be generated at the SPBE of our auctions along with the expected theoretical standard deviations.

Table 3

A major finding in Table 3 is that the $RTC(4,4,2)$ and $RTC(4,3,2)$ raised substantially more revenue than predicted by the theory. Not only did the $RTC(4,4,2)$ outperform the $GBG(4,2)$, *it raised significantly more revenue than the theoretical optimal auction*, the $OGBG(4,2)$. The mean revenue of the $RTC(4,4,2)$ is 203.7, whereas the mean revenue of the $OGBG(4,2)$ is only 178.8. Mann-Whitney tests ran on the sample of revenues generated by these auctions indicates that this difference is significant at less than the 1% level.[11] The mean revenue of the GBG auction with no reserve price is only 145.1, which is significantly less than that raised by the optimal auction at the 1% level.

The data in Table 3 confirms our theoretical result that quantity restriction leads to more aggressive bidding in each of the first three phases of the RTC auction. This is evident by noting that in each of these phases the revenue raised by the $RTC(4,3,2)$ is greater than that raised in the $RTC(4,4,2)$. Because bids in the $RTC(4,4,2)$ were already higher than those predicted, the bid increment induced by quantity restriction was

[11] All Mann-Whitney test results use independent average data per group as observations and groups with bankruptcies are not used.

not sufficient to completely offset the fact that only three goods were sold. Indeed, the difference in revenues between the $RTC(4, 4, 2)$ and the $RTC(4, 3, 2)$ is not statistically significant. However, since quantity restriction clearly leads to more aggressive bidding, it may be very well be the case that for different values of K and n quantity restriction will lead to higher revenues. In particular, our example from Section 3 with $K = 2$ and $n = 1$ illustrates a situation in which it is clearly better to restrict quantity.

Table 3 also supports our assertion that a right-to-choose auction with quantity restriction is capable of raising significantly more revenue than a good-by-good auction with no reserve prices. In particular, the mean revenue of the $RTC(4, 3, 2)$ is 188.7, whereas the mean revenue of the corresponding good-by-good auction is only 145.1. According to a Mann-Whitney test, the difference between these two amounts is significant at less than the 1% level. Furthermore, the mean revenue of the $RTC(4, 3, 2)$ is strictly higher than that of the $OGBG(4, 2)$, though this difference is not statistically significant. This suggests that a seller, who does not have enough information to set optimal reserve prices, may benefit from conducting a RTC auction with quantity restriction.

The expected revenue generated by an auction format is not the only criterion we might use to judge its desirability. All other things being equal, most sellers would probably prefer auction formats in which the variability of revenue is low. Theoretically, both the restricted and the unrestricted right-to-choose auctions are expected to be less volatile than the standard good-by-good auction. For example, as we see in Table 3, at the equilibrium the variance of the revenues generated by the $RTC(4, 4, 2)$ auction is 33.7 while it is 52.9 for the $GBG(4, 2)$ auction. Although the right-to-choose auctions actually produced a higher variation of revenues than expected in theory, each generated a lower variance than both GBG auctions. A series of pair-wise F-tests reveal that the $RTC(4, 3, 2)$ is least volatile at the 1% level. However, the difference between the variance of the $RTC(4, 4, 2)$ and those of the two GBG auctions (the $GBG(4, 2)$ and the $OGBG(4, 2)$) is not statistically significant at the 5% level. These findings provide further support that a seller faced with unknown demand may benefit from conducting RTC auctions with quantity restriction.

5.2 Bidding Behavior

Different auction mechanisms require different levels of cognitive ability from their users by asking them to perform different strategic operations. For example, a second price auction run with two people only requires people to be able to understand what a dominant strategy is and use it. First price auctions used in the same setting require a more enriched set of cognitive tools and ask bidders to make more elaborate calculations. Since the theory assumes that it is costless for social agents to perform these cognitive tasks, it predicts a revenue equivalence between these two auctions. But from a behavioral point of view, one might clearly expect otherwise.

Likewise, the strategic decision task required of subjects in our RTC auctions is more elaborate than that asked of subjects in our GBG and OGBG auctions. More precisely, RTC auctions require subjects to deal not only with a static trade-off between the probability of winning and profits, but to also understand and cope with a more subtle inter-temporal trade-off. The latter trade-off arises because, in theory, the price of the good decreases over time while the probability of it still being available also decreases. This induces a bidder to think about when is the right time to buy the good, weighing the price at which it could be bought with the likelihood that it might be available at a later stage.

In the symmetric equilibrium of our model bidders use a linear bid function in each phase. I.e., within each phase bidders bid a constant fraction of their values. The inter-temporal trade-off mentioned above is resolved in the theory by having the linear bid coefficients increase over time.

It is our contention that in RTC auctions bidders resolve the inter-temporal trade-off by using a rule of thumb that departs from the perfect Bayes Nash equilibrium strategy. While bidding behavior does change over phases, it also changes within each phase where subjects tend to bid in a discontinuous manner using a bid strategy with a structural break or cutoff. Below the cutoff, bidding behavior tends to be linear but with a bid coefficient greater than that prescribed by the equilibrium (actually they almost bid their value) while, above the cutoff, the bid function is approximately quadratic and subjects tend to shade their bid more (but at a decreasing rate).

To explain this behavior consider the first phase of the $RTC(4,4,2)$ auction. Here, subjects realize that if they draw only a moderate value it is unlikely that they will win in any phase. So their only hope of walking away from the auction with something positive in their pockets is to bid close to their value and hope that the person, who also values their good, is bidding low now believing that he could win the good at a lower price later in the auction. Hence, when a subject draws a moderate value he realizes that in order to win he will have to outbid all other bidders - including those that want other goods - and do so early in the auction before they raise their bids. This is exactly the market thickening trick described in the introduction and it works most directly on moderate-value bidders.

For example, in the first phase of the $RTC(4,4,2)$ auction a moderate-value bidder may conclude that in order to win and receive any surplus he must beat all seven of his opponents. This is strategically equivalent to being in an auction where there are four goods with eight bidders desiring each good, i.e., a RTC$(4,4,8)$. Using the formula presented in Proposition 1 we see that this would lead to a phase-1 bid function of $b_1(v) = \frac{28}{31}v = 0.9(v)$ (instead of $0.57(v)$ as predicted by the theory), which is remarkably close to the 0.86 bid coefficient found in our regressions below. In fact we can not reject the hypothesis that they are equal at the 5% level. [12]

Receiving a high value, however, focuses a bidder's attention on the fact that there is only one other bidder amongst the remaining seven who also values the same good and who, most probably, has a value lower than his. Therefore, a high valuation bidder can afford to lose in the first (and maybe second) phase of the auction, and try to win the later phases where prices are low.

The same logic applies for later phases of the $RTC(4,4,2)$ auction as well as for the $RTC(4,3,2)$ auction but in even starker terms: here, moderate-value bidders face even more competition since they need to outbid seven other bidders for only three (as opposed to four) goods.

Note that if bidders use a relatively high cutoff to differentiate between "moderate" and "high" values, then the above rule-of-thumb would generate significantly more revenue than that predicted by the theory. In particular, if bidders bid their value for a relatively large range of values, then the price paid by winners would most likely be the *value* of the second highest bidder (instead of that bidder's shaded bid). This means that even if winners are high bidders who shade their own bid, the prices paid in the RTC auction would rise significantly above the predicted prices. Moreover, since there are more order statistics in each non-terminal phase of the RTC auction than in each separate auction in the GBG design, the RTC auction may potentially generate a higher revenue than the OGBG when winners always pay the *value* of the second highest bidder. The RTC auction therefore accomplishes its objective of raising revenues by fostering competition, especially for moderate value bidders.

The strategy described above is a discontinuous moderate-high strategy that specifies a different behavior for each class of values. In contrast, equilibrium behavior suggests a continuous strategy, which is constant across values. Note that our explanation of the

[12] The 95% confidence interval around the slope of the bid function for low values is $0.8096 - 0.9160$ which includes 0.90.

moderate-high rule-of-thumb suggests that if cutoffs change across phases, they should move downward. The intuition for this is that the winner in each phase was most likely the highest valuation bidder in that phase. Hence, a value that was considered only moderate in phase 1 may be among the high values of phase 2.

To test our hypothesis regarding the subjects' bidding behavior, we estimated a set of bid functions for our $RTC(4,4,2)$ and $RTC(4,3,2)$ auctions. Using a random effects model we tested for structural breaks in the aggregate bidding behavior of subjects (aggregated over all subjects and periods) in each phase and period of the auction. The exact regression run was of the form,

$$bid_{itjp} = a + b_1 v_{itjp} + b_2 (v_{itjp})^2 + Db_3 + Db_4 v_{itjp} + Db_5 (v_{itjp})^2 + u_{ijt} + \varepsilon_{itjp}$$

where the variables are defined as follows:

- bid_{itjp} is the bid of subject i in treatment t, phase j and period p,
- v_{itjp} is the value of subject i in the corresponding treatment, phase and period,[13]
- D is a dummy variable taking a value of 1 if the value is above the cut-off value of the bidder,
- u_{ijt} is an error term associated with individual i's bid in phase j and treatment t, and which does not change across periods, and
- ε_{itjp} is a disturbance term for i, which is allowed to vary across periods.

We tested for a structural break at any particular value by testing jointly whether the coefficients on both the slope and intercept dummy variables in the regression have coefficients different from zero. We test for linearity below and above the break by testing whether coefficients b_2 and b_5 are significantly different from zero, respectively. We then search over the set of break points to find the point, which maximizes the R^2 of the regression.[14] If we reject our joint hypothesis at every value of the domain of the bidding function, we conclude that the regression has no significant break and present the results

[13] Regressions with higher order terms were also run but no terms of order three or more were found to be consistently significant at the 5% level.

[14] We search over the interval $[20, 85]$ to insure sufficient data for estimation.

of that regression without a break. If the bid function is linear on either side of the break, we only present the coefficient on the linear term (no bid function was quadratic to the left of the estimated break point).

Our estimation results for the $RTC(4,4,2)$ and $RTC(4,3,2)$ are presented in Table 4a and Table 4b respectively.

Tables 4a-4b

We depict the estimated aggregate bid functions for each phase in the $RTC(4,4,2)$ and $RTC(4,3,2)$ in Figures 1a and 1b respectively.[15]

Figures 1a-1b

Notice that the estimated bid functions in both Figures 1a and 1b are strictly higher than the equilibrium bid functions posited by the theory (except in the last phase where subjects are supposed to bid theory vales). This suggests that the RTC auctions would raise significantly more revenue than predicted.

Our estimation results indicate that there is a structural break for all phases of the auctions except for the last phase of the $RTC(4,4,2)$ and $RTC(4,3,2)$ where we would expect subjects to bid their values.[16] Looking at figure 1a we can easily see the change in behavior on either side of the structural break in Phases 1 through 3 of the $RTC(4,4,2)$. Note that all break points are approximate at 71 and that all bid functions to the right of the break point are concave with a discontinuous step-down at the break point. The fact that the break point occurs at a relatively high cutoff helps explain why the $RTC(4,4,2)$ raised significantly more revenue than the $OGBG(4,2)$.

Such discontinuous bidding behavior is seen quite often in the data of all-pay auctions.

[15] The bidding function of phase 4 in the $RTC(4,4,2)$ was estimated after we eliminated four bids that deviated from the private value by more than forty points (e.g., a bid of 70 made by a bidder with a value of zero). Since it is weakly dominating to bid one's value in the final phase, and since bidding one's value strictly dominates bidding *above* one's value, these four bids are inexplicable, and we suspect that they are genuine mistakes.

[16] In the $RTC(4,3,2)$ case, however, this structural break is eliminated only after we eliminate the bids of six outliers who substantially bid below their value in phase 3 in a manner that is inexplicable. For example, one subject received a value of 97 and bid 4 in the last phase instead of bidding his or her value as a dominant strategy.

Despite these outliers it is evident that in this last phase almost all subjects bid their value. For example, of the 512 bids made in phase 3 of the $RTC(4,3,2)$ auction, the mean (median) deviation from value was $0.054(0)$ while 90% of the bids made did not deviate from value by more than five. In addition, over the interval $[0,84]$ the slope of the estimated bid function is 0.970 which is insignificantly different from one.

For example, Mueller and Schotter (2004) experimentally investigate a model of contests where subjects make bids by investing effort in order to win prizes. They find that when subjects receive a negative productivity shock such that it becomes very unlikely that they will win a prize, they tend to "drop-out" and exert zero effort (bid zero) rather than exert the equilibrium effort suggested by the theory.

We note that the bid functions of phases 1-3 in the $RTC(4,4,2)$ are not monotonic to the right of the break point: they reach an interior maximum at approximately a value of 92. This non-monotonicity can be explained by bidding behavior, which is consistent with our rule-of-thumb hypothesis. Recall that subjects with high values focus their attention on two things: (i) the rival subject who values their good, and (ii) the phase in which they hope to beat him. When a subject receives an extremely high value he is almost certain that this value is higher than that of his rival. Such subjects may believe that most likely, the highest value below their own does not belong to the subject who values their good. Hence, they can afford to lose in the first and even second phase of the auction. Consequently, in these two phases subjects with extremely high values may bid lower than subjects with lower values (yet still above the cutoff). One extreme example of this as observed in our data is a bidder with a value of 96. He bid bid zero in the first and second phase, but bid his value in the third phase and won. When one investigates the shape of the aggregate bid functions in phases 1 and 2 they would appear quadratic and non-monotonic because the estimated bid function would be pulled down at the right end by this extreme waiting strategy. In contrast, if a subject's value is high, but not extremely so, he may shade his bid, but only slightly since the highest value below his may belong to the subject who wants the same good as he; hence, bidding too low may ruin his chances of winning.

Figure 1b confirms our earlier intuition that if the cutoff of the bidding rule-of-thumb changes across phases, it should decrease. Indeed, our estimation of the $RTC(4,3,2)$ bid function yields that the cutoff drops from 81 to 54 as we move from phase 1 to phase 2. However, as evident from Figure 1a, the cutoff of the estimated bid function in the $RTC(4,4,2)$ remains approximately 71 across the first three phases. It is unclear to us why the estimated cutoffs behave differently in the two auctions.

Because competition is enhanced in a RTC auction with quantity restriction, we would expect that the slopes of the estimated functions are higher in the $RTC(4,3,2)$ than in the $RTC(4,4,2)$. To verify this intuition, for each of the first three phases, we compared the estimated bid function in the $RTC(4,3,2)$ with the corresponding bid function in the

$RTC(4,4,2)$. These comparisons are depicted in Figures $2a - 2c$.

Figure 2a-2c

As Figures $2a-2c$ indicate, phase by phase the bid functions in the $RTC(4,3,2)$ experiment are everywhere above those of the $RTC(4,4,2)$ experiments.

As a final note to this section it is important to point out that our RTC auctions outperform the GBG auctions (run with and without optimal reserve prices) despite the fact that, at least in those auctions, subjects bid according to the dictates of theory. Hence, the relatively good performance of our RTC auctions was not the result of sub-optimal behavior by subjects in our $GBG(4,2)$ or $OGBG(4,2)$ experiments. To illustrate this point remember that in these auctions, given their second price nature, subjects are expected to bid their value. From the bid data generate we see that this was true in the sense that the median difference between bid and value in both of these auctions was zero while the mean difference between bid and value in the $GBG(4,2)$ and $OGBG(4,2)$ auctions were 1.21 and 0.40, respectively. Figures $3a$ and $3b$ present histograms where the variable on the x axis is the difference between the value a bidder received and his or her bid in the GBG and OGBG auctions respectively.

Figure 3a-3b

Note that a clear majority of the bidders basically bid their values in the two auction formats. There are only very few bids that deviate substantially from value. Such behavior indicates that bidders in these auctions clearly saw that it was dominant to bid one's value. That is, the cognitive task presented to subjects in the OGBG auction was one they could easily understand. We interpret this to mean that our experimental RTC auctions outperformed the OGBG auctions when the latter was performing at its best.

5.3 Learning

The description of behavior offered in the previous section is based on the aggregate data generated by our experiment, i.e., data generated over all 16 periods. However, it is possible that while this behavior was exhibited in the first half of the experiment, before subjects learned what was going on, in the later half of the experiment they might have learned the behavior predicted by the equilibrium and behaved that way. Hence, our results may be biased by the, perhaps severe, behavior of subjects in the beginning of the experiment. The implication is that if this auction were run repeatedly we might expect behavior to converge to something closer to that predicted by the theory.

To address the learning issue we repeated the regression analysis above but used data only from the last eight rounds of the experiment. Clearly, if behavior changed over the length of the experiment we would expect that these regressions would reflect that change. Our results are presented in Figures $4a - 4b$ and in the right hand side of Table 4.

Figure 4a-4b

The results are striking. Rather than learning the equilibrium behavior for the auction, our subjects actually converged in their behavior to the behavior described above in Section 5.2. In other words, the behavior characterized in that section is more characteristic of the behavior of subjects over the last eight periods of the experiment than the first eight.

To illustrate this, consider Figures $4a$ and $4b$. In these figures we superimpose the estimated bid functions for the last eight periods on top of those estimated using the data generated by periods $1-16$. As we can see, there is little difference either in the qualitative or quantitative features of subject behavior. Subjects continued to exhibit structural breaks in the last eight periods and continued to have a significant quadratic section to the right of the break and a linear portion to the left. While in the $RTC(4,4,2)$ treatment the structural breaks occurred in approximately the same place, they were significantly higher in the $RTC(4,3,2)$ treatment and exhibited a break in phase 3 which was not detected before. Further, except for some significant differences in where the breaks occurred, the confidence intervals for the coefficients of the two regressions overlap at the 95% level for all estimated coefficients indicating that these regressions are reflecting identical bid behavior. Finally, note that the bid functions for the last eight periods are everywhere above those of the overall regressions indicating that the behavior described above had no tendency to abate as time went on.

It is also important to note that the revenues in periods 9-16 are virtually identical to those raised in all 16 periods. For example, the revenues raised by the $RTC(4,4,2)$ and the $RTC(4,3,2)$ in periods 9-16 were 206.5 and 192.3 respectively, while those raised in all 16 periods were 203.7 and 188.7 respectively. Similarly, the revenue raised by the $OGBG(4,4,2)$ in periods 9-16 was 179.1, whereas the average revenue over all 16 periods was 178.8. When comparing revenues in the last eight periods across auction designs, the difference between the $OGBG(4, 2)$ revenue and that of the $RTC(4, 2)$ in periods 9-16 is statistically significant at the 5% level.

On the basis of these results we can only conclude that the behavior we described above is the behavior that subjects converged to in the later portions of the experiment and which was not eliminated by learning.

5.4 Risk aversion

The previous subsection offered an explanation for why the subjects in our experimental RTC auctions submitted significantly higher bids than those predicted by our theoretical analysis. Since our theoretical analysis assumed that bidders are risk-neutral, it is important to verify that risk-aversion alone cannot explain the bidding behavior of our subjects. To do this we used experimental data gathered from a variant of our RTC auction. The key difference between this RTC format and our original format is that losing bidders were not told which good was won in each phase. The winner in each phase paid the second highest bid, and his preferred good was not available, he picked some good at random. In contrast to our original RTC format, in this type of auction a bidder may end up paying for a good he does not value.[17] We shall refer to this type of auction as a right-to-choose auction with no information, or NIRTC for short.

The experimental design of the NIRTC treatment followed the design described in Section 4. In particular, there were four objects, each demanded by two distinct buyers, and the set of values used for the Amsterdam group and for the New York group were the same as those used in the RTC and GBG auctions. Appendix B contains a more detailed description of the NIRTC together with the theoretical predictions for risk-neutral agents.

While in RTC auctions a risk averse bidder would bid higher than his risk neutral cohort, in NIRTC auctions just the opposite is true. More precisely, as long as one does not bid above one's value in a RTC auction a bidder can avoid incurring a loss. Hence, if a bidder decides to bid above the prescribed risk neutral equilibrium bid, he or she is simple trading off an increased probability of winning a good against the profit to be made conditional on winning. As is true of auctions in general, risk averse subjects deal with this trade-off by raising their bids so we would expect that risk averse subjects in RTC auctions would do exactly as we have observed them doing and bid above the predictions of the risk neutral equilibrium bid function. As we have mentioned above, this might explain

[17] This feature is also shared by pooled auctions. See Iachini and Salmon (2003).

why we observed larger than expected revenues in the RTC auctions we ran. Therefore, with respect to RTC auctions risk aversion does offer an explanation of observed behavior.

Risk aversion fails, however, to explain behavior and revenues in NIRTC auctions. Here, as the auction progresses, the probability that your good has been selected increases as does the probability that you will incur a loss if you win. In such circumstances risk aversion suggests that a bidder lower his bid below his risk neutral bid. As a result, we would expect lower revenues in NIRTC auctions when bidders are risk averse.

To help illustrate this point consider an auction where all bidders have the same CARA utility function of the form $U(x) = \frac{1-e^{-rx}}{r}$ and let $r = 0.07$.[18] Using this utility function we can immediately see how risk aversion alone fails to explain explain why the revenues raised by the experimental RTC auctions are significantly above the risk-neutral prediction. Table 5 presents the revenues that can be expected for a CARA model with $r = 0.07$, together with the observed revenues and the revenues predicted by the risk neutral model. Although risk-aversion leads to higher revenue than those predicted with risk-neutrality, the increase in revenue is not as dramatic as that observed in our experiment. Furthermore, risk aversion fails to explain the fact that revenues also rise in the NIRTC auctions. In those auctions, risk aversion implies that revenues should fall in comparison to the risk neutral model.

Table 5

5.5 Efficiency

We focus on two measures of efficiency. First, we investigate the allocative efficiency of these auctions by looking at whether the *available* goods are allocated to the bidders who value them the most. To measure allocative efficiency we simply count the fraction of times during the auction that the goods were sold to the highest valuation bidders. We call this measure *Ordinal Efficiency* since it only takes into account whether an optimal trade was made or not but not the value of the trade.

One drawback of ordinal efficiency is that it disregards the magnitude of welfare losses. In particular, this measure of efficiency does not distinguish between a good that is not sold to any buyer and a good that is not sold to the highest valuation buyer. We therefore propose a second measure of efficiency, called *Cardinal Efficiency*, which is defined as the

[18] Goeree and Offerman (2003) use the same utility function to analyze second-price private value auctions with value uncertainty. They report maximum likelihood estimates for r in the range of $[0.06, 0.08]$. Therefore, we chose to use $r = 0.07$.

ratio of "realized surplus" to "maximal surplus". By "realized surplus", we mean the sum of values of the winning bidders, whereas "maximal surplus" refers to the sum of values of the highest valuation bidders for each good. While ordinal efficiency reports whether or not a welfare loss was incurred, Cardinal Efficiency reports the magnitude of this loss. Table 6 presents our efficiency results.

Table 6

Several features of this table are noteworthy. First, notice that the $RTC(4,4,2)$ and the $GBG(4,2)$ auctions are close to being fully efficient in the cardinal sense. Each practically achieves 100% efficiency. Second, note that they are less efficient in ordinal terms. This clearly indicates that if an inefficient allocation occurred it was more likely to have occurred when the subjects valuing the same good had similar values since all missed allocations have equal weight when measuring ordinal efficiency but a mistaken allocation occurring between subjects with very similar values is unlikely to decrease our cardinal measure significantly (i.e. it matters little for cardinal efficiency if a good is allocated to a subject who values it at 76 while his pair member values it at 77).

We used Mann-Whitney tests to determine whether or not differences in efficiency scores are statistically significant. Under both notions of efficiency, the difference between the score of the $RTC(4,4,2)$ and those of $RTC(4,3,2)$ and $OGBG(4,2)$ is statistically significant at the 1% level. As expected, the two efficiency scores of the $RTC(4,4,2)$ are not statistically different than those of the $GBG(4,2)$, while those of the latter are significantly higher than the ordinal and cardinal scores of the $OGBG(4,2)$ (at the 1% level).

Table 4 highlights an interesting experimental finding. A recurring theme in the mechanism-design literature is the trade-off between efficiency and optimality. In contrast, we find that the highest revenue is in fact generated by an efficient auction: the unrestricted RTC auction. This suggests an additional important advantage of RTC auctions.

6 Concluding Remarks

This paper examines a detail-free mechanism for selling multiple goods. This mechanism proceeds sequentially where in each phase the seller auctions a right-to-choose one of the available goods. If all goods are sold, then this right-to-choose auction is efficient. We propose a detail free way of introducing inefficiency into the RTC auction: restrict the number of right-to-choose that are sold. This quantity restriction intensifies competition and leads to higher prices. In the model we analyze, quantity restriction also leads to higher revenues.

The RTC auction has the desirable property that it can be executed independently of the distribution of buyers' valuations and regardless of whether or not the goods are substitutes for some of the buyers. In this paper we highlight another desirable feature of this auction: the RTC auction induces an aggressive bidding behavior that generates a substantially higher revenue than that predicted by the theory. Moreover, in an experimental setting where the theoretical optimal auction is known, the RTC auction raised on average more revenue than the theoretical optimal auction.

This finding has the following implications. First, sellers with multiple goods, each having a thin market, may benefit from thickening those markets via a RTC auction. In particular, in an environment where the optimal auction is difficult to compute (either because there is not enough information or because of multi-dimensionality), the RTC auction seems especially desirable. Second, our findings may explain the popularity of the RTC format in the real-estate market even though it is not the theoretical optimal auction in these environments. Our experimental results suggest that perhaps the RTC format was ultimately adopted because it proved to be better in terms of revenues than alternative procedures.

Finally, our findings suggest a new *behavioral* approach to mechanism-design: the design of mechanisms that take advantage of certain behavioral regularities that individuals exhibit. In our case, the difficulty of resolving an inter-temporal trade-off, coupled with the availability of a simple rule-of-thumb, shifts the behavior of buyers in the direction of more aggressive bidding. By understanding how different biases can be manipulated by various procedural aspects, one may be able to design simple, robust mechanisms that perform better than what our standard theory predicts.[19]

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Appendix A: The Right-to-Choose Auction with No Information

A NIRTC is a sequential RTC auction, which proceeds in exactly the same manner as the RTC design described in Section 3, except for the following difference: At the end of each phase, *bidders are not informed of which good was taken by the winner of that phase*. That is, a bidder who wins a phase, first pays the second highest bid, and then gets to choose one of the available goods. Therefore, a bidder who wins a phase, but whose good was taken by a winner of a previous phase, may pay a positive price but receive no value. Thus, just as in the pooled auction in which a bidder may win a good whose value is less than the price he pays, bidders may also incur a loss in a NIRTC auction. However, in equilibrium, it is ex-ante individually rational to participate in a NIRTC.

We assume that when a winner of some phase does not find the good he values, he picks one of the remaining goods at random. We interpret this assumption as saying that a bidder does not want to leave an auction empty handed (especially if he wins and pays).[20] We could therefore view a NIRTC auction as a form of an "all-pay-if-you win auction" in which winners pay a positive price even if they have a zero value for the good they buy, while buyers who fail to win pay zero and get nothing for it. A NIRTC with K goods, $K-q$ phases and n bidders per good is denoted $NIRTC(K, K-q, n)$. The equilibrium bids of this auction are derived using arguments similar to those made in Proposition 5.

Proposition 6 *For all $q < K$, the $NIRTC(K, K-q, n)$ auction has a SPBE in which bidders use a linear bid function in every round. The bidding coefficients $(\beta_k)_{k=1}^{K-q}$ are given by*

$$\beta_{K-q} = \frac{n\,(q+1)}{(n-1)\,K + q + 1}$$

and the unique solution to the following system of difference equations

$$\frac{N\,(n-1)}{N_k N_{k+1}} = \beta_k - \frac{N_{k+1} - (k+1)}{N_{k+1}} \beta_{k+1}$$

where $k = 1, \ldots, K-q-1$.

When all rights-to-choose are sold (i.e., when $q = 0$), the SPBE of a NIRTC has the following simple closed-form solution:

$$\beta_k = \frac{N(n-1)}{N_{k+1}} \sum_{i=k}^{K} \frac{1}{N_i}$$

Note that in contrast to our original *RTC* auction, individuals bid a smaller and smaller fraction of their value as the auction progresses. The intuition for this is clear. In the first phase of the auction all goods are available, and so the winner of this phase will get a good he or she values positively. This explains why the first phase bid is the highest. Still, bidders do not bid their value in the first phase because although they are certain to receive their good if they win, they would

[20] We could formalize this interpretation by assuming that for each bidder there is only one good from which his value is drawn from a uniform distribution on $[\varepsilon, 1]$, while for the other $K-1$ goods his value is $0 < \mu < \varepsilon$, and where ε is arbitrarily close to zero.

pay a high price. As the auction progresses, the chance of not finding one's good increases, and so the willingness-to-pay decreases.

The following are the theoretical predictions for the NIRTC auction in our set up.

Proposition 7 *The NIRTC auction with K phases raises more expected revenue than the GBG auction or the unrestricted RTC auction only if the market for each good is very thin, i.e., when $n = 2$; otherwise, it generates a lower expected revenue than the two other auction types.*

Proposition 8 *Quantity restriction lowers the expected revenue in the NIRTC auction: for all $q < K$, the expected revenue in the SPBE of $NIRTC(K, K-q-1, n)$ is less than the expected revenue in the SPBE of $NIRTC(K, K-q, n)$.*

We tested both the $NIRTC(4,4,2)$ and the $NIRTC(4,3,2)$ in the laboratory using exactly the same procedures of Section 4. In contrast to the RTC auctions, after each phase of the NIRTC auctions, no information was offered as to which good was chosen. All that subjects were told was that some good had been chosen. Bidding then continued as it did in the previous phase until all rights were sold. If the good preferred by a winning bidder had been selected previously, the bidder was assigned a good at random (a good for which he or she has a zero value). At the end of the period the earnings of each subject was placed on the screen as was the cumulative earnings of the subject up until that period.

Appendix B: Proofs

For ease of exposition, in what follows bidders will be labeled by their values, e.g., a bidder with value v will be referred to as "bidder v". We shall also use the notation $Y_i^{(m)}$ to denote the i-th order statistic of m independent draws from $[0,1]$ (e.g., $Y_1^{(N_k)}$ denotes the highest order statistic among the remaining bidders in phase k).

Proof of Proposition 1. We need to show that the $RTC(K, K, n)$ auction has a SPBE of the following form:

$$b_k(v) = \frac{N_k - (K-k+1)}{N_k - 1} v \qquad \text{(A1)}$$

where N_k denotes the number of bidders in round k.

Clearly, the final phase of the auction, phase K, is simply a standard second-price auction. Hence, $b_K(v)$ should be equal to v. Indeed, by letting $k = K$ in equation $(A1)$ we obtain

$$b_K(v) = \frac{N_k - 1}{N_k - 1} v = v$$

Next we show that there is no $k < K$ and no value v, such that bidder v would want to deviate from $b_k(v)$ to $b_k(x)$, where $x \neq v$. We proceed in two steps. First, we verify that a "downward" deviation (i.e., $x < v$) is not profitable. Then we check that an "upward" deviation (i.e., $x \geq v$) is also not profitable.

STEP 1. We begin by making two observations. First, by the one-deviation property for extensive games with imperfect information (see Osborne and Rubinstein (1994), p.227), a profile of bid functions $(b_k(v))_{k=1}^{K}$ is a SPBE if and only if the following is true: For every round k, each bidder v has no incentive to deviate from $b_k(v)$, assuming that at every $k' > k$ bidder v bids according to $b_{k'}(v)$. Second, a downward deviation by a bidder affects the outcome only if that bidder has the highest valuation.

With these two observations in mind, consider the highest valuation bidder in phase k. Label this bidder by the index h and denote his value by v. If all bidders follow $(A1)$, then bidder h would win in phase k and pay the bid of the next highest valuation bidder, which we denote by j. The expected value of bidder j, conditional on v being the highest value in phase k, is the expected value of the highest order statistic of $N_k - 1$ independent draws from $[0, v]$. This expected value is equal to $\frac{N_k-1}{N_k}v$. Therefore, by following the proposed equilibrium bidding strategy, bidder h expected to gain an expected payoff of

$$v - \beta_k \left(\frac{N_k - 1}{N_k}\right) v \tag{A2}$$

where β_k denotes the coefficient of the linear bidding function given by $(A1)$.

Suppose bidder h deviates from $(A1)$ and bids $\beta_k x < \beta_k v$. Then h loses in round k, while bidder j wins. Bidder h enters phase $k+1$ of the auction only if j did not take his good. The probability of this event occuring, denoted p_{k+1}, is given by

$$\frac{\binom{N_k-2}{n-1}}{\binom{N_k-1}{n-1}} = \frac{N_k - n}{N_k - 1} = \frac{n(K-k)}{n(K-k+1)-1}$$

If h makes it into phase $k+1$, then he will win for sure. However, the price he pays upon winning, depends on which bidders demand the same good as bidder j who won in phase k. It follows that bidder h's expected payoff from bidding $\beta_k x < \beta_k v$ is

$$\frac{\binom{N_k-2}{n-1}}{\binom{N_k-1}{n-1}} \left[v - \beta_{k+1} \sum_{i=1}^{n} \frac{\binom{N_k-2-i}{n-i}}{\binom{N_k-2}{n-1}} \left(\frac{N_k - i - 1}{N_k}\right) v \right] \tag{A3}$$

where

$$\sum_{i=1}^{n} \frac{\binom{N_k-2-i}{n-i}}{\binom{N_k-2}{n-1}} \left(\frac{N_k - i - 1}{N_k}\right) = \frac{\binom{N_k-3}{n-1}}{\binom{N_k-2}{n-1}} \frac{(N_k-1)(N_k-2)}{N_k(N_k-n)}$$

Hence, bidder h has no incentive to bid as if his value is lower than v if $(A2)$ is at least as large as $(A3)$. Because a bidder with a value of zero cannot pretend to have a lower value, $(A2)$ is at least as large as $(A3)$ if

$$(1-\beta_k) + \frac{\beta_k}{N_k} \geq \frac{\binom{N_k-2}{n-1}}{\binom{N_k-1}{n-1}} - \beta_{k+1} \frac{\binom{N_k-3}{n-1}}{\binom{N_k-1}{n-1}} \frac{(N_k-1)(N_k-2)}{N_k(N_k-n)} \tag{A4}$$

Using the fact that

$$\frac{\binom{N_k-3}{n-1}}{\binom{N_k-1}{n-1}} = \frac{(N_k-n)(N_k-n-1)}{(N_k-1)(N_k-2)}$$

we can rewrite inequality ($A4$) as follows

$$(1-\beta_k)+\frac{\beta_k}{N_k}\geq\frac{\binom{N_k-2}{n-1}}{\binom{N_k-1}{n-1}}-\beta_{k+1}\frac{(N_k-n-1)}{N_k} \tag{A5}$$

To show that inequality ($A5$) holds, we first note that the bid function in ($A1$) satisfies the following difference equation for all $k<K$:

$$1-\beta_k=\frac{\binom{N_k-2}{n-1}}{\binom{N_k-1}{n-1}}-\beta_{k+1}\left(\frac{N_k-n-1}{N_k-1}\right) \tag{A6}$$

Substituting ($A6$) into ($A5$) and cancelling common terms, we get

$$\begin{aligned}\frac{\beta_k}{N_k} &\geq \beta_{k+1}(N_k-n-1)\left(\frac{1}{N_k-1}-\frac{1}{N_k}\right)\\ &= \beta_{k+1}\frac{N_k-n-1}{N_k\,(N_k-1)}\end{aligned}$$

$$\Updownarrow$$

$$\beta_k\geq\beta_{k+1}\frac{N_k-n-1}{N_k-1}=\frac{(n-1)(K-k)}{n(K-k+1)-1} \tag{A7}$$

where the last inequality follows from ($A1$) and the fact that $N_k=n(K-k+1)$. But by ($A1$) we have that $\beta_k=\frac{(n-1)(K-k+1)}{n(K-k+1)-1}$, hence ($A7$) is satisfied.

Step 2. Consider bidder v in phase k of the auction. Suppose this bidder considers bidding as if his value is $x\geq v$. Assume this deviation is profitable. Then it must be profitable when $v<Y_1^{(N_k-1)}$. Conditional on this event, bidder v's expected payoff in phase k is

$$v-\beta_k E\left[Y_1^{(N_k-1)}|v<Y_1^{(N_k-1)}<x\right]$$

Consider an alternative strategy for bidder v in which he bids $\beta_k v$ in phase k and $\beta_{k+1}x$ in phase $k+1$. His expected payoff from this strategy, conditional on $v<Y_1^{(N_k-1)}<x$, is

$$p_{k+1}v-p_{k+1}\beta_{k+1}E\left[Y_1^{(N_{k+1}-1)}|v<Y_1^{(N_k-1)}<x\right]$$

Noting that the equilibrium indifference condition implies

$$Y_1^{(N_k-1)}-\beta_k Y_1^{(N_k-1)}=p_{k+1}\left[Y_1^{(N_k-1)}-\beta_{k+1}E\left(Y_1^{(N_{k+1}-1)}|Y_1^{(N_{k+1}-1)}<Y_1^{(N_k-1)}\right)\right]$$

we obtain that

$$E\left[v-\beta_k Y_1^{(N_k-1)}-p_{k+1}v+p_{k+1}\beta_{k+1}E\left(Y_1^{(N_{k+1}-1)}|Y_1^{(N_{k+1}-1)}<Y_1^{(N_k-1)}\right)|v<Y_1^{(N_k-1)}<x\right]<0 \tag{A8}$$

Inequality (A8) implies that for any phase k, in which $v<Y_1^{(N_k-1)}$ and $x\geq v$ is a profitable

deviation, bidder v prefers to win with x in phase $k+1$ than in phase k. But this cannot be true since a bidder who has a profitable deviation in the phase before the last would not want to deviate in the last and final phase. $\|$

Proof of Proposition 2. The proof is essentially the same as the proof of Proposition 1. The only change is that $b_{K-1}(v) = v$, given that round $K-1$ is the final round. $\|$

Proof of Proposition 3. When bidders bid their value in the $GBG(K,n)$, and when they follow the SPBE of the $RTC(K,K,n)$, as given in Proposition 1, then (1) the allocation of goods in both auctions is efficient, and (2) the expected payment of a bidder with value zero is zero. Hence, by the Revenue-Equivalence-Theorem, both auctions yield the same expected revenue.[21] $\|$

Proof of Proposition 4. We begin by introducing a few helpful notations:

- $\mathcal{P}$ denotes the set of all partitions of nK order statistics into K groups of n. An element in $\mathcal{P}$ (i.e., a particular partition) is denoted P. If the i-th order statistic is in the same group as the j-th order statistic, then we say that the i-th and j-th order statistics demand the same good.
- Let $\beta_k^K v$ be the equilibrium bid function for $RTC(K,K,n)$.
- Let $\beta_k^{K-1} v$ be the equilibrium bid function for $RTC(K,K-1,n)$.

Fix some partition $P \in \mathcal{P}$. Suppose we run both a $RTC(K,K,n)$ and a $RTC(K,K-1,n)$ for this given P. Because the partition is held fixed, the set of order statistics who win is exactly the same in $RTC(K,K,n)$ and in $RTC(K,K-1,n)$. Moreover, the order statistic, whose bid is paid in each of the first $K-1$ rounds, is also the same in both auctions. Let $B^K(P)$ be the set of expected prices paid in the first $K-1$ rounds of $RTC(K,K,n)$. Similarly, let $B^{K-1}(P)$ be the set of expected prices paid in the first $K-1$ rounds of $RTC(K,K-1,n)$. Then

$$\begin{aligned} B^K(P) &= \left\{\beta_1^K\left(\frac{N-\theta_1^P}{N+1}\right), \ldots, \beta_{K-1}^K\left(\frac{N-\theta_{K-1}^P}{N+1}\right)\right\} \\ B^{K-1}(P) &= \left\{\beta_1^{K-1}\left(\frac{N-\theta_1^P}{N+1}\right), \ldots, \beta_{K-1}^{K-1}\left(\frac{N-\theta_{K-1}^P}{N+1}\right)\right\} \end{aligned}$$

where $\frac{N-\theta_k^P}{N+1}$ is the θ_k^P+1 order statistic, such that $\theta_1^P=1$ and $n \geq \theta_{k+1}^P - \theta_k^P \geq 1$. We define $\Theta(P) \equiv \left(\theta_1^P, \ldots, \theta_{K-1}^P\right)$.

For the given partition P, the total expected gain from quantity restriction is equal to the following sum:

$$\sum_{k=1}^{K-1} \left(\beta_k^{K-1} - \beta_k^K\right)\left(\frac{N-\theta_k^P}{N+1}\right) \tag{A9}$$

[21] See, for example, Klemperer, 1999, who shows that revenue-equivalence results in multi-unit private value auctions where bidders have unit-demand

While the expected loss is equal to $\frac{N-\theta_K}{N+1}$. From Propositions 2.1 and 2.2 it follows that $\beta_k^{K-1} - \beta_k^K = \frac{1}{N_k - 1}$. Hence, expression $(A9)$ can be rewritten as follows:

$$\sum_{k=1}^{K-1} \left(\frac{1}{N_k - 1}\right)\left(\frac{N - \theta_k^P}{N+1}\right) \tag{A10}$$

Given P, quantity restriction *raises* expected revenue if and only if

$$\sum_{k=1}^{K-1} \left(\frac{1}{N_k - 1}\right)\left(\frac{N - \theta_k^P}{N+1}\right) > \frac{N - \theta_K^P}{N+1} \tag{A11}$$

Multiplying both sides of $(A11)$ by $N+1$, we obtain that inequality $(A11)$ holds if and only if

$$\sum_{k=1}^{K-1} \left(\frac{N - \theta_k^P}{N_k - 1}\right) > N - \theta_K^P \tag{A12}$$

We denote the LHS and RHS of $(A12)$ by $G(P)$ and $L(P)$ respectively. Hence, given P, the expected gain from quantity restriction is at least as large as the expected loss if and only if $G(P) \geq L(P)$.

Let $\mathcal{P}^j \subset \mathcal{P}$ be the set of partitions with the property that $\theta_K^P = K + j$ for every $P \in \mathcal{P}^j$, where $j \in \{0, 1, \ldots, (K-1)\,n+1\}$. Note that for any j every $P \in \mathcal{P}^j$ satisfies $\theta_k^P \leq k + j$ for $1 \leq k \leq K-1$. This implies that for any j and any $P \in \mathcal{P}^j$, we have

$$\begin{aligned} G(P) &\geq \sum_{k=1}^{K-1} \left[\frac{N - (k+j)}{N_k - 1}\right] = \sum_{i=1}^{K-1} \frac{nK - (K - i + j)}{(i+1)\,n - 1} \\ L(P) &= N - K - j = (n-1)K - j \end{aligned}$$

Hence, $G(P) \geq L(P)$ if and only if

$$\sum_{i=1}^{K-1} \frac{(n-1)K - j}{(i+1)\,n - 1} + \sum_{i=1}^{K-1} \frac{i}{(i+1)\,n - 1} \geq (n-1)K - j$$

Let $\Psi\,(K, n) \equiv \left[\sum_{i=1}^{K-1} \frac{1}{(i+1)n-1}\right] - 1$. Then the above inequailty holds if and only if

$$[(n-1)K - j]\,\Psi\,(K, n) + \sum_{i=1}^{K-1} \frac{i}{(i+1)\,n - 1} \geq 0 \tag{A13}$$

Define $K^*\,(n)$ to be the smallest positive integer that satisfies $\Psi\,(K, n) \geq 1$. Such an integer exists because $\Psi\,(K, n)$ increases with K and tends to infinity as $K \to \infty$. It follows that inequality $(A13)$ holds for all $K \geq K^*\,(n)$. Note that because $\Psi\,(K, n)$ decreases with n, the integer $K^*(n)$ increases with n.

We now turn to show that when $n = 2$ (as is the case in our experimental design) quantity restriction raises expected revenue for all $K > 2$. First, it is straightforward to verify that for $K = 3$, selling two out of three goods raises more expected revenue than selling all three goods

($R(3,2,2) = \frac{38}{35}$, while $R(3,3,2) = 1$). It remains to show that quantity restriction raises expected revenue for all $K > 3$. We begin by identifying the set of partitions for which quantity restriction lowers expected revenue.

Lemma A1. $G(P) < L(P)$ only if $P \in \mathcal{P}^0$.

Proof. Assume not. Then there exists a partition $P \in \mathcal{P} \backslash \mathcal{P}^0$ for which $G(P) < L(P)$. First, we claim that for every $P \in \mathcal{P}$ and $1 \leq k \leq K-1$ the following must hold: $\theta_k^P \leq 2k-1$. This follows from the fact that $\theta_1^P = 1$ and $n \geq \theta_{k+1}^P - \theta_k^P \geq 1$ for every P and k. Next, we claim that for all $P \in \mathcal{P}$ and $1 \leq k \leq K-1$ we have $\frac{N-\theta_k^P}{N_k-1} \geq 1$. Suppose not. Then $2K-\theta_k^P < 2(K-k+1)-1$, which implies that $\theta_k^P > 2k-1$, a contradiction. Therefore, $G(P) \geq K-1$ for all $P \in \mathcal{P}$, where equality holds if $K = 2$. Because each $P \in \mathcal{P} \backslash \mathcal{P}^0$ satisfies $L(P) \leq K-1$, we have that $G(P) \geq L(P)$, in contradiction to our initial assumption. $\|$

By Lemma A1, there are exactly $(K-1)!$ partitions for which $G(P) < L(P)$. Moreover, for each of these partitions, $L(P) - G(P) = K - G^0$. Because each partition P is equally likely, $E_{P \in \mathcal{P}}[G(P) - L(P)] > 0$ if there exist at least $(K-1)!$ partitions for which $G(P) - L(P) \geq K - G^0$, where the inequality is strict for at least one of these partitions.

Let $\mathcal{P}^* \subset \mathcal{P}^K$ be a set of partitions with the property that for every $P \in \mathcal{P}^*$, $\Theta(P) = (1, \ldots, K-1)$. This set contains $(K-1)!$ partitions. In addition, for every $P \in \mathcal{P}^*$, $G(P) - L(P) = G^0 - 1$. Thus, $G^0 - 1 > K - G^0$ if and only if $G^0 > \frac{1}{2}(K+1)$. If $K > 3$, then the latter inequality holds because $G^0 \geq K-1$. $\|$

Proof of Proposition 5. Consider the market for good k. Our assumptions on the demand in this market satisfy the assumptions made in Riley and Samuelson (1981). A straightforward application of their model implies that the optimal mechanism in the market for good k can be implemented by a second price auction with a reserve price of $\frac{1}{2}$.

Now consider any general mechanism Γ for selling all the K goods. Because all bidders draw the value for their preferred good independently from the same distribution, and because the number of buyers who value each good is the same, the expected revenue obtained by Γ must be equal to nK times the expected payment made by a single bidder in that mechanism. This means that by an appropriate choice of a reserve price - perhaps a random one - one can generate $\frac{1}{K}$ of the expected revenue obtained by Γ by using a second-price auction for good k. But this means, by the argument in the previosu paragraph, that the highest expected revenue is obtained by conducting K separate second-price auctions with a reserve-price of $\frac{1}{2}$. $\|$

Proof of Proposition 6. We need to show that for all $q < K$, the $NIRTC(K, K-q, n)$ auction has a SPBE in which bidders use a linear bid function in every round, where the bidding coefficients $(\beta_k)_{k=1}^{K-q}$ are given by

$$\beta_{K-q} = \frac{n(q+1)}{(n-1)K + q + 1} \tag{A14}$$

and the unique solution to the following system of difference equations

$$\frac{N(n-1)}{N_k N_{k+1}} = \beta_k - \frac{N_{k+1}-1}{N_{k+1}}\beta_{k+1} \tag{A15}$$

The proof proceeds in four steps. First, we derive p_k, the probability that a bidder's good is still available in round k. Second, we use the expression for p_{K-q} to derive the bid in the last round. Third, we show that if the $NIRTC(K, K-q, n)$ has a SPBE, then the equilibrium bid functions must satisfy a system of difference equations, the solution of which is given by $(A15)$. Finally, we verify that if all bidders follow $(A15)$, then there is no subgame in which some bidder has an incentive to deviate downward or upward.

STEP 1. Consider some bidder i in round k of the auction. There were $k-1$ winners in previous rounds. For each one of the $k-1$ winners, there are $n-1$ bidders among the remaining $nK-k+1$ bidders whose good was taken away.[22] There are $\binom{nK-k+1}{(n-1)(k-1)}$ possible combinations of selecting from among the $nK-k+1$ bidders in round k, the $k-1$ bidders whose good was taken away. Of these combinations, $\binom{nK-k}{(n-1)(k-1)}$ do not include bidder i. Therefore, the probability that bidder i's good is still present in round k is

$$p_k = \frac{\binom{nK-k}{(n-1)(k-1)}}{\binom{nK-k+1}{(n-1)(k-1)}} = \frac{n(K-k+1)}{nK-k+1} \tag{A16}$$

STEP 2. The subgame that begins with the final round of the auction (round n) is a one-shot second price auction. Therefore, it is a weakly dominating strategy for each bidder to bid his expected value, i.e., his value v multiplied by the probability that his good is still available. Using $(A16)$, we have

$$b_{K-q}(v) = p_{K-q}v = \frac{n(q+1)}{(n-1)K+q+1}v \tag{A17}$$

STEP 3. Suppose there exists a SPBE in $NIRTC(K, K-q, n)$ with the property that for every $k \in \{1, \ldots, K-q\}$, bidders use a continuous increasing bid function $b_k(v)$ that maps $[0,1]$ onto $[0,1]$. Consider the subgame that begins in some round k of the auction. The SPBE bid in round k, $b_k(v)$, is the highest price a bidder is willing to pay, conditional on winning that round. The willingness to pay of the highest valuation bidder in round k must therefore be equal to the difference between the expected value of winning in round k and the expected value of winning in the next round. Hence,

$$p_k v - b_k(v) = p_{k+1}v - b_{k+1}\left[E(Y_1^{(nK-k-1)}|Y_1^{(nK-k-1)} < v\right] \tag{A18}$$

From Step 1 and our assumption that bidders' values are independently drawn from the uniform distribution on $[0,1]$, it follows that equation $(A18)$ can be rewritten as follows:

$$\left[\frac{n(K-k+1)}{nK-k+1}\right]v - b_k(v) = \left[\frac{n(K-k)}{nK-k}\right]v - b_{k+1}\left(\frac{nK-k-1}{nK-k}v\right) \tag{A19}$$

[22] To see why, suppose $K=3$ and $n=2$. Consider the winner in the first round. Suppose this winner takes the good of the winner in the next round. Then the winner in round 2 will necessarily take the good of two remaining bidders. Thus, the winners in the first two rounds have taken away a good desired by two other bidders.

Given Step 2, we can solve for $b_{K-q-1}(v)$ by substituting $(A17)$ for $b_{K-q}(v)$. Proceeding inductively, we can solve for $b_k(v)$ for $k = 1, \ldots, K-q-1$. It is easy to see that for every k, the bidding function has the linear form $b_k(v) = \beta_k v$. Moreover, for every k the coefficient β_k is obtained by solving a linear equation, hence there is a unique solution for each β_k. By substituting $\beta_k v$ for $b_k(v)$ in $(A19)$ and rearranging we obtain equation $(A15)$.

Step 4. There are two cases to consider.

Case 1: "Downward" deviation.

Using the same argument as in Step 1 of the proof of Proposition 1, we consider the highest valuation bidder in round k. Label this bidder by the index h and denote his value by v. Conditional on v being the highest of the values in round k, bidder h has no incentive to bid $\beta_k x < \beta_k v$ if the following inequality holds:

$$p_k v - \beta_k \left(\frac{nK-k}{nK-k+1} \right) v \geq p_{k+1} v - \beta_{k+1} \left(\frac{nK-k-1}{nK-k+1} \right) v \tag{A20}$$

Since a bidder with value 0 cannot pretend to be a lower type, we can divide both sides of (A20) by v and rearrange this inequality as follows:

$$p_k - p_{k+1} \geq \beta_k \left(\frac{nK-k}{nK-k+1} \right) - \beta_{k+1} \left(\frac{nK-k-1}{nK-k+1} \right) \tag{A21}$$

Using $(A16)$, the LHS of $(A21)$ becomes:

$$p_k - p_{k+1} = \frac{n(K-k+1)}{nK-k+1} - \frac{n(K-k)}{nK-k} = \frac{Kn(n-1)}{(nK-k+1)(nK-k)} \tag{A22}$$

Using $(A15)$, we can rewrite the RHS of $(A21)$ as follows:

$$\left(\frac{nK-k}{nK-k+1} \right) \left[\frac{Kn(n-1)}{(nK-k+1)(nK-k)} + \left(\frac{nK-k-1}{nK-k} \right) \beta_{k+1} \right] - \left(\frac{nK-k-1}{nK-k+1} \right) \beta_{k+1} \tag{A23}$$

Rearranging $(A23)$, the RHS of $(A21)$ becomes:

$$\frac{Kn(n-1)}{(nK-k+1)^2} \tag{A24}$$

Clearly, $(A24)$ is strictly smaller than $(A22)$. Hence, $(A21)$ must hold, which proves that downward deviations are not profitable.

Case 2: "Upward" deviation.

The proof for this case is essentialy the same as in Step 2 of the proof of Proposition 1. ‖

Proof of Proposition 7. We begin by introducing a few helpful notations. Let $R^N(K, K-q, n)$ denote the expected revenue generated in the $SPBE$ of the $NIRTC\,(K, K-q, n)$. Similarly, define $R^{GBG}(K, n)$ to be the expected revenue in the $SPBE$ of $GBG(K, n)$. Finally, let β_k^N denote the coefficient of the linear bidding strategies employed in the SPBE of $NIRTC(K, K, n)$.

By Proposition 3,

$$\beta_k^N = \frac{N(n-1)}{N_{k+1}} \sum_{i=k}^{K} \frac{1}{N_i}$$

Therefore,

$$R^N(K,K,n) = \sum_{k=1}^{K} \beta_k^N \left(\frac{N-k}{N+1}\right) = \frac{N(n-1)}{N+1} \sum_{k=1}^{K} \frac{k}{N_k} \tag{A25}$$

Hence, $R^N(K,K,2) > R^{GBG}(K,2)$ if and only if

$$\frac{N(n-1)}{N+1} \sum_{k=1}^{K} \frac{k}{N_k} > \left(\frac{n-1}{n+1}\right) K \tag{A26}$$

Consider first the case of $n = 2$. Then $(A26)$ becomes:

$$\frac{2K}{2K+1} \sum_{k=1}^{K} \frac{k}{2K-k+1} > \frac{K}{3} \tag{A27}$$

The above inequality holds if and only if

$$\sum_{k=1}^{K} \frac{k}{2K-k+1} > \frac{K}{3} + \frac{1}{6} \tag{A28}$$

Lemma A2. Inequality $(A28)$ holds for all $K \geq 2$.

Proof of Lemma A2. We first rewrite the LHS of $(A28)$ as follows:

$$\begin{aligned} \sum_{k=1}^{K} \frac{k}{2K-k+1} &= \sum_{k=1}^{K} \frac{(-2K+k-1)+(2K+1)}{2K-k+1} \\ &= -K + (2K+1) \sum_{k=1}^{K} \frac{1}{2K-k+1} \end{aligned} \tag{A29}$$

Using (2) we obtain that inequality $(A28)$ holds if and only if

$$\sum_{k=1}^{K} \frac{1}{2K-k+1} > \frac{8K+1}{6(2K+1)}$$

The above inequality can be simplified further as follows:

$$\sum_{j=K+1}^{2K} \frac{1}{j} > \frac{2}{3} - \frac{1}{4K+2} \tag{A30}$$

We now proceeds by induction on K. It is easy to verify that $(A30)$ holds for $K = 2$. Assume it holds for some $K > 2$. We wish to show that this inequality also holds for $K+1$. In order to show this it suffices to prove that when K is raised to $K+1$, the net increase to the LHS is greater than the net increase to the RHS. Thus, to prove the inductive step we need to establish that

$$\frac{1}{2K+2} + \frac{1}{2K+1} - \frac{1}{K+1} > \frac{-1}{4K+6} + \frac{1}{4K+2}$$
$$\Updownarrow$$
$$\frac{1}{2(2K+1)} - \frac{1}{2(K+1)} > \frac{-1}{2(2K+3)}$$
$$\Updownarrow$$
$$\frac{1}{2}\left(\frac{1}{2K+1} + \frac{1}{2K+3}\right) > \frac{1}{2K+2} \tag{A31}$$

Because the function $\frac{1}{2K+i}$ is convex for $i = 1, 2, \ldots$, it follows that inequality $(A31)$ must hold. $\|$

The lemma above establishes that $R^N(K, K, 2) > R^{GBG}(K, 2)$ for all $K \geq 2$.

Consider the case of $n > 2$. To show that in this case $R^N(K, K, n) < R^{GBG}(K, n)$ we find an upper bound on $R^N(K, K, n)$, and show that this upper bound is strictly less than $R^{GBG}(K, n)$. Using expression $(A25)$, note that $R^N(K, K, n)$ can be bounded above as follows:

$$\begin{aligned}
\frac{N(n-1)}{N+1}\sum_{k=1}^{K}\frac{k}{N_k} &= \left(\frac{nK}{nK+1}\right)\left[(n-1)\sum_{k=1}^{K-1}\frac{k}{nK-k+1} + \frac{(n-1)K}{(n-1)K+1}\right] \\
&< \left(\frac{nK}{nK+1}\right)\left[\frac{n-1}{(n-1)K+2}\sum_{k=1}^{K-1}k + \frac{(n-1)K}{(n-1)K+1}\right] \\
&= \left(\frac{nK}{nK+1}\right)\left[\left(\frac{n-1}{(n-1)K+2}\right)\left(\frac{K-1}{2}\right)K + \frac{(n-1)K}{(n-1)K+1}\right] \\
&= \frac{n(n-1)K^2}{nK+1}\left(\frac{K-1}{2(n-1)K+4} + \frac{1}{(n-1)K+1}\right) \\
&= \frac{n(n-1)K^2}{nK+1}\left[\frac{(n-1)K^2+nK+3}{(2(n-1)K+4)((n-1)K+1)}\right]
\end{aligned}$$

We wish to show that the following inequality holds for all $K \geq n \geq 3$:

$$\frac{n(n-1)K^2\left[(n-1)K^2+nK+3\right]}{(nK+1)\left[2(n-1)K+4\right]\left[(n-1)K+1\right]} < \left(\frac{n-1}{n+1}\right)K$$

Because $K > 0$ and $n > 2$, necessary and sufficient condition for the above inequality is the following:

$$n(n+1)K\left[(n-1)K^2+nK+3\right] < (nK+1)\left[2(n-1)K+4\right]\left[(n-1)K+1\right] \tag{A32}$$

Rearranging ($A32$), we obtain the following inequality:

$$0 < (n^3 - 4n^2 + 3n)K^3 + (-n^3 + 7n^2 - 10n + 2)K^2 + (-3n^2 + 7n - 6)K + 4 \tag{A33}$$

Because $K \geq 3$ we have that $(n^3 - 4n^2 + 3n)K^3 \geq (n^3 - 4n^2 + 3n)K^2$, and so

$$\begin{aligned} (n^3 - 4n^2 + 3n)K^3 + (-n^3 + 7n^2 - 10n + 2)K^2 &\geq \left(3n^2 - 7n + 2\right) K^2 \\ &\geq \left(9n^2 - 21n + 6\right) K \end{aligned}$$

Hence, inequality ($A33$) holds if

$$\begin{aligned} \left(9n^2 - 21n + 6\right) K + (-3n^2 + 7n - 6)K + 4 &> 0 \\ 2n(3n - 7)K + 4 &> 0 \end{aligned}$$

Because $n \geq 3$, the last inequality holds, implying that ($A33$) holds, which in turn implies that inequality ($A32$) holds. This completes the proof. ‖

Proof of Proposition 8. Let $b_k^q(v)$ denote the symmetric equilibrium bidding function in phase k of $NIRTC(K, K-q, n)$. Because the highest $K-q$ bidders win in $NIRTC(K, K-q, n)$, we have that $R(K, K-q-1, n) - R(K, K-q, n)$ is equal to the following expression:

$$\sum_{k=1}^{K-q-1} \left(\beta_k^{q+1} - \beta_k^q\right) \left(\frac{N_k - 1}{N+1}\right) - \beta_{K-q}^q \left(\frac{N - (K-q)}{N+1}\right) \tag{A34}$$

where β_k^{q+1} and β_k^q are the bidding coefficients of the linear bidding functions $b_k^q(v)$ and $b_k^{q+1}(v)$. We now show that expression ($A34$) is negative.

By (A18), β_{K-q-1}^q and β_{K-q}^q must satisfy the following equation:

$$p_{K-q-1}v - \beta_{K-q-1}^q v = p_{K-q}v - \beta_{K-q}^q \left[\frac{N_{K-q} - 1}{N_{K-q}}\right] v \tag{A35}$$

By ($A17$) it follows that $p_{K-q-1} = \beta_{K-q-1}^{q+1}$ and $\beta_{K-q}^q = p_{K-q}$ Thus, equation ($A35$) can be rewritten as follows:

$$\beta_{K-q-1}^{q+1} - \beta_{K-q-1}^q = \frac{\beta_{K-q}^q}{N_{K-q}} \tag{A36}$$

Hence,

$$\left(\beta_{K-q-1}^{q+1} - \beta_{K-q-1}^q\right)(N_{K-q} - 1) = \left(\frac{N_{K-q} - 1}{N_{K-q}}\right) \beta_{K-q}^q$$

Using (A18) we have that for $k < K - q - 1$,

$$p_k v - \beta_k^q v = p_{k+1}v - \beta_{k+1}^q \left(\frac{N_{k+1} - 1}{N_{k+1}}\right) v \tag{A37}$$

$$p_k v - \beta_k^{q+1} v = p_{k+1}v - \beta_{k+1}^{q+1} \left(\frac{N_{k+1} - 1}{N_{k+1}}\right) v \tag{A38}$$

Thus, subtracting (2) from (2) we obtain

$$\beta_k^{q+1} - \beta_k^q = \left(\beta_{k+1}^{q+1} - \beta_{k+1}^q\right)\left(\frac{N_{k+1} - 1}{N_{k+1}}\right) \tag{A39}$$

Hence, using $(A36)$ we have that for all $k \leq K - q - 1$,

$$\beta_k^{q+1} - \beta_k^q = \left(\frac{\beta_{K-q}^q}{N_{K-q}}\right)\Pi_{l=k+1}^{K-q-1}\left(\frac{N_l - 1}{N_l}\right) \tag{A40}$$

Note that both the $NIRTC(K, K-q, n)$ and the $NIRTC(K, K-q-1, n)$ share the following property for $k \leq K-q-1$: $N_k - 1 = N_{k+1}$. Using this property, equation $(A40)$ becomes:

$$\beta_k^{q+1} - \beta_k^q = \frac{\beta_{K-q}^q}{N_{k+1}} \tag{A41}$$

Multiplying both sides of equation $(A41)$ by $N_k - 1$ and summing from $k = 1$ to $k = K - q - 1$ we obtain

$$\begin{aligned} \sum_{k=1}^{K-q-1}\left(\beta_k^{q+1} - \beta_k^q\right)(N_k - 1) &= \sum_{k=1}^{K-q-1}\left(\frac{N_k - 1}{N_{k+1}}\right)\beta_{K-q}^q \\ &= (K - q - 1)\,\beta_{K-q}^q \\ &< [(n-1)K + q]\,\beta_{K-q}^q \end{aligned} \tag{A42}$$

By dividing both sides of the last inequality in (2) by $N + 1$ we obtain $(A34)$. ‖

Appendix C: Instructions RTC(4,3,2,)

Welcome to this experiment on decision-making! You can make money in this experiment. Read the instructions carefully. There is paper and a pen on your table. You can use these during the experiment. Before the experiment starts, we will hand out a summary of the instructions and there will be one practice period.

THE EXPERIMENT

You will earn points in the experiment by purchasing a good you value in a market. At the end of the experiment your points will be exchanged to dollars. Each 15 points will yield 1dollar. At the beginning of the experiment you will receive a starting capital of 150 points that you will not have to pay back at the end of the experiment. You will also be able to earn more money as the experiment progresses. The experiment consists of 16 periods. Your total earnings in the experiment will be equal to the sum of the starting capital and your earnings in all 16 periods.

Each period you will be allocated to the same group of eight persons and within each period there will be three phases. Your earnings will be determined by your own choices and the choices of the other participants in your group. In each group four fictitious goods will be available for sale in each period: good A, good B, good C and good D.

VALUES OF THE GOODS

Each participant will want to buy only one of the goods in a period: the value for this good to him or her will lie between 0 points and 100 points, and each number between 0 and 100 is equally likely. That is, the value of the good is equally likely to be 25 as it is to be 100 as it is to be 51 etc. The other goods have no value (=0 points) to the participant. Each participant will receive a different value for her or his preferred good (that is, the one good for which she or he has a positive value). The value of the preferred good of the one participant does not depend on the values of the preferred goods of the other participants. The value of your preferred good is therefore (very) likely different from those of others. At the start of a period you will get to know which one is your preferred good and how much you value it. You will not know the values of the preferred goods of other participants, other participants will not know the value of your preferred good. Among the seven other participants in the group there will be one other who also values the same good as you. This means that there is exactly one other person in your group who wants to buy the same good as you do and his or her value is also determined randomly from the interval between 0 and 100.

Which good a participant prefers changes (randomly) from period to period. This implies that the person who prefers the same good as you do changes (randomly) from period to period. Each participant also receives a new value for the preferred good in each period. The value for a preferred good in the one period will not depend on the value for the preferred good in any other period.

SALE OF THE GOODS

Rather than sell the goods one by one, the market you participate in will, in each period, sell 'rights to choose' one by one. If in any period you win one of the rights to choose you will be able to choose which of the goods remaining at that time you want. To be more precise, each period consists of three phases. In each phase a 'right to choose' is sold to the highest bidder. In the first phase all eight bidders in a group will submit a bid for the first right to choose. The highest of these eight bidders wins the first right to choose and chooses the good that she or he prefers. At the end of the first phase, every bidder will be informed whether she or he won the first phase or not. The winner of the first phase and the person that prefers the same good as the winner will no longer participate in the remaining phases of this period and will have to wait until the next period starts.

Then the second phase starts, where the remaining six bidders (whose goods are still unsold) submit a new bid for the second right to choose. At the end of this phase, each bidder is informed whether he or she was the winner. The highest bidder wins and chooses the good that he or she likes. This winner and the one other buyer who wants the same good as the winner will no longer participate in the remaining phase of this period and will have to wait until the start of the next period. In the third and final phase the remaining four bidders submit a new bid for the third right to choose. The highest bidder wins the third right to choose and selects the good that he or she likes.

Notice that only three of the available four goods are sold in a period. Which goods are sold depends on the bids of the participants.

PRICES OF THE GOODS

In each phase, the winner of a good pays a price that depends on the bids of that phase. Each participant submits a 'drop out price': this amount reflects what the participant maximally wants to pay for the right to choose in that phase. This drop out price has to be an integer number between 0 and 100 points. The winner and winning price in any phase is determined as follows: First, the computer raises the price from 0 to 100 points. If the price reaches the 'drop out price' of a participant, this participant drops out and will not win the right to choose in the current phase. This process continues until the level where all but one participant have dropped out. The remaining bidder wins the right to choose and pays a price equal to this level. Notice that in this way the price will be equal to the second highest submitted drop out price.

If two (or more) participants have submitted the same drop out price which happens to be the highest, then one of these bidders will be randomly selected. Only in this case the winner pays a price equal to the own submitted drop out price.

The buyer of a right to choose will automatically receive the preferred good. The profit to the bidder from winning will be equal to her or his value minus the price she or he pays, so profit = (value-price). The only person who is told the price at which a good was sold in a particular phase is the winner of that phase. Participants that do not buy a right in a period receive a profit of 0 in that period.

Notice that the highest bidder in a phase can make a loss if she or he pays a price higher than her or his value for the good. This can only happen if the bidder submits a drop out price higher than her or his value, because in that case the second highest drop out price may also be higher than this value. For example, suppose Bob whose value for good A is 10 submits a drop out price of 15. If 15 is the highest drop out price and the second highest drop out price is 12, Bob wins goods A but incurs a loss of 2.

EXAMPLE

The procedure to sell the goods is now illustrated with an example. THE NUMBERS IN THE EXAMPLE ARE ARBITRARILY CHOSEN.

Assume that the winners of the first two phases have selected goods A and C. Then the four bidders who prefer either good B or good D bid in the third phase for the third and final right to choose. Assume that Bob submits a drop out price of 44, Arthur submits a drop out price of 23, Lisa submits a drop out price of 39, and Susan submits a drop out price of 59. Say that Bob and Arthur value the good B while Susan and Lisa value good D. Then the result will be as follows. The computer raises the price from 0. At a price of 23 Arthur drops, at a price of 39 Lisa drops and at a price of 44 Bob drops. The remaining bidder Susan wins her good and pays a price equal to 44. If Susan happened to value the good at 70 her profit would be 70-44=26.

PROCEDURE TO SUBMIT A BID

In the upper middle part of the screen you see how you can make your decision in a phase. The cursor on the bar reflects the drop out price that you are willing to submit. By pushing the 'right arrow' key on your keyboard, you can increase your drop out price and by pushing the 'left arrow' key you can decrease your drop out price. Alternatively, you can use the mouse to drag the slider to your preferred drop out price. Once you are satisfied with your drop out price, you push the 'CONFIRMATION' button. Then you will be asked whether you are sure. If you answer 'NO' then you get the possibility to reconsider your drop out price. Once you answer 'YES' your decision is final. In the upper left part of the screen you see the good you want listed after 'Type'. You also see the balance of your total earnings listed after 'Earnings'.

QUESTION ABOUT THE PRICE OF A GOOD

Assume that in the second phase your drop out price equals 61, while the drop out prices of the other five remaining bidders equal 23, 35, 47, 49 and 55. What is the price that you will have to pay for your good?

[Answer: 55. The computer raises the price until the level where all but one participant have dropped out. This way the price will be equal to the second highest submitted drop out price, which in the example is 55.]

FINAL PAYOFFS

When a period is over the next one will begin. Here each participant will be assigned a new good to value and that value will be randomly determined. Hence, the person who values your good in this period will probably not be the same one who valued it in the previous period - that person will be determined randomly in each period. The rules for this period will be the same as those before it and the final payoff you receive at the end of the experiment will be equal to the sum of what you have earned in all periods plus the starting capital. There will be a total of 16 periods in the experiment.

END

You have reached the end of the instructions. If you want to read some parts of the instructions again, push the button BACK. When you are ready, push the button READY. When all participants have pushed READY, the experiment will start. When the experiment has started, you will NOT be able to return to these instructions. Before the experiment is started, a summary of the instructions will be handed out and a practice period will be carried out. Your earnings during the practice period will NOT be added to your total earnings.

If you still have questions, please raise your hand!

Table 1
Experimental Design

treatment	number of groups	subjects per group	total
RTC(4,4,2)	8	8	64
RTC(4,3,2)	8	8	64
NIRTC(4,4,2)	8	8	64
NIRTC(4,3,2)	8	8	64
GBG(4,2)	8	8	64
OGBG(4,2)	8	8	64
Total			384

Table 2
Equilibrium Bid Coefficients and Expected Revenues

auction format	phase 1	phase 2	phase 3	phase 4	expected revenue *theory*	expected revenue *values*
RTC(4,4,2)	4/7	3/5	2/3	1	133.3	136.2
RTC(4,3,2)	5/7	4/5	1	NA	152.1	157.7
NIRTC(4,4,2)	1599/2205	214/315	44/75	2/5	152.1	155.2
NIRTC(4,3,2)	115/147	47/63	2/3	NA	147.6	149.4
GBG(4,2)	1	NA	NA	NA	133.3	140.1
OGBG(4,2)	1	NA	NA	NA	166.7	174.0

Table 3
Revenues

	phase 1	Phase 2	phase 3	phase 4	total
RTC(4,4,2) actual	72.1 (14.4)	61.2 (15.8)	47.9 (18.2)	22.6 (15.1)	203.7 (51.1)
Nash	45.2 (7.3)	39.2 (9.6)	32.6 (12.4)	19.1 (13.3)	136.2 (33.7)
RTC(4,3,2) actual	74.2 (14.3)	64.9 (15.9)	49.7 (18.8)	NA	188.7 (42.4)
Nash	56.4 (9.4)	52.5 (12.7)	48.7 (18.9)	NA	157.7 (35.5)
GBG(4,2) actual	NA	NA	NA	NA	145.1 (55.2)
Nash	NA	NA	NA	NA	140.1 (52.9)
OGBG(4,2) actual	NA	NA	NA	NA	178.8 (52.4)
Nash	NA	NA	NA	NA	174.0 (55.9)

Note: standard deviations are listed in parentheses.

Table 4a
Bidding Functions RTC(4,4,2)

	periods 1-16			periods 9-16		
phase 1	**coeff (s.e.)**	**Z**	**P>|Z|**	**coeff (s.e.)**	**z**	**P>|Z|**
value	0.86 (0.03)	31.78	0.00	0.92 (0.04)	21.35	0.00
value R(71)[65]	7.71 (2.21)	3.49	0.00	5.48 (1.74)	3.14	0.00
$(\text{value})^2$ R(71)[65]	-0.05 (0.01)	-3.64	0.00	-0.04 (0.01)	-3.39	0.00
constant R(71)[65]	-315.38 (93.06)	-3.39	0.00	-212.77 (71.01)	-3.00	0.00
constant	2.16 (1.64)	1.32	0.19	0.93 (2.23)	0.42	0.68
	n=1018, R^2=0.65			n=509, R^2=0.60		
phase 2	**coeff (s.e.)**	**Z**	**P>|Z|**	**coeff (s.e.)**	**z**	**P>|Z|**
value	0.89 (0.24)	37.69	0.00	0.92 (0.04)	26.18	0.00
value R(71)[65]	7.85 (2.39)	3.29	0.00	4.58 (1.68)	2.72	0.01
$(\text{value})^2$ R(71)[65]	-0.05 (0.01)	-3.37	0.00	-0.28 (0.01)	-2.72	0.01
constant R(71)[65]	-322.35 (99.76)	-3.23	0.00	-187.49 (67.66)	-2.77	0.01
constant	1.86 (1.21)	1.54	0.12	1.60 (1.61)	0.99	0.32
	n=762, R^2=0.82			n=381, R^2=0.77		
phase 3	**coeff (s.e.)**	**Z**	**P>|Z|**	**coeff (s.e.)**	**z**	**P>|Z|**
value	0.92 (0.02)	47.95	0.00	0.94 (0.03)	34.39	0.00
value R(71)[71]	7.73 (2.37)	3.26	0.00	11.97 (3.13)	3.83	0.00
$(\text{value})^2$ R(71)[71]	-0.05 (0.01)	-3.30	0.00	-0.07 (0.02)	-3.92	0.00
constant R(71)[71]	-319.82 (98.46)	-3.25	0.00	-494.38 (131.8)	-3.75	0.00
constant	2.58 (0.95)	2.71	0.01	1.63 (1.09)	1.49	0.13
	n=508, R^2=0.90			n=255, R^2=0.90		
phase 4	**coeff (s.e.)**	**Z**	**P>|Z|**	**coeff (s.e.)**	**z**	**P>|Z|**
value	0.93 (0.02)	38.50	0.00	0.99 (0.03)	38.65	0.00
constant	3.40 (0.98)	2.41	0.02	1.42 (1.06)	1.34	0.18
	n=253, R^2=0.85			n=128, R^2=0.79		

Notes: R(.) indicates the break in periods 1-16; [.] indicates the break in periods 9-16.

Table 4b
Bidding Functions RTC(4,3,2)

	periods 1-16			periods 9-16		
phase 1	**coeff (s.e.)**	**z**	**P>|Z|**	**coeff (s.e.)**	**z**	**P>|Z|**
value	0.95 (0.01)	63.20	0.00	0.94 (0.02)	50.60	0.00
value R(81)[85]	7.35 (4.68)	1.57	0.12	25.44 (10.25)	2.48	0.01
$(\text{value})^2$ R(81)[85]	-0.05 (0.03)	-1.58	0.12	-0.14 (0.06)	-2.46	0.01
constant R(81)[85]	-336.94 (211.51)	-1.59	0.11	-1185 (471.46)	-2.51	0.01
constant	0.08 (1.04)	0.77	0.44	1.74 (1.20)	1.44	0.15
	n=1024, R^2=0.83			n=512, R^2=0.85		
phase 2	**coeff (s.e.)**	**z**	**P>|Z|**	**coeff (s.e.)**	**z**	**P>|Z|**
value	1.00 (0.02)	43.03	0.00	0.98 (0.02)	59.41	0.00
value R(54)[85]	1.85 (0.41)	4.53	0.00	16.33 (11.35)	1.44	0.15
$(\text{value})^2$ R(54)[85]	-0.01 (0.00)	-5.17	0.00	-0.09 (0.06)	-1.48	0.14
constant R(54)[85]	-63.24 (15.25)	-4.15	0.00	-735.26 (520.91)	-1.41	0.16
constant	0.13 (0.90)	0.15	0.88	0.31 (0.99)	0.31	0.76
	n=768, R^2=0.90			n=384, R^2=0.90		
phase 3	**coeff (s.e.)**	**z**	**P>|Z|**	**coeff (s.e.)**	**z**	**P>|Z|**
value	0.97 (0.01)	100.25	0.00	1.02 (0.02)	46.46	0.00
value R[59]	NA	NA	NA	1.39 (0.72)	1.92	0.06
$(\text{value})^2$ R[59]	NA	NA	NA	-0.01 (0.00)	-1.81	0.07
constant R[59]	NA	NA	NA	-58.66 (27.63)	-2.12	0.04
constant	1.57 (0.64)	2.46	0.01	0.52 (0.74)	0.07	0.48
	n=506, R^2=0.94			n=255, R^2=0.96		

Notes: R(.) indicates the break in periods 1-16; [.] indicates the break in periods 9-16.

Table 5
Total Revenues

	RTC(4,4,2)	RTC(4,3,2)	NIRTC(4,4,2)	NIRTC(4,3,2)
risk neutral	136.2	157.7	155.2	149.4
CARA, r=0.07	162.1	164.0	122.9	117.4
actual	203.7	188.7	196.7	187.1

Table 6
Efficiency

	ordinal efficiency		cardinal efficiency	
	predicted	actual	predicted	actual
RTC(4,4,2)	100.0%	92.6%	100.0%	98.2%
RTC(4,3,2)	75.0%	69.7%	84.9%	83.4%
GBG(4,2)	100.0%	92.4%	100.0%	98.3%
OGBG(4,2)	78.9%	73.8%	87.7%	87.9%

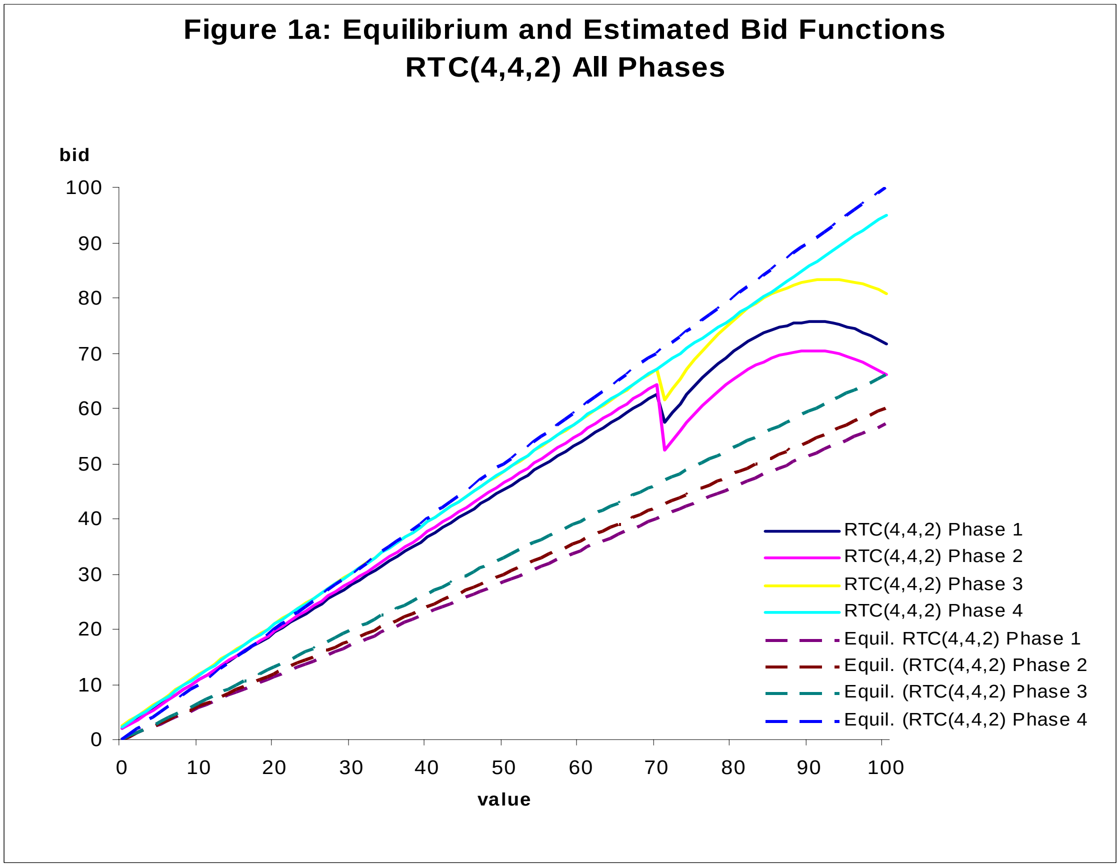


Figure 1a: Equilibrium and Estimated Bid Functions
RTC(4,4,2) All Phases
bid
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
value
RTC(4,4,2) Phase 1
RTC(4,4,2) Phase 2
RTC(4,4,2) Phase 3
RTC(4,4,2) Phase 4
Equil. RTC(4,4,2) Phase 1
Equil. (RTC(4,4,2) Phase 2
Equil. (RTC(4,4,2) Phase 3
Equil. (RTC(4,4,2) Phase 4

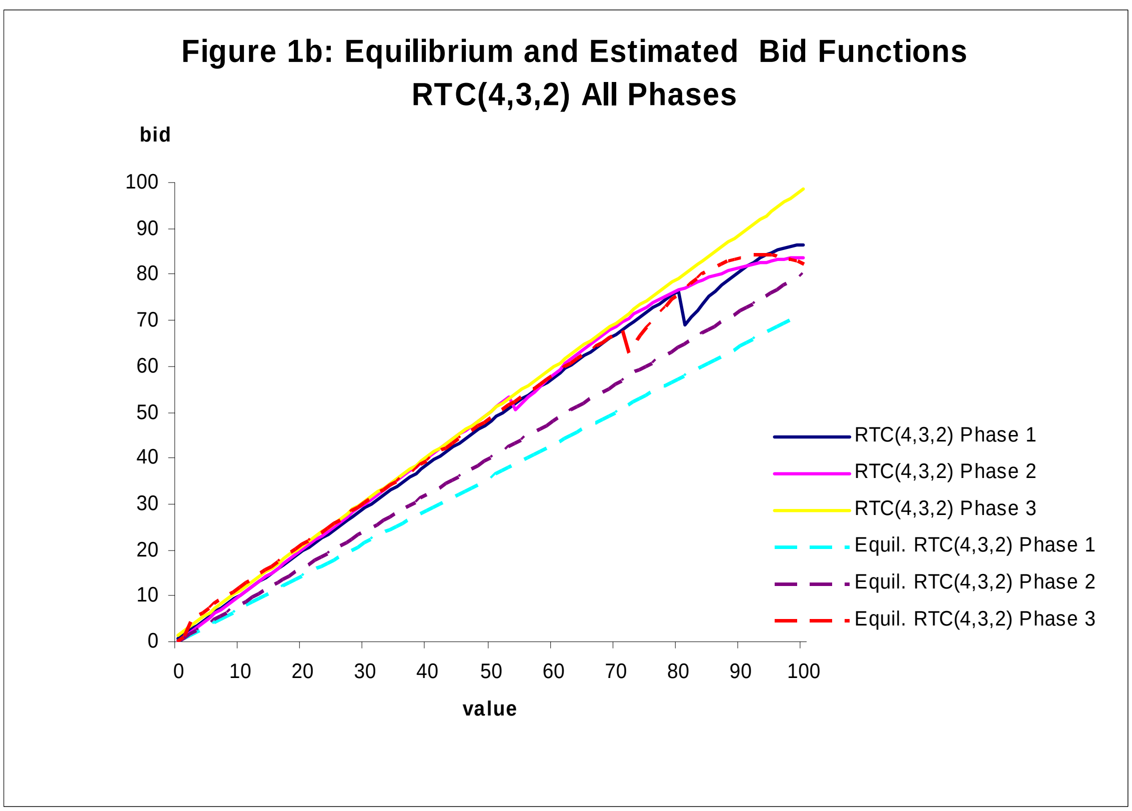


Figure 1b: Equilibrium and Estimated Bid Functions
RTC(4,3,2) All Phases
bid
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
value
RTC(4,3,2) Phase 1
RTC(4,3,2) Phase 2
RTC(4,3,2) Phase 3
Equil. RTC(4,3,2) Phase 1
Equil. RTC(4,3,2) Phase 2
Equil. RTC(4,3,2) Phase 3

Figure 2

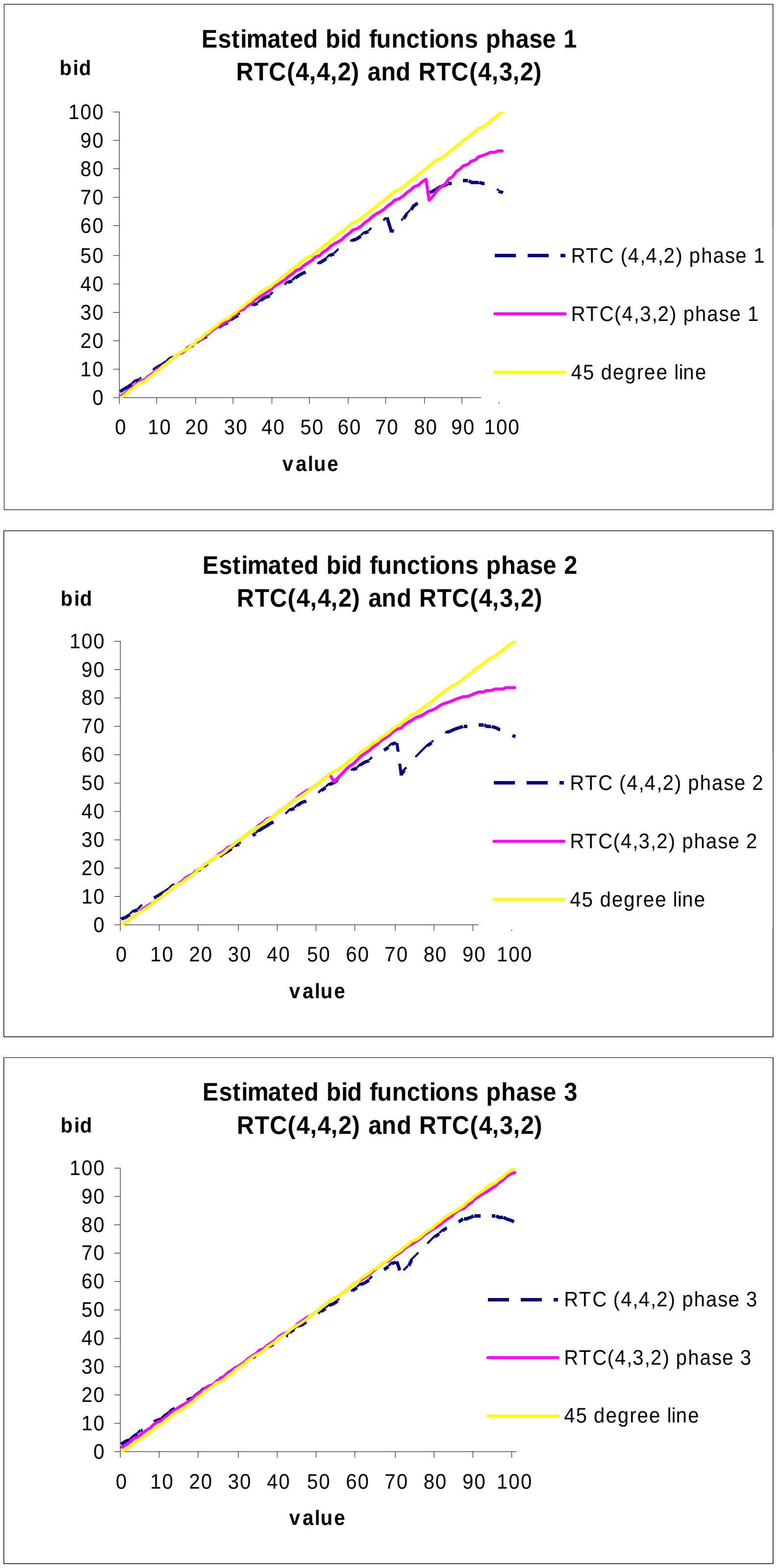

Estimated bid functions phase 1
RTC(4,4,2) and RTC(4,3,2)
bid
value
RTC (4,4,2) phase 1
RTC(4,3,2) phase 1
45 degree line
Estimated bid functions phase 2
RTC(4,4,2) and RTC(4,3,2)
bid
value
RTC (4,4,2) phase 2
RTC(4,3,2) phase 2
45 degree line
Estimated bid functions phase 3
RTC(4,4,2) and RTC(4,3,2)
bid
value
RTC (4,4,2) phase 3
RTC(4,3,2) phase 3
45 degree line

Figure 3

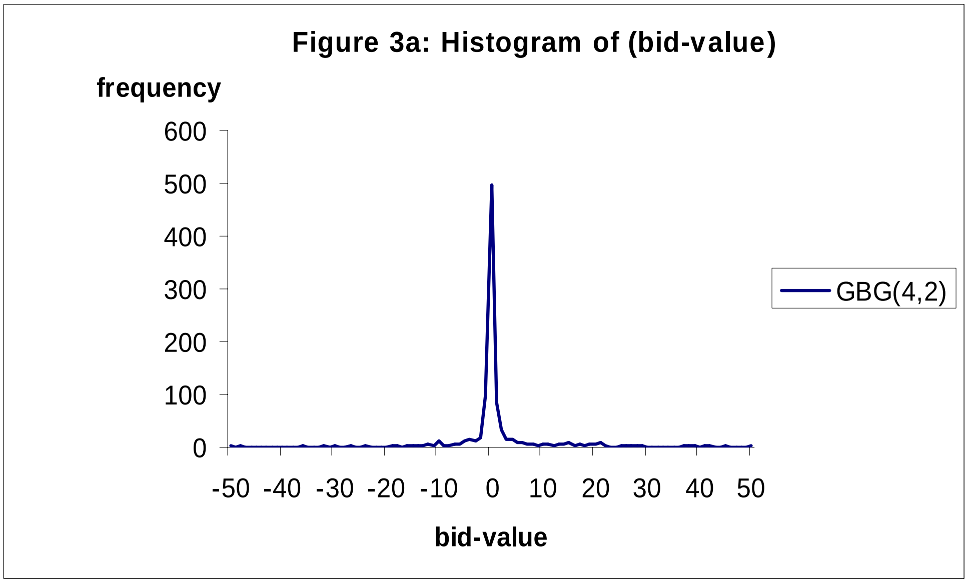


Figure 3a: Histogram of (bid-value)
frequency
600
500
400
300
200
100
0
-50 -40 -30 -20 -10 0 10 20 30 40 50
bid-value
GBG(4,2)

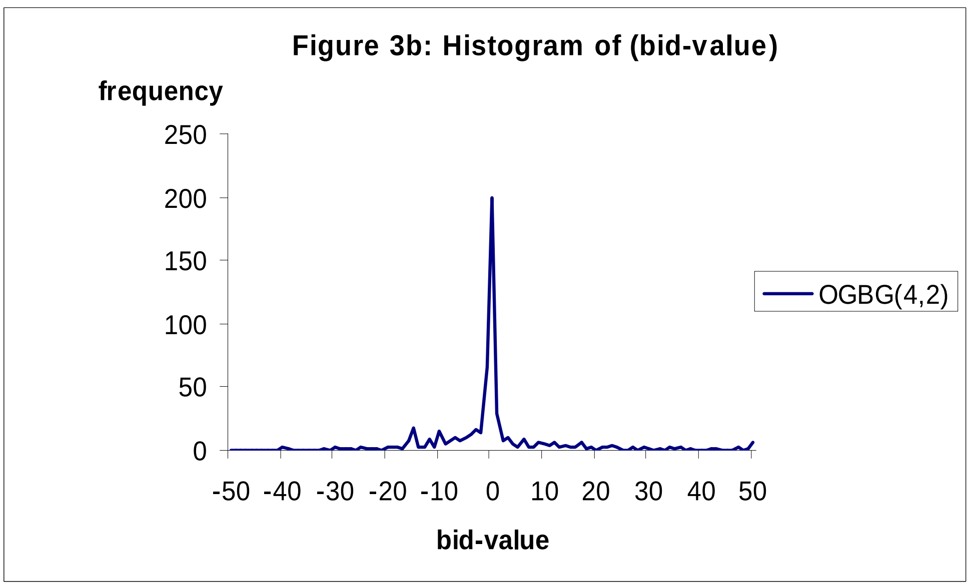


Figure 3b: Histogram of (bid-value)
frequency
250
200
150
100
50
0
-50 -40 -30 -20 -10 0 10 20 30 40 50
bid-value
OGBG(4,2)

Figure 4a

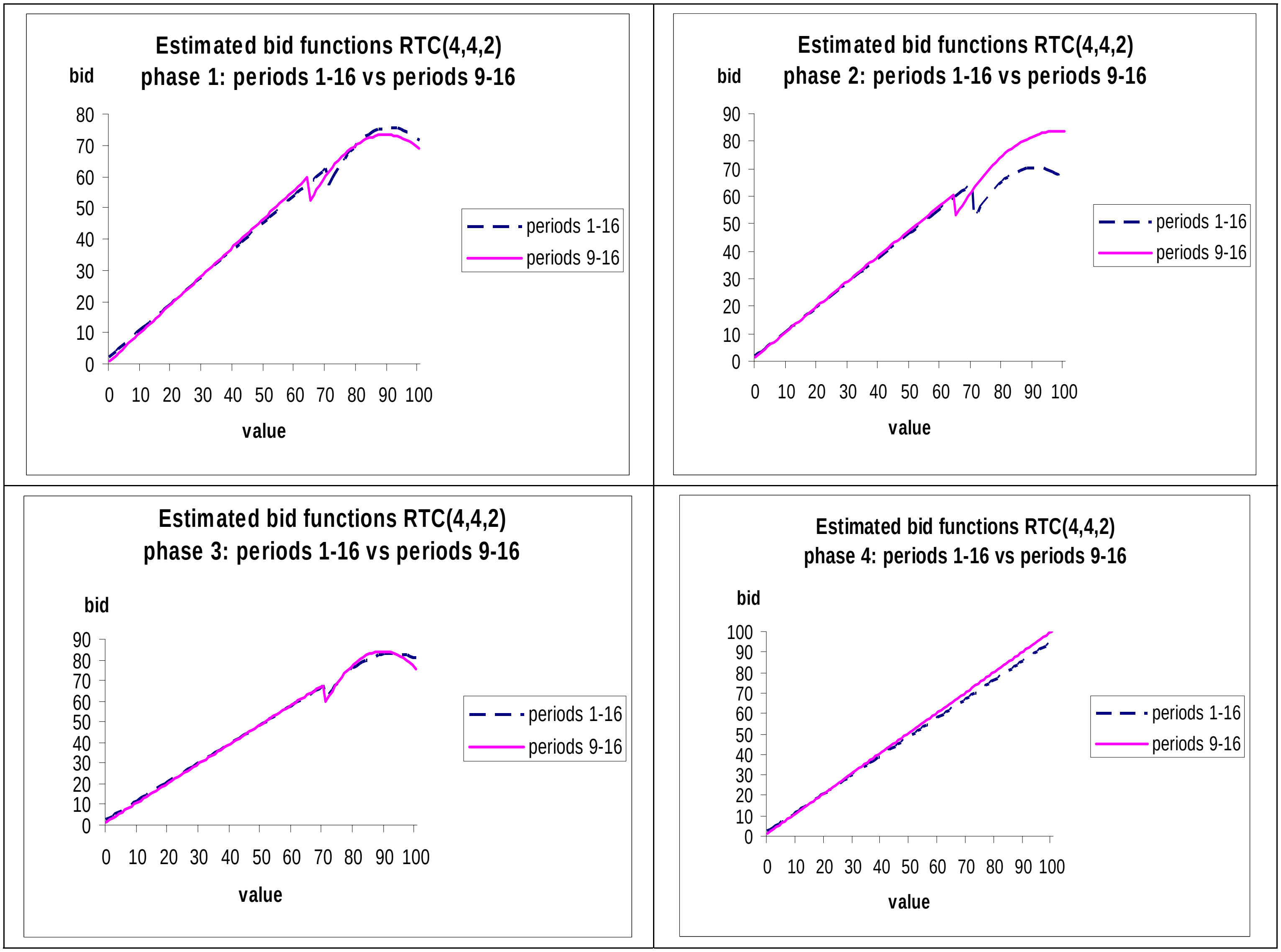
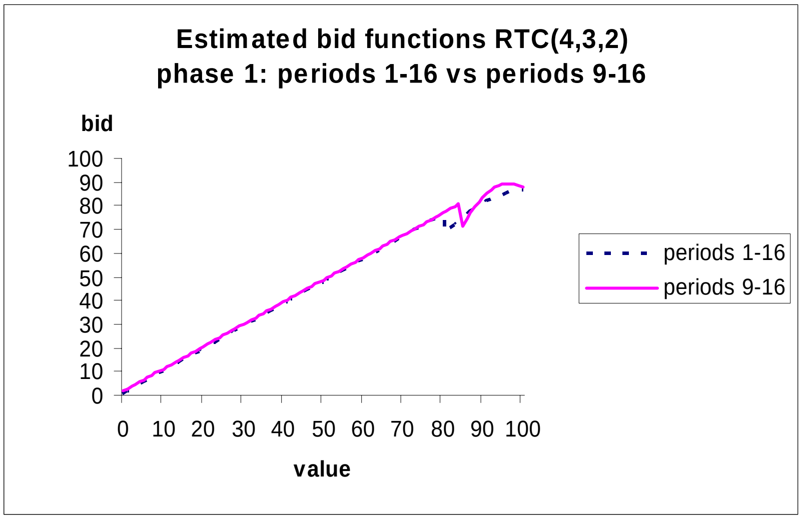
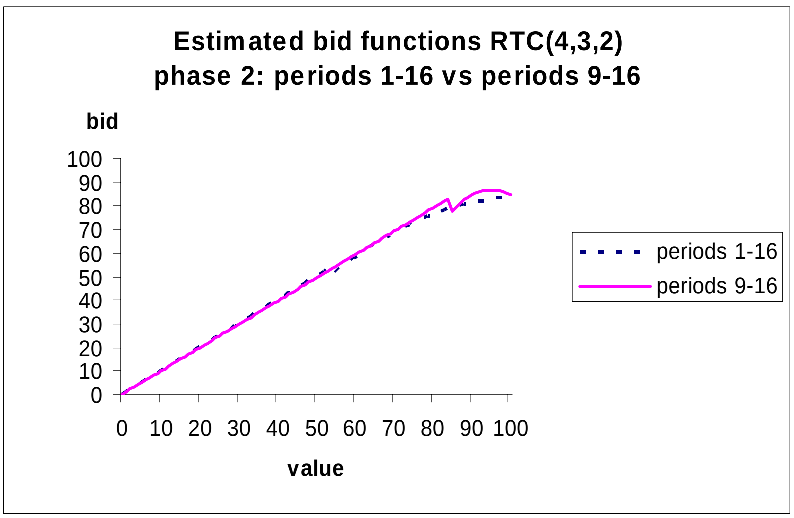
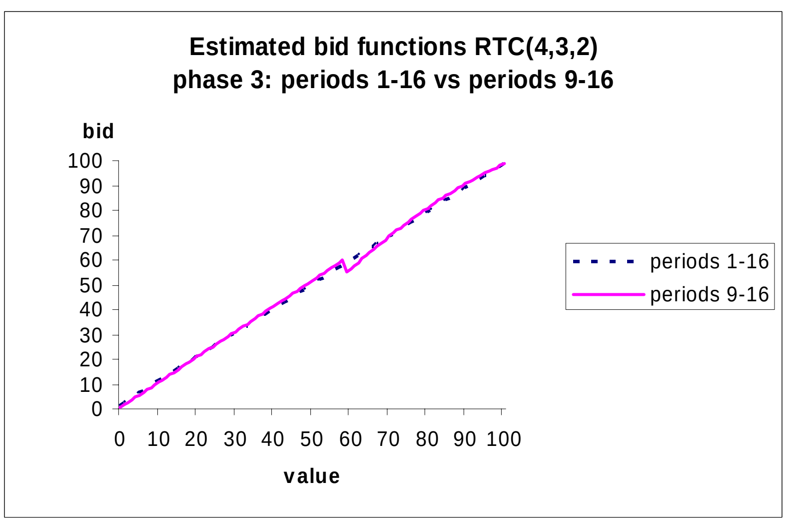

Estimated bid functions RTC(4,4,2)
phase 1: periods 1-16 vs periods 9-16
bid
value
periods 1-16
periods 9-16
Estimated bid functions RTC(4,4,2)
phase 2: periods 1-16 vs periods 9-16
bid
value
periods 1-16
periods 9-16
Estimated bid functions RTC(4,4,2)
phase 3: periods 1-16 vs periods 9-16
bid
value
periods 1-16
periods 9-16
Estimated bid functions RTC(4,4,2)
phase 4: periods 1-16 vs periods 9-16
bid
value
periods 1-16
periods 9-16

Figure 4b


Estimated bid functions RTC(4,3,2)
phase 1: periods 1-16 vs periods 9-16
bid
100
90
80
70
60
50
40
30
20
10
0
0 10 20 30 40 50 60 70 80 90 100
value
periods 1-16
periods 9-16


Estimated bid functions RTC(4,3,2)
phase 2: periods 1-16 vs periods 9-16
bid
100
90
80
70
60
50
40
30
20
10
0
0 10 20 30 40 50 60 70 80 90 100
value
periods 1-16
periods 9-16


Estimated bid functions RTC(4,3,2)
phase 3: periods 1-16 vs periods 9-16
bid
100
90
80
70
60
50
40
30
20
10
0
0 10 20 30 40 50 60 70 80 90 100
value
periods 1-16
periods 9-16