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INCUMBENTS? LEARNING VERSUS
RE-ELECTION CONCERNS**

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ABSTRACT

Do Elections Always Motivate Incumbents? Learning versus Re-election Concerns*

This Paper studies a principal-agent model of the relationship between officeholder and an electorate, where everyone is initially uninformed about the officeholder's ability. If office-holder effort and ability interact in the determination of performance in office, then an office-holder has an incentive to learn, i.e., raise effort so that performance becomes a more accurate signal of their ability. Elections reduce the learning effect, and the reduction in this effect may more than offset the positive 're-election concerns' effect of elections on effort, implying higher effort with appointment. When this occurs, appointment of officials may welfare-dominate elections.

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1. Introduction

In recent years, economists and political scientists have applied principal-agent theory to study the relationship between voters and elected officials. An early and important contribution is by Ferejohn (1986), who assumed a pure moral hazard (hidden action) problem between voters and the incumbent: the incumbent can improve the outcome for the voters by exerting higher effort, but such effort is costly and unobservable, or at least non-contractible. He also supposed that the voters could commit to a retrospective voting strategy of voting the incumbent out if his performance was below some cutoff level. In this setting, the incumbent has a static incentive to minimize effort, but a dynamic incentive to provide effort in order to get re-elected, implying a maximum incentive-compatible level of effort. If voters can coordinate, they can set the cutoff to induce the incumbent to provide this maximum. So, in Ferejohn's model, electoral discipline clearly motivates the incumbent.

Ferejohn's classic paper has stimulated an extensive literature. For example, Persson, Roland and Tabellini (1997) build on this basic model, in combination with the legislative bargaining model of Baron and Ferejohn (1989) in their influential analysis of presidential and parliamentary regimes (see also Persson and Tabellini (2000)). Many other applications, both theoretical and empirical, of the basic Ferejohn model can be found in more recent literature. To take only two examples, Aidt and Magris (2003) show how the dynamic incentives provided by retrospective voting can partially solve the well-known "capital levy" problem, and Besley and Burgess (2002) use a version of the model to generate some testable predictions about the determinants of government responsiveness to falls in food production and crop flood damage in India.

The theme of this literature is that elections play a positive role in mitigating the moral hazard problem between incumbents and voters, by inducing them to supply more effort, or to divert less rent to their own pocket, than they would in the absence of elections. The purpose of our paper is to demonstrate that in an environment with both moral hazard *and* symmetric but incomplete information about the ability of the incumbent¹, elections do *not* always have this motivating feature. Moreover, our paper is the first²,

¹That is, initially, both (potential) incumbents and voters are uncertain about the ability of potential incumbents.

²The only possible exception (to our knowledge) is the career concerns model of Chapter 4.5 of Persson and Tabellini (2000). However, in that model, there is no noise in the function mapping ability and effort to performance, so that incumbents can perfectly observe their ability from performance at the end of the first period of office, and consequently, there is no learning effect.

to our knowledge, to explore the implications of this information structure in a model of interaction between voters and incumbents.

Our setting is very simple. A committee (or electorate) has to select a representative to undertake a binary project in each of two periods. All members of the committee care equally about the outcome, preferring success to failure. The probability of success depends on the effort exerted by the incumbent representative, times an ability parameter. As argued below, the effort variable can also be interpreted as a decision of how much rent to divert from a budget which funds the project. Initially, all agents have the same prior beliefs about their own ability and that of others. Effort is costly (and unobservable by all other members of the committee), but the incumbent is rewarded by either some material benefit from office, or some ego-rent. When there is no uncertainty about ability, our model is simply a special case of Ferejohn's.

In this setting, we consider two institutional arrangements. The first, appointment, does not allow for any replacement of the initial incumbent. The second, election, allows selection of a challenger to contest an election with the incumbent at the beginning of the second period. All members of the committee vote, and the winner takes office in the second period.

We show that in equilibrium, elections may *demotivate*: that is, with elections, in the first period, the incumbent may supply *less* effort than with appointment. The intuition is the following. When ability and effort of the office-holder interact positively, the office-holder can learn more about his ability by supplying more effort in the first period, and moreover, this information will be valuable in the second period. We call this the *learning* incentive for supplying effort. However, if he is exposed to the possible future loss of office, his motive to learn will be lessened. This diminution in the learning motive may more than offset the increase in effort induced by the desire to win the election (the *re-election concerns* incentive). In this event, the agent will supply less effort than he would were he simply permanently appointed to the job³.

We also study the welfare properties of the two institutional forms. First, we show that if effort is higher with elections, then voter⁴ utility will be higher. This is because relative to appointment, elections have both incentive and selection effects, in the terminology of Besley and Smart, (2003). The selection effect allows the replacement of an incompetent

³One way of interpreting the diminution is as *short-termism*; the incumbent underinvests, in information acquisition, anticipating he will lose power (see also Besley and Coate, 1998, for examples of this type).

⁴We focus on the utility of the members of the committee other than the incumbent and challenger. Analysis of the welfare levels of the latter is more complex.

candidate (as revealed by a failed project) and thus always increases voter utility. So, if the incentive effect of elections on effort is also higher, then voters will gain overall from elections.

So, a *necessary* condition for appointment to yield higher voter utility is that effort is higher under appointment than under elections in order to offset the selection effect. We also show via numerical example that effort can be sufficiently higher under appointment to make voter utility higher with appointment than with elections.

In the wider literature on incentive effects of elections, this kind of finding is not new. For example, in the model of Rogoff (1990), where there is an adverse selection problem between voters and politicians (i.e., politicians know their competency but voters do not) then it may be better to abolish elections. The intuition there is that elections induce distortive signalling in fiscal policy, which must be weighed against a positive selection effect. However, our welfare result is new, as far as we know, in the class of models that study a pure moral hazard problem between voters and politicians.

The rest of the paper is structured as follows. Section 2 describes the model and Section 3 presents the basic results on effort levels. Section 4 is devoted to normative analysis. Section 5 discusses extensions. Finally, Section 6 concludes and discusses related literature.

2. The Model

2.1. The Set-Up

There are two periods $t = 1, 2$, and a set of agents is $N = \{1, \dots, n\}$, with $n \geq 3$. There is an office or post to which one of the n agents can be appointed or elected in either period. The responsibility of the office-holder (incumbent) at period t is to implement a discrete project. The outcome is x_t , where $x_t = 0, 1$ denotes failure and success respectively. If the incumbent is $i \in N$, the probability of success in either time period is $p_t = \theta^i e_t$, where $e_t \in [0, 1]$ denotes effort, and θ^i measures ability, with $\theta^i \in \{\theta_l, \theta_h\}$, $0 \leq \theta_l \leq \theta_h \leq 1$. Initially, all agents believe that $\theta^1, \dots, \theta^n$ are independent draws from the same distribution, where $\Pr(\theta^i = \theta_h) = \pi$.

Every agent values a successful project at 1, and an unsuccessful one at 0. If no project is implemented, all agents, including the incumbent, get zero. The cost of effort for the incumbent is $c(e)$, with c increasing, strictly convex and⁵ $c'(0) = c(0) = 0$. The incumbent is motivated to hold office, in spite of the cost of effort, by a rent $R > 0$ from office. This

⁵This condition ensures that the constraint $e \geq 0$ is never binding in equilibrium.

may be psychological (e.g., an “ego rent” as in Rogoff, 1990), or capture some material benefit from office. [For example, in universities, heads of departments are often rewarded by lower teaching loads!]

We consider two possible institutional forms in this paper:

Appointment - At the beginning of $t = 1$, an agent i is drawn at random from N , and instructed to implement the project in both periods.

Election - an agent i is drawn at random from N , and instructed to implement the project in period 1. At the beginning of period 2, an agent j is randomly selected from $N/\{i\}$. Incumbent i and challenger j then simultaneously decide whether or not to stand for election. All agents then vote on i vs. j , (if both stand) or one candidate versus the status quo (if one stands). The winner is determined by majority vote. Otherwise, the status quo is implemented. All agents vote having observed the outcome of the project at $t = 1$. If at least one candidate stands, the winner is instructed to implement the project in period 2.

Note that the only difference between the two institutional forms⁶ is that appointment does not allow any mechanism for replacement of the incumbent, whereas election does. So, one way of thinking of appointment is that it involves a precommitment not to replace the appointee.

2.2. Discussion

Some comments are in order here. First, it should be noted that the effort decision can also be interpreted as a decision of how much rent to divert from a budget which finances the project. Interpret e as the amount of money (or more generally, some purchased input) actually spent on the project by the incumbent. Also, suppose that the available budget for the project is normalized to unity. So, the rent diverted by the incumbent is $r = 1 - e$. Then $c(1 - r) = u(r)$ can be interpreted as the utility of rent for the incumbent. The assumptions on c imply that u is increasing and concave.

Second, this model nests the pure moral hazard model of the Ferejohn (1986) type as a special case. To see this, set $\theta_h = \theta_l = 1$ so that there is nothing to be learnt about ability. Then, with appointment, the incumbent sets effort level in both periods to equate the incremental probability of success from higher effort, 1, equal to the cost, i.e., $c'(e_A) = 1$.

⁶The “election” institutional form is not a full description of what happens with elections, as it does not fully model candidate entry. However, we do ensure that the incumbent and challenger both have the option of not standing for election, and so a basic individual rationality constraint, that no-one can be forced to stand, is respected. The case of fully endogenous candidate entry is discussed in Section 4.

With elections, there is an equilibrium where the incumbent makes a higher effort than e_A in the first period (say e_E), enforced by the threat of losing office if the project is a failure. In this equilibrium, the incremental cost of effort is equated to one plus the net ego-rent from retaining office next period, i.e., $c'(e_E) = 1 + \delta(R - c(e_2))$, where e_2 is second-period effort and is thus equal to e_A . Moreover, $R - c(e_A) > 0$: otherwise, the incumbent would not want to take office, even if he wins the election. So, it follows that $e_E > e_A$. This is not, however, the only equilibrium, as at the time of election, voters are indifferent between the incumbent and challenger⁷. But, there cannot be an equilibrium where the incumbent puts in lower effort with election than with appointment. In what follows, we abstract from this special case by assuming $\theta_h > \theta_l$, thus creating a learning motive.

Finally, a key feature of the model is that the incumbent cares about the outcome of the project - in fact, he cares as much as the rest of the electorate, although this is not essential. If the incumbent did not care at all, he would anticipate supplying zero effort in the second period under either institutional form, and so have no incentive to learn about his ability in the first period. Thus, the model is in the “citizen-candidate” tradition of Besley and Coate (1997) and Osborne and Slivinski (1996) rather than the type of model where the incumbent is solely motivated by ego-rent or monetary gain.

3. Appointment

3.1. The Second Period

Let π_2 be the belief on the part of the incumbent that he is high-ability at the end of $t = 1$, having observed the outcome of the project in the first period. He chooses e to maximize

$$[\pi_2\theta_h + (1 - \pi_2)\theta_l]e + R - c(e)$$

So, e solves

$$\pi_2\theta_h + (1 - \pi_2)\theta_l = c'(e) \tag{3.1}$$

Let this value of e be $e(\pi_2)$. From (3.1), we see that $e'(\pi_2) = (\theta_h - \theta_l)/c'' > 0$. Also define

$$v(\pi_2) = [\pi_2\theta_h + (1 - \pi_2)\theta_l]e(\pi_2) - c(e(\pi_2)) \tag{3.2}$$

⁷For example, there is also an equilibrium where the voters always re-elect the incumbent, and so he puts in effort e_A in the first period, as well as the second. As we will see, this problem of multiple equilibria does not arise in the model with learning.

to be second-period utility of the incumbent, excluding any ego-rent. Note from (3.2) that v is strictly increasing and convex in π_2 ;

$$v'(\pi_2) = (\theta_h - \theta_l)e(\pi_2) > 0, \quad v''(\pi_2) = (\theta_h - \theta_l)e'(\pi_2) > 0$$

3.2. The First Period

Generally, the incumbent's posterior belief that he is able, π_2 , will depend on (i) the success ($x = 1$) or failure ($x = 0$) of the project at $t = 1$; (ii) the effort made e_1 at $t = 1$. Indeed, by Bayes' rule, we can calculate:

$$\pi_2(1, e_1) = \frac{\pi\theta_h e_1}{\pi\theta_h e_1 + (1 - \pi)\theta_l e_1}, \quad \pi_2(0, e_1) = \frac{\pi(1 - \theta_h e_1)}{\pi(1 - \theta_h e_1) + (1 - \pi)(1 - \theta_l e_1)} \quad (3.3)$$

where π is the prior belief that ability is high. Note that $\pi_2(1, e_1)$ is in fact independent of e_1 , and so we write $\pi_2(1, e_1) \equiv \pi_2(1)$. This is because an increase in e_1 will not change the probability that the project is a success, conditional on high ability, *relative to* the probability that the project is a success, conditional on low ability, as this relative probability is simply $\theta_h e_1 / \theta_l e_1 = \theta_h / \theta_l$. On the other hand, $\pi_2(0, e_1)$ is decreasing⁸ in e_1 . So, conditional on project success and failure respectively, second-period payoffs are $v(\pi_2(1))$, $v(\pi_2(0, e_1))$.

Moreover, $\bar{\theta}e_1$, with $\bar{\theta} = \pi\theta_h + (1 - \pi)\theta_l$, is the probability that the project is a success in period 1, given the information available at the beginning of period 1, i.e., before the outcome of the first-period project is observed. Then from the point of view of the beginning of period 1, the expected second-period payoff to the incumbent (excluding ego-rent) can be written

$$V(e_1) = v(\pi_2(1))\bar{\theta}e_1 + v(\pi_2(0, e_1))(1 - \bar{\theta}e_1) \quad (3.4)$$

Given our definition of $V(e_1)$, the expected discounted sum of payoffs to the appointee in period 1 is

$$\bar{\theta}e_1 + R - c(e_1) + \delta(V(e_1) + R) \quad (3.5)$$

The optimal choice of e_1 , denoted e_1^A , maximizes (3.5) subject to $e_1 \in [0, 1]$. Note from (3.3),(3.4) that V is differentiable. Assuming an interior solution, i.e., $e_1^A < 1$, the first-order condition for e_1 is:

$$\bar{\theta} - c'(e_1) + \delta V'(e_1) = 0 \quad (3.6)$$

⁸See equation (A.3) in the Appendix. This is because the probability that the project is a failure, conditional on high ability, *relative to* the probability that the project is a failure, conditional on low ability, namely $(1 - \theta_h e_1) / (1 - \theta_l e_1)$, is decreasing in e_1 .

The first term $\bar{\theta} - c'(e_1)$ on the left-hand side is the first-period (myopic) gain from a small increase in effort. The second term $\delta V'(e_1)$ is the *dynamic effort incentive*.

What is the sign and magnitude of the dynamic effort incentive? An increase in e_1 will have two effects on V . First, it will increase the probability of project success, raising V , and second, it will decrease the second-period posterior belief by the incumbent that he is high-ability, lowering V . It can be shown that the first effect dominates: formally, $V'(e_1) > 0$.

The intuition for this result is that an increase in e_1 will *increase the information contained in the observation of the project outcome* x_1 , and thus increase second-period expected utility. To see this, note that an increase in e_1 will increase the “spread” or difference between the posteriors, i.e., $\pi_1(1) - \pi_2(0, e_1)$, as $\pi_2(0, e_1)$ is decreasing in e_1 , so that the project outcome becomes a more accurate signal of her ability. For this reason, we will call the $V'(e_1)$ the *learning incentive*.

The result that V is increasing in e_1 can easily be shown diagrammatically, as shown in Figure 1 below.

Figure 1 in here

In Figure 1, for a fixed value of e_1 , $V(e_1)$ is the convex combination of points $v(\pi_2(1))$, $v(\pi_2(0, e_1))$ on the graph of the function $v(\pi_2)$, with weights $\bar{\theta}e_1$, $1 - \bar{\theta}e_1$. Note that weighted sum of the posteriors, $\pi_2(1)\bar{\theta}e_1 + \pi_2(0, e_1)(1 - \bar{\theta}e_1)$, is equal to π , the incumbent’s own prior belief that he is high-ability: this fact follows directly from Bayes’ rule and can be verified by direct calculation. So, by construction, $V(e_1)$ is the value generated by the prior π , given a mapping from π_2 to V described by the line AB. When e_1 increases to e'_1 , as shown, $\pi_2(0, e_1)$ falls, so that $V(e'_1)$ must be the value generated by the prior π , given a mapping from π_2 to V described by the line AC. As AC lies above AB (as $v(\cdot)$ is strictly convex), it follows that $V(e'_1) > V(e_1)$.

As the learning incentive is strictly positive, it follows from (3.6) that $e_1^A > e^*$, where $e^* < 1$ is the *myopic* effort level that solves $\bar{\theta} = c'(e^*)$. What determines the size of the learning incentive and thus $e_1^A - e^*$? In the case of quadratic costs, i.e., $c(e) = \frac{e^2}{2}$, it is easily checked (see Appendix for details) that the first-order condition (3.6) reduces to

$$e_1^A = \bar{\theta} + \delta \frac{\pi^2(1 - \pi)^2(\theta_h - \theta_l)^4}{2\bar{\theta}(1 - \bar{\theta}e_1^A)^2} \quad (3.7)$$

so that the second term on the right of (3.7) is the learning incentive. Other things equal, it is larger (i) the larger the initial uncertainty about the ability parameter, as measured by $\theta_h - \theta_l$; (ii) the closer π is to 0.5, i.e., the more “uniform” the prior.

4. Elections

4.1. The Second Period

Once someone is chosen as representative in the second period, the analysis is the same whether the institution is appointment or election, and so the analysis of Section 3.1 applies. So, it remains to characterize the outcome of the second-period election.

Suppose that i and j are candidates, with i being the incumbent. First, if the incumbent stands and wins, his payoff is $v(\pi_2(x_1, e_1)) + R$, while all other agents get $w(\pi_2(x_1, e_1))$, where

$$w(\pi) = [\pi\theta_h + (1 - \pi)\theta_l] e_2(\pi)$$

is simply the probability that the project will be a success in the second period, given belief π . (Note also for future reference that $v(\pi) = w(\pi) - c(e(\pi))$, so $w(\pi) > v(\pi)$). Second, if the challenger j stands for election and wins, his payoff is $v(\pi) + R$, while all other agents get $w(\pi)$. Otherwise everybody gets 0.

Noting that $w(\pi)$ is strictly increasing in π , and $\pi_2(1) > \pi > (\pi_2(0, e_1))$ for all $e_1 \in [0, 1]$, all agents other than i, j strictly prefer the incumbent to the challenger if the project is success, and vice versa if the project is a failure. Moreover, as $w(\pi)$ is strictly positive, all agents other than i, j prefer either candidate over the status quo. So, all agents other than i, j have the following unique weakly undominated voting strategies: (i) if i, j both stand, vote for the incumbent (challenger) if $x_1 = 1$ ($x_1 = 0$); (ii) if only one candidate stands, vote for that candidate. We will assume that all agents other than i, j play these strategies. On the other hand, whenever i or j wishes to stand, his unique weakly undominated strategy is to vote for himself. So, as $n \geq 3$, the voting behavior of those not standing for election determines the outcome⁹. The outcome is therefore that if the incumbent and challenger both stand, the incumbent (challenger) wins if $x_1 = 1$ ($x_1 = 0$), and if only one stands, he is elected.

Given this outcome, when deciding whether or not to stand for office, i, j are clearly playing the following 2×2 matrix game, where S denotes the decision to stand, and N the decision not to stand, and the incumbent (challenger) chooses rows (columns):

i/j	S	N
S	a, b	$v(\pi_2(x_1, e_1)) + R, w(\pi_2(x_1, e_1))$
N	$w(\pi), v(\pi) + R$	$0, 0$

⁹Note that unlike in the Ferejohn model, Nash equilibrium in voting strategies is unique, given the rather weak requirement that the Nash equilibrium be weakly undominated. So, our results do not require picking one favourable equilibrium at the voting stage.

where
$$a = \begin{cases} v(\pi_2(1, e_1)) + R & \text{if } x_1 = 1 \\ w(\pi) & \text{if } x_1 = 0 \end{cases}, \quad b = \begin{cases} w(\pi_2(1, e_1)) & \text{if } x_1 = 1 \\ v(\pi) + R & \text{if } x_1 = 0 \end{cases}$$

The following Lemma describes the possible weakly undominated Nash equilibria of this game. The proof is straightforward and is omitted.

Lemma. *Whatever x_1 , there is (generically¹⁰) exactly one undominated Nash equilibrium in the above game. If $x_1 = 1$, then, if $v(\pi_2(1)) + R > w(\pi)$, it is S, S , and if $v(\pi_2(1)) + R < w(\pi)$, it is N, S . If $x_1 = 0$, then, if $v(\pi) + R > w(\pi_2(0, e_1))$, it is S, S , and $v(\pi) + R < w(\pi_2(0, e_1))$, it is S, N .*

It is sensible to focus on continuation equilibria where the both candidates contest the election, i.e., the outcome is S, S . Other continuation equilibria are possible, i.e., there are parameter values where they can arise, but they are rather perverse. For example, if $v(\pi_2(1)) + R < w(\pi)$, the incumbent does not stand if the project is a success. From the Lemma, to ensure an outcome S, S , we need to assume that

$$v(\pi_2(1)) + R > w(\pi), \quad v(\pi) + R > w(\pi_2(0, e_1)) \quad (4.1)$$

Condition (4.1) says (i) in the event that the project is a success, the incumbent must strictly prefer to continue in office rather than be replaced by the challenger ($v(\pi_2(1)) + R > w(\pi)$), and (ii) in the event that the project is a failure, that the challenger must strictly prefer to take office rather have the incumbent continue ($v(\pi) + R > w(\pi_2(0, e_1))$).

Effectively, (4.1) says that R is "large enough" to motivate both incumbent and challenger to stand. We can thus express it more concisely, and incorporate the assumption that $R > 0$, as follows:

$$A1. \quad R > \max\{-\Delta_I, -\Delta_C, 0\}$$

where

$$\Delta_I = v(\pi_2(1)) - w(\pi), \quad \Delta_C = v(\pi) - w(\pi_2(0, e_1))$$

are the gains to standing for office for incumbent and challenger respectively, excluding ego-rent, given (i) that the incumbent (resp. challenger) wins; (ii) the other agent also stands for office.

¹⁰If $v(\pi_2(1)) + R = w(\pi)$, both S, S and N, S are equilibria. If $v(\pi) + R = w(\pi_2(0, e_1))$, both S, S and S, N are equilibria. Assumption 1 below rules out these non-generic cases.

4.2. The First Period

If the first-period project is a success, which occurs with probability $\bar{\theta}e_1$, the incumbent gets $v(\pi_2(1)) + R$, and if it is a failure, the incumbent gets $w(\pi)$. So, the expected discounted sum of payoffs for the incumbent at the beginning of period 1 is

$$\bar{\theta}e_1 - c(e_1) + R + \delta[(v(\pi_2(1)) + R)\bar{\theta}e_1 + w(\pi)(1 - \bar{\theta}e_1)] \quad (4.2)$$

The optimal choice of e_1 , denoted e_1^E , maximizes (4.2) subject to $e_1 \in [0, 1]$. Assuming an interior solution, the first-order condition for e_1 is therefore:

$$\bar{\theta} - c'(e_1) + \delta\bar{\theta}[\Delta_I + R] = 0 \quad (4.3)$$

The dynamic effort incentive is now $\delta\bar{\theta}[\Delta_I + R]$ and is strictly positive by A1. This incentive is composed of two parts. The term $\bar{\theta}\delta R$ is the present value of future ego-rent δR , times the probability of getting that rent. The term in Δ_I may be positive or negative. To see this, using the fact that $w(\pi) = v(\pi) + c(e(\pi))$ for any π , it is helpful to rewrite Δ_I as

$$\Delta_I = [w(\pi_2(1)) - w(\pi)] - c(e(\pi_2(1))) \quad (4.4)$$

In (4.4), the first term in the square brackets is always positive, and captures the fact that the incumbent knows that he will only win the election if the project is a success, in which case he believes that he is higher-ability than the challenger, and thus gets a higher payoff if he wins, ignoring any cost of effort. But subtracted from that effect is the cost of effort, which is only borne by the incumbent if he wins the election. We will call the overall dynamic effort incentive $\delta\bar{\theta}[\Delta_I + R]$ the *re-election incentive*.

5. Comparing Appointment and Elections

5.1. Effort Levels

We can now turn to the main topic of the paper, the comparison of effort levels under appointment and election. In the final period, conditional on posterior belief about type, the same effort level $e(\pi_2)$ occurs under both institutions. The interesting comparison is therefore in the first period. The difference between the two dynamic effort incentives in the first period can be written as follows. First, differentiating (3.4), we see that

$$V'(e_1) = \bar{\theta}(v(\pi_2(1)) - v(\pi_2(0, e_1))) + (1 - \bar{\theta}e_1)v'(\pi_2(0, e_1))\frac{\partial\pi_2(0, e_1)}{\partial e_1} \quad (5.1)$$

Using (5.1), (4.4), and $v(\pi) = w(\pi) - c(e(\pi))$, we can write

$$\begin{aligned} V'(e_1) - \bar{\theta}[\Delta_I + R] &= \bar{\theta}[(w(\pi) - v(\pi_2(0, e_1)))] \\ &\quad + (1 - \bar{\theta}e_1)v'(\pi_2(0, e_1))\frac{\partial\pi_2(0, e_1)}{\partial e_1} - \bar{\theta}R \end{aligned} \quad (5.2)$$

So, the incremental dynamic effort incentive under appointment relative to election can be divided into three terms. The first, which is always positive (as w is increasing, $w > v$, and $\pi > \pi_2(0, e_1)$) is due to the increased *penalty to failure* to the incumbent with appointment. Under appointment, in the event of project failure, the incumbent is not replaced, but must carry out his task in the next period, even though his ability is revealed to be low, and moreover, must personally incur the effort cost of doing so. By contrast, with elections, in the event of failure, the incumbent hands over the reins to someone who is of average ability, and moreover, avoids any cost of effort in the second period. So, the future payoff¹¹ in the event of project failure is lower with appointment than elections, providing a stronger incentive to avoid project failure, and thus put in high effort.

Set against this positive penalty to failure term are two terms that are negative. The first negative term is due to the fact that with appointment, a higher first-period effort lowers the probability belief $\pi_2(0, e_1)$ in the event of failure - this belief is irrelevant with elections, as the incumbent is replaced. Finally, with elections, in the event of failure, the incumbent loses ego-rent R which provides an additional effort incentive.

When can the right-hand side of (5.2) be positive, bearing in mind that A1 must also hold? A useful benchmark here is when $\theta_h = \theta_l$, so there is no learning in the model. In this case, the increased penalty to failure is simply that with appointment, the incumbent will continue to hold office, and so incurs cost $c(e_2)$ where e_2 is second-period effort. Moreover, the second term the right-hand side of (5.2) is zero. So, overall, we see that the incremental effort incentive is $\bar{\theta}(c(e_2) - R)$. But with $\theta_h = \theta_l$, it is easily checked that A1 reduces to $R > c(e_2)$. So, without learning, the incremental effort incentive with appointment is *always* negative.

With a learning motive, (5.2) provides, we think, a useful conceptual understanding of the forces that cause the dynamic effort incentives to differ with appointment and election, but it cannot be directly used to find conditions under which the dynamic effort incentive is higher under appointment (although we would expect a sufficient condition to involve a “low” R , which as we shall see, is one of the requirements).

¹¹Conditional on fixed beliefs, and ignoring ego-rent R .

We can, however, argue as follows. First, A1 can be rewritten $\Delta_I + R > \max\{0, \Delta_I - \Delta_C, \Delta_I\}$. So, it is clear that if $\Delta_I \leq \min\{0, \Delta_C\}$ the electoral incentive $\Delta_I + R$ can be made arbitrarily small subject to A1 being satisfied by appropriate choice of R . But then as $V'(e_1)$ is bounded above zero on $[0, 1]$, R can always be chosen to make $e_1^A > e_1^E$, because $V'(e_1)$ is independent of R .

Moreover, it is possible to choose parameter values such that $\Delta_I \leq \min\{0, \Delta_C\}$. For example, it is shown in the Appendix that if costs are quadratic, i.e., $c = \frac{e_1^2}{2}$, and $\pi = 1/\sqrt{2}$, $\theta_l = 0$, $\theta_h \geq \sqrt{2} \frac{\rho}{(\sqrt{2}-1)} \simeq 0.91$, then this condition is satisfied. So, to conclude, in the quadratic cost case, there are certainly parameter values for which $\Delta_I \leq \min\{0, \Delta_C\}$. We have thus proved:

Proposition 1. *If $\Delta_I \leq \min\{0, \Delta_C\}$, then a “low enough” ego-rent R can be found for which effort is higher under appointment ($e_1^A > e_1^E$). Moreover, if the cost of effort is quadratic, other parameter values can be chosen so that $\Delta_I \leq \min\{0, \Delta_C\}$.*

Thus, we have the possibility that elections can demotivate. This is the key result of the paper. The interpretation of $\Delta_I \leq \min\{0, \Delta_C\}$ is that it is sufficient for the first term to dominate the second in (5.2).

Table 1 below illustrates this result with some numerical simulations with a quadratic cost-of-effort function. In particular it shows how much higher e_1^A can be compared to e_1^E . As can be seen, the difference can be fairly significant (e.g., in the first row, e_1^A is shown to be ten percent larger than e_1^E). The Table also shows the effect of changes in effort levels (i.e., changes in the ability spread, changes in the ego rent from office, changes in the prior belief regarding ability). Δ_I and Δ_C are also reported to confirm that A1 is satisfied in all cases, and also the myopic effort level e^* is reported. Finally, in the last column, expected voter utilities are presented: these are discussed in the next section.

Table 1. Equilibrium Effort Levels e_1^A, e_1^E

Parameters	e_1^A, e_1^E	$-\Delta_I, -\Delta_C$	e^*	U_A, U_E
$\theta_h = 0.99, \theta_l = 0, \pi = \frac{1}{\sqrt{2}}, R = 0.15$	0.878, 0.794	-0.32, -0.22	0.700	1.18, 1.24
$\theta_h = 0.99, \theta_l = 0, \pi = \frac{1}{\sqrt{2}}, R = 0.005$	0.878, 0.703	-0.25, -0.04	0.700	1.18, 1.15
$\theta_h = 0.92, \theta_l = 0, \pi = \frac{1}{\sqrt{2}}, R = 0.15$	0.727, 0.738	-0.30, -0.15	0.650	0.91, 1.04
$\theta_h = 0.92, \theta_l = 0, \pi = \frac{1}{\sqrt{2}}, R = 0.005$	0.727, 0.653	-0.21, -0.01	0.650	0.91, 0.97
$\theta_h = 0.99, \theta_l = 0, \pi = 0.9, R = 0.15$	0.606, 0.670	-0.66, -0.19	0.495	0.61, 0.77
$\theta_h = 0.99, \theta_l = 0, \pi = 0.9, R = 0.005$	0.606, 0.606	-0.55, -0.04	0.495	0.61, 0.72
$[\delta = 0.9 \text{ in all cases}]$				

5.2. Welfare Levels

Consider the expected payoff to a voter who is not selected for office in either period under appointment or election. These payoffs, under appointment or election respectively are:

$$U_A = \bar{\theta}e_1^A + \delta[\bar{\theta}e_1^A w(\pi_2(1)) + (1 - \bar{\theta}e_1^A)w(\pi_2(0, e_1^A))] \quad (5.3)$$

$$U_E = \bar{\theta}e_1^E + \delta[\bar{\theta}e_1^E w(\pi_2(1)) + (1 - \bar{\theta}e_1^E)w(\pi)] \quad (5.4)$$

Then using (5.3), (5.4), we can write

$$U_A = \bar{\theta}e_1^A + \delta W(e_1^A) \quad (5.5)$$

$$U_E = \bar{\theta}e_1^E + \delta W(e_1^E) + \delta(1 - \bar{\theta}e_1^E)(w(\pi) - w(\pi_2(0, e_1^E))) \quad (5.6)$$

where

$$W(e) = \bar{\theta}ew(\pi_2(1)) + (1 - \bar{\theta}e)w(\pi_2(0, e))$$

and recalling that $w(\pi) = (\pi\theta_h + (1 - \pi)\theta_l)e(\pi)$. Now, note that by definition,

$$w'(\pi) = (\theta_h - \theta_l)e(\pi) + (\pi\theta_h + (1 - \pi)\theta_l)e'(\pi) > 0$$

$$w''(\pi) = 2(\theta_h - \theta_l)e'(\pi) + (\pi\theta_h + (1 - \pi)\theta_l)e''(\pi)$$

So, if $e''(\pi) \geq 0$, then $w''(\pi) > 0$, and then $W'(e) > 0$ by the same argument that established $V'(e) > 0$ above.

Now, we can write the gain to election over appointment as

$$U_E - U_A = \bar{\theta}(e_1^E - e_1^A) + \delta[W(e_1^E) - W(e_1^A)] + \delta(1 - \bar{\theta}e_1^E)(w(\pi) - w(\pi_2(0, e_1^E))) \quad (5.7)$$

The gain to elections over appointment thus decomposes neatly into three different effects. The first, $\bar{\theta}(e_1^E - e_1^A)$, is the first-period benefit from higher effort. The second is the second-period benefit from higher effort which (assuming $e''(\pi) \geq 0$) is positive if $e_1^E > e_1^A$, as W is increasing in e . In the terminology of Besley and Smart (2003), these two effects are both *incentive effects* of elections. The third is the *selection effect* of elections, and is always positive as $w(\pi) > w(\pi_2(0, e_1^E))$. It captures the fact that elections allow de-selection of incompetent incumbents.

From (5.7), and this discussion, the following result follows immediately:

Proposition 2. *Assume $e''(\pi) \geq 0$. If $e_1^E \geq e_1^A$, then election welfare-dominates appointment, i.e., $U_E > U_A$.*

This is because the incentive and selection effects work in the same direction. So, a necessary condition for appointment to dominate is that $e_1^E < e_1^A$. Table 1 presented above, shows that, in the case of a quadratic cost-of-effort function, $U_E < U_A$ is indeed possible. The table also reports e_1^E, e_1^A and it can be confirmed that $e_1^E < e_1^A$ is indeed required, but not sufficient, for $U_E < U_A$. Finally, note that $e''(\pi) \geq 0$ is not a strong assumption. It is satisfied for example, by $c(e) = \frac{1}{\alpha}e^\alpha$, all $1 < \alpha \leq 2$, which includes the quadratic.

6. Some Extensions

6.1. More General Interaction between Ability and Effort

So far, we have assumed that the probability of project success is of the form $p = \theta e$. This is sufficient to generate a positive value of information about the project outcome in the first period, which is also increasing in e_1 . This is the key feature of the model which drives our main results. However, it is easily verified that this key feature is also generated by a more general mapping $p = p(\theta, e)$ as long as $p_{\theta e} > 0$: that is, some multiplicative interaction between ability and effort is required.

6.2. Endogenous Candidate Entry

It is possible to write down a version of the model with fully endogenous candidate entry where the above conclusions are substantially unchanged. Suppose for simplicity that only two members of the committee, 1 and 2, are competent to hold office. Then, at the beginning of each period, both simultaneously decide whether to stand or not: standing for office is costless. If no-one stands, or a single candidate for office is defeated in favour of the status quo, then the status quo is implemented. Then, the analysis of Section

3.4 above applies exactly from the point where a first-period incumbent is elected. With endogenous entry, the only change is that, given the restrictions on electoral incentives required to show that $e_1^A > e_1^E$, it may be that at the beginning of $t = 1$, each of the two potential candidates may prefer the *other* to stand for office. But, under the assumptions made so far, each would prefer to take office at $t = 1$ rather than have the status quo implemented. In this case, each will randomize over the entry decision, implying a further source of inefficiency with elections; the project may not be implemented with some probability.

6.3. Many Project Outcomes

One simplifying assumption of the model is that the project outcome is binary, i.e., $x_t \in \{0, 1\}$. In Le Borgne and Lockwood (2000), the more general case where x_t is a real number is studied: this paper also allows for a wider class of interactions between ability and effort. This more general case is much less tractable, and so only numerical results comparing appointment to elections can be established. We briefly sketch this extension here. If i is the incumbent, the project outcome at t , x_t is given by

$$x_t = (1 - \mu)(\theta^i + e_t) + \mu\theta^i e_t + \omega_t \quad (6.1)$$

where θ^i is the ability parameter as before, $e_t \in [0, \infty)$ his effort level in period t , ω_0, ω_1 are i.i.d. mean zero random shocks, and finally $\mu \in (0, 1]$, to generate a learning motive. In either period, the office-holder chooses e_t without observing ω_t , which is assumed to have a continuous distribution with probability density function f , cumulative distribution function F , and has full support on \Re . Conceptually, the analysis of appointment and elections is much as before. Due to additional complexity of the model, fewer analytical results could be proved, and so we relied more on numerical simulations. These simulations, reported in Le Borgne and Lockwood (2000), show that it is possible that $e_1^A > e_1^E$, and also that voter welfare under appointment may be higher.

7. Related Literature and Conclusions

7.1. Related Literature

There are a number of related papers other than those mentioned in the introduction. First, our results have implications for the very general results obtained by Banks and Sundaram (1998) on optimal retention in agency problems. They consider a very general principal-agent model where (i) information about ability is asymmetric, i.e., only known

by the agent, and (ii) the agent can only be controlled by the (credible) threat of firing, i.e., non-retention. This model includes almost all¹² existing political agency models as special cases, as well as having many other applications. In this setting, they show that the ability of the principal to fire the agent unambiguously raises the agent's effort¹³. Indeed, under some very weak regularity conditions, the threat of (electoral) dismissal induces agents of *all* types to supply more effort than they would otherwise in their first term of office (Proposition 3.3). As our model (with their information structure) is a special case of theirs, our paper shows that this otherwise very general result is *not* robust to a change in the information structure.

Second, Persson and Tabellini (2000, Chapter 4.5), have a two-period electoral model with both adverse selection and moral hazard, where, as in this paper, initially the incumbent does not know his type. In their model, given an incumbent with ability θ , the technology for supplying the public good is $g = \theta(\tau - r)$ where g is output of the public good, τ is exogenous tax revenue, and r are rents misappropriated from tax revenues. So, having observed g and r at the end of the first period, the incumbent can perfectly infer his productivity. Therefore, learning is complete, whatever the level of rent diversion (or effort), and so there is no learning effect, as we have defined it.

Third, there is a link to the “career concerns” literature initiated by Holmstrom’s classic paper (Holmström (1982, 1999)). This literature makes the same informational assumptions as us, while the economic model is rather different (the wage of the manager (incumbent) is endogenous; there is no possibility of being fired). Our information structure (although not the model of the principal-agent relationship¹⁴) is the same as the career concerns literature of Holmström (1982, 1999), and Dewatripont, Jewitt, and Tirole (1999). This literature - to our knowledge - has not noted the existence of learning effects. This is because the existing literature assumes either (i) an additive technology, where information has no value (Holmström, 1982, 1999); (ii) one period only, in which case information acquired currently cannot be used in the future (Dewatripont et al.,

¹²The exception is Coate and Morris (1995), where asymmetric information is two-dimensional: the incumbent not only privately observes his type, but the type of the public project.

¹³This follows from Proposition 3.2 of their paper, which shows that the lower bound of the support of the random effort in the first period of the agent’s life is higher than effort in the second (last) period, when the threat of firing has no force. Given their information structure, second-period effort is the same that would be supplied in the first period if the principal had no power to fire the agent.

¹⁴The career concerns literature focuses on the labour market, not the relationship between the electorate and public officials. Specifically, in the career concerns literature, pay of the agent is not exogenous (as in our model) but depends on the employer’s belief about the marginal/average product of the agent, and that belief depends in turn on past performance.

1999).

Finally, there is a related paper¹⁵ by Swank and Visser (2003), which we saw only after the first version of this paper was complete (Le Borgne and Lockwood (2000)). In their model, an incumbent has to make two decisions about a discrete project: whether to design it, and whether to evaluate the benefits (to the voter) of the design. Both these activities are costly. Voters can precommit to a retrospective voting rule. The main result is that elections per se do not provide any incentives to agents evaluate projects, only to design them. Unlike our model, effort is two-dimensional. So, in this sense, our results and theirs are complementary. We show that even if effort is one-dimensional, elections may not incentivise the agent. They show that with two dimensions, elections may distort the pattern of effort levels.

7.2. Conclusions

We have shown that when the informational assumptions of the political agency literature are changed (by supposing that candidates for office are less than certain about their abilities), a learning motive for choice of effort comes into play. This motive is weakened by elections, and so if the learning motive is strong enough, elections may demotivate office-holders (relative to appointment). The intuition behind our results are, however, more general and applies to other labour markets: as long as the agent has some positive probability of being “fired” by the principal, that the model is dynamic and the technology the agent uses is at least partly multiplicative in talent and effort then both re-election concerns and learning will be present. Extension of the analysis of this paper to other labour markets is a topic for further work.

8. References

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¹⁵A somewhat less related paper is by Biglaiser and Mezzetti (1997). There, in the first period, the incumbent chooses an observable discrete project, but where the value of the project depends on the incumbent’s ability (initially unknown to everybody) and a random shock. The paper focuses on the issue of whether undertaking the project is a good or bad signal to the electorate about the incumbent’s ability.

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A. Appendix

A.1. Derivation of e_1^A when $c = e^2/2$

It is easily checked that

$$e_2(\pi) = [\pi\theta_h + (1 - \pi)\theta_l], \quad w(\pi) = [\pi\theta_h + (1 - \pi)\theta_l]^2, \quad v(\pi) = \frac{1}{2}w(\pi) \quad (\text{A.1})$$

From (3.6) and (5.1), the FOC defining e_1^A is:

$$\bar{\theta} - e_1 + \delta\bar{\theta}[v(\pi_2(1)) - v(\pi_2(0, e_1))] + \delta v'(\pi_2(0, e_1)) \frac{\partial \pi_2(0, e_1)}{\partial e_1} (1 - \bar{\theta}e_1) = 0 \quad (\text{A.2})$$

Differentiating (3.3), we have

$$\frac{\partial \pi_2(0, e_1)}{\partial e_1} = -\frac{\pi(1 - \pi)(\theta_h - \theta_l)}{(1 - \bar{\theta}e_1)^2} < 0 \quad (\text{A.3})$$

Combining (A.1),(A.2), (A.3), we get:

$$\begin{aligned} & (\bar{\theta} - e_1) + \frac{\delta\bar{\theta}}{2} [\pi_2(1)\theta_h + (1 - \pi_2(1))\theta_l]^2 - [\pi_2(0, e_1)\theta_h + (1 - \pi_2(0, e_1))\theta_l]^2 \\ & - \delta \frac{\pi(1 - \pi)(\theta_h - \theta_l)^2}{(1 - \bar{\theta}e_1)} [\pi_2(0, e_1)\theta_h + (1 - \pi_2(0, e_1))\theta_l] = 0 \end{aligned}$$

After straightforward but lengthy simplification, available on request, this rearranges to

$$e_1 = \bar{\theta} + \delta \frac{\pi^2(1 - \pi)^2(\theta_h - \theta_l)^4}{2\bar{\theta}(1 - \bar{\theta}e_1)^2}$$

as shown in the main text.

A.2. Example With $\Delta_I \leq \min\{0, \Delta_C\}$.

Set $\theta_l = 0$, $\pi = \frac{1}{\sqrt{2}}$. Then we have

$$\Delta_I = v(\pi_2(1)) - w(\pi) = 0.5 \frac{\pi\theta_h^2 + (1 - \pi)\theta_l^2}{\bar{\theta}} - \bar{\theta}^2 = \theta_h^2(0.5 - \pi^2) = 0$$

So, for $\Delta_I \leq \min\{0, \Delta_C\}$ we need only $0 \leq \Delta_C$. But

$$\Delta_C = v(\pi) - w(\pi_2(0, e_1^E)) = \frac{1}{2}(\pi\theta_h)^2 - (\pi_2(0, e_1^E))^2(\theta_h)^2$$

so $0 \leq \Delta_C$ reduces to

$$\frac{1}{\sqrt{2}}\pi \geq \pi_2(0, e_1^E) = \frac{\pi(1 - \theta_h e_1^E)}{\pi(1 - \theta_h e_1^E) + 1 - \pi} \quad (\text{A.4})$$

Also, $\pi_2(0, e_1)$ is decreasing in e_1 , and in equilibrium, $e_1^E > e^* = \bar{\theta} = \pi\theta_h$. So, given $\pi = 1/\sqrt{2}$, (A.4) is certainly satisfied if

$$\frac{1}{2} \geq \pi_2(0, \pi\theta_h) = \frac{1 - \theta_h^2/\sqrt{2}}{1 - \theta_h^2/\sqrt{2} + \sqrt{2} - 1} \implies \theta_h \geq \sqrt{2} \frac{1}{\sqrt{2} - 1} \simeq 0.91.$$

Figure 1 –The Learning Effect

