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Sylvester C W Eijffinger
and Matteo Governatori

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Sylvester C W Eijffinger, Tilburg University and CEPR
Matteo Governatori, Tilburg University

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Fiscal and Monetary Interaction: The Role of Asymmetries of the Stability and Growth Pact in EMU*

The Paper builds a simplified model describing the economy of a currency union with decentralized national fiscal policy, where the main features characterizing the policy-making are similar to those in EMU. National governments choose the size of deficit taking into account the two main rules of the Stability and Growth Pact on public finance. Unlike previous literature the asymmetric working of those rules is explicitly modeled in order to identify its impact on the Nash equilibrium of deficits arising from a game of strategic interaction between fiscal authorities in the union.

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Sylvester C W Eijffinger
Department of Economics
CentER, Tilburg University
PO Box 90153
5000 LE Tilburg
THE NETHERLANDS
Tel: (31 13) 466 2411
Fax: (31 13) 466 3042
Email: s.c.w.eijffinger@uvt.nl

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Matteo Governatori
Department of Economics
CentER, Tilburg University
PO Box 90153
5000 LE Tilburg
THE NETHERLANDS
Tel: (31 13) 2757 4221
Fax: (31 13) 3466 3360
Email: m.governatori@uvt.nl

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=158335

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1 Introduction and aim of the paper

An Economic and Monetary Union between twelve European countries has been created in 1999. One of its crucial feature is the asymmetry between monetary and fiscal policy. The former is fully centralized in the hands of the European Central Bank, while the national governments keep their control over the latter. Nonetheless a system of fiscal rules to which all participating countries are subject has been set up: the Stability and Growth Pact (SGP).

It implies three major requirements:

Running a nominal budget deficit not larger than 3% of the country's GDP.

Having an outstanding stock of government debt not larger than 60% of the country's GDP

Running a structural budget close to balance or in surplus[1].

In case a country does not fulfill the 3% threshold an institutional procedure is supposed to be triggered by the ECOFIN (the EU council of Finance Ministers), culminating in the payment of fines by the country violating the rule, in case no corrective measures are taken.

The basic rationale underneath the set-up of the pact is the fact that in a monetary union with decentralized fiscal policy, cross-country spillovers of government deficit increase since, on one hand, it has a higher direct positive impact on the partner's output because of stronger trade links and, on the other, a higher indirect negative impact on the same variable since higher national deficit means higher union-wide inflation to which the bank reacts with higher interest rates for all the countries in the Union. This clearly leads to a more severe problem of cross-country fiscal externality than in an environment with national monetary policy[2], thereby calling for some disciplining devices to help countries internalizing it.

This paper builds up a simple model which provides a description of the economy of a currency union and which incorporates the constraints of the SGP in the objective function of the national fiscal policy-makers. The formulation follows the one used in Governatori-Eijffinger (2004a) adding a crucial feature: the *asymmetric working of the pact's rules*, i.e. the fact that the two constraints imply sanctions for the country only when it *overcomes* the corresponding threshold levels for deficit, while no loss is incurred when deficit is lower than those thresholds. This set-up is used to investigate what is the

[1] The European Commission clarified that this actually means the structural, or cyclically adjusted, deficit. This in turn means the balance net of the part due to the economic cycle, i.e. to the working of the automatic stabilisers.

[2] In a normal setting of national monetary and fiscal policy, this cross-country spillovers are strongly softened: the impact of higher deficit on foreign output is indirect because of trade restrictions, exchange rate risk, restriction on free movement of capital, which typically vanish or are at least strongly lowered in a monetary union. As far as the inflation/interest rate spillover is concerned, it is also lowered, since foreign monetary policy will just care about foreign inflation and not the overall one of the Union.

impact of this system of rules and sanctions on the budget deficits of member countries. In particular the aim is to identify how the SGP shapes the strategic interaction between individual countries' fiscal policy choices.

The paper is structured as follows.

The second section outlines the model with two countries making up a currency union, specifying how the SGP asymmetric rules enter the optimization problem faced by each fiscal policy authority.

The third section outlines the procedure for solving the model and determining the potential equilibrium choices of deficits by each country.

The fourth section rules out unfeasible equilibria and finds the fiscal best response functions of the two countries, i.e. the equilibrium deficit choice of one country given each of the possible moves of the other.

The fifth section finds the Nash equilibria of deficit.

The sixth section performs some simulations and robustness analysis of the findings previously identified.

The seventh section concludes.

2 The model

The model considers a two countries monetary union in which monetary policy is decided by a single institution, while fiscal policy authority is retained by each of the participating countries. The two countries are identical in every respect except for their shares of the aggregate GDP of the union, which are ω for the former and $1-\omega$ for the latter, with $0<\omega<1$.

The model includes a demand curve which states that the output gap (meaning actual minus potential GDP) of the whole currency union, y^D, depends positively on the weighted average of the government deficits of the two countries (g_1, g_2), expressed in terms of their national GDPs, and negatively on the currency union wide real interest rate, $i-p_e$ where the latter term is the expected inflation rate.

$$y^D = \alpha\left[\omega g_1 + (1-\omega)\, g_2\right] - \beta\,(i-p_e) \tag{1}$$

The supply curve is a Phillips curve whereby surprise inflation in the union leads to an increase in the output gap, the rationale being the nominal rigidities which are assumed to characterize the union's markets:

$$y^S = \varphi\,(p-p_e) \tag{2}$$

We add the usual equilibrium condition:

$$y^D = y^S = y \tag{3}$$

The monetary policy of the common central bank is represented by a Taylor rule whereby the interest rate is exclusively set in order to reach the inflation target of the bank, which for simplicity is set to $p_0 = 0$.

$$i = \delta p \tag{4}$$

The last equation links the total and structural budget deficit of each country (the latter being b_i). Following Buti-Giudice (2002), the latter is the cyclically adjusted deficit, i.e. the total deficit net of the part due to the cyclical situation of the economy (hence, to the working of automatic stabilizers)[3]:

$$g_i = b_i - \phi y \tag{5}$$

Where ϕ measures the cyclical sensitivity of the budget and it is assumed to be equal for both countries.

All parameters are assumed to be between 0 and 1 except $\delta > 1$ in order to rule out negative real interest rate.

Our aim is to solve the model for the optimal deficit of the individual country.
The first step is to identify the policy instrument which the fiscal authorities can use, which is the structural budget deficit. Once the latter is set by both countries the total deficits will be determined through the equilibrium output gap of the Union. So we first express all total deficits in the demand equation as functions of the structural ones and the output gap, then we replace the Taylor rule into the demand equation.
Then, the inflation rate is found through the output market equilibrium condition:

$$\alpha\left[\omega\left(b_1 - \phi y\right) + \left(1-\omega\right)\left(b_2 - \phi y\right)\right] - \beta\left(\delta p - p_e\right) = \varphi\left(p - p_e\right) \tag{6}$$

Solution is :

[3] In fact, we consider the Union's economy to be in recession whenever the output gap is negative ($y < 0$), in that case $-\phi y$ is positive due to the working of automatic stabilisers, hence the total deficit will be *higher* than the structural one. The symmetric reasoning holds when the economy is in upturn.

$$p = \frac{\alpha\omega b_1 + \alpha b_2 - \alpha\phi y - \alpha\omega b_2 + \beta p_e + \varphi p_e}{\varphi + \beta\delta} \tag{7}$$

The equilibrium output gap is found plugging inflation on the supply curve:

$$y = \varphi\left(\frac{\alpha\omega b_1 + \alpha b_2 - \alpha\phi y - \alpha\omega b_2 + \beta p_e + \varphi p_e}{\varphi + \beta\delta} - p_e\right) \tag{8}$$

, Solution is:

$$y = \varphi\frac{\alpha\omega b_1 + \alpha b_2 - \alpha\omega b_2 + \beta p_e - p_e\beta\delta}{\varphi + \beta\delta + \alpha\varphi\phi} \tag{9}$$

This shows the positive impact of both deficits on the equilibrium output gap, i.e. the positive spillovers of each country's deficit onto the other country. The equilibrium inflation is found plugging y in the expression previously identified:

$$p = \frac{\varphi p_e + \varphi p_e\alpha\phi + \alpha\omega b_1 + \alpha(1-\omega)b_2 + \beta p_e}{\varphi + \beta\delta + \alpha\varphi\phi} \tag{10}$$

which instead highlights the negative externality of either country's deficit through its effect on common inflation. The structural deficits are the only endogenous variables still to be determined. Hence, we proceed outlining the framework for the fiscal policy optimization problem.

The fiscal policy of each of the two countries is set in order to minimize a loss function. We assume that the fiscal authorities care about stabilizing the Union

output toward its potential level[4] and about fulfilling the first two requirements of the SGP we outlined above. Therefore they incur a loss whenever they run a structural budget deficit, equal to a share η of this deficit $(0 < \eta < 1)$ and a further one whenever they run a total deficit larger than the threshold set by the SGP, t. In case the threshold is overcome a proportion $0 < \rho < 1$ of the excess deficit must be paid by the country as a fine[5].

[4] We assume that each country cares about stabilising the **currency union wide** output gap instead of the national one, as one would expect. The problem is that in this set-up the national output gap cannot be calculated from the aggregate one.

[5] This set-up is a simplified version of what is actually foreseen in the Pact, in fact the fine is actually made up of a fixed proportion of the country's GDP plus a variable part which is a constant fraction of the excess deficit and there is an upper threshold beyond which the fine cannot be raised (0.5% of GDP).

Finally, the fine is actually paid in reference to past and not current amounts of deficit. Our set-up is, though, in line with the literature, see Bolt (1998) and Beetsma-Uhlig (1999).

Choosing the optimal deficit, the country trades off the use of fiscal policy to stabilize the output toward its structural level with the sanctions it has to bear if the deficit it runs overcomes two parallel but partly independent upper thresholds.

Intuitively the country is faced by the risk that in order to achieve the desired output level it has to run a structural deficit, i.e. to violate directly the first constraint of the pact. It has a slightly more limited control on the second constraint since it concerns the total deficit, which depends on the output gap that will eventually arise, which in turn depends also on the other country's structural deficit. This means that the optimal equilibrium fiscal choice will also depend on the other country's choice.

The optimization strategy of the fiscal authorities can therefore fall into four possible scenarios (we consider country 1, for country two the reasoning is clearly the same):

1. The country runs a structural budget in balance or surplus, so it fulfills the first constraint, moreover it runs a total deficit not higher than the threshold, thereby fulfilling also the second constraint, which means that the output gap is high enough to avoid an excessive working of the automatic stabilizers. The loss function is then composed only by the term capturing deviations of the output gap from zero and the optimization problem is as follows:

$$\begin{aligned} &\underset{b_1}{Min}\left\{\sigma y^2\right\} \\ &s.t. \\ b_i &\leq 0; \quad g_i - t \leq 0 \end{aligned} \tag{11}$$

2. The country runs a structural deficit thereby violating the first constraint and suffering the corresponding loss. On the other hand the total deficit is still lower than the threshold implying fulfillment of the second rule. The loss function is therefore composed by the term on output gap deviations and that capturing the sanctions for first constraint's violation. The optimization problem becomes the following:

$$\begin{aligned} &\underset{b_1}{Min}\left\{\sigma y^2 + \eta\left(b_1\right)^2\right\} \\ &s.t. \\ b_1 &> 0; \quad g_i - t \leq 0 \end{aligned} \tag{12}$$

3. The country runs a structural budget in balance or surplus, fulfilling the first requirement, but violates the second, running a total deficit higher than the threshold. This is a pretty unlikely scenario but cannot be ruled out

since a structural surplus may be consistent with a total excessive deficit if the output gap is negative and large enough to produce a huge automatic stabilizers-driven total deficit. The loss function is then composed by the terms referring to output gap stabilization and the payment of fines for running an excessive deficit, respectively. The opimization problem becomes:

$$Min\left\{\sigma y^2+\rho^2\left(g_i-t\right)^2\right\} \tag{13}$$
$$s.t.$$
$$b_1 \leq 0; \quad g_i-t>0$$

4. The country is a "full sinner": it violates both constraints since it runs a structural deficit and a total deficit higher than the threshold. The loss function is then made up of three terms: output gap deviations from target and sanctions for violation of both constraints respectively.

$$Min\left\{\sigma y^2+\eta\left(b_i\right)^2+\rho^2\left(g_i-t\right)^2\right\}$$
$$s.t.$$
$$b_1 > 0; \quad g_i-t>0$$

3 Solving the model

We now need to solve the optimization problem of the individual country's fiscal policy-maker.

We assume that in this two countries monetary union the two fiscal authorities *independently* and *simultaneously* set the size of their structural budget deficits, incorporating the working of this simple economy and the interest rate rule followed by the central bank.

We need to determine the optimal deficit of the first country taking the other country's deficit as given and considering that corner solutions can also arise.

In fact in the first and second scenarios, as far as the first constraint is concerned, the country can choose to play exactly a zero structural deficit, the maximum level allowed, or may optimize with the constraint being not binding: i.e. running a negative structural deficit.

In the remaining scenarios this would not be possible since by definition they imply the violation of the constraint, so necessarily a strictly positive structural deficit.

Equally, in the first and third scenarios, the country can choose a corner solution with respect to the second constraint: it can play a structural deficit that, *given what the other country does* and the resulting output gap from both countries' fiscal moves, leads to a total deficit exactly equal to the threshold, so that no fines are paid. In the remaining scenarios this option is not possible since in that case an excessive deficit is run by definition. Figure 1 synthetises the possible moves within each scenario.

	Scenario 1 Structural balance/surplus No excessive total deficit	Scenario 2 Structural deficit, No excessive total deficit	Scenario 3 Structural balance/surplus Excessive total deficit	Scenario 4 Structural deficit, Excessive total deficit
Case 1: Structural budget in balance	$b(1,1) \Rightarrow$ $b=0, g<t$	Impossible $\Rightarrow$ falls in $b(1,1)$	$b(3,1) \Rightarrow$ $b=0, g>t$	Impossible $\Rightarrow$ falls in $b(3,1)$
Case 2: Total deficit equal to threshold	$b(1,2) \Rightarrow$ $b<0, g=t$	$b(2,2) \Rightarrow$ $b>0, g=t$	Impossible $\Rightarrow$ falls in $b(1,2)$	Impossible $\Rightarrow$ falls in $b(2,2)$
Case 3: No constraint binding	$b(1,3) \Rightarrow$ $b<0, g<t$	$b(2,3) \Rightarrow$ $b>0, g<t$	$b(3,3) \Rightarrow$ $b<0, g>t$	$b(4,3) \Rightarrow$ $b>0, g>t$

Figure 1

The optimization problem is solved using the following procedure:

1. In the rest of this section, we calculate all the possible deficit choices of country one according to the lines of the above figure, *as functions of the other country's deficit*. Those of country two are the same, the only change concerns reversing the economic weights of the countries in the resulting formulas. No immediate conclusions can be drawn on whether these equilibria are feasible. Feasibility of the boundary solution requires that also the other constraint is satisfied and not with equality sign, whereas for unconstrained optima it requires that both constraints are satisfied and none of them with equality sign. This feasibility check cannot be carried on at this stage since equilibria still depend on the other country's deficit which could fall in any of the four scenarios and in any case within each scenario. As a consequence, we cannot identify already the move that minimizes the loss function and, so, will be chosen by the first country. For that purposes we need the following steps.
2. We find the best fiscal responses of country one with respect to each

of the possible moves of country two. In that case we end up with just two numbers (and not anymore functions) for both countries' deficits. This means that we can perform the feasibility checks for any equilibrium and once we are left with the feasible ones we pick the one leading to the lowest loss. This step will be performed in section 4

3. Since the results on best responses of the second country will be symmetric, for they are identical in everything but the economic weights, we get the deficit reaction functions of both countries, hence we can determine the Nash equilibria. This will be done in section 5.

3.1 First scenario: the "virtuous" country

If the country chooses the first scenario, the problem, after replacing the output gap with its equilibrium expression and expressing the second constraint on total deficit in terms of the structural one, becomes the following:

$$\begin{aligned} & \underset{b_1}{Min}\left\{\sigma\left(\varphi\frac{\alpha\omega b_1+\alpha b_2-\alpha\omega b_2+\beta p_e-p_e\beta\delta}{\varphi+\beta\delta+\alpha\varphi\phi}\right)^2\right\} \\ s.t \quad b_1 & \leq 0 \\ b_1 & \leq \frac{t\left(\varphi+\beta\delta+\alpha\varphi\phi\right)+\phi\varphi\left[\alpha b_2\left(1-\omega\right)+\beta p_e\left(1-\delta\right)\right]}{\varphi+\beta\delta+\alpha\varphi\phi-\varphi\phi\alpha\omega} \end{aligned} \tag{14}$$

Based on our previous discussion, we distinguish four possible cases:

case 1: *structural criterion binding* and *total criterion not binding.* In this situation the country chooses to use all the room left by the first constraint and, so, plays a zero structural deficit. This can be played only under the assumption that the second constraint is also satisfied by such a move:

$$b_1 = 0 \tag{15}$$

case 2: *second constraint binding* and *first not binding*. In this case the country chooses the structural deficit leading to a total deficit exactly equal to the threshold, assuming that this is consistent with a structural deficit in balance or surplus.

$$b_1 = \frac{t\left(\varphi+\beta\delta+\alpha\varphi\phi\right)+\phi\varphi\left[\alpha b_2\left(1-\omega\right)+\beta p_e\left(1-\delta\right)\right]}{\varphi+\beta\delta+\alpha\varphi\phi\left(1-\omega\right)} \tag{16}$$

case 3: *no constraint binding.* In this case the country does not find it optimal to use all the room for fiscal manoeuvre left by any of the two constraints, so it just plays the unconstrained optimum of this scenario. From the first order condition we then get the following solution, assuming it is consistent with fulfillment of both constraints (and not with equality sign):

$$\left\{ b_1 = \frac{-\alpha b_2 + \alpha b_2 \omega - \beta p_e + \beta p_e \delta}{\alpha \omega} \right\} \tag{17}$$

3.2 Second scenario: structural deficit

The problem in the second scenario becomes:

$$\underset{b_1}{Min} \left\{ \sigma \left(\varphi \frac{\alpha \omega b_1 + \alpha b_2 - \alpha \omega b_2 + \beta p_e - p_e \beta \delta}{\varphi + \beta \delta + \alpha \varphi \phi} \right)^2 + \eta \left(b_1 \right)^2 \right\} \tag{18}$$

$$s.t.$$

$$\begin{aligned} b_1 &> 0 \\ b_1 &\leq \frac{t \left(\varphi + \beta \delta + \alpha \varphi \phi \right) + \phi \varphi \left[\alpha b_2 \left(1 - \omega \right) + \beta p_e \left(1 - \delta \right) \right]}{\varphi + \beta \delta + \alpha \varphi \phi - \varphi \phi \alpha \omega} \end{aligned}$$

we again divide the different cases. The 0 structural deficit is not feasible now (see Figure 1). The equilibrium leading to total deficit exactly equal to the threshold is the same as before. We just need to find the optimum in which both constraints are not binding, which is, once the first order condition is solved:

$$b_1 = \frac{-\sigma \varphi^2 \alpha \omega \left[\alpha b_2 \left(1 - \omega \right) + \beta p_e \left(1 - \delta \right) \right]}{\sigma \varphi^2 \alpha^2 \omega^2 + \eta \left(\varphi + \beta \delta + \alpha \varphi \phi \right)^2} \tag{19}$$

3.3 Third scenario: output-driven excessive deficit

In this scenario the problem becomes:

$$
\begin{aligned}
&\underset{b_1}{Min}\left\{\begin{array}{c}\sigma\left(\varphi\dfrac{\alpha\omega b_1+\alpha b_2-\alpha\omega b_2+\beta p_e-p_e\beta\delta}{\varphi+\beta\delta+\alpha\varphi\phi}\right)^2+\\ \rho^2\left(b_1-\dfrac{t\left(\varphi+\beta\delta+\alpha\varphi\phi\right)+\phi\varphi\left[\alpha b_2\left(1-\omega\right)+\beta p_e\left(1-\delta\right)\right]}{\varphi+\beta\delta+\alpha\varphi\phi-\varphi\phi\alpha\omega}\right)^2\end{array}\right\}\\
&s.t.\\
b_1 &\leq 0 \qquad (20)\\
b_1 &> \frac{t\left(\varphi+\beta\delta+\alpha\varphi\phi\right)+\phi\varphi\left[\alpha b_2\left(1-\omega\right)+\beta p_e\left(1-\delta\right)\right]}{\varphi+\beta\delta+\alpha\varphi\phi-\varphi\phi\alpha\omega}
\end{aligned}
$$

The equilibrium exactly matching the SGP threshold for total deficit is ruled out. The zero deficit equilibrium, instead, is still possible.

Solving for the unconstrained optimum we get[6];

$$
b_1=\frac{-\varphi\left[\alpha b_2\left(1-\omega\right)+B\right]\left(\rho^2\phi J^2+\sigma\varphi\alpha\omega\left(-J+\phi\varphi\alpha\omega\right)\right)}{\left[Y\alpha^2\omega^2+\rho^2J^2\right]\left(-J+\phi\varphi\alpha\omega\right)} \qquad (21)
$$

3.4 Fourth scenario:

In the last scenario the problem becomes:

$$
\begin{aligned}
&\underset{b_1}{Min}\left\{\begin{array}{c}\sigma\left(\varphi\dfrac{\alpha\omega b_1+\alpha b_2-\alpha\omega b_2+\beta p_e-p_e\beta\delta}{\varphi+\beta\delta+\alpha\varphi\phi}\right)^2+\eta\left(b_1\right)^2\\ +\rho^2\left(b_1-\dfrac{t\left(\varphi+\beta\delta+\alpha\varphi\phi\right)+\phi\varphi\left[\alpha b_2\left(1-\omega\right)+\beta p_e\left(1-\delta\right)\right]}{\varphi+\beta\delta+\alpha\varphi\phi-\varphi\phi\alpha\omega}\right)^2\end{array}\right\}\\
&s.t.\\
b_1 &> 0 \qquad (22)\\
b_1 &> \frac{t\left(\varphi+\beta\delta+\alpha\varphi\phi\right)+\phi\varphi\left[\alpha b_2\left(1-\omega\right)+\beta p_e\left(1-\delta\right)\right]}{\varphi+\beta\delta+\alpha\varphi\phi-\varphi\phi\alpha\omega}
\end{aligned}
$$

The equilibria in which either constraint is exactly binding are unfeasible. The unconstrained optimum is, after solving the FOC:

[6] where:
$J=\varphi+\beta\delta+\alpha\varphi\phi$
$B=\beta p_e\left(1-\delta\right)$
$Y=\sigma\varphi^2$

$$b_1 = \frac{-\varphi\left[\alpha b_2\left(1-\omega\right)+B\right]\left(\rho^2\phi J^2+\sigma\varphi\alpha\omega\left(-J+\phi\varphi\alpha\omega\right)\right)}{\left[Y\alpha^2\omega^2+\left(\rho^2+\eta\right)J^2\right]\left(-J+\phi\varphi\alpha\omega\right)} \tag{23}$$

4 Best response functions of deficit

From previous sections we conclude that there are 6 possible deficit choices for each country, meaning 36 possible equilibria; the two equilibria corresponding to either constraint being exactly binding, can in principle be played, as we saw, in two scenarios, but from how the problem is structured we immediately see that both of them are feasible only in one scenario, given what the other country plays. In fact, playing a zero structural deficit leads automatically either to non excessive or excessive deficit, once the move of the other country is specified, so that it is a feasible move only on the first or third scenario respectively. Analogously, the structural deficit consistent with total deficit equal to threshold can be either negative/zero, leading to the first scenario, or positive, leading to the second one. That been said, we proceed with the second step of the procedure previously outlined:

We consider each of the six possible moves of the second country and we calculate all the final *potential* equilibria for deficit, corresponding to all the possible moves by the first country. This allows us to verify for each of them its feasibility. Nonetheless no conclusion could be made on the latter *in general, i.e. for any possible value of parameters.* In order to have unambiguous conclusions we focused on reasonable value ranges: expected inflation rate is considered to be close to 0 (which is the target of the bank), the deficit threshold close to 3 (given the 3% limit actually applied in the Euro-Area) and all the parameters close to 0.5 (including the size parameter which implies assuming a low degree of asymmetry between the two countries). The exception is δ which is assumed to be slightly larger than 1 ($\simeq 1.2$).

Once we ruled out unfeasible solutions, we find the equilibrium minimizing the first country's loss function.

In order to simplify notations, we label from now onwards the deficit choices of the two countries according to the two relevant dimensions: the scenario and the case in which it may arise:

so in general we will write

$$b_1\left(i,j\right) \tag{24}$$

meaning the equilibrium deficit choice of country 1 arising in the scenario $i=1,...,4$ and in the case $j=1,2,3$, we already used this notation in figure 1.

In the rest of the analysis, to ease exposition and avoid too heavy notations, we will not show all the equilibrium expressions, but just limit ourselves to the best responses. One further remark on the procedure concerns the fact that once an equilibrium $[b_1(i,j), b_2(h,k)]$ is ruled out as unfeasible, the same conclusion is drawn for the symmetric one, i.e. for $[b_1(h,k), b_2(i,j)]$, since our assumption that the economic sizes of the two countries are close to equal implies that the two equilibria are very similar as well.

a) Best response to $b_2 = 0 = b_2(i,1), i = 1,3$, i.e. to country 2 playing zero structural deficit:

The first country may choose to be in the first scenario, hence it would have the three options we outlined previously.

1. $b_1(1,1) = 0$

This is feasible, since both the first and second constraints are respected. As far as the latter is concerned we have in fact:

$$\frac{t(\varphi + \beta\delta + \alpha\varphi\phi) + \phi\varphi\beta p_e(1-\delta)}{\varphi + \beta\delta + \alpha\varphi\phi(1-\omega)} > 0 = b_1$$

this means that it will not be a feasible equilibrium in the third scenario which implies violation of the second fiscal constraint: so we can already rule out $b_1(3,1)$.

2. $b_1(1,2)$, i.e. it plays the structural deficit that brings the total deficit size exactly to the threshold of second constraint. This leads to an unfeasible equilibrium since b_1 will be positive contradicting the first constraint of the scenario, i.e. structural deficit being negative or 0. We therefore can already infer that this equilibrium will be feasible only in the second scenario, which implies positive structural deficit.

3. We replicate this reasoning for every potential equilibrium: in this way we see that playing $b_1(1,3)$ is unfeasible.

If the first country chooses to be in the second scenario, we already know $b_1(2,2)$ is feasible, while $b_1(2,1)$ is not, then we need to check $b_1(2,3)$ which leads to a feasible equilibrium.

If the third scenario is chosen we need only to check $b_1(3,3)$, which leads to an unfeasible equilibrium since the second constraint is not respected. If the fourth scenario is played we just need to check the choice $b_1(4,3)$, which leads to an unfeasible equilibrium since the first country would violate the first constraint.

So the potential equilibria are $[b_1(2,2), 0]$, $[b_1(2,3), 0]$ and $[b_1(1,1), 0]$ leading, after calculations, to the following ranking of losses for country one's fiscal authority:

$$L_{23}(0) < L_{11}(0) < L_{22}(0)$$

so the best response to $b_2 = 0 = b_2(i,1)$ is:

$$b_1(2,3) = \frac{-\sigma\varphi^2\alpha\omega\beta p_e(1-\delta)}{\sigma\varphi^2\alpha^2\omega^2 + \eta(\varphi + \beta\delta + \alpha\varphi\phi)^2} \tag{25}$$

meaning that first country plays structural deficit but without running excessive total deficit.

For all the other moves of country two we perform the same analysis, reaching the conclusions that follow.

b) Best response to the second country playing $b_2(i,2), i = 1,2$, i.e. total deficit equal to threshold*:*

$$b_1(1,3) = \frac{-\alpha t(1-\omega) + p_e\beta(\delta - 1)}{\alpha\omega} \tag{26}$$

meaning that the first country plays a structural surplus and does not overcome the threshold for total deficit. This equilibrium leads to a 0 loss for the first country $(L_{13}(b_2(i,2)) = 0)$, while all the other options lead to strictly positive losses.

c) Best response to deficit not violating constraints, i.e. to second country playing $b_2 = \dfrac{-\alpha\omega b_1 - \beta p_e + p_e\beta\delta}{\alpha(1-\omega)} = b_2(1,3)$:

$b_1(2,2) = t$, meaning that the first country plays a structural deficit leading total deficit to be exactly equal to the threshold. This equilibrium leads to the loss $L_{22}(b_2(1,3)) = \eta t^2$[7].

d) Best response to second country playing $b_2(2,3)$ i.e. structural deficit without violating the constraint on excessive total deficit:

$$b_1(2,3) = \frac{-Y\alpha\omega[\alpha b_2(2,3)(1-\omega) + B]}{Y\alpha^2\omega^2 + \eta J^2} = -\frac{\omega Y\alpha B}{\eta J^2 + Y\alpha^2\omega} \tag{27}$$

meaning that the first country plays a structural deficit but runs a total deficit strictly lower than the threshold.

e) Best response to second country playing $b_2(3,3)$, i.e. total deficit violating the second constraint while running a structural budget surplus.

[7] One remark concerns the case $[b_1(1,3), b_2(1,3)]$: solving the corresponding system we see that it is undetermined so we rule out this equilibrium.

In this case we end up with two feasible equilibria: $[b_1(3,3), b_2(3,3)]$ and $[b_1(2,2), b_2(3,3)]$. Nonetheless, some further remarks can be done for both of them.

The equilibrium $[b_1(3,3), b_2(3,3)]$ is feasible in principle but it is very unlikely to arise, since if both countries use a structural surplus they get a strongly negative output gap, which basically means hurting themselves, since they get a recession which is so deep to revert the sign of total deficit (due to automatic stabilizers) to positive and bigger in absolute value than the pact's threshold. This strategy could be optimal only with very high inflation expectations and with demand more strongly affected by interest rate than by fiscal stimulus: in that case negative structural deficit lowers inflation and interest rate leading to a positive impact on the output gap more than offsetting the negative one implied by the direct demand channel. This is intuitively a very unlikely event so we exclude this equilibrium.

Hence, we are left with $b_1(2,2)$, but we can exclude this strategy too. In fact, if the other country plays $b_2(3,3)$, the output gap must be strongly negative, since, although it plays a structural surplus, it runs an excessive total deficit. Now, since the size of automatic stabilizers is by assumption equal for the two countries, the first country, which runs a structural deficit, must end up *a fortiori* with an even higher total deficit than the second one, so it is not possible that it avoids violating the second constraint.

We conclude that there is no feasible response to the second country playing $b_2(3,3)$.

f) Best response to second country playing $b_2(4,3)$, i.e. a deficit violating both constraints:

In this case all possible equilibria turn out to be unfeasible. Therefore, we

conclude that there is no feasible response to country 2 making this move.

5 The Nash equilibrium

We can now sum up the previous results through the following figure:

	Structural deficit = 0	Total deficit = t	$b_2\,(1,3)$	$b_2\,(2,3)$	$b_2\,(3,3)$	$b_2\,(4,3)$
Zero structural deficit: $b_1(1,1)$	feasible	Feasible	Unfeasible: Symm of $b_1(1,3);b_2(i,1)$	feasible	Unfeasible: Symm of $b_1(3,3);b_2(i,1)$	Unfeasible Symm of $b_1\,(4,3);b_2(i,1)$
Z.s.d. in third scenario: $b_1\,(3,1)$	Unfeasible $g_1< t$	Unfeasible $g_1< t$		Unfeasible $g_1< t$		
Def. = t in first scenario $b_1(1,2)$	Unfeasible $b_1 >0$	Unfeasible $b_1 >0$	Unfeasible $b_1 >0$	Unfeasible $b_1 >0$	Unfeasible $b_1 >0$	Unfeasible Symm of $b_1\,(4,3);b_2(i,2)$
D. = t, second scenario $b_1(2,2)$	feasible	Feasible	Feasible **Best response**	feasible	Unfeasible	
$b_1\,(1,3)$	Unfeasible $b_1 >0$	Feasible **Best response**	undetermined	Unfeasible Symm of $b_1(2,3);b_2(1,3)$	Unfeasible Symm of $b_1(3,3);b_2(1,3)$	Unfeasible Symm of $b_1(4,3);b_2(1,3)$
$b_1\,(2,3)$	Feasible **Best resp.**	feasible	Unfeasible $b_1=0$	Feasible **Best response**	Unfeasible Symm of $b_1(3,3);b_2(2,3)$	Unfeasible Symm of $b_1(4,3);b_2(2,3)$
$b_1\,(3,3)$	Unfeasible $g< t$	feasible	unfeasible $g< t$	unfeasible $g< t$	unfeasible	Unfeasible Symm of $b_1(4,3);b_2(3,3)$
$b_1\,(4,3)$	Unfeasible $b_1<0$	Unfeasible $b_1<0$	Unfeasible $b_1=0$	Unfeasible $b_2<0$	Unfeasible $b_1<0$	Unfeasible $b_1<0$

Figure 2

Summing up, there are only *three possible moves for the first country,* which can be represented by the following set:

$$\Theta_1 = \left\{ \begin{array}{c} b_1\,(1,3) = f\,(b_2\,(i,2))\,;b_1\,(2,3) = f\,(b_2\,(i,1) = 0, b_2\,(2,3))\,; \\ b_1\,(2,2) = t = f\,(b_2\,(1,3)) \end{array} \right\} \quad (28)$$

We call this set the *fiscal reaction function* of the first country.

This result holds symmetrically, for the reasons previously outlined, for the second country. Its fiscal reaction function therefore is:

$$\Theta_2 = \left\{ \begin{array}{c} b_2\,(1,3) = f\,(b_1\,(i,2))\,;b_2\,(2,3) = f\,(b_1\,(i,1) = 0, b_1\,(2,3))\,; \\ b_2\,(2,2) = t = f\,(b_1\,(1,3)) \end{array} \right\} \quad (29)$$

We can represent graphically the best responses or *fiscal reaction functions* of the two countries, taking, for simplification, the case of symmetric countries ($\omega = \frac{1}{2}$):

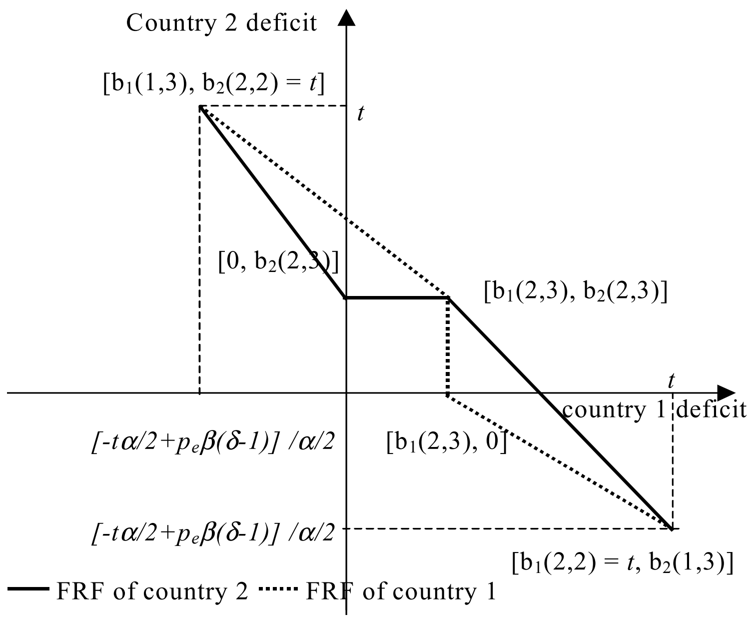

Figure 3

Since the two countries are just playing Nash with each other we end up with three possible Nash equilibria:

$$\Sigma = \{[b_2(1,3), b_1(2,2)]; [b_2(2,3), b_1(2,3)], [b_2(2,2), b_1(1,3)]\} \tag{30}$$

Clearly the first and the last equilibria are just the symmetric of each other.

We now close the model determining the values of economic fundamentals in the Union in all three equilibria and interpreting the results.

If the equilibrium is $\left[b_1(2,2) = t, b_2(1,3) = \dfrac{-\alpha t\omega + p_e\beta(\delta-1)}{\alpha(1-\omega)}\right]$

the equilibrium output gap is:

$$\begin{aligned} y[b_1(2,2), b_2(1,3)] &= \qquad (31) \\ \varphi\frac{\alpha\omega b_1(2,2) + \alpha b_2(1,3) - \alpha\omega b_2(1,3) + \beta p_e - p_e\beta\delta}{\varphi + \beta\delta + \alpha\varphi\phi} &= 0 \end{aligned}$$

The equilibrium inflation is determined by

$$p = \frac{\varphi p_e + \varphi p_e\alpha\phi + \alpha\omega b_1(2,2) + \alpha(1-\omega)b_2(1,3) + \beta p_e}{\varphi + \beta\delta + \alpha\varphi\phi} \Longrightarrow p = p_e \tag{32}$$

The intuition behind this equilibrium is then the following. One country runs a structural deficit up to the point in which it exactly hits the threshold for the total deficit, so it violates the first constraint of the Pact, and *exploits all the room for fiscal expansion left by the second constraint*, without violating it. The other country reacts with a structural fiscal surplus, in order to offset the impact of the other country's policy on output gap, untill it brings the latter to 0, as the fiscal objective function implies.

In a sense there is a division of labour between one country spending a lot and the other compensating for that.

Since the output gap is zero, structural and total deficit coincide, so the first country chooses a deficit exactly equal to the SGP threshold and incurs a loss due to violation of first constraint and proportional to the threshold: $L_1\left(b_1\left(2,2\right)\right) = \eta t^2$.

The other country, instead, is better off, since it runs a surplus and so it does not violate any of the constraints, besides it gets a zero output gap. As a consequence it is in the best possible situation with a zero loss. We therefore observe that the higher the SGP threshold (i.e. the softer the second constraint is), the higher the loss for the first country, since it will violate the first constraint to a larger extent.

It is important to observe that in this case the output gap goes to 0 *whatever the inflation expectations of agents are.* In fact the second country, besides offsetting the partner's policy, also increases its deficit proportionally to inflation expectations in order to undo completely the negative effect of the latter on actual inflation, which in turn would lead to higher interest rates set by the bank and, so, to lower output gap. Yet, inflation expectations turn out to be rational by construction, i.e. the inflation rate is equal to the expected one simply through the inflation equilibrium expression, without need of assuming rational expectations from the start.

As a consequence, this situation is consistent with any possible inflation rate: the output gap will always be 0 and the losses of the two countries end up unchanged since they just care for the output stabilization and not at all about inflation, which is of course a very strong assumption.

In order to close the model we therefore can reasonably say that, since all inflation rates are indifferent for the countries, the central bank, which by mandate must keep inflation as close as possible to its target (0), will choose a zero nominal interest rate. In fact, from the demand side we have:

$$y^D = \alpha\left[\omega\frac{-\alpha t\left(1-\omega\right)+p\beta\left(\delta-1\right)}{\alpha\omega}+\left(1-\omega\right)t\right]-\beta\left(i^*-p\right) =$$
$$\alpha\left[\omega\frac{-\alpha t\left(1-\omega\right)+p\beta\left(\delta-1\right)}{\alpha\omega}+\left(1-\omega\right)t\right]+\beta p = 0$$

Solution is:

$$p = 0 \tag{33}$$

therefore the equilibrium deficit of country two becomes:

$$b_2(1,3) = \frac{-\alpha t\omega + p\beta(\delta-1)}{\alpha(1-\omega)} = \frac{-t\omega}{1-\omega} \tag{34}$$

Finally, we found that also the reversed equilibrium, meaning the second country playing the deficit threshold and the first country compensating, $\left[b_1(1,3) = \frac{-\alpha t(1-\omega) + p\beta(\delta-1)}{\alpha\omega}, b_2(2,2) = t\right]$, is also a Nash equilibrium. All the previous results hold in this case as well.

If the last equilibrium is played:

$$\left[b_1(2,3) = -\frac{\omega\alpha\beta p_e(1-\delta)\sigma\varphi^2}{\eta J^2 + \alpha^2\sigma\varphi^2\omega}, b_2(2,3) = \frac{-\beta p_e(1-\delta)\sigma\varphi^2\left(\eta J^2 + \alpha^2\omega\sigma\varphi^2(1-\omega)\right)}{(\eta J^2 + \alpha^2\sigma\varphi^2\omega)\left[\alpha^2\sigma\varphi^2(1-\omega)^2 + \eta J^2\right]}\right]$$

we get:

$$y\left[b_1(2,3), b_2(2,3)\right] = \frac{\varphi\beta p_e(1-\delta)\eta J\left[\eta J^2 + \omega\sigma\varphi^2(1-\omega)\alpha^2\right]}{\left[\eta J^2 + \sigma\varphi^2\alpha^2\omega\right]\left[\sigma\varphi^2\alpha^2(1-\omega)^2 + \eta J^2\right]} < 0 \tag{35}$$

Both countries violate the first constraint running a structural deficit, meaning that the burden of fiscal expansion to stimulate the aggregate economy is shared. Yet *they do not do that up to the point of exactly hitting the threshold for total deficit*, so the room left by the second constraint is not entirely exploited. They both end up in a second best solution since the output gap turns out to be negative and that implies, together with sanctions for first constraint's violation, positive loss for the two of them. A free-riding problem occurs: the countries do not manage to split the fiscal burden to get a zero output gap. Each country is afraid that the other runs a too low deficit in order to enjoy economic upturn paying a lower price than its partner in terms of sanctions for violation of the first constraint.

On top of that we observe that both countries' deficit is just proportional to inflation expectations: the rationale is, again, that of offsetting the negative impact on output gap of higher interest rates, driven by higher inflation expectations.

Not surprisingly, the two deficit expressions just depend on the sanctions for the violation of the first constraint (η), while those linked to the second one (ρ) do not appear, since it is not binding.

6 Simulations and robustness analysis

In this section we perform some simulations on our model to check whether our solutions, which, as we stressed, are actually valid for a limited range of values for parameters, are robust to some changes in the numbers associated to them.

We first plugged in values close to the baseline scenario we referred to in the above analysis: all values of structural parameters equal to 0.5 except δ set to 1.2. This corresponds to the values used (except for δ) in one of the simulations performed in Beetsma-Debrun-Klassen (2001) for a similar model. The values of the parameters attached to the two constraints of the pact are set to 0.5 as well, meaning a pretty high degree of stringency of those requirements (they can be interpreted as implying that 50% of the excess with respect to the two thresholds for deficits is paid as a fine). In this scenario we distinguish two cases based on the value of inflation expectations: they are set to 0 in the former and to 2% in the latter. Finally we do not assume symmetry of the two countries in terms of economic size to make our results more realistic with respect to the situation in the Euro-Area. Therefore we set $\omega = \frac{2}{3}$, i.e. the first country is twice as large as the second one. We then recalculated the deficit Nash equilibria in both cases, corresponding to the first and second column of the following figure respectively.

In case of 2% inflation expectation (second column) we find the same three equilibria we previously found. We see that the symmetric equilibrium (i.e. both countries playing the same deficit strategy) basically implies very low deficit rates (0.01% and 0.005% of GDP for the two countries respectively). In the zero inflation expectation case (first column) we see that we do not have anymore the equilibrium $b_1(2,3), b_2(2,3)$: it is replaced by the one in which both countries play 0 structural deficits: this is no surprise, since the symmetric equilibrium has both deficits linear in inflation expectation, so when the latter is 0 both deficits go to 0 as well. In the fourth row we show the output gaps corresponding to the different deficit equilibria. In the lower rows we performed an exercise of *welfare ranking* of the three equilibria, based on an utilitarian welfare function: we found the sum of the two countries' losses associated to each of the three equilibria and, in the last row, we identified the equilibrium which minimizes that sum. In both cases the symmetric equilibrium is the best one, moreover, with zero inflation expectation, it leads to the first best situation of zero cumulative loss. These results fully confirm our previous finding.

In the third and fourth columns the values are the same as before except for the two fiscal rules' parameters which are both set to 0,1. This is a more realistic situation since, in the actual implementation of the SGP, the first constraint is not very strictly adhered to, while the rules on excessive deficit can be roughly approximated by assuming that 10% of the excess of deficit over threshold is actually paid as a fine (see Bolt (1999)). So the pact is now supposed to be less strict.

The three Nash equilibria are the same as in the previous scenario, with the same distinction between 0 and 2% inflation expectations. In the latter case the symmetric equilibrium implies higher deficits than in the similar case with stricter pact, this is no surprise: when the sanction is softer both countries violate the first constraint to a larger extent. The asymmetric equilibria are instead identical to the case of tougher pact since their expressions, as we saw, do not depend on the rules' parameters on the fiscal loss function but just on the deficit threshold. The symmetric equilibria are still those leading to the highest welfare.

In the last two columns we make a more significant change: we use the values for structural parameters taken from the baseline simulation performed in Buti-Van den Noord (2003), who check a model similar to ours. Surprise inflation as well as deficit have now a higher impact on output while fiscal authorities care much more about output stabilization. The parameters on the Pact rules are kept to 0.1. Two cases are distinguished: inflation expectations equal to 2% and to 6% respectively.

The picture changes then substantially: no more symmetric equilibria arise and in both cases $b_1(2,3), b_2(1,1)$ becomes a Nash equilibrium, so the first country violating the first constraint and the second (smaller) playing zero deficit. The usual asymmetric equilibria stay in the case of 2% inflation expectation, while in the 6% case only one is kept: $b_1(2,2), b_2(1,3)$ but not the reversed one. Intuitively when inflation is higher there is need that the big country (which is the first one) and not the small one stimulates the economy to offset the tough monetary policy stance due to high inflation expectations. $b_1(2,3), b_2(1,1)$ is in both cases the equilibrium with the highest welfare, moreover it implies a first country's structural deficit significantly higher than in the best equilibria of the other cases: 0.51% and 1.54% of GDP with 2 and 6% inflation expectation respectively. It is interesting to notice that in this scenario either, the option of violating the second constraint of the pact is never adopted.

	B-D-K values, strict SGP, p_e=0	B-D-K values, strict SGP, p_e=2	B-D-K values, soft SGP, p_e=0	B-D-K values, soft SGP, p_e=2	B-vdN values, soft SGP, p_e=2	B-vdN values, soft SGP, p_e=6
1st NE	$b_1(1,1)$, $b_2(1,1)$= (0,0)	$b_1(2,3)$, $b_2(2,3)$= (0.01,0.005)	$b_1(1,1)$, $b_2(1,1)$= (0,0)	$b_1(2,3)$, $b_2(2,3)$= (0.05,0.025)	$b_1(2,3)$, $b_2(1,1)$= (0.51,0)	$b_1(2,3)$, $b_2(1,1)$= (1.54,0)
2nd NE	$b_1(1,3)$, $b_2(2,2)$ = (-1.5,3)	$b_1(1,3)$, $b_2(2,2)$ =(-0.9,3)	$b_1(1,3)$, $b_2(2,2)$= (-1.5,3)	$b_1(1,3)$, $b_2(2,2)$= (-0.9,3)	$b_1(1,3)$, $b_2(2,2)$= (-0.75,3)	$b_1(2,2)$, $b_2(1,3)$ =(3,-1.5)
3rd NE	$b_1(2,2)$, $b_2(1,3)$= (3,-6)	$b_1(2,2)$, $b_2(1,3)$= (3,-4.8)	$b_1(2,2)$, $b_2(1,3)$= (3,-6)	$b_1(2,2)$, $b_2(1,3)$ =(3,-4.8)	$b_1(2,2)$, $b_2(1,3)$ =(3,-4.5)	
Output gap	(0,0)⇒0 (-1.5,3)⇒ $6*10^{-6}$ (3,-6)⇒ $-1.2*10^{-5}$	(0.01,0.005) ⇒-0.08 (-0.9,3)⇒ $3*10^{-6}$ (3,-4.8)⇒ $-1.26*10^{-5}$	(0,0)⇒0 (-1.5,3)⇒ $6*10^{-6}$ (3,-6)⇒ $-1.2*10^{-5}$	(0.05,0.025) ⇒-0.073 (-0.9,3)⇒ $3*10^{-6}$ (3,-4.8)⇒ $-1.26*10^{-5}$	(0.51,0)⇒ -0.09 (-0.75,3)⇒ $1.75*10^{-5}$ (3,-4.5)⇒0	(1.54,0)⇒ -0.27 (3,-1.5)⇒0
L_1+L_2	(0,0)⇒0 (-1.5,3)⇒ 4.5 (3,-6)⇒4.5	(0.01,0.005) ⇒0.00644 (-0.9,3)⇒ 4.5 (3,-4.8)⇒ 4.5	(0,0)⇒0 (-1.5,3)⇒ 0.9 (3,-6)⇒0.9	(0.05, 0.025)⇒ 0.00566 (-0.9,3)⇒ 0.9 (3,-4.8)⇒ 0.9	(0.51,0)⇒0. 05 (-0.75,3) ⇒0.9 (3,-4.5)⇒ 0.9	(1.54,0) ⇒0.456 (3,-1.5)⇒ 0.9
Best NE	(0,0) L_1=0 L_2=0	(0.01,0.005) L_1=0.0032 L_2=0.00324	(0,0) L_1=0 L_2=0	(0.05,0.025) L_1=0.0029 L_2=0.0027	(0.51,0) L_1=0.1 L_2=0.347	(1.54,0) L_1=0.1 L_2=0.347

Figure 4

7 Conclusions

We can now sum up the results of the exercise we carried on.

We built up a simple model with a two-countries monetary union in which the two fiscal policy-makers simultaneously choose their deficit taken that chosen by the other one as given. They are sanctioned if they do not fulfill two fiscal rules close to those of the Stability and Growth Pact and concerning the structural and total deficit respectively.

The (very few) analysis carried on in the literature neglected the fact that those rules are *asymmetric*, i.e. the sanction/fine is raised whenever the deficit is *higher* than a threshold, but not when it is lower. In the current paper this issue is analyzed allowing explicitly for this asymmetry in the fiscal policy-makers' loss function.

We therefore divided the fiscal optimization problem in four scenarios, based on whether either, both or none of the two SGP constraints is violated by each country. Then we calculated the three Nash equilibria of deficit.

The first two are each the symmetric of the other, and they correspond to the case in which one country runs a structural deficit up to the point in which it exactly hits the threshold for the total deficit, so it violates the first constraint of the Pact and *exploits all the room for fiscal expansion left by the second constraint*. On the other hand the other country responds with a structural fiscal surplus in order to offset the impact of the other country's policy on output gap, untill it brings the latter to 0. Moreover it sets its fiscal policy so that it undoes the negative effect of higher inflation expectations on the output gap. The zero output gap, in turn, implies rational inflation expectations. In a sense there is a division of labour between one country spending a lot and the other compensating for that. The country playing structural deficit bears a loss because of violation of the first constraint while the other gets to the first-best situation with zero loss.

In the third equilibrium the two countries play the same fiscal strategy: they both violate the first constraint running a structural deficit, but they do not exploit all the room for manoeuvre left by the second constraint. They share equally the burden of fiscal expansion to stimulate the aggregate economy. Nonetheless they end up in a second best solution since the output gap turns out to be negative. The first best is prevented by free-riding: each country is afraid that the other runs a too low deficit in order to reap the benefits of higher output paying a lower price in terms of sanctions for violation of the first constraint.

In the first equilibrium we see that we end up with zero output gap and zero inflation, but we could get exactly the same results with both countries playing 0 deficits and so both enjoying zero losses (see appendix). The Pact does not let this first best outcome arise: the uncoordinated setting of fiscal policies under the pact's constraints lead either to fiscal expansion by one country, totally undone by the other, leading to an inefficient loss born by the former, or by a symmetric but softer fiscal expansion which leads to an inefficiently low fiscal stimulus which is not enough to offset the negative impact on output of positive inflation expectations.

The simulations and robustness analysis performed in the last section show that the above results are quite robust to changes in the extent to which the two fiscal rules are binding, but less robust with respect to changes in the values of structural parameters of the model: the symmetric deficit equilibrium seems to disappear if we deviate from the baseline scenario.

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8 Appendix

8.1 Symmetric Nash equilibrium with rational expectations

We now reconsider the equilibrium $[b_2(2,3), b_1(2,3)]$ adding the assumption that the private agents have rational expectations, i.e. they are able to perfectly foresee (in this context where no stochastic component is present) the inflation rate, so we have $p = p_e$ which means that also the output gap (through the supply function) will end up being 0. Moreover, replacing p_e with p in the equilibrium inflation expression we see that it can hold only if $p = 0$,which leads

to: $y[b_1(2,3), b_2(2,3)] = b_1(2,3) = b_2(2,3) = 0$

since they are all linear in p. This means that this equilibrium, unlike the asymmetric one, leads to the first-best solution when inflation rational expectations are assumed: zero output gap and zero deficits lead to zero losses for both countries.

8.2 The first-best solution: zero deficits by both countries

In this sub-section we consider the *first best* situation in a general framework where we do not make any assumption on inflation expectations.

First-best means zero losses for both countries, which in turn implies zero output gap, which in turn means that structural and total deficit coincide.

$$y[b_1, b_2] = \varphi \frac{\alpha\omega b_1 + \alpha b_2 - \alpha\omega b_2 + \beta p_e - p_e \beta\delta}{\varphi + \beta\delta + \alpha\varphi\phi} = 0, \text{ Solution is:}$$

$$b_2 = \frac{1}{\alpha - \alpha\omega}(-\beta p_e - \alpha\omega b_1 + \beta\delta p_e) \tag{36}$$

Zero losses for both countries also mean no violation whatsoever of the constraints of the stability and growth pact, i.e. negative or zero structural deficit (which implies automatically no violation of the second constraint either)

Zero output gap also implies $p = p_e$ from the supply side of the economy, so rational expectations are again endogenously determined by the model with no need of assuming them exogenously in the first place.

Than conditions for first best are summarized by the following system:

$$\begin{aligned} b_2 &= \frac{-\alpha\omega b_1 + \beta p(\delta - 1)}{\alpha - \alpha\omega} \\ b_i &\leq 0 \quad i = 1, 2 \end{aligned} \tag{37}$$

One solution is clearly $b_1 = b_2 = 0$, which leads in turn, from equilibrium inflation expression, to $p = 0$

Is there any possible solution with $p \neq 0$?

if $p > 0 \Rightarrow b_2 \leq 0 \Leftrightarrow b_1 > 0$ so this solution is ruled out
if $p < 0 \Rightarrow b_2 = 0 \Leftrightarrow b_1 = \frac{\beta p(\delta - 1)}{\alpha\omega} < 0$ or $b_2 < 0 \Leftrightarrow b_1 > \frac{\beta p(\delta - 1)}{\alpha\omega}$

so solutions are:

$$\{b_1 = b_2 = p = 0\}, \left\{b_2 = 0, p < 0, b_1 = \frac{\beta p(\delta - 1)}{\alpha\omega}\right\}, \tag{38}$$
$$\left\{b_1 = 0, p < 0, b_2 = \frac{\beta p(\delta - 1)}{\alpha(1 - \omega)}\right\}, \left\{b_2 < 0, p < 0, \frac{\beta p(\delta - 1)}{\alpha\omega} < b_1 \leq 0\right\}$$
$$\left\{b_1 < 0, p < 0, \frac{\beta p(\delta - 1)}{\alpha(1 - \omega)} < b_2 \leq 0\right\}$$

So either there is no inflation and zero deficits from both countries or there can be deflation with either country running structural surplus while the other runs a balanced budget or with both countries running a surplus.

That comes from the fact that the two countries do not care about inflation *per se,* so deflation can lead to zero loss since they can undo the effects on output gap, which occurs through a negative interest rate, using a budget surplus, which they can do indefinitely without running any punishment from the pact.

Nonetheless, if we replicate the reasoning on the zero inflation target of the bank we outlined commenting the asymmetric Nash equilibrium, we can rule out all the deflationary first-best cases:

In fact from the demand channel we must have:

$$y^D = \alpha \left[\omega b_1 + (1-\omega) \frac{\beta p (\delta - 1) - \alpha\omega b_1}{\alpha - \alpha\omega} \right] - \beta (i - p) = 0$$

if the bank then sets $i = 0$, foreseeing zero inflation, that will actually arise:

$$y^D = \alpha \left[\omega b_1 + (1-\omega) \frac{\beta p (\delta - 1) - \alpha\omega b_1}{\alpha - \alpha\omega} \right] + \beta p = 0$$
$$\Rightarrow \beta\delta p = 0 \Rightarrow p = 0$$

which is consistent, as we saw, only with zero deficits by both countries.

We conclude that the general first-best equilibrium is: $\{b_1 = b_2 = p = 0 \Longrightarrow L_1 = L_2 = 0\}$