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**"SECRET" BUY - BACKS OF LDC  
DEBT**

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**INTERNATIONAL  
MACROECONOMICS**



**Centre for Economic Policy Research**

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Centre for Economic Policy Research  
6 Duke of York Street  
London SW1Y 6LA  
Tel: (44 71) 930 2963

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## ABSTRACT

### "Secret" Buy-Backs of LDC Debt\*

We analyse the buy-back of its debt by an LDC. Contrary to the analyses that were previously done on this subject, we assume that the debtor can hide its transactions behind the veil of a fictitious operator: the banks, we assume, cannot discriminate intra-bank transactions from buy-backs by the debtor itself. With this assumption, the lenders set the price by (rationally) taking account of the country's incentive to repurchase its debt. Will the debtor undertake a buy-back of its debt? The answer is a qualified yes. Large buy-backs will take place. With a continuum of debtors (whose cash constraints are not perfectly known to the banks), the rich debtors will attempt to repurchase as much of their debt as their cash constraint allows them to. This is shown to be Pareto-improving (both the banks and the country like it).

JEL classification: 110, 430

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Daniel Cohen  
CEPREMAP  
142 rue du Chevaleret  
75013 Paris  
FRANCE  
Tel: (33 1) 40778459

Thierry Verdier  
DELTA  
48 Boulevard Jourdan  
75014 Paris  
France  
Tel: (33 1) 48814302

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Submitted 23 August 1990

## NON-TECHNICAL SUMMARY

Recent controversies on LDC buy-backs of their own debt have questioned the idea that the buy-backs may be a good business for the debtors. The argument against buy-backs may be stated as follows. One more dollar of a country's debt yields two different dividends to an (outside) investor: the first is that (perhaps in some good states of nature) it raises the net payments that the country will pay to its creditors; the second is that it nominally inflates the property right of the *one* investor who purchases the extra dollar to the detriment of the others. When it buys back one dollar on the secondary market, a country pays for the two dividends (the average price of the debt) when it is obviously interested only in the first (the marginal price of the debt, as Bulow and Rogoff put it).

This discrepancy explains why marginal buy-backs are not likely to be profitable to the debtor. As far as *large* buy-backs are concerned, things get even worse since the debtor must pay for a third term, which represents the capital gain that it offers to its creditors by reducing its overall debt. As one sees, however, the argument against large buy-backs hinges on the assumption that the buy-back is known to the creditors.

Assume rather that the debtor succeeds in dissimulating its transaction behind the veil of a fictitious outside operator. If well hidden, the false outside investor can very well buy back all of the country's debt at the *ex ante* price. When Citicorp sells its claims on Brazil to Morgan, the transaction does not change the price on the secondary market (if the markets are efficient and the investors well diversified). It is only when the buyer is identified as the country itself that the price of the debt rises, since only in that case is the solvency of the country improved by the purchase of the debt. When the country succeeds in hiding its identity, a buy-back can turn out to be a good deal: even though it overpays for the 'first' dollar that it buys back, it may end up underpaying for the 'last' dollar if the transaction is large enough.

The best set of circumstances under which the debtor will be induced to buy back its debt is obviously when its purchases are neither observed nor expected by the creditors. Assume, however, that the lenders are rational and lack no information on the country's characteristics (resources and preferences). Even though they may not observe the actual transactions of the debtor on the secondary market, they may foresee its incentive to transact (and consequently raise the price at which they are ready to sell). Do such assumptions preclude that buy-backs will be undertaken by a utility-maximizing debtor? It is this question that the paper addresses.

Intuitively, one sees immediately why – at least under certain circumstances – some buy-backs will take place. Assume rather that no buy-backs will take place,

hence that they are not expected by rational investors: a large buy-back would then offer the country an opportunity to repurchase all its debt at a cheap price. But, on the other hand, nor can it be the case that the large buy-back is fully anticipated by the creditors: otherwise the buy-back would be fully acknowledged, and we have seen why this would not be rewarding for the debtor.

One consequently sees that an equilibrium must be somehow 'in between' these two polar cases. Assume, for instance, that the creditors do not perfectly identify the cash constraints faced by the debtors. We show that a positive fraction of the debtors (in the rich segment) will certainly undertake the buy-back. Furthermore, we show that the 'secret' buy-backs raise the welfare of the debtor that undertakes them *and* raise the profits of the banks.

'Secret' buy-backs appear indeed to be a way of alleviating the free-riding problem that keeps the banks from reaching an efficient agreement with the debtors. This result may help to close the gap between the normative literature (arguing against buy-backs) and the positive literature, which shows that countries like to undertake buy-backs and, indeed, most often do so secretly.

## I - INTRODUCTION

Recent controversies on the buy-back of LDC debt have questioned the idea that buy-backs may be a good business for the debtors (see Bulow and Rogoff (1988) and Sachs (1988) for the main features of the controversy, and the useful survey by Claasens and Diwan (1989) for an overall view on the debate).

The argument against buy-backs may be stated as follows. One more dollar of a country's debt yields two different dividends to an (outside) investor. The first one is that (perhaps in some good states of nature) it raises the net payments that the country will pay to its creditors; the second component is that it nominally inflates the property right of the one investor who purchases the extra-dollar to the detriment of the others. When it buy-backs one dollar on the secondary market, a country pays for the two dividends, (the average price of the debt) when it is obviously interested by the first one only (the marginal price of the debt, in Bulow and Rogoff's words).

This discrepancy explains why marginal buy-backs are not likely to be profitable to the debtor. As far as large buy-backs are concerned, things get even worse since the debtor must pay, for a third term which represents the capital gain which it offers to its creditors by reducing its overall debt. As one sees, however, the argument against large buy-back critically hinges on the assumption that the buy-back is known to the creditors. Assume instead that the debtor succeeds in dissimulating its transaction behind the veil of a fictitious outside operator. If well hidden, the false outside investor can very well buy-back all of the country's debt at the ex-ante price. When Citicorp sells its claims on Brazil to Morgan, the transaction does not change the price on the secondary market (if the markets are efficient and the investors well diversified). It is only when the buyer is identified to be the country itself that the price of the debt rises, since, only in that case, the solvency of the country is improved by the purchase of the debt. When the country succeeds to hide its identity a buy-back can turn out to be a good deal: even though it overpays the "first" dollar

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that it buys back, it may end up underpaying the "last" dollar if the transaction is large enough.

The best set of circumstances under which the debtor will be induced to buy-back its debt is obviously when its purchases are neither observed nor expected by the creditors. Assume however that the lenders are rational and lack no informations on the country's characteristics (ressources and preferences). Even though they may not observe the actual transactions of the debtor on the secondary market, they may foresee its incentive to do so (and raise consequently the price at which they are ready to sell). Are such assumptions ruling out that buy-backs will be undertaken by a utility-maximizing debtor? It is this question which this paper attempts to answer.

Intuitively, one sees immediately why -at least under certain circumstances- the answer must be that, yes, some buy-backs will take place. Indeed, assume instead that no buy-backs take place, hence that they are not expected by rational investors. Large buy-back would then offer the country with an opportunity to repurchase all its debt at a cheap price. But, on the other hand, it cannot either be the case that the large buy-back is fully anticipated by the creditors: otherwise we would be back to the case when the buy-back is fully acknowledged and we have seen why this would not be rewarding for the debtor. One consequently sees that an equilibrium must be somehow "in between" these two polar cases. Technically, we shall see indeed that there is an equilibrium in the space of mixed strategies: the debtor randomizes its purchases of debt so that, in "average", a positive amount of debt is bought back, but the creditors can never know if it will occur for sure.

A (perhaps) more appealing version of this result is spelled out in section IV of the paper. Rather than assuming that the debtors' characteristics are perfectly known by the creditors, we assume that there is a continuum of them (some rich, some poor) which the creditors cannot perfectly identify. We then show that a positive fraction of the debtors (in the rich segment) will undertake the buy-back for sure.



In the last section of the paper, we assess the welfare implications of secret buy-backs. We first show that they are always Pareto-improving. In the first case, however, (when there is only one type of debtor), we show that all the benefits accrue to the creditors (while the debtor is ex-post indifferent). On the other hand, in the case with a continuum of debtors, we shall see that all debtors which are in the rich segment (and can afford to buy-back their debt) do make a positive gain (and share with the creditors the benefit of reducing the face value of the debt; the poor debtors are obviously indifferent). This result may help closing the gap between the normative literature (arguing against buy-backs) and the positive one (see Eichengreen and Portes (1989)) which shows that countries do like to undertake them, and most often secretly).

## II - A FRAMEWORK OF ANALYSIS

### 1. Technology of production

We assume that the country's output in periods 1 and 2 can be characterized as follows. In period 1, the country can produce  $Q_1$  units of a good that can be indifferently used for investment, consumption or repurchasing the debt. In period 2, the output is a number  $\tilde{y} + f(I)$  in which:  $\tilde{y}$  is a random variable distributed over the interval  $(y, \bar{y})$  with a cumulative differentiable distribution  $G(y)$ ;  $I$  is the amount of investment undertaken in period 1.  $f$  is a concave differentiable function. (The model and notations are borrowed from Claasens and Diwan (1989); see also Froot (1989), Helpman (1989) and Williamson (1988) for a related apparatus).

### 2. The debt

The country owes initially an external debt  $D_0$  which must be serviced at time  $t=2$ . (We set the discount-factor equal to 1 for simplicity). The country pays at most a fraction  $\underline{a}$  of its output to the creditors (for the Eaton-Gersovitz (1981)'s reason that it may repudiate the debt; see Cohen (1990) for details and extensions). The repayment of

a debt whose contractual value is  $D_0$  in period 2 is therefore equal to the amount:

$$R_2 = \text{Min} \left\{ D_0, a [y+f(I)] \right\}$$

We assume that the debt is traded in period 1 on a secondary market. Assuming that the country repurchases a stock  $X$  of its external debt at a price  $p$  (see below on how the price  $p$  is set), the two periods stream of consumption of the country can be written as follows

$$(1) \begin{cases} C_1 = Q_1 - p X - I \\ C_2 = y + f(I) - \text{Min} \left\{ D_0 - X, a [y+f(I)] \right\} \end{cases}$$

or equivalently

$$(2) \quad C_2 = \int_{\underline{y}}^{y^*(I,X)} (1-a)[y + f(I)] dG(y) + \int_{y^*(I,X)}^{\bar{y}} [y + f(I) + X - D_0] dG(y)$$

in which  $y^*(I,X) = \frac{D_0 - X}{a} - f(I)$  is the threshold above which the (remaining) contractual value of the debt,  $D_0 - X$ , is repaid in full.

### 3. The secondary market price of the debt

Contrarily to the previous analysis of a debt-buy back (such as surveyed in Claasens and Diwan) we are interested here in analyzing the mechanics of a "secret" buy-back. We consequently assume the following structure. At time  $t=1$ , The price  $p$  at which the banks agree on selling their debt is set without them observing how much is intra-bank operations and how much is actually sold to the country. Furthermore, we must (equivalently) also assume that the investment  $I$ , which is chosen by the debtor, is unobservable to the banks. Hidden behind a fictitious operator, the country repurchases  $X$  dollars of its own debt at the price  $p$ . Obviously, the price  $p$  is set by the banks so as to take account of

the opportunities that it opens to the debtor. To the extent however that, we assume, the banks cannot discriminate intra-banks operations from buy-backs by the debtor, the price at which all transactions are made is a constant that the country can take to be independent of its actual transactions (see below).

#### 4. The country's welfare

We assume that the country's welfare is measured by  $U(C_1, C_2) = C_1 + C_2$ . For simplicity, we assume that the investment and buy-back decisions are (simultaneously) chosen by a social planner. Given equation (2), the welfare of the country which is associated to a decision  $(X, I)$  (when the secondary market price is  $p$ ) can be written as :

$$(3) \quad U[X, p, I] = Q_1 - pX - I + \int_{\bar{y}}^{y^*(I, X)} (1-a)[y + f(I)] dG(y) + \int_{y^*(I, X)}^{\bar{y}} [y + f(I) + X - D_0] dG(y)$$

$$\text{with } y^*(I, X) = \frac{D_0 - X}{a} - f(I).$$

One can readily show the following:

Lemma - Let  $V(p, X) \equiv \text{Max}_I U [X, p, I]$ .  $V(p, X)$  is a convex function of  $X$ .

Proof - The first order condition which characterizes the country's investment decision is :

$$(4) \quad \frac{\partial U}{\partial I} = 0 \Leftrightarrow f'(I) \left\{ 1 - a G \left[ \frac{D_0 - X}{a} - f(I) \right] \right\} = 1$$

Let  $I(X)$  be the investment decision associated to the decision  $X$ .

$$\text{Call } G^* = \hat{G} \left[ \frac{D_0 - X}{a} - f(I^*) \right] ; g^* = G' \left[ \frac{D_0 - X}{a} - f(I^*) \right]$$

One can write (through the envelope theorem) that

$$\frac{\partial V}{\partial X} = \frac{\partial U}{\partial X} [X, p, I(X)]$$

so that

$$\frac{\partial^2 V}{\partial X^2} = \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial X \partial I} [X, p, I(X)].$$

$$\text{From (4), one can see that } \frac{dI}{dX} = - \frac{g^* f'}{f'' [1-G^* a] + a g^* f'^2} > 0$$

if and only if  $f''$  is large small enough (as required by the second order optimality condition in (4)).

Furthermore, one can also see that

$$\frac{\partial V}{\partial X} = - p + [1-G^*]$$

$$\frac{\partial^2 U}{\partial X^2} = g^* \frac{1}{a} ; \quad \frac{\partial^2 U}{\partial X \partial I} = f' g^*$$

$$\text{so that } \frac{\partial^2 V}{\partial X^2} = g^* \left[ \frac{1}{a} + f' \frac{dI}{dX} \right] > 0 . \text{ QED}$$

We shall call  $I_0$  (resp.  $I_{D_0}$ ) the investment that is undertaken when the country makes no buy-back (resp. when it repurchases all its debt).

### III - THE EQUILIBRIUM ON THE SECONDARY MARKET

#### 1. The banks' strategy

Let us assume, here, the banks anticipate that the country will buy-back an amount  $X^e$  on the secondary market (and let  $I^e$  be the corresponding expectation for  $I$ ).

The market value of the remaining  $D_0 - X^e$  is estimated by the banks to be :

$$V^e = (D_0 - X^e)[1 - G^*] + \int_y^{y^*(I^e, X^e)} a [y + f(I)] dG(y)$$

The price at which the banks are consequently willing to sell the debt on the secondary market is :

$$(5) \quad p = 1 - G^* + \frac{a}{D_0 - X^e} \int_y^{y^*(I, X^e)} [y + f(I^e)] dG(y)$$

One can readily check that

$$\frac{dp}{dX^e} = \frac{a}{(D_0 - X^e)^2} \int_y^{y^*(I, X)} [y + f(I^e)] dG(y) + \frac{a}{D_0 - X^e} G^* \frac{\partial I}{\partial X^e} f' > 0$$

Let  $p_0$  be the price of the debt when  $X^e=0$ .

## 2. The country's strategy

Let us now investigate how much buy-back the country will undertake when the price at which it can repurchase its debt is  $p$ . With the preceding notations one simply looks for

$$X^* = \underset{X}{\text{Argmax}} V(p, X).$$

Now, to the extent that we have shown that  $V(p,X)$  is a convex function of  $X$ , we must necessarily have a corner solution :

$$X = 0$$

or  $X = \bar{X}(Q_1,p)$  in which  $\bar{X}$  is a solution to  $\bar{X} = \text{Max}[X_1, D_0]$ , with  $X_1$  itself a solution to  $I(X) + p X = Q_1$  (i.e.,  $C_1=0$ ).

Since  $-\frac{\partial V}{\partial X} \Big|_{X=0} = -p + [1-G^*] \leq -p_0 + (1-G^*) < 0$ , one can see that the geometry of the problem can be depicted as in figure 1.

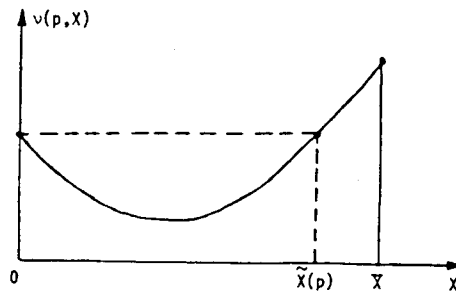


FIGURE 1

Let  $\tilde{X}(p)$  be the solution, if it exists, to the equation :

$$(6) \quad V[p,0] = V[p,\tilde{X}(p)] .$$

In order to characterize more specifically the function  $\tilde{X}(p)$  let us prove the following lemma :

Lemma 1 - The function  $\tilde{X}(p)$  (the solution to equation (6)) exists if and

only if  $p \leq \bar{p}$  with  $\bar{p} = p_0 + \frac{[f(I_{D_0}) - I_{D_0}] - [f(I_0) - I_0]}{D_0}$  and one has :

$$\tilde{X}'(p) > 0 .$$

The lemma's intuitive content is simply the following: in order to have a solution to (6) it must be that repurchasing all the debt is

better than doing nothing, non withstanding the non-negativity constraint on  $C_1$ .

Proof - The left-hand side of equation (6) can be written as

$$V[p,0] = Q_1 - I_0 + E(y) + f(I_0) - p_0 D_0$$

and (obviously) does not depend upon  $p$ .

The right-hand side of equation (6) is:

$$V[p,\tilde{X}] = Q_1 - p \tilde{X} - I(\tilde{X}) + E(y) + f [I(\tilde{X})] - p[D_0 - \tilde{X}]$$

Given the shape of  $V(p,.)$ ,  $\tilde{X}$  is well defined if and only if  $V[p,D_0] \geq V[0,p]$ . The necessary and sufficient condition for  $\tilde{X}$  to exist is therefore that  $p \leq \bar{p}$ , in which  $\bar{p}$  is the solution to  $V[\bar{p},D_0]=V(.,0)$ . From the comparison between the LHS and the RHS of equation (6), one sees that  $\bar{p}$  is a solution to  $f(I_0) - I_0 - p_0 D_0 = f(I_{D_0}) - I_{D_0} - \bar{p} D_0$ . QED.

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Subject to the condition that  $p \leq \bar{p}$ , one can consequently write the country's best response (in the space of mixed strategy) as follows:

$$(7) \left\{ \begin{array}{l} X(p) = 0 \text{ if } \tilde{X}(p) > \bar{X} \\ X(p) = \bar{X}(Q_1,p) \text{ if } \tilde{X}(p) < \bar{X}(p,Q_1) \\ X(p) = 0 \text{ with probability } q \\ X(p) = \bar{X}(Q_1,p) \text{ with probability } 1-q \end{array} \right\} \text{ if } \tilde{X}(p) = \bar{X}(p,Q_1).$$

(with  $q$  any number  $\in [0,1]$ )

### 3. Existence of an equilibrium on the secondary market

In order to prove the existence of an equilibrium, let us now show the following lemma:

Lemma 2 - If the following condition is satisfied:

$$(8) \quad I [\tilde{X}(p_0)] + p_0 \tilde{X}(p_0) < Q_1$$

then there exists a unique price  $p(Q_1)$  which satisfies :

$$\tilde{X}(p) = \bar{X}(p, Q_1) \text{ and one has } p(Q_1) > p_0.$$

Proof - Let us consider the following function :

$$(9) \quad \varphi(p) = \bar{X}(p, Q_1) - \tilde{X}(p).$$

One has that:

$$\varphi'(p) = \frac{\partial \bar{X}}{\partial p} - \tilde{X}'(p) < 0$$

(since  $\partial \bar{X} / \partial p < 0$  and  $\tilde{X}'(p) > 0$  cf. lemma 1).

Inequality (8) is equivalent to:  $\bar{X}(p_0, Q_1) > \tilde{X}(p_0)$  (the country has an incentive to repurchase its debt if it is not expected to do so by the creditors). One therefore has that  $\varphi(p_0) > 0$ . On the other hand, when  $p = \bar{p}$ , one has that  $\tilde{X}(\bar{p}) (= D_0) \geq \bar{X}(\bar{p}, Q_1)$  (the country will never to repurchase more than all of its debt). One therefore has that  $\varphi(\bar{p}) < 0$ . There consequently exists a unique  $p(Q_1) \in ]p_0, \bar{p}]$  such that  $\tilde{X}(p) = \bar{X}(p, Q_1)$ . QED.

Subject to inequality (8) (which we now assume is satisfied), one can now fully characterize the country's response as follows:

$$(10) \quad \left. \begin{array}{l} X(p) = 0 \quad \text{if } p > p(Q_1) \\ X(p) = X(p, Q_1) \quad \text{if } p < p(Q_1) \\ X(p) = 0 \quad \text{with probability } q \\ \quad = \bar{X}(p, Q_1) \quad \text{with probability } 1-q \end{array} \right\} \text{ if } p=p(Q_1), \text{ for any } q \in [0, 1].$$



Geometrically, the country's response comes as in figure 2.

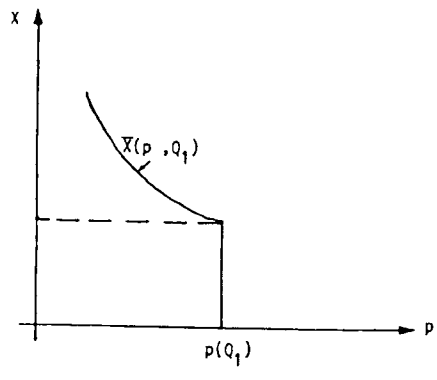


FIGURE 2

We may now characterize the Nash equilibrium to prevail on the secondary market as a pair  $(X^*, p^*)$  for which the banks and the country's response are consistent one with the other.

The geometry of the problem can readily be shown to be as in figure 3.

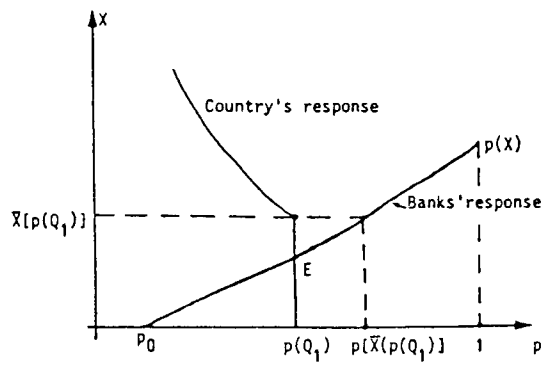


FIGURE 3

One can see that the banks' and the country's responses must intersect each other at a point on the vertical part of the country's response. Indeed, on the one hand one knows that  $p_0 < p(Q_1)$ . On the

other hand, one also knows that the price  $p(Q_1)$ , (which is the solution to  $X[p(Q_1)] = \bar{X}[p(Q_1)]$ ) is such that  $\left. \frac{dV}{d\bar{X}} [p(Q_1), X] \right|_{X=\bar{X}} > 0$  so that  $p(Q_1) < p[\bar{X}[p(Q_1)]]$ .

We can now show:

Proposition - There exists a unique equilibrium on the secondary market. The equilibrium price is  $p=p(Q_1)$  such as defined in Lemma 2. The country's best strategy is a randomized buy-back with a probability :

$$(11) \quad q = \frac{p(Q_1) - p_0}{p[X(p_1)] - p_0} \in ]0,1[$$

Proof - The only point which is left to check is the equilibrium value of  $q$ .  $q$  must be such that the return which is expected by the bankers yields a price  $p(Q_1)$  which is exactly that for which the country is indifferent between making no buy-back or going to the other extreme  $\bar{X}[p(Q_1)]$ .  $q$  must consequently be a solution to  $p(Q_1) = q X(p(Q_1)) + (1-q)p$ , which is equation (11). QED.

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An equilibrium in mixed strategy is perhaps unappealing. Intuitively, it does show however that "some" buy-backs are likely to occur. Indeed, if none of them were expected (by the bankers), then the country would certainly find it optimal to undertake some of them. On the other hand, if the bankers were to think that the country would always undertake the full package  $\bar{X}[p_1(Q_1)]$ , then the country would not want to do it. A more intuitive way to analyze the equilibrium is to change slightly the framework of analysis above and to assume that there exists some (unobservable) disparity among a continuous group of debtors.

## IV - AN EQUILIBRIUM WITH A CONTINUUM OF DEBTORS

Let us change our framework of analysis as follows. Assume that there exists a continuum of debtors which are characterized by a distribution of initial endowments  $Q_1$  along an interval  $[\underline{Q}_1, \bar{Q}_1]$  with a cumulative distribution  $F(Q_1)$ . All other characteristics of the debtors are identical. For any debtor (characterized by an initial endowment)  $Q_1$ , let  $\bar{X}(Q_1, p)$  be the solution to  $I(X, p) + pX = Q_1$ . We know (from the analysis above) that  $\tilde{X}'(p) > 0$ ,  $\frac{\partial \bar{X}}{\partial p} < 0$  and  $\frac{\partial \bar{X}}{\partial Q_1} > 0$ . Call  $Q_1^C(p)$  the threshold point such that  $\tilde{X}(p) = \bar{X}(p, Q_1)$ . One can therefore see that  $\frac{dQ_1^C}{dp} = \frac{\tilde{X}'(p) - \frac{\partial \bar{X}}{\partial p}}{\frac{\partial \bar{X}}{\partial Q_1}} > 0$ . For each individual country, the optimal response to a secondary market price  $p$  comes as in the previous analysis:

$$(12) \begin{cases} X(p) = 0 & \text{if } p > \bar{p} \\ X(p) = 0 & \text{if } \tilde{X}(p) > \bar{X}(p, Q_1) \Leftrightarrow Q_1 < Q_1^C(p) \\ X(p) = \bar{X}(p, Q_1) & \text{if } \tilde{X}(p) < \bar{X}(p, Q_1) \Leftrightarrow Q_1 > Q_1^C(p) \end{cases}$$

For each price  $p$  set on the secondary market, one consequently knows that the following buy-backs will be undertaken:

$$X = \int_{Q_1^C(p)}^{\bar{Q}_1} \bar{X}[p, Q_1] dF(Q_1)$$

Call  $\hat{p}[Q_1, p]$  the "true" price of country  $Q_1$ 's debt, when the secondary market price  $p$  prevails:  $\hat{p}$  is defined as in equation (5) with  $X^e = \bar{X}[p, Q_1]$ .

The equilibrium on the secondary market, if it exists, is therefore a solution to :

$$(13) \quad p = \int_{Q_1^C(p)}^{\bar{Q}_1} \hat{p}[Q_1, p] dF(Q_1) + \left\{ 1 - F[Q_1^C(p)] \right\} p_0.$$

We can prove :

Proposition 2 - There always exists a unique equilibrium price  $p^*$  on the secondary market (and a unique cut off point  $Q_1^C(p^*)$ ) which is a solution to (13).

Proof -

$$\text{Let us define } g(p) \equiv \int_{Q_1^C(p)}^{\bar{Q}_1} \hat{p}[Q_1, p] dF(Q_1) + p_0 \left\{ 1 - F[Q_1^C(p)] \right\}$$

One can see that  $g$  is a continuous mapping from  $[p_0, 1]$  into itself. It has therefore necessarily a fixed point  $g(p)=p$ . One also sees that  $g(p)$  is a decreasing function of  $p$  ; the fixed point is therefore unique.

#### V - WELFARE IMPLICATIONS

Let us now investigate what are the welfare implications of "secret" buy-backs. Is one of the parties losing from their allowance, or are they Pareto-improving? We shall now see that the latter holds.

First, by a revealed preference argument, one knows that the debtor country can never lose. More specifically, in the framework set up in section III, we know that the country is indifferent, at equilibrium, between undertaking or not the buy-back. In the context of a model with a continuum of debtors, on the other hand, we also know that the debtor with  $Q_1 \leq Q_1^C$  does not care, while those with  $Q_1 > Q_1^C$  are better off thanks to their ability to repurchase their debt. The only aspect that is left to analyze is whether the banks may lose from the buy-back.

Call  $G(p, X)$  the value that is extracted by the banks from the country when an amount  $X$  is actually brought-back by the debtor at a price  $p > p_0$  (the price with zero buy-backs). One has:

$$G(X, p) = pX + \int_0^y \frac{1}{a} (D-X) - f(I) a[y+f(I)] dG(y) + [1-G^*][D-X].$$

with  $G^* = G[\frac{D-X}{a} - f(I)]$ .

First note that that  $G(X, p) + V(X, p) = - I(X) + f[I(X)]$ . To the extent that any buy-back raises the incentive of the country to invest (and since  $f'(I) > 1$  as long as  $D_0 - X > 0$ ), one sees that buy-backs are a positive sum game, since they reduce the "debt-overhang". On the other hand one has :

$$\frac{\partial G}{\partial p} = X \geq 0$$

and

$$\frac{\partial G}{\partial X} = p - (1-G^*) + a f'(I) \frac{dI}{dX} \cdot G^*$$

The geometry of the banks' profits can therefore be represented as in figure 4.

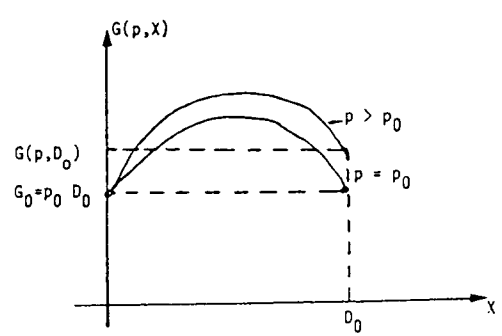


FIGURE 4

1

In order to show that any buy-back undertaken at a price  $p > p_0$  is Pareto-improving for the banks, one simply needs to note that  $G(p,0) = p_0 D_0$  (with no buy-backs, the banks get the original market value of the debt), and that  $G(p,D_0) = p D_0$ . One therefore has that  $G(p,0) < G(p,D_0)$  if and only if  $p_0 < p$ . With this assumption, the geometry of the banks' returns show that their pay-off is always improved by the countries' buy-backs.

The intuition underlying these results is simple. Both parties, debtor and creditors, could Pareto-improve the equilibrium with no-buy-backs by agreeing to trade-off a reduction of the face value of the debt against some down payment (since this would raise investment efficiently). Because lenders act non-cooperatively, no such transaction can directly take place. When there is a continuum of debtors, the inability of the creditors to screen perfectly the behavior of the debtor help enhancing the collective rationality of the creditors (by weakening their chances to free-ride one on the others). As a result, buy-backs are Pareto improving.

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