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ABSTRACT

A Theory of Wages and Labour Demand with Intra-firm Bargaining and Matching Frictions*

Firms are the field of several strategic interactions that standard neo-classical analysis often ignores. Such strategic considerations concern relations between capital owner and labour, relations between marginal employees and incumbents and more generally all relations between different groups within the firm with different bargaining positions. This Paper provides a synthetic model of the labour market equilibrium with search frictions in a dynamic framework where wage bargaining is influenced by within-firm strategic interactions, with explicit closed form solutions. We then explore systematically within-firm strategic interactions and shed new light on the micro- and macroeconomic consequences of conflicts on wages, unemployment and capital accumulation. First, we recover the partial equilibrium over-employment phenomenon put to the fore by Stole and Zwiebel (1996a,b) at the firm level, according to which the bargaining power of workers increases employment. Further, with heterogeneous labour, higher relative bargaining power for some groups leads quite generally to over-employment relative to other groups, those other groups being under-employed if they have a lower relative bargaining power. The over-employment results do not necessarily hold at the macroeconomic level, however. Quantitative exercises suggest that the bargaining power of workers is actually detrimental to employment when labour is considered as an homogeneous input. Finally, the hold-up problem between capital owners and employees does not necessarily lead to under-investment in physical capital as it is usually the case. Actually, strategic over-employment can induce over-investment when employees substitutable to capital have strong bargaining power.

JEL Classification: J30 and J60
Keywords: intra-firm bargaining, over-employment, search and matching, strategic wage bargaining and unemployment

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NON-TECHNICAL SUMMARY

Beyond the neo-classical analysis, firms are the field of several strategic interactions between capital owners and labor, between marginal employees and incumbents and more generally between different groups with different bargaining positions within the firm. It is usually considered that employees' bargaining power allows them to get rents but destroys jobs. In that respect, Lars Stole and Jeffrey Zwiebel's (1996a and b) papers are very provocative. Stole and Zwiebel provide a rigorous intrafirm bargaining model where employees and the firm engage in *individual* wage negotiation. Under the assumption that contract incompleteness does not enable either party to commit to future wages and employment decisions, they show that intrafirm, e.g. individual, bargaining yields no rent for employees and gives rise to over-employment compared to a competitive labor market. The basic idea of Stole and Zwiebel is that a firm, with a diminishing marginal productivity of labor, can decrease the bargained wage, which is proportional to the marginal productivity of labor, by increasing employment. In Stole and Zwiebel's setting, firms over-employ strategically, up to the point where workers get their reservation wage.

This result is in sharp contrast with the predictions of the now the standard framework to analyze unemployment, namely search and matching models of Christopher A. Pissarides (2000). In such models, any increase in the bargaining power of workers raises unemployment and allows employees to get rents. Further, the related RBC literature which incorporates search frictions (Monika Merz 1995, David Andofalto 1996) has typically excluded the strategic interactions between firms and workers emphasized by Stole and Zwiebel. Several questions thus naturally arise here and most notably, what are the good predictions and what is the relevant model for macroeconomists? We will argue that both approaches are complementary as they illustrate different mechanisms at work in employment relationships. The approach of Stole and Zwiebel yields a good account of strategic interactions within the firm, but it is static and does not go beyond the analysis of the equilibrium of the firm. The approach of Pissarides is explicitly dynamic, offers a very elegant analysis of labor market equilibrium in the presence of search frictions that give rise to rent sharing problems, but does not analyze in depth the strategic interactions within the firm.

The first contribution of our paper is therefore to provide a synthetic model of the labor market equilibrium with search frictions in a dynamic framework where the wage bargaining is influenced by within firm strategic interactions between employers and heterogeneous workers. As we will show, the derivation of wages in this context cannot be, in the general case, the simple bargaining solution derived in the one worker-one firm model where the wage is simply a weighted average of the marginal product of labor and of the reservation wage of workers. The associated solutions are more complex. We also show the existence and uniqueness of the solutions for wages and employment. This contribution is important because we solve a difficult analytical problem, exhibit a general strategy of solving it in using spherical coordinates and find closed form solutions that can be directly exploited by both labor economists and macroeconomists in their quest of a theory of wages in large firms in presence of search frictions. This quest started quite a while ago. Several papers of the macro-labor literature had acknowledged the theoretical difficulties implied by the use of a large firm model with intrafirm bargaining and had recognized that decreasing returns to scale in labor inputs would create complications and notably over-employment meant as a situation in which the marginal product of labor is below the labor cost.[1] The typical way to overcome the problem notably in presence of heterogeneous labor (a case typically implying decreasing returns in each labor factor) has been to introduce several sectors (a final good sector and a sector for intermediate goods, specific to each type of labor) in order to generate a separate and standard wage and employment determination for each type of labor, at the cost of increasing the complexity of models.[2]

[1] The earliest paper acknowledging this result was Giuseppe Bertola and Ricardo Caballero (1994), more recent models with such feature being Ramon Marimon and Fabrizio Zilibotti (2000), Giuseppe Bertola and Pietro Garibaldi (2001) and Monique Ebell and Christian Hafke (2003). Even in the absence of search frictions, individual bargaining generates additional insights about labor hoarding and cyclical movements in real wages and consumption. See for instance the paper by Julio Rotemberg (1998).

[2] See e.g. Daron Acemoglu (2001). Another route is to use the large firm matching model and let the firm be wage taker (see e.g. Merz and Eran Yashiv, 2003, page 7) which is less satisfactory in a macroeconomic

Although useful as such, our contribution is not limited to purely analytical and methodological considerations. We then go on exploring systematically the implications of our general theory of wage determination along several dimensions. First, in line with the research agenda in the literature, we examine the robustness of the striking analysis of Stole and Zwiebel with respect to the introduction of the labor market equilibrium feedback in a dynamic framework.[3] This focus is particularly relevant as it allows us to make a synthesis between the macroeconomic analysis with search frictions and the new insights generated by Stole and Zwiebel in the industrial organization literature and organizational design. Along these lines, we show that within firms strategic interactions shed new light on the macroeconomic consequences of conflicts between employers and employees. Indeed, when employers manipulate wages through their employment policies, an increase in the bargaining power of workers does not necessarily lead to a decrease in equilibrium employment. It may induce firms to raise employment so as to reduce enough the marginal product of workers and thus attenuate the wage increase. We show that, in theory, this phenomenon can imply that increases in the bargaining power of workers raise employment. However, quantitative exercises suggest that the bargaining power of worker has a negative impact on employment for relevant values of the parameters of the model when labor is homogeneous.

Second, in generalizing the analysis to several labor factors, we find different, richer and somehow unanticipated interactions between the firm and employees and between heteroge-

perspective.

[3] Recent papers have questioned the robustness of the basic mechanism put forward by Stole and Zwiebel according to which firms could employ strategically up to the point where workers get their reservation wage. Stole and Zwiebel's static analysis as been enriched by Asher Wolinsky (2000) in a dynamic partial equilibrium setting. Wolinsky shows that multiple equilibria occur if employers can use complex dynamic strategies. Namely, Wolinsky shows that there exists, among others, an equilibrium, sustained by trigger strategies, where employment is efficient (*i.e.* the marginal productivity of labor equates the reservation wage) and the wage exhibits a mark-up over the reservation wage. However, Wolinsky (2000, p. 875) stresses that his analysis "confirms earlier results derived by Stole and Zwiebel (1996a,b) in the context of a static model and shows that they are very robust". In contrast, Catherine de Fontenay and Joshua Gans (2003) have argued instead that, allowing firms to replace insiders by block by outsiders would undo the main result, in the sense that even if the outsiders received the Stole and Zwiebel's wage, this wage would be in the limit equal to workers' reservation wage.

neous employees themselves. Notably, changes in the bargaining power of an employee entail changes in the whole wage and employment structure of the firm. It is shown that these changes are related to the distribution of bargaining power across employees and to the properties of the technology of the firm. It turns out that an increase in the bargaining power of a type of employees can decrease some wages and increase some others, in both partial and general equilibrium. In some cases, that appear to be empirically relevant, a rise in the bargaining power of some employees leads to overall employment increases. Interestingly, we show that the over-employment result in general equilibrium is more pervasive in the multi-labor extension of the model, when labor inputs have different and independent bargaining power. We notably show that a larger bargaining power for a given group of workers raises employment of this group if firms can easily substitute these workers to other workers with low bargaining power.

Third, a striking set of results is further obtained when modelling the demand for physical capital. When the analysis is extended to the case in which the stock of capital is predetermined with a standard constant returns to scale technology, we obtain drastically different results from a previous paper (Cahuc and Wasmer 2001) in which we recovered the solutions in Pissarides (2000) in the large firm matching model with physical capital.[4] In the perhaps more realistic case in which capital is predetermined, one usually obtains a typical hold-up problem i.e. workers appropriate part of the rent of employment and thus discourage firms from investing in capital. Accordingly, the hold-up problem always entails under-investment when labor is homogeneous, which is also what we obtain in this paper. However, when labor is heterogeneous and workers have different bargaining powers, the hold-up problem can give rise to *over*-investment in physical capital, which is a new but not totally unexpected result as the intuition of over-employment

[4] In that paper, we clarified the partial equilibrium properties of wage determination in the large firm matching model when capital and labor are associated into production. In the presence of constant returns to scale in production, the solution for the wage is the 'static' bargaining solution, i.e. the weighted average of the reservation wage and the marginal product if capital is not a predetermined variable. In Cahuc and Wasmer (2002) (which this paper nests and generalizes), we have studied some general equilibrium properties of the matching model with intrafirm bargaining in the single factor case.

still carries through the demand for physical capital. In fact, this situation can emerge when workers substitutable to capital have a strong bargaining power with respect to those who are complementary to capital. Calibration exercises suggest that this situation can indeed emerge for relevant values of the parameters of the model (e.g. when unskilled workers for instance, more substitutable to capital, have the ability to capture stronger rents). This may contribute to explain why the hold-up problem, at the heart of the macroeconomic literature inspired by Paul Grout (1984), fails to explain the fact that capital-labor ratio are still pretty high in Europe compared to the US.

The paper is organized as follows. In section 1, we expose the setup and the environment of the firm. In section 2 we derive partial equilibrium results. In section 3 we determine the general equilibrium, which is extended to capital in section 4. In section 5 we derive three sets of quantitative implications. Section 6 concludes.

1 A general model

1.1 Setup

We consider an economy with a numeraire good produced thanks to $n \geq 1$ labor types. Each type of labor, $i = 1, .., n$, is supplied by a continuum of infinitely lived workers of size normalized to one. Each worker supplies one unit of labor. Production of the numeraire good is obtained thanks to a concave production function denoted $F(N_1, N_2, ...)$, where $N_i \geq 0$, $i = 1, ...n$, stands for employment of type i. $\mathbf{N} = (N_1, .., N_n)$ denotes the vector of N_i. In our framework, employment $\mathbf{N}$ is a state variable that cannot be increased instantaneously. To recruit, the firm has to post vacancies. It incurs a group-specific hiring cost γ_i per unit of time and per vacancy posted. Vacancies are matched to the pool of unemployed workers according to a technology h_i determining the mass of aggregate contacts between the mass of vacancies, denoted by V_i, and unemployed $u_i = 1 - N_i$ where the population of workers is normalized to one.

Functions $h_i(u_i, V_i)$ are assumed to be constant returns to scale, increasing and concave in each argument. $\theta_i = V_i/u_i$ denotes the group-specific ratio of vacancies to unemployed workers (the tightness of the labor market). The probability to fill a vacant slot per unit of time is given by $h_i(u_i, V_i)/V_i = q_i(\theta_i)$ with $q_i'(\theta_i) < 0$, $q_i(0) = +\infty$,while $p_i = h_i(u_i, V_i)/u_i = \theta_i q_i(\theta_i)$ with $d\left[\theta_i q_i(\theta_i)\right]/d\theta_i > 0$. Note that θ_i is exogenous to the firms' decisions.

As the wage is continuously negotiated, it is potentially a function of employment. We denote by $w_i(\mathbf{N})$ the wage of the type-i workers which potentially depends on employment of all types of labor. At this stage, it is assumed that this function is continuous and differentiable. It will be shown later on that this assumption is fulfilled. Note that the wage w_i is *common to the group i of workers by symmetry but nevertheless it is individually bargained.*[5]

1.2 Labor demand

Denoting employment of group i at date t by N_i, and employment at date $t + dt$ by N_i^+, the value function of the firm, $\Pi(\mathbf{N})$, solves the Bellman equation:

$$\Pi(\mathbf{N}) = \underset{\mathbf{V}}{\text{Max}} \left(\frac{1}{1+rdt}\right) \left\{ \left[F(\mathbf{N}) - \sum_{j=1}^{n} w_j(\mathbf{N}) N_j - \gamma_j V_j \right] dt + \Pi(\mathbf{N}^+) \right\}, \quad (1)$$

$$\text{subject to } N_i^+ = N_i(1 - s_i dt) + V_i q_i dt \quad (2)$$

where the constraint (2) is the law of motion of jobs and where $\mathbf{V} = (V_1, .., V_n)$ denotes the vector of vacancies, the control variable of firms. We denote by $J_i(\mathbf{N})$ the marginal value of a additional worker of type i, i.e. $J_i(\mathbf{N}) = \frac{\partial \Pi(\mathbf{N})}{\partial N_i}$. The first-order and the envelope conditions for an optimal choice of V_i are derived in Appendix A in equations (A1) and (A2). We exclusively focus on stationary solutions where $\mathbf{N} = \mathbf{N}^+$. One finds that the steady state employment level, $\mathbf{N}^*$, satisfies:

$$J_i(\mathbf{N}^*) = \frac{\gamma_i}{q_i} \quad (3)$$

[5] We follow the approach of Stole and Zwiebel (1996a, 1996b) who provided a simple strategic bargaining game where identical workers get the same wage through invididual bargaining.

while marginal profit of employment N_i is obtained as

$$J_i(\mathbf{N}) = \frac{\frac{\partial F(\mathbf{N})}{\partial N_i} - w_i(\mathbf{N}) - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i}}{r + s_i} \tag{4}$$

The marginal profit appears above as the discounted marginal product, net of the individual wage and net of the effect of the marginal hire on the wage bill of all employees. Equation (3) states that the marginal product at the optimal level of employment is precisely equal to the expected recruitment costs of workers.

One can then re-express the marginal product of labor at the firm's optimal employment in combining the two equations (3) and (4). The marginal value of a job hinges not only on its marginal productivity and wage: there is in addition the potential impact of an extra-unit of labor N_i on wages of all groups, as the third term in equation (5) indicates.

$$F_i(\mathbf{N}) = \underbrace{w_i(\mathbf{N})}_{\text{Wage}} + \underbrace{\frac{\gamma_i(r + s_i)}{q_i}}_{\text{Turnover costs}} + \underbrace{\sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i}}_{\text{Employment effect on wages}} \tag{5}$$

We thus obtain a relation between the marginal productivity and the labor cost for each category of job. Only the first two terms are found in the large firm analysis in Pissarides (2000). Let us now look at the determination of the wage $w_i(\mathbf{N})$.

1.3 Wage determination

Wages are continuously and instantaneously negotiated. Since firms need time to hire workers, employment is considered as a state variable during the negotiation process. The spirit of wage determination in this at-will firm is that the individual wage in a group i is determined through a split of the surplus in shares $\beta_i \in (0, 1)$, where β_i is an index of the bargaining power of workers *specific* to their type.

The surplus of a type$-i$ worker is $E_i - U_i$ where E_i and U_i denote the expected discounted utility of an employed and an unemployed type$-i$ worker respectively. At steady state, E_i solves:

$$rE_i = w_i(\mathbf{N}) + s_i(U_i - E_i) \tag{6}$$

where b_i stands for the income flow of unemployed type-i workers. The surplus of the worker only depends on its own individual wage w_i: using (6), we have $E_i - U_i = \frac{w_i(\mathbf{N}) - rU_i}{r+s_i}$. The surplus of the firm, computed in (4), instead depends on the wage of all groups and their derivatives. The usual Nash-sharing rule

$$\beta_i J_i(\mathbf{N}) = (1-\beta_i)(E_i - U_i), \tag{7}$$

thus yields a system of partial first order differential equations[6]:

$$w_i(\mathbf{N}) = (1-\beta_i)rU_i + \beta_i \left(\frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} \right), \text{ for } i = 1,..,n. \tag{8}$$

where the summation over j reflects that one additional unit of labor i has potential consequences on the wages of all the groups $j = 1,..,n$. In partial equilibrium, U_i is treated as a parameter. It can be noticed that the wage of labor i depends not only on its own level of employment N_i and the effect of an additional unit of N_i on the wage bill of labor i, but more generally on the effect of an additional unit of N_i on the wage of all labor inputs j, $j = 1,...n$. As it will be shown, this mechanism is *a priori* dependent of the structure of substitutability/ complementarity of the different labor inputs in production.

2 Partial equilibrium

In a partial equilibrium, rU_i and q_i are taken as given by firms. In this context, we can obtain a solution for wages and labor demand. Both are parametrized by a quantity reflecting the extent to which the marginal product of labor differs from labor costs. This quantity, depending on the properties of the production function and on the bargaining power of workers, captures the extent to which there is over-employment.

[6] to our knowledge, this system has not been solved in the case $n > 1$. Several authors, willing to deal with multiple labor in matching model and large firms, have thus to adopt a multi-sector model with intermediate and final goods, such as Acemoglu (2001). As we will show, once calculated, the solutions of our model are quite simple and allow us to account for strategic interactions that are usually neglected.

2.1 Solutions

The system of differential equations (8) is solved in appendices B.1 to B.4. One obtains the following expression for wages:

$$w_i(\mathbf{N}) = (1-\beta_i)rU_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z))dz \tag{9}$$

where F_i stands for the partial derivative of F with respect to the argument i and the term $\mathbf{A}_i(z)$ stands for a diagonal matrix defined in appendix B.4.[7] It reflects the distortion of labor inputs in the outcome of wage bargaining and is such that the vector $\mathbf{N}\mathbf{A}_i(z)$ reads

$$\mathbf{N}\mathbf{A}_i(z) = (N_1 z^{\frac{\beta_1}{1-\beta_1}\frac{1-\beta_i}{\beta_i}}, N_2 z^{\frac{\beta_2}{1-\beta_2}\frac{1-\beta_i}{\beta_i}}, ..., N_n z^{\frac{\beta_n}{1-\beta_n}\frac{1-\beta_i}{\beta_i}}). \tag{10}$$

The wage equation generalizes Stole and Zwiebel (1996 a and b). It implies that the solution for wages is a weighted average of rU_i and of an additional term in which infra-marginal returns show up with weights depending on β_i and other β_j. For convenience, the exact intuition of this second term is described later on, first in more specific cases and then in general cases.

Determining the value of $\sum_{j=1}^n N_j \frac{\partial w_i(\mathbf{N})}{\partial N_i}$ from equation (8) and replacing it into (5), one obtains straight away the following expression for labor demand

$$F_i(\mathbf{N}) = \underbrace{w_i(\mathbf{N}) + \frac{\gamma_i(r+s_i)}{q_i}}_{\text{Labor costs}} + \underbrace{\sum_{j=1}^n N_j \int_0^1 z^{\frac{1-\beta_j}{\beta_j}(\frac{\beta_i}{1-\beta_i}+1)} F_{ji}(\mathbf{N}\mathbf{A}_j(z))dz}_{\text{Empl. effect on wages}} \tag{11}$$

where it can be seen that over-employment, defined here as a marginal product below the labor costs, can arise when the last term above is negative, i.e. when the negative effect of F_{ii} dominate over the other, possible positive effects, of F_{ij}.

We can simplify the exposition of the two main equations (9) and (11) of partial equilibrium, namely the wage equation and labor demand, after an integration by part in equation (11), as

[7] See Appendix B for further details on the existence of the integral and the solution of the wage differential equation.

follows:

$$OE_iF_i(\mathbf{N}) = w_i(\mathbf{N}) + \frac{\gamma_i(r+s_i)}{q_i} \tag{12}$$

$$w_i(\mathbf{N}) = (1-\beta_i)rU_i + OE_i\beta_iF_i(\mathbf{N}), \text{ for } i = 1,..,n. \tag{13}$$

where both equations are parametrized by OE_i defined as an over-employment factor equal to

$$OE_i = \frac{\int_0^1 \frac{1}{\beta_i} z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z))dz}{F_i(\mathbf{N})} > 0 \text{ and } \lesseqgtr 1 \tag{14}$$

Indeed, it appears that, since the right hand-side in equation (12) is again the labor cost of group i, including the turnover cost, the marginal product of labor i is below its labor cost and there is over-employment if and only if $OE_i > 1$. If $OE_i < 1$, the marginal product is above the labor cost and there is instead under-employment of labor i.

Equations (12) and (13) fully characterize the partial equilibrium and wage determination at the firm's level. To get a sense of the intuitions of the various concepts introduced here, notably OE_i and $\mathbf{A}_i(z)$, let us first study specific cases.

2.2 Specific cases

2.2.1 The single labor case

The wage equations (9) can be reduced to:

$$w(N) = (1-\beta)rU + \int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz. \tag{15}$$

while labor demand (11) is

$$F'(N) = w(N) + \frac{\gamma(r+s)}{q} + N\int_0^1 z^{\frac{1-\beta}{\beta}+1} F''(Nz)dz$$

When returns to labor are constant, i.e. when $F'(N) = y$ for all N and thus $F'' = 0$, one obtains the standard value of wages and labor demand in the matching literature. The wage is a weighted average of the reservation wage rU and of the marginal product of labor, the latter being now independent of N, $w = (1-\beta)rU + \beta y$. Labor demand is such that the marginal product of labor is equal to the labor cost. In Cahuc and Wasmer (2001), we showed that this case also corresponds to the case with instantaneous capital adjustment and constant returns to scale in total factors, as discussed in Pissarides (1990).

When there are strictly decreasing returns to scale, i.e. as soon as $F'' \leq 0$ with strict inequality over some sub-part of $(0, N)$, things are different. The first equation above shows, as discussed above and in Stole and Zwiebel, that *infra-marginal* products of labor contribute to wages with a weight represented by the factor $z^{(1-\beta)/\beta}$ in the integrals which is decreasing with the distance to N, i.e. distance to the margin. The second equation above has a strictly negative integral. This clearly means that *firms exploit decreasing returns to scale to reduce workers' wages in expanding employment.*

2.2.2 Homogeneous production functions with identical bargaining power

When all groups are endowed with the same bargaining power, equation (10) shows that $\mathbf{NA}_i(z)$ is proportional to $\mathbf{N}$ and actually equal to $z\mathbf{N}$. If in addition the production function is homogeneous of degree $1 - \lambda$, $0 \leq \lambda < 1$, the OE_i term writes

$$OE_i = (1 - \beta\lambda)^{-1} \geq 1 \tag{16}$$

The marginal productivity is always below the marginal cost of labor when labor has decreasing returns to scale ($\lambda > 0$). When $\lambda = 0$, the OE factor goes to one and one is back to the standard matching analysis with no over-employment.

The value of the OE term in equation (16) in fact indicates that workers' bargaining power and returns to scale interact complementarily with each other. The further away from a constant returns to scale world, the larger the impact of the bargaining power of workers on the propensity of firms to raise employment. A partial and quite surprising conclusion here is that over-employment is a feature of high bargaining power.

This result and the one obtained with a single type of labor seem to be a good illustration of Stole and Zwiebel's convincing story that intra-firm bargaining leads to over-employment. In those cases, our model as Stole and Zwiebel's model always predict over-employment as OE is larger than 1. However, the generalization we offer with several labor inputs and different bargaining power shows that the single factor case or the identical bargaining power cases can be misleading. When the bargaining power parameters are different, the terms OE_i can be either larger or smaller than one.

2.2.3 Homogeneous production functions with different bargaining power

When β_i differ across groups, there is a *geometric* interpretation of $\mathbf{NA}_i$: different bargaining power distort the metric of the space of employment, and the distortion is such that groups with higher bargaining power than group i count more in infra-marginal products of equation (9), as they are given an exponent larger than 1. The converse holds for groups with a smaller bargaining power than i. In the limit, a group j with no bargaining power is not given any weight in the integral. In wage determination, the *distortion factor* can be measured by the quantity

$$\chi_{ij} = \frac{1-\beta_i}{\beta_i}\frac{\beta_j}{1-\beta_j} \tag{17}$$

with $\chi_{ii} = 1$ and $\chi_{ij} = \chi_{ji}^{-1}$. This distortion factor will show up in the value of OE_i: with a Cobb-Douglas technology $F(\mathbf{N}) = \prod_i N_i^{\alpha_i}$, and denoting by $\lambda = 1 - \Sigma_j \alpha_j$ to keep consistent notations throughout, one obtains:

$$OE_i = [1 - \beta_i \lambda + \beta_i \Sigma_{j \neq i} \alpha_j (\chi_{ij} - 1)]^{-1} \tag{18}$$

The first interesting new finding here is that the distortion factor χ_{ij} interacts with the individual returns to scale specific to each group. Notably, for a given level of β_i, a higher β_j implies a higher χ_{ij} which means a lower OE_i, i.e. a lower over-employment of group i. This means that over-employment of a group can be reduced if other groups have a high bargaining power, leading even to a situation of under-employment, in which the marginal productivity of type$-i$ workers is higher than their marginal cost ($OE_i < 1$). The intuition is that decreasing the employment of type$-i$ workers allows the firm to decrease the marginal productivity of type$-j$ workers (because F_{ij} is positive here), and then their wages. Therefore, the firm faces a trade-off: recruiting less type$-i$ workers increases their marginal productivity and then their wage, but decreases the wages of the other workers. The latter effect dominates if the bargaining power of the other workers is relatively strong. This trade-off explains why workers with strong bargaining powers can be over-employed whereas workers with weak bargaining power can be under-employed. It should be noticed that this result cannot hold if $F_{ij} < 0$, for all (i, j) because, in that case, there is necessarily over-employment for all labor types, as shown by equation (12).

The second interesting effect is that, in absence of decreasing returns to scale, i.e. when $\lambda = 0$ or $\Sigma_j \alpha_j = 1$, OE_i can still be greater or smaller than 1. The distortion factor χ_{ij} on over-employment has still the same effect on over-employment and, if larger than 1, leads to under-employment. OE_i is also influenced by the returns to scale of the other group α_j. The direction of this effect depends on whether χ_{ij} is greater or lower than 1: indeed, a group with stronger decreasing returns to scale (lower α_j) will lead to over-employment of group i only if $\beta_j > \beta_i$. Take for instance the two factors case with $\beta_1 = 1/3$ and $\beta_2 = 0.5$, we have the distortion parameter $\chi_{12} = 2$ and $\chi_{21} = 1/2$, and thus

$$OE_1 = (1 + 2\alpha_2/3)^{-1} < 1 \; ; OE_2 = (1 - \alpha_1/4)^{-1} > 1$$

This example suggests that increases in the bargaining power of one type of worker can raise their employment level and might have ambiguous effects on aggregate employment, a possibility explored later on in section 5.2.

2.3 Partial equilibrium results on wages

A change in β_j has a direct impact on *all* bargained wages. Consider the differentiation of equation (9), for a given level of $\mathbf{N}$, with respect to β_j. One obtains, still at constant rU_i:

$$\frac{\partial w_i(\mathbf{N})}{\partial \beta_j} = \int_0^1 \frac{z^{\frac{1-\beta_i}{\beta_i}} \log(z)}{(1-\beta_j)^2} N_j \frac{1-\beta_i}{\beta_i} z^{\frac{1-\beta_i}{\beta_i}\frac{\beta_j}{1-\beta_j}} F_{ij}(\mathbf{N}\mathbf{A}_i(z))dz \text{ if } i \neq j$$

Given that $\ln z < 0$ for all z between 0 and 1, this equation indicates that an increase in β_j entails a drop of the wages of workers i who are complement to type-j workers (such that $F_{ij} > 0$) but an increase in the wage of substitute workers (such that $F_{ij} < 0$). It is worth noting that this mechanism can lead to conclusions that are very different from those obtained from a standard wage equation, that posits $w_i = (1-\beta_i)rU_i + \beta_i F_i(\mathbf{N})$. This effect relies on the assumption that wages are continuously renegotiated. Indeed, in case of disagreement with a type-i worker, the worker leaves the firm and the other wages are renegotiated. Let us suppose that $F_{ij} < 0$. In this case, the departure of a type-i worker leads to an increase in the marginal productivity of type-j workers who will get a wage increase that will be larger the larger is their bargaining power β_j. Therefore, the loss of the firm in case of disagreement with a type-i worker is larger, the larger the bargaining power of type-j workers, which implies that the wage of substitutable type-i workers increases with β_j.

This result has the consequence that substitutable workers might support together more than complementary workers. It is reminiscent of the result of Henrik Horn and Asher Wolinsky (1988) who analyzed the relation between the pattern of unionization between two unions, each union representing one type of labor, and the degree of substitutability between the two types of labor. Horn and Wolinsky argued that when the two types of labor are close substitute (the marginal revenue product of one type is decreasing in the quantity of the other), then the equilibrium form of unionization is an encompassing union. When the two types of labor are sufficiently strong complement, the two types are likely to be organized in two separate unions.

Obviously, our result holds true only for a given level of $\mathbf{N}$, i.e. among the workers who are within the firm. The attitude of employees towards potential entrants might be very different, as insiders might be opposed to the recruitment of workers substitutable to them but support the hiring of workers who instead increase their own marginal productivity.[8]

3 Labor market equilibrium

At labor market equilibrium, the reservation wage, rU_i, and the labor market tightness, θ_i, are endogenous variables.

The equilibrium value of rU_i is given by

$$rU_i = b_i + \theta_i q_i(\theta_i)(E_i - U_i) = b_i + \gamma\theta_i \frac{\beta_i}{1-\beta_i}$$

where the first equality is the arbitrage equation, in which b_i is the flow income of unemployed workers of group i and $\theta_i q_i(\theta_i) = h(u_i, V_i)/u_i$ is their exit rate from unemployment, and the second equality combines the first one with Nash-bargaining (7) and the optimal value of marginal employment in (3).

Now, eliminating wages from (12) and (13), one obtains n relations between $\theta = (\theta_1, \theta_2..)$ and $\mathbf{N}$ which are vacancy curves denoted by (VC)

$$OE_i F_i(\mathbf{N}) = \frac{1}{\beta_i}\int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z))dz_i = b_i + \frac{\beta_i}{1-\beta_i}\gamma_i\theta_i + \frac{1}{1-\beta_i}\frac{\gamma_i(r+s_i)}{q_i(\theta_i)} \qquad \text{(VC)}$$

The right-hand side is without ambiguity increasing in θ_i. Therefore, this equation provides a unique θ_i as a function of all $\mathbf{N}$.[9] On the other hand, the (VC) curve do not characterize the

[8] This point shows up in calculating $\partial w_i/\partial\beta_i$ from equation (9). First, there is a standard positive term: workers get a share of their marginal product $F_i(\mathbf{N}\mathbf{A}_i(z))$ that increases with their bargaining power. However, the interesting new terms here are the cross derivatives F_{ij}. It turns out that workers who are substitutes to i reduce the positive impact of β_i on wages. When the firm has more possibility to substitute some workers to the type-i worker, the loss of the firm in case of disagreement with the type-i worker is smaller. Accordingly, for every category of worker, the presence of other, closed substitute workers limits their possibility to take advantage of their own bargaining power.

[9] In the space (θ_i, N_i), equation (SS) implies that N_i has limits zero (resp. one) as θ_i goes to zero (resp. infinity). Thus, if there exists a θ_i when $N_i = 1$ for a given set of N_j, $j \neq i$, which corresponds to the usual viability assumption on the sub-market i, there is a single intersection between the two curves.

equilibrium in each single segment: all demand functions are interdependent, as labor market tightness depends on other groups' employment.

The steady-state condition on flows and stocks on each labor market $i = 1, .., n$, implies that $(1 - N_i)\theta_i q_i(\theta_i)$ exits from unemployment and $N_i s_i$ entry into unemployment per unit of time compensate each other. Accordingly, the flow equilibrium implies in steady-state that:

$$N_i s_i = (1 - N_i)\theta_i q_i(\theta_i) \tag{SS}$$

This equation implicitly defines N_i as an upwards sloping function of θ_i. Inverting $\theta_i(N_i)$ from equation (SS) and plugging into (VC), we have a system of n equations and n unknowns $\mathbf{N}$.

We can prove that if the production function is Cobb-Douglas and has constant or decreasing returns to scale, an equilibrium $\mathbf{N}$ exists and is unique. The proof is in Appendix C. Uniqueness can be shown in the Cobb-Douglas case because the OE_i factors are constant and depend only on β_i and exponents of the Cobb-Douglas function.

Interestingly, the strategy of the proof remains valid in imposing $OE_i \equiv 1$ for all i, i.e, neglecting all strategic interactions at the firm's level. This result means that, in the large firm matching model *à la* Pissarides, there is also existence and uniqueness which, to our knowledge, had never been established except in the case $n = 1$. When the production function is not Cobb-Douglas but still exhibits constant or decreasing returns to scale, we have never been able to find numerical examples of multiple solutions. It seems that the multiplicity of solutions would require very strong and rapid variations of the sign of F_{ij} which CES function generalizing Cobb-Douglas functions do not have.

We also have some general results for the existence of a solution. Indeed, we can show that a solution always exists when the production is not Cobb-Douglas under some conditions on the value of unemployment benefits. If we define I_n the hypercube $(0, 1) \times (0, 1)...(0, 1)$ to which a solution $\mathbf{N}$, must belong, and $\underline{I_n}$ the hypercube minus its surface (thus excluding 0 and 1 for all components of $\mathbf{N}$), if we define $\mu_i = \inf_{\mathbf{N} \in I_n} \frac{\partial F}{\partial N_i}(N)$, a solution always exists if

$$\inf_i \mu_i \geq \sup_i b_i$$

which generalizes the usual viability rule stating that the marginal product is larger than unemployment benefits. Further, it is interior, i.e. solutions belong to $\underline{I_n}$. Note however that for a Cobb-Douglas production functions, $\mu_i = 0$ which would impose $b_i = 0$ which is a too restrictive assumption.

4 Extension to capital

Pissarides (1990, 2000) stressed that the large firm model is equivalent to the one-firm one-job model even in the presence of capital and labor. However, this result relies on the assumption of *instantaneous adjustment of the stock of capital*, as Pissarides discusses and as we emphasized in Cahuc and Wasmer (2001). Since it is conventionally believed that firms actually face severe adjustment costs adopting new capital or selling old capital, a general model of intrafirm bargaining in presence of such adjustments costs is necessary to have a better understanding of the effect of distortions due to search frictions and wage bargaining on employment and real wages. From this point of view, it is interesting to deal with the case where physical capital is a predetermined variable, that cannot change instantaneously when workers quit the firm.

4.1 Firm's program

Let us extend the previous analysis by the integration of capital into the model. We assume that the production function now reads $F(\mathbf{N}, K)$, where K denotes the stock of capital and $\mathbf{N}$ the vector of labor inputs. We denote by $w(\mathbf{N}, K)$ the wage, that is potentially a function of the capital and the vector of jobs. A straightforward extension of our previous analysis indicates that the expression of the negotiated wage will be similar to the one found before, as the presence of capital, which is a fixed input during the bargaining process, does not modify the previous analysis. Accordingly, one can write

$$w_i(\mathbf{N}, K) = (1-\beta_i)rU_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{NA}_i(z), K)dz,\ \ i = 1, .., n. \tag{19}$$

where $F_i(\mathbf{NA}_i(z), K)$ denotes the partial derivative of F with respect to the ith coordinate of the vector $\mathbf{NA}_i(z)$.

Denoting by I the investment, by δ the depreciation rate of capital, the value function of the firm, $\Pi(\mathbf{N}, K)$, solves the Bellman equation:

$$\Pi(\mathbf{N}, K) = \underset{\{V_i, I\}}{\text{Max}} \left(\frac{1}{1+rdt}\right) \left\{ \left[F(\mathbf{N}, K) - \sum_{j=1}^{n} w_j(\mathbf{N}, K)N_j - \gamma_j V_j - I \right] dt + \Pi(\mathbf{N}^+, K^+) \right\}, \tag{20}$$

subject to $K^+ = K(1-\delta dt) + Idt$; $N^+ = N(1-sdt) + Vq(\theta)dt$

The first-order and the envelope conditions for an optimal choice of V_i and I are presented in appendix. We obtain an identical first order condition on employment (see Equation (D25) in Appendix D) and the following first order condition on capital:

$$F_K(\mathbf{N}, K) = r + \delta + \underbrace{\sum_{i=1}^{n} N_i \frac{\partial w_i(\mathbf{N}, K)}{\partial K}}_{\text{Cap. effect on wages}} \tag{21}$$

It can be seen that there is a term that does not show up usually in the capital demand equation (21): the marginal product of capital in equilibrium has to be equal to the sum of discount factors plus a "capital effect on wages" term reflecting that raising capital affects the marginal product of labor, which will thus affect the wage bill, and so the optimal stock of capital has to incorporate these effects.

4.2 Capital and labor demand

Using the wage equation (19), we can rewrite the capital and labor demand as:

$$F_i(\mathbf{N},K) = \underbrace{w_i(\mathbf{N},K) + \frac{\gamma_i(r+s_i)}{q_i(\theta_i)}}_{\text{Labor costs}} + \underbrace{\sum_{j=1}^{n} N_j \int_0^1 z^{\frac{1-\beta_j}{\beta_j}(\frac{\beta_i}{1-\beta_i}+1)} F_{ji}(\mathbf{N}\mathbf{A}_j(z),K)dz}_{\text{Empl. effect on wages}} \tag{22}$$

$$F_K(\mathbf{N},K) = r + \delta + \underbrace{\int_0^1 \sum_{i=1}^{k} N_i z^{\frac{1-\beta_i}{\beta_i}} F_{iK}(\mathbf{N}\mathbf{A}_i(z),K)dz}_{\text{Cap. effect on wages}} \tag{23}$$

The employment equation (22) has the same features as in the absence of capital in equation (11): the marginal productivity of labor is different than the labor cost. Equation (23) shows that the sign of the impact of an increase in the capital stock on the wage bill depends on the sign of the cross-derivatives of capital with the labor inputs and the bargaining power of every group of workers. Let us remark that this term is necessarily positive if there is only one type of labor input, because the concavity of the production function together with the constant returns assumption implies that the cross derivative between capital and labor is necessarily positive.[10] In this case, strategic interactions between capital and labor necessarily imply under-investment. The bargaining power allows the workers to take a share of the returns of capital and then diminishes the returns of investment to the firm. This is the standard hold-up problem, stressed by Grout (1984) and many others, which entails under-investment.

This properties show up clearly with a Cobb-Douglas technology, $F(\mathbf{N},K) = N^{1-\alpha}K^{\alpha}$, if one re-introduces the convenient notation for the over-employment factor $OE = (1-\alpha\beta)^{-1}$, which yields the capital and labor demand:

$$(1-\beta)OE \cdot F_K(\mathbf{N},K) = r + \delta \tag{24}$$

$$OE \cdot F_N(\mathbf{N},K) = w(N,K) + \frac{\gamma(r+s)}{q} \tag{25}$$

[10] If $F(N,K)$ if homogeneous of degree one, Euler's theorem implies that $F_N(N,K)$ is homogeneous of degree zero and thus $NF_{NN}(K,N) = -KF_{KN}(K,N)$. Since the concavity of F implies that $F_{NN} < 0$, one gets $F_{KN} > 0$.

where, using the notation $k = K/N$, marginal products are equal to $F_K(\mathbf{N}, K) = \alpha k^{\alpha-1}$ and $F_N(\mathbf{N}, K) = (1-\alpha)k^{\alpha}$. Equation (24) shows that the marginal product of capital differs from its cost $r+\delta$ by two terms, $(1-\beta)$ and OE. On the one hand, $(1-\beta)$ characterizes the hold-up problem, in the sense that firms obtain only a share $1-\beta$ of marginal profits. On the other hand, over-employment (aiming at reducing the wage bill) raises the marginal product of labor and partly compensate under-investment. One can easily see that under-investment dominates as $OE(1-\beta)$ is smaller than one.

However, the "capital effect on wages" term can be either positive or negative when there are multiple labor inputs. It is obviously negative if $F_{iK} < 0$ for all i. More generally, it is negative if the bargaining power of the workers who are substitutable to capital is large, relatively to the workers who are complement to capital, because workers with larger β_i have larger weight in the term[11] $\int_0^1 \sum_{i=1}^n N_i z^{\frac{1-\beta_i}{\beta_i}} F_{iK}(\mathbf{N}\mathbf{A}_i(z), K)dz$. If this term is negative, strategic interactions between capital and multi-labor inputs entail over-investment. The firm over-invests in order to lower the wage of the workers who are substitutable to capital.

5 Quantitative implications

The theory of labor demand and wages, extended to capital investments, has many quantitative implications. We develop a few of them and try to answer the following questions. First, how large is the impact of intrafirm bargaining on the level of employment and on wages in a "reasonably parametrized economy"? Second, do workers' bargaining power always reduce labor demand and capital investments? Third, what are the size of rents in this economy, i.e. how wages compare to marginal products and reservation wages? We answer these questions either analytically or numerically, after calibration.

[11] because the derivative of $z^{\frac{1-\beta_i}{\beta_i}}$ with respect to β_i is $-(1/\beta_i^2)z^{\frac{1-\beta_i}{\beta_i}} \ln z > 0$.

5.1 Order of magnitude

We begin by calibrating an economy with one type of labor and capital. The production function is $F(K,N) = AK^{\alpha}N^{1-\alpha}$, $\alpha = 1/3$, and the value of the parameters is chosen as follows: the size of the labor force is normalized to one, the interest rate has a quarterly value of 1%, the depreciation rate of capital, denoted by δ, is 4% per quarter, the job destruction rate is 5% per quarter, the matching technology implies that $q(\theta) = \theta^{-1/2}$, and we set $\beta = 0.5$.

First, we assume that there is no intra-firm bargaining (Table 1, column I).[12] We choose the value of the unemployment benefits b, of the cost of job vacancy γ, and of the parameter A of the production function in order to get an unemployment rate that amounts to 6%, a value of the replacement ratio b/w of 0.3 and an endogenous capital stock K that exactly amounts to one. One obtains $\gamma = 0.107$, $A = 0.15, b = 0.03$. In equilibrium, the average duration of unemployment is slightly above 15 weeks and the production per worker is 0.16.

Then, we look at the consequence of intra-firm bargaining in this economy, keeping the same value of the parameters and of the capital stock. In column II, we apply the $OE = (1-\beta\alpha)^{-1} = 1.2$ coefficient to both the wage curve and the labor demand curve and obtain an increase in employment by 0.95 percentage point, or a 12% decline in the unemployment rate which is now 5.26%. Wages are pushed up by 20% between column I and II because labor market tightness is increased. This exercise suggests that within firms strategic interactions can have a significant impact on employment and wages. It shows that the over-employment strategy of firms can lead to higher employment *and* wages in equilibrium.

However, this result gives only a partial account of within firm strategic interactions, since the adjustment of capital has been neglected. In column III, we let the capital be endogenous. We let intrafirm bargaining take place (i.e. $OE = 1.2$) in implementing the solution of equation (24).

[12] This assumption boils down to assume that there is a final good produced thanks to capital and an intermediate good utilized in quantity X, with the technology $F(K,X)$; the intermediate good is produced by one-job firms thanks to a constant returns to scale technology using labor only, which writes $X = N$, as in the approach of Acemoglu (2001).

	I	II	III
Unemployment rate (percentage)	6.0 (c)	5.26	6.32
Capital stock	1	1 (c)	0.55
Unemployment spell (weeks)	15.31	13.31	16.20
Wage	0.102	0.122	0.094
OE	1	1.2	1.2

Table 1: Values of endogenous variables in the economy without intrafirm bargaining (I), with intrafirm bargaining and exogenous capital (II), with intrafirm bargaining and endogenous capital (III).
(c) means constrained.

We indeed obtain the conventional hold-up problem in which returns to capital are expropriated by labor: firms anticipate that wages will depend on capital accumulation and reduce the wage bill by reducing capital accumulation. Table 1 displays a huge decrease in the capital stock that is reduced by 45%. This reduces the marginal product of labor and thus labor demand Unemployment is thus increased up to 6.32%. The drop in capital also decreases the equilibrium wage by more than 6% with respect to the case without intrafirm bargaining.

This quantitative analysis suggests two things. First, *intrafirm bargaining is conducive to under-employment rather than over-employment in labor market equilibrium when there is a single labor input*. From this point of view, the results displayed in Table 1 are robust to changes in the value of the bargaining power parameter β, since intrafirm bargaining always lead to lower employment when capital is endogenous, whatever the value of β. However, the under-employment phenomenon is more pronounced when the value of β is large: the unemployment rate amounts to 10.07% when $\beta = .9$ and to 6.01% when $\beta = .1$.

The second implication of this quantitative analysis is that the role played by capital is crucial since *in absence of capital adjustment, the opposite conclusion, namely over-employment, arises.* In the remainder of our quantitative analysis, we explicitly separate the impact of parameters with and without capital adjustment.

5.2 Effect of β_i on employment and wage with a fixed capital stock

In sub-section 2.3 we showed the effect on β_i on wages while in sub-section 2.2 we saw their effect through OE_i on labor demand. Ignoring capital first, we can investigate these two effects together (still in partial equilibrium) in eliminating wages from equations (12) and (13) to get:

$$OE_i F_i(\mathbf{N}, K) = \underbrace{rU_i + \frac{1}{1-\beta_i}\frac{\gamma_i(r+s_i)}{q_i}}_{\text{Labor costs}}$$

A larger β_i raises labor costs in the last term above which tends to reduce employment, but also raises incentives to over invest with a larger OE_i. This two effects that play in opposite directions suggest that employment and labor market tightness θ_i may have non monotonic variations with respect to the index of bargaining power of worker β_i.

Our calibration exercises show that the positive employment effect of increases in the bargaining power of worker does not dominate in the single labor case for relevant values of the parameters. However, a very different conclusion is reached when there are several labor types.

In the single labor-type case, assuming that the capital stock is given and that the production function is the same as in Section 5.1, increases in the bargaining power of workers lead to employment drops. As displayed in the Figure 1 that compares the impact of variations in the bargaining power parameter β on unemployment in the standard search and matching model (one-job firm) and in the intrafirm bargaining model[13], it turns out that increases in the bargaining power of workers always rise unemployment. This result happens to be very robust for relevant values of the parameters. Figure 1 shows that increases in the bargaining power of the workers have a larger impact on unemployment in the one-job firm, but that the difference between the two models is not very large in this realm. Unreported simulations show that it is only when the returns to scale become very decreasing ($\alpha > 3/4$) that the bargaining power of workers reduces unemployment.

[13] The value of the parameters is defined in the benchmark calibration of section 5.1.

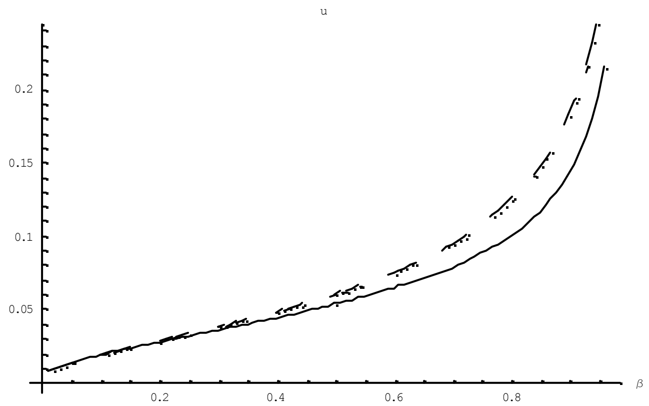

Figure 1: **Single-labor case with fixed capital.** The effects of changes in the bargaining power parameter on unemployment with intrafirm bargaining (continuous line) and without intrafirm bargaining (dotted line). The value of the parameters is defined in the benchmark model of section 5.1: $F(K,N) = AK^{1/3}N^{2/3}, \gamma = 0.101,\ A = 0.15, b = 0.03, q(\theta) = \theta^{-1/2}, s = 0.05, \delta = 0.04, r = 0.01$.

The positive employment effect can dominate in the case of several labor types. This phenomenon is illustrated in Figure 2 which displays the effect of changes in β_2 on the employment of each type of labor and on the unemployment rate with a Cobb-Douglas technology $F(\mathbf{N}) = AN_1^{1/3}N_2^{1/3}$. The two left-hand side graphs show that the employment of each category of worker increases with β_2 when β_2 is small. The employment of the type-1 workers can increase because their bargaining power diminishes with β_2 as it has been shown previously. The employment level of the type-2 workers increases because the over-employment margin is raised by their bargaining power. These two effects push the employment of both type of workers upwards for sufficiently low values of β_2. However, the strong bargaining power of the type-2 workers exerts a negative impact on profits, which is detrimental to labor demand and it turns out that there is a non-monotonous relation between employment and β_2 for each group of worker. The right-hand side graph of Figure 2 shows that bargaining powers hikes of one category of worker can entail significant drops of the overall unemployment rate.

We can also investigate the role of the bargaining power of workers on physical capital. On

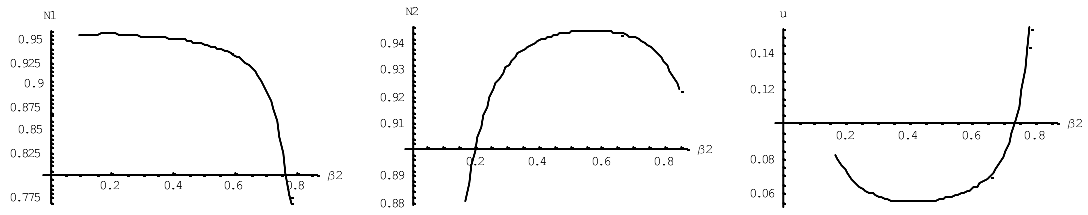

Figure 2: **Multi-labor case without capital.** The consequence of changes in β_2 on employment and the aggregate unemployment rate $u = (2 - N_1 - N_2)/2$. The production function is $F(\mathbf{N}) = AN_1^{1/3}N_2^{1/3}$ and the value of the parameters has been chosen to be comparable with the single labor type benchmark calibration in which unemployment rate amounts to 6% when $\beta_1 = \beta_2 = 1/2$ and the overall returns to scale to labor is 2/3. The value of the parameters is defined as follows $A = 0.15, \gamma_i = 0.02, b_i = 0.045, \beta_1 = 1/2, h_i(u_i, V_i) = \sqrt{u_i \cdot V_i}, s_i = 0.05, r = 0.01, i = 1, 2.$

this issue, we also find that the multi-labor case yields conclusions that are very different from those that are obtained from the single labor case.

5.3 Effect of β_i on employment and wage with adjustment in the capital stock

Now, to contrast with the fixed capital case, we simulate the single worker's type Cobb-Douglas case of Section 4. An increase in the bargaining power of workers has indeed strong adverse effects on employment, because they diminish the capital-labor ratio. This phenomenon is illustrated on Figure 3 which displays the impact of changes in the bargaining power parameter β on the unemployment rate and the capital-labor ratio in the economy with predetermined capital (continuous line) and in the economy in which the capital can be adjusted instantaneously (dotted lines, in that case there is no over-employment and the capital-labor ratio satisfies: $F_K(K/N, 1) = r + \delta$. If capital is predetermined, as it is assumed in Section 4, bargaining power increases diminish the capital-labor ratio, that goes to zero when the bargaining power of workers goes to one. This implies that the unemployment rate is much more sensitive to changes in the bargaining power than in the economy without adjustment cost of capital, in which the capital-labor ratio is not influenced by β.

Can we conclude that intra-firm bargaining is always associated with under-investment in

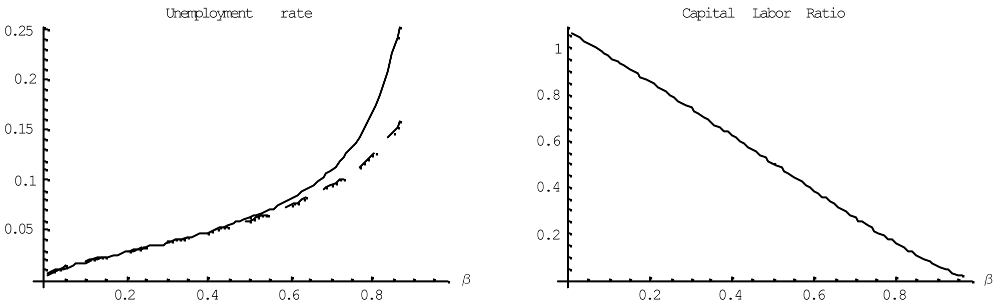

Figure 3: **Single-labor case with endogenous capital.** The impact of changes in the bargaining power parameter β on the unemployment rate u and the capital-labor ratio k, in the economy with endogenous predetermined physical capital (continuous line) and in the economy with no adjustment cost of capital (dotted line). The value of the parameters is defined in the benchmark model of section 5.1: $F(K,N) = AK^{1/3}N^{2/3}, \gamma = 0.101$, $A = 0.15, b = 0.03, q(\theta) = \theta^{-1/2}, s = 0.05, \delta = 0.04, r = 0.01$.

capital which always lead under-employment? The answer is in fact no. Let us indeed show that one can get very different results when labor force is heterogeneous. We consider a case with two labor types and the following production function:

$$F(\mathbf{N}, K) = AN_1^{1-\alpha}\left[aK^{\frac{\sigma-1}{\sigma}} + (1-a)N_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\alpha\sigma}{\sigma-1}}$$

Let us assume, for the sake of simplicity, that $\beta_1 = 0$ and $\beta_2 > 0$. If $\beta_1 = 0$, one has $w_1 = b_1$. It can be checked that $F_{K2} < 0$ iff $\sigma > 1/(1-\alpha)$. Accordingly, equation (23) indicates that the marginal productivity of capital is smaller than $r + \delta$ iff $\sigma > 1/(1-\alpha)$.

In this framework increases in β_2 can increase physical investment. Assuming $\alpha = 1/3$, $\sigma = 2$ and $a = 1/2$, $s_i = 0.2$, the figure 4 shows that the capital stock *increases* with β_2 in equilibrium. In this context, the increase in the bargaining power of type-2 workers, who are substitutable to capital, leads the firm to over-invest in order to decrease their marginal productivity and then limit their wage increase. The total impact of changes in β_2 on the employment of type-2 workers stems from the composition of three effects: first, over-employment of type-2 workers; second, over-investment; these two effects, which play positively on employment, aim at decreasing the marginal productivity of type-2 workers in order to exert a downward pressure on their wage when β_2 increases. But, as β_2 increases, there is also less surplus accruing to the firm; this third effect leads to less job creation. Figure 4 shows that the positive effects dominate when β_2 is sufficiently small. Figure 4 also shows that increases in β_2 can lower the overall unemployment rate.

To conclude this part, we have examined the interactions between physical capital and employment in presence of intrafirm bargaining and adjustment costs to both labor and capital. We have notably shown that under-investment in capital usually arises due to the standard hold-up problem when labor is homogeneous. We have also shown that is not necessarily the case when labor is heterogeneous. This is notably the case when there is over-employment of workers who are substitutable to capital since firms increase capital accumulation when the bargaining power of such workers is increased. It is striking that this phenomenon can entail an increasing relation between investment and the bargaining power of workers who are substitutable to capital at the firm level, but also at labor market equilibrium for relevant values of the parameters of the model.

5.4 Wages, reservation wage and rents

Equations (12) and (13) indicate that the wage is in the general case above[14] rU_i. It is interesting to remark that rU_i is a reservation wage: indeed, from (6), it is clear that workers are indifferent between employment and unemployment i.e. $E_i = U_i$ if and only if $w_i = rU_i$. Thus, as long as turnover costs γ_i are positive, workers obtain a rent over their reservation wage. This result is also in contrast with the analysis of Stole and Zwiebel who look at the case in which there is no hiring cost ($\gamma_i = 0$), which in fact implies that the employed workers are paid their reservation wage in their setup as well as in our setup (we cannot however prove this result before reaching

[14] Eliminating $OE_iF_i(\mathbf{N})$ from (12) and (13) one gets

$$w_i(N) = rU_i + \frac{\beta_i}{1-\beta_i}\frac{\gamma_i(r+s_i)}{q_i}.$$

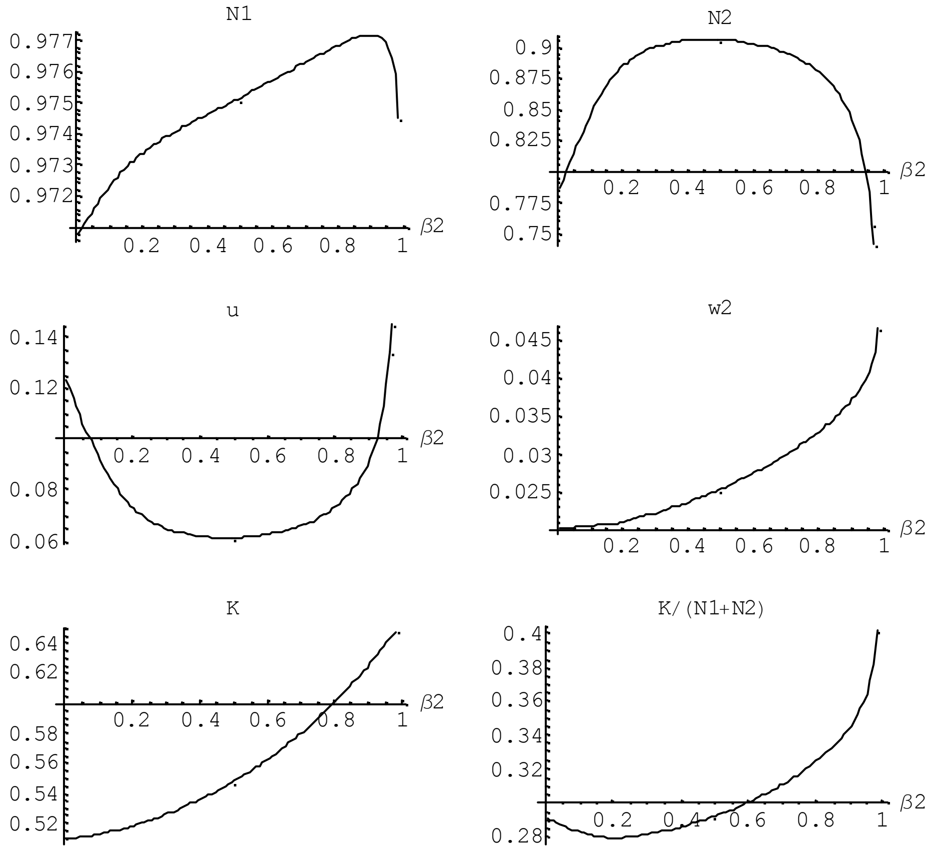

Figure 4: **Multi-labor case with endogenous capital.** The consequence of changes in β_2 on employment, the aggregate unemployment rate u, the wage w_2, the capital stock K and the capital-labor ratio $K/(N_1 + N_2)$. The production function is $F(K, \mathbf{N}) = AN_1^{1-\alpha}\left[aK^{\frac{\sigma-1}{\sigma}} + (1-a)N_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\alpha\sigma}{\sigma-1}}$ and the value of the parameters has been chosen to get an unemployment rate of 6% when $\beta_2 = 0.5$ as in the benchmark calibration of section 5.1. The value of the parameters is: $A = 0.15, \gamma_1 = 0.15, \gamma_2 = 0.02, b_1 = 0.07, b_2 = 0.02, \beta_1 = 0, h_i(u_i, V_i) = \sqrt{u \cdot V}, s_i = 0.05, r = 0.01, \delta = 0.04, \alpha = 1/3, a = 1/2, \sigma = 2, i = 1, 2.$

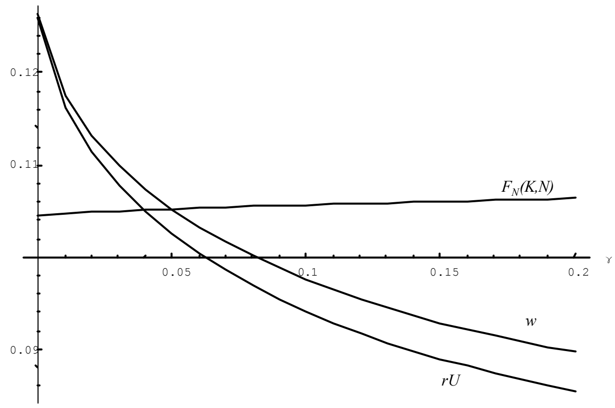

Figure 5: **Wage, marginal product and reservation wage.** Impact of changes in the cost of vacant jobs γ on the marginal productivity of labor $F_N(K, N)$, on the reservation wages rU and on the wage w in the economy with intrafirm bargaining and fixed capital. The value of the parameters is chosen to get the same unemployment rate and the same unemployment duration as in the benchmark calibration of section 5.1: $F(K, N) = AK^{1/2}N^{2/3}, K = 1, \beta = 0.3$, $A = 0.15, b = 0.03, q(\theta) = \theta^{-1/2}, s = 0.05, \delta = 0.04, r = 0.01$.

the general equilibrium, as tightness of labor market i and thus q_i depend on γ_i).

One can show on simulations that when γ_i are small but positive, we have $F_i(\mathbf{N}) < rU_i < w_i$, i.e. turnover costs are sufficiently small for firms to over-employ and reduce the marginal product of labor below the reservation wages. As γ_i increase, this over-employment possibility becomes more costly, and the marginal product of labor becomes larger than the reservation wage but smaller thant the wage: $rU_i < F_i(\mathbf{N}) < w_i$. Eventually, for sufficiently high values of the cost of vacant jobs, one can get the standard situation, $rU_i < w_i < F_i(\mathbf{N})$, met in matching models where the wage is a convex combination of the marginal product of labor and of the workers' reservation wage. This property is illustrated on Figure 5 in the benchmark calibration of the single labor case with fixed capital, in which the bargaining power parameter β has been set to 0.3 to be able to get a situation where the marginal productivity of labor becomes larger than the wage as the cost of vacant jobs is raised – the marginal productivity of labor is always smaller than the wage when $\beta = 0.5$ for the value of the parameters of the benchmark model.

6 Concluding comments

We showed that the consequence of intrafirm bargaining in a model with labor market search frictions is precisely the existence of over-employment, as firms indeed recruit up to a point where the marginal productivity of labor is smaller than marginal labor cost. However, the presence of search frictions and of physical investment decisions implies that this phenomenon does not necessarily give rise to a positive impact of the bargaining power of workers on employment in the single labor case. Actually, quantitative exercises suggest that the opposite holds true when labor is considered as an homogeneous input: the strategic interactions involved in intrafirm bargaining contribute to increase unemployment. Moreover, increases in the bargaining power of workers are more detrimental to employment when firms use employment and physical investment than employment only as strategic instruments to manipulate wages. From this point of view, it would seem that the over-employment phenomenon put to the fore by Stole and Zwiebel (1996a,b) does not play an important role at the macroeconomic level.

However, this conclusion has been reached only in a framework with homogeneous labor. Now, the analysis of the multi labor inputs case allows us to highlight some non-trivial mechanisms that mitigate this conclusion. First, it turns out that the firm can under-employ some low bargaining power workers in order to over-employ the high-bargaining power workers. Second, this phenomenon implies that increases in the bargaining power of one type of workers can lead to increases in overall employment and physical investment for relevant set of values of the parameters of the model.

Our contribution is a first step towards a better understanding of the consequences of within firm strategic interactions on wages and employment in labor markets with frictions. From this point of view, many extensions would be necessary. First, the continuous renegotiation of wages is a strong assumption. It would be interesting to look at other cases where the wages would not be renegotiated continuously. Second, our approach neglects job to job mobility. Introducing job to job mobility would allow us to account for the strategic interactions within the firm and between different firms (as it is the case in Cahuc *et al.*, 2003, and Shimer, 2004). Third, our analysis of the interactions between heterogeneous workers could be a promising way to study the emergence of collective versus individual bargaining.

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Appendix

A Demand for factors

First-order conditions to the problem of the firm in equation (1) subject to constraint (2) are

$$-\gamma_i + q_i J_i(\mathbf{N}^+) = 0, \tag{A1}$$

$$\left[\frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} - w_i(\mathbf{N})\right] dt + (1 - s_i dt) J_i(\mathbf{N}^+) = J_i(\mathbf{N})(1 + r dt). \tag{A2}$$

and lead, in a steady-state where $N^+ = N$, to (3) and (4).

B Wage determination with multiple labor inputs

We now solve the system of differential equations (8) by first considering specific cases and then, excluding those cases, proceed to the general solution.

B.1 Case $\beta = 0$

In this case, $w_i(\mathbf{N}) = rU_i$: the wage is equal to the equity value of unemployment, i.e. the reservation wage of workers. We now assume $\beta > 0$.

B.2 Case $n = 1$

With one type of labor, equation (8) becomes

$$w(N) = (1 - \beta) rU + \beta \left(\frac{\partial F(N)}{\partial N} - N \frac{\partial w(N)}{\partial N} \right) \tag{B3}$$

which reads, without the constant $(1 - \beta) rU$:

$$\frac{dw}{dN} + \frac{w}{\beta N} - \frac{F'(N)}{N} = 0 \tag{B4}$$

The solution of the homogeneous equation, $\frac{dw}{dN} + \frac{w}{\beta N} = 0$, reads

$$w(N) = C N^{1/\beta} \tag{B5}$$

where C is a constant of integration of the homogeneous equation. Assuming that C is a function of N and deriving (B5) with respect to N, one gets

$$\frac{dw}{dN} = \frac{dC}{dN} N^{-1/\beta} - \frac{1}{\beta} C N^{-1-\left(\frac{1}{\beta}\right)} \tag{B6}$$

Substituting (B5) and (B6) into (B4) yields

$$\frac{dC}{dN} = N^{\frac{1-\beta}{\beta}} F'(N)$$

or, by integration

$$C = \int_0^N z^{\frac{1-\beta}{\beta}} F'(z) dz + D$$

where D is a constant of integration. This last equation implies that the solution of equation (B4) satisfies

$$w(N) = \int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz + DN^{-1/\beta}$$

or equivalently:

$$w\left(N\right) = \frac{1}{N}\int_0^1 z^{(1/\beta)-2} zNF'(zN)dz + DN^{-1/\beta} \tag{B7}$$

Since $1/\beta - 2 > -1$ for all $\beta < 1$, it is sufficient to show that $NF'(N)$ is continuous in zero to obtain the convergence of the integral $\int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz$.

Assumption 1: Assume that $NF'(N)$ is continuous in zero and more generally, $N^p F^{(p)}(N)$ is continuous in zero where $F^{(p)}(.)$ is the p-th derivative of F.

As a consequence of this assumption, the integral $\int_0^1 z^{\frac{1-\beta}{\beta}} F'(Nz)dz$ is defined for all positive N and is actually smooth and can be differentiated at all orders for all strictly positive N.

At this stage, an additional assumption is needed to determine the constant of integration Din equation (B7). Henceforth, it will be assumed that $\lim_{N\to 0} Nw(N) = 0$.Thus, equation (B7) implies that $D = 0$.

Accordingly, the solution of equation (B3) is:

$$w(N) = (1-\beta)rU + N^{-1/\beta}\int_0^N z^{\frac{1-\beta}{\beta}} F'(z)dx. \tag{B8}$$

B.3 Case $n \geq 1$ and $\beta_i \equiv 1$

Equation (8) is now:

$$w_i(\mathbf{N}) = \frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^n N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i}, \text{ for } i = 1, .., n. \tag{B9}$$

Remarking that

$$w_i(\mathbf{N}) + \sum_{j=1}^n N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} = \frac{\partial \sum_{j=1}^n N_j w_j(\mathbf{N})}{\partial N_i},$$

the system is described by

$$\frac{\partial(\sum_{j=1}^n N_j w_j(\mathbf{N}) - F(\mathbf{N}))}{\partial N_i} = 0,$$

for all $i = 1, 2, ...n$. This implies that $\sum_{j=1}^n N_j w_j(\mathbf{N}) - F(\mathbf{N})$ is a constant of $\mathbf{N}$, i.e. of N_i for all i, that is:

$$\sum_{j=1}^n N_j w_j(\mathbf{N}) = F(\mathbf{N}) \tag{B10}$$

under the assumption that

$$\lim_{N_i \to 0} w_i(\mathbf{N}) N_i = 0$$
$$F(0) = 0$$

Equation (B10) does not provides the individual wage of each labor input, but tells us that profits are equal to zero. Since firms have to pay a vacancy cost to hire workers, no firm wishes to open a vacancy and thus, there is no general equilibrium solution with $N_i > 0$ to this problem.

B.4 General Case ($n \geq 1$ and $0 < \beta_i < 1$), but with different β's

To solve for this system, we need to take the partial derivative of equation (8) for a given i with respect to a labor input l, $i \neq l$. We obtain,

$$\frac{\partial w_i}{\partial N_l} + \beta_i \frac{\partial w_l}{\partial N_i} = \beta_i \left(\frac{\partial^2 F(\mathbf{N})}{\partial N_i \partial N_l} - \sum_{j=1}^{n} N_j \frac{\partial^2 w_j}{\partial N_i \partial N_l} \right)$$

which rewrites, denoting by $(E_i)'_l$ the second-order differential equation :

$$\frac{\partial w_i}{\partial N_l}(1-\beta_i) = \beta_i \frac{\partial^2}{\partial N_i \partial N_l} \left(F(\mathbf{N}) - \sum_{j=1}^{n} N_j w_j \right) \tag{$(E_i)'_l$}$$

for all $i, l = 1, ...n$. One can then note that the difference between $\beta_l (E_i)'_l$ and $\beta_i (E_l)'_i$ eliminates the symmetric terms, which, given that $0 < \beta_i < 1$, leads to

$$\frac{\partial w_l}{\partial N_i} = \frac{\beta_l}{1-\beta_l} \frac{1-\beta_i}{\beta_i} \frac{\partial w_i}{\partial N_l} \tag{B11}$$

for all $i, l = 1, ...n$. Let us denote by

$$\chi_{il} = \frac{\beta_l}{1-\beta_l} \frac{1-\beta_i}{\beta_i} \tag{B12}$$

It implies that

$$\sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N})}{\partial N_i} = \sum_{j=1}^{n} \chi_{ij} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j} \tag{B13}$$

which allows to conveniently rewrite (8) as

$$w_i(\mathbf{N}) = (1-\beta_i) r U_i + \beta_i \left(\frac{\partial F(\mathbf{N})}{\partial N_i} - \sum_{j=1}^{n} \chi_{ij} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j} \right) \tag{B14}$$

Let us first assume identical β_i's: $\beta_i \equiv \beta$. Then, $\chi_{ij} \equiv 1$. To proceed with the system (B14) for $i = 1, ...n$, , one can remark that $\sum_{j=1}^{n} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j}$ has a simple expression in another system of coordinates. Indeed, using the generalized spherical coordinates $\rho, \phi_1, ..., \phi_{n-1}$, where ρ is the distance to the origin such that $\sum_{j=1}^{n} N_j^2 = \rho^2$ and ϕ_i are the angles of projection in different sub-planes, one can write:

$$\begin{aligned}
N_1 &= \rho \cos\phi_1 ... \cos\phi_{n-2} \cos\phi_{n-1} \\
N_2 &= \rho \cos\phi_1 ... \cos\phi_{n-3} \sin\phi_{n-2} \\
N_2 &= \rho \cos\phi_1 ... \cos\phi_{n-2} \sin\phi_{n-3} \\
&\vdots \\
N_{n-1} &= \rho \cos\phi_1 \sin\phi_2 \\
N_n &= \rho \sin\phi_1
\end{aligned}$$

Then, we have that, using the notation $\phi = (\phi_1, ..., \phi_{n-1})$ for convenience,

$$\sum_{j=1}^{k} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j} = \rho \frac{\partial w_i(\rho, \phi)}{\partial \rho}.$$

Note that the economic interpretation of ρ is the scale of the use of labor inputs, while ϕ reflects the proportions in which different labor inputs are used. For instance, $\phi = (0, 0, ..0)$ means that the firm employes only labor of type 1. Then, (B14) in this system of coordinates is simplified to

$$\beta \frac{\partial w_i(\rho, \phi)}{\partial \rho} + w(\rho, \phi) = (1-\beta) r U_i + \beta \frac{\partial F(\rho, \phi)}{\partial N_i} \tag{B15}$$

which is similar to equation (B3) and follows the same resolution. Therefore, one gets:

$$w_i(\rho, \phi) = (1-\beta) r U_i + \rho^{-1/\beta} \left(\kappa_i(\phi) + \int_0^\rho z^{\frac{1-\beta}{\beta}} \frac{\partial F(z, \phi)}{\partial N_i} dz \right). \tag{B16}$$

where $\kappa_i(\phi)$ is a constant which is function of ρ. Using again that $\rho w_i(\rho, \phi)$ goes to zero when ρ goes to zero, we have that the constant $\kappa_i(\phi)$ is identically equal to zero. Further, noticing that if $\mathbf{N} = (\rho, \phi)$, then $(z\rho, \phi) = (zN_1, zN_2, .., zN_n) = z\mathbf{N}$, one can eliminate the spherical coordinates by rewriting this solution as

$$w_i(\mathbf{N}) = (1-\beta) r U_i + \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(z\mathbf{N})}{\partial N_i} dz. \tag{B17}$$

Now, in the case β_i different from each other, one simply need to introduce a new variable $\mathbf{M}_i = (M_{i1}, M_{i2}, ...M_{in})$ such that

$$\sum_{j=1}^n M_{ij} \frac{\partial v_j(\mathbf{M}_i)}{\partial M_{ij}} = \sum_{j=1}^n \chi_{ij} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j}$$

with $v_i(\mathbf{M}_i) = w_i(\mathbf{N})$. Note that this new notation $\mathbf{M}_i$ is indexed on i since variable change needs to be done for each equation (E_i). Denote further $G(\mathbf{M}_i) = F(\mathbf{N})$ the production function. To find the right $\mathbf{M}_i$ as a function of $\mathbf{N}$,denoted by $\mathbf{M}_i(\mathbf{N})$, let us assume a simple form: assume that $M_{il} = M_{il}(N_l)$, then, we only need to have

$$M_{il} \frac{\partial v_j(\mathbf{M}_i)}{\partial M_{ij}} = \chi_{ij} N_j \frac{\partial w_i(\mathbf{N})}{\partial N_j}$$

Since by definition,

$$\frac{\partial w_i(\mathbf{N})}{\partial N_j} = \frac{\partial v_j(\mathbf{M}_i)}{\partial M_{ij}} \frac{dM_{ij}}{dN_j}$$

one obtains a differential equation for M_{ij} which is

$$M_{ij} = \chi_{ij} N_j \frac{dM_{ij}}{dN_j}$$

One needs one solution only, the simplest being

$$M_{ij} = N_j^{1/\chi_{ij}} = N_j^{\chi_{ji}},$$

remarking that $1/\chi_{ij} = \chi_{ji}$. Then, using $\partial F/\partial N_j = \chi_{ji} N_j^{\chi_{ji}-1} \partial G/\partial M_{ij}$ and notably $\partial F/\partial N_i = \chi_{ii} N_j^{\chi_{ii}-1} \partial G/\partial M_{ii} = \partial G/\partial M_{ii}$ since $\chi_{ii} = 1$, the system (8) can be rewritten as

$$v_i(\mathbf{M}_i) = (1-\beta_i) r U_i + \beta_i \left(\frac{\partial G(\mathbf{M}_i)}{\partial M_{ii}} - \sum_{j=1}^n M_j \frac{\partial v_i(\mathbf{M}_i)}{\partial M_{ij}} \right)$$

which is formally equivalent to (B14). Hence, we now a solution for v_i which is

$$v_i(\mathbf{M}_i) = (1-\beta_i) r U_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}} \frac{\partial G(z\mathbf{M}_i)}{\partial M_{ij}} dz$$

Coming back to the initial notations, we thus obtain the equation (13):

$$w_i(\mathbf{N}) = (1-\beta_i)rU_i + \int_0^1 z^{\frac{1-\beta_i}{\beta_i}} F_i(\mathbf{N}\mathbf{A}_i(z))dz,\ i = 1,..,n. \tag{B18}$$

with

$$\mathbf{A}_i(z) = \begin{pmatrix} z^{\frac{\beta_1}{1-\beta_1}\frac{1-\beta_i}{\beta_i}} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & z^{\frac{\beta_j}{1-\beta_j}\frac{1-\beta_i}{\beta_i}} & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

is a diagonal matrix with $z^{\frac{\beta_j}{1-\beta_j}\frac{1-\beta_i}{\beta_i}}$, $j = 1,..,n$, on its main diagonal.

It should be noticed that, like in the single labor input case, equation (B18) and the existence of $w_i(\mathbf{N})$ rely on two assumptions:

- First: $\lim_{N_i \to 0} w_i(\mathbf{N})N_i = 0$
- Second: $F(\mathbf{N})$ is continuous for all $N_i \geq 0$ and infinitely differentiable for all $N_i > 0$. Assume in addition that $N_i \frac{\partial F}{\partial N_i}(\mathbf{N})$ is continuous in zero and further, that the quantity $N_1^{m_1}...N_n^{m_n} \frac{\partial^{\overline{m}} F}{\partial N_1^{m_1}...\partial N_n^{m_n}}$ (denoted for simplicity by $\mathbf{N}^m F^{(m)}(\mathbf{N})$) with $\overline{m} = \sum m_i$ is continuous in zero.

These assumptions imply that $w_i(\mathbf{N})$ exists and is smooth for all $N_i > 0$.

C Existence and uniqueness of the decentralized equilibrium

C.1 Single factor: existence and uniqueness

With a single factor ($n = 1$) and $0 < \beta < 1$, the system writes

$$\frac{1}{\beta}\int_0^1 z^{\frac{1}{\beta}-1} F'(zN)dz = b + \frac{\beta}{1-\beta}\gamma\theta + \frac{\gamma(r+s)}{1-\beta}\frac{1}{q(\theta)}$$

or with simplifying notations,

$$a(N) = v(\theta)$$

with $v' > 0$ and goes from b to $+\infty$. Further, $a(N) \geq 0$ and $a' \leq 0$ since $F'' \leq 0$. Given that $F'(N)$ is decreasing, $a(N) \geq \frac{1}{\beta}\int_0^1 z^{\frac{1}{\beta}-1}dz \times F'(1) = F'(1)$ so that a sufficient condition for the existence and uniqueness of $\theta(N)$is $F'(1) \geq b$. The other equation (SS) indicates that $N = \frac{p(\theta)}{s+p(\theta)}$ which is strictly increasing with $N = 0$ for $\theta = 0$ and $N = 1$ for $\theta = +\infty$. By continuity, there is then a unique decentralized equilibrium (θ^*, N^*).

C.2 Existence with $n \geq 1$

C.2.1 General case

In the case $n \geq 1$ et $0 < \beta_i < 1$, equation (VC) writes with similar notations

$$a_i(N_i) = v_i(\theta_i)$$

We denote by $I_n = (0,1)^n$ and

$$\mu_i = \inf_{\mathbf{N}\in I_n} \frac{\partial F}{\partial N_i}(N) \geq 0$$

Simple calculations indicates that $a_i(N) \geq \mu_i$. Further, $v_i(\theta_i)$ increases with θ_i with $v_i(0) = b_i$ and $v_i(+\infty) = +\infty$. As before, existence and uniqueness of $\theta_i = \theta_i(N) \geq 0$ is obtained when $\mu_i \geq b_i$. A sufficient condition for the existence of all θ_i is

$$\inf_i \mu_i \geq \sup_i b_i \tag{C19}$$

which is a generalization of the viability condition of labor markets, as μ_i is the marginal product of workers of type i at zero employment. Equations (SS$_i$) $N_i = \frac{p_i(\theta_i)}{s_i + p_i(\theta_i)}$ generate a strictly increasing link between N_i and θ_i for all i. Let us denote by

$$\hat{N}_i = (N_1, \ldots, N_{i-1}, N_{i+1}, \ldots, N_n)$$

the $(n-1)$ array of employment levels of all labor inputs but i. One can eliminate N_i from (VC$_i$) in combining it with (SS$_i$) and create a set of solutions denoted with a superscript $^\times$, so that

$$\theta_i^\times = \theta_i^\times(\hat{N}_i)$$

and then using (SS$_i$) again,

$$N_i^\times(\hat{N}_i) = \frac{p_i(\theta_i^\times(\hat{N}_i))}{s_i + p_i(\theta_i^\times(\hat{N}_i))}$$

The couple $(\theta_i^\times,\ N_i^\times)$ is uniquely obtained from $\hat{N}_i$. Given that N belongs to I_n, the system $N_i^\times(\hat{N}_i)$ implies n hypersurfaces in the hypercube I_n of dimension n.

To obtain an intuition, let us consider the case $n = 2$. We have

$$N_1^\times(N_2) = \frac{p_1(\theta_1^\times(N_2))}{s_1 + p_1(\theta_1^\times(N_2))}$$

and similarly, $N_2^\times(N_1)$. In the square I_2, the equation $N_1^\times(N_2)$ defines a curve going continuously from the bottom of the square to its top. In the same way, $N_2^\times(N_1)$ defines a curve going continuously from the left part of the square to its right part. There is thus at least one intersection $N^* = (N_1^*, N_2^*)$ and thus $\theta^* = (\theta_1^*, \theta_2^*)$. See the figure.

As it indicates, in the general case, there is no reason for having a uniquene interection to the system of n hypersurfaces.

C.2.2 Cobb-Douglas case

Note that condition (C19) is very restrictive and is not suitable to the Cobb-Douglas case as $\mu_i = 0$. However, in the Cobb-Douglas case, we can exhibit a less restrictive condition. Indeed, when $F(N) =$

$\Pi N_i^{\alpha_i}$, equation (VC_i) is in the Cobb-Douglas case

$$OE_i \alpha_i \frac{F}{N_i} = v_i(\theta_i) = b_i + a_i\theta_i + c_i \frac{1}{q_i(\theta_i)}$$

For a given $\hat{N}_i = (N_1, \ldots, N_{i-1}, N_{i+1}, \ldots, N_n)$ with $N_j > 0$, $i \neq j$, a sufficiently small N_i makes the left-hand side sufficiently large. In other words, this equation always define a $\theta_i(N)$ given that $v_i(\theta_i)$ is strictly increasing in θ_i.

C.3 Uniqueness with $n \geq 1$ in the Cobb-Douglas case

When $F(N) = \Pi N_i^{\alpha_i}$, equation ($\mathrm{VC}_i$) is in the Cobb-Douglas case

$$OE_i \alpha_i \frac{F}{N_i} = b_i + a_i\theta_i + c_i \frac{1}{q_i(\theta_i)}$$

with OE_i a constant defined in (18) and $a_i = \frac{\beta_i}{1-\beta_i}\gamma_i$ and $c_i = \frac{\gamma_i(r+s_i)}{1-\beta_i}$. Let us define $t_i = \ln N_i$. Equation (VC_i) is then

$$\sum_j \alpha_j t_j = t_i + g_i(\theta_i) - \ln(OE_i\alpha_i) \tag{C20}$$

where

$$g_i(\theta_i) = \ln\left(b_i + a_i\theta_i + c_i\frac{1}{q_i(\theta_i)}\right)$$

Note that $g_i()$ is strictly increasing in θ_i since $a_i, c_i > 0$ and $q_i(\theta_i)$ is increasing. Further, given (SS), we have

$$p_i(\theta_i) = \frac{s_i N_i}{1 - N_i}$$

with $N_i = e^{t_i}$ so that one can define n functions $\theta_i = \theta_i(t_i)$ that are strictly increasing in t_i. It follows that in the right hand-side of equation (C20), one has a function $H_i(t_i)$ with

$$H_i(t_i) = t_i + g_i(\theta_i(t_i)) - \ln(OE_i\alpha_i)$$

which is strictly increasing in t_i. Define $t_i^{\sup} = \sup_{N_i^\times} t_i \in (-\infty; 0)$ and $J_n = \prod_i) - \infty; t_i^{\sup})$ on which functions $H_i(t_i)$ are defined simultaneously. In such a space J_n, we thus have, remarking that the left-hand side of (C20) is independent of i, for all i,

$$H_i(t_i) = H_1(t_1)$$

which implies that, given the fact that $H_i$are strictly increasing, there is one and only one solution $t_i(t_1)$ denoted by $h_i(t_i)$, such that

$$t_i = h_i(t_1) \text{ with } \frac{dh_i}{dt_1}(t_1) = \frac{H_1'(t_1)}{H_i'(t_i)} > 0$$

where the derivative using the theorem of implicit functions. Summing up all $t_i$in the left hand-side of (C20), there is an equation defining t_1 which is

$$K(t_1) = \alpha_1 t_1 + \sum_{i>1} \alpha_i h_i(t_1) - H_1(t_1) = 0$$

One can now easily verify that $K(t_1)=0$ defines a unique solution. Indeed,

$$\begin{aligned} K'(t_1) &= \alpha_1 + \sum_{i>1} \alpha_i \frac{H_1'(t_1)}{H_i'(t_i)} - H_1'(t_1) \\ &= \sum_{i=1}^{n} \alpha_i \frac{H_1'(t_1)}{H_i'(t_i)} - H_1'(t_1) \\ & H_1'(t_1)\left[\sum_{i=1}^{n} \frac{\alpha_i}{H_i'(t_i)} - 1\right] \end{aligned}$$

Now, $H_i^{'}(t_i) = 1 + g_i'(\theta_i)\frac{\partial \theta_i}{\partial t_i} > 1$ because g_i and θ_i are strictly increasing. Thus $K'(t_1) < H_1'(t_1)\left[\Sigma\alpha_i - 1\right] < 0$. There is thus at most one solution in t_1. Since there is necessarily one solution in Nas shown in Appendix B.2.2, there is at least one solution for t_1 and thus t_1 is unique. This implies uniqueness of all $t_i$and thus of N_i. The decentralized equilibrium is thus unique.

D Physical capital

The resolution of the program of the firm leads to the first order conditions:

$$\left[F_i(\mathbf{N},K) - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N},K)}{\partial N_i} - w_i(\mathbf{N},K)\right] dt + (1 - s_i dt)\frac{\partial \Pi(\mathbf{N}^+,K^+)}{\partial N_i} = \frac{\partial \Pi(\mathbf{N}^+,K^+)}{\partial N_i}(1 + r dt), \tag{D21}$$

$$\left[F_K(\mathbf{N},K) - \sum_{j=1}^{n} N_j \frac{\partial w_j(\mathbf{N},K)}{\partial K}\right] dt + (1 - \delta dt)\frac{\partial \Pi(\mathbf{N}^+,K^+)}{\partial K} = \frac{\partial \Pi(\mathbf{N},K)}{\partial K}(1 + r dt) \tag{D22}$$

where $F(\mathbf{N},K)$ denotes the partial derivative of F with respect to the ith coordinate of the vector N.The envelop conditions write as

$$-\gamma_i + q_i \frac{\partial \Pi(\mathbf{N}^+,K^+)}{\partial N_i} = 0, \tag{D23}$$

$$-1 + \frac{\partial \Pi(\mathbf{N}^+,K^+)}{\partial K} = 0 \tag{D24}$$

Looking for the steady state solution and eliminating $\partial\Pi(\mathbf{N},K)/\partial N_i$ (that is henceforth denoted as $J_i(\mathbf{N},K)$) and $\partial\Pi(\mathbf{N},K)/\partial K$ from these four equations, one obtains

$$J_i(\mathbf{N},K) = \gamma_i/q_i, \text{with } J_i(\mathbf{N},K) = \frac{F_i(\mathbf{N},K) - w_i(\mathbf{N},K) - \sum_{j=1}^{n} N_j \frac{\partial w_i(\mathbf{N},K)}{\partial N_i}}{r + s_i} \tag{D25}$$

and equation (21) in the text.