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ABSTRACT

The 'Genome' of NEG Models with Vertical Linkages: A Positive and Normative Synthesis*

This Paper takes a broader look at how vertical linkages can trigger the spatial agglomeration of economic activity in a ‘new economic geography’ (NEG) set-up. First, it formally establishes the key positive features of a wide class of vertical-linkage models without resorting to numerical simulations. Second, it proposes an analytically solvable model of this class. Third, it addresses the important though neglected issue of whether in such models market forces yield too much or too little agglomeration. It shows that, in terms of positive implications, vertical-linkage models are identical to migration models once considered in their ‘natural’ state space. Important differences arise, however, in terms of normative implications in the absence of interregional transfers: in migration models agglomeration is necessarily bad for people stuck in lagging regions; in the vertical-linkage models it can be good for everybody as it delivers richer product variety.

JEL Classification: F12, F15, F21 and R12
Keywords: new economic geography, vertical linkages and welfare analysis

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*This Paper merges and extends unpublished work of ours that we developed independently: the mimeo by Ottaviano (2002) and the manuscript note due to Robert-Nicoud on the structure of NEG models with vertical linkages and their welfare properties. Some of the analysis of sections 2 and 3 in this Paper is also sketched in Baldwin *et al.* (2003, chapter 8). We are especially grateful to Sylvie Charlot, Carl Gaigné and Jacques Thisse for discussions regarding the welfare analysis of NEG models.

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1 INTRODUCTION

Peaks and troughs in the spatial distributions of populations, employment and wealth are a universal phenomenon in search of a general theory (Ottaviano and Thisse, 2004). More than a decade ago the 'new economic geography' (henceforth, NEG) set out to explain the observed spatial unbalances as the result of economic interactions in imperfectly competitive markets. NEG has now reached its first theoretical consolidation. As pointed out by Ottaviano (2003), this has been achieved by the appearance of two books that retrospectively systematize what NEG has attained so far. On the one hand, Fujita, Krugman and Venables (FKV, 1999) expose the techniques of NEG models and survey the positive insights they provide when applied to urban, regional, and international issues.[1] On the other hand, Fujita and Thisse (2002) assess the relative merits of NEG insights within the rich tradition of regional and urban economics.

The key insight of NEG is that, due to cumulative causation in NEG models even small transitory asymmetric shocks across locations can have large permanent effects on the economic landscape. This way, the economic development ('second nature') of locations is disconnected from their nature-given attributes ('first nature'). These results are also obtained by urban and regional models based on localized technological externalities (see, Fujita and Thisse, 2002). However, NEG models link the appearance of cumulative causation to the values of a set of microeconomic parameters describing, among others, the fundamental features of preferences and technology in specific sectors.

NEG models highlights three forces whose composition gives rise to cumulative effects in the presence of trade costs and plant-level increasing returns to scale: 'market access', 'market crowding', and 'cost-of-living/producing' effects. The first effect stems from the fact that firms like to locate close to their customers as this decreases their shipping costs ('backward linkages'). The second effect stems from the fact that firms do not like to locate close to their competitors as this reduces their market share and/or bid input prices up and output prices down. As to the third effect, it is due to the fact that consumers/firms like to

[1] Fujita et al. (1999) derive many properties of their models using numerical simulations and informal stability analysis. As to the former, Puga's (1999) analytical derivation of one of the key statisics (the break point –see below) came early enough to be introduced in the book. However, a thorough method for deriving the full set of long run equilibria in the simplest versions of the present model came later (Robert-Nicoud, 2005) and hence is not incorporated in Fujita et al. (1999). As to stability analysis, Baldwin (2001) establishes that the informal methods adopted by Fujita et al. (1999) are technically valid. He also extends Krugman's (1991) seminal model to the case of forward looking migrants. Ottaviano (2001) makes similar points in the Forslid-Ottaviano (2003) setting.

locate close to their suppliers as this decreases their living/production costs ('forward linkages'). Accordingly, the evolution of the spatial landscape is related to microeconomic parameters: agglomeration is more likely to take place in sectors where increasing returns are intense, market power is strong, and trade costs are low. The reason is that more intense returns to scale strengthen the market access effect while stronger market power weakens the market crowding effect. On the other hand, lower trade costs reduce all effects, but the market crowding effect more than the others.

As stressed by Puga (1999), NEG models achieve these results by allowing for some elasticity of factor supply in the agglomerating sectors. In Krugman (1991) it is the geographical mobility of labour that allows firms and workers to cluster due to demand and supply linkages. In Venables (1996) it is the intersectoral reallocation of factors within the same location that allows firms to cluster due to reciprocal vertical linkages. This model has been popularised by Krugman and Venables (1995). Its most simple version appears in Fujita et al. (1999, chapter 14, section 2). Of these two classes of models the latter, sometimes dubbed 'vertical-linkage (VL) models' is more difficult to deal with analytically than the former, also known as 'core-periphery (CP) models'.[2] Thus, notwithstanding their empirical appeal, the user unfriendliness of VL models has someway hampered the exploitation of their full potential.

Our aim is to reduce such a technical barrier by making life easier when using VL models. In so doing, we start with developing a setup that encompasses the model by Krugman and Venables (1995) – henceforth, simply 'CPVL model' – as a special case. We show that such setup also encompasses a simpler analytically solvable model recently put forth in a manuscript due to Ottaviano (2002) – henceforth, 'FEVL model' - as another special case. We then prove that most of the crucial properties (such as the number and stability of equilibria) of the encompassing setup are independent from the specific parameter values that generate the CPVL and FEVL models as special cases. The reason of such equivalence is that the two models as well as the encompassing setup are all 'isomorphic' in the sense of Robert-Nicoud (2005): they can be entirely characterized by the same set of equations in the appropriate state-cum-parameter space.

This is a first remarkable result. When crossed with Robert-Nicoud's (2005) results on CP models, it implies that, even though a general NEG theory is not yet available (and might

[2] Throughout the paper, the acronyms of the different models are borrowed from Baldwin et al (2003).

never be),[3] all simple models that are using the Dixit-Stiglitz-Samuelson paradigm share the same qualitative properties. An alternative non-Dixit-Stiglitz-Samuelson framework exists too (see Ottaviano, Tabuchi and Thisse, 2002). Fortunately, its properties are very similar. We can therefore be confident that the results derived in the NEG literature do have some degree of generality.

Another important implication of our analysis is that, given their fundamental equivalence, the solvable FEVL model should be used instead of the original non-solvable CPVL model (and, even more so, instead of the encompassing set-up). The advantage of analytical solvability is not only about theoretical elegance. Most of all, it is about the insights it may deliver on some open issues in the literature. To illustrate such potential, we provide a detailed welfare analysis of VL models in the wake of Charlot et al. (2004). Our central result is that, while VL and CP models are identical in terms of their positive implications (and are thus observationally equivalent), they may yield quite different normative implications. Specifically, when interregional transfers are not available, in CP models agglomeration is always a win-lose outcome that rewards core regions and penalizes peripheral ones. Differently, even without transfers agglomeration can be a win-win outcome in VL models provided that vertical linkages are strong and trade barriers are low. The reason is that agglomeration enriches product variety in VL models, while leaving it unaffected in CP models. We see this as a second remarkable result since, as argued by Neary (2001) about NEG, “[t]he field’s potential to throw light on policy is undoubtedly part of its appeal”. This is clearly exemplified by the few applications of NEG insights to the debate on European regional policies such as Martin (1999) and Puga (2002).

The paper is organized in four additional sections after the introduction. The first presents the encompassing setup. The second describes the short run and the long run equilibrium conditions. The third characterises the number and the stability of equilibria. The fourth presents the welfare analysis. The fifth concludes.

2 THE MODEL

This section develops a setup that encompasses both the CPVL and the FEVL models as special cases. It shows that such setup identifies a class of isomorphic models that can be entirely characterized by the same set of equations in the appropriate state-cum-parameter space.

[3] A paper similar to Jones (1965) for the Heckser-Ohlin paradigm awaits to be written for the NEG.

2.1 Endowments and geography

Consider a two-region, two-sector, one-factor environment. We label regions with a subscript $r \in \{1,2\}$ and aggregate values with the superscript 'w' for 'world'. The two sectors are a traditional sector *Z* and manufacturing *M*. Labour is the only productive factor. It is freely mobile between sectors within regions but immobile between regions. Each region is symmetrically endowed with $L^w/2$ workers.

2.2 Technology

The traditional sector produces a homogenous good under constant returns to scale and perfect competition. One unit of output requires one unit of labour, hence profit maximisation implies $p_r^Z = w_r$, where w_r denotes the wage prevailing in region *r*. Inter-regional trade in *Z* is costless and we assume that the parameters of the model are such that no region ever specialises in the production of good M only, hence $p_1^Z = p_2^Z$.[4] We choose this good as the numéraire, hence $w_r = p_r^Z = 1$ for all *r*.

The manufacturing sector, labelled *M*, produces a continuum of horizontally differentiated varieties, indexed by *j*, under increasing returns to scale and Dixit-Stiglitz monopolistic competition. The total cost of producing x_j units of variety *j* in region *r* is given by

(1)
$$\begin{aligned} C_r(x_j) &= F w_r^{1-\mu} P_r^{\mu} + a^M w_r^{1-\alpha} P_r^{\alpha} x_j \\ &= F P_r^{\mu} + a^M P_r^{\alpha} x_j \end{aligned}$$

where the second equality stems from the fact that w_r=1. Turning to the parameters, *F* and a^M are scale parameters related to the fixed and variable costs, respectively, and P_r is the manufacturing price index defined as

(2)
$$P_r = \left(\int_{k=0}^{n^w} p_r(k)^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}}$$

where $p_r(k)$ is defined as the delivered price of variety *k*.

[4] This requires that the labour force in each region is large enough to serve the world demand of M and yet still produce some Z. A sufficient condition requires $\mu < ½$. If this condition is violated, a richer set of results emerges. See FKV (1999, chapter 14) for details.

The interpretation of the cost function in (1) is as follows. The first term in the right-hand side is the fixed cost of production; it involves a composite of manufacturing goods (a share μ of the fixed cost) and labour (a share 1-μ). The second term is the variable cost, which also uses a Cobb-Douglas composite of manufacturing goods (a share α of the variable cost) and labour (a share 1-α) The manufacturing input is itself a CES composite of the n^w varieties of the manufacturing good available and σ>1 is the elasticity of substitution between any two pair of varieties. In this section, we treat n^w as a parameter. It will be determined endogenously in section 3 by the free-entry and exit of firms. Thus, firms active in sector *M* use each other's output as intermediate inputs.[5]

Because (2) displays increasing returns to scale but no scope economies, each variety is produced by a single firm. We assume that varieties are freely traded within regions but face 'iceberg' trade barriers between regions. Specifically, it takes τ>1 units to be shipped from region *r* for one unit of the good to arrive at destination in region $s \neq r$. The rest 'melts' in transit (hence the name).

2.3 Preferences

Define Y_r as the income of the representative resident of region *r*. Preferences over the composite manufacturing good *M* and the numéraire *Z* are described by the following indirect utility function:

(3) $$V_r = Y_r P_r^{-\mu} \left(p_r^Z \right)^{-1+\mu}$$

implying that the representative consumer spends a share μ on manufacturing goods and the rest on the traditional good. Note that the composite manufacturing final good is the same as the composite manufacturing input in (2). This standard assumption simplifies the analysis considerably. Also, $w_r = p_r^Z = 1$ implies $V_r = Y_r P_r^{-\mu}$. Moreover, for reasons that will become obvious soon, it proves convenient to write the manufacturing price index in a multiplicatively separable form that emphasizes the influence of n^w (the total number or mass of firms). Specifically, we define Δ such that *P* may be rewritten as

(4) $$P_r^{1-\sigma} = \Delta_r \left(n^w \right)^{\frac{1}{1-\alpha}}$$

[5] Strictly speaking, the linkages are more horizontal than vertical, but we stick with the now-established convention and refer to vertical linkages in the sequel. See Venables (1996) and Fujita et al (1999, chapter 16) for models with 'true' vertical linkages and Baldwin et al. (2003, chapter 8) for a discussion.

2.4 Demand

Applying Shephard's lemma to (1), the typical firm established in region r producing the quantity $x(r)$ demands the following quantity of variety j as intermediate inputs:

(5) $$d_r(j) = \frac{p_r(j)^{-\sigma} P_r^{\mu}}{(n^w)^{1/(1-\alpha)} \Delta_r} \mu F + \alpha a^M x(r) \frac{p_r(j)^{-\sigma} P_r^{\alpha}}{(n^w)^{1/(1-\alpha)} \Delta_r}$$

Let the representative resident of region r own all labour and firms in r. By Roy's identity, her demand for variety j is given by

(6) $$c_r(j) = \frac{p_r(j)^{-\sigma}}{(n^w)^{1/(1-\alpha)} \Delta_r} \mu Y_r$$

where Y_r is her income. By the same token, her demand for the traditional good is given by $c_r^Z = (1-\mu)Y_r$, $r = 1,2$.This income is made up of the labour wage and profits, namely,

(7) $$Y_1 = w_1 \frac{L^w}{2} + s_n n^w \Pi_1, \quad Y_2 = w_2 \frac{L^w}{2} + (1-s_n) n^w \Pi_2$$

where $s_n \in [0,1]$ is the share of firms operating in region 1 (this is treated as exogenous for now but will be endogenously determined in section 3) and Π_r is the pure profit of the average firm producing in r, namely

(8) $$\Pi_r(x_r) \equiv \pi_r - F P_r^{\mu}; \quad \pi_r \equiv (p_r^{mill} - a^M P_r^{\alpha}) x_r$$

where π_r is the operating profit and p_r^{mill} is the mill price of the typical variety produced in r.

The market-clearing condition for each variety of M implies

(9) $$x_r = (d_r + n_r d_r) + \tau(d_s + n_s d_s); \quad s \neq r, \quad n_1 \equiv s_n n^w, n_2 \equiv (1-s_n) n^w$$

where the output supplied (on the left-hand side of the above expression) must match final and intermediate demand from both domestic consumers and firms (the terms in the first bracket on the right-hand side) as well as from consumers and firms from region $s \neq r$, inclusive of transportation costs (the term involving the second bracket).

2.5 Prices

Under Dixit-Stiglitz monopolistic competition transportation costs are fully passed onto consumers. As a result, firms charge a unique mill price, irrespective of the destination market. Using (5), (6), and (9), maximization of Π_r yields

(10) $$p_r^{mill}(1-1/\sigma) = a^M P_r^\alpha$$

where we have dropped the variety indices as they are all symmetric within each region. The term on the right-hand side of the expression above is the marginal cost. Unless α=0 (in which case P_r=1 by (4) and the choice of numéraire), it is region-specific. The term (1-1/σ) on the left-hand side is the inverse of the markup. Indeed, each firm is a monopolist over her own variety, and σ is the demand elasticity she perceives. Note that this elasticity is constant and equal to the elasticity of substitution (a parameter in the model), regardless of the region in which the variety is produced and consumed. We choose units of M so that $a^M = 1-1/\sigma$, hence $p_r^{mill} = P_r^\alpha$. By (8) and (10), this implies $\pi_r = P_r^\alpha x_r/\sigma$.

With mill pricing, the delivered prices are then equal to

(11) $$p_r = P_r^\alpha, \quad p_s = \tau P_{s\ r}^\alpha, \quad s \neq r$$

which, by (2) implies a recursive definition for P_r. When α=0, (11) simplifies to p_r=1 and $p_s=\tau$, $s \neq r$, namely, prices are function of the parameters only. This is what makes the model of subsection 4.1 so much more tractable than the original CPVL model.

Before proceeding, we define ϕ over the unit interval as the freeness of trade, to be precise $\phi \equiv \tau^{1-\sigma}$. When $\tau=\infty$, inter-regional trade in manufacturing products is prohibitive and ϕ=0. When τ=1, the two regions are perfectly integrated and as a result the freeness of trade is perfect, i.e. ϕ=1.

Using (4) and the fact that all varieties produced in each region are symmetric, we can now provide a simpler recursive definition of Δ_r :

(12) $$\Delta_1 = s_n \Delta_1^\alpha + \phi(1-s_n)\Delta_2^\alpha, \quad \Delta_2 = \phi s_n \Delta_1^\alpha + (1-s_n)\Delta_2^\alpha$$

This way, n^w directly impacts on P and s_n influences P via Δ only. Disentangling the respective impacts of n^w and s_n will prove really helpful later on.

2.6 Special cases

Simple substitutions reveal the our model encompasses both the CPVL and the FEVL models as sub-cases. Indeed, the former or the latter materializes after imposing $\alpha=\mu$ or α=0 respectively.

3 SHORT RUN AND LONG RUN

Fujita et al (1999) distinguish between a 'short-run' and a 'long-run' equilibria. In the short run entry and exit of firms are restricted, so both n^w and s_n are given. Accordingly, consumers maximise utility, firms maximise profits, all markets clear for given number and geographical distribution of firms. Labour is instead freely mobile between sectors, so all workers earn the same wage. In the long run firms are free to enter and exit, so no active firm makes pure profits or losses in equilibrium.[6]

3.1 Short-run equilibrium

In a short-run equilibrium, given n^w and s_n consumers maximize utility, firms maximize profits, all markets clear, and workers are indifferent between working for sector *Z* or *M* as they both pay the same nominal wages.

We start by defining μE_1 as expenditure on manufacturing products in region 1 (μE_2 is defined symmetrically), which essentially combines the numerators of (5) and (6). By (1) and (3), this yields $\mu E_1 = \mu[Y_1 + s_n n^w P_1^{\mu} F + a^M s_n n^w P_1^{\alpha} x_1]$. Using (7), (8), (10) and $\pi_r = P_r x_r / \sigma$, this yields

(13)
$$E_1 = \frac{L^w}{2} + s_n n^w \left(1 + (\sigma - 1)\frac{\alpha}{\mu}\right)\pi_1, \quad E_2 = \frac{L^w}{2} + (1 - s_n) n^w \left(1 + (\sigma - 1)\frac{\alpha}{\mu}\right)\pi_2$$

Using these results together with (9), we can write the following expression for the operating profits π_1 and π_2:

(14)
$$\pi_r = \frac{bE^w}{n^w} B_r, \quad r = 1,2; \quad b \equiv \frac{\mu}{\sigma}$$

where $E^w \equiv E_1 + E_2$,

(15)
$$B_1 \equiv \Delta_1^{\alpha}\left(\frac{s_E}{\Delta_1} + \phi\frac{1 - s_E}{\Delta_2}\right), \quad B_2 \equiv \Delta_2^{\alpha}\left(\phi\frac{s_E}{\Delta_1} + \frac{1 - s_E}{\Delta_2}\right)$$

and $s_E \equiv E_1 / E^w$. Accordingly, operating profits are higher in the region exhibiting a higher share of expenditures and a lower share of firms. The more so the higher the freeness of trade.

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[6] Our definition of short run differs from Fujita et al (1999, chapter 14) who assume that in the short run firms enter and exit freely while labour cannot move between sectors. Accordingly, the short run would be characterized by no pure profits and intersectoral wage divergence. It turns out that this setup gives exactly the same results as ours in terms of long run equilibrium analysis (see Puga, 1999).

Using (12), (14) and (15), it is straightforward to see that the aggregate operating profits in this economy is proportional to aggregate expenditure: $n^w[s_n\pi_1+(1-s_n)\pi_2]=bE^w$ –or, equivalently, $s_nB_1+(1-s_n)B_2=1$. Plugging this into (13) gives an expression for E^w, namely,

(16) $$E^w=\frac{L^w}{1-\beta},\quad \beta\equiv\alpha+\frac{\mu-\alpha}{\sigma}$$

that is, E^w is a function of the parameters of the model only. This result stems from the specificities of the model (in particular the Cobb-Douglas specification for the preferences and the costs). To make economic sense, E^w must be strictly positive so we restrict the parameters to obey the condition $(1-\alpha)\sigma>\mu-\alpha$.[7]

Using this and (13)-(14), we find that the expenditure shares obey the simple arithmetic average:

(17) $$s_E=\frac{1-\beta}{2}+\beta s_nB_1,\quad 1-s_E=\frac{1-\beta}{2}+\beta(1-s_n)B_2$$

In addition, the world production of Z is equal to the world demand of Z: $Z_1+Z_2=(1-\mu)Y^w$. Inter-regional trade balances regional excess demands. Finally, labour markets clear by Walras' law.

To sum-up, the short-run equilibrium is characterized by an expression for the expenditure shares of each region (17) and an expression for the 'biases' in operating profits (15). Note that we have said nothing about what pure profits look like. This will be the object of the next section.

3.2 Long-run equilibrium

In the long-run free entry and exit of firms drive all pure profits to zero.

Therefore, a long run equilibrium has to fulfil all the properties of a short-run equilibrium plus the requirement that no active firm makes pure profits.

We will assume that new firms are created (closed) in each region as long as profits are positive (negative) or, which is the same, as long as q_r is larger than unity, where

(18) $$q_r\equiv\frac{\pi_r}{FP_r^{\mu}},\quad r=1,2$$

[7] This parameter restriction is always satisfied under the so-called 'no-black-hole' condition (Krugman, 1991) that we shall impose below.

which bears some resemblance with Tobin’s q. Specifically, we assume that firms enter and exit each market according to the following ad-hoc laws of motion:

(19) $$\dot{n}_1 = n_1(q_1 - 1), \quad \dot{n}_2 = n_2(q_2 - 1)$$

The idea is that firms are myopic and that they enter the market as long as operating profits are larger than the fixed cost.

If instead of n_1 and n_2 we use $s_n \equiv n_1/(n_1+n_2)$ and $n^w \equiv n_1+n_2$ as the state variables, then the laws of motion take the following intuitive form, which readily follows from (19):

(20) $$\dot{s}_n = s_n(1-s_n)(q_1 - q_2), \quad \dot{n}^w = n^w[s_n q_1 + (1-s_n)q_2 - 1]$$

The first differential equation in (20) says that, unless all firms are clustered in either region, the share of firms active in a region increases whenever local firms are more profitable than firms in the other region. The second equation in (20) says that the aggregate number (mass) of firms is increasing as long as the aggregate profits in sector *M* are positive.

A long run equilibrium is reached whenever $\dot{s}_n = 0$ and $\dot{n}^w = 0$. Given (20), two cases can occur. Either the system reaches an interior steady-state, in which case $q_1=q_2=1$ and $s_n \in (0,1)$, or the system is at a corner solution, namely $s_n=1$ and $q_1=1$.[8]

4 EQUILIBRIUM ANALYSIS

4.1 Agglomeration and dispersion

As is standard in the NEG literature, we use stability criteria to eliminate some of the long-run equilibria mentioned above. These methods are informal but Baldwin (2001) shows that they give the same answer as formal methods. We start with equilibria that entail either full agglomeration or even dispersion of manufacturing.

Core-periphery

The first question we want to address is under which circumstances the agglomeration of the manufacturing sector into a single region is sustainable. Without loss of generality, we assume that region *1* is the potential manufacturing ‘core’ whereas region *2* is the candidate ‘periphery’. Formally, this involves imposing $s_n=1$ and $q_1=1$. This equilibrium is stable if $q_2<1$. Indeed, if all these conditions hold simultaneously, by (20) the system reaches a long-

[8] By the symmetry of the system, $s_n=0$ and $q_2=1$ are also part of a long run equilibrium. Without loss of generality, we concentrate on the case in the text from now on.

run equilibrium in which no firm is active in *2*, firms active in *1* just break even, and no firm has any incentive to start producing in region *2* since, by (18), this would involve making negative pure profits.

Let

(21) $$\theta \equiv \frac{\mu}{\sigma - 1} + \alpha \quad \text{and} \quad \chi \equiv \frac{1-\beta}{1+\beta}$$

represent combinations of parameters that respectively capture the extent of forward and backward linkages. Specifically, θ=0 means that forward linkages are turned off and a larger θ is associated with stronger forward linkages. Conversely, χ=1 means that backward linkages are nil whereas χ=0 means that they are at their strongest.

From (12), it is obvious that s_n=1 implies Δ_1=1 and $\Delta_2=\phi$. Using (14), (15), (17), (18), and (21), in turn this implies

(22) $$q_1 = \frac{bE^w}{F}\left(n^w\right)^{-\frac{1-\theta}{1-\alpha}}, \quad q_2 = \phi^{\theta}\left(\phi s_E + \frac{1-s_E}{\phi}\right)q_1; \quad s_E = \frac{1+\beta}{2}$$

Two points are noteworthy in (22).

First, irrespective of n^w, $q_2 \le q_1$ if and only if ϕ is large enough. This means that n^w has no effect on profitability of firms established in either region relative to the other (this remarkable feature holds for any s_n). We will make this statement more precise below.

Second, q_1=1 if n^w is equal to n^0, where, by (16),

(23) $$n^0 \equiv \left(\frac{bL^w}{F(1-\beta)}\right)^{\frac{1-\theta}{1-\alpha}}$$

If n^w is larger than n^0, then all firms make losses because there are too many competitors (‘market crowding effect’). To be sure, more competitors also mean more input suppliers and hence smaller costs as well as more demand for intermediates (‘forward and backward linkages’), but the former effect dominates the latter if $0<\theta<1$.[9] In this case n^0 is increasing and convex in the term in the brackets in (23).

It is beyond doubt that θ is positive (indeed, $\theta>\alpha$ must hold for any admissible value of the structural parameters). But what happens if $1\le\theta$? In this case, the forward and

[9] We discuss the nature of the ‘market crowding’ dispersion force and of the ‘backward’ and ‘forward linkages’ in detail in Appendix D.

backward linkages are so strong that adding a new competitor increases the profitability of all competitors already present. In mathematical terms, we write $\partial q_1/\partial n^w>0$. By the law of motion for n^w in (20), n^w and q_1 grow beyond n^0 and 1, respectively. This growth is bounded by the capacity of the labour market only. The so-called 'no black hole condition' $\theta<1$ (Krugman 1991), which we impose from now on, rules out this 'pathological' case.

As we wrote a couple of paragraphs above, agglomeration of the manufacturing sector within a single location is sustainable whenever transportation costs are low enough (which corresponds to a ϕ large enough). In mathematical terms, $q_1>q_2$ and $s_n=1$ simultaneously if $\phi\in(\phi^S,1)$, where ϕ^S is implicitly defined as the smallest root of the polynomial

(24) $$(1+\chi)\phi^\theta-\phi^2-\chi=0$$

To derive this expression, set $q_2=q_1$ in (22) and use (16) to get an expression for s_E. We shall refer to ϕ^S as the 'sustain point' because the core-periphery pattern is 'sustainable' beyond that value of ϕ.

Eq. (24) reveals two interesting facts. Firstly, when τ is low (ϕ large), the agglomeration forces, made of the forward and backward linkages, dominate the market crowding force (in which case the left-hand side of (24) is positive). It is easy to see that agglomeration is more likely (i.e. takes place over a larger interval) when χ is low (a low χ is associated with strong backward linkages) and when θ is large (which is associated with strong forward linkages).

Note that, if forward linkages are so strong that $\theta>1$, then agglomeration is sustainable for the whole range of parameters, that is, $\phi^S=0$. The no black hole condition rules out this extreme and uninteresting case and thus $\phi^S>0$. Such condition also ensures that n^w is finite and equal to n^0.

Dispersion

A second type of long run equilibrium that can arise is one in which the manufacturing sector is spread across regions and $q_1=q_2=1$. We focus on the symmetric equilibrium first: $s_n=½$ and $q_1=q_2=1$. As is obvious by the symmetry of the model, such a steady-state always exists. Indeed, since the two regions are identical in terms of the exogenous variables of the model, all active firms are equally profitable if the manufacturing sector is evenly split between the two regions.

In the symmetric equilibrium, free entry and exit pin down the aggregate mass of firm to $n^w=n^{½}$, where

(25) $$n^{1/2} \equiv n^0 \left(\frac{1+\phi}{2}\right)^{\frac{\theta-\alpha}{1-\theta}}$$

Note that $n^{½}$ is lower than n^0 and is an increasing function of ϕ if (and only if) the no black hole condition $1>\theta$ holds. In other words, when the two regions become more integrated, the market becomes more fragmented.

Let us comment briefly on this. The fact that more firms enter the market when ϕ increases results from the interplay of two different forces. On the one hand, when ϕ increases the market crowding effect increases. This tends to decrease profits and hence n^w at equilibrium. But this also decreases production costs (more firms means more input suppliers) and increases expenditure on intermediates (more firms means more customers purchasing intermediates), hence boosting profits. The net effect is unambiguously positive as (25) reveals.

The symmetric equilibrium always exists but might not be stable. To asses the stability of this equilibrium, NEG typically makes the following thought experiment: let a shock perturb the number of firms in either region, say the number of firms in region *1* increases. In the 'short run', this shocks feeds into all endogenous variables. If, as a result of the dynamics (20), the system goes back to the initial equilibrium, then the symmetric equilibrium is said to be stable. If, instead, the system moves further away, then the symmetric equilibrium is unstable. As we show in the appendix, the symmetric equilibrium is stable if ϕ is low enough, that is, if

(26) $$\phi \le \phi^B, \quad \phi^B \equiv \chi \frac{(1-\theta)}{(1+\theta)}$$

We shall refer to ϕ^B as the 'break point' because symmetry is 'broken' beyond that value of ϕ.

Four comments are in order. First, when trade is so free that $\phi>\phi^B$, the symmetric equilibrium is unstable. If, for example, a plant closes down in region *2*, then the forward and backward linkages are so strong relative to the market crowding dispersion force that firms remaining in *2* face a fall in their profitability while the profits of firms in region *1* increase. As a result, some firms shut down in region *2* and new plants open in region *1*. By cumulative causation, the cycle repeats and the system moves away from the symmetric equilibrium.

Second, when $\phi>\phi^B$ the system ends up in the nearest stable long run equilibrium. See Baldwin (2001) for a formalisation of this claim.

Third, when agglomeration forces are stronger, the symmetric equilibrium is unstable over a larger range of ϕ. In particular, ϕ^B is increasing in χ (which inversely captures the strength backward linkages) and decreasing in θ (which captures the strength of forward linkages). Also, agglomeration forces are stronger in the CPVL model ($\alpha=\mu$) than in the FEVL model ($\alpha=0$).[10] This was to be expected: in the former model intermediate inputs are needed both for the fixed and the variable components of the cost function, whereas in the latter model intermediates are used in the fixed component only.

Finally, when the no-black-hole condition is violated, forward and backward linkages (then agglomeration forces) are so strong relative to the market crowding effect (the dispersion force) that the symmetric equilibrium is never stable.[11] In this case, the core-periphery outcome is the unique stable long run equilibrium.[12] As already stated, we rule this case out by imposing $\theta<1$.

4.2 Partial agglomeration

The previous subsection has characterized the existence and stability of two types of long-run equilibria such that manufacturing is either evenly distributed between regions ('dispersion') or fully agglomerated in one region ('core-periphery'). The present subsection establishes the existence and stability of all other long-run equilibria. In particular, it proves that additional equilibria exist only when ϕ falls between the break and sustain points. These additional equilibria are two. They are associated with an uneven spatial distribution of manufacturing, though short of full agglomeration. They are always unstable.

Mathematically, we show that there exist some s_n in (½,1) and some $n^w>0$ such that $q_1=q_2=1$ and that q_1 (q_2) is increasing (decreasing) in the neighbourhood of that s_n.[13] In so doing, we proceed in two steps. First, we show that in the specific case where $\alpha=0$ our (FEVL) model allows us to get closed form solutions for nearly all variables of interest. This, in turn, allows us to completely characterise the set of steady states *as well as their stability properties*. Second, we show that these apply to the general case, the reason being that the exact value of α does not affect the fundamental qualitative properties of the model. This second step requires indirect and longish arguments as closed form solutions are available for a subset of spatial configurations only. A key implication is that, given the fundamental

[10] By (14) and (16), β is increasing in α.
[11] In this case, $\phi^B=0$.
[12] This is a direct consequence of Propositions 1 and 2 below.
[13] By the symmetry of the model, we can concentrate on the interval [½,1] for s_n without loss of generality.

equivalence among all variants of our general model, the solvable FEVL model should be used instead of the original non-solvable CPVL model. The advantage of analytically solvability is not only about theoretical elegance. Most of all, it is about potential insights as the next section will substantiate in terms of welfare analysis.

A (not so) special case

Consider the particular case α=0. This gives rise to the so-called FEVL model in which firms use intermediate inputs in the fixed component of (1) only. The simplifying effect of such assumption is paramount. The definition of the price indices is no-longer recursive. Hence, (4) and (12) reduce to:

(27) $$P_r = \left(\Delta_r n^w\right)^{-\frac{\mu}{\sigma-1}}, r = 1,2; \quad \Delta_1 = s_n + \phi(1-s_n), \quad \Delta_2 = \phi s_n + (1-s_n)$$

Similarly, we can get a closed form solution for s_E from (15) and (17):

(28) $$s_E = \frac{1}{2} + \frac{\phi b}{[s_n + \phi(1-s_n)][\phi s_n + (1-s_n)] - b(1-\phi^2)s_n(1-s_n)}\left(s_n - \frac{1}{2}\right)$$

which makes it obvious that region *1* is larger than region *2* (in terms of expenditure) if and only if most firms are located there. In turn, the expression for the operating profits (14) becomes:

(29) $$\pi_1 = \frac{bE^w}{n^w}\left[1 - \frac{(1-s_n)(s_n - \frac{1}{2})}{s_n(1-s_n) + \dfrac{1-\psi^2}{4\psi(\psi - b)}}\right], \quad \pi_2 = \frac{bE^w}{n^w}\left[1 + \frac{s_n(s_n - \frac{1}{2})}{s_n(1-s_n) + \dfrac{1-\psi^2}{4\psi(\psi - b)}}\right]$$

where $\psi \equiv (1-\phi)/(1+\phi)$ and, as before, bE^w/n^w is the average operating profit in the economy. Also, the terms in the square brackets above correspond to the *B*'s in section 2.

Two features of the expressions above are noteworthy. First, there are two cases in which the prevailing operating profits in region *1* are equal to the world average bE^w/n^w: when s_n=1 (in which case π_1 is the average operating profit by definition) and when s_n=½ (in which case $\pi_1 = \pi_2$ by the symmetry of the model). Second, the market crowding effect always dominates the backward linkage, and thus $\pi_1 < \pi_2$, if (and only if) s_n>½. In other words, being close to one's competitors is bad for operating profits. Without another agglomeration force, firms would always choose to disperse and s_n=½ would be the unique stable equilibrium.

But there are forward linkages. These operate by reducing the fixed cost. Indeed, the closed form solution for (18) when α=0 is

(30)
$$q_1 = \frac{\pi_1}{F\left[n^w\left(s_n + \phi(1-s_n)\right)\right]^{-\frac{\mu}{\sigma-1}}}, \quad q_2 = \frac{\pi_2}{F\left[n^w\left(\phi s_n + (1-s_n)\right)\right]^{-\frac{\mu}{\sigma-1}}}$$

with the expressions for the operating profits substituted for from (29).

The number of interior long-run equilibria can be directly assessed from (29)-(30) by looking for values of s_n such that q_1=q_2=1. Since n^w enters all expressions multiplicatively, the problem can be separated in two sub-problems: finding s_n such that q_1-q_2=0, and then n^w such that q_r=1 (r=1,2) conditional on s_n. Since q_r is monotonic, the solution of the second sub-problem is unique: there is one and only one n^w for any s_n. On the contrary, the first sub-problem may have multiple solutions. Their maximum number can, nonetheless, be readily determined. To see this, take the derivative of ln(q_1/q_2) with respect to s_n. Since the numerator of the resulting expression is a third-order polynomial in s_n, ln(q_1/q_2) admits at most three flat points on [0,1] and hence q_1-q_2=0 admits at most four solutions. As we already know that s_n=½ is always one solution, the symmetry of the model implies that also a second solution must equal s_n=½. This leaves at most two additional twin solutions s_n≠½ ('asymmetric equilibria'): if they exist, they exist together; one solution is smaller and the other is larger than s_n=½; their distance from s_n=½ is the same. As a result, there are at most three different values such that q_1-q_2=0. By continuity, the twin solutions s_n≠½ exist if ϕ is strictly between the break and sustain points; they are stable if (and only if) $\phi \in (\phi^B, \phi^S)$ and unstable if $\phi \in (\phi^S, \phi^B)$. These two cases are mutually exclusive. We show in the Appendix that $\phi^S < \phi^B$ holds for all admissible parameter values. Therefore, if it exists at all, no interior long-run equilibrium with s_n≠½ is ever stable.

To summarise:

Proposition 1(Equilibrium in the FEVL model). Letα=0. The model displays at most three interior long run equilibria (whereby $q_1=q_2=1$). In particular, the symmetric equilibrium (s_n=½) always exists and is stable whenever $\phi \leq \phi^B$. The asymmetric equilibria ($s_n \neq$½) exist whenever $\phi \in (\phi^S, \phi^B)$; however, they are unstable. Finally, there always exist two core-periphery equilibria whereby $s_n \in \{0,1\}$. These equilibria are stable whenever $\phi \geq \phi^S$.

The general case

In this subsection we argue that a result similar to Proposition 1 holds for the general case. This is an important result: for the first time it allows to formally establish the dynamic properties of the much used CPVL model ($\alpha=\mu$). It also generates a corollary that will prove important for both empirical and applied theoretical work. The proof (in the Appendix) is based on the procedure used by Robert-Nicoud (2005) to prove the dynamic properties of the migration model by Krugman (1991).

We thus have established our second important result:

Proposition 2. The model displays at most three interior long run equilibria. In particular, the symmetric equilibrium s_n=½ always exists and is stable whenever $\phi \leq \phi^B$. Two asymmetric interior equilibria exist whenever $\phi \in (\phi^S, \phi^B)$; moreover, they are unstable. Finally, there always exist two core-periphery equilibria whereby $s_n \in \{0,1\}$. These equilibria are stable whenever $\phi \geq \phi^S$.

An important corollary to this proposition is the following.

Corollary 2.1. The value taken by α has no influence on the number and nature of long run equilibria. Similarly, the nature of the instantaneous equilibrium is independent of α..

This corollary has three far-reaching implications. The first implication of is that the CPVL model ($\alpha=\mu$) and the simpler FEVL model (α=0) are isomorphic (see equation (54) in Appendix C). The former model is cumbersome to work with because the instantaneous equilibrium is described by a system of simultaneous, implicit, and non-linear equations. The latter, by contrast, expresses the endogenous variables that are key to the dynamics of the system, namely q_1 and q_2, as a function of the variable of interest in any NEG model, that is, the spatial allocation of manufacturing activity s_n. Since both models put forth the same agglomeration mechanism (the interaction between vertical linkages, increasing returns at the plant level, and transportation costs) and are mathematically equivalent in a deep sense, our results suggest that *theoretical work that uses this framework should impose* $\alpha=0$.

The second implication is that, since the two models are observationally equivalent, it would be futile to try to distinguish them empirically. Hence, practitioners could go for the easiest model at no cost.

The third implication is that the present paper completes the main result in Robert-Nicoud (2005). It shows that NEG models whereby the agglomeration force is the result of embodied factor mobility (Krugman, 1991; Forslid and Ottaviano, 2003), of disembodied factor mobility and vertical linkages (Fujita et al. 1999, chapter 14; Robert-Nicoud 2002; Ottaviano, 2002; Baldwin et al. 2003, chapter 8), or of factor accumulation (Baldwin, 1999) are all isomorphic: just like the general model in this paper, their short-run equilibrium is given by expression (54) in Appendix C and their sustain and break points by (24) and (26), which we repeat for convenience:[14]

(31) $$(1+\chi)\left(\phi^S\right)^{1-\theta}-\left(\phi^S\right)^2-\chi=0, \quad \phi^B=\chi\frac{1-\theta}{1+\theta}$$

These are remarkable results. All these models have in common a Dixit-Stiglitz monopolistic competition sector that faces Samuelsonian iceberg costs of transportation. The parameters ϕ, θ and χ have the same economic interpretation throughout. Yet, the agglomeration mechanisms differ across models.

What does all this mean? It means that, even though a general NEG theory is not yet available (and might never be),[15] all simple models that are using the Dixit-Stiglitz-Samuelson paradigm yield the same qualitative results. An alternative non-Dixit-Stiglitz-Samuelson framework exists too (see Ottaviano, Tabuchi and Thisse, 2002). Fortunately, its qualitative results are very similar to those developed here. We can therefore be somewhat confident that the results derived in the NEG literature have some degree of generality.

5 WELFARE ANALYSIS

The previous section has proven the fundamental isomorphism of the widely used CPVL and the simpler FEVL models: they provide equivalent insights on the process of spatial

[14] Robert-Nicoud (2005) shows how to derive these expressions directly from (54).

[15] A paper similar to Jones (1965) for the Heckser-Ohlin paradigm awaits to be written for the NEG.

agglomeration. However, while the FEVL model is analytically solvable, the CPVL is not. Hence, the latter can be used instead of the former with no cost in terms of potential insights.

In this section we use this result to tackle an important issue that is largely neglected in the literature: the welfare implications of NEG models. Notable exceptions are Baldwin et al. (2003, chapter 11), Charlot et al. (2004), and Ottaviano and Thisse (2002) and Robert-Nicoud (2002). Baldwin et al. compare the market equilibrium with the utilitarian planner’s first best. Their approach is incomplete under two respects. First, their focus is on CP models only. Second, their utilitarian approach is questionable when people face different price indices in different regions (more on this below). For this reason, Charlot et al. (2004) supplement the utilitarian analysis with additional tools, such as Hicks’s and Kaldor’s criteria, borrowed from standard public economics. Also their investigation, however, is limited to CP models.

The main contribution of the present section is to apply the comprehensive approach of Charlot et al. (2004) to VL models. In so doing, it builds on two results from the previous section. First, as already recalled, the encompassing model (and *a fortiori* the CPVL model) behaves as the FEVL model, so our analysis can focus on the latter without loss of generality. Second, when they exist, equilibria other than full agglomeration (‘core-periphery’) and even dispersion are always unstable, so they can be disregarded. This allows us to confine our attention to the conditions that make full agglomeration socially preferable to even dispersion.

Before we proceed, an important remark is in order. By comparing full agglomeration and even dispersion only, we are not pursuing a first-best welfare analysis; rather, we compare the only two market outcomes. We do this on the following grounds. On the one hand, unless the planner has the means to enforce the whole vector of optimal resource and production allocations, the first best is a purely theoretical outcome with no chance of being implemented in a decentralized economy. On the other hand, as we shall see, even lump-sum transfers dramatically alter the nature of the long-run equilibria, hence the ‘ceteris paribus’ type of argument has no practical bite. As Little (1949) forcefully argues, it is not enough to look at the desirability of compensations; they must also be feasible at the corresponding equilibrium prices and wages. That is, paraphrasing Charlot et al. (2004), we must check that the compensations may be effectively paid and that all markets clear at the incomes net of compensations.

5.1 Pareto improvements

We start our welfare analysis by evaluating the indirect utility levels of workers in the two regions at the two alternative long-run equilibrium configurations. This can be done by using (4) while remembering that pure profits are driven to zero in the long run and that in the FEVL model α=0. In particular, from (23) and (25) we get:

(32) $$n^{1/2} = n^0\left(\frac{1+\phi}{2}\right)^{\frac{a}{1-a}},\ n^0 = \left(\frac{b}{1-b}\frac{L^w}{F}\right)^{1-a},\ a \equiv \frac{\mu}{\sigma-1}, b \equiv \frac{\mu}{\sigma}$$

Under dispersion (s_n=½ and n^w=$n^{½}$), perfect symmetry implies that all workers enjoy the same indirect utility V^D=$w_rP_r^{-\mu}$. Since their nominal income is w_r=1 and they all face the same price index P_r, we can write

(33) $$V^D = \left(n^{1/2}\right)^a\left(\frac{1+\phi}{2}\right)^a = \left(n^0\right)^a\left(\frac{1+\phi}{2}\right)^{\frac{a}{1-a}}$$

Under agglomeration (s_n=1 and n^w=n^0), individuals in region *1* (‘core’) benefit from lower consumer prices on all varieties. Conversely, individuals in region *2* (‘periphery’) have to pay for the transportation of all varieties. Hence, indirect utility is higher in the core than in the periphery:

(34) $$V_1^A = \left(n^0\right)^a,\ V_2^A = \left(n^0\right)^a\phi^a$$

By (32), under the no-black-hole condition n^0 is larger than $n^{½}$, so workers in region *1* are unambiguously better off under agglomeration than under dispersion.

While worse off than workers in region *1*, it is *a priori* uncertain whether workers in region *2* should prefer agglomeration to dispersion. On the one hand, they benefit from the larger set of varieties that is available under agglomeration than under dispersion (n^0>$n^{½}$). On the other hand, under agglomeration they have to pay transport costs on all varieties rather than only on half the number available under dispersion. For this reason, agglomeration is preferred also by workers in region *2* if the associated variety expansion is pronounced and transport costs are low. By (32) this happens if forward and backward linkages are strong (large a).

Formally, *agglomeration Pareto dominates dispersion* if $V_1^A \geq V^D$, which is always strictly the case, and $V_2^A > V^D$ i.e.

(35) $$\phi^a - \left(\frac{1+\phi}{2}\right)^{\frac{a}{1-a}} > 0$$

where we have used (33) and (34). By inspection, (35) holds when a is large (strong linkages) and ϕ is large (low trade costs). To see this, note that $a<1$ holds under the no-black-hole condition. Then the left hand side of (35) is a concave continuous function of ϕ reaching its unique maximum at $\phi=[2(1-a)^{(1-a)/a}-1]$. Since it is always negative at $\phi=0$ and decreasing and equal to unity at $\phi=1$, there exists a threshold value $\phi^{Pareto}<[2(1-a)^{(1-a)/a}-1]$ such that (35) holds for $\phi>\phi^{Pareto}$ and it is violated otherwise. Larger a reduces $[2(1-a)^{(1-a)/a}-1]$ and ϕ^{Pareto} (as two lines of standard algebra reveal). Thus, we have:

> *Proposition 3 (Pareto improvements). Agglomeration Pareto-dominates dispersion if trade costs are low ($\phi>\phi^{Pareto}$), otherwise the two configurations cannot be Pareto-ranked.*

This result differs markedly from what has been established by Charlot et al. (2004) in CP models; there agglomeration is always worse than dispersion for people left behind in the periphery. The reason is that in those models the total mass of varieties is always the same whatever the spatial distribution of firms and workers, thus the price index is always larger in the periphery under agglomeration than even dispersion.

5.2 Social welfare function

Proposition 3 establishes the existence of a conflict between core and periphery when linkages are weak ($a<½$) and trade costs are large. In the remainder of this section, therefore, we work under the assumption $a<½$ but in some places we remind the reader what would happen under the alternative $a\geq½$. In this case, social welfare functions can be used to weigh the gains and losses of the two regions albeit this approach is debatable (Fleurbaey and Hammond, 2004). Among the many possible alternatives here we follow Charlot et al. (2004) and adopt the symmetric CES-type social welfare function (henceforth SWF):

(36) $$SWF^j(\phi;\iota) \equiv \begin{cases} \frac{1}{1-\iota}\left[\frac{L^w}{2}\left(V_1^j(\phi)\right)^{1-\iota} + \frac{L^w}{2}\left(V_2^j(\phi)\right)^{1-\iota}\right], & \iota \neq 1 \\ \frac{L^w}{2}V_1^j(\phi) + \frac{L^w}{2}\ln V_2^j(\phi), & \iota = 1 \end{cases}$$

in which $\iota \geq 0$ measure the degree of societal aversion to inequality and $j=A,D$ denotes the state of the economy (agglomeration or dispersion). In particular, when $\iota=0$ (zero aversion towards inequality) then SWF is identical to the utilitarian social welfare function. Conversely, when ι tends toward infinity then (36) is identical to the Rawlsian welfare function.

To fix ideas, we start by adopting the utilitarian SWF as in Baldwin et al. (2003) and Robert-Nicoud (2002), therefore defining social welfare as

(37) $$W^j(\phi) \equiv \frac{L^w}{2} V_1(\phi) + \frac{L^w}{2} V_2(\phi)$$

Under the utilitarian criterion, agglomeration is preferred to dispersion whenever $W^A(\phi)=(L^w/2)V_1^A+(L^w/2)V_2^A$ is larger than $W^D(\phi)=L^w V^D$. By (33) and (34), that happens whenever:

(38) $$\frac{1+\phi^a}{2} - \left(\frac{1+\phi}{2}\right)^{\frac{a}{1-a}} > 0$$

Since ϕ is smaller than one, (38) is clearly less stringent than (35). Thus, if agglomeration Pareto-dominates dispersion, also the utilitarian criterion favours agglomeration. The converse, of course, is not true: there is an intermediate range of trade costs such that the utilitarian approach selects agglomeration even though that does not Pareto-dominate dispersion. To see this, note that the left hand sides of (35) and (38) have the same concave shape and coincide at $\phi=1$. The latter is nonetheless larger for any other ϕ. Moreover, it is negative at $\phi=0$ since $a<½$. In this case, (38) shares the same property as (35): there exists a threshold value ϕ^u such that (35) holds for $\phi>\phi^u$ and is violated otherwise. Such threshold is reduced by any increase in a. Since (38) is larger than (35) unless $\phi=1$, it must be $\phi^u<\phi^{Pareto}$.[16]

[16] When $½<a<1$, (38) always holds, so agglomeration is always selected under the utilitarian approach.

Proposition 4 (Utilitarian welfare function). The utilitarian planner prefers agglomeration to dispersion when the former is Pareto-dominant ($\phi > \phi^{Pareto}$). When the two configuration cannot be Pareto-ranked ($\phi < \phi^{Pareto}$), it still favours agglomeration provided vertical linkages are strong and trade costs are low enough ($\phi > \phi^{u}$). In other words, the utilitarian criterion is biased against dispersion ($\phi^{u} < \phi^{Pareto}$).

When comparing this result with the market outcome, there is no presumption whether ϕ^{u} is larger than ϕ^{B} or ϕ^{S}. Three cases may occur. First, the utilitarian solution coincides with the market outcome when this entails dispersion if linkages are weak and trade is costly (a<½ and $\phi < \min(\phi^{S}, \phi^{u})$). Second, the utilitarian solution coincides with the market outcome when this delivers agglomeration if linkages are strong and trade is cheap (a>½ and $\phi \geq \phi^{B}$, or if a<½ and $\phi > \max(\phi^{B}, \phi^{u})$). Third, in all other cases (intermediate linkage and trade costs), the market solution and the planner's preferred outcome do not coincide.

These results are comparable with those derived by Baldwin et al. (2003) and Charlot et al. (2004) for CP models. When agglomeration forces are strong, there exists an intermediate range of openness where the market delivers agglomeration while the planner implements dispersion. The opposite is true for weak agglomeration forces.

Another interesting benchmark is ι=2 (Fleurbaey and Michel, 2001). Indeed, for degrees of aversion to inequality beyond 2, the simultaneous reduction of 1% of the utility of the better off and increase of 1% of the well being of the worse off is judged favourably by the SWF. In colloquial words, destroying millions of dollars of Bill Gates' fortune to improve the lot of somebody as poor as Job by a few cents would increase social welfare in such a case. Thus, when ι=2, (36) implies

(39) $$SWF^{A}(\phi;2) - SWF^{D}(\phi;2) = \frac{L^{w}}{2}\left[-(1+\phi^{-a}) + 2\left(\frac{1+\phi}{2}\right)^{-a/(1-a)}\right]$$

which is positive if (and only if) ϕ is larger than ϕ^{FM}, where ϕ^{FM} is defined as the unique root (different than unity) of

(40) $$F(\phi) = -(1+\phi^{-a})\left(\frac{1+\phi}{2}\right)^{a/(1-a)} + 2 > 0$$

this root is in the relevant [0,1) range if $a \leq ½$. Using (38) and (39), it is easy to check that the condition in the former holds if the expression in the latter is positive. In other words, $\phi > \phi^{FM}$ implies $\phi > \phi^{u}$.

More generally, the range of parameters over which agglomeration is said to yield larger social welfare than dispersion is decreasing in the degree of aversion to inequality. To see this, note that (36) implies, for general ι's:

(41) $$SWF^{A}(\phi;\iota) - SWF^{D}(\phi;\iota) = \frac{L^{w}}{1-\iota}\left[\frac{1+\phi^{a(1-\iota)}}{2} - \left(\frac{1+\phi}{2}\right)^{\frac{a(1-\iota)}{1-a}}\right]$$

which is larger than zero for any ϕ larger than $\phi(\iota)$, where $\phi(\iota)$ is implicitly defined as the solution to

(42) $$G\left(\phi(\iota)\right) \equiv \frac{1+\phi^{a(1-\iota)}}{2} - \left(\frac{1+\phi}{2}\right)^{\frac{a(1-\iota)}{1-a}} = 0$$

and is positive unless $a > ½$. The method of proof is the same as for the special case $\iota = 0$. Numerical simulations suggest that $\phi(\iota)$ is increasing in ι. This very intuitive result can be summarised as follows:

Proposition 5 (CES social welfare functions). Agglomeration is preferred to dispersion over a lower range of parameter values if society judges inequality more severely ($\partial\phi(\iota)/\partial\iota > 0$). In particular, the utilitarian planner might prefer agglomeration to dispersion even when a planner whose aversion to inequality is equal to 2 prefers dispersion to agglomeration ($\phi^{u} < \phi^{FM}$).

5.3 Potential Pareto improvements

Social welfare functions involve interpersonal comparisons. This is notoriously problematic, especially when different people face different price indices as in the present model (Wildasin, 1986).[17] By contrast, compensation criteria based on the prevailing equilibrium prices and wages do not suffer from this caveat. Accordingly, we make two compensation experiments. First, following Kaldor (1939), we evaluate the minimum transfer from core to periphery that under agglomeration would let the periphery reach the same welfare level as

[17] See Charlot et al. (2004) for a detailed discussion of this issue in CP models.

under dispersion. Second, following Hicks (1940), we evaluate the maximum transfer from periphery to core that under dispersion would let the core reach the same welfare level as under agglomeration. In both cases, in the wake of Little (1949), we check the feasibility of transfers at the corresponding equilibrium prices.

Kaldor’s approach

We start with assessing the conditions under which agglomeration corresponds to a potential Pareto improvement with respect to dispersion. In so doing, initially we compute the transfer needed to compensate the peripheral region *2*. By (33) and (34), people in the periphery are exactly compensated if they get additional income T_A such that

(43) $$(1+T_A)V_2^A = V^D \quad \leftrightarrow \quad (1+T_A)\phi^a = \left(\frac{1+\phi}{2}\right)^{\frac{a}{1-a}}$$

Since core and periphery host the same number of people, each resident in the core pays T_A. By Proposition 3, T_A is positive unless a>½ and $\phi>\phi^{Pareto}$.

We now check that the above compensation scheme is indeed feasible by verifying that the material balance conditions still hold at the consumption pattern corresponding to the compensated incomes, $Y_1=(1-T_A)L^w/2$ and $Y_2=(1+T_A)L^w/2$. Since $\Delta_2=\phi$ when $s_n=1$, (15) implies $B_1=1$ for all s_E. These incomes leave the demand that each firm faces unchanged. As prices have not changed either, output remains the same too. Thus, the initial market clearing conditions still hold and the proposed compensation scheme is indeed feasible.

Lastly, we determine under which conditions region *1* residents prefer agglomeration with compensation to dispersion. Given (33) and (34), that is the case if and only if:

(44) $$(1-T_A)V_1^A > V^D \quad \leftrightarrow \quad (1-T_A) > \left(\frac{1+\phi}{2}\right)^{\frac{a}{1-a}}$$

By substituting the value of T^A that solves (43), condition (44) can be rewritten as:

(45) $$K(\phi) \equiv 2-(1+\phi^{-a})\left(\frac{1+\phi}{2}\right)^{\frac{a}{1-a}} > 0$$

Under the no-black-hole condition (a<1), the fact that ϕ is smaller than one implies that (45) is less stringent than (35). Thus, most naturally, if agglomeration Pareto-dominates dispersion, then Kaldor’s criterion favours agglomeration, too. The converse, of course, is not true: there is an intermediate range of trade costs such that Kaldors’ criterion selects agglomeration even though this outcome does not Pareto-dominate dispersion. To see this, note that the left hand sides of (35) and (45) have similar concave shapes and are both equal to zero at $\phi=1$. While the latter is larger for any other ϕ, they are both always negative at $\phi=0$: there exists a threshold value ϕ^{Kaldor} such that (45) holds for $\phi>\phi^{Kaldor}$ and is violated otherwise. Such threshold is reduced by any increase in a. Since (40) is larger than (35) unless $\phi=1$, it must be $\phi^{Kaldor}<\phi^{Pareto}$: thanks to compensation agglomeration is preferable over a wider parameter range.

Note also that $K(\phi)$ is identical to $F(\phi)$, that is, Kaldor's criterion gives the same answer as the SWF when aversion to inequality is equal to two! This surprising result is an artefact of our functional forms that combine the Dixit-Stiglitz-Krugman model with a CES SWF.

We have thus established:

Proposition 6 (Kaldor improvements). Kaldor's criterion prefers agglomeration to dispersion when the former is Pareto-dominant ($\phi^{Kaldor}<\phi^{Pareto}$). When the two configuration cannot be Pareto-ranked ($\phi<\phi^{Pareto}$), it still favours agglomeration provided vertical linkages are strong and trade costs are low enough ($\phi>\phi^{Kaldor}$, $\phi^{Kaldor}<\phi^{Pareto}$). Finally, Kaldor's criterion favours agglomeration over dispersion over a larger parameter range than the SWF if (and only if) $\iota>2$.

In other words, agglomeration with compensation from core to periphery can make both regions better off when trade costs are low enough. Moreover, stronger linkages make compensation easier, for given trade costs.

The intuition behind this result is straightforward. Agglomeration enhances product variety. This has a positive impact on consumer surplus both in the core and the periphery. If linkages are strong, the impact is large, which makes it easier to compensate the periphery. The more so, the lower the transport costs.

It is interesting to contrast this result with its counterpart derived by Charlot et al. (2004) for CP models. These authors show that, also in CP models, compensation from core to periphery is possible (provided that transport costs are low enough). The underlying reason is, nonetheless, very different. Indeed, in CP models product variety is independent from the spatial distribution of economic activities. Hence, agglomeration cannot be a Pareto improvement with respect to dispersion. On the other hand, migration-driven agglomeration implies that the number of residents in the core is larger than the number of residents in the periphery. Hence, compensation is possible for given product variety.

Hicks’s approach

Following Hicks, under dispersion we have to check for the existence of an appropriate redistribution of income from periphery to core that would let the latter region reach the same welfare as under agglomeration. Clearly, for the problem to be nontrivial, (35) must be violated.

It is easy to see that a compensation scheme à la Hicks cannot exist. The payment of any compensation makes the spatial distribution of expenditures uneven (s_E>½). Unlike in the agglomerated outcome, under dispersion B_1 does depend on s_E, so firms’ demands are functions of the spatial distribution of expenditures. This implies that with any transfer supply no longer matches demand at the dispersed market prices, hence even dispersion is no longer an equilibrium. Accordingly, the would-be periphery is unable to compensate the would-be core at the prevailing market prices without destroying even dispersion.

Therefore we have:

> *Proposition 7 (Hicks improvements). Hicks’s criterion always prefers agglomeration to dispersion.*

This is reminiscent of Charlot et al. (2004), who obtain the same result for CP models.

Scitovsky’s indetermination

The foregoing analysis has shown that, if trade costs are small and linkages are strong (i.e. condition (45) holds), then agglomeration is preferred under both Kaldor’s and Hicks’s criteria. Otherwise, if trade costs are large and linkages are weak, the former criterion favours dispersion while the latter still supports agglomeration. This is a case of indetermination in the sense of Scitovsky (1941): neither outcome is preferred to the other with respect to both criteria.

To summarize, we write:

> *Proposition 8 (Potential Pareto improvements). Agglomeration is always socially desirable according to Hicks. It is the socially desirable also according to Kaldor only when transport is cheap and forward and backward linkages are strong. In this case, the market might deliver inefficient dispersion. Otherwise, the market outcome cannot be improved upon.*

This result is reminiscent of Charlot et al. (2004), who find the same sort of indeterminacy in CP models and consider it "as the synthesis of the very contrasted views that prevail in a domain in which the two tenets have many good reasons to be right (p. 4)."

6 CONCLUSION

Through an encompassing setup we have shown the fundamental isomorphism of a broad class of VL models based on Dixit-Stiglitz-Samuelson monopolistic competition. When crossed with Robert-Nicoud's (2005) results on CP models, it implies that all simple Dixit-Stiglitz-Samuelson models share the same qualitative properties. Since also the non-Dixit-Stiglitz-Samuelson framework by Ottaviano, Tabuchi and Thisse (2002) exhibits similar properties, we can therefore be confident that the results derived in the NEG literature have some degree of generality.

Our encompassing setup includes the most widely used VL model by Krugman and Venables (1995) as a special case. It also comprises the simpler VL model by Ottaviano (2002). Since this model turns out to be the only analytically solvable one within the equivalence class, for practical purposes it should be used instead. In this spirit, we have used the simpler model to investigate the much neglected welfare properties of VL models. We have shown that the market outcome is efficient only in extreme situation: when forward and backward linkages are either very strong or very weak, and when trade costs are either very small and very large. For intermediate situation, the market is usually inefficiently biased against agglomeration.

While these insights match those obtained by Charlot et al. (2004) for CP models, there exists a crucial difference. When interregional transfers are not available, in CP models agglomeration is always a win-lose outcome that rewards core regions and penalizes peripheral ones. Differently, agglomeration can be a win-win outcome in VL models even without transfers. The reason is that agglomeration enriches product variety in VL models, while it does not affect it in CP models.

7 APPENDIX

A. Stability of the symmetric equilibrium

To assess the stability of the symmetric long-run equilibrium, we linearise system (20) around that s_n=½ and n^w=$n^{½}$ (see (25)). In matrix notation we get:

(46)
$$\begin{bmatrix} d\ln(q_1) \\ d\ln(q_2) \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} ds_n \\ d\ln(n^w) \end{bmatrix}$$

where the matrix $J \equiv [J_{ij}]$ is the Jacobian. The system is locally stable if (and only if) J is negative definite.

To get an expression for J we proceed in three steps. First, by the symmetry of the system, it must be that an increase in the aggregate mass of varieties n^w has the same impact on the profitability of firms in either region, hence J_{12}=J_{22}. Conversely, the impact of an increase in the share of varieties produced in region *1*, s_n, has opposite and symmetric effects on q_1 and q_2, hence J_{21}=-J_{11}.

Second, when linearised around the symmetric equilibrium the system given by Eqs. (12), (14), (15) and (17) can be rewritten as

(47)
$$\begin{bmatrix} -1 & \alpha-\psi & 2\psi \\ -\frac{1}{2}\beta & 0 & 1 \\ 0 & 1-\alpha\psi & 0 \end{bmatrix} \begin{bmatrix} d\ln(\pi) \\ d\ln(\Delta) \\ ds_E \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \beta & \frac{1}{2}\beta \\ 2\psi & 0 \end{bmatrix} \begin{bmatrix} ds_n \\ d\ln(n^w) \end{bmatrix}$$

where $\psi\equiv(1-\phi)/(1+\phi)$ and the variables without subscripts refer to region *1*'s. For instance, dln(π)=dln(π_1). Due the symmetry of the equilibrium under consideration, dln(π_2)=-dln(π), dln(Δ_2)=-dln(Δ), etc. Hence, the number of unknowns and equations is in effect reduced by half at the symmetric equilibrium.

Last, by the same token, using (12) and (4), the linearisation of (18) gives:

(48)
$$d\ln(q) = d\ln(\pi) + (\theta-\alpha)\, d\ln(\Delta) + \frac{\theta-\alpha}{1-\alpha}\, d\ln(n^w)$$

Using Cramer's rule in (47), we can get an expression for dlnΔ and dlnπ as functions of ds_n and dln(n^w). Since dlnq_1=dln$q$ by definition and dlnq_2=-dlnq_1 at the symmetric equilibrium, (47) and (48) give the following expressions for the Jacobian:

(49) $$J_{11} = -J_{21} = 2\psi \frac{(1-\beta\psi)\left(\frac{\mu}{\sigma-1}+\alpha\right)+\beta(1-\psi)}{(1-\beta\psi)(1-\alpha\psi)}, \quad J_{22} = J_{12} = -\frac{1-\theta}{1-\alpha}$$

As stated above, the symmetric equilibrium is locally stable if (and only if) J is negative definite, which is the case if (and only if) J_{11}, J_{22}<0 and $J_{11}J_{22}$>abs($J_{12}J_{21}$). Given the structure of J, this is the case if J_{11}<0. By inspection, this holds if $0<\psi<[\beta+\theta]/[1+\beta\theta]$. Alternatively, expressed in terms of ϕ, J_{11} is negative if $\phi\in[0,\phi^B]$, where the 'break point' ϕ^B is defined as in (26).

B. Instability of asymmetric equilibria[18]

The model exhibits at most three interior long-run equilibria: the 'symmetric equilibrium' with s_n=½ (which is always stable whenever $\phi<\phi^B$) and two 'asymmetric equilibria', one with s_n smaller and the other with s_n larger than ½, but both at the same distance from ½. Since two stable (unstable) equilibria are always separated by one unstable (stable) equilibrium, if (and only if) $\phi^B>\phi^S$, then the two asymmetric equilibria area unstable. Following Robert-Nicoud (2005), here we prove that is always the case. We proceed in two steps.

Step1: Let $h(\phi)=(\phi^2+\chi)/[\phi^{1-\theta}(1+\chi)]$. Then, by (31), ln$h(\phi)$=0 defines ϕ^S. Standard algebra reveals that ln$h$(.) has a unique zero on [0,1) and that h(.) is increasing (respectively decreasing) if, and only if, $\phi>\phi^S$ ($\phi<\phi^S$). Therefore, to prove that $\phi^B>\phi^S$ it is necessary and sufficient to show that ln$h(\phi^B)$<0.

Define the function G(θ,χ) as:

(50) $$G(\theta,\chi)\equiv \ln h(\phi^B)$$

We can easily infer from (31) that the break and sustain points coincide when θ=0, that is, G(0,χ)=0 for all $\chi\in[0,1]$. So we are left with the task of showing that the presence of vertical linkages makes the break point strictly larger than the sustain point. A sufficient condition for this to be true is that $\partial G/\partial\theta\leq 0$ holds for all χ and θ in [0,1].

Step 2: To this aim, note that G(.,χ) is concave:

[18] 18 Further details on the proof can be found in Robert-Nicoud (2005).

(51) $$\frac{d^2}{d\theta^2}G(\theta,\chi) = -4\frac{\left(\frac{1+\theta}{1-\theta}-\chi\right)\left[\left(\frac{1+\theta}{1-\theta}\right)^2-\chi\right]}{(1+\theta)\left[\left(\frac{1+\theta}{1-\theta}\right)^2+\chi\right]} < 0$$

because $0<\chi,\theta<1$. Hence, a sufficient condition for $G(.,\chi)\leq 0$ to hold is $G(0,\chi)=0$ (which we know to be true) and the first derivative of G with respect to θ to be non-positive when evaluated at $\theta=0$. This is indeed the case, as we now show. Define:

(52) $$g_0(\chi) \equiv \frac{d}{d\theta}G(\theta,\chi)\bigg|_{\theta=0} = \frac{\partial}{\partial\phi}\ln h(\phi;\theta)\bigg|_{\phi=\phi^{break}} \frac{d}{d\theta}\phi^{break}\bigg|_{\theta=0} + \frac{\partial}{\partial\theta}\ln h(\theta;\phi)\bigg|_{\phi=\phi^{break},\theta=0}$$

Standard algebra then reveals that $g_0(\chi)\leq 0$ holds for all χ:

(53) $$g_0(\chi) = 2\frac{1-\chi}{1+\chi}+\ln\chi \leq 0$$

These facts imply that $G(\theta,\chi)$ is non-positive over the whole parameter space. The point of all this is that the upper bound of $G(\theta,\chi)$, and thus the upper bound of $h(\phi^B)$, is zero. We know, therefore, that for all permissible values of χ and θ, $\phi^S \leq \phi^B$, as claimed.

C. Proof of Proposition 2

The proof follows the procedure devised by Robert-Nicoud (2005) for migration-driven NEG models. First, we rewrite the model in a particular state-cum-parameter space. Second, we establish that there is a one-to-one mapping from the original model to the rewritten one. Thus, key properties of the latter readily apply to the former. Third, we stress the fact that in the rewritten model the aggregate mass of varieties has no effect on the relative endogenous variables. Fourth, we claim that such model exhibits at most three interior equilibria. Finally, we argue that in that model the sustain point is smaller than the break point.

Step 1 – New state-cum-parameter space. Fix $n^w>0$ and define $\eta\equiv s_n\pi_1/(1-s_n)\pi_2$ as the ratio of the sum of operating profits arising in region *1* over the sum of operating profits arising in region *2*. As we know, $s_n\pi_1+(1-s_n)\pi_2=bE^w/n^w$, that is the sum of operating profits is equal to a constant. This is equivalent to writing $s_nB_1+(1-s_n)B_2=1$. Hence we can write $s_nB_1=\eta/(1+\eta)$ and $(1-s_n)B_2=1/(1+\eta)$. Moreover, η is strictly increasing in s_n as we shall see, hence we can treat η as the short run exogenous variable instead of s_n.

As it turns out, η has a simple, meaningful economic interpretation as the relative 'mobile' expenditure. Indeed, when some workers enter the manufacturing sector in a region new firms appear that increase the local demand for manufacturing products and thus change the local market size. As we will see, such a mobile expenditure plays a crucial role in rewriting the model in an alternative state space. Accordingly, the variable η can also be understood as the ratio of the 'mobile' factor's expenditure in *1* to 'mobile' factor's expenditure in *2* . Therefore, with this interpretation in mind, η is a measure of the relative market size of region *1*.

By the same token, we can redefine the key endogenous variables as ratios. As long as there is a one-to-one relation between the ratio variable and its parent, the short-run equilibrium of the model can as well be described using these instead of the expressions in section 2. Thus, we define the ratio of expenditures as $\varepsilon \equiv s_E/(1-s_E)$, the ratio of the inverse measure of the price indices as $\delta \equiv \Delta_1/\Delta_2$, and the relative profitability of firms operating in each region as $\omega \equiv q_1/q_2$. Also, define $B \equiv B_1/B_2$.

With all this at hand, we can rewrite the short-run equilibrium conditions –the one for the relative expenditures in (17), those for the price indices in (4) and (12), and those for the relative profitability in (14) and (15)– as follows:

(54) $$\varepsilon = \frac{\eta + \chi}{\eta\chi + 1}, \quad \delta^{\theta}\eta = \omega\frac{\delta - \phi}{1 - \delta\phi}, \quad \omega = \delta^{\theta}\,\frac{\varepsilon + \phi\delta}{\phi\varepsilon + \delta}$$

where χ and $\theta<1$ are defined in the text.

<u>Step 2 – One-to-one mapping</u>. We want to show that there is a one-to-one relationship between the system as defined at the beginning of this section (i.e. written in the structural variables and parameters) and the system in (54), given n^w. This is the case if (and only if) the mapping from s_n to η is a bijection. By construction, there corresponds only one η to each s_n. Thus, we need to show that there corresponds only one s_n to each η. This is easily established by contradiction. Take two tuples $\{s_n, B, \eta, \varepsilon, \delta\}$ and $\{s_n', B', \eta', \varepsilon', \delta'\}$ that satisfy (54). Assume $s_n \neq s_n'$ and $\eta = \eta'$. These imply $B \neq B'$ but $\varepsilon = \varepsilon'$ (by the first expression in (54)) and $\delta = \delta'$ (by the second and third expressions). Since B is equal to $\delta^{uI}(\varepsilon + \phi\delta)/(\varepsilon\phi + \delta)$, $\varepsilon = \varepsilon'$ and $\delta = \delta'$ imply $B = B'$, which contradicts $B \neq B'$. Hence, given n^w, the instantaneous equilibrium of the model is fully described by (54).

<u>Step 3 – Relative and absolute numbers of firms</u>. Note that n^w does not enter (54), not even indirectly. That is, the aggregate mass of varieties n^w has no effect on the relative

endogenous variables. Hence, s_n (or η) and n^w play quite different roles: the former influences the relative profitability of firms established in region *1* vis-à-vis the firms in region *2* while the latter pins down the aggregate profitability of the manufacturing sector. To see this, note that $q_1=q_2$ if (and only if) ω=1, by definition of ω. As can be seen in (54), this is or is not the case irrespective of the (strictly positive) value taken by n^w.

But ω=1 does not suffice to define an interior long run equilibrium. We also need q_r=1 to hold, by the free-entry and exit condition. Given η (and hence s_n) such that $q_1=q_2$, it is clear from (14) and (18) that there always exists a unique n^w that solves the condition q_r=1.[19]

Step 4 – Number of equilibria. We want to show that the system admits at most three interior long run equilibria, that is,

(55) $$\#\left\{(s_n, n^w)\in[0,1]\times\Re_+ : q_1 = q_2 = 1\right\}\leq 3$$

By steps 2 and 3, this is the case if, and only if,

(56) $$\#\left\{\eta\in\Re_+ : \omega = 1\right\}\leq 3$$

To see this, note first that (56) holds in the special case α=0 by Proposition 1. Hence, the system in (54) admits at most three interior steady states in that special case. But the special case was nowhere needed to get the results derived in this subsection, so it must be that (56) holds for *any* admissible α. By implication, we have shown that this crucial property also holds in the CPVL model (for which $\alpha=\mu$).

Step 5 – Break and sustain points. We have already established that $\phi^S<\phi^B$ for α=0 (see Appendix B). It is easy to see that this is true irrespective of the value taken by α.[20]

D. Forces at work

The forces at work in the model can be neatly illustrated using (54) in Appendix C.

Backward linkages. The parameter χ is to be interpreted as follows. When all active firms operate in region *1*, the relative expenditure is maximised there and equal to $(1+\beta)E^w/2$. Conversely, expenditure is minimised in the other region and equal to $(1-\beta)E^w/2$, so $\chi<1$ captures the penalty of being in the periphery that results from lower demand. Indeed, as can be seen from the first expression in (54), ε is increasing in η. As a result, everything else equal, firm profitability is maximised in region *1* because demand for each manufacturing

[19] This is also true in the migration-driven models that Robert-Nicoud (2005) studies in depth, but this condition is more trivially satisfied for the latter than for the VL models.
[20] As Robert-Nicoud (2005) shows, the crucial condition for this to be true is χ, $\theta\in(0,1)$.

variety is largest there. Indeed, the third expression in (54) establishes that ω is increasing in ε.

Forward linkages. As to the parameter θ, the manufacturing price-index (and hence production costs) is lowest where Δ_r is highest. The parameter θ captures the extend to which the manufacturing price index plays an important role in the cost function. If forward linkages are important, that is firms use each other's output as intermediate a lot, then θ is large and the relative profitability (including fixed costs) of the typical firm established in either region depends to a large extend to δ. Keeping the term $(\varepsilon+\phi\delta)/(\varepsilon\phi+\delta)$ in the last expression of (54) constant (which captures the market crowding effect), it is obvious that ω is increasing in δ, the more so if θ is large.

Clearly, δ is increasing in η, that is, the manufacturing price index is lowest where there are more firms. To see this, note first that ϕ and s_n belong to the unit interval, hence (12) implies $\phi<\delta<1/\phi$. Moreover, differentiating the second expression in (54) shows:

(57) $$d\ln\delta\left(\frac{1}{\delta-\phi}+\frac{\phi}{1-\delta\phi}-\theta\right)-d\ln\eta+d\ln\omega=0$$

To illustrate the forward linkage, it proves convenient to impose $d\omega=0$ above.

From (12), it is clear that

(58) $$\Delta_1-\phi\Delta_2=(1-\phi^2)s_n\Delta_1^{\mu I}\geq 0 \quad \text{and} \quad \Delta_2-\phi\Delta_1=(1-\phi^2)(1-s_n)\Delta_2^{\mu I}\geq 0$$

hold, hence, by definition of δ,

(59) $$\delta\in\left[\phi,\frac{1}{\phi}\right], \quad \text{all } s_n\in[0,1]$$

As a result, it is readily established by contradiction that $\max\left\{\frac{1}{\delta-\phi},\frac{\phi}{1-\delta\phi}\right\}>1$. Because $\theta<1$, the term in the bracket in (57) is positive. Hence, $d\ln\delta/d\ln\eta>0$. This, together with $d\omega/d\delta>0$, implies that forward linkages make it more profitable to locate near input suppliers, ceteris paribus.

Market crowding. It turns out that δ is also a measure of the market crowding dispersion force. Indeed, δ is increasing in the share of firms established in region *1*, η (and s_n), at least when ω is kept constant.

Recalling that a firm that establishes itself in a region where there are fewer other firms faces less competitors, most naturally market crowding culminates when all firms are established within the same region. Everything else equal, this implies that ω should be decreasing in δ. This is exactly what the term $(\varepsilon+\phi\delta)/(\varepsilon\phi+\delta)$ in the last expression of (54) captures. This time keeping Δ^{θ} constant because this term captures forward linkages, we find $\frac{d\ln\omega}{d\ln\delta}=-\frac{(1-\phi^2)\varepsilon}{(\varepsilon+\phi\delta)(\varepsilon\phi+\delta)}<0$ as was to be expected.[21]

[21] Note that ∂ω/∂δ may or may not be negative, depending on whether the market crowding force or the forward linkage dominates.

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