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ABSTRACT

Optimal Monetary Policy with Imperfect Common Knowledge*

This Paper studies optimal nominal demand policy in a flexible price economy with monopolistic competition and inattentive firms (Shannon). Inattentiveness gives rise to idiosyncratic information errors and imperfect common knowledge about the shocks hitting the economy. Strategic complementarities in the price-setting game between firms then strongly amplify the effects of information frictions and the real effects of monetary policy. Therefore, strategic complementarities make it optimal to stabilize the output gap by nominally accommodating shocks to firms' desired mark-up. As mark-up shocks become more persistent, however, optimal policy is again increasingly characterized by price level stabilization. Shocks to the natural rate of output are found not to generate a policy trade-off.

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"The peculiar character of the problem of rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess."

Friedrich A. Hayek (1945)

1 Introduction

Decentralized economic activity, as it takes place in modern market economies, tends to generate decentralized knowledge, i.e., information which is not necessarily shared among all agents that might find it potentially relevant.

Most economic models, however, derive policy recommendations under the assumption that private agents share a common information set. In the realm of monetary policy, for example, information asymmetries between private agents have not yet received much attention, and the literature has mainly focused on asymmetries between the private sector and the policymaker, e.g., Svensson and Woodford (2002, 2003).

This paper considers optimal monetary policy when private agents do not share a common information set, seeking to close in part this gap in the monetary policy literature. Presented is a simple dynamic model with imperfectly competitive firms, flexible prices, and a benevolent policymaker controlling nominal demand. The novel feature of the model is that firms possess private information about the shocks hitting the economy and that for such a setting optimal nominal demand policy is determined.

As pointed out earlier by Keynes (1936) and Phelps (1983), disparate information sets coupled with the assumption that agents hold rational expectations generate substantial difficulties: optimal decision making would require that agents formulate so-called higher order beliefs, i.e., beliefs about the beliefs of others and beliefs about what the others believe about others, and so on ad infinitum.[1] This is the case because agents' optimal decisions typically depend on the choices of other agents, thus, on other agents' beliefs.[2]

Despite these difficulties, a number of recent papers successfully pioneered methods for determining rational expectations equilibria in imperfect common

[1] Morris and Shin (2000) have shown that agents do not necessarily have to formulate such higher order beliefs. In binary action games, optimal decisions can be generated by holding simple uniform beliefs about other agents' *actions.*

[2] In the present model such dependencies arise from price competition between firms.

knowledge environments, most notably Townsend (1983b, 1983a), Sargent (1991), Binder and Pesaran (1998), Kasa (2000), Woodford (2002), Pearlman and Sargent (2004), and the recent literature on global games, see Morris and Shin (2000).

While the present setup in many respects is simpler than in these earlier contributions it adds to the literature by solving an optimal policy problem for a private sector rational expectations equilibrium with imperfect common knowledge.

Imperfect common knowledge arises in the model because firms' capacity to process information is assumed to be limited. Such limitations may arise, e.g., because of the presence of a finite number of managerial staff collecting and processing information and taking the corresponding decisions, see Radner (1992). As a result of these processing limitations, firms make idiosyncratic errors since processing all information perfectly would require unlimited processing capacity.

Firms' processing limitations are modeled by assuming that firms receive information through information channels with finite channel capacity, following Shannon (1948) and Sims (2003). There are several advantages in modelling information frictions with the help of information channels. First, the information structure turns out endogenous since firms *choose* which variables to observe through their channels. The information structure, therefore, reacts to the policy pursued by the monetary policymaker. Second, information channels preserve the linear quadratic nature of the policy problem considered in the paper thereby allowing for closed form solutions, even in a dynamic setup.[3]

The main result of this paper is that the presence of differential information has stark implications for the effectiveness of monetary policy to influence real variables. In particular, strategic complementarities in the price setting game between firms strongly amplify the effects of information frictions. Nominal demand policy then has considerable real effects, even if information frictions are relatively weak.

Private information renders coordination between firms difficult because firms are uncertain about the decisions of other firms that base their price decisions on (slightly) different information sets. In the presence of strategic complementarities this causes firms' prices to underreact to private information.

[3] This would not be the case if the variance of the observation error was taken as the primitive parameter characterizing information frictions, as is common in the literature on global games.

Strategic complementarities and differential information, thus, imply that monetary policy has the ability to stabilize the output gap. Consequently, optimal policy tends to nominally accommodate shocks to firms' desired mark-up, even when information frictions are relatively weak. When mark-up shocks display persistence, however, the incentives to accommodate are weakened and it becomes optimal to place more emphasis on price stabilization. The optimal policy response to mark-up shocks thus depends on the relative importance of shock persistence and strategic complementarities.

This paper also analyzes optimal policy in response to shocks to the natural rate of output. It turns out optimal to fully nominally accommodate these shocks. Firms then *choose* not to process any information about natural rate shocks, implying that their prices do not respond. Nominal demand policy then induces real output movements that cause output to follow its natural rate. Mark-up shocks, therefore, do not generate a trade-off between price and output gap stabilization.

In related papers Morris and Shin (2003) and Amato and Shin (2003, 2004) also derive normative implications for imperfect common knowledge settings but focus on the welfare effects of disclosing public information. Ui (2003) shows the non-neutrality of money in a Lucas island economy with private information. Bacchetta and van Wincoop (2003) study the impact of public and private information in an asset pricing context. Ball et al. (2004) analyze optimal monetary policy with disparate information by assuming that some agents set prices based on lagged information. The assumed information lags, however, do not generate imperfect common knowledge. In related work Hellwig (2002) derives impulse responses to a large range of shocks for an economy with monopolistic competition and imperfect common knowledge.

This paper is organized as follows. Section 2 outlines a simple static economy with monopolistically competitive firms. As a benchmark, section 3 derives optimal policy when there is common knowledge among firms. Imperfect common knowledge and information channels are introduced in section 4, which summarizes relevant results from information theory. Section 5 determines the rational expectations equilibrium with imperfect common knowledge and section 6 characterizes optimal monetary policy. Results are extended to a dynamic setup in section 7. A conclusion summarizes the main findings while technical details are in an appendix.

2 The model economy

This section introduces a representative agent economy with flexible prices, a continuum of monopolistically competitive firms, and a central bank controlling nominal demand. The economy is subject to labor supply shocks, inducing variations in the efficient level of output, and shocks to the price elasticity of demand, generating variations in firms' desired mark-up.

2.1 Households

The household sector is described by a representative consumer choosing aggregate consumption Y and labor supply L to maximize

$$\begin{aligned} & U(Y) - \nu V(L) \\ & s.t. \\ 0 = & WL + \Pi - T - PY \end{aligned} \tag{1}$$

where W denotes a competitive wage rate, Π monopoly profits from firms, T lump sum transfers, and P the price index of the aggregate consumption good. The parameter $\nu > 0$ is a stochastic labor supply shifter with $E[\nu] = 1$, which induces variations in the efficient level of output. The consumer observes all prices in the economy. Furthermore, $U' > 0$, $U'' < 0$, $\lim_{Y\to\infty} U'(Y) = 0$, $V' > 0$, $V'' > 0$ and $V'(0) < U'(0)$.

2.2 Firms

The production sector consists of a continuum of monopolistically competitive firms $i \in [0, 1]$. Firm i produces an intermediate good Y^i with labor input L^i according to a linear production function of the form

$$Y^i = L^i,$$

and aggregate labor demand is $L = \int L^j \mathrm{dj}$. Intermediate goods enters into the aggregate output good Y according to a Dixit-Stiglitz aggregator

$$Y = \left(\int_{[0,1]} (Y^j)^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}} \tag{2}$$

where the demand elasticity $\theta > 1$ is stochastic with mean $E[\theta] = \overline{\theta}$.

Linearizing the first order condition of firm i delivers a standard pricing equation of the form

$$p(i) = E\left[p + \xi(y - y_n) + \varepsilon | I^i\right], \tag{3}$$

where $p(i)$ denotes the profit maximizing price and I^i the information set available to firm i, see appendix A.1.[4] The profit maximizing price depends on the expected values of the average price level $p = \int p_t(j)dj$, the output gap $y - y_n$, where y denotes the average output across firms and y_n the natural output level, and the mark-up shock ε. Fluctuations in the natural rate y_n are induced by the labor supply shock ν, while the mark-up fluctuation ε is due to the elasticity shock θ. I assume $y_n \sim N(0, \sigma_y^2)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

The parameter $\xi > 0$ in equation (3) determines whether firms' prices are strategic complements or substitutes. This can be seen by defining nominal spending q as

$$q = y + p \tag{4}$$

and using it to substitute y in equation (3):

$$p(i) = E\left[(1-\xi)p + \xi q - \xi y_n + \varepsilon | I^i\right] \tag{5}$$

For $\xi \leq 1$ prices are strategic complements, since the optimal price is (weakly) increasing in the average price level for a given level of nominal demand q. For $\xi > 1$ prices are strategic substitutes because the optimal price decreases in the average price level.[5] Throughout the paper, it is assumed that prices are strategic complements, i.e., $\xi \leq 1$, which appears to be the case of greatest economic interest. Analytical results, however, hold as long as $\xi < 2$.

2.3 Monetary Policy

The monetary policymaker has preferences over the output gap and the price level and adjusts nominal demand to maximize its objective. More specifically, the monetary policy problem is

$$\max_q -E\left[(y - y_n)^2 + \lambda p^2\right] \tag{6}$$

$$s.t.$$

$$p(i) = E\left[(1-\xi)p + \xi q - \xi y_n + \varepsilon | I^i\right]$$

$$p = \int p(j)\mathrm{dj}$$

$$y = q - p$$

where the price level target is normalized to zero and $\lambda > 0$ is a parameter indicating the relative weight given to price stability.

[4] Lower case letters indicate variables that are expressed as percentage deviations from deterministic steady state.

[5] Note that an increase in the price level reduces real demand. For $\xi > 1$ the demand shortfall reduces production costs by so much that a single firm finds it optimal to reduce prices.

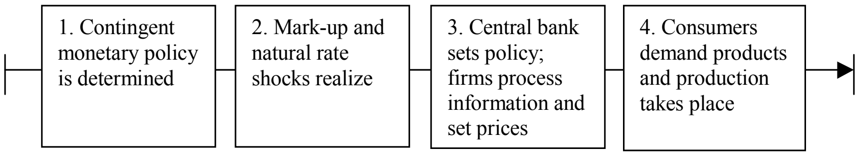

Figure 1: Sequence of events

Appendix A.2 shows that the underlying economy allows to interpret the policy objective (6) as a second order approximation to the utility function of the representative agent for an appropriate choice of λ.[6] Alternatively, objective (6) may be interpreted as describing the policy objectives pursued by real-world central banks. Explicit reference to measures of price stability and real activity is made, for example, in the policy objectives stated by the European Central Bank and the Bank of England.[7]

In the present model a trade-off between price stability and output gap stability can arise. This trade-off is generated by the pricing behavior of firms, as summarized by the constraints of problem (6). The goal of this paper is to show that the nature of this trade-off depends critically on the information sets I^i attributed to firms.

To determine first best policies I consider a central bank acting under commitment and choosing policy at time zero, i.e., before shocks realize. For completeness, the paper also briefly addresses the implications of choosing policies after the realization of shocks.[8]

Figure 1 illustrates the sequence of events. After monetary policy has been determined, the stochastic disturbances realize. The central bank then implements its policy, firms process information about the shocks and the policy choice, as will be explained in section 4, and simultaneously determine prices. Finally, consumers demand products for consumption and production takes place.

[6] Intuitively, when firms cannot process information perfectly, price level variability generates idiosyncratic information errors and (inefficient) price dispersion between firms.

[7] This is not to say that objective (6) precisely captures the declared objectives of these institutions. The exact policy objectives can be found at http://www.ecb.int/mopo/intro/html/objective.en.html and http://www.bankofengland.co.uk/framework.htm

[8] This turns out to be less interesting because it does not generate a policy trade-off.

3 Optimal policy in two benchmark settings

This section considers two common knowledge settings with rather extreme informational assumptions. In the first setting firms observe shocks perfectly; in the second firms do not observe shocks at all. The optimal policies for these settings will serve as useful benchmarks when analyzing policy in environments where firms have imperfect common knowledge about these shocks.

3.1 Benchmark I: Perfectly observable shocks

Suppose each firm and also the policymaker perfectly observe the shocks y_n and ε and suppose this is common knowledge. Based on equation (5) firm i's optimal price can be expressed as

$$p(i) = E\left[(1-\xi)p|I\right] + \xi q - \xi y_n + \varepsilon, \tag{7}$$

which uses the fact that all firms share the same information set and assumes that q is a function of the shocks y_n and ε, thus, perfectly observed by agents.

Integrating equation (7) over $i \in [0,1]$ and taking conditional expectations with respect to I delivers

$$E\left[p|I\right] = q - y_n + \frac{1}{\xi}\varepsilon, \tag{8}$$

which shows that agents can determine average expectations as a function of the observable shocks y_n and ε. Substituting equation (8) into (7), integrating over i, and using one more time the definition of q, allows to determine how prices and the output gap depend on shocks and policy decisions:

$$p = q - y_n + \frac{1}{\xi}\varepsilon \tag{9}$$

$$y - y_n = -\frac{1}{\xi}\varepsilon. \tag{10}$$

Not surprisingly, nominal demand policy affects the price level only but has no effect on the output gap. In the absence of nominal rigidities and information asymmetries monetary neutrality holds, see Lucas (1972). Optimal policy then stabilizes the price level. Equation (9) implies that this is achieved by setting

$$q = y_n - \frac{1}{\xi}\varepsilon, \tag{11}$$

i.e., by accommodating natural rate shocks and by appropriately contracting in response to mark-up shocks.

3.2 Benchmark II: Unobservable shocks

Consider the case where firms have no information about shocks and policy reactions to shocks. The policymaker continues to observe shocks perfectly.

Firms' expectations about the shocks equal the unconditional mean values of shocks. Equation (5) and the definition of q then imply

$$p = 0 \tag{12}$$

$$y = q. \tag{13}$$

Nominal demand policy now has real effects only and no effects on the price level, which is the reverse situation when compared to the case with fully observable shocks. Clearly, since firms fail to observe policy decisions, nominal demand variations come as a 'surprise'.

With nominal demand policy having real effects only, optimal policy stabilizes the output gap. This is achieved by nominally accommodating natural rate shocks, as in the case of observable shocks, i.e.,

$$q^* = y_n.$$

Unlike in the case with observable shocks, however, policy does not have to react to mark-up shocks. Since firms fail to observe these shocks, prices and the output gap both fail to react to them.

4 Imperfect common knowledge and information channels

From here on I consider the more realistic case of economic agents that do not share a common information set.

I assume that each firm receives information through a so-called information channel that is contaminated with idiosyncratic noise.[9] The presence of idiosyncratic noise will generate private information about the shocks hitting the economy.

Information channels allow for the transmission of information from the source to the receiver in a similar way as telephone or modem lines do. However, due to the presence of noise, the information arriving to the receiver (the channel output) does not perfectly reveal the information at the source (the channel

[9] Information channels have been introduced by Shannon (1948). See Cover and Thomas (1991) for a textbook treatment and Sims (2003) and Moscarini (2004) for applications in macroeconomics.

input). Noise may arise, e.g., from limited attention on the part of the receiver due to a finite number of staff processing the information, or from interpretation errors due to background noise in the channel. I will refer to such information noise simply as processing errors and interpret them as errors due to limitations in firms' processing of information.

Firms are assumed to operate separate and independent information channels. Thus, each firm has its own 'window on the world' and aggregates and processes information individually. As a result, firms make idiosyncratic processing errors and have private information about the state of the economy. This captures Hayek's view that information exists only in the form of dispersed bits of incomplete and frequently contradictory knowledge. The case where all firms use the same information channel is discussed in Adam (2004).

Information channels are characterized by their capacity $K \geq 0$. The capacity places an upper bound on the amount of information that can be transmitted via the channel, as will be made precise below. Channel capacity is a simple technology parameter, like the TFP-parameter in a production function, that depends on technical features of the channel such as the number of signals the channel can transmit per period of time, the number of letters in the channel's alphabet, the probability with which the respective letters are transmitted correctly, etc..

An attractive feature of information channels is that they limit only the overall amount of information flowing to agents while these decide which random variables to observe with what precision. Agents, thus, *choose the information structure* subject to the constraint imposed by the capacity limit. This causes the information structure to be endogenous since agents' information choices depend on the stabilization policy pursued by the central bank and the parameters characterizing the economy. Moreover, since the capacity of firms' channels is limited, the information noise in the channel will depend on the variability of the objects agents seek to observe. In particular, more volatile variables are informationally more demanding and, for given capacity, generate larger processing errors.

Readers familiar with Kalman filtering may think of this situation as one where agents *choose* their observation equation and where the variance of the observation noise is determined by the capacity of the available information channel.

In the next section I provide a brief introduction to the real-valued Gaussian information channel. Such a channel will be used in the latter part of the paper.[10]

4.1 The real-valued Gaussian channel

Suppose a firm must choose a price $p \in R$ to maximize a quadratic profit function of the form

$$\max_p -E\left[(p - \zeta' Z)^2 |I\right], \tag{14}$$

where $Z \sim N(0, V)$ is a vector of shocks driving the economy and the profit maximizing price $\zeta' Z$ depends on these shocks. The vector ζ' may thereby be a function of the parameters of the underlying economic model, the policy pursued by the central bank, and other factors that the agent takes as given.

The profit function (14) can be interpreted as a quadratic approximation to the firm's profit function. Such a quadratic approximation is convenient because it leads to linear decision rules that mimic the linearized first order conditions in equation (3).

Suppose the information set I is exogenous. The solution to the above problem is then trivially given by

$$p^* = E[\zeta' Z|I],$$

and the expected loss equals

$$-Var(\zeta' Z|I). \tag{15}$$

Now suppose instead that the firm can choose the information structure I but must receive information about $\zeta' Z$ through an information channel with capacity $K \in [0, \infty)$.

The channel coding theorems (e.g. theorem 8.7.1 in Cover and Thomas (1991)) state that channel capacity K places a limit on the amount of entropy reduction that can be achieved by the channel.[11] Formally,

$$H(\zeta' Z) - H(\zeta' Z|s) < K, \tag{16}$$

where $H(\zeta' Z)$ denotes the entropy of the random variable $\zeta' Z$ prior to observing the channel output signal s and $H(\zeta' Z|s)$ the entropy after observing the signal.

Intuitively, entropy is a measure of the uncertainty about a random variable. Stated in these terms, equation (16) provides a bound for the maximum

[10] Readers interested in a more detailed treatment may consult the textbook of Cover and Thomas (1991) or the, very accessible, original contribution of Shannon (1948).

[11] The entropy $H(X)$ of a continous random variable X is defined as $H(X) = -\int \ln(x)p(x)dx$ where $p(x)$ is the probability density function of X and where the convention is to take $ln(x)p(x) = 0$ when $p(x) = 0$.

uncertainty reduction that can be achieved by observing the channel output s. Since the entropy $H(\zeta' Z)$ is determined by the distribution of $\zeta' Z \sim N(0, \zeta' V \zeta)$, thus, taken as given, equation (16) simply implies that $H(\zeta' Z|s)$ must lie above a certain threshold.

A relevant question is then: What is the optimal information structure that fulfills this entropy constraint? Equation (15) shows that the expected loss associated with any information structure is equal to $Var(\xi' Z|s)$. Thus, choosing the optimal information structure is identical to minimizing this conditional variance subject to the constraint that $H(\zeta' Z|s)$ is above the threshold.

Shannon (1948) shows that Gaussian variables minimize the variance for a given entropy.[12] Thus, if possible, the observation noise should be Gaussian such that the posterior distribution $\xi' Z|s$ is Gaussian and has the minimum variance property.

I will assume that the coding allows for such kind of Gaussian noise, i.e., there exists a way to map the realizations of $\zeta' Z$ into a sequence of input signals from the channel's alphabet such that the observation noise generated by the incorrect transmission of signals is Gaussian and independent across the realization of the input signal.

When this is the case, optimal use of the channel implies that the channel output signal s has a simple representation of the form

$$s = \zeta' Z + \eta, \tag{17}$$

where $\eta \sim N(0, \sigma_\eta^2)$ is the Gaussian observation noise and σ_η^2 is the infimum variance satisfying the channel capacity constraint

$$\ln Var(\zeta' Z) - \ln Var(\zeta' Z|s) < 2K. \tag{18}$$

Constraint (18) follows from equation (16) and the fact that the entropy of a Gaussian random variable is equal to one half its log variance plus a constant.

From the updating formula for normal random variables[13] and the capacity constraint (18) it follows that the observation noise has (infimum) variance

$$\sigma_\eta^2 = \frac{1}{e^{2K} - 1} Var(\zeta' Z). \tag{19}$$

Firms' expectations after observing the signal are then given by

$$E[\zeta' Z|s] = k \cdot s, \tag{20}$$

[12] Shannon solves the dual problem of maximizing entropy for a given variance.

[13] $Var(\zeta' Z|s) = Var(\zeta' Z) - Var(\zeta' Z)^2 / (Var(\zeta' Z) + \sigma_\eta^2)$

where the Kalman gain k is

$$k = \frac{Var(\zeta' Z)}{Var(\zeta' Z) + \sigma_\eta^2} = (1 - e^{-2K}). \tag{21}$$

The Kalman gain k is a useful summary statistic indicating agents' ability to process information. For $k = 0$ firms receive no information since $\sigma_\eta^2 = \infty$. Conversely, with $k = 1$ firms observe perfectly since $\sigma_\eta^2 = 0$. These two cases, thus, generate the benchmark settings considered in section 3. At intermediate values of k the variance of the observation noise is positive and decreases in k.

One should note that the information channel endogenizes the information structure along two relevant dimensions. First, it endogenizes the size of firms' processing errors. As revealed by equation (19) these errors are proportional to the variability of the variables agents seek to observe. To the extent that policy affects the variability of these variables it also affects the size of agents' processing errors. Second, the channel endogenizes the amount of 'attention' agents allocate to different variables, as expressed by the relative weights in the vector ζ. Policy affects these weights and thereby influences the variables agents seek to observe.

5 Private sector equilibrium

In this section I endow each firm with an information channel of given capacity and solve for a rational expectations equilibrium (REE) with imperfect common knowledge (ICK) in which firms choose profit maximizing prices *and* optimal information structures.

Solving for the rational expectations equilibrium is not trivial. Since there is 'information dispersion' in the economy, firms do not know what other firms have observed and must formulate beliefs about other agents' beliefs. This leads to a system of so-called higher order beliefs that affects the equilibrium outcome.

Section 5.1 determines how profit-maximizing prices depend on firms' higher-order beliefs, and section 5.2 derives the rational expectations equilibrium.

5.1 Price setting with ICK

I first have to introduce some notation to be able to refer to firms' expectations of various order.

Let $x^{(m)}(i)$ denote firm i's m-th order expectation of x and let

$$x^{(m)} = \int x^{(m)}(j)dj$$

denote the average expectations of order m. Firms' expectations of order zero are given by the variable itself, i.e.

$$x^{(0)}(i) = x.$$

Firms' expectations of order $m+1$ are their expectations of the average m-th order expectation, i.e.,

$$x^{(m+1)}(i) = E[x^{(m)}|I^i].$$

Therefore, $x^{(1)}(i)$ denotes the familiar (first order) expectation $E[x_t|I^i]$; the second order expectations $x^{(2)}(i)$ denote i's expectations of the average (first order) expectations; the third order expectations $x^{(3)}(i)$ denote i's expectations of the average second order expectations, etc.

With this notation the price setting equation (5) can be expressed as

$$p(i) = (1-\xi)p^{(1)}(i) + \xi q^{(1)}(i) - \xi y_n^{(1)}(i) + \varepsilon^{(1)}(i). \tag{22}$$

Iterating on equation (22) by taking repeatedly the average and conditional expectations, one obtains

$$p(i) = E\left[\sum_{m=0}^{\infty}(1-\xi)^m \left(\xi q^{(m)} - \xi y_n^{(m)} + \varepsilon^{(m)}\right)|I^i\right]. \tag{23}$$

Firms' optimal price depends on the first and higher order expectations of q, y_n, and ε. For $\xi = 1$, i.e., without strategic interactions among firms, only first order expectations matter. For smaller values of ξ, i.e., in the presence of strategic complementarities, higher order expectations increasingly influence the optimal price. Intuitively, this is due to the fact that strategic elements between firms cause them to put more weight on the beliefs about other firms' beliefs.

5.2 REE and optimal information structure

This section determines agents' optimal information structure and characterizes the rational expectations equilibrium with optimal information gathering.

Equation (23) and the discussion in section 4 imply that agents wish to observe the term

$$\sum_{m=0}^{\infty}(1-\xi)^m \left(\xi q^{(m)} - \xi y_n^{(m)} + \varepsilon^{(m)}\right) \tag{24}$$

as precisely as possible through their information channels.

Equation (24) suggests that firms' seek to observe a combination of the fundamental shocks and agents' higher-order expectations about these shocks.

The latter implies that to construct a rational expectations equilibrium one has to determine a fixed point in the space of beliefs of infinite order. A much simpler way to proceed, however, is to let agents observe only the fundamentals

$$\xi q - \xi y_n + \varepsilon. \tag{25}$$

They can then construct the higher order beliefs in (24) using their (noisy) observation of these fundamentals. As shown in appendix A.3, this leads to the same equilibrium outcome, but equilibrium is much simpler to derive and easier to interpret.

Firms' observation equation is, thus, given by

$$s^i = (\xi q - \xi y_n + \varepsilon) + \eta^i, \tag{26}$$

where η^i is an idiosyncratic observation error that I take to be normally distributed for the reasons discussed in section 4. From equation (20) it then follows that

$$E[\xi q - \xi y_n + \varepsilon | I^i] = ks^i, \tag{27}$$

where $k = 1 - e^{2K}$, see equation (21).

Integrating equation (27) over i, using equation (26) to substitute s^i, and taking the expectations $E[\cdot|I^i]$ delivers

$$E[\xi q^{(1)} - \xi y_n^{(1)} + \varepsilon^{(1)} | I^i] = k^2 s^i.$$

Applying the same operations m times one obtains

$$E[\xi q^{(m)} - \xi y_n^{(m)} + \varepsilon^{(m)} | I^i] = k^{m+1} s^i. \tag{28}$$

Expectations of higher order, thus, react less strongly to the signal s^i than expectations of lower order. This is rational because firms are increasingly uncertain about the expectations of higher order, which require to repeatedly average (integrate) over information sets that differ from their own.

Substituting expression (28) into equation (23) delivers

$$p(i) = \frac{k}{1-(1-\xi)k} \left(\xi q - \xi y_n + \varepsilon + \eta^i\right). \tag{29}$$

Averaging over firms yields an expression for the equilibrium price level

$$p = \frac{k}{1-(1-\xi)k} \left(\xi q - \xi y_n + \varepsilon\right). \tag{30}$$

The equilibrium output gap follows from equation (4):

$$y - y_n = \frac{1-k}{1-(1-\xi)k} q - \frac{1-k}{1-(1-\xi)k} y_n - \frac{k}{1-(1-\xi)k} \varepsilon. \tag{31}$$

As one would expect, nominal demand policy has real effects as long as firms process information only with limited capacity, i.e., $\partial y/\partial q > 0$ for $k < 1$. These real effects, however, are considerably amplified in the presence of strategic complementarities, i.e., when $\xi < 1$. In the limiting case $\xi \to 0$, for example, nominal demand policy has real effects only but no effect on prices.[14]

Strategic complementarities imply that expectations of higher order are given more weight in the price setting decisions, see equation (23). As shown in section 5.1, however, expectations of higher order react more sluggishly to information than expectations of lower order. Prices, therefore, react more sluggishly to nominal demand variations and monetary policy increasingly affects output. Note that for the limiting cases $k = 1$ and $k = 0$, respectively, equations (30) and (31) reduce to the benchmark expressions for the common knowledge case derived in section 3.

6 Optimal stabilization policy

This section determines optimal monetary policy when firms possess imperfect common knowledge about shocks.

Using the results from section 5, the policy problem (6) can be expressed as

$$\max_q -E\left[(y - y_n)^2 + \lambda p^2\right] \tag{32a}$$

$$s.t.$$

$$p = \frac{k}{1-(1-\xi)k}\left(\xi q - \xi y_n + \varepsilon\right) \tag{32b}$$

$$y - y_n = \frac{1-k}{1-(1-\xi)k}q - \frac{1-k}{1-(1-\xi)k}y_n - \frac{k}{1-(1-\xi)k}\varepsilon \tag{32c}$$

which is a simple linear-quadratic maximization problem.[15] Since certainty equivalence applies, the results derived below do not depend on the assumption that the central bank observes shocks perfectly.

The solution to (32) is readily calculated to be

$$q = y_n + a \cdot \varepsilon \tag{33}$$

[14] This holds for any value $k < 1$.

[15] This is the case because the Kalman gain k in equations (32b) and (32c) is independent of policy. If the variance of observation noise was specified exogenously, this property would be lost and closed form solutions would be unavailable, even for the relative simple policy problem at hand.

where

$$a = \frac{(1-k)k - \lambda\xi k^2}{(1-k)^2 + \lambda\xi^2 k^2}. \tag{34}$$

The economic forces shaping the optimal monetary policy function (33) are discussed below.

6.1 Policy reaction to natural rate shocks

Optimal policy is to nominally accommodate natural rate shocks y_n. This holds independently of the degree of strategic complementarity, the extent to which firms can process information, and the relative weight given to price stabilization in the policy objective. The response thus remains unchanged if compared to the common knowledge benchmarks analyzed in section 3.

The optimal response to natural rate shocks implies that firms *choose* not to observe any information about these shocks, see equation (26). Consequently, firms' prices fail to react and the nominal demand adjustment comes as a (deliberate) surprise, affecting real output only. Output then follows its natural rate and prices remain stable at the target level. Therefore, natural rate shocks do not induce a trade-off between output gap and price level stabilization.

6.2 Policy reaction to mark-up shocks

The situation differs notably when considering mark-up shocks. The optimal reaction coefficient a then depends on the degree of strategic complementarity, the extent to which firms can process information, and the relative weight given to the price level objective.

Mark-up shocks also generate a trade-off between output and price stabilization. Consider the case of a central bank pursuing output gap stabilization only ($\lambda = 0$). The optimal reaction coefficient (34) is then given by

$$a_y = \frac{k}{1-k} > 0. \tag{35}$$

Shocks are nominally accommodated and accommodation increases in the processing index k. Higher values of k imply that firms receive more information, which reduces the real effects of monetary policy and requires stronger policy reactions.[16] Note that the reaction coefficient is independent of the degree of strategic complementarity. While lower values of ξ increase the real effects of monetary policy, they also increase the real effects of mark-up shocks, see equation (32c).

[16] Policy successfully stabilizes output at the target as long as $k < 1$.

Next, consider the case of pure price level stabilization ($\lambda \to \infty$). The optimal reaction coefficient (34) is then given by

$$a_p = -\frac{1}{\xi} < 0. \tag{36}$$

Nominal contraction in response to mark-up shocks is required in this case, indicating that there exists a trade-off between price level and output gap stabilization. Moreover, stronger complementarities (lower values of ξ) require a stronger policy reaction: strategic complementarities increase the importance of higher-order beliefs, causing firms' prices to respond more sluggishly to policy.

For $0 < \lambda < \infty$ the optimal reaction coefficient (34) is a convex combination of the optimal reaction coefficients for the single objectives and can be written as

$$a = \omega a_y + (1 - \omega) a_p$$

where the weight on the output coefficient is

$$\omega = \frac{(1-k)^2}{1 - 2k + k^2 + \xi^2 k^2}$$

As is easily seen, ω increases as strategic complementarities become stronger and in the limit $\omega \to 1$ as $\xi \to 0$.[17] Strategic complementarities thus shift the trade-off between output and prices in favor of output gap stabilization. Strategic complementarities imply that price stabilization is rather costly in terms of its output implications: with prices responding sluggishly, policies aiming at price stabilization have to be rather aggressive and would generate large output gaps.

This fact is illustrated in figure 2. Depicted is the optimal reaction coefficient (34) as a function of the processing index k for various degrees of strategic complementarity.[18] As ξ decreases, i.e., as strategic complementarities become stronger, the policy reaction at intermediate degrees of information imperfections is increasingly characterized by nominal accommodation, as suggested by equation (35). At $\xi = 0.15$, for example, which Woodford (2001) suggests to be a plausible parameter value for the U.S. economy, policy is accommodative in response to mark-up shocks as long as $k < 0.869$, i.e., over a large range of information frictions.

6.3 Discretionary policy

For completeness I now briefly consider a discretionary policymaker determining policy after or simultaneously with firms, i.e., after the shocks have realized. Such a policymaker takes firms' pricing decisions and, thus, the aggregate price

[17] This holds for all $k < 1$.

[18] The figure assumes $\lambda = 1$, i.e., equal weights for the output gap and the price level target.

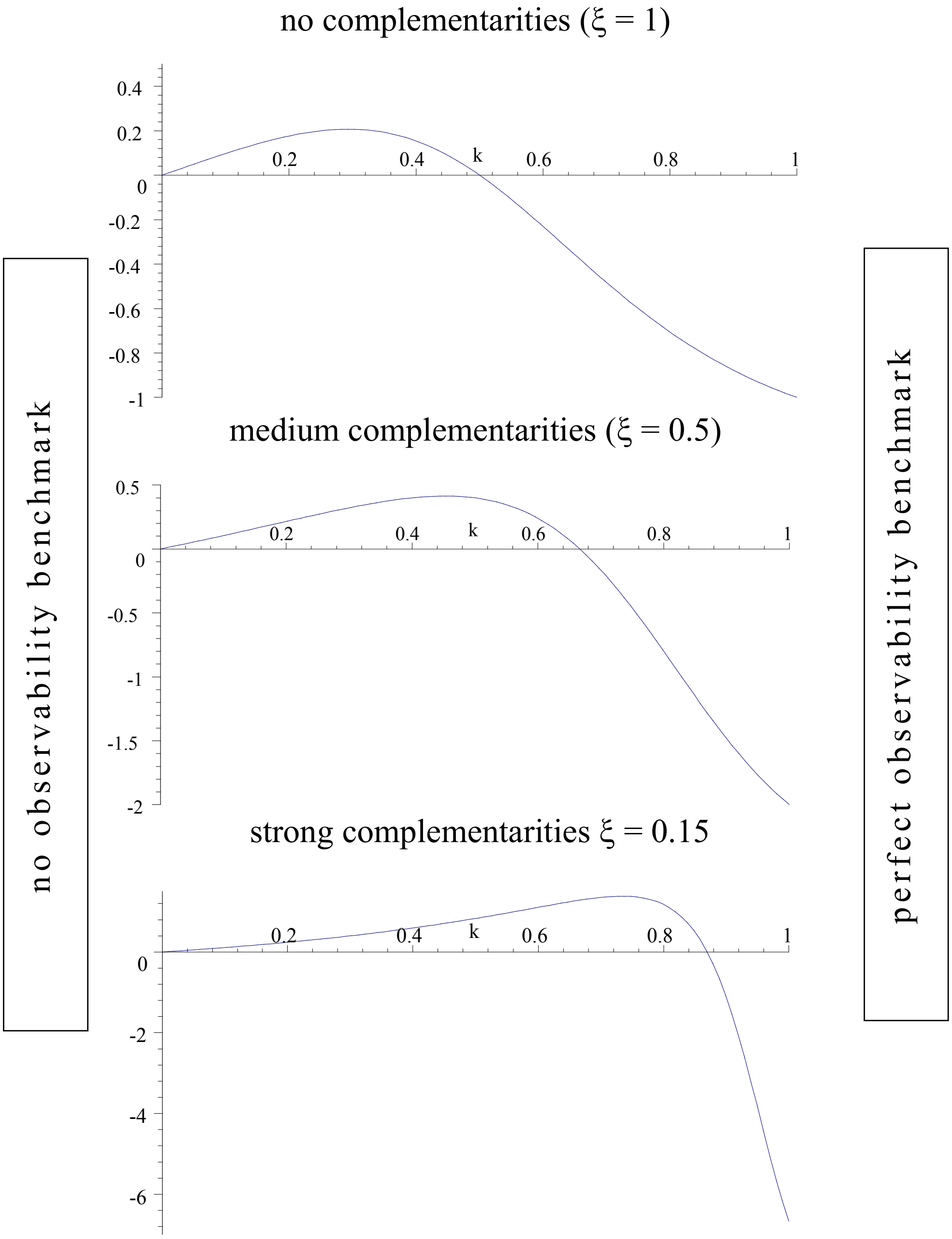

Figure 2: Optimal reaction coefficient (mark-up shocks)

level as given and the output gap remains the only policy objective. Optimal discretionary policy is thus given by

$$q = y_n + a_y \cdot \varepsilon$$

where a_y is defined in equation (35). Clearly, discretionary policy generates (inefficiently) large price level volatility.

7 Optimal policy in a dynamic economy

This section extends the static setting considered so far to an infinite horizon economy.

A dynamic setting is of interest because it allows to analyze persistent shock disturbances, which generate additional policy incentives. In particular, monetary policy decisions then not only affect current economic outcomes but also the prior beliefs with which firms enter into the next period. This intertemporal aspect of policy is absent in a static economy or when shocks are white noise.

Time is discrete and indexed by $t = 0, 1, 2, \ldots$The policymaker commits to a policy rule at the beginning of period zero and in each period steps 2 to 4 depicted in figure 1 take place. To simplify on notation I abstract from natural rate shocks and consider mark-up shocks only.[19] The mark-up shocks now evolve according to

$$\varepsilon_t = \rho\varepsilon_{t-1} + v_t \tag{37}$$

where $v_t \sim iiN(0, \sigma_v^2)$ and $\rho \in (-1, 1)$. For $\rho = 0$ the economy reduces to a sequence of independent static economies.

The policymaker is assumed to maximize

$$-E\left[y^2 + \lambda p^2\right] \tag{38}$$

where $E[\cdot]$ denotes the unconditional expectations operator.[20] The natural rate of output has been normalized to zero.

Consider a policy rule of the form

$$q_t = a \cdot \varepsilon_t \tag{39}$$

[19] It is not difficult to show that natural rate shocks still do not generate a policy trade-off and should be nominally accommodated in the same way as in the static setup.

[20] Maximizing (38) is identical to maximizing $-E_0[\sum_{t=0}^{\infty} \beta^t(y_t^2 + \lambda p_t^2)]$ for the limiting case $\beta \to 1$.

where the reaction coefficient a remains to be determined. Following Woodford (2002), I conjecture an equilibrium law of motion of the form

$$X_t = MX_{t-1} + mv_t, \tag{40}$$

where

$$M = \begin{pmatrix} \rho & 0 \\ M_{21} & M_{22} \end{pmatrix}, \quad m = \begin{pmatrix} 1 \\ m_2 \end{pmatrix}$$
$$X_t = \begin{pmatrix} \varepsilon_t \\ f_t \end{pmatrix} \quad with \quad f_t = \sum_{m=0}^{\infty} (1-\xi)^m \varepsilon_{t|t}^{(m)},$$

and where M_{21}, M_{22}, and m_2 are unknown parameters. Using (23), (39), and (4), the equilibrium price level and output level can be expressed as linear functions of X_t:

$$p_t = (\xi a + 1) f_t \tag{41}$$
$$y_t = a\varepsilon_t - (\xi a + 1) f_t \tag{42}$$

Thus, once the equilibrium law (40) has been determined, the objective function (38) is readily evaluated.

Appendix A.4 derives analytical expressions for the equilibrium values of M and m in equation (40). The equilibrium values must satisfy the following fixed point property: given the law of motion (40), optimal belief updating by firms must exactly generate (40) again.

The appendix shows that the equilibrium law (40) depends on the parameters ξ, k, and ρ, but that it is independent of the policy parameter a.[21] This is the case because the optimal Kalman gains in agent's filtering equations happen to be independent of the policy function, as in the simple static case. The optimal policy problem, therefore, retains its simple linear quadratic structure and results do not depend on the variance of the innovations σ_v^2.

The upper panel of figure 3 illustrates the effects of mark-up shock persistence on the optimal reaction coefficient, assuming $\lambda = \xi = 1$. It shows that shock persistence causes the optimal reaction coefficient to be on average closer to its perfect observability benchmark $a = -1/\xi$. Persistence thus induces a policy shift towards more aggressive price stabilization.

This shift is optimal because persistent mark-up shocks imply that f_t follows (on average) more closely the pattern of ε_t. In particular, simulations

[21] This does not imply that firms' beliefs about p_t are independent of policy, see equation 41.

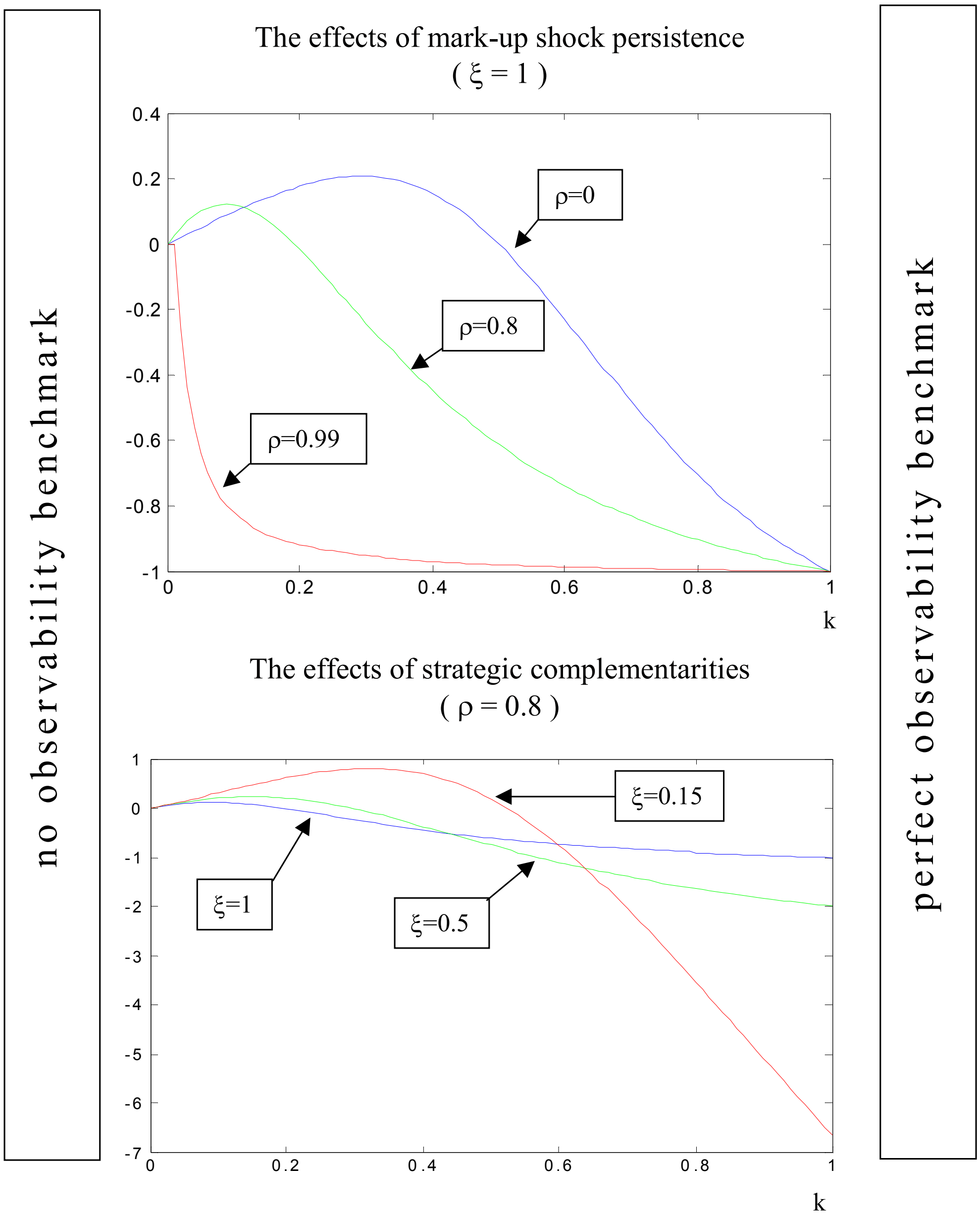

Figure 3: Optimal policy reaction coefficients

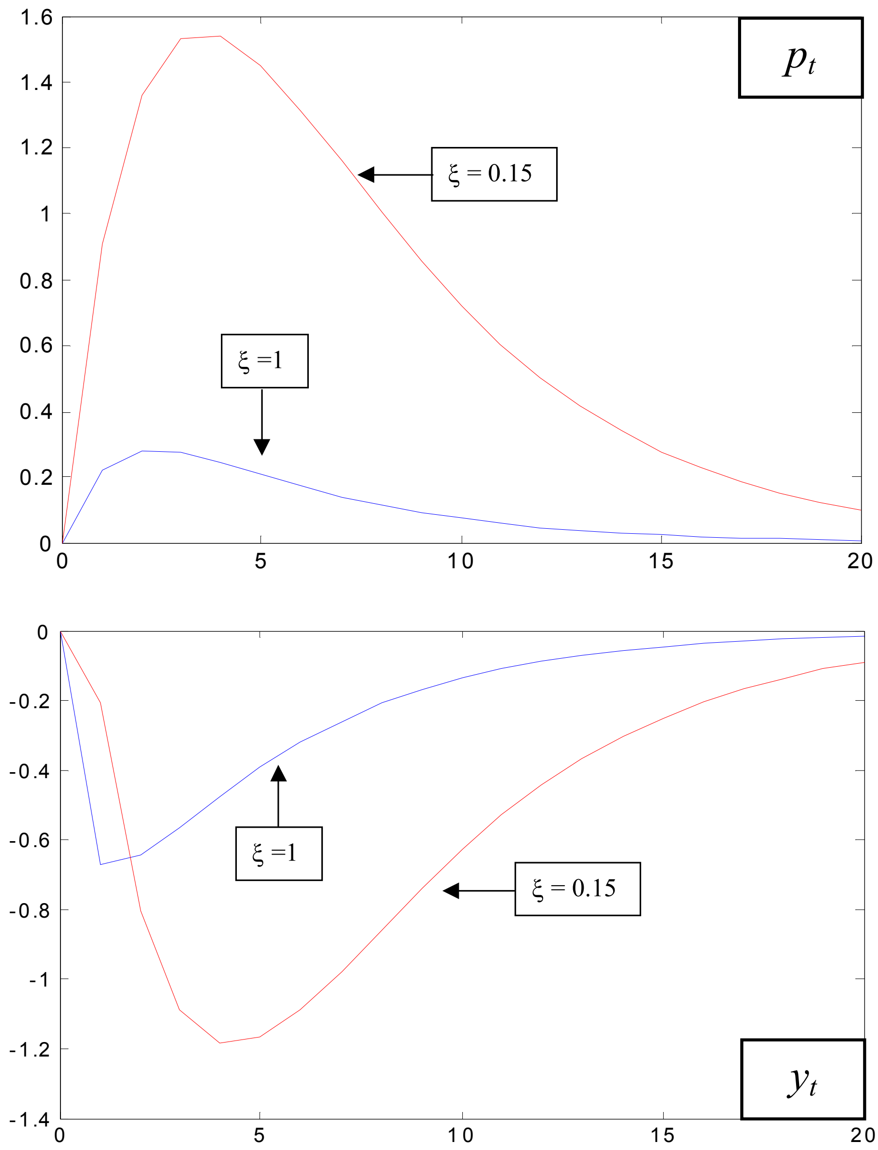

Figure 4: Impulse responses ($\rho = 0.8$, $k = 0.4$)

shows that for the limiting case $\rho \to 1$, one has $var(f_t)/var(\varepsilon_t) \to (1/\xi)^2$ and $cov(f_t, \varepsilon_t)/\sqrt{var(f_t)var(\varepsilon_t)} \to 1$. This suggests that one can take the approximation $f_t \approx (1/\xi)\varepsilon$ in equations (41) and (42). These then imply that $a = -(1/\xi)$ is optimal.[22]

The lower panel of figure 3 illustrates the effects of strategic complementarities on the optimal reaction coefficient when mark-up shocks are persistent.[23] The findings resemble closely the ones for the static economy. In particular, strategic complementarities induce policy to accommodate mark-up shocks over a wider range of values for the processing index k. As before, strategic complementarities amplify the real effects of monetary policy and induce a more accommodative policy stance.

Figure 4 depicts the impulse responses of the price level and output in response to a persistent mark-up shock.[24] Without strategic complementarities ($\xi = 1$) output drops immediately in response to the shock and prices display a hump-shaped pattern.[25] With strategic complementarities ($\xi = 0.15$) the response of these variables is more sluggish, i.e., the peak of output and inflation is reached with a larger delay. Initially, policy is rather successful in stabilizing output because higher order beliefs, which become important in the presence of complementarities, react less strongly to information than beliefs of lower order.[26] The peak response, however, is more pronounced because complementarities amplify the effects of shocks.

8 Conclusions

It has been shown by Phelps (1970) and Lucas (1973) that monetary policy has real effects when firms are only imperfectly informed about the shocks hitting the economy.

Considering firms that can process information only at a finite rate, this paper shows that information dispersion, i.e., imperfect common knowledge about shocks, may significantly enhance the real effects of nominal demand policy in a flexible price economy.

[22] This holds independently of the value of the weight λ in (38).

[23] The lower panel assumes equal weights to price and output targets ($\lambda = 1$) and persistent mark-up shocks ($\rho = 0.8$).

[24] The figure assumes equal weights to price and output targets ($\lambda = 1$), persistent mark-up shocks ($\rho = 0.8$), and $k = 0.4$, which implies that firms' Kalman gain for estimating ε_t is 0.4.

[25] For other parameterizations the maximum drop in output is not necessarily immediate but may occur also with some delay.

[26] Equation (41) implies that p_t is proportional to f_t.

When firms' prices are strategic complements, prices respond rather sluggishly to private information about shocks and policy decisions. This gives rise to substantial real effects of monetary policy and makes it optimal to stabilize the output gap. When mark-up shocks are persistent, however, the ability to affect the output gap is reduced and optimal policy is increasingly characterized by price level stabilization.

Whether it is optimal to stabilize the output gap or the price level thus depends on the importance of strategic complementarities, on the degree of information frictions, and on the persistence of the shocks that hit the economy. Empirical work seeking to quantify these various elements should therefore be of great interest to monetary policymakers.

A Appendix

A.1 Derivation of the price setting equation

Consider the nonlinear economy outlined in section 2. The product demand function associated with the Dixit-Stiglitz aggregator (2) is

$$Y^i(P^i) = \left(P^i/P\right)^{-\theta} Y \tag{43}$$

where P^i is the price charged by firm i and

$$P = \left(\int \left(P^i\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}} \tag{44}$$

The profit maximization problem of firm i is given by

$$\max_{P^i} E\left[(1+\tau)P^iY^i(P^i) - WY^i(P^i)|I^i\right] \tag{45}$$

where τ denotes an output subsidy, and I^i firm i's information set, containing information about the labor supply shock ν, the demand shock θ, and monetary policy decisions. Using (43) one can derive the first order condition of the firm's problem (45):

$$P^i = E\left[\frac{1}{1+\tau}\frac{\theta}{\theta-1}W|I^i\right] \tag{46}$$

The household's first order condition is

$$W = \frac{\nu V'(\widehat{Y})}{U'(Y)}P \tag{47}$$

where

$$\widehat{Y} = \int Y^j \mathrm{dj} \tag{48}$$

Combining (46) and (47) delivers

$$P^i = E\left[\frac{1}{1+\tau}\frac{\theta}{\theta-1}\frac{\nu V'(\widehat{Y})}{U'(Y)}P|I^i\right] \tag{49}$$

Assuming

$$\tau = \frac{1}{\overline{\theta}-1}$$

there exists a symmetric deterministic steady state with $P^i = \overline{P}$, $Y^i = \overline{Y}$, $\theta = \overline{\theta}$, and $\nu = 1$ where $\overline{Y}$ solves

$$\frac{V'(\overline{Y})}{U'(\overline{Y})} = 1 \tag{50}$$

and $\overline{P}$ is any value chosen by the central bank. For a given labor supply shock ν, the first best output level Y_n solves

$$\frac{\nu V'(Y_n)}{U'(Y_n)} = 1 \tag{51}$$

Linearizing this equation around the steady state delivers

$$\nu - 1 = -\frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{\left(U'(\overline{Y})\right)^2}\overline{Y}\left(\frac{Y_n - \overline{Y}}{\overline{Y}}\right) \tag{52}$$

Linearizing (49) around the deterministic steady state and using (52) delivers (3) where:

$$\begin{aligned}
\varepsilon_t &= -\frac{1}{\overline{\theta}-1}\frac{(\theta-\overline{\theta})}{\overline{\theta}} \\
\xi &= \frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{\left(U'(\overline{Y})\right)^2}\overline{Y} \\
&= \frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})} - \frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})}
\end{aligned}$$

A.2 A welfare based monetary policy objective

Consider the nonlinear economy outlined in section 2. A second order expansion of the utility Ω of the representative agent around the steady state level $\overline{\Omega}$ is given by

$$\begin{aligned}
\Omega - \overline{\Omega} = {} & U'(\overline{Y})(Y-\overline{Y}) - V'(\overline{Y})(\widehat{Y}-\overline{Y}) \\
& + \frac{1}{2}U''(\overline{Y})(Y-\overline{Y})^2 - \frac{1}{2}V''(\overline{Y})(\widehat{Y}-\overline{Y})^2 \\
& - V'(\overline{Y})(\widehat{Y}-\overline{Y})(v-1) + O(2) + t.i.p
\end{aligned} \tag{53}$$

where $\widehat{Y}$ is defined in (48), *t.i.p.* denotes (first and higher order) terms that are independent of policy, and $O(2)$ summarizes endogenous terms of order larger than two.

Substituting (52) into (53), using (50) and the fact that $Y = \widehat{Y} + O(1) + t.i.p.$, one obtains

$$\begin{aligned}\Omega - \overline{\Omega} &= U'(\overline{Y})(Y - \overline{Y}) - V'(\overline{Y})(\widehat{Y} - \overline{Y}) \\ &+ \frac{1}{2}\left(U''(\overline{Y}) - V''(\overline{Y})\right)(\widehat{Y} - \overline{Y})^2 \\ &- \left(U''(\overline{Y}) - V''(\overline{Y})\right)(\widehat{Y} - \overline{Y})(Y_n - \overline{Y}) + O(2) + t.i.p\end{aligned}$$

Adding $\frac{1}{2}\left(U''(\overline{Y}) - V''(\overline{Y})\right)(Y_n - \overline{Y})^2$, which is a term independent of policy, one obtains

$$\begin{aligned}\Omega - \overline{\Omega} &= U'(\overline{Y})(Y - \overline{Y}) - V'(\overline{Y})(\widehat{Y} - \overline{Y}) \\ &- \frac{1}{2}\left(V''(\overline{Y}) - U''(\overline{Y})\right)((\widehat{Y} - \overline{Y}) - (Y_n - \overline{Y}))^2 + O(2) + t.i.p\end{aligned} \tag{54}$$

A second order expansion of (2) yields after some tedious but straightforward calculations

$$Y - \overline{Y} = (\widehat{Y} - \overline{Y}) - \frac{1}{2}\frac{1}{\overline{\theta Y}}\int (Y^j - \widehat{Y})^2 \mathrm{dj} + O(2) + t.i.p \tag{55}$$

A first order approximation to (43) delivers

$$Y^i = Y - \frac{\overline{\theta Y}}{\overline{P}}(P^i - P) + O(1) + t.i.p. \tag{56}$$

Integrating (56) over all firms, subtracting the result from (56), and taking squares delivers

$$(Y^i - \widehat{Y})^2 = \frac{\overline{\theta Y}}{\overline{P}}(P^i - \widehat{P}) + O(2) + t.i.p. \tag{57}$$

where

$$\widehat{P} = \int P^j \mathrm{dj}$$

Using (55) and (57) one can express (54) as

$$\begin{aligned}\Omega - \overline{\Omega} &= -\frac{1}{2}U'(\overline{Y})\overline{\theta Y}\int \left(\frac{P^j - \overline{P}}{\overline{P}} - \frac{\widehat{P} - \overline{P}}{\overline{P}}\right)^2 \mathrm{dj} \\ &- \frac{1}{2}\left(V''(\overline{Y}) - U''(\overline{Y})\right)\overline{Y}^2\left(\frac{(\widehat{Y} - \overline{Y})}{\overline{Y}} - \frac{(Y_n - \overline{Y})}{\overline{Y}}\right)^2 + O(2) + t.i.p\end{aligned} \tag{58}$$

Maximizing (58) is, thus, equivalent to maximizing

$$-\left(y - y_n\right)^2 - \gamma \int (p(j) - p)^2 dj \tag{59}$$

for

$$\gamma = \frac{\overline{\theta}}{\frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})} - \frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})}} > 0$$

Equation (29) implies

$$\int (p(j) - p)^2 dj = \left(\frac{k}{1-(1-\xi)k} \right)^2 Var(\eta^i) \tag{60}$$

where

$$VAR(\eta^i) = \frac{1-k}{k} E[(\xi q - \xi y_n + \varepsilon)^2] \tag{61}$$

by equation (19). Using (60), (61), and (30) the welfare-based objective (59) can be written as

$$\begin{aligned} &-(y-y_n)^2 - \gamma \frac{k(1-k)}{(1-(1-\xi)k)^2} E[(\xi q - \xi y_n + \varepsilon)^2] \\ &= -(y-y_n)^2 - \lambda p^2 \end{aligned} \tag{62}$$

for $\lambda = \gamma \frac{1-k}{k}$. Equation (62) is the central bank's objective assumed in the text.

A.3 The optimal observation equation

The text assumes that agents receive a signal about (25). Below I show that the rational expectations equilibrium (REE) derived under this assumption is unaffected when allowing agents to receive a signal about (24) instead.

Let f denote the infinite sum of equation (24). Equation (28) implies that in the rational expectations equilibrium where agents observe (25):

$$f = \sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y_n + \varepsilon) \tag{63}$$

and

$$\begin{aligned} E[f|s^i] &= k \sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y_n + \varepsilon + \eta^i) \\ &= k \sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y_n + \varepsilon) + \frac{k}{1-(1-\xi)k} \eta^i. \end{aligned} \tag{64}$$

Next, suppose that agents instead observe

$$\widetilde{s}^i = f + \widetilde{\eta}^i.$$

Expectations are then given by

$$\begin{aligned} E[f|\widetilde{s}^i] &= k\widetilde{s}^i \\ &= kf + k\widetilde{\eta}^i \\ &= k \sum_{m=0}^{\infty} ((1-\xi)k)^m (\xi q - \xi y_n + \varepsilon) + k\widetilde{\eta}^i. \end{aligned} \tag{65}$$

where the last line uses the fact that in the REE under consideration equation (63) holds. The expectations in (65) are identical to ones in (64) if

$$\widetilde{\eta}^i = \frac{1}{1-(1-\xi)k}\eta^i,$$

which follows from the fact that the agent faces the same channel capacity constraint, independently of which object is observed. Equation (23) together with $E[f|\widetilde{s}^i] = E[f|s^i]$ then implies that agents set the same profit maximizing price, independently of whether they observe (24) or (25).

A.4 Equilibrium in a dynamic economy

With just one shock, agents' optimal observation equation is trivially given by

$$s_t^i = h' X_t + \eta_t^i$$

where $h' = (1, 0)$. Let $X_{t|t}$ denote agents' average believe about X_t based on information up to t. The Kalman filter equations then imply

$$X_{t|t} = (I - gh')MX_{t-1|t-1} + gh'(MX_{t-1} + mv_t) \tag{66}$$

where

$$g = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$$

is a vector of Kalman gains that remains to be determined.

Now note that

$$f_t = \overline{\xi} X_{t|t} \tag{67}$$

where $\overline{\xi} = (1, 1-\xi)$. Using (66), equation (67) can be expressed as

$$\begin{aligned} f_t = {} & ((1-z)\rho + (1-\xi)M_{21})\varepsilon_{t-1|t-1} \\ & + (1-\xi)M_{22} f_{t-1|t-1} \\ & + z\rho\varepsilon_{t-1} + zv_t \end{aligned} \tag{68}$$

where

$$z = g_1 + (1-\xi)g_2 \tag{69}$$

Using equation (67) for $t-1$ to substitute $f_{t-1|t-1}$ in (68) delivers

$$\begin{aligned} f_t = {} & ((1-z)\rho + (1-\xi)M_{21} - M_{22})\varepsilon_{t-1|t-1} \\ & + M_{22} f_{t-1} + z\rho\varepsilon_{t-1} + zv_t \end{aligned} \tag{70}$$

Equation (70) is consistent with the second line of the conjectured equilibrium law (40) when

$$M_{21} = \rho z \tag{71}$$

$$M_{22} = \rho(1 - \xi z) \tag{72}$$

$$m_2 = z \tag{73}$$

The previous equations determine M and m in (40) up to z, which depends on the Kalman gains, see (69). To determine the Kalman gain note that the Kalman filter updating equations imply that

$$g = P_{t|t-1}h(h'P_{t|t-1}h + \sigma_\eta^2)^{-1} \tag{74}$$

where σ_η^2 is the variance of the private sector observation error and $P_{t|t-1}$ denotes the prior uncertainty about X_t. By equation (19)

$$\sigma_\eta^2 = \frac{1-k}{k}h'P_{t|t-1}h \tag{75}$$

where k is defined in (21). Substituting this result into (74) delivers

$$\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} k \\ k\frac{P_{t|t-1}^{21}}{P_{t|t-1}^{11}} \end{pmatrix} \tag{76}$$

where $P_{t|t-1}^{ij}$ denotes the (i,j)-th element of $P_{t|t-1}$. Note that (76) already determines g_1. To find g_2 one has to compute the steady state values of $P_{t|t-1}$. The Kalman filter updating equations for P are

$$\begin{aligned} P_{t|t} &= P_{t|t-1} - P_{t|t-1}h(h'P_{t|t-1}h + \sigma_\eta^2)^{-1}h'P_{t|t-1} \\ &= \begin{pmatrix} (1-k)P_{t|t-1}^{11} & (1-k)P_{t|t-1}^{11} \\ (1-k)P_{t|t-1}^{11} & P_{t|t-1}^{22} - k\frac{P_{t|t-1}^{21}P_{t|t-1}^{12}}{P_{t|t-1}^{11}} \end{pmatrix} \end{aligned} \tag{77}$$

Using (77) and

$$P_{t+1|t} = MP_{t|t}M' + mm'\sigma_v^2$$

one can solve for the steady state values of $P_{t+1|t}^{11}$ and $P_{t+1|t}^{21}$. These are given by

$$P_{t+1|t}^{11} = \frac{\sigma_v^2}{1-\rho^2(1-k)} \tag{78}$$

$$P_{t+1|t}^{21} = \frac{z\sigma_v^2}{1-\rho^2(1-k)(1-z\xi)}\left(1+\rho^2\frac{1-k}{1-\rho^2(1-k)}\right) \tag{79}$$

Substituting (78) and (79) into the second line of (76) and using (69) one can analytically determine z. For $\rho = 0$ or $k = 1$:

$$z = \frac{k}{1-k+k\xi},$$

otherwise:

$$z = \frac{\left(1 + k\xi - k + \rho^2(-1 + k - k\xi + k^2\xi) - \sqrt{A}\right)}{2\rho^2\xi(k-1)},$$

where

$$
\begin{aligned}
A = {} & 1 + k^2 + 2k\xi - 2k^2\rho^2 + 2k^2\rho^2\xi + 4k\rho^2 - 2\rho^2 - 2k - 2k^2\xi \\
& - 2k^3\xi\rho^2 + 4k^2\rho^4\xi - 2k^3\rho^4\xi - 2\rho^4 k + \rho^4 k^2 + k^2\xi^2 - 2k^2\xi^2\rho^2 \\
& + 2k^3\xi^2\rho^2 - 2\rho^4 k\xi + k^2\rho^4\xi^2 - 2k^3\rho^4\xi^2 + k^4\rho^4\xi^2 + \rho^4
\end{aligned}
$$

This together with (71)-(73) completes the determination of the equilibrium law.

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