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ABSTRACT

Optimal Monetary Policy Under Discretion with a Zero Bound on Nominal Interest Rates*

We determine optimal discretionary monetary policy in a New Keynesian model when nominal interest rates are bounded below by zero. Nominal interest rates should be lowered faster in response to adverse shocks than in the case without bound. Such 'pre-emptive easing' is optimal because expectations of a possibly binding bound in the future amplify the effects of adverse shocks. Calibrating the model to the US economy we find the easing effect to be quantitatively important. Moreover, the lower bound binds rather frequently and imposes significant welfare losses. Losses increase further when inflation is partly determined by lagged inflation in the Phillips curve. Targeting positive inflation rates reduces the frequency of a binding lower bound, but tends to reduce welfare compared to a target rate of zero. The welfare gains from policy commitment, however, appear significant and are much larger than in the case without lower bound.

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Non-Technical Summary

In the recent past nominal interest rates in major world economies have reached historically low levels. The inability to further lower nominal interest rates can lead to higher than desired real interest rates and it is often feared that the economy might then embark on a deflationary path, often referred to as a 'liquidity trap'.

This paper uses a standard monetary policy model with nominal rigidities, the so-called New Keynesian model, and determines optimal monetary policy, taking explicitly into account that nominal interest rates cannot be set to negative values. We thereby focus on the case of discretionary policy making where decisions are taken in a day-by-day fashion. In particular, the paper determines the quantitative implications of the zero lower bound for discretionary monetary policy in the U.S., using estimates of the shock processes that hit the economy during the period 1983-2002.

With discretionary policy the welfare losses inflicted by the zero lower bound appear significant. This differs notably from the case with policy commitment, i.e., the case where the policymaker can engage in credible promises about its own behavior in the future.

We show that when adverse shocks threaten to push the economy into a situation with zero nominal interest rates, it turns out optimal to lower nominal rates more aggressively in advance, i.e., already before hitting the bound. Such 'preemptive' action is optimal because agents anticipate the possibility of binding shocks in the future and thereby tend to amplify the effects of adverse shocks via an adjustment of their expectations. A stronger policy response counteracts this amplification.

Optimal discretionary policy that targets an average inflation rate of zero implies that shocks to the so-called 'natural' real rate of interest cause the lower bound to become binding rather frequently in the U.S. economy. Therefore, we consider whether it is possible to improve upon this situation by targeting

a positive average inflation rate. This increases average nominal interest rates and thereby reduces the risk of hitting the zero lower bound. Such a policy has been frequently suggested in the literature. We find that positive target rates do reduce the likelihood with which the lower bound is reached but also tend to reduce welfare because they increase the average inflation rate. This suggests that one should be careful in judging the welfare consequences of monetary policies by looking solely at the frequency with which the lower bound is reached.

Besides addressing substantive economic questions, this paper also implements a new approach to numerically solving discretionary nonlinear optimal policy problems with forward-looking constraints that might be of wider interest.

1 Introduction

The relevance of the zero lower bound on nominal interest rates for the conduct of monetary policy is a much debated topic among both policymakers and academics. Clearly, the economic experience of Japan during the last decade as well as the low levels of nominal interest rates prevailing in Europe and the United States contribute to the renewed interest in this topic.[1]

While deflationary pressures seem eventually to be subsiding, a systematic investigation of how to conduct monetary policy when interest rate decisions are subject to the zero lower bound remains an open question of considerable interest. This knowledge would be relevant for dealing with similar policy problems should they reemerge in the future.

This paper studies optimal monetary policy under discretion in a canonical New Keynesian model featuring monopolistic competition and sticky prices in the product market (see Clarida, Galí and Gertler (1999) and Woodford (2003)). The contribution made here is to analyze a fully stochastic setup that takes explicitly into account the zero lower bound on nominal interest rates. The paper also introduces a new numerical algorithm for solving discretionary nonlinear optimal policy problems with endogenous state variables. The algorithm is complementary to first order based approaches, but has the crucial advantage that one can numerically verify in a simple way whether second order conditions are actually satisfied.

Studying a fully stochastic setup is of economic interest for a number of reasons. First, it allows us to calibrate the model to the U.S. economy and to study the quantitative importance of the zero lower bound. Second, we can assess how policy should react in a situation where interest rates are low but still positive and future adverse shocks may drive the economy into a situation where the lower bound is binding. This appears to be especially important in the

[1] For recent discussions see Auerbach and Obstfeld (2003), Coenen and Wieland (2003), Eggertsson and Woodford (2003), and Svensson (2003).

current era of low nominal interest rates and inflation rates. Earlier literature instead has focused exclusively on situations where the zero lower bound is currently binding but never returns to being binding again some time onwards in the future.[2]

As a benchmark, we analyze a purely forward-looking model that is calibrated to the U.S. economy. We find that the lower bound is reached frequently, inflicting sizeable welfare losses.[3] Based on our estimates of the historical U.S. shock processes for the period 1983-2002, the welfare losses are roughly 16% higher than those generated if nominal interest rates were allowed instead to become negative.[4] In a hybrid specification, where inflation is also partly determined by lagged inflation, the welfare losses tend to be even larger.

These results differs considerably from the case with policy commitment, which is analyzed in a companion paper of ours, see Adam and Billi (2004). In a purely forward-looking model the additional welfare losses generated by the zero lower bound are then below 1%. The existence of a lower bound entails that there are important welfare gains from policy commitment.

We find that nominal interest rates should be lowered faster in response to a fall in the natural real rate than if nominal interest rates were allowed to become negative.[5] The required 'preemptive easing' of policy is quantitatively important for the U.S. economy. For our baseline model we find that (depending on the state of the economy) one should set nominal interest rates as much as 75 basis points below the level suggested by a model abstracting from the zero lower bound.

This result emerges because expectations significantly reinforce the effects of adverse shocks. A binding lower bound implies deflation and output losses in

[2] See, for example, Eggertsson and Woodford (2003), Svensson (2003), or Jung et al. (2001).

[3] For our baseline calibration zero nominal rates would occur in about one quarter every five years on average.

[4] Losses are then due to nominal price rigidities only.

[5] The natural real rate is the real interest rate of the (efficient) flexible price equilibrium.

equilibrium. Agents anticipate this possibility even before the bound is reached, therefore, they reduce output and inflation expectations correspondingly. Since lower expected output and inflation lead to lower current values of these variables, it is optimal to reduce nominal interest rates.

The deflationary pressure generated by the lower bound causes a so-called 'deflation bias', i.e., a drop of the average inflation rate below its target level. This effect, however, is quantitatively small in the order of less than 10 basis points for our baseline calibration. Moreover, the lower bound does not generate an 'output bias', i.e., a downward distortion of average output. While a binding bound does lead to output losses, the lower nominal interest rates implemented before the bound is reached generate positive output gaps that compensate for these losses on average.

Since targeting positive inflation rates is frequently suggested as a remedy to overcome the constraints imposed by the zero lower bound, we investigate the welfare consequences of such policies. We find that small positive target levels for inflation, e.g., 10 basis points annually, have the potential to increase welfare. Policies that cause the zero bound to be significantly less binding, e.g., inflation targets of about 50 basis points annually, generate large additional welfare losses. This suggests that one should be careful in judging the welfare consequences of monetary policies by looking solely at the frequency with which the zero lower bound is reached.

The remainder of this paper is structured as follows. Section 2 discusses the related literature. Section 3 introduces the economic model and the policy problem. Section 4 defines the rational expectations equilibrium with discretionary monetary policy, and section 5 presents the calibration of the model for the U.S. economy. Section 6 analytically determines the perfect foresight equilibrium. Section 7 discusses the numerical results for the stochastic equilibrium and analyzes the welfare effects of targeting positive inflation rates. Section 8 checks for the robustness of our findings to alternative parameterizations and model specifications, e.g., the introduction of lagged inflation in the Phillips curve. Section 9 briefly concludes.

2 Related Literature

The literature on monetary policy under discretion was initiated by the seminal contributions of Kydland and Prescott (1977) and Barro and Gordon (1983). While their models lacked explicit microfoundations, the recent development of general equilibrium models with monopolistic competition and sticky prices allows to extend earlier analyses to fully microfounded models, see Clarida et al. (1999) and Woodford (2003).[6]

Overall, the literature following Kydland and Prescott (1977) and Barro and Gordon (1983) has tended to stress the 'inflationary bias' associated with discretionary monetary policy and its potential solutions, see Persson and Tabellini (1994).

Krugman (1998) seems to have been the first to note that when taking into account the zero lower bound on nominal interest rates the credibility problem may equally generate a 'deflation bias'. This emerges because the policy maker cannot engage in credible promises about the conduct of future monetary policy, which is the only policy instrument left once the zero lower bound is binding.[7] Eggertsson (2003) and Jeanne and Svensson (2004) build upon this idea and discuss potential solutions to the credibility problem.

Nakov (2004) compares the performance of simple rules to that of optimal discretionary policy and finds that simple rules perform almost as good as optimal discretionary policy. Following simple rules, however, requires commitment power. We show that the welfare gains from commitment to an optimal rule

[6] Using general equilibrium models with sticky prices, Albanesi et al. (2002), and King and Wolman (2003) recently highlighted that when monetary authorities act under discretion there is the possibility of having multiple steady states.

[7] Any monetary expansion implemented during a time of zero nominal interest rates is expected to be reversed once the lower bound ceases to be binding.

are considerable by confronting the results of this paper with those derived in Adam and Billi (2004).

3 The Model

We consider a simple and well-known monetary policy model of a representative consumer and firms in monopolistic competition facing restrictions on the frequency of price adjustments (Calvo (1983)). Following Rotemberg (1987), this is often referred to as the 'New Keynesian' model, that has frequently been studied in the literature, e.g., Clarida, Galí and Gertler (1999) and Woodford (2003).

We augment this otherwise standard monetary policy model by explicitly imposing the zero lower bound on nominal interest rates. We thus consider the following problem:

$$\max_{\{y_t,\pi_t,i_t\}} -E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha y_t^2\right) \tag{1}$$

s.t.:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda y_t + u_t \tag{2}$$

$$y_t = E_t y_{t+1} - \varphi\left(i_t - E_t \pi_{t+1}\right) + g_t \tag{3}$$

$$i_t \geq -r^* \tag{4}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t} \tag{5}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \tag{6}$$

behavior of future monetary authorities is given (7)

u_0, g_0 given (8)

where π_t denotes the inflation rate, y_t the output gap, and i_t the nominal interest rate expressed as deviation from the interest rate consistent with the zero inflation steady state.

Assuming that monetary policy cannot commit to future plans, one solves problem (1)-(8) period by period. In other terms, the policymaker rationally anticipates its inability to commit, therefore, treats the behavior in future periods as given. This is captured by constraint (7).

The monetary policy objective (1) is a quadratic approximation to the utility of the representative household, where the weight $\alpha > 0$ depends on the underlying preference and technology parameters. Equation (2) is a forward-looking Phillips curve summarizing, up to first order, profit-maximizing price setting behavior by firms, where $\beta \in (0, 1)$ denotes the discount factor and $\lambda > 0$ depends on the underlying utility and technology parameters.[8] Equation (3) is a linearized Euler equation summarizing households' intertemporal maximization, where $\varphi > 0$ denotes the interest rate elasticity of output. The shock g_t captures the variation in the 'natural' real interest rate and is usually referred to as a real rate shock, i.e.,

$$g_t = \varphi(r_t - r^*) \tag{9}$$

where the natural real rate r_t is the real interest rate consistent with the flexible price equilibrium and $r^* = 1/\beta - 1$ is the real rate of the deterministic zero inflation steady state.[9] The requirement that nominal interest rates have to remain positive is captured by constraint (4). Finally, equations (5) and (6) describe the evolution of the stochastic shock processes, where $\rho_j \in (-1, 1)$ and $\varepsilon_{j,t} \sim iiN(0, \sigma_j^2)$ for $j = u, g$.[10]

[8] The case with a hybrid Phillips curve where inflation depends also on lagged inflation is considered in section 8.

[9] The shock g_t summarizes all shocks that under flexible prices generate time variation in the real interest rate, therefore, it captures the combined effects of preference shocks, productivity shocks, and exogenous changes in government expenditure.

[10] As shown in Adam and Billi (2004), this specification of the shock processes is sufficiently general to describe the historical sequence of shocks in the U.S. economy for the period 1983:1-2002:4 that we consider.

3.1 Discussion

3.1.1 Relation to earlier work

The new feature of our policy problem is both the presence of the lower bound (4) and of the stochastic disturbances $\varepsilon_{u,t}$ and $\varepsilon_{g,t}$. These elements together render the policy problem nonlinear, since the disturbances will cause the lower bound to be occasionally binding, see Christiano and Fisher (2000).

The model without lower bound is analyzed in Clarida, Galí and Gertler (1999). Without lower bound the policy problem is linear quadratic, so one can solve for the equilibrium dynamics analytically using standard methods. Jung, Teranishi, and Watanabe (2001) consider a model with lower bound but assume perfect foresight. In their model the lower bound may be binding in $t = 0$, but never returns to being binding again some time onwards in the future. As shown below, the equilibrium of a stochastic economy differs considerably from such a perfect foresight solution, because shocks may always drive the economy into a situation with a binding lower bound.

3.1.2 Policy instruments

It should be stressed that here the interest rate is assumed to be the only available policy instrument. We thereby abstract from a number of alternative policy instruments that might be important in a situation of zero nominal interest rates, most notably fiscal policy, exchange rate policy, and quantity-based monetary policies. Our setup, thus, tends to give prominence if not overemphasize the policy implications of the zero nominal interest rate bound.

While the omission of fiscal policies clearly constitutes a shortcoming that ought to be addressed in future work, ignoring exchange rate and money policies may be less severe than one might initially think.[11]

Clarida, Galí and Gertler (2001), e.g., show that one can reinterpret the

[11] Eggertsson (2003) considers discretionary monetary and fiscal policy with a lower bound in a perfect foresight economy.

present setup as an open economy model and that there exists a one-to-one mapping between interest rate policies and exchange rate policies. It is then inessential whether policy is formulated in terms of interest rates or exchange rates.

Similarly, ignoring quantity-oriented monetary policies in the form of open market operations during periods of zero nominal interest rates seems to be of little relevance. Eggertsson and Woodford (2003) show that in the present model such policies have no effect on the equilibrium, unless they influence the future path of interest rates. Under discretion, however, monetary policy cannot commit to a future path, thereby base money policies are irrelevant for the equilibrium outcome.

We recognize that alternative policy instruments may still be relevant in practice.[12] Focusing on interest rate policy in isolation is nevertheless of interest, since it allows one to assess what interest rate policy alone can achieve in alleviating the negative effect of the zero bound. This seems important for one to know, given that alternative instruments are often subject to (potentially uncertain) political approval by external authorities and may therefore not be readily available.

3.1.3 How much non-linearity?

We now briefly comment on the fact that we use linear approximations to the first order conditions of households and firms, i.e., equations (2) and (3), and a quadratic approximation to the objective function, i.e., equation (1), instead of the fully nonlinear model. Doing so means that the only nonlinearity that we take account of is the one imposed by the zero lower bound (4).[13]

[12] See Eggertsson (2003) on how other policy instruments, e.g., nominal debt policy, may be used as a commitment device.

[13] Technically, this approach is equivalent to linearizing the first order conditions of the nonlinear Ramsey problem around the first best steady state except for the non-negativity constraint for nominal interest rates that is kept in its original nonlinear form. This holds because deriving first order conditions and linearizing thereafter is equivalent to linearizing first and then taking derivatives.

Clearly, this approach has advantages and disadvantages. One disadvantage is that for the empirically relevant shock support and the estimated value of the discount factor the linearizations (2) and (3) may perform poorly at the lower bound. However, this depends on the degree of nonlinearity present in the economy, an issue about which relatively little seems to be known.

A paramount advantage of focusing on the nonlinearities induced by the lower bound alone is that we do not have to parameterize higher order terms in our empirical application later on.[14] In addition, one can economize in the dimension of the state space. A fully nonlinear setup would require instead an additional state variable to keep track over time of the higher-order effects of price dispersion, as shown by Schmitt-Grohé and Uribe (2003).

A positive by-product of all this is that the results remain easily comparable to the standard linear-quadratic analysis without lower bound, as the only difference consists of imposing equation (4).

4 Discretionary Equilibrium

We restrict attention to stationary Markov perfect equilibria in which the policy functions depend on the current predetermined states u_t and g_t only.[15]

A Markov perfect equilibrium consists of policy functions $y(u_t, g_t)$, $\pi(u_t, g_t)$, and $i(u_t, g_t)$ that solve problem (1)-(8) when the expectations in equations (2) and (3) are given by

$$E_t \pi_{t+1} = \int \pi(\rho_u u_t + \varepsilon_{u,t+1}, \rho_g g_t + \varepsilon_{g,t+1}) f(\varepsilon_{u,t+1}, \varepsilon_{g,t+1}) \mathrm{d}(\varepsilon_{u,t+1}, \varepsilon_{g,t+1}) \tag{10}$$

$$E_t y_{t+1} = \int y(\rho_u u_t + \varepsilon_{u,t+1}, \rho_g g_t + \varepsilon_{g,t+1}) f(\varepsilon_{u,t+1}, \varepsilon_{g,t+1}) \mathrm{d}(\varepsilon_{u,t+1}, \varepsilon_{g,t+1}) \tag{11}$$

[14] Calibrating the higher order terms would probably amount to choosing relatively arbitrary values given the available knowledge of the economy under exam.

[15] When considering a model with lagged inflation in the Phillips curve, as in section 8, policy functions also depend on lagged inflation rates.

where $f(\cdot,\cdot)$ is the probability density function of $(\varepsilon_u, \varepsilon_g)$.

Equations (10) and (11) show that the solution to problem (1)-(8) enters the constraints (2) and (3). Solving for the equilibrium, thus, requires finding a fixed point in the space of policy functions.

We numerically solve for the fixed point as follows. We guess initial policy functions and then compute the associated expectations in equations (10) and (11). Given the expectations, problem (1)-(8) is a simple static one-period maximization problem, where the first order conditions can be used to determine updated policy functions.[16] We iterate in this manner until convergence. The numerical procedure is described in detail in appendix A.1.

5 Calibration to U.S. Economy

To calibrate the model to the U.S. economy we use the parameterization from Adam and Billi (2004), that is based on the results of Rotemberg and Woodford (1998) and our estimates of the U.S. shock processes for 1983:1-2002:4. The parameter values are summarized in table 1 and serve as the baseline calibration of the model. The implied steady state real interest rate for this parameterization is 3.5% annually. In section 8 we check the robustness of our results to various changes in this baseline parameterization.

6 Perfect Foresight

To gain intuition for our numerical findings, this section analytically determines the Markov perfect equilibrium under perfect foresight. For simplicity, we abstract from time variations in the mark-up shock u_t and focus on variations of the real rate shock g_t instead.[17]

[16] In the purely forward-looking model considered here second order conditions hold because the discretionary maximization problem is static and the one-period return function is concave (quadratic).

[17] Once we calibrate the model to the U.S. economy, mark-up shocks turn out to be empirically less relevant.

To characterize the equilibrium define for the real rate shock a critical value

$$g^c = -\varphi r^*$$

and partition the real line into a number of non-intersecting intervals

$$I^0 = [g^c, +\infty)$$
$$I^j = \left[g^c / (\rho_g)^j, g^c / (\rho_g)^{j-1}\right) \qquad for\ j = 1, 2, 3 \ldots$$

Under perfect foresight these intervals have the convenient property that if $g_t \in I^j$ then $g_{t+1} \in I^{j-1}$ for all $j > 0$. The interval I^0 is an absorbing interval that is reached in finite time for any initial value g_0.

In appendix A.2 the following result is shown:

Proposition 1 *Suppose $\varepsilon_{u,t} = \varepsilon_{g,t} = 0$ and $u_0 = 0$. There exists a Markov perfect equilibrium with perfect foresight such that*

$$i = \begin{cases} \frac{1}{\varphi} g & for\ g \geq g^c \\ -r^* & for\ g < g^c \end{cases} \tag{12}$$

and in which the output gap and inflation are continuous functions of g. For $g \in I^0$ the output gap and inflation are equal to zero. For $g \in I^j$ $(j > 0)$ the output gap and inflation are negative and linearly increasing in g at a rate that increases with j.

Figure 1 illustrates the equilibrium for the case with lower bound (solid line) and without lower bound (dashed line with circles) when using the U.S. baseline calibration from table 1.[18]

Without lower bound real rate shocks do not generate any policy trade-off. The policymaker neutralizes variations in the natural real rate by adjusting nominal interest rates appropriately.

Instead, with lower bound it remains optimal to mimic this policy as long as the lower bound is not reached, but to set nominal interest rates to zero once

[18] For this calibration $g^c \approx -5.47$.

the natural real rate drops below the critical value g^c. Output then falls short of potential and inflation becomes negative.

Figure 1 also shows that, as stated in proposition 1, the effects of a marginal reduction of g on output and inflation are increasing as the real rate becomes more negative. More negative values for g also imply more negative values of expected future output and inflation. This reinforces the downward pressure on current output and inflation stemming from low values of the real rate shock, see equations (2) and (3).

7 Stochastic Equilibrium

This section presents the stochastic Markov perfect equilibrium for the baseline parameterization in table 1.[19] The sensitivity of our results to alternative model specifications and parameterizations is discussed in section 8.

7.1 Impact on Average Values

We first discuss the effect of the nonlinearities on average output and inflation. Since we have a nonlinear stochastic model the average values of endogenous variables will generally differ from their steady state values due to a breakdown of certainty equivalence.

The perfect foresight solution presented in section 6 suggests that both average output and inflation fall short of their steady state value, i.e., zero. Our stochastic simulations show, however, that the downward bias for average inflation is rather small, i.e., in the order of less than 8 basis points annually. In addition, average output displays a slight upward distortion of about 0.6 basis points. Therefore, biases for average output and inflation are relatively small and not even of the sign suggested by the perfect foresight solution.[20]

[19] All variables are expressed in terms of percentage point deviations from steady state values; interest rates and inflation rates are expressed in annualized percentage deviations; the real rate shock and the mark-up shock are expressed as their quarterly percentage values.

[20] Overall, the findings parallel those for the case of policy commitment, see Adam and Billi

The reason for these results is clarified in the next section, which looks at the stochastic equilibrium in greater detail and presents the optimal policy functions.

7.2 Optimal Policy Response to Shocks

We first discuss the optimal policy reaction to mark-up shocks and then discuss that to real rate shocks.

We find that in equilibrium the zero lower bound does not impose a binding constraint on dealing with mark-up shocks. The empirical variability of these shocks is simply too small for the policy constraint to matter. The left-hand panels of figure 2 display the optimal response of output, inflation, and nominal interest rates to mark-up shocks.[21] The solid line corresponds to the reaction function if the bound is imposed, while the dashed line with circles refers to the case where nominal interest rates are allowed to become negative. The figure shows that the optimal reaction to mark-up shocks is virtually unaffected by the presence of the zero lower bound. Moreover, the lower bound remains far from being binding, even for very negative values of the mark-up shock.

The situation differs notably when considering real-rate shocks. The right-hand side panels of figure 2 depict the optimal response of output, inflation, and nominal rates to real rate shocks. Again, the solid line corresponds to the case where the lower bound is imposed while the dashed line with circles denotes the equilibrium response when nominal rates are allowed to become negative. This figure reveals a number of interesting features, especially when compared to the perfect foresight solution depicted in figure 1.

First, while large negative values of the real rate shock result in negative output gaps and deflation, these effects are now much more pronounced than

(2004), and suggest that empirically observed average biases are unlikely to be informative about whether policy acts under discretion or commitment.

[21] The figure depicts policy responses over a range of ± 4 unconditional standard deviations. The values of state variables not shown on the x-axis is set equal to zero.

under perfect foresight. In particular, the maximum output loss approximately doubles and the maximum deflation is about triple.

Second, as illustrated in figure 3 in greater detail, the zero lower bound now binds much earlier than under perfect foresight, since interest rates are lowered more aggressively in response to negative real rate shocks. For our calibration the presence of the lower bound might require setting nominal interest rates up to 75 basis points lower than if nominal rates were allowed to become negative or than under perfect foresight.

Third, figure 3 reveals that the output gap becomes slightly positive and inflation slightly negative well before the zero lower bound starts to be binding. Thus, real rate shocks generate a policy trade off between output and inflation stabilization even before the zero lower bound is reached.

All these features emerge because shocks may drive the economy from a situation with positive nominal interest rates into one where the lower bound is binding. Since output and inflation are negative once the lower bound is reached, the possibility of a binding lower bound in the future generates a downward bias in expected output and expected inflation well before interest rates hit the lower bound.[22]

This reduction in expected future output and inflation is isomorphic to a negative mark-up shock and a negative real rate shock in equations (2) and (3), respectively. To both shocks the policymaker reacts by lowering nominal interest rates, this explains the 'preemptive easing' of interest rates that can be observed in the lower panel of figure 3.

Negative mark-up shocks, however, generate a policy trade-off and policy reacts to them by letting output rise and inflation fall, see figure 2. The downward bias in expectations, therefore, also explains the output boom that can

[22] Technically: since the policy functions of output and inflation depicted in figure 1 are concave, Jensen's inequality implies a downward bias once we allow for uncertainty about the future value of the natural real rate.

be observed in the 'run-up' to a binding lower bound in figure 3, i.e., before g_t enters the binding area.

Finally, the downward bias of expected future values due to the presence of shocks generates a downward bias for actual values of output and inflation. This in turn justifies even lower expectations. This complementarity between expectations and outcomes explains the large differences in magnitudes implied by the perfect foresight equilibrium and the stochastic equilibrium.

7.3 Welfare Losses and Frequency of Zero Nominal Rates

In this section we discuss how often the zero lower bound is binding and assess the welfare implications of the zero lower bound.

Figure 2 already indicates that the lower bound is reached quite often.[23] Our simulations show that under optimal discretionary policy zero nominal interest rates occur about one quarter every 5.5 years on average. Yet, zero nominal rates persist for only 1.67 quarters on average, i.e., a relatively short period of time.

Zero nominal rates, therefore, emerge much more frequently than in the case with policy commitment, where the bound was reached instead in about one quarter every 17 years on average, see Adam and Billi (2004). The average persistence of zero nominal rates under discretionary policy, however, is roughly comparable to the case with policy commitment.[24]

Table 2 presents the welfare losses that arise by imposing the zero bound on nominal interest rates.[25] The table reports losses for the cases with and without

[23] The figure shows policy reactions over the range of ± 4 unconditional standard deviations of the shocks. Already a -2 standard deviation value of the real rate shock leads to a binding lower bound.

[24] The average persistence under commitment is about 1.37 quarters.

[25] The table reports the average discounted losses over random initial draws of u_0 and g_0 from their stationary distribution. To compute the average we take 1000 simulations with a length of 1000 periods each.

bound and with and without commitment.[26]

Due to the strong forward-looking elements in the underlying economic model, there are considerable welfare gains from commitment even if the lower bound is not imposed. These gains increase even further once the lower bound is taken into account.

When the lower bound is reached, commitments about the future path of policy are the only available monetary policy instrument. Since this instrument is unavailable to a discretionary policy maker, the lower bound increases welfare losses roughly by 15% under discretionary policy, while the same figure is about 1% for the case with commitment. This suggests that one may significantly underestimate the welfare gains from policy commitment if ignoring the zero lower bound on nominal interest rates.

7.4 Positive Inflation Targets

Given the results shown so far, it seems rather unlikely that monetary authorities in the United States maximize a social welfare function under discretion. It would imply that nominal interest rates are more often at zero and inflation is lower than was the case during the last two decades.

One plausible explanation for this is that U.S. monetary authorities target an inflation rate larger than zero. Many central banks (apart from the Federal Reserve) explicitly state positive target levels for inflation. Moreover, academics and policymakers alike frequently argue in favor of slightly positive inflation levels, partly as a way to overcome the adverse consequences of a binding zero lower bound on nominal rates, e.g., Coenen, Orphanides, and Wieland (2004), Bernanke (2002), and Trichet (2003).

Here we take up this issue and evaluate the effectiveness of positive inflation targets in achieving welfare superior outcomes.[27] From the theory of the second

[26] In the case without lower bound losses are generated by nominal price rigidities only.

[27] As pointed out by Vestin (2002) and Wolman (2003) it might be better to assign a price

best it follows that adding an additional distortion in the form of positive inflation targets may potentially improve upon the discretionary policy outcome. In particular, the policy objective is now given by

$$-E_0 \sum_t \beta^t \left((\pi_t - \pi^*)^2 + \alpha y_t^2 \right)$$

where $\pi^* \geq 0$ denotes the target rate for inflation.

Figure 4 illustrates the effects of positive inflation targets for average inflation, the frequency with which zero nominal rates are reached, and the average persistence of zero nominal interest rates. Positive inflation targets raise average inflation rates and are quite effective in reducing the frequency with which the lower bound binds. Already modest target rates of 50 basis points annually significantly reduce the likelihood of zero nominal interest rates. Positive targets rates also slightly reduce the average persistence of zero nominal rates.

While this suggests that positive inflation targets might be a useful tool to ameliorate the policy constraints imposed by the zero lower bound, these results ignore the adverse welfare consequences of positive inflation rates.

Figure 5 illustrates the welfare losses for various levels of the inflation target.[28] Modest target rates of about 10 basis points have the potential to increase welfare slightly. Average inflation is then approximately equal to zero, see figure 4. Target rates that significantly reduce the likelihood of hitting the lower bound, e.g., an inflation target of 50 basis points, lead to very large welfare losses, up to 50% above those that would be achieved with a zero inflation target. This suggests that the frequency with which zero nominal rates are binding is not necessarily indicative of the welfare losses associated with a given inflation target.

level target instead of an inflation target. As shown by these authors, this would make the inflation rate history dependent in a way that mimics the commitment solution.

[28] To evaluate these losses we use the utility-based welfare function, i.e., $-E_0 \sum \beta^t \left({\pi_t}^2 + \alpha y_t^2 \right)$.

8 Sensitivity Analysis

In this section we report the results of robustness exercises regarding the model specification and parameterization.

8.1 Hybrid Phillips Curve

In the benchmark model considered thus far inflation is assumed to be purely forward-looking. A number of econometric studies, however, suggest that inflation is at least partly determined by lagged inflation rates, e.g. Galí and Gertler (1999).

This section studies the implications of allowing inflation to depend on lagged inflation rates. In particular, we consider the policy problem (1)-(8) when the forward-looking Phillips curve (2) is replaced by its 'hybrid' version

$$\pi_t = \frac{1}{1+\beta\gamma}\left[\beta E_t \pi_{t+1} + \gamma \pi_{t-1} + \lambda y_t + u_t\right] \tag{13}$$

where $\gamma \geq 0$ is an 'indexation parameter' that indicates the degree to which firms automatically adjust their prices to lagged inflation rates when they do not fully reoptimize prices, see Woodford (2003).[29]

For $\gamma = 0$ equation (13) reduces to the forward-looking Phillips curve (2). For $\gamma > 0$ inflation is partly determined by lagged inflation, which becomes an endogenous state variable of the system. Solving the policy problem is then more involved, since the discretionary maximization problem fails to be static. In particular, the optimal policy functions now depend on the exogenous shocks (u, g) and also on the lagged inflation rate π_{-1}. We assume that the current

[29] As shown in Woodford (2003), lagged inflation appearing in the Phillips curve affects the second order approximation of the welfare function, i.e., equation (1), which is then given by

$$-E_0 \sum_{t=0}^{\infty} \beta^t \left((\pi_t - \gamma \pi_{t-1})^2 + \alpha y_t^2 \right)$$

When employing this objective function instead of equation (1) we obtain qualitatively very similar results.

policymaker behaves as a Stackelberg leader, rationally anticipating the reaction of future authorities to its own actions.

Appendix A.3 describes the numerical algorithm used to solve the model, which can be seen as a generalization to a nonlinear setup of the algorithm described in Söderlind (1999). The algorithm is based on a value function representation of the policy problem, and has the advantage that it allows to verify numerically whether second order conditions actually hold for the solution derived.

From the economic viewpoint, introducing lagged inflation gives rise to two opposing effects. On the one hand, it alleviates the problems of discretionary policy making: by influencing current inflation rates the policymaker can affect future policy decisions since these depend on lagged inflation. On the other hand, the presence of lagged inflation is potentially damaging because inflation now displays more persistence than in a purely forward-looking model; this may cause real interest rates to remain undesirably high for a longer period of time.

We find that for our calibration to the U.S. economy the second effect strongly dominates. For small degrees of indexation, e.g., $\gamma = 0.15$, the additional welfare losses generated by imposing the zero lower bound are about 37%. The additional losses in our baseline model ($\gamma = 0$) are approximately 15%. Moreover, higher degrees of inflation indexation lead to a rapid increase in the welfare losses. For example, increasing γ just slightly to 0.16 raises the additional welfare losses to 49% already. For larger values of γ our numerical algorithm then fails to converge.

In addition, we find that endogenous inflation persistence significantly increases the amount of deflation and the size of output losses associated with negative values of the real rate shock. Also, the policymaker has to ease monetary policy even more aggressively than in a purely forward-looking specification. Average inflation rates and output losses, however, seem to be affected only slightly.[30]

Overall, these results suggest that endogenous inflation persistence significantly amplifies the welfare costs of the zero lower bound, and that most of the quantitative results are rather sensitive to the presence of lagged inflation in the Phillips curve. Qualitatively, however, the findings parallel those of the purely forward-looking model.

8.2 More Variable Shocks

We also consider the sensitivity of our results to the benchmark parameterization of the shock processes in table 1. In particular, we assess the effects of an increased variability of the disturbances, motivated by the fact that the period 1983-2002 that we use to estimate the shock processes is generally considered to be a relatively 'calm' period, e.g., compared to the 1970s.

Results are remarkably stable with respect to changes in the variance of mark-up shocks. This holds even if we double the variance σ_u^2 of the mark-up shock innovations. Instead, results are rather sensitive to the parameterization of the real rate process, i.e., to changes in the persistence parameter ρ_g and the variance σ_g^2 of the real rate shock innovations. A slight increase in one of these parameters considerably increases the welfare losses generated by the zero lower bound.[31] More persistent and more variable natural real rate shocks increase the likelihood of a binding bound in the future and thereby increase the downward bias of output and inflation expectations, see the discussion in section 7.2. As a result, interest rates have to be lowered even faster and the lower bound constrains policy more often.

[30] For $\gamma = 0.16$ the average annual inflation rate drops to about -17 basis points and the average output gap increases to +1.2 basis points.

[31] Increasing the variance σ_g^2 by only 10% raises the additional welfare losses from the zero lower bound to 43%. Raising the persistence of ρ_g to 0.81 increases the additional losses to 45%. For the baseline calibration this number was roughly 16%.

8.3 Lower Interest Rate Elasticity of Output

Our benchmark calibration of table 1 assumes an interest rate elasticity of output of $\varphi = 6.25$, which seems to lie on the high side for plausible estimates of the intertemporal elasticity of substitution.[32] Therefore, we also consider a calibration with $\varphi = 1$, that corresponds to log utility in consumption, and constitutes the usual benchmark parameterization in the real business cycle literature. This calibration is taken from our companion paper, see Adam and Billi (2004), and is summarized in table 3.

With this calibration, the lower bound is now reached even more frequently, namely about once every three quarters on average. The average inflation rate drops to -0.38% annually and the additional welfare losses from the lower bound surge to 67%. However, these features emerge mainly because the calibration in table 3 implies a slightly more variable natural real rate process than the one implied by the baseline calibration of table 1.

9 Conclusions

When U.S. monetary authorities maximize social welfare in a discretionary way, the zero lower bound seems to inflict significant welfare losses. Once the zero lower bound is binding the inability to commit to future policies deprives monetary policymakers of their policy instruments.

Uncertainty about the future natural real rate has a significant impact on the equilibrium outcome. The anticipation of possibly binding shocks in the future lowers expectations of future output and inflation, thereby leads to an increase in real interest rates already before the zero lower bound is reached. Policymakers react to such pressures by reducing nominal rates more aggressively, causing the lower bound to be reached even earlier.

[32] As argued by Woodford (2003), a high elasticity value may capture non-modeled interest-rate-sensitive investment demand.

While positive inflation targets may reduce the constraints imposed upon policy by the zero lower bound, the welfare gains of positive inflation targets seem rather limited. Indeed, considerable welfare losses may be associated with even moderately positive target levels.

A Appendix

A.1 Numerical algorithm (forward-looking Phillips curve)

We define a grid of N interpolation nodes over the state space (u, g) and evaluate functions at intermediate values resorting to linear interpolation. The expectations defined in equations (10) and (11) are evaluated at each interpolation node using an M node Gaussian-Hermite quadrature scheme.[33] Our numerical algorithm consists of the following steps:

Step 1: Choose N and M and assign the interpolation and quadrature nodes. Guess initial values for the policy functions y_0, π_0, and i_0 at the interpolation nodes.

Step 2: At each interpolation node compute the expectations (10) and (11) implied by the current guess y_k, π_k, and i_k. Then employ the first order conditions of (1)-(8) to derive a new guess for the policy functions in the following way. At each interpolation node, first assume $i_t > -r^*$. The first order conditions then imply the 'targeting rule'

$$\pi_t = -\frac{\alpha}{\lambda} y_t \tag{14}$$

This together with (2) delivers the implied values for y_t and π_t. Plugging these into (3) delivers a value for i_t. If $i_t > -r^*$, as initially conjectured, one has found a solution. Otherwise, set $i = -r^*$ and solve (2) and (3) for y_t and π_t. Performing this at each node delivers a new guess y_{k+1}, π_{k+1}, and i_{k+1}.

[33] See chapter 7 in Judd (1998) for details.

Step 3: Stop if $\max\{|y_k - y_{k+1}|_{\max}, |\pi_k - \pi_{k+1}|_{\max}, |i_k - i_{k+1}|_{\max}\} < \tau$ where $|\cdot|_{\max}$ denotes the maximum absolute norm and $\tau > 0$ the tolerance level. Otherwise continue with step 2.

In our application we set $N = 275$ and $M = 9$. Relatively more nodes are placed in areas of the state space where the policies display a higher degree of curvature, i.e., at negative values of g where the lower bound is reached. The support of the interpolation nodes is chosen to cover ± 4 unconditional standard deviations for each of the shocks. The tolerance level is $\tau = 1.49 * 10^{-8}$, i.e., the square root of machine precision. Our initial guess is given by the policy that is optimal in the absence of the zero lower bound.

A.2 Proof of proposition 1

Suppose $g \in I^0$, then $g' \in I^0$ where g' denotes the value of g in the subsequent period. Given the interest rate policy (12), equations (2) and (3) imply that $\pi = y = 0$ constitutes a perfect foresight equilibrium for all $g \in I^0$. Clearly, the interest rate policy (12) is optimal for all $g \in I^0$.

Now suppose $g \in I^1$. Since this implies that $g' \in I^0$, we can solve the problem by backward induction: $g' \in I^0$ implies that the private sector's expectations are given by $E\pi' = Ey' = 0$. It then follows from equations (2), (3), and (12) that

$$y = -g^c + g \tag{15}$$

$$\pi = -\lambda g^c + \lambda g \tag{16}$$

Note that output and inflation are continuous in the transition from I^1 to I^0 and linear in g for $g \in I^1$. One can iterate in this manner to obtain output and inflation for $I^2, I^3, \ldots$. Continuity and linearity of all equations involved thereby implies that output and inflation are continuous functions of g. Moreover, for the stated interest rate policy output and inflation in each interval I^j are linear in g and can be represented as

$$y = c_y^j + s_y^j g \tag{17}$$

$$\pi = c_\pi^j + s_\pi^j g \tag{18}$$

Equations (15) and (16) imply

$$\begin{pmatrix} s_y^1 \\ s_\pi^1 \end{pmatrix} = \begin{pmatrix} 1 \\ \lambda \end{pmatrix}.$$

Using equations (17) and (18) and the law of motion for g to construct expectations in the interval I^{j+1}, equations (2) and (3) and the interest rate policy imply

$$\begin{pmatrix} s_y^{j+1} \\ s_\pi^{j+1} \end{pmatrix} = \begin{pmatrix} s_y^1 \\ s_\pi^1 \end{pmatrix} + A \begin{pmatrix} s_y^j \\ s_\pi^j \end{pmatrix} \qquad where$$

$$A = \begin{pmatrix} \rho_g & \varphi\rho_g \\ \rho_g\lambda & \rho_g\left(\lambda\varphi + \beta\right) \end{pmatrix}$$

Iterating on this equation implies that

$$\begin{aligned} s^2 &= s^1 + As^1 \\ s^3 &= s^2 + A^2 s^1 \\ s^4 &= s^3 + A^3 s^1 \\ &\vdots \end{aligned}$$

where

$$s^j \equiv \begin{pmatrix} s_y^j \\ s_\pi^j \end{pmatrix}.$$

Since $s^1 > 0$ and all entries in A are positive, this shows that the slopes s^j are increasing in j. Since output and inflation are negative for $g \in I^1$ it follows from continuity and the values of s^j that they are negative for all $g \in I^j$ with $j > 1$. Therefore, zero nominal interest rates are optimal for $g \in I^j$ with $j > 1$, since positive nominal interest rates would generate even lower output levels and inflation rates.

A.3 Numerical algorithm (hybrid Phillips curve)

We define a grid of N interpolation nodes over the state space (u, g, π_{-1}). Associated with the policy functions $\pi_k(u, g, \pi_{-1})$ and $y_k(u, g, \pi_{-1})$ are the expectation functions

$$E_k\pi_{+1} = \int \pi_k(\rho_u u + \varepsilon_{u,+1}, \rho_g g + \varepsilon_{g,+1}, \pi) f(\varepsilon_{u,+1}, \varepsilon_{g,+1}) \mathrm{d}(\varepsilon_{u,+1}, \varepsilon_{g,+1}) \quad (19)$$

$$E_k y_{+1} = \int y_k(\rho_u u + \varepsilon_{u,+1}, \rho_g g + \varepsilon_{g,+1}, \pi) f(\varepsilon_{u,+1}, \varepsilon_{g,+1}) \mathrm{d}(\varepsilon_{u,+1}, \varepsilon_{g,+1}) \quad (20)$$

where $f(\cdot, \cdot)$ is the probability density function of $(\varepsilon_u, \varepsilon_g)$. The expectations (19) and (20) are evaluated at each interpolation node using an M node Gaussian-Hermite quadrature scheme.[34] Our numerical algorithm then performs the following steps:

Step 1: Choose N and M and assign the interpolation and quadrature nodes. Guess initial values for the policy functions y_0, π_0, and i_0 at the interpolation nodes.

Step 2: At each interpolation node compute the expectations (19) and (20) implied by the current guess y_k, π_k, and i_k. For given expectation functions, the Lagrangian of problem (1), (13), (3)-(8) can be written as a recursive saddle point problem

$$V_k(u, g, \pi_{-1}) = \max_{(y,\pi,i)} \min_{(m^1,m^2)} h_k(u, g, \pi_{-1}, y, \pi, i, m^1, m^2, E_k\pi_{+1}, E_k y_{+1}) + \beta E V_k(u_{+1}, g_{+1}, \pi) \quad (21)$$

$$s.t.:$$

$$u_{+1} = \rho_u u + \varepsilon_{u,+1}$$

$$g_{+1} = \rho_g g + \varepsilon_{g,+1}$$

where

$$h_k(\cdot) = -\pi^2 - \alpha y^2 + m^1[\pi - \frac{1}{1+\beta\gamma}(\beta E_k\pi_{+1} + \gamma\pi_{-1} + \lambda y + u)] + m^2 [y - E_k y_{+1} + \varphi (i - E_k\pi) - g]$$

[34] See chapter 7 in Judd (1998) for details.

and m^1 and m^2 are Lagrange multipliers. Using the collocation method one can numerically solve for the fixed point of (21) and the associated optimal policy functions y_{k+1}, π_{k+1}, i_{k+1}, m^1_{k+1}, m^2_{k+1}. Details of this procedure are described, e.g., in appendix A.2 in our companion paper, see Adam and Billi (2004).

Step 3: Stop if the maximum of $|y_k - y_{k+1}|_{\max}$, $|\pi_k - \pi_{k+1}|_{\max}$, $|i_k - i_{k+1}|_{\max}$, $\left|m^1_k - m^1_{k+1}\right|_{\max}$ and $\left|m^2_k - m^2_{k+1}\right|_{\max}$ is smaller than τ where $|\cdot|_{\max}$ denotes the maximum absolute norm of these functions evaluated at the interpolation nodes and $\tau > 0$ the tolerance level. Otherwise continue with step 2.

In our application we set $N = 1375$ and $M = 9$. Relatively more nodes are placed in areas of the state space where the policies display a higher degree of curvature, i.e., at negative values of g where the lower bound is reached. The support of the interpolation nodes is chosen to cover ± 4 unconditional standard deviations for each of the shocks, and to insure that all values of π_t lie inside the state space when using the solution to simulate one million model periods. Since this can only be verified after the solution is obtained, some experimentation is necessary. The tolerance level is $\tau = 1.49 * 10^{-8}$, i.e., the square root of machine precision. Our initial guess is given by the policy that is optimal in the absence of the zero lower bound.

To check whether second order conditions hold, we numerically verify if the right-hand side of (21) is a saddle point, i.e., a maximum with respect to (y, π, i) and a minimum with respect to (m^1, m^2), respectively, at the conjectured optimal policy. As is well known, e.g., chapter 14.3 in Silberberg (1990), the saddle point property is a sufficient condition for having found a constrained optimum. Technically, we verify the saddle point property by considering a large number of simultaneous deviations from the conjectured optimum for (y, π, i) and (m^1, m^2), respectively, at a large number of points in the state space. Due to the recursive structure of the problem it thereby suffices to verify the saddle point property for one-period deviations only.

References

Adam, Klaus and Roberto M. Billi, "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Center for Financial Studies, Frankfurt, Germany, Working Paper 13/2004*, 2004.

Albanesi, Stefania, V. V. Chari, and Larry Christiano, "Expectation Traps and Monetary Policy," *NBER Working Paper No. 8912*, 2002.

Auerbach, Alan J. and Marurice Obstfeld, "The Case for Open-Market Purchases in a Liquidity Trap," *NBER Working Paper No. 9814*, 2003.

Barro, Robert and David B. Gordon, "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 1983, *91*, 589–610.

Bernanke, Ben S., "Deflation: Making Sure 'It' Doesn't Happen Here," 2002. Remarks before the National Economists Club, Washington. D.C., November 21, 2002, available at www.federalreserve.gov.

Calvo, Guillermo A., "Staggered Contracts in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12*, 383–398.

Christiano, Lawrence J. and Jonas Fisher, "Algorithms for Solving Dynamic Models with Occasionally Binding Constraints," *Journal of Economic Dynamic and Control*, 2000, *24(8)*, 1179–1232.

Clarida, Richard, Jordi Galí, and Mark Gertler, "The Science of Monetary Policy: Evidence and Some Theory," *Journal of Economic Literature*, 1999, *37*, 1661–1707.

___ , ___ , and ___ , "Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach," *American Economic Review Papers and Proceedings*, 2001, *91*, 248–252.

Coenen, Günter and Volker Wieland, "The Zero-Interest-Rate Bound and the Role of the Exchange Rate for Monetary Policy in Japan," *Journal of Monetary Economics*, 2003, *50*, 1071–1101.

——, **Athanasios Orphanides, and Volker Wieland**, "Price Stability and Monetary Policy Effectiveness When Nominal Interest Rates are Bounded at Zero," *Advances in Macroeconomics*, 2004, *4(1) article 1.*

Eggertsson, Gauti, "How to Fight Deflation in a Liquidity Trap: Committing to Being Irresponsible," *IMF Working Paper 03/64*, 2003.

—— **and Michael Woodford**, "Optimal Monetary Policy in a Liquidity Trap," *NBER Working Paper No. 9968*, 2003.

Galí, Jordi and Mark Gertler, "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 1999, *44*, 195–222.

Jeanne, Olivier and Lars E.O. Svensson, "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank," *Princeton University mimeo*, 2004.

Judd, Kenneth L., *Numerical Methods in Economics*, Cambridge: MIT Press, 1998.

Jung, Taehun, Yuki Teranishi, and Tsutomu Watanabe, "Zero Bound on Nominal Interest Rates and Optimal Monetary Policy," 2001. Kyoto Institute of Economic Research Working Paper No. 525.

King, Robert G. and Alexander L. Wolman, "Monetary Discretion, Pricing Complementarity and Dynamic Multiple Equilibria," *NBER Working Paper No. 9929*, 2003.

Krugman, Paul R., "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 1998, *49(2)*, 137–205.

Kydland, Finn E. and Edward C. Prescott, "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 1977, *85*, 473–492.

Nakov, Anton, "The Zero Bound on Nominal Interest Rates and Optimal Monetary Policy under Discretion," *Universitat Pompeu Fabra mimeo*, 2004.

Persson, Thorsten and Guido Tabellini (Eds), *Monetary and Fiscal Policy, Volume 1: Credibility*, Cambridge, USA: MIT Press, 1994.

Rotemberg, Julio J., "The New Keynesian Microfoundations," *NBER Macroeconomics Annual*, 1987, *2*, 69–104.

____ **and Michael Woodford**, "An Optimization-Based Econometric Model for the Evaluation of Monetary Policy," *NBER Macroeconomics Annual*, 1998, *12*, 297–346.

Schmitt-Grohé, Stephanie and Martín Uribe, "Optimal Simple and Implementable Monetary and Fiscal Rules," *Duke University Mimeo*, 2003.

Silberberg, Eugene, *The Structure of Economics: A Mathematical Analysis*, New York: McGraw-Hill, 1990.

Söderlind, Paul, "Solution and Estimation of RE Macromodels with Optimal Policy," *European Economic Review*, 1999, *43*, 813–823.

Svensson, Lars E. O., "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others," *Journal of Economic Perspectives*, 2003, *17(4)*, 145–166.

Trichet, Jean-Claude, "The ECB's Monetary Policy Strategy After the Evaluation and Clarification of May 2003," 2003. Speech delivered November 20, 2003 at the Center for Financial Studies' key event, available at www.ecb.int.

Vestin, David, "Price-Level Targeting versus Inflation Targeting in a Forward-Looking Model," 2002. working paper.

Wolman, Alexander L., "Real Implications of the Zero Bound on Nominal Interest Rates," *forthcoming Journal of Money Credit and Banking*, 2003.

Woodford, Michael, *Interest and Prices*, Princeton: Princeton University Press, 2003.

Parameter	**Economic interpretation**	**Assigned value**
β	quarterly discount factor	$\left(1+\frac{3.5\%}{4}\right)^{-1} \approx 0.9913$
α	weight on output in the loss function	$\frac{0.048}{4^2} = 0.003$
λ	slope of the AS curve	0.024
φ	real rate elasticity of output	6.25
ρ_u	AR-coefficient mark-up shocks	0
ρ_g	AR-coefficient real rate shocks	0.8
σ_u	s.d. mark-up shock innovations (quarterly %)	0.154
σ_g	s.d. real rate shock innovations (quarterly %)	1.524

Table 1: Parameter values (baseline calibration)

	Discretion	Commitment	Welfare loss due to discretion
Without zero bound	-2.297	-1.770	29.8%
With zero bound	-2.656	-1.786	48.7%
Welfare loss due to lower bound	15.6%	0.9%	

Table 2: Welfare losses (baseline calibration)

Parameter	**Economic interpretation**	**Assigned value**
β	quarterly discount factor	$\left(1+\frac{3.5\%}{4}\right)^{-1} \approx 0.9913$
α	weight on output in the loss function	0.007
λ	slope of the AS curve	0.057
φ	real rate elasticity of output	1
ρ_u	AR-coefficient mark-up shocks	0.36
ρ_g	AR-coefficient real rate shocks	0.8
σ_u	s.d. mark-up shock innovations (quarterly %)	0.171
σ_g	s.d. real rate shock innovations (quarterly %)	0.294

Table 3: Parameter values (RBC calibration)

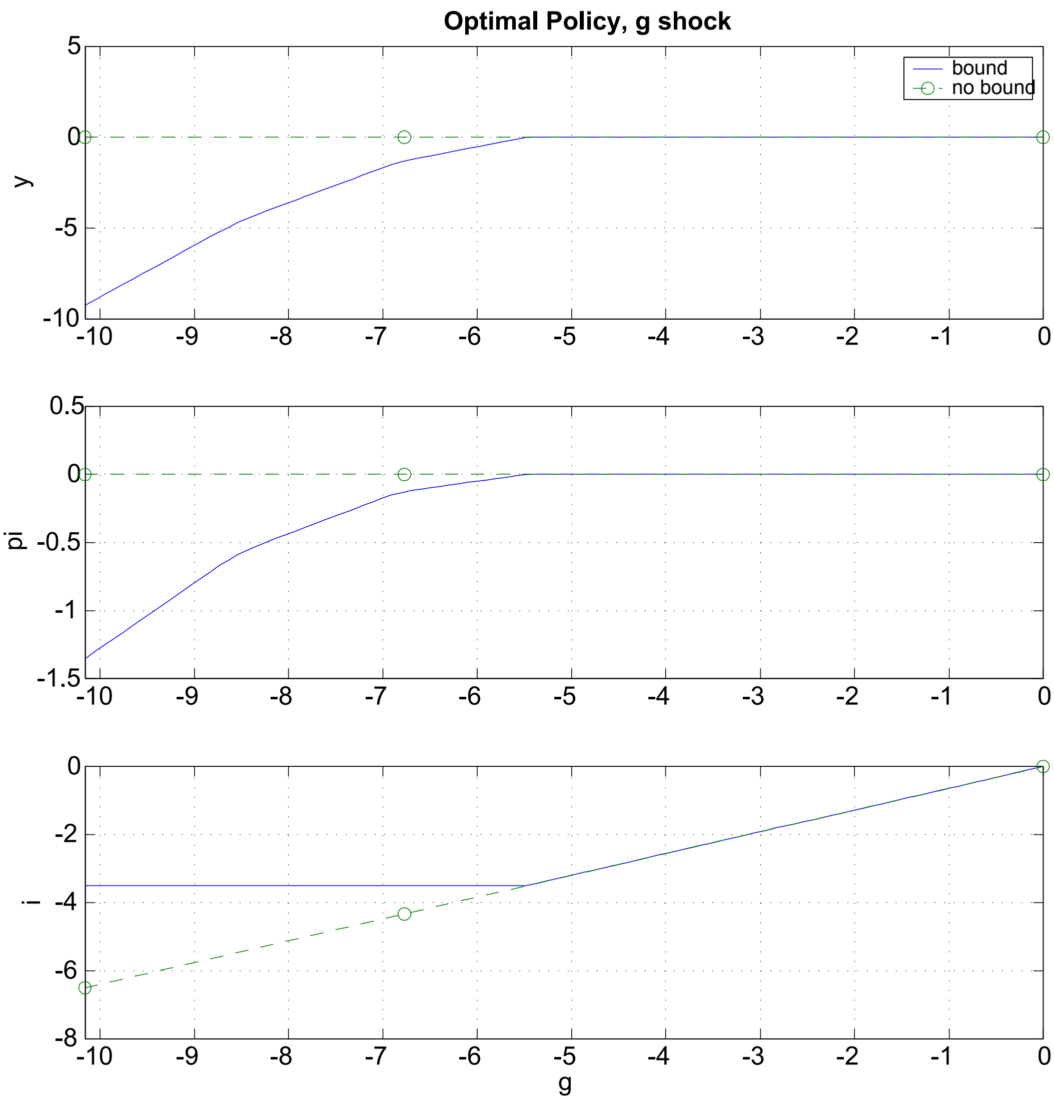

Figure 1: Perfect foresight equilibrium

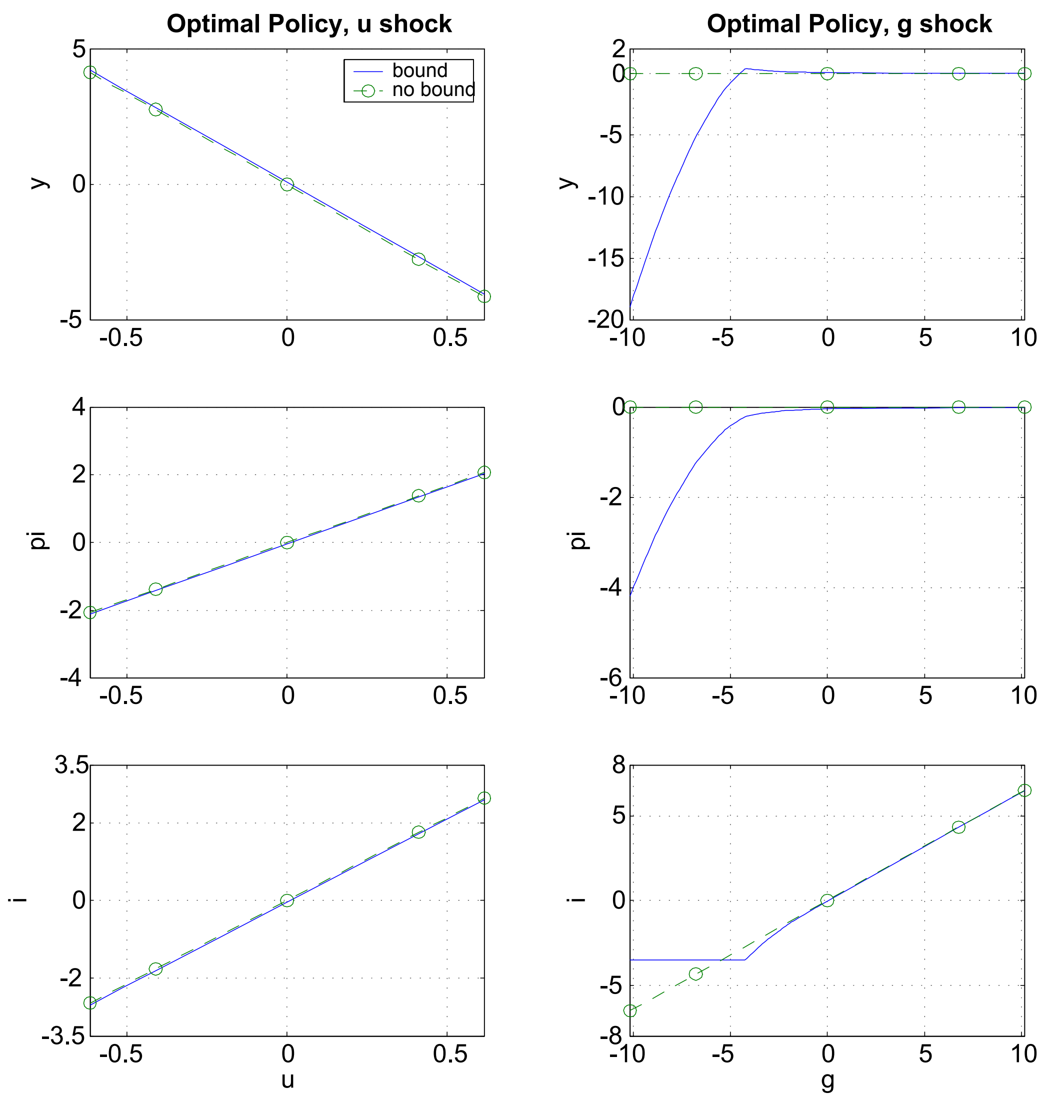

Figure 2: Optimal policy response

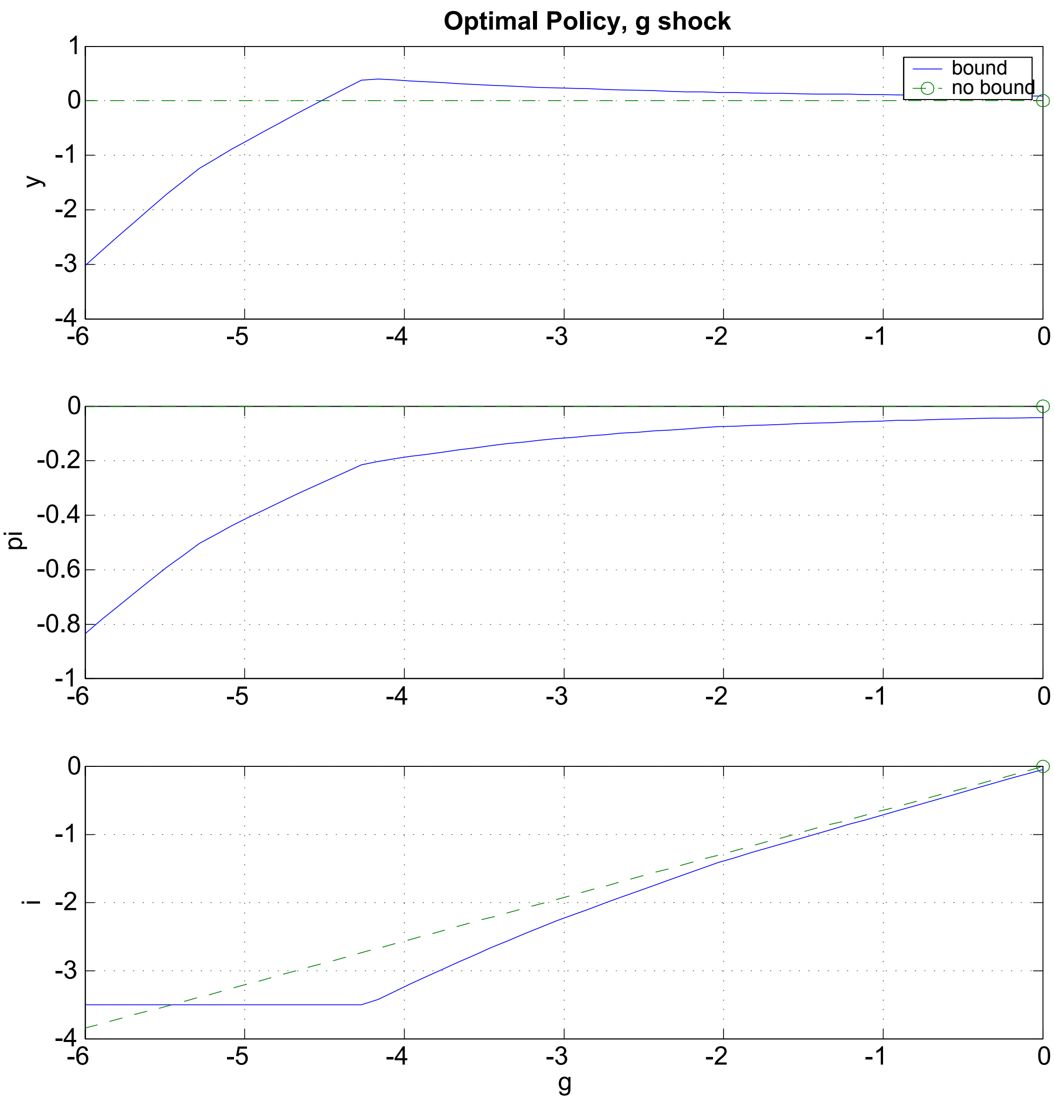

Figure 3: Optimal policy response to negative real rate shocks

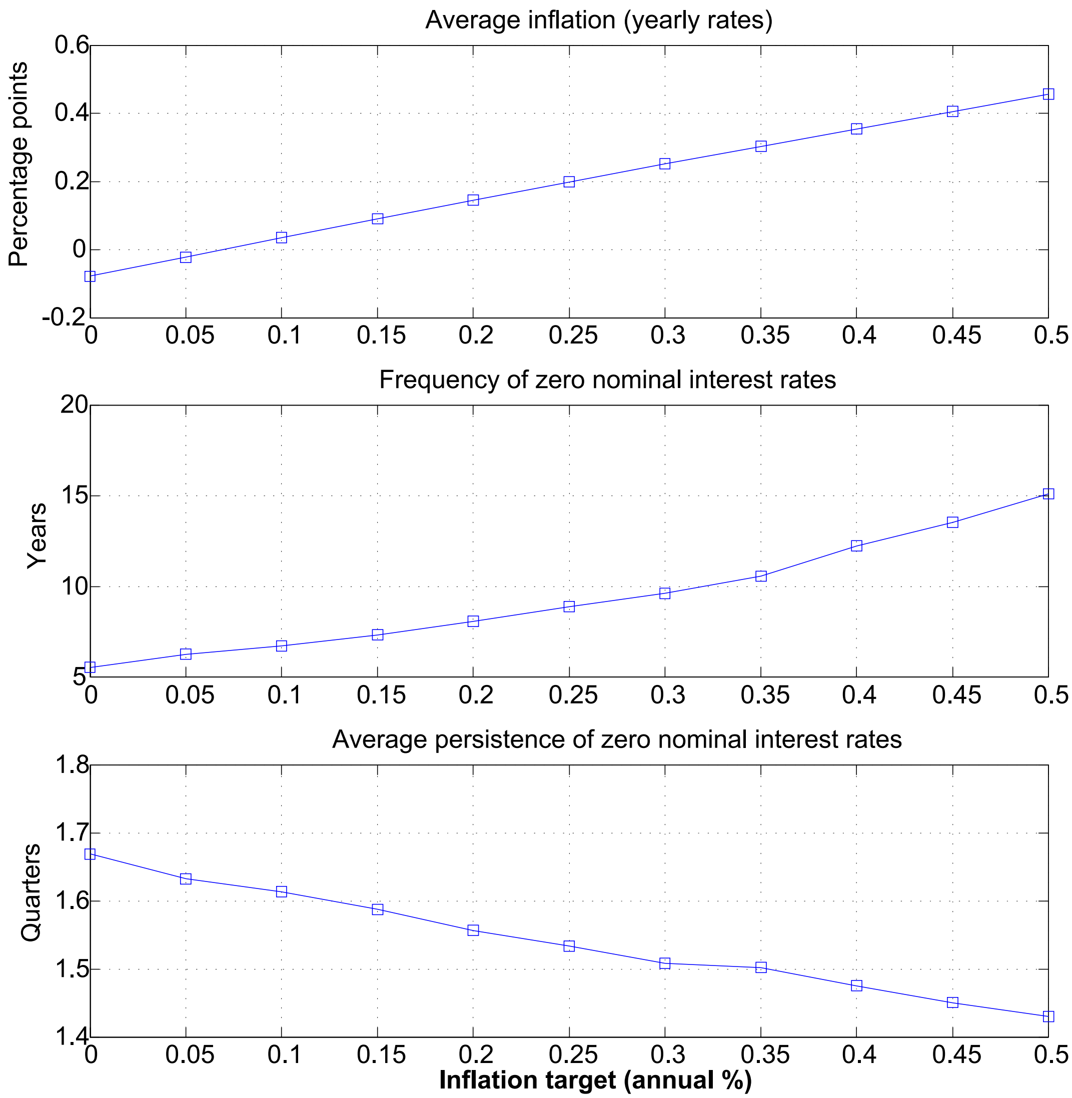

Figure 4: Positive inflation targets

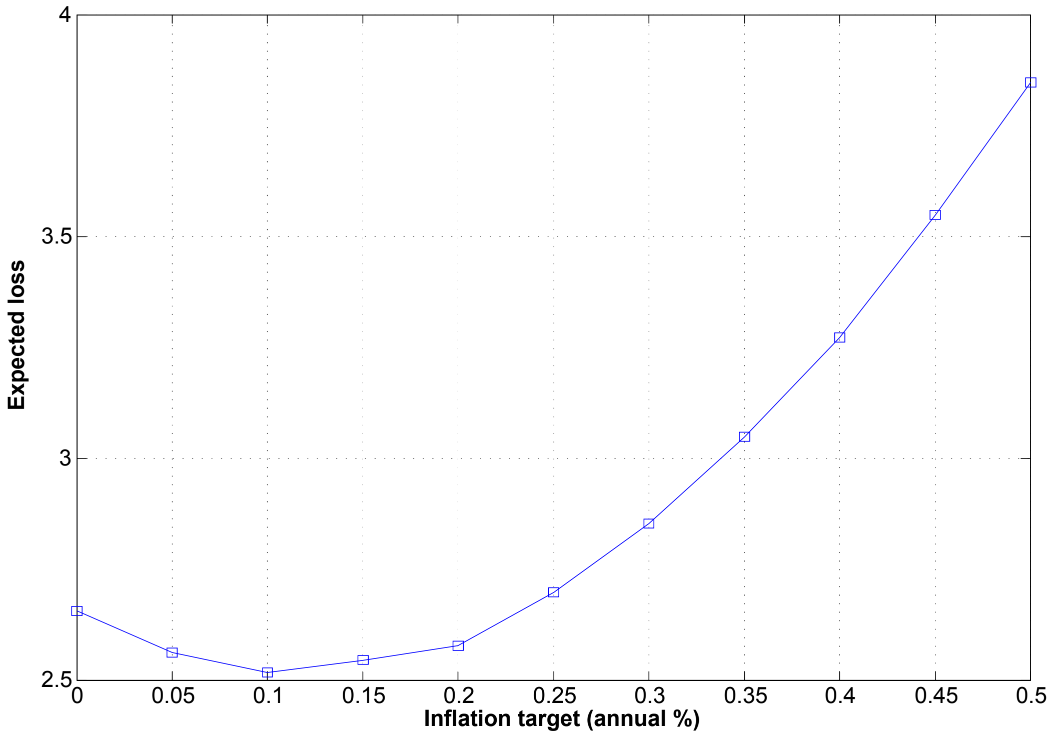

Figure 5: Welfare effects of positive inflation targets