DISCUSSION PAPER SERIES

No. 4575

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PUBLIC POLICY

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ISSN 0265-8003

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Discussion Paper No. 4575
August 2004

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CEPR Discussion Paper No. 4575

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ABSTRACT

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In this Paper, we study the role of subsidies to fertility in ensuring the political viability of unfunded social security (SS). In our model, agents are heterogeneous in age and income. Young generations confront promises made previously by older generations, and in turn choose current levels of fertility subsidies, and future levels of social security benefits. We find that subsidies to the costs of children expand the set of equilibria, making social security viable where it would otherwise have to be abandoned. Moreover, the model successfully captures the observed evolution of social security and family support systems during the demographic transition. Our results indicate that the seemingly explosive evolution of SS taxes will be curbed once the underlying demographic transition is completed, after which the SS system will converge to a steady state lower than simple extrapolation of current trends would imply, and fertility will rebound with the aid of higher subsidy levels.

JEL Classification: E62, H20, H30, H55, J13 and J14
Keywords: endogenous fertility, OLG models, political economy, redistribution and social security

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*We wish to thank Gerhard Glomm, Steve Dowrick and Abinay Muthoo for useful suggestions to improve the manuscript. All remaining errors are ours.

Submitted 27 July 2004

1 Introduction

The ageing of the generation known as the baby boomers has stimulated much recent interest in the viability of unfunded social security (SS) schemes, which are a feature of most developed countries. As a consequence of the higher dependency ratios brought about by the demographic transition in the postwar era, the tax burden necessary to fund the system's liabilities is expected to grow further, raising concerns that social security will impose large welfare losses to current workers and future participants in the system.

Among the policy options, the most drastic is to overhaul the social security system and convert it to a fully funded system. Social security reforms of this type have been undertaken to different degrees in Chile, Argentina, and Australia, among others. Another possibility is to overcome the generational imbalance by increasing fertility rates. Subsidies destined to increase fertility rates are in place in a number of developed countries. Beyond the existence of free schooling, these subsidies take a number of forms: free or subsidized pre-schooling, a lump sum payment for each child, and paid maternity leave, among others. There is strong evidence that such policies affect the fertility decisions of couples (see Hotz et al. [1997] and references therein), although there is some disagreement on the quantitative importance of these effects.

In the two decades from 1970 to 1989, OECD countries experienced a 10% increase in the costs of children, as measured by the gender wage gap. This increase was accompanied by a 27% decrease in projected lifetime fertility rates over the same period. The size of pension benefits expanded from representing 4.4% of GDP in 1970, to 7.7% in 1989. Fertility subsidy rates remained roughly constant over this period, representing 2.4% of annual earnings for the first child, 2.9% for the second child, and 3.8% for the third, but increased by 43%, 39% and 12% respectively over the next ten years. The relative size of the family benefits system, which includes subsidies other than those mentioned above, also grew moderately from 1970 to 1989 [1]. Although the size of the social security system increased, the internal rate of return on social security contributions did not, at least for the US. Caldwell et al. [1998] reports that such rates decreased for all income groups from cohorts 1945 on, were consistently higher for poorer households, and became negative for the highest income group for all cohorts born after 1970.

In this paper, we examine the joint evolution of unfunded social security and fertility policies as being generated through a political economy mechanism, and we use this framework to address the questions of the viability of unfunded social security during and after the demographic transition, and the role of fertility policies in ensuring it.

For our purposes, a number of recent contributions on the political economy of SS (see Galasso and Profeta [2002] for a survey) are especially relevant. First, Casamatta et al. [2000] and Tabellini [2000] study SS as a device that not only redistributes income from young to old, but also from wealthy to poor households, and therefore SS arrangements can be sustained as a

[1] See appendix A.1 for a description of the dataset used

political equilibrium without resorting to intergenerational (e.g. dynamic efficiency) considerations. Then, Cooley and Soares [1999] and Galasso [1999] study SS as an institution with inherited rules that are costly to change, but their approaches are quite different from the one taken here. The paper that is closer to ours is Boldrin and Rustichini [2000]. In that model, agents are confronted with an existing promise of paying a SS tax to old agents, and may either abandon the SS system altogether, or pay the SS tax and vote for a level of SS benefits in the next period. The analysis in Boldrin and Rustichini [2000] gives an explicit dynamic dimension to the problem of sustaining a SS system, and provides a clear interpretation of it as one of unfunded SS liabilities, in line with most of the public policy debate.

A small number of studies examine the joint determination of SS along with another dimension of the welfare system, as we do. In particular, Conde-Ruiz and Galasso [2003] separate the redistributive aspect of SS into a within and a between cohort components, and consider the circumstances under which both aspects arise as an equilibrium. Boldrin and Montes [2002] show that, when human capital investments are limited by borrowing constraints, public education *plus* old age SS are necessary to implement the first best allocation. We are aware of no paper that studies the political economy of fertility policies.

In this paper, we motivate the existence of fertility subsidies as a voting equilibrium that arises in the presence of an unfunded social security system. Social Security is valued as a device that redistributes wealth from tomorrow's wealthy young to today's poor young, and a positive level of fertility subsidies is chosen by the median voter as they will sustain higher levels of SS benefits in the future. When voting on fertility subsidies, young households take into account the distortionary cost of these subsidies. They also take into account the negative effect that higher fertility will have on their political influence when they are old.

Our analysis contributes to the literature in three ways. First, we extend the dynamic framework of Boldrin and Rustichini [2000] by allowing for heterogeneity in individual productivities, so that SS emerges in equilibrium as a redistributive institution. Second, we clarify the role of beliefs over the willingness by future generations to pay SS taxes in determining the political equilibrium. Finally, the main contribution of this paper is to extend the literature on the political economy of SS systems to incorporate the endogenous determination of fertility policies.

The main findings of this paper can be summarized as follows. The inclusion of fertility subsidies as a policy tool expands the set of equilibria where SS is valued, making it viable where it would otherwise have to be abandoned. Moreover, the existence of fertility subsidies is a necessary condition for the existence of a unique, positive and globally stable steady state level of SS taxes. We find that strategic setting of fertility subsidies to limit the political influence of the newborn generation in the future decreases the levels of fertility subsidies, but shifts up the dynamic path of SS taxes. Finally, our model successfully reproduces the dynamics of fertility, social security, and fertility subsidies during the demographic transition that is described above. After the underlying change in the costs of fertility that drive the demographic transition is over, our model suggests that the rate of growth of the SS system will be curbed, converging to a steady state much lower than simple extrapolation would indicate, and fertility will rise, driven by an increase in subsidy levels

This paper has four other sections. In the next section we present the framework with the policy variables set exogenously. In section 3 we present the political economy environment, and define the equilibrium in which we are interested. Then section 4 derives the results using two examples differentiated by the assumptions on the distribution of labor productivities. Section 5 concludes.

2 The competitive equilibrium

We study an overlapping generations economy where households live for two periods. Households have preferences defined over their consumption when young c^y, the number of children to be born at the end of the first period n, and their consumption when old c^o. Labor supply is inelastic, with each household working one unit of time in period one and then retiring.

Households are heterogeneous in their labor productivities (α), so even though everyone supplies the same amount of labor, household i's labor income is indexed by α_i. The distribution of productivities over young households is summarized by the CDF $G(\alpha)$, with mean θ and support Θ. Since we are not concerned with the transmission of earnings abilities across generations, we assume that young households get a random draw from this distribution.

There are two policy instruments, SS taxes τ^{ss} and fertility subsidies τ^{f}. The SS system is unfunded, or pay-as-you-go, so the young households pay a proportional tax on labor income that finances the unique level of SS benefits received by old households at every period. Fertility subsidies (taxes) are designed to reduce (increase) the cost of having children bn, where b is the cost per child, and are financed by a lump sum tax (rebate) T. Note that n is normalized so that the unit of measurement of the population is the young (two person) household, so $n = 1$ implies 2 children per couple. Household i solves the problem

$$\begin{aligned} \max_{\{c^y, c^o, n\}} \quad & c_t^y + \beta c_{t+1}^o \quad + \gamma \ln n_t \\ s.t. \quad & (1 - \tau_t^{ss}) w_t \alpha_i = s_t + c_t^y + b(1 - \tau_t^f) n_t + T_t \\ & c_{t+1}^o = s_t(1 + r_{t+1}) + ss_{t+1}. \end{aligned} \tag{1}$$

where β is a discount factor, w_t is the wage rate per efficiency unit, s_t is savings, r_{t+1} is the interest rate paid next period and ss_{t+1} represents SS benefits at time $t + 1$. The use of quasilinear preferences will ensure that, for individual wage incomes $w\alpha_i$ above a minimum level, the choice of n is independent of wealth.

Firms hire capital and labor and produce using the Cobb Douglas technology $F(K_t, L_t) = AK_t^{\rho} L_t^{1-\rho}$, with $L_t = N_t\theta$ where N_t is the population of young households at time t. We can write the production function in efficiency units of labor as $f(k_t) = Ak_t^\rho$. Optimality conditions for the firm, imply that interest rates and wage rates satisfy:

$$\begin{aligned} r_t &= f'(k_t) - \delta && (2) \\ w_t &= f(k_t) - f'(k_t)k_t. && (3) \end{aligned}$$

where δ is the rate of depreciation of capital. The government raises taxes and provides subsidies and SS benefits under a restrictive rule of budget balance: SS benefits are financed with SS contributions, and fertility subsidies are financed through a lump sum tax. These conditions are formalized below.

$$\begin{aligned} N_{t+1}\tau_{t+1}^{ss}\theta w_{t+1} &= N_t ss_{t+1} && (4) \\ bn_t\tau_t^f &= T_t && (5) \end{aligned}$$

The model is closed by specifying laws of motion for capital and population. Capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{6}$$

where I_t is aggregate investment at t. Working population (N_t) evolves according to

$$N_{t+1} = n_t N_t. \tag{7}$$

We are now ready to define an equilibrium.

Competitive equilibrium: A competitive equilibrium is a collection of prices $\{w_t, r_t\}_{t=0}^{\infty}$ and allocations $\{c_{i,t}^y, c_{i,t+1}^o, n_t\}_{t=0}^{\infty}$, all i, and $c_{i,0}^o$ for the old at time 0 which, together with a sequence of tax and benefit levels $\{\tau_t^f, \tau_{t+1}^{ss}, T_t, ss_t\}_{t=0}^{\infty}$, and a distribution of types over households $G(\alpha_i)$, satisfy:

1. Given prices and taxes/benefits, the allocation $\{c_{i,t}^y, c_{i,t+1}^o, n_{i,t}\}$ solves the household problem for all generations, all i.

2. Given prices, firms maximize profits, which implies conditions (2) and (3) above.

3. The government budget is balanced according to (4) and (5).

4. The capital and labor markets clear, so that

$$N_t \int s_t dG = k_{t+1} N_{t+1},$$
$$L_t = N_t \int \alpha dG.$$

We now describe the properties of this equilibrium, starting with the household's problem given by (1). Decision rules for the household depend on next period discounted interest rate $\beta(1 + r_{t+1})$ and disposable income $((1 - \tau_t^{ss}) w_t \alpha_i + ss_{t+1}/(1 + r_{t+1}) - T_t)$. When $\beta(1 + r_{t+1}) = 1$, all households choose the same number of children, and the remaining income is consumed, with the consumption path c^y/c^o being indeterminate [2]:

$$n_t = \frac{\gamma}{b(1 - \tau_t^f)}, \tag{8}$$
$$c_t^y + \beta c_{t+1}^o = (1 - \tau_t^{ss}) w_t \alpha_i + ss_{t+1}/(1 + r_{t+1}) - T_t - \gamma. \tag{9}$$

[2] If income is 'too low', total consumption is zero, and all resources are used in raising kids. Since this possibility is not of interest, we restrict the parameters of the model accordingly. A formal solution of the household's problem is presented in appendix A.2.

If $\beta(1+r_{t+1}) < 1$, fertility n_t is also represented by (8), but all the remaining income will be consumed when young. When $\beta(1+r_{t+1}) > 1$, fertility will decrease by a factor of $\frac{1}{\beta(1+r_{t+1})}$ and the remaining income will be saved and consumed when old. Due to this unwelcome feature of the model off the steady state, we will focus on economies that start with a steady state level of capital. The steady state level of capital k^s per efficiency units of labor is

$$k^s = (\frac{1}{\rho A}(\frac{1}{\beta} - 1 + \delta))^{\frac{1}{\rho-1}}$$

and is implied by $\beta(1+r(k)) = 1$. Note that k^s does not depend on taxes [3].

Since tax rates affect consumption and fertility choices, households can be seen as having preferences over the levels of these taxes. To obtain a derived preference ordering over tax rates for young households we solve for the value function in problem (1). For young household i at k^s this function is

$$V_i^Y(\tau_t^{ss}, \tau_{t+1}^{ss}; \tau_t^f) = (1-\tau_t^{ss})w\alpha_i + \frac{\theta\beta w\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \frac{\gamma}{1-\tau_t^f} + \gamma \ln \frac{\gamma}{(1-\tau_t^f)b}. \quad (11)$$

Equation (11) shows that household i's utility is increasing in labor income $w\alpha_i$, aggregate labor productivity θw, and next period SS tax rate τ_{t+1}^{ss}, and decreasing in the current period's SS tax τ_t^{ss}.

[3] If the capital stock is larger, young households will increase current consumption c^y until next period's capital satisfies $\beta(1+r(k)) = 1$. If the capital stock is lower, young households will postpone consumption until either tomorrow's capital stock equals k^s, or $c^y = 0$. Capital then displays so called bang-bang dynamics, it follows

$$k_{t+1} = \min\{k^s, (1-\tau_t^{ss})(1-\rho)b(1-\tau_t^f)\beta(1+r(k_t))f(k_t)/\gamma - 1/\theta\}. \quad (10)$$

So k^s is attained in a finite number of periods. In particular, whenever $k_{t+1} < k^s$ we have $c^y = 0$

Note that, since the consumption path c^y/c^o is indeterminate, the value function for old households is also indeterminate. For our purposes, it will be useful to take the level of individual saving as a parameter ($\overline{s}$), and consider the function

$$V^o(\tau_t^{ss}) = \overline{s}_{t-1}(1+r) + \frac{\theta\beta w\tau_t^{ss}\gamma}{(1-\tau_{t-1}^f)b}. \tag{12}$$

At time t, old households prefer higher levels of τ_t^{ss}. However, since k^s -and therefore prices- do not depend on $\{\tau_{t+1}^{ss}, \tau_t^f\}$, old households are indifferent among different levels of these taxes [4] . In the next section, we present a political mechanism that is used to determine the levels of tax rates at every period.

3 Voting over taxes

3.1 The game

We now extend the framework to allow for the determination of whether the SS system is abandoned or continued, and in the latter case what levels of fertility subsidies and SS taxes are implemented. In this majority voting model, such decisions are set to maximize the utility of the median voter. It is helpful to think of this arrangement as a game that takes the following form: Successive cohorts of players play the game twice, when young and when old. Since players are indexed by their date of birth and productivity

[4]When τ_t^{ss} is above a critical level, aggregate savings can be insufficient to ensure $k_{t+1} = k^s$. In what follows we focus on equilibria with SS where τ_t^{ss} is restricted to lie below this critical value.

level, we can represent the set of players as

$$P = \{(i, t) \in \mathbb{R}^2 : i \in \Theta, t \in \mathbb{N}\}$$

At each period t, young households are confronted with a SS tax τ_t^{ss}, promised to the current old at $t-1$ and earmarked to finance a unique level of retirement benefits. Young and old households then vote over three policy dimensions.

First, households vote on λ_t, expressing their preference for honoring the promise τ_t^{ss} or defaulting. If a majority of voters chooses to default, the young lose their right to obtain SS payments next period. If on the other hand voters choose not to default, households proceed to vote for their preferred levels of future SS tax τ_{t+1}^{ss} and current level of fertility subsidy τ_t^f.

As is apparent from (12), old households have preferences for maintaining the SS system. However, because the old die at the end of the period, they are indifferent as to the levels of SS taxes or population levels in the future. In the same vein, should voters have initially chosen to default, the SS system is abandoned and young households are indifferent with respect to the promised levels of future SS taxes. We assume that, when voters are indifferent over policy choices, they abstain from voting. Abstention then needs to be included as a possibility in the set of actions a_t.

$$a_t = \{\lambda_t, \tau_{t+1}^{ss}, \tau_t^f\}$$

where $\lambda_t \in \{1, 0\}$ is a vote for keeping (1) or rejecting (0) SS, $\tau_{t+1}^{ss} \in$

$[0,1] \cup \{abst\}$ and $\tau_t^f \in (-\infty, 1) \cup \{abst\}$. The timing of these decisions is illustrated in figure 1.

Consider now the problem of voting over $\{\tau_{t+1}^{ss}, \tau_t^f\}$ once a majority of young and old households has voted for continuation of the SS system. As mentioned above, the old abstain from voting over tax levels. We may think of the young household's problem as first finding the optimal τ_t^f as a function of τ_{t+1}^{ss}, and then choosing the highest τ_{t+1}^{ss} that will not induce tomorrow's median voter [5] to reject SS. Note that the optimal value of τ_{t+1}^{ss} will depend on the level of SS benefits that the young born at $t+1$ (and in particular the median voter) may in turn expect for $t+2$. This implies that today's young voters need to have a belief about the level of SS benefits that the generation born tomorrow will achieve.

Definition: A belief $\widehat{\tau}_{t+2}^{ss}$ is the level of SS taxes that the young born at t predict the young born at $t+1$ will vote for as the SS tax for period $t+2$, conditional on not abandoning SS.

Households born at t may be informed of the entire sequence of beliefs from $t+2$ on, or only of a subset of it. At a minimum, they must know $\widehat{\tau}_{t+2}^{ss}$, or what they believe the next generation will achieve as the social security tax rate for $t+2$. We begin by using this latter assumption.

Assumption 1 *Households born at t know only $\widehat{\tau}_{t+2}^{ss}$.*

It is important to emphasize that rationality and full information about the game are not sufficient to pin down beliefs, only to restrict them, as will

[5] "Median voter" always refers to the vote on λ

be seen below. In that sense, beliefs play an independent role in determining the equilibrium outcomes, giving rise to an indeterminacy in the equilibrium path. It is sensible to restrict beliefs to sequences of values that can be optimal choices by households:

Definition: A belief $\widehat{\tau}^{ss}_{t+2}$ *is rational if, given all* $\widehat{\tau}^{ss}_{t+j+2}$ *that are known at* t*, it can be part of an optimal choice by the young at* t*.*

If only $\widehat{\tau}^{ss}_{t+2}$ is known at t, rationality eliminates only the values such that SS would be abandoned by the young at $t+2$ for all possible beliefs $\widehat{\tau}^{ss}_{t+2}$. In the next section we substitute assumption 1 for the assumption that the sequence of beliefs is common knowledge, and show that it removes all indeterminacy, up to an initial condition.

The payoff function for old households maps the choice of λ by the median voter to the value function $V^o(\tau^{ss}_t)$ in (12), with the convention that $\tau^{ss}_t = 0$ whenever $\lambda^m_{t+1} = 0$. Deriving the payoff function for young households is a less straightforward problem, which we now discuss.

Young households first cast a vote on rejecting versus keeping the SS system. If a majority of voters (young and old) choose to abandon the SS system ($\lambda = 0$), the payoff for all young households is $V^y_i(0, 0; 0)$. If on the other hand a majority of voters choose to keep SS, young households pay τ^{ss}_t of their wage and vote on $\{\tau^f_t, \tau^{ss}_{t+1}\}$. The choice of tax rates is the solution to the following problem:

$$\max_{\{\tau^f_t, \tau^{ss}_{t+1}\}} V^y_i(\tau^f_t, \tau^{ss}_{t+1}; \tau^{ss}_t) \tag{13}$$

$$s.t. \quad V^m_{t+1}(\tau^f_{t+1}, \widehat{\tau}^{ss}_{t+2}; \tau^f_t, \tau^{ss}_{t+1}) \geq V^m_{t+1}(0, 0; \tau^f_t, 0) \quad if \ \tau^f_t > 1 - \gamma/b \tag{14}$$

$$\tau_{t+1} \leq 1 \qquad \qquad \textit{otherwise} \qquad (15)$$

where m stands for median voter when voting on λ_{t+1}. The objective function (13) is the indirect utility function (11), and the constraint states that the tax rates chosen must be incentive compatible for tomorrow's voters, who might otherwise choose to abandon the SS system. The discontinuity in the constraint reflects an interesting, if awkward, possibility: If the fertility rate is less than or equal to one, something that can always be achieved by setting $\tau_t^f \leq 1 - \gamma/b$, the current young form a larger constituency at $t+1$ than their offspring (see equations 7 and 8). Young households can therefore set tomorrow's SS taxes at any rate up to one, and be assured that it will not be rejected by tomorrow's voters, of whom they will be a majority.

The alternative is to choose fertility subsidies so that fertility is higher than one. In this case $\{\tau_t^f, \tau_{t+1}^{ss}\}$ are chosen so that tomorrow's median voter -who will now be a young household- will (weakly) prefer to keep rather than abandon the SS system. We have stressed in constraint (14) both that the value functions refer to tomorrow's $(t+1)$ median voter, and that who this median voter is depends on current fertility subsidies (τ_t^f), through its impact on the relative size of tomorrow's young versus old constituencies. Indeed, if $n > 1$, the median voter at $t+1$ is such that a fraction $\frac{n_t - 1}{2n_t}$ of the young vote along with the old, this fraction being a function of τ_t^f. Note that, since τ_{t+2}^{ss} is not known by the young at t, it is replaced by their belief $\widehat{\tau}_{t+2}^{ss}$.

The solutions to this problem are policy functions of the form

$$\tau_t^f = \begin{cases} \tau^f(\widehat{\tau}_{t+2}^{ss}) & \text{if } \widehat{\tau}_{t+2}^{ss} > \tau_0 \\ 1 - \gamma/b & \text{otherwise} \end{cases} \tag{16}$$

$$\tau_{t+1}^{ss} = \begin{cases} \tau^{ss}(\widehat{\tau}_{t+2}^{ss}) & \text{if } \widehat{\tau}_{t+2}^{ss} > \tau_0 \\ 1 & \text{otherwise} \end{cases} \tag{17}$$

for τ_0 a constant. The value of keeping the SS system $J^i(\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss})$ can be derived from these policy functions and the objective function 13.

The next problem is that of voting on the continuation of the SS system. Young households will vote for continuing the SS system if the payoff from doing so $J^i(\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss})$ is larger than that of abandoning the system $V_i(0, 0; 0)$. More formally, households choose λ by solving the problem

$$\max_{\lambda \in \{0,1\}} J^i(\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss})\lambda + V_i(0, 0; 0)(1 - \lambda) \tag{18}$$

The policy function for this problem has the form

$$\lambda_t^i = \begin{cases} 1 & \text{if } g(\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss}, \alpha_i) \leq 0 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

where the function g is defined by $J^i - V_i(0, 0; 0)$. If we let λ^m be the choice of the median voter, the payoff function for young voters maps the set $\{\lambda_t^m, \tau_t^f, \tau_{t+1}^{ss}\}$ into the utility index

$$V_i(\tau_t^f, \tau_{t+1}^{ss})\lambda^m + V_i(0, 0; 0)(1 - \lambda^m). \tag{20}$$

Initial conditions in this model are a sequence of beliefs $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty}$ and an initial level of SS tax τ_0^{ss}. This completes the description of the game, now we just need an equilibrium concept.

The type of equilibrium relevant to this game is a voting equilibrium in tax rates that satisfies three conditions: The policy choices solve problems (13)-(15) and (18), the allocation is a competitive equilibrium given tax rates, and beliefs are rational.

Voting Equilibrium: A voting Equilibrium with beliefs is a collection of sequences for votes $\{\tau_{t+1}^{ss}, \tau_t^f, \lambda_t^m\}_{t=0}^{\infty}$, beliefs $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty}$, and the n-tuple $\{c_t^y, c_{t+1}^o, n_t, r_t, w_t, N_t, G(\alpha)\}$, that satisfy:

1. Given $\{\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss}\}$, young and old households at time t choose $\{\lambda_t, \tau_{t+1}^{ss}, \tau_t^f\}$ to maximize their payoff functions V^y and V^o.

2. Given $\{\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss}, \tau_t^f, \lambda_t^m\}_{t=0}^{\infty}$, the n-tuple $\{c_t^y, c_{t+1}^o, n_t, r_t, w_t, N_t, G(\alpha)\}$ is a competitive equilibrium.

3. Beliefs are rational.

Note that this is a Nash equilibrium, and is (trivially) subgame perfect, as no threats can be made to tomorrow's young households who are not yet born. Before proceeding to study the voting equilibrium of our model, we briefly consider the role that beliefs play in determining the features of this equilibrium.

3.2 The role of beliefs

When assumption 1 holds, the sequence of beliefs $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty}$ is uniquely pinned down (to zero) only whenever the SS tax rate becomes larger than one in finite time, so that SS is not sustainable. In this case, beliefs have a largely independent role in determining the equilibrium. At time $t+1$, the median voter will be faced with $\tau_{t+1}^{ss}(\widehat{\tau}_{t+2}^{ss})$, and will choose $\tau_{t+2}^{ss}(\widehat{\tau}_{t+3}^{ss})$. Depending then on how large is the belief $\widehat{\tau}_{t+3}^{ss}$ with respect to $\widehat{\tau}_{t+2}^{ss}$, the SS system will be continued or abandoned. An arbitrary sequence of beliefs may then give rise to periods of functioning SS alternating with periods where the system is abandoned.

The fact that beliefs are unrestricted in this way is not however completely satisfying. Although it is natural in our discussion to take into account current taxpayer's expectations about future taxpayer's willingness to be taxed, we believe that a form of the social contract places restrictions on such expectations. We will implement these restrictions by assuming that the sequence of beliefs $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty}$ is common knowledge

Assumption 2 *The sequence of beliefs* $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty}$ *is common knowledge.*

Common knowledge plus rationality implies that the equilibrium is unique up to an initial condition $\widehat{\tau}_{2}^{ss}$: Once $\widehat{\tau}_{2}^{ss}$ is given, only one sequence of beliefs is consistent with rationality. This sequence is self fulfilling:

$$\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty} = \{\tau_{t+2}^{ss}\}_{t=0}^{\infty}.$$

To see this, suppose first that $\widehat{\tau}_{t+2}^{ss} < \tau_{t+2}^{ss}$. Households at time $t+1$ set τ_{t+2}^{ss} given their belief $\widehat{\tau}_{t+3}^{ss}$, so it must be that households at t could have obtained a higher payoff by increasing τ_{t+1}^{ss} by a small amount. Since households at t know the entire sequence of beliefs, and in particular $\widehat{\tau}_{t+3}^{ss}$ it was not rational for them to believe $\widehat{\tau}_{t+2}^{ss}$, so the sequence of beliefs is not rational. By a similar argument, if $\widehat{\tau}_{t+2}^{ss} > \tau_{t+2}^{ss}$ the median voter at $t+1$ would choose to abandon SS, so $\widehat{\tau}_{t+2}^{ss}$ is not rational for households at t.

A natural implication is that when SS has not been implemented, so that $\tau_0^{ss} = 0$, the belief $\widehat{\tau}_2^{ss}$ determines the starting level of SS benefits, and therefore the 'rents' enjoyed by the first generation that does not pay SS taxes. In the next section we examine the properties of the equilibrium with two examples.

4 Properties of the equilibrium: Two examples

In this section we examine the properties of the voting equilibrium under two different distributions of productivities $G(\alpha)$. These examples illustrate the two main tradeoffs present in the voting model. The first tradeoff involves the gains from increased fertility in the form of a larger tax base in the future, weighted against the deadweight loss from subsidizing fertility. The first example, with a binomial distribution of types, allows us to isolate this feature of the problem. The second tradeoff in our model involves weighting the net gains from increased fertility via a larger tax base, as discussed above, against a lower level of SS taxes that tomorrow's median voter will be willing to pay. Indeed, higher fertility shifts the identity of the median voter towards someone who is wealthier, and therefore willing to remain in the SS system only if paying a lower SS tax rate. The second example, that uses a uniform distribution of types, allows for a discussion of this tradeoff.

4.1 First example: Two types

In this example we take $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$ with $prob(\alpha_i = \overline{\alpha}) = .5$, so that $\theta = \frac{\overline{\alpha}+\underline{\alpha}}{2}$. It will be instructive to first consider a version of the problem where there is no recourse to fertility subsidies. The value of keeping the SS promise for young household i is

$$V_i(.;.) - V_i(0;0) = \frac{\theta\beta w\gamma\tau^{ss}_{t+1}}{b} - \tau^{ss}_t w\alpha_i \tag{21}$$

This is higher for low productivity types ($\alpha = \underline{\alpha}$) than for high productivity types. Note that SS is valued because of its redistributive aspect (and possibly because of the economy being dynamically inefficient, as discussed later), and net redistribution occurs between tomorrow's wealthy young and today's poor young. Thus, the SS system is always sustained by a coalition of the old (who are always beneficiaries) and the poor young voting $\lambda_t = 1$.

We proceed backwards by first deriving the tax rates chosen if SS is not abandoned, and then considering the choice of keeping the SS system. Young households choose $\{\tau^{ss}_{t+1}\}$ by solving a problem analogous to (13)-(15), which in this case takes the form

$$\max_{\tau^{ss}_{t+1}} \quad (1-\tau^{ss}_t)w\alpha_i + \tfrac{\theta\beta w\gamma\tau^{ss}_{t+1}}{b} - \gamma \quad +\gamma\ln\frac{\gamma}{b} \tag{22}$$

$$s.t. \quad -\tau^{ss}_{t+1}w\underline{\alpha} + \tfrac{\theta\beta w\gamma\hat{\tau}^{ss}_{t+2}}{b} \geq 0 \qquad \text{if } \gamma/b > 1 \tag{23}$$

$$\tau^{ss}_{t+1} \leq 1 \qquad \text{otherwise.} \tag{24}$$

Note that in an interior solution ($\gamma/b > 1$) the median voter in the next period is always a low productivity young household ($\alpha_m = \underline{\alpha}$): If $\lambda_{t+1} = 1$

for young households such that $\alpha = \overline{\alpha}$, the low productivity young voters will also choose $\lambda_{t+1} = 1$ (by (21)), so τ_{t+1}^{ss} can be increased by a small amount without SS being abandoned next period, and therefore cannot be an optimal choice in (22)-(23). The policy function is

$$\tau_{t+1}^{ss} = \begin{cases} \frac{\theta}{\underline{\alpha}} \frac{\gamma}{b} \beta \widehat{\tau}_{t+2}^{ss} & \text{if } \gamma/b > 1 \\ 1 & \text{otherwise} \end{cases} \tag{25}$$

We can decompose the coefficient in the policy function into three parts: $\frac{\theta}{\underline{\alpha}}$, $\frac{\gamma}{b}$, and β. Note that average income over the income of the richest household that votes for SS, denoted by $\theta/\alpha_m > 1$, is a measure of how redistributive the SS system is. In the binomial case $\theta/\alpha_m = \theta/\underline{\alpha}$ is also a measure of pure income inequality. Note also that the ratio γ/b is both the fertility rate and the dependency ratio in the next period. The intuition behind (25) is as follows: The larger income inequality is, the more the low productivity young households at $t+1$ have to gain from redistribution for any given SS tax rate $\widehat{\tau}_{t+2}^{ss}$. Consequently, they in turn can be taxed more and still want to preserve SS. A similar argument holds for the fertility rate, as a higher rate implies larger SS benefits for any given tax rate, since there are more young households to be taxed. Finally, since costs τ_{t+1}^{ss} are paid one period before SS benefits for the median voter at $t+1$, these benefits need to be discounted by β.

The value for a young household from continuing the SS system is

$$J^i(\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss}) = (1 - \tau_t^{ss}) w \alpha_i + (\frac{\theta \beta \gamma}{b})^2 \frac{w}{\underline{\alpha}} \widehat{\tau}_{t+2}^{ss} - \gamma + \gamma \log(\gamma/b) \tag{26}$$

which increases with wages, as expected. It also increases with fertility and inequality, higher levels of which make SS more valuable for next period's median voter, from whom more rents in the form of SS contributions can then be extracted.

While old households will unanimously choose $\lambda_t = 1$, young households will only choose to keep the SS system if $J^i(\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss}) \geq V_i^Y(0,0;0)$, which implies

$$\lambda_t = \begin{cases} 1 & \text{if } (\frac{\theta\beta\gamma}{b})^2 \frac{1}{\alpha_i \underline{\alpha}} \widehat{\tau}_{t+2}^{ss} - \tau_t^{ss} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

Under assumption 1, SS will always be implemented at time 1 if condition (21) holds for $\alpha_i = \underline{\alpha}$, and in particular when $\tau_0^{ss} = 0$, and will be continued in the next period if $\tau_2^{ss} \geq \widehat{\tau}_2^{ss}$ (which is equivalent to $\frac{\theta\beta\gamma}{\underline{\alpha}b}\widehat{\tau}_3^{ss} \geq \widehat{\tau}_2^{ss}$) and abandoned otherwise. The dynamic path of an economy will depend on how consistent will be $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty}$ with the equilibrium realizations of this variable.

Assumption 2 ensures that beliefs are self fulfilling. In this case, the dynamics of SS taxes can be obtained by inverting (25), using the fact that $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty} = \{\tau_{t+2}^{ss}\}_{t=0}^{\infty}$.

$$\tau_{t+1}^{ss} = \begin{cases} \tau_t^{ss} \frac{\underline{\alpha}b}{\theta\beta\gamma} & \text{if } \gamma/b > 1 \\ 1 & \text{otherwise} \end{cases} \tag{28}$$

Without fertility taxes, the dynamics of the system depend on $x \equiv \frac{\underline{\alpha}b}{\theta\beta\gamma}$. If $x > 1$, no positive levels of SS benefits can be sustained as part of an equilibrium, as the economy reaches a point where tax rates higher than one are necessary to sustain the system. If $x = 1$, the economy is stable at any positive level of τ^{ss}, and if $x < 1$, SS tax rates converge monotonically to zero, the only steady state level of SS taxes. This result is similar to those in Boldrin and Rustichini [2000], who find that zero is the only stable steady state, and that therefore any SS system will gradually shrink as time progresses.

In this version of our model, a high fertility rate, high inequality, and high degree of patience (β close to one), all contribute to making SS implementable ($x < 1$) since they imply that any level of future SS taxes is associated with higher levels of benefits. Median voters may therefore prefer to continue the SS system even if the sequence of SS tax rates is decreasing.

Let us now allow for fertility subsidies and return to the original problem of choosing $\{\tau^{ss}_{t+1}, \tau^f_t, \lambda_t\}$. We proceed backwards, and begin with the choice of $\{\tau^{ss}_{t+1}, \tau^f_t\}$. The value of honoring the SS tax promised in the previous period is given by the value function in problem (13). In this example, the problem takes the form

$$\max_{\{\tau^f_t, \tau^{ss}_{t+1}\}} \quad (1-\tau^{ss}_t)w\alpha_i + \tfrac{\theta\beta w\gamma\tau^{ss}_{t+1}}{(1-\tau^f_t)b} - \tfrac{\gamma}{1-\tau^f_t} \quad +\gamma \ln \frac{\gamma}{(1-\tau^f_t)b} \tag{29}$$

$$\begin{aligned} s.t. \quad & -\tau^{ss}_{t+1}w\underline{\alpha} + \tfrac{\theta\beta w\gamma\widehat{\tau}^{ss}_{t+2}}{(1-\tau^f_{t+1})b} - \tfrac{\gamma}{1-\tau^f_{t+1}} \\ & +\gamma + \gamma \ln \tfrac{1}{1-\tau^f_{t+1}} = 0 \qquad \text{if } \tau^f_t > 1-\gamma/b & (30) \\ & \tau^{ss}_{t+1} \leq 1 \qquad \text{otherwise} & (31) \end{aligned}$$

Figure (2) is a graphic representation of this problem, where the first part of the constraint (equation (30)) has been rearranged to highlight that τ^{ss}_{t+1} is a function of $\widehat{\tau}^{ss}_{t+2}$. The problem has an interior solution only for $\widehat{\tau}^{ss}_{t+2}$ above a critical value. In the figure, $\widehat{\tau'}^{ss}_{t+2}$ is above this critical value, but $\widehat{\tau}^{ss}_{t+2}$ is not. In an interior solution, the choice of $\{\tau^f_t, \tau^{ss}_{t+1}\}$ is given by the first order conditions

$$(\tau_{t+1}^{ss}) \qquad \frac{\theta\beta w\gamma}{(1-\tau_t^f)b} - \mu w\underline{\alpha} = 0 \tag{32}$$

$$(\tau_t^f) \quad -\frac{\gamma}{(1-\tau_t^f)^2} + \frac{\theta\beta w\gamma\tau_{t+1}^{ss}}{b(1-\tau_t^f)^2} + \frac{\gamma}{(1-\tau_t^f)} = 0 \tag{33}$$

$$\begin{aligned}(\mu) \qquad & -\tau_{t+1}^{ss} w\underline{\alpha} + \frac{\theta\beta w\gamma\widehat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b} - \frac{\gamma}{1-\tau_{t+1}^f} \\ & +\gamma + \gamma \ln \frac{1}{1-\tau_{t+1}^f} = 0 \end{aligned} \tag{34}$$

where μ is the lagrange multiplier associated with (30). The above first order conditions suggest a sequential solution: First obtain τ_t^f as a function of τ_{t+1}^{ss} from (33), then substitute in (34) to solve for the optimal SS tax. This is only possible because current fertility does not affect optimal choices by tomorrow's median voter whenever it is a young household, a consequence of using the binomial distribution.

Note that the first order conditions governing the choice of tax rates do not depend on the household type, so all voters choose the same fertility subsidy and future SS tax. The first term represents the cost of paying for higher subsidy levels for each kid, and the second and third represent utility gains from higher future SS benefit levels, and from having more kids respectively. The optimal level of subsidies is positive and increases linearly in tomorrow's SS tax levels.

$$\tau_t^f = \tau_{t+1}^{ss}\frac{\theta\beta w}{b} \tag{35}$$

The optimal policies in this problem take the form

$$\tau_t^f = \begin{cases} -\frac{\theta\beta\gamma}{b\underline{\alpha}}\log(1-\frac{\theta\beta w}{b}\widehat{\tau}_{t+2}^{ss}) & \text{if } \widehat{\tau}_{t+2}^{ss} > \tau_0 \\ 1-\gamma/b & \text{otherwise} \end{cases} \tag{36}$$

$$\tau_{t+1}^{ss} = \begin{cases} -\frac{\gamma}{w\alpha}\log(1 - \frac{\theta\beta w}{b}\widehat{\tau}_{t+2}^{ss}) & \text{if } \widehat{\tau}_{t+2}^{ss} > \tau_0 \\ 1 & \text{otherwise} \end{cases} \tag{37}$$

for τ_0 a constant (see appendix A.3). From this discussion of the household problem it should be clear that, even though households vote over three policy dimensions, there is no ambiguity as to who is the median voter. The reason is that households will share preferences over $\{\tau_t^f, \tau_{t+1}^{ss}\}$, and will only disagree on their preferences over λ.

In the corner solution, young households effectively impose net taxes on fertility to reduce the dependency ratio to one. By doing this, they reduce the number of taxpayers in the future in exchange for imposing a confiscatory SS tax of one, after which the economy ends. Households are more likely to prefer this alternative the lower are beliefs about future SS benefits, and the higher is the no-tax fertility rate γ/b. For a level of γ/b above a threshold, no corner solution will be chosen at any level of beliefs.

The discontinuity that this alternative imposes is naturally due to the fact that SS taxes do not distort labor supply. Other than that, this corner solution suggests that the current young may set fertility taxes in order to reduce fertility, and in that way improve their bargaining position in the future. Since we doubt that more can be said about this feature of the solution, we restrict the model parameters so as to eliminate it from the numerical examples.

To vote on λ_t, households then compare the value function of this problem, $J^i(\tau_t^{ss}, \widehat{\tau}_{t+2}^{ss})$ with $V_i(0, 0; 0)$. As in the problem without fertility taxes, the dynamics of $\{\tau_t^f, \tau_{t+1}^{ss}\}$ depend on the sequence of beliefs. We will resort again to assumption 2 in order to provide a sharper characterization of these dynamics. In this case, the dynamic paths of $\{\tau_t^f, \tau_{t+1}^{ss}\}$ are obtained from (36) and (37), and using the fact that beliefs are self fulfilling:

$$\tau_t^f = \begin{cases} 1 - \exp(-\tau_t^{ss}\frac{w\alpha}{\gamma}) & \text{if } \tau_t^{ss} > \tau_0 \\ 1 - \gamma/b & \text{otherwise} \end{cases} \tag{38}$$

$$\tau_{t+1}^{ss} = \begin{cases} \frac{b}{\theta\beta w}\{1 - \exp(-\tau_t^{ss}\frac{w\alpha}{\gamma})\} & \text{if } \tau_t^{ss} > \tau_0 \\ 1 & \text{otherwise} \end{cases} \tag{39}$$

The expression in (39) gives τ_{t+1}^{ss} as a (locally) concave and increasing function of τ_t^{ss}, with a discontinuity at τ_0. Under further Inada-like conditions, a unique, positive and locally stable steady state level of SS and fertility taxes is guaranteed.

Figures 3A to 3C show these dynamics for chosen parameter values. Each figure shows a discontinuous 45 degree line. Then, in descending order, the figures present the dynamics of SS taxes in the cases where no fertility subsidies are available, the dynamics of SS taxes with endogenous fertility subsidies, as well as the dynamics of these subsidies in the latter case. Note that there are three possibilities: SS is viable without fertility subsidies, in which case it is also viable with subsidies and the common stable steady state is zero taxes (figure 3C). SS may not be sustainable under either arrangement (figure 3B), and SS may not be sustainable without fertility subsidies, but be sustainable and converge to a unique positive SS when these subsidies are available (figure 3A).

In figure 4 we simulate the benchmark economy (see table 1) that starts with a SS tax of 7%, where the cost of children (b) increases 1% for the first ten generations up to the benchmark level of .22, and stays constant thereafter. We calibrate the model as follows: In the OECD countries, around 1970, the dependency ratio was 3 if all persons aged over 65 are counted, and surely higher if only those entitled to old age SS are counted, so we choose a value of 3.7. To measure the cost of children, we use equivalence scales. These scales only measure direct costs, and therefore tend to underestimate actual costs, but at the same time kids only live for a fraction of a period with their parents, so we hope that both effects balance out. The OECD and "square root of n" equivalence scales give a cost of 16.7% and 18.4% of household consumption for the first kid respectively, and lower numbers for subsequent kids. We use a benchmark of 16% for the measure $\frac{b}{\theta w}$. We set β so that the annual interest rate is 6.6%. Data on the income share of the 80th percentile over that of the 20th may used as a guide to pin down $\overline{\alpha}/\underline{\alpha}$. This ratio was 2.65 in the US in 1974, 2.6 in Spain in 1980 and 2.51 in Italy in 1986. It was lower for most other OECD countries in the nineties [6]. We set $\overline{\alpha}/\underline{\alpha} = 2.7$.

We are interested in the question of the evolution of the SS system during and after the demographic transition. Note first that fertility subsidies are not sufficient to reverse the trend of decreasing fertility that is brought about by the higher cost of children (figure 4B). During this demographic transition

[6] Data on income inequality is from the Luxembourg Income Study, at http://www.lisproject.org/keyfigures/ineqtable.htm

we observe that increasing levels of SS taxes and fertility subsidies, as well as an expanding SS system, coexist with decreasing fertility rates. At the same time, the rates of return of SS are positive across the board for the first generation that implements SS (not shown in the figure, as their contributions -the denominator- are zero), but much lower for subsequent generations, and even negative for the highest productivity households.

This characterization of the behavior of the economy during the demographic transition corresponds closely to the stylized facts discussed in the introduction. Our model suggests that once the underlying demographic transition is completed, both the rate of growth of the SS system will be dampened, and fertility rates will rebound with the help of larger subsidies. In this simple simulation, the economy converges to a steady state where SS transfers represent about 18% of GDP (up from 5%), SS tax rates are about 27% of labor income (up from 7%), fertility costs are subsidized at 23% (up from 6%), and fertility is actually higher than it was before the transition.

In summary, the main result of this subsection is that fertility subsidies can be used to ensure the long term viability of the SS system. Moreover, the set of equilibria is extended in a meaningful sense: If the SS system is implemented in an economy without recourse to fertility subsidies, it can also be implemented in an economy with subsidies, and the steady state level of SS benefits is zero in both cases. On the other hand, if SS is not viable in an economy without fertility subsidies, it may be viable if there is recourse to such policy tool, and in that case the SS system will converge monotonically to a positive level of steady state contributions, given $\widehat{\tau}_{t+2}^{ss} > \tau_0$.

Fertility subsidies can be used to avoid the curse of exploding SS tax levels by making SS benefits (ss_{t+1}) a strictly convex function of SS tax rates τ_{t+1}^{ss}. Note indeed that

$$\frac{\partial V_{t+1}^m}{\partial \tau_{t+1}^{ss}} = \theta \beta w \frac{\gamma}{b(1-\tau_t^f)} \tag{40}$$

is the gain in utility from increasing τ_{t+1}^{ss} marginally. Since fertility $n_t = \frac{\gamma}{b(1-\tau_t^f)}$ is convex in the subsidy level, we have $\frac{\partial^2 V_{t+1}^m}{\partial (\tau_{t+1}^{ss})^2} > 0$: For an exogenous increase in τ_t^{ss}, the amount by which τ_{t+1}^{ss} has to be increased in order to keep the median voter indifferent is lower at higher levels of τ_t^{ss}, because fertility subsidies can be used together with increases in τ_{t+1}^{ss} to obtain a compensating change in SS benefits ss_{t+1}.

Before turning to the second example, we briefly consider the welfare implications of the model. Dynamic efficiency plays an important role in the literature on the existence of SS. In our model, the economy is dynamically inefficient whenever $\beta \frac{\gamma}{b} > 1$. This is a sufficient, but not necessary condition for the existence of SS, as SS can be politically sustained by redistribution only. Indeed, in all three examples of figure 3 the economy is dynamically efficient.

In this model -as in Boldrin and Rustichini [2000]- the first generation is the only one to unanimously benefit from the institution of SS. For all subsequent generations, only households with productivities lower than the median voter benefit from the system. Using the Hicksian compensation mechanism, we compute the extra income necessary to make households indifferent between keeping the SS system or abandoning it, or compensating variation (CV) measure. In our model, the expression for the CV is

$$CV_i = \tau_t^{ss} w(\alpha_i - \underline{\alpha}). \tag{41}$$

This expression is zero for low productivity households and positive for high productivity households. Welfare losses increase with SS taxes as the deadweight loss of fertility subsidies becomes higher, and its burden falls entirely on the high productivity households.

4.2 Second example: A continuum of types

We now turn to a slightly more general version of the previous model, where we allow fertility choices to affect the identity (productivity level) of the median voter at $t+1$ [7]. We use a uniform distribution of productivities:

$$G(\alpha_i) = \begin{cases} 0 & \text{if } \alpha_i < \underline{\alpha} \\ \frac{\alpha_i - \underline{\alpha}}{\overline{\alpha} - \underline{\alpha}} & \text{if } \alpha_i \in [\underline{\alpha}, \overline{\alpha}] \\ 1 & \text{if } \alpha_i > \overline{\alpha} \end{cases} \tag{42}$$

To see how τ_t^f has an effect on who will be the future median voter, note that old voters will unanimously choose to honor the SS promise ($\lambda_{t+1} = 1$), since they are the beneficiaries. If we normalize the number of old households to 1, then the mass of voters is $1 + n_t$. With $n_t > 1$, the median voter is such that a proportion $\frac{n_t - 1}{2n_t}$ of young households will vote $\lambda_{t+1} = 1$. Together with the fact that the lowest productivity young households are the ones to

[7] A version of this mechanism appears in the previous example in a discrete form, as young households can choose τ^f so that the median voter next period is an old household.

vote for continuing SS, this implies that the median voter at time $t+1$ will have a productivity level

$$\alpha_{m,t+1} = \underline{\alpha} + (\overline{\alpha} - \underline{\alpha})\frac{n(\tau_t^f) - 1}{2n(\tau_t^f)}. \tag{43}$$

with $\frac{d\alpha_{m,t+1}}{d\tau_t^f} > 0$: Because with higher fertility the young will form a larger constituency, a larger proportion of them will be needed to form a majority pro SS along with the old. This is illustrated in figure 5. In this figure, productivity is measured on the vertical axis. The horizontal axis represents the number of voters, where the number of old households is normalized to one and young households are ranked from lowest to highest productivity. As fertility increases from n to n', the productivity level of the median voter increases from α_m to α_m'.

As in the previous example, we begin by considering the choice of $\{\tau_t^f, \tau_{t+1}^{ss}\}$, conditional on the SS system being continued. Using the above expression for α_m, and with n_t given by (8), the problem of choosing $\{\tau_t^f, \tau_{t+1}^{ss}\}$ for young households becomes:

$$\max_{\{\tau_t^f, \tau_{t+1}^{ss}\}} \quad (1-\tau_t^{ss})w\alpha_i + \tfrac{\theta\beta w\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} \quad + \gamma \ln \frac{\gamma}{(1-\tau_t^f)b} \tag{44}$$

$$s.t. \quad -\tau_{t+1}^{ss} w\{\tfrac{(1-(1-\tau_t^f)b/\gamma)(\overline{\alpha}-\underline{\alpha})}{2} + \underline{\alpha}\} + \tfrac{\theta\beta w\gamma\widehat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b}$$

$$-\tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0 \qquad \text{if } \tau_t^f > 1-\gamma/b \tag{45}$$

$$\tau_{t+1}^{ss} \leq 1 \qquad \text{otherwise} \tag{46}$$

This problem is equivalent to (29)-(31) except that the identity of the median voter is now a function of fertility choices in the previous period.

Figure 6 illustrates the problem of choosing $\{\tau^{ss}_{t+1}, \tau^f_t\}$. In an interior solution, and letting μ be the multiplier assigned to constraint (45), tax rates are obtained from the following first order conditions

$$
\begin{aligned}
&(\tau^{ss}_{t+1}) && \tfrac{\theta\beta w\gamma}{(1-\tau^f_t)b} - \mu w\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau^f_t)b/\gamma)\} = 0 && (47)\\
&(\tau^f_t) && -\tfrac{\gamma}{(1-\tau^f_t)^2} + \tfrac{\theta\beta w\gamma\tau^{ss}_{t+1}}{b(1-\tau^f_t)^2} + \tfrac{\gamma}{(1-\tau^f_t)} - \mu\tau^{ss}_{t+1}w\tfrac{b}{\gamma}\tfrac{(\overline{\alpha}-\underline{\alpha})}{2} = 0 && (48)\\
&(\mu) && -\tau^{ss}_{t+1}w\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau^f_t)\tfrac{b}{\gamma})\} && (49)\\
& && +\tfrac{\theta\beta w\gamma\hat{\tau}^{ss}_{t+2}}{(1-\tau^f_{t+1})b} - \tfrac{\gamma}{1-\tau^f_{t+1}} + \gamma + \gamma\ln\tfrac{1}{1-\tau^f_{t+1}} = 0 &&
\end{aligned}
$$

The first condition represents the fact that increasing τ^{ss}_{t+1} implies a constant marginal gain of $\frac{\theta\beta w\gamma}{(1-\tau^f_t)b}$ for the household, and a constant marginal cost of $w\alpha_m$ for the median voter at $t+1$.

The second first order condition, equation (48), governs the choice of τ^f_t. Note that the first three terms are the same as in the previous example (equation (33)). The fourth term adds a further effect from increasing τ^f_t. As fertility in one period increases, the tax base in the following period will increase, because there will be a larger pool of workers, as in the previous example. However, the productivity level of tomorrow's median voter will also increase. Because the young now form a larger constituency, a larger proportion of them is needed to form a majority pro SS together with the old. This means that the median voter will now be someone with a higher productivity level than before (as figure 5 illustrates), and a higher productivity median voter obtains less net gains from participating in SS, so they will be indifferent between maintaining or abandoning the SS system at a *lower* SS tax τ^{ss}_{t+1}.

Once the levels of tax rates are chosen, the household votes on λ. As in the previous example, we impose some constraints, in the form of assumption 2, to the sequence of beliefs $\{\widehat{\tau}^{ss}_{t+2}\}^{\infty}_{t=1}$. With this assumption we derive the dynamics of tax rates. Since these dynamics turn out to be intractable, we present a numerical example in figure 7.

In this figure, we compare the solution of the problem given by (48)-(50) (labelled the 'rational' solution), to the solution of the same problem where households do not take into account the effect of τ^{f}_{t} on the identity of next period's median voter (the 'myopic' solution). As expected, when accounting for this effect fertility subsidies are lower, but note that SS taxes are higher, as future median voters need to be compensated for the lower level of fertility.

Some commentators view unfunded SS schemes as potentially invoking a war of the generations, in which the new old extract a disproportionate share of the labor income of the working young (see, for instance, Kotlikoff and Burns [2004]). The present example suggests, first, that the baby boom generation could promise itself very high benefit levels *because* of the demographic transition, that ensured its political influence today, and not in spite of it. These results also suggest that foresight of the effects of fertility on future political influence would have limited the incentives to develop policies aimed at increasing fertility rates.

Before concluding, it is useful to consider how sensitive the results in this paper are to the use of quasilinear preferences. Note that the feature of the solution that drives the dynamics is that the median voter is indifferent between keeping and abandoning SS. Fertility subsidies bring about a stable interior steady state of SS taxes because they are valuable to the median voter, who then will be indifferent with a level of future SS taxes lower than if he had no access to fertility subsidies. These mechanics are largely independent of preferences over consumption streams.

5 Conclusion

In this paper, we have examined a political economy model of social security and fertility subsidies, where young generations confront promises made previously by older generations, and in turn promise themselves future levels of SS benefits. We found that subsidies to the costs of children play a vital role in ensuring the viability of the SS system. Moreover, our results indicate that the seemingly explosive evolution of SS taxes will be curbed once the underlying demographic transition is completed, after which the SS system will converge to a steady state lower than simple extrapolation of current trends would indicate, and fertility will rebound with the aid of higher subsidy levels. Finally, we highlighted that, when choosing to subsidize fertility, young generations are not only increasing the tax base in the future, but are at the same time also limiting their own political influence. If they were to take into account this latter effect, fertility subsidies would be lower and SS taxes higher than otherwise.

Our model suggests that unfunded SS should survive the demographic transition, and beyond. It does not say that this is a desirable outcome. Even without any deadweight loss from SS taxes, in this model a majority of households is worse off, from a life cycle perspective, with SS than without it. The SS system survives only because, for the old, these costs are sunk. Including a labor-leisure choice, so that SS taxes are distortionary, would only make this point stronger. To be sure, unfunded SS systems fulfill an important redistributive role in most modern economies, but it is unclear that redistribution cannot be achieved more efficiently within, rather than between generations. We leave this question for future work.

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A Appendix

A.1 Data

We assembled a data set using the "Comparative welfare states data set" from Northwestern University and the University of North Carolina, and the "Comparative Family Benefits Database", version 2, assembled at the University of Calgary. Observations for all variables used here was available from 1970 to 1999, except for fertility tax rates (1972-1999), for which we extrapolated the 1972 values for the year 1970.

A.2 Household problem

We solve for the interior solution of the household problem in example 1. The problem is

$$\begin{aligned} \max_{\{c^y, c^o, n\}} \quad & c_t^y + \beta c_{t+1}^o + \gamma \ln n_t \\ s.t. \quad & (1 - \tau_t^{ss}) w_t \alpha_i = s_t + c_t^y + b(1 - \tau_t^f) n_t + T_t \\ & c_{t+1}^o = s_t(1 + r_{t+1}) + ss_{t+1}. \end{aligned}$$

And the Kuhn-Tucker conditions

$$c_t^y \geq 0 \quad \mu_1 \geq 0 \quad c_t^y \mu_1 = 0$$
$$c_{t+1}^o \geq 0 \quad \mu_2 \geq 0 \quad c_{t+1}^o \mu_2 = 0$$

Where μ_1 and μ_2 are Kuhn-Tucker multipliers. We can form the lagrangian and obtain the First Order Conditions (FOC's):

$$\begin{aligned}\mathcal{L} &= c_t^y + \beta c_{t+1}^o + \gamma \ln n_t + \chi((1-\tau_t^{ss})w_t\alpha_i - \\ &\quad s_t - c_t^y - b(1-\tau_t^f)n_t - T_t) + c_t^y\mu_1 + c_{t+1}^o\mu_2\end{aligned}$$

The FOC's are

$$\begin{array}{lcr} (c_t^y) & 1-\chi-\mu_1 & =0 \\ (c_{t+1}^o & \beta - \frac{\chi}{1+r_{t+1}} + \mu_2 & =0 \\ (n_t) & \frac{\gamma}{n_t} - \chi b(1-\tau_t^f) & =0 \\ (\chi) & (1-\tau_t^{ss})w_t\alpha_i - s_t - c_t^y - b(1-\tau_t^f)n_t - T_t & =0 \end{array}$$

Together with the Kuhn-Tucker conditions above. There are four cases of interest.

Case 1: $c_t^y > 0$ and $c_{t+1}^o > 0$.

In this case $\mu_1 = \mu_2 = 0$, and the FOC's imply:

$$\begin{aligned} r_{t+1} &= \frac{1}{\beta} - 1 \\ n_t &= \frac{\gamma}{b(1-\tau_t^f)} \\ c_t^y + \beta c_{t+1}^o &= (1-\tau_t^{ss})w_t\alpha_i + \frac{ss_{t+1}}{1+r_{t+1}} - T_t - \gamma \end{aligned}$$

Case 2: $c_t^y > 0$, $c_{t+1}^o = 0$.

In this case, we have $\mu_1 = 0$ and $\mu_2 \geq 0$. The FOC's imply:

$$
\begin{aligned}
r_{t+1} &\leq \frac{1}{\beta} - 1 \\
n_t &= \frac{\gamma}{b(1-\tau_t^f)} \\
c_t^y &= (1-\tau_t^{ss})w_t\alpha_i + \frac{ss_{t+1}}{1+r_{t+1}} - T_t - \gamma
\end{aligned}
$$

Case 3: $c_t^y = 0$ and $c_{t+1}^o > 0$.

This implies $\mu_1 \geq 0$ and $\mu_2 = 0$. From the FOC's we obtain:

$$
\begin{aligned}
r_{t+1} &\geq \frac{1}{\beta} - 1 \\
n_t &= \frac{\gamma}{\beta(1+r_{t+1})b(1-\tau_t^f)} \\
c_{t+1}^o &= ss_{t+1} + (1+r_{t+1})(1-\tau_t^{ss})w_t\alpha_i - \gamma/\beta - T_t(1+r_{t+1})
\end{aligned}
$$

Case 4: $c_t^y = c_{t+1}^o = 0$

In this case, we have

$$
n_t = \frac{ss_{t+1}/(1+r_{t+1}) + (1-\tau_t^{ss})w_t\alpha_i - T_t}{b(1-\tau_t^f)}
$$

The condition on the parameters is that disposable income is lower than γ: $\gamma > ss_{t+1}/(1+r_{t+1}) + (1-\tau_t^{ss})w_t\alpha_i - T_t$. As mentioned in the text, we restrict the model parameters to rule out this (uninteresting) possibility.

A.3 Derivation of optimal policies

We derive the optimal policies (36) and (37). The first step is to obtain the optimal choices given an interior solution ($\tau_t^f > 1 - \gamma/\beta$, $\tau_{t+1}^{ss} < 1$).

From (35) we have $\tau_t^f = \tau_{t+1}^{ss}\frac{\theta\beta w}{b}$. Leading τ_t^f one period, and substituting τ_{t+2}^{ss} by its belief, we obtain an expression for τ_{t+1}^f that can be used to eliminate τ_{t+1}^f from (34). We obtain:

$$-\tau_{t+1}^{ss} w\underline{\alpha} + \frac{\frac{\theta\beta\gamma}{b}\widehat{\tau}_{t+2}^{ss} - \gamma}{1 - \frac{\theta\beta\gamma}{b}\widehat{\tau}_{t+2}^{ss}} + \gamma - \log(1 - \frac{\theta\beta\gamma}{b}\widehat{\tau}_{t+2}^{ss}) = 0$$

After cancelling out the two terms in the middle, and rearranging, we obtain:

$$\tau_{t+1}^{ss} = -\frac{\gamma}{w\underline{\alpha}}\log(1 - \frac{\theta\beta w}{b}\widehat{\tau}_{t+2}^{ss}).$$

Substitution of this expression in (35) yields the first part of expression (37):

$$\tau_t^f = -\frac{\theta\beta\gamma}{b\underline{\alpha}}\log(1 - \frac{\theta\beta w}{b}\widehat{\tau}_{t+2}^{ss})$$

Substitution of these optimal choices in the objective function (29) gives the value of choosing an interior solution, which we call $\mathcal{Z}^I$.

$$\mathcal{Z}^I = (1 - \tau_t^{ss})w\alpha_i - \gamma + \gamma\log\gamma - \gamma\log(1 + \frac{\theta\beta\gamma}{b\underline{\alpha}}\log(1 - \frac{\theta\beta w}{b}\widehat{\tau}_{t+2}^{ss}))$$

On the other hand, the value of choosing the corner solution $\{\tau_t^f = 1 - \gamma/\beta, \tau_{t+1}^{ss} = 1\}$ is $\mathcal{Z}^C$:

$$\mathcal{Z}^C = (1 - \tau_t^{ss})w\alpha_i + \theta\beta - b$$

The household will choose an interior solution whenever $\mathcal{Z}^I > \mathcal{Z}^C$, which a after some algebra is equivalent to

$$\begin{aligned}\widehat{\tau}_{t+2}^{ss} &> \frac{b}{\theta\beta w}\{1 - \exp(\frac{b\alpha}{\theta\beta\gamma}(\exp(\frac{b - \gamma - \theta\beta + \gamma\log\gamma}{\gamma}) - 1))\}\\ &\equiv \tau_0\end{aligned}$$

	Example 1	Example 2
β	0.15	0.15
γ	0.7	0.7
A	4	4
ρ	0.35	0.35
δ	0.8	0.8
b	0.22	0.12
$\underline{\alpha}$	0.6	0.4
$\overline{\alpha}$	1.6	2.6

Table 1: Benchmark parameters

Figure 1: Timing of decisions

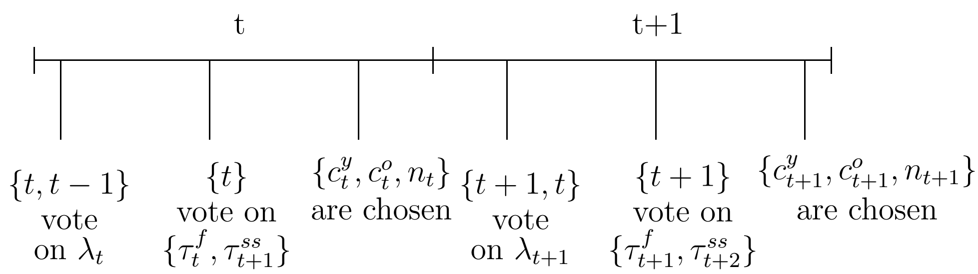

Figure 2: Example 1. Choice of tax rates

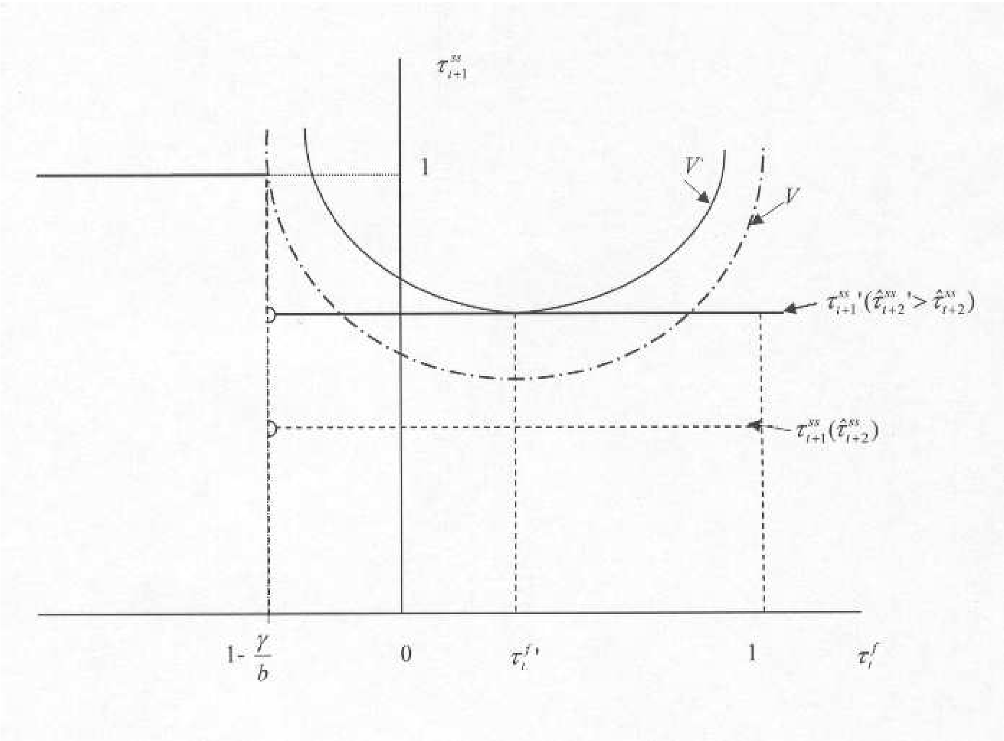

Figure 3: Example 1. Tax rate dynamics

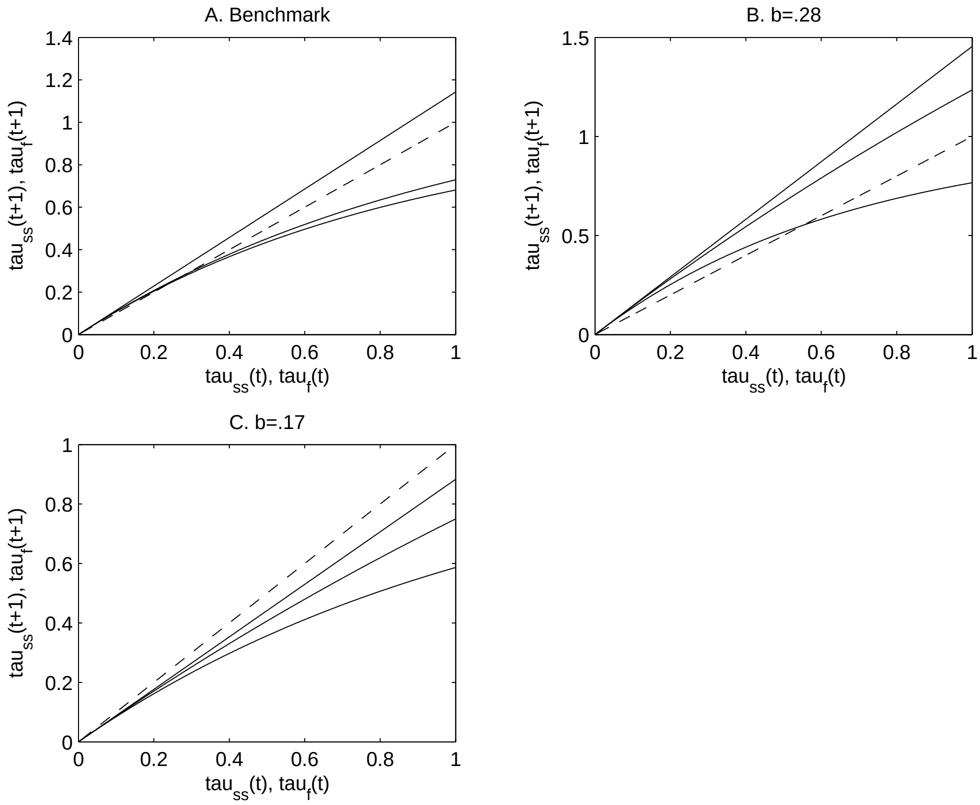

Figure 4: Example 1. Simulated paths

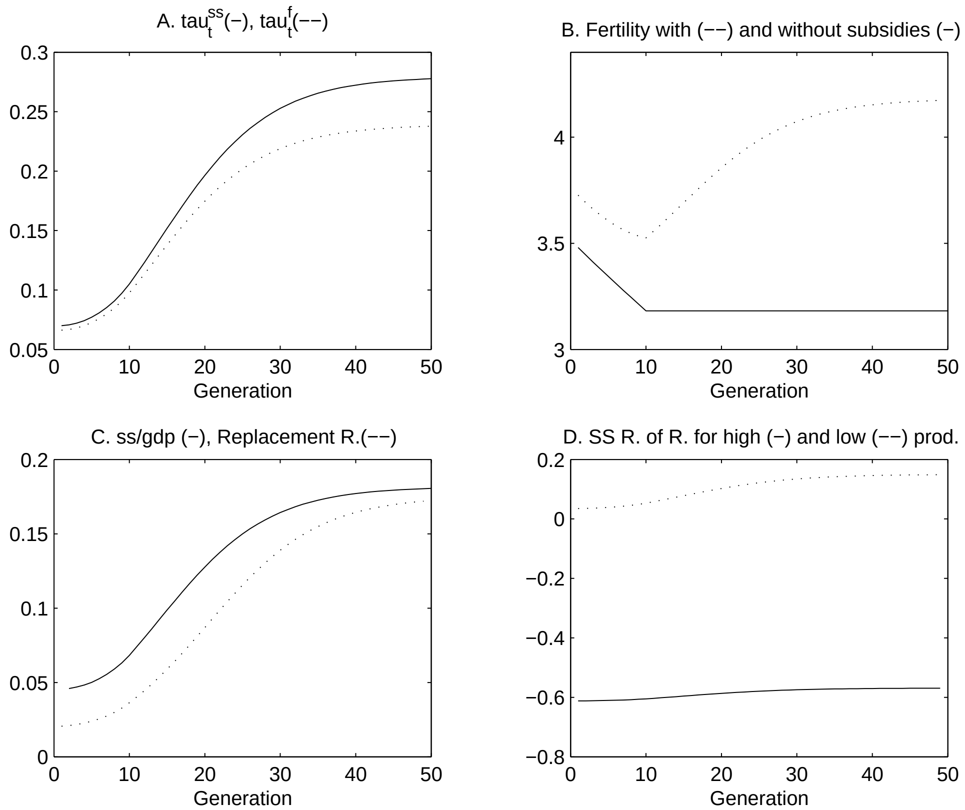

Figure 5: Who is the median voter

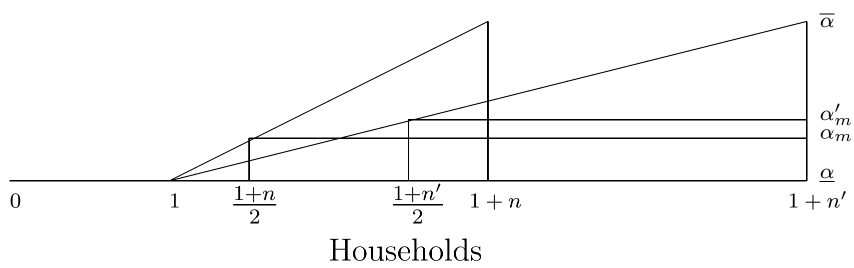

Figure 6: Example 2. Choice of tax rates

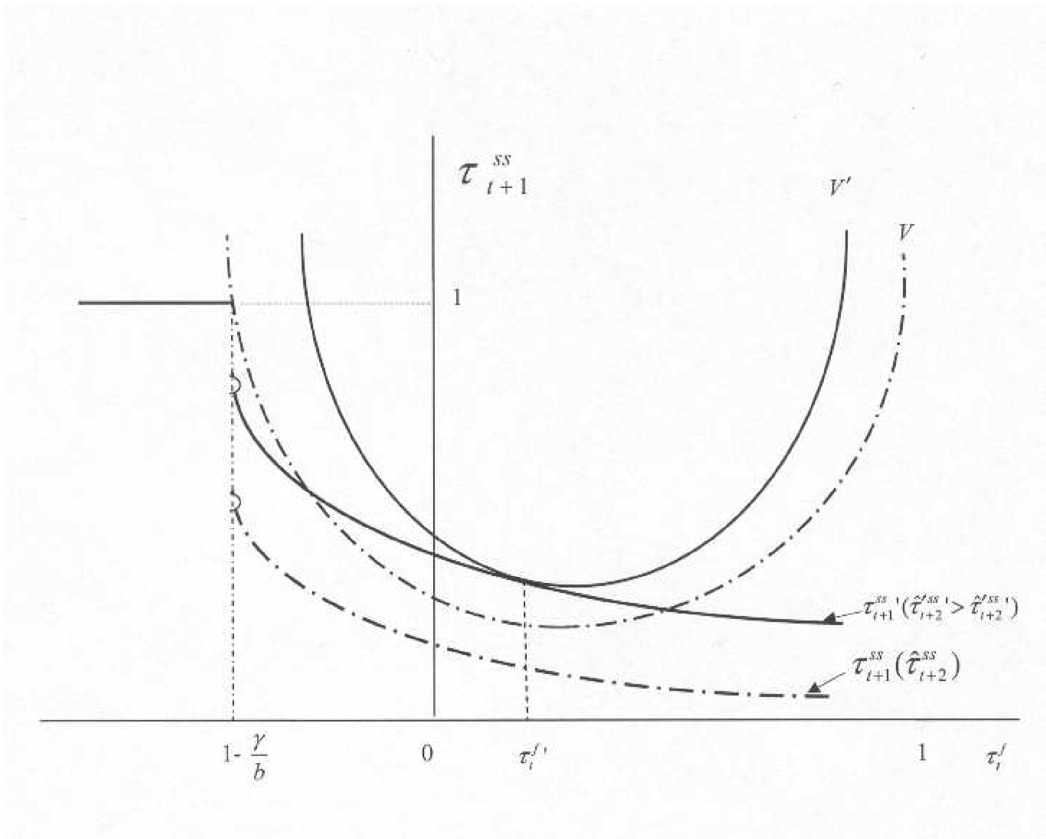

Figure 7: Example 2. Tax rate dynamics