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ABSTRACT

Productivity Improvements in Public Organizations*

In this Paper we examine the possibilities a principal in a public organization has to motivate agents for productivity improvements where standard stick and carrot incentives cannot be used. The principal's only incentive device is a reallocation of budgets and tasks across agents depending on the extent of productivity improvements revealed by each agent. We first show that as long as agents do not collude, the principal can use rotation and tournament schemes to eliminate all slack in the organization. Second, to break collusion between agents, the principal must use discriminatory tournament schemes. In some cases, however, there does not exist an incentive scheme to overcome collusion.

JEL Classification: D82, L31 and M12
Keywords: collusion, incentive schemes, public organizations and tournament and rotation schemes

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1. Introduction

In many organizations the principal typically depends on the help of agents to improve the productivity of an organization. For instance, agents have accumulated specific knowledge and can reveal unproductive tasks to the principal. However, such activities may reduce the budget and the sphere of control of the agents, and hence may in turn also reduce their utility. In many public organizations, standard stick (e.g. firing) or carrot (e.g. wage increases) incentives cannot be used to motivate agents to reveal unproductive tasks, because of guaranteed life-time employment and a rigid and predetermined wage structure. Moreover, implicit incentives due to career concern may be limited or even nonexistent, as can be the case in state-owned universities. In addition, the output of agents may not be verifiable or may only be verifiable with considerable delay, thus limiting the use of incentive devices based on output, as is the case in research and health care. As an example we may take the restructuring of German universities in the late nineties. Departments were forced to cut costs by forfeiting a fixed portion of their budget, irrespective of their past performance, because no performance measures were available. After the fixed budget cuts, it remained unclear whether less productive activities had indeed been eliminated.[1]

In this paper, we examine the possibilities a principal in a public organization has to motivate agents for productivity improvements when all the standard stick and carrot incentives cannot be used. Therefore, our central assumption is that the principal's only incentive device is a reallocation of existing budgets and tasks across agents. Our major conclusion is that even if agents collude, the principal can eliminate in many cases all unproductive tasks in the organization by appropriate mechanisms, such as tournaments or job rotation.

We consider a model with a principal and a number of agents characterized by the following information asymmetry. The agents become informed about the extent of unproductive tasks in the organization. The principal, however, does not observe the productivity of the tasks. We assume that the principal can verify and eliminate a set of unproductive tasks if this set is revealed by an agent and communicated to the principal in the form of hard information. Since the agents derive utility from the size of their unit, and

[1] See e.g. Ministerium für Wissenschaft, Forschung und Kunst Baden-Württemberg (1998). More examples are discussed in section 3.

therefore from the unproductive tasks in their unit, the principal must provide the agents with incentives to reveal their knowledge.

We consider two scenarios distinguished with respect to the knowledge agents can supply to the principal in the form of hard information. In the first case, agents have knowledge specific to the organization (organization specific knowledge), i.e. they are able to reveal the slack in all the units of the organization to the principal. In the second case, which appears more plausible, agents have knowledge specific to their unit (unit specific knowledge) and are only able to provide hard information on the unproductive tasks in the unit they govern.

For both cases, we show that a first-best solution in which the principal eliminates all unproductive tasks in the organization can be achieved by appropriate mechanisms involving the revelation of unproductive tasks by agents. If collusion is possible, a first-best solution can only be implemented if knowledge is organization specific. In particular, we establish the following results.

First, if agents act noncooperatively, a tournament guarantees the first-best solution in the organization specific knowledge scenario. The prize of the tournament is the award of extra tasks. In the case of unit specific knowledge however, a tournament does not work, since the agent with the largest productivity improvement potential only has to match the agent with the second highest share of unproductive tasks. The principal, however, can realize a first-best solution through partial job rotation. Under such a scheme, the principal assigns each agent a part of his traditional tasks and a part of the other agent’s traditional tasks. The assigned shares depend on the announcements of unproductive tasks.

Second, if agents can collude but cannot transfer utility among themselves via side payments, only tournaments in which the principal does not treat agents equally will yield a first-best solution in the organization specific knowledge scenario. Such discriminatory tournament schemes reward agents differently, even if agents reveal the same share of unproductive tasks. If agents are treated unequally, it is impossible for less well-treated agents to offer collusion contracts without side payments. Otherwise, collusion would be feasible and productivity improvements would not occur. The necessity to treat equal agents unequally represents a fundamental trade-off between fairness and efficiency. In the case of unit specific knowledge, there does not exist an incentive scheme which implements a first-best solution.

Our analysis suggests that in practice task and budget assignments may be used as an incentive device motivating unit managers to improve productivity in public organizations. In the past, uniform cost-cutting across all departments in public organizations such as in German universities, has probably eliminated both unproductive tasks and productive tasks. In recent years, universities have tried to reward departments by using a tournament-like scheme with new budgets and possibilities to expand into other areas in exchange for the revelation of less productive tasks.

The paper is organized as follows. Section 2 discusses related literature. We introduce the model in section 3. In section 4, we specify the contracts and the game for the two scenarios, differing with respect to the extent of hard information agents can supply to the principal. In section 5, we derive the optimal incentive schemes if agents act non-cooperatively. In section 6, we consider the game where collusion is possible. Our conclusions are presented in section 7.

2. Relation to the Literature

Many contributions in recent organization and productivity literature have emphasized the role of the organization of functions and tasks as a key factor in explaining productivity differences between organizations (e.g. McKinsey (1993), Milgrom and Roberts (1992), Womack et al. (1990)). Benchmark studies reveal that more than half of the productivity differences between German and Japanese car manufacturers are attributable to differences in the way functions and tasks are organized (e.g. Baily and Gersbach (1995)). The organization of functions and tasks includes the way companies organize internal communication and design incentive schemes to make better use of the special local knowledge that their workers alone possess (e.g. Milgrom and Roberts (1992), Baily and Gersbach (1995), Imai (1989)). The underlying incentive problem was first addressed by Carmichael and MacLeod (1993). They suggest that multiskilling of workers will enable a firm to reap the gains from enlisting the full cooperation of workers for labor saving techniques. Our paper is complementary to this line of work. We discuss how improvements in the organization of functions and tasks in public organizations can be achieved by appropriate incentive schemes, such as tournament and rotation schemes.

Whereas a broad range of insights has been derived for private organizations using explicit and implicit incentive schemes with agents acting noncooperatively[2] or colluding[3], only a few contributions deal with adequate incentive schemes for public organizations. Private and public organizations differ with respect to the set of possible incentive instruments (e.g. Wilson (1989)). Dewatripont, Jewitt and Tirole (1999a, 1999b) provide insight into the role of implicit incentives in the form of career concern within a generalized version of the career concern model developed by Holmström (1982b), which can also be applied to public organizations.

In our paper we focus on task and budget assignment as a tool for motivating agents in public organizations, where career concerns cannot provide sufficient incentives. Examples are the education and health services. We provide insight into the role of job rotation and the use of tournaments in motivating agents to reveal unproductive tasks and to improve productivity in the organization. Our analysis may also have some bearing on private organizations when authority to allocate budgets and to determine compensation differs. For instance, a manager in an R&D division can allocate budgets, but is not authorized to change his worker compensation, and thus may act under similar constraints as in public organizations.

3. Model

We consider an organization with *n* units $n > 1$, indexed by *i*, *j* or *k*. Each unit is managed by an agent and performs a variety of tasks. The agent is indexed by the unit he is governing. Therefore, we call agent *i* the unit manager *i*. To simplify the analysis, we assume that there are only two types of tasks. Tasks can be either productive or unproductive. The return of a task is either r^g or r^b with $r^g > 0$ and $r^b < 0$. The share of unproductive tasks is denoted by A_i and we assume that half of the tasks of a unit may be unproductive at the most, i.e. $A_i \in [0, \frac{1}{2}]$. Depending on the share of unproductive tasks A_i, the average productivity or return of a unit is:

2 See e.g. the surveys by Hart and Holmström (1987), Holmström and Tirole (1989) or Prendergast (1999). Important contributions to this literature include Holmström (1979, 1982a), Grossman and Hart (1983), Lazear (1989), Holmström and Milgrom (1991).

3 See. e.g. Tirole (1986, 1992), Holmström and Milgrom (1990), Faure-Grimaud, Laffont and Martimort (1998).

$$\overline{r_i} = A_i r^b + (1 - A_i) r^g$$

Unproductive tasks can be beneficial for unit managers however, because of a desire for power, control, empire building or because of private benefits from tasks that may include perquisites for the job, the acquisition of human capital and the possibility of signaling ability (see Aghion and Tirole (1997))[4]. Thus, unproductive tasks can occur when individuals want to expand as much as possible or when agents focus on generating private benefits from tasks which may make such tasks unproductive for the principal. Therefore, there are a variety of reasons why unproductive activities are beneficial for managers. Naturally, we think that unit managers also have an influence on the productivity of their units. Our model can capture this in the following way. Suppose that managers have an unobservable talent or ability to impact on productivity in their unit. Very talented managers will make most activities productive, i.e., A_i will be low. Less talented managers end up with a large share of unproductive tasks, and thus have a high value of A_i. Such an interpretation naturally implies that the principal neither observes the ability of unit managers, nor the share of unproductive tasks A_i. This will be precisely the central assumption explored in the next few paragraphs.

The principal is responsible for the whole organization. We assume that the principal cannot use the average productivity in each unit to set up incentive schemes. Either he does not directly observe $\overline{r_i}$ because output is not measurable, or because it is only measurable with long delays, as is the case in the education or the health sector. To derive the potential for task reallocation, our central assumption is that standard stick and carrot incentive schemes cannot be used at all. Therefore, the principal's instruments are restricted to budget or task assignments. This assumption, which rules out any monetary incentives, is crucial for our analysis and deserves further comment.

First, there are real-world organizations where monetary incentives play little or no role. In many employee categories in the public sector in Germany, individuals cannot increase their wages through excellent performance. Examples of this are all those civil servants who have reached their highest possible career position. In many cases, since the wage profiles are set

[4] Technological change may also make certain activities obsolete and thus create unproductive activities too.

by law, civil servants reach the highest possible position quite early in their careers. Therefore, monetary incentives play a limited role for these employees. Predominantly, wages depend on job descriptions, the university degrees necessary to get the job and seniority. Detailed examples can be found in the pay regulations for the public sector in Germany[5]. Anecdotal evidence suggests that similarly rigid wage structures also operate in the public sector in France. However, even in the rigid wage structures of public organizations, there are career opportunities associated with an increase of wages. But, as mentioned above, wage increases for promotion are rather small in public organizations and seniority determines wage rises. In some cases, wages do not respond to promotion at all. The latter is the case in Germany,[6] when a professor becomes the head of a faculty or even of the university. Therefore, we believe that monetary incentives in public organizations, e.g. in Germany or France, play only a limited role for most of the employees.

Second, the absence of monetary incentives can be justified by infinite risk aversion in wealth or by noncontractability of the principal's benefit as discussed extensively in Aghion and Tirole (1997) and Tirole (2001).

Concern is appropriate that the absence of monetary incentives in a public organization may be due to unmodelled behavior that also acts as a constraint on the schemes presented in this paper. Such unmodelled behavior might be connected with multi-task issues and the influence of unions. In the first case, we know from Holmström and Milgrom (1991) that measurement problems severely hamper the use of monetary incentives. In the second case, unions may be prompted to restrict incentive schemes in order to compress the wage distribution of its members (see e.g. Fitzenberger and Franz (1999)). We cannot exclude the possibility that such constraints affect the ability of the principal to implement the schemes presented in this paper.

In recent years universities in Germany have tried to use tournament-like schemes to allocate tasks and budgets in accordance with the revelation of less productive tasks[7]. From the perspective of industrial organization literature, we interpret such moves as a response to

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[5] See e.g. Bundesbesoldungsgesetz, Bundesangestelltentarifvertrag in the survey of Bundesministerium des Innern (2002).

[6] This has been confirmed in private communication.

[7] For instance, departments that have revealed a significant share of less productive tasks may be rewarded by the allocation of funds in future investment programs, or by temporal guarantees for their remaining budgets. See also Frankfurter Allgemeine Zeitung, 11.2.2002.

increasing global competition, in the sense of exposure to best practice in areas where public ownership plays, and continues to play, a large role in continental Europe (e.g. Baily and Gersbach (1995)). Examples are universities, health care and vocational training schools.

To sum up, monetary incentives play a very limited role in many public sector organizations. Moreover, we suggest that lack of competition has permitted such organizations to refrain from using incentive schemes such as those indicated in this paper. Therefore, our analysis has both a positive and a normative flavor.

A further assumption of our analysis is that tasks can be reallocated with no productive loss. There are various potential justifications for this assumption. First, the role of managers initially is to organize a unit and to identify unproductive tasks. After units have been organized and unproductive tasks have been identified, tasks can be reallocated across unit managers without costs. Second, the assumption is made for simplicity of exposition and serves as a benchmark case. In principle, we think that managers can add values continuously to their unit, which will be lost when tasks are reallocated. In such a setting, the application of the schemes in our paper requires that such losses due to reallocations are not too large. Third, we could imagine that each manager leads a unit consisting of standard and special operations, e.g. teaching and research activities at universities. The budgets for the special operations are distributed equally among managers initially. Our schemes are then applied to the special operations whose budgets can be freely redistributed. Each unit always requires a person to manage the standard operations, but some tasks can be reallocated with no productive loss.

The principal faces the following informational asymmetry. The unit manager *i* observes the productivity of the tasks performed in his unit. Therefore, he could in principle eliminate unproductive tasks and raise productivity in the organization. The principal, however, does not observe the productivity of the tasks. He has a priori knowledge about the extent of unproductive tasks in unit *i*. From the perspective of the principal, A_i is assumed to be uniformly distributed over the interval $\left[0, \frac{1}{2}\right]$. Hence, unit *i* may have a small or a large share of unproductive tasks. We further assume that A_i and A_j are uncorrelated across units for any $i, j,\ i \neq j$. An important assumption is that the principal can verify and eliminate a set of unproductive tasks if this set of tasks and its productivity is revealed by the unit manager. Hence, we assume that the unit manager can provide the principal with hard information about the productivity of tasks.[8] For instance, he may directly reveal to the principal the inefficiencies involved in certain stages of activities in his unit. Our assumption is in line with the practices in management consulting where it is discussed that slack reductions in business units require information by the employees of the unit (e.g. Baily and Gersbach (1995)).

We use α_i to denote the share of tasks revealed as being unproductive by the unit manager of unit *i*. The principal can verify whether these tasks are indeed unproductive. Hence, announcements of a portion of unproductive tasks larger than A_i could be verified and only unproductive tasks would be eliminated. Thus, it is never worthwhile for the unit manager to announce productive tasks. The principal cannot, however, observe whether the unit manager has disclosed all the unproductive tasks in his unit, because he does not observe the productivity of the residual tasks. The rationale for this assumption stems from the productivity and organization literature on continuous improvements (e.g. Womack et al. (1990), Baily and Gersbach (1995)). Only workers and managers directly involved in the execution of activities acquire knowledge enabling them to increase productivity.

The principal's utility, denoted by U_P, is monotonically increasing in the net returns. The principal is assumed to be risk neutral. Thus, U_P is given by:

$$U_P = \sum_i \overline{r_i}$$

The manager of unit *i* derives utility from two sources: the income and the size of the organizational unit. Since flexible wages play a limited role in public organizations, we assume that wages are fixed exogenously by a predetermined and rigid wage structure. We normalize the utility from the fixed wage to zero. We assume that the utility unit managers derive from the size of their unit is positive, regardless of whether the tasks are productive for the principal. Examples are the desire for power, control or empire building often mentioned in the organization literature or the possibility of the unit manager using some of the resources in his unit for activities that are not in the interest of the organization. The unit manager's utility before contracting depends solely on the size of his unit and is normalized to 1, i.e. $U_i = 1$.

[8] The hard information paradigm was introduced by Tirole (1986) and (1992) and used in Laffont and Tirole (1993), Laffont and Meleu (1997), and Kofman and Lawaree (1993).

Since unit managers derive utility from the size of their unit, and hence also from tasks that are unproductive, the principal must provide his unit managers with incentives to reveal their knowledge.

4. The Game

In the following, we consider two cases distinguished by the unit manager's knowledge about unproductive tasks. Knowledge refers to the extent to which a unit manager can provide hard information on unproductive tasks.

In the first case, the unit managers are supposed to have organization specific knowledge. In this case, unit managers are able to reveal the share of unproductive tasks of each unit in the organization to the principal and therefore the overall slack in the whole organization. This assumption can be motivated by the similarity of the units or by cross-sectional training of the unit managers at the beginning of their careers. In the second case, we assume that the unit managers have only unit specific knowledge, i.e. they know only the share of unproductive tasks in their own unit and thus they are only able to provide hard information on unproductive tasks in that unit. Which case is more plausible is not a priori clear. In the example of German universities one could argue that unit specific knowledge is more plausible. However, organization specific knowledge cannot be excluded because of communication and observations among agents prior to restructuring.

4.1. Organization Specific Knowledge

If the unit managers have organization specific knowledge, they are able to report the unproductive tasks in each unit. α_i^j denotes the share of unproductive tasks in unit j revealed by the manager of unit i. Thus, the unit manager i announces a vector $\left(\alpha_i^1,\ldots,\alpha_i^n\right)$ containing the share of unproductive tasks in each unit he wants to reveal to the principal. The unproductive tasks will be eliminated by the principal. The principal offers a contract $D^i\left(\beta_{ii}\left(\left(\alpha_1^1,\ldots,\alpha_1^n\right),\ldots,\left(\alpha_n^1,\ldots,\alpha_n^n\right)\right),\beta_{ij}\left(\left(\alpha_1^1,\ldots,\alpha_1^n\right),\ldots,\left(\alpha_n^1,\ldots,\alpha_n^n\right)\right)\right)$ with the following interpretation. Depending on the revealed knowledge, unit manager i receives a share β_{ii} of the residual tasks in his unit, i.e. tasks that are not eliminated by the principal, and a share β_{ij} of the residual tasks in unit j $\left(j \neq i\right)$.

Two possible interpretations of β_{ij} fit our model. First, β_{ij} is the amount of tasks (or the budget) not deleted in unit *j* and reallocated to unit *i*. For this interpretation, the residual activities of unit *j* needs to be broken up, since we do not consider joint control over units. In a second interpretation, the shares β_{ij} could be probabilities of obtaining control over the undeleted activities in unit *j*. As unit managers' utility is assumed to be linear in the size of their units and $\sum_i \beta_{ij} = 1$, such an interpretation is equivalent. We think that the second interpretation is somewhat less plausible from an empirical point of view, at least when considering restructuring efforts in public organizations in Germany. Complete reallocations of units occur but in most cases control over units remains in place, with some redistribution of activities or budgets[9].

For simplicity of presentation, we assume that the announcements are nested, i.e., the maximum share of unproductive tasks in unit *i* the principal can eliminate is given by:

$$\max\{\alpha_1^i, \ldots, \alpha_n^i\}$$

The assumption can be justified technologically by assuming that there is a natural order in revealing unproductive tasks to the principal. Or the principal may only need to use $\max\{\alpha_1^i, \ldots, \alpha_n^i\}$ in all of our suggested incentive schemes. As we will see, the latter is true and, hence, we proceed directly on the nested assumption that the maximal share of unproductive tasks the principal eliminates in unit *i* is given by $\max\{\alpha_1^i, \ldots, \alpha_n^i\}$.

Under this contractual arrangement, the utility of unit manager *i* is given by:

$$U_i = \beta_{ii}\left(1 - \max\{\alpha_1^i, \ldots, \alpha_n^i\}\right) + \sum_{j \neq i} \beta_{ij}\left(1 - \max\{\alpha_1^j, \ldots, \alpha_n^j\}\right)$$

The game between the principal and the unit managers is given as follows:

Stage 1: The principal offers the incentive scheme

$$D^i\left(\beta_{ii}\left(\left(\alpha_1^1, \ldots, \alpha_1^n\right), \ldots, \left(\alpha_n^1, \ldots, \alpha_n^n\right)\right), \beta_{ij}\left(\left(\alpha_1^1, \ldots, \alpha_1^n\right), \ldots, \left(\alpha_n^1, \ldots, \alpha_n^n\right)\right)\right).$$

[9] See e.g. Ministerium für Wissenschaft, Forschung und Kunst Baden-Württemberg (1998).

Stage 2: Each unit manager announces a vector of the shares $\left(\alpha_i^1,\ldots,\alpha_i^n\right)$ of unproductive tasks across all units.

Stage 3: The principal and all unit managers observe the announcements. Task reallocation represented by $\beta_{ii},\beta_{ij},\ \forall i,j,\ i\neq j$ is executed.

The principal's objective is to eliminate all unproductive tasks. Thus, he must choose the incentive coefficients β_{ii},β_{ij} such that at least one unit manager will fully reveal his private knowledge on the slack in the organization. The principal maximizes the sum of the average productivities subject to the individual incentive constraints $\left(IC_i\right)$ and the task constraints $\left(TC_j\right)$. Note that we can neglect the participation constraints since we assume that wages are sufficiently high to motivate managers to work in the public organization. Our problem is an exercise in implementation theory (see Moore (1992) for a comprehensive survey). We examine the implementation of productivity improvements in strictly or weakly dominant strategies, where possible, and for Nash implementation otherwise.

Hence, the principal's problem is given by:

$$\max_{\{\beta_{ij}\}} U_P=\sum_i\left\{\bar{r}_i-\max\left\{\alpha_1^i,\ldots,\alpha_n^i\right\}r^b\right\}$$

$$\text{s.t.}\quad IC_i:\quad U_i\left(\left(\alpha_1^1,\ldots,\alpha_1^n\right),\ldots,\left(\alpha_i^1,\ldots,\alpha_i^n\right),\ldots,\left(\alpha_n^1,\ldots,\alpha_n^n\right)\right)\geq$$

$$U_i\left(\left(\alpha_1^1,\ldots,\alpha_1^n\right),\ldots,\left(\hat{\alpha}_i^1,\ldots,\hat{\alpha}_i^n\right),\ldots,\left(\alpha_n^1,\ldots,\alpha_n^n\right)\right)\ \forall i$$

$$TC_j:\quad \sum_i\beta_{ij}=1\ \forall j$$

$$0\leq\hat{\alpha}_i^j\leq\alpha_i^j\leq A_j\leq\tfrac{1}{2}\ \forall i,j$$

The principal maximizes expected returns by eliminating unproductive tasks. The incentive constraints require that an announcement vector $\left(\alpha_i^1,\ldots,\alpha_i^n\right)$ for unit manager i provide utility that is weakly higher than any other feasible announcement vector $\left(\hat{\alpha}_i^1,\ldots,\hat{\alpha}_i^n\right)$, given the announcement vectors of the other unit managers. The task constraints represent the fact that no more than all residual tasks can be distributed to the unit managers.

4.2. Unit Specific Knowledge

If the unit managers can only reveal the share of unproductive tasks in their own unit, the principal offers the contract $D^i\left(\beta_{ii}(\alpha_1,\ldots,\alpha_i,\ldots,\alpha_n),\beta_{ij}(\alpha_1,\ldots,\alpha_i,\ldots,\alpha_n)\right) \ \forall i, j,\ j \neq i$. α_i denotes the share of unproductive tasks in unit *i* revealed by unit manager *i* with $\alpha_i \leq A_i$.

Under this contractual arrangement, the utility of unit manager *i* is given by:

$$U_i = \beta_{ii}(1-\alpha_i) + \sum_{j\neq i}\beta_{ij}(1-\alpha_j) \text{ with } \sum_i \beta_{ij} = 1$$

The structure of the game is similar to the case of organization specific knowledge. The only difference is the contract offered by the principal.

The principal's problem in this case is given by:

$$\max_{\{\beta_{ij}\}} U_P = \sum_i \left\{\bar{r}_i - \alpha_i r^b\right\}$$

$$\text{s.t.} \quad IC_i: \quad U_i(\alpha_1,\ldots,\alpha_i,\ldots,\alpha_n) \geq U_i(\alpha_1,\ldots,\hat{\alpha}_i,\ldots,\alpha_n) \quad \forall i$$

$$TC_j: \quad \sum_i \beta_{ij} = 1 \ \ \forall j$$

$$0 \leq \hat{\alpha}_i \leq \alpha_i \leq A_i \leq \tfrac{1}{2} \ \ \forall i$$

4.3. First-Best

We complete our model by the characterization of the first-best solution, which follows immediately from the principal's profit function.

Proposition 1:

(i) An incentive scheme under organization specific knowledge is first-best if for every i $\max\left\{\alpha_1^i,\ldots,\alpha_n^i\right\} = A_i$.

(ii) An incentive scheme under unit specific knowledge is first-best if for every i $\alpha_i = A_i$.

At this point it is obvious that the first-best solution can be achieved by standard incentive schemes if the principal is able to use monetary incentives or the threat of firing (see e.g. the surveys by Hart and Holmström (1987) or Prendergast (1999)). Our novel element is the design of non-monetary incentive schemes to achieve first-best solutions when standard stick and carrot instruments cannot be used.

5. Optimal Incentive Schemes without Collusion

In this section, we derive the incentive schemes for implementing the first-best solution when unit managers do not collude.

5.1. Organization Specific Knowledge

If the unit managers have organization specific knowledge, the principal can implement a first-best solution by a tournament as shown in the next proposition.

Proposition 2:

Suppose the unit managers have organization specific knowledge. The principal can implement a first-best solution as a unique equilibrium of the announcement game with the following tournament where m denotes the number of unit managers revealing the highest amount of unproductive tasks:

$$\beta_{ii} = \beta_{ij} = \begin{cases} 1 & \text{if} \quad \sum_{k=1}^{n} \alpha_i^k > \sum_{k=1}^{n} \alpha_j^k \quad \text{for all } j \neq i \\ p & \text{if} \quad \sum_{k=1}^{n} \alpha_i^k = \max_{j \neq i} \left\{ \sum_{k=1}^{n} \alpha_j^k \right\} \\ & \quad \text{for } m\,(m>1) \text{ unit managers with } p = \begin{cases} 0 & \text{with probability } 1-\frac{1}{m} \\ 1 & \text{with probability } \frac{1}{m} \end{cases} \\ 0 & \text{if} \quad \sum_{k=1}^{n} \alpha_i^k < \sum_{k=1}^{n} \alpha_j^k \quad \text{for some } j \end{cases}$$

Every unit manager i announces $(A_1, \ldots, A_n)$ *and expects utility*

$$U_i = \tfrac{1}{n}\left(n - \sum_i A_i\right).^{10}$$

[10] Note that only one manager will be chosen from the urn since tasks can only be allocated once.

The proof is given in the appendix. The tournament scheme implements a first-best solution since unit managers only have the chance to obtain tasks if they reveal as much as other unit managers. This induces unit managers to reveal the maximum possible.

Besides the tournament scheme there are other possibilities of implementing the first-best solution. The principal can also use incentive schemes that eliminate the unit managers' costs of announcements instead of rewarding large-scale announcements of unproductive tasks with additional tasks and utility as discussed above. For instance, the principal can use a specific form of job rotation.

Proposition 3:

Suppose the unit managers have organization specific knowledge, then the principal can implement a first-best solution through job rotation:

$$\beta_{ii} = 0, \quad \beta_{ij} = \begin{cases} 1 & if \quad j = i+1 \\ 1 & if \quad i = n, j = 1 \\ 0 & otherwise \end{cases}$$

Proof:

Under job rotation, the unit managers' utility after rotation does not depend on the announcements of unproductive tasks in their own unit. Thus, the unit managers are indifferent with respect to the revelation of unproductive tasks in their own unit. Hence, every combination of announcements $\alpha_i^i \in [0, A_i] \quad \forall i$ is part of an equilibrium of the announcement game. In particular, the announcement vectors $\left(0,\ldots,0,\alpha_i^i = A_i,0,\ldots,0\right)$, $i = 1,\ldots,n$ constitute a Nash equilibrium of the announcement game. (q.e.d.)

The main disadvantage of this scheme is the non-uniqueness of equilibria in the announcement game[11]. Hence, the principal can only implement the first-best solution by job rotation with certainty if indifferent unit managers act in the interests of the principal. In contrast to the tournament scheme, in which the unit managers completely reveal their private knowledge, the rotation scheme may not induce unit managers to reveal the maximum share

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[11] The same problem occurs if the principal simply asks each unit manager about unproductive tasks in other units without using an incentive scheme at all.

of unproductive tasks in each unit, since unit managers could lose by rotating to the next unit.

5.2. Unit Specific Knowledge

We now consider unit specific knowledge where the unproductive tasks in the manager's own unit can only be demonstrated to the principal as hard information. We will show that the principal can always design an incentive scheme, which implements a first-best solution. This scheme requires that unit managers must be compensated for the loss of tasks in their own unit by a transfer of tasks from other unit managers.

Proposition 4:

Suppose n>2. Suppose the unit managers have unit specific knowledge. Then, an incentive scheme exists which uniquely implements a first-best solution. The scheme is given by:

$$\beta_{ii} = 1 - \varepsilon \sum_{j \neq i} \alpha_j \qquad \textit{with } \frac{1}{n-1} < \varepsilon \leq \frac{2}{n-1} \textit{ and } j \neq i$$
$$\beta_{ij} = \varepsilon \alpha_i$$

The proof is given in the appendix. Under the proposed incentive scheme, unit managers have a strict incentive to reveal their knowledge given the announcements of the other unit managers, because the loss of tasks in their traditional unit is compensated by a higher share of tasks obtained from other units. Since unit managers receive a share of their traditional tasks and a share of the other unit managers' traditional tasks, such schemes can be characterized as partial job rotation.

Note that the announcement of all unproductive tasks is a dominant strategy. Therefore, the incentive scheme works independently of whether unit managers have the same limited information about unproductive tasks in other units as the principal or whether unit managers may observe the extent of unproductive tasks in other units but at the same time be unable to provide hard information to the principal. Therefore, the limitations of the revelation principle (see Haller (1992)) do not apply in our context.

The schemes in proposition 4 and in propositions 2 and 3 imply that some managers end up with more tasks and others with less. This could occur through yearly reallocations of budgets without influencing the job status and the pay of unit managers. For instance, professors in German universities are endowed with a certain budget, which can change every year, and thus professors can undertake more or fewer tasks. Allocating more tasks can also be interpreted as promotion, while fewer tasks could mean demotion. Such an interpretation is more problematic, since in Germany, for example budget reductions for managers in public organizations are widespread, but they are not real demotion with associated pay reductions. As in the case of organization specific knowledge, job rotation can yield a first-best solution.

6. Collusion between Unit Managers

In this section, we allow for collusion between unit managers. We believe that unit managers have strong incentives to go for collusion agreements. Since, under the incentive schemes in propositions 2 and 4 for instance, activities will be eliminated, the aggregate utility of managers declines. If the unit managers have the same information structure, they can write binding side contracts which can hurt the principal. This is obvious in the case of organization specific knowledge. In the case of unit specific knowledge, the possibility of collusion through contracts requires unit managers to have the same information about the proportion of unproductive tasks across units. However, as they do not share the same knowledge, they can only give hard information about unproductive tasks in their own unit. For instance, the unit managers know the extent of unproductive tasks in their own unit but cannot indicate the unproductive tasks in other units to the principal because their knowledge is not sufficiently detailed. Without this assumption, collusion under unit specific knowledge does not occur and therefore our scheme in proposition 4 can be used to achieve the first-best solution. In the following, we assume that collusion is possible in both cases in order to explore how collusion can be broken.

6.1. Side Contracts between Unit Managers

We examine collusion in which agents are not able to transfer utility via side payments in order to remain consistent in excluding monetary incentives. Hence, a side contract is only characterized by the announcements of the unit managers, i.e. the announcements unit managers agree to supply to the principal as hard information. In the following we introduce side contracts for unit specific knowledge without side payments. In sections 6.2. and 6.3. we discuss collusion for organization specific and unit specific knowledge in detail.

Let us denote side contracts under unit specific knowledge by $C(\alpha_1,\ldots,\alpha_n)$. $(\alpha_1,\ldots,\alpha_n)$ is the vector of announcements all n unit managers agree to deliver to the principal. Collusion agreements for a subset of unit managers are defined accordingly. Obviously, the set of side contracts the unit managers will write under a given incentive scheme fulfills the condition that no other side contract $C(\alpha_1^0,\dots,\alpha_n^0)$ exists with

$$U_i(\alpha_1^0,\dots,\alpha_n^0)\geq U_i(\alpha_1,\dots,\alpha_n) \quad \forall i \text{ and}$$

$$U_j(\alpha_1^0,\dots,\alpha_n^0)> U_j(\alpha_1,\dots,\alpha_n) \text{ for some } j\neq i.$$

As a tie-breaking rule, we assume that unit managers do not engage in side contracts if they are indifferent between the noncooperative outcome and the outcome under a side contract as defined above. That is, if they are indifferent unit managers will act in the principal's interests by not colluding. In the case of unit specific knowledge, the game between the principal and the unit managers when side contracts can be written is given as follows:

Stage 1: The principal offers the incentive scheme $D^i(\beta_{ii}(\alpha_1,\dots,\alpha_i,\dots,\alpha_n),\beta_{ij}(\alpha_1,\dots,\alpha_i,\dots,\alpha_n))$.

Stage 2: Unit managers may write side contracts $C(\alpha_1,\dots,\alpha_n)$.

Stage 3: Unit managers make their announcements.

Stage 4: The principal and all unit managers observe the announcements. Task reallocation as determined by β_{ii},β_{ij} is executed.

6.2. Organization Specific Knowledge

If agents can collude, the tournament scheme in the last section is not collusion-proof. The unit managers can raise utility with a side contract in which they commit themselves to revealing nothing. Such a side contract $C((0,\dots,0),\dots,(0,\dots,0))$ implies

$$U_i^{Co}=\tfrac{1}{n}(n)=1>\tfrac{1}{n}\left(n-\sum_i \alpha_i\right)=U_i^{NC},$$

where U_i^{Co} is the utility when collusion is possible and U_i^{NC} the utility under no collusion when the principal uses a tournament scheme.

However, the principal can modify the tournament scheme to make it collusion proof. The principal can determine ex ante a unit manager who will be treated specially by the incentive scheme if other unit managers reveal the same share of unproductive tasks. Such an incentive scheme is discriminatory, but it helps to prevent collusion and to implement a first-best solution. To simplify notation we introduce the set of managers L who reveal the largest improvement in productivity:

$$L = \left\{ i \;\middle|\; \sum_{k=1}^{n} \alpha_i^k \geq \sum_{k=1}^{n} \alpha_j^k \quad for\, all\;\; j \neq i \right\}.$$

We denote the number of elements of L by m. We obtain:

Proposition 5

Suppose collusion is feasible and that the unit managers have organization specific knowledge. Then the principal can implement a first-best solution through a discriminatory incentive scheme. The incentive coefficients are given by:

$$\beta_{ij} = \begin{cases} 1 & if \quad i \in L \; and \;\; m = 1 \\ 1 - \varepsilon \alpha_i^j & if \quad i \in L,\, m > 1 \, and \;\, no \;\, k \in L \;\, with \;\, k < i \\ \dfrac{\varepsilon \alpha_i^j}{m-1} & if \quad i \in L,\, m > 1 \, and \;\, there \; exists \;\, k \in L \;\, with \;\, k < i \\ 0 & if \quad i \notin L \end{cases}$$

with $0 < \varepsilon < 2$

Collusion does not occur and every manager i announces $(A_1, \ldots, A_n)$. *Utilities are:*

$$U_1 = \sum_{i=1}^{n} (1 - \varepsilon A_i)(1 - A_i) = n + \sum_{i=1}^{n} A_i (\varepsilon A_i - \varepsilon - 1)$$

$$U_j = \frac{1}{n-1} \sum_{i=1}^{n} \varepsilon A_i (1 - A_i) = \frac{\varepsilon}{n-1} \sum_{i=1}^{n} A_i (1 - A_i) \;\; for \;\; j = 2, \ldots, n$$

The proof is given in the appendix. Note that the scheme is discriminatory as the manager with the lowest index is treated differently if several managers reveal the highest productivity improvements. As before, the unit managers have an incentive to win the tournament. To prevent collusion, the principal must, however, modify the tournament in two ways. First, he has to provide a positive share of tasks for all unit managers who reveal their knowledge completely. Thus, subcoalitions entering collusion agreements are not attractive. Second, the principal has to single out one of the unit managers under consideration with respect to the design of the incentive coefficients. In the tournament scheme, this is achieved by assigning a special share of tasks to the unit manager with the lowest index value if more than one unit manager reveals the maximum amount of unproductive tasks. The manager with the lowest index value in this case is called special unit manager. Moreover, while the utility of the special unit manager decreases with the announcement level in the discriminatory incentive scheme, the other unit managers' utility increases with a higher share of tasks revealed to the principal. Therefore, collusion by the grand coalition is not attractive either. The special unit manager obtains the largest share of tasks if the principal sets ε very small. If ε is higher and closer to 2, the special unit manager can be better or worse off than the other unit managers depending on $(A_1,\ldots,A_n)$.

6.3. Unit Specific Knowledge

Given the noncooperative solution derived in section 5.2., the unit managers can increase their payoffs through a side contract under unit specific knowledge. For a given difference in the shares of unproductive tasks across two units, the unit managers have higher utilities if they commit themselves to announcements with a lower absolute level but without changing the difference between them. To illustrate the possibility of collusion in this case, we consider two unit managers. Suppose that the shares of unproductive tasks, A_1, A_2, are given by $A_1 = A_2 + \Delta$ with $\Delta > 0$ and are sufficiently small. Since utility under the incentive scheme in section 5.2. depends on the relative announcements of the unit managers, the unit managers can raise their payoffs by lowering the absolute level of the announcements, leaving the relative levels unchanged. Suppose the unit managers write a side contract in which they commit themselves to the announcements $\alpha_1 = \Delta, \alpha_2 = 0$. Then, the utilities under the side contract are higher than without collusion for both unit managers because:

$$U_1^{NC} = 1 - \alpha_2 - \Delta + \varepsilon\Delta \leq 1 - \Delta + \varepsilon\Delta = U_1^{Co}$$

$$U_2^{NC} = 1 - \alpha_2 - \varepsilon\Delta \leq 1 - \varepsilon\Delta = U_2^{Co} \qquad 0 \leq \alpha_2 < \tfrac{1}{2} - \Delta$$

To implement a first-best solution, the principal must design an incentive scheme, which satisfies the following intuitive requirements. First, in order to prevent collusion, at least one unit manager must gain a higher utility from the noncooperative outcome than from a situation with side contracts. In the following, we denote this individual no-collusion condition for unit manager *i* by NC_i. Second, the individual incentive constraint must hold for each unit manager, which is most difficult to fulfill in the case of the unit manager who would benefit from side contracting. As demonstrated in the following proposition, a first-best incentive scheme that can overcome the collusion threat does not exist.

Proposition 6:

Suppose collusion is feasible and that the unit managers have unit specific knowledge. Then, no incentive scheme exists which implements a first-best solution.

The proof is given in the appendix. Colluding unit managers can agree to reduce their announcements without changing the order of revealed announcements, which under any feasible incentive scheme make the managers better off.[12]

7. Conclusion

In this paper we have analyzed incentives schemes in public organizations aimed at improving the productivity by motivating agents such as unit managers to reveal unproductive tasks. Since these organizations are often characterized by nonverifiable output, wage rigidities, lifetime employment guarantees, and limited career perspectives, other incentive schemes than standard stick and carrot incentives need to be applied. If an agent's utility is influenced by the sphere of control, as is often the case for unit managers, task or budget assignments are the principal's last resort in motivating agents.

Depending on the knowledge unit managers can give to the principal as hard information, simple tournament and rotation schemes implement a first-best solution if unit managers act noncooperatively. In both schemes, unit managers must be compensated for the loss of the announced tasks by being given tasks from other units. To prevent collusion in the case of organization specific knowledge, stronger requirements need to be fulfilled. In the case of organization specific knowledge, the principal must reward unit managers differently even if they make the same announcements. Thus, there is a trade-off between efficiency and

[12] One could use outside managers in order to break collusion by assigning tasks to them.

fairness. In the case of unit specific knowledge, an incentive scheme that can break collusion does not even exist.

Potential extensions of the paper include changes in the assumption on the symmetry of the units. While the incentive schemes for the case of organization specific knowledge will still hold, adjustments must be made for incentive schemes in the case of unit specific knowledge. Suppose, for instance, that the units differ with respect to their size. This would reduce incentives to reveal unproductive tasks for the manager of the larger unit because he cannot be compensated sufficiently by the residual tasks in the smaller unit. Overall, asymmetry of units tends to lower the power of partial rotation schemes.

A further fruitful extension of our paper is to enrich the framework by drawing upon the model and the considerations in Aghion and Tirole (1997). In particular, we could allow agents to screen projects on behalf of the principal where each project is associated with a verifiable monetary gain for the principal and private benefit for the agent. The agent can communicate a project proposal to the principal. The principal could either overrule an agent or order another agent to take over. In such a framework, the principal must provide incentives to search for information and its communication. The real question is whether our schemes remain optimal in such circumstances. While it appears that schemes such as the one in proposition 4 tend to generate appropriate incentives for the search of beneficial projects, a full-fledged analysis of this point would be a fruitful task for future research.

Although the derived schemes implement a first-best solution in our model, the use of such schemes must be complemented by additional considerations. In particular, the use of discriminatory tournament schemes could be problematic. Several authors emphasize that there is a positive relationship between fairness and agents' motivation (e.g. Akerlof and Yellen (1990)), since agents compare their performances and rewards with those of other agents. Moreover, the use of discriminatory incentive schemes may result in lobbying activities aimed at influencing the choice of the special unit manager, which may harm the principal in other respects (e.g. Milgrom and Roberts (1988, 1990)). Hence, the question whether discriminatory incentive schemes are applicable in public organizations merits further research. Another useful line of research would be the investigation of the possibilities to achieve productivity improvements by job design. In particular, to which extent public organizations should ensure that unit managers know the fine details of the production process in other areas than their own units. Our analysis shows a trade-off between the gains of specialization and the possibilities of providing incentives if people are specialized.

Our analysis suggests that, in general, task and budget assignments can be used as an incentive device motivating unit managers to improve productivity in public organizations. However, due to the difficulties to prevent collusion, task and budget assignments may not always be an equivalent substitute for standard stick and carrot incentives. Nevertheless, their use is indispensable in public organizations to achieve productivity improvements.

Appendix

Proof of proposition 2:

(i) Since the principal can verify the announcements of the unit managers, the only deviations that need to be checked are those downward of the equilibrium announcements. If one unit manager announces fewer than $\sum_i A_i$ unproductive tasks for the whole organization, given that everybody else announces $\sum_i A_i$, his utility is $U_i = 0$ and hence smaller than utility in equilibrium. Hence, the announcement vectors $\left(A_i^1, \dots, A_i^n\right)$, $i = 1, \dots, n$ are a Nash equilibrium of the announcement game under the tournament scheme.

(ii) In the next step, we show the uniqueness of the Nash equilibrium in the announcement game. Suppose there is a Nash equilibrium with $\max_i \left\{\sum_{j=1}^{n} \alpha_i^j\right\} < \sum_i A_i$. For at least one unit manager, say *k*, the payoff in this equilibrium cannot be larger than $\frac{1}{n}\left(n - \max_i \left\{\sum_{j=1}^{n} \alpha_i^j\right\}\right)$ because all unit managers either announce an aggregate share $\sum_{j=1}^{n} \alpha_i^j$ or some unit managers announce fewer and therefore receive nothing. The unit manager under consideration, however, can announce $\max_{i \neq k} \left\{\sum_{j=1}^{n} \alpha_i^j\right\} + \varepsilon$ and will receive utility $U_i = n - \max\left\{\sum_{j=1}^{n} \alpha_i^j\right\} - \varepsilon$, which is larger than in the candidate equilibrium for sufficiently small ε. Hence, no other Nash equilibria of the announcement game exist. (q.e.d.)

Proof of proposition 4:

Unit managers have a strict incentive to reveal all unproductive tasks in their unit if and only if

$$\frac{\partial U_i}{\partial \alpha_i} = -\beta_{ii} + \beta'_{ii}(1-\alpha_i) + \sum_{j \neq i} \beta'_{ij}(1-\alpha_j) > 0 \,.$$

Inserting β_{ii} and β_{ij} from the proposed incentive scheme yields

$$\frac{\partial U_i}{\partial \alpha_i} = -1 + \varepsilon \sum_{j \neq i} \alpha_j + \varepsilon \left(\sum_{j \neq i} 1 - \alpha_j \right) = -1 + \varepsilon(n-1) > 0$$

which holds if and only if $\frac{1}{n-1} < \varepsilon$ which determines the lower bound on ε.

The upper bound of ε is derived from the assumption that $A_i \in [0, \frac{1}{2}]$. The incentive scheme must guarantee that $\beta_{ii}, \beta_{ij} \in [0,1]$. Suppose that the share of unproductive tasks in all units is $A_i = \frac{1}{2}$ $\forall i$, i.e. the share of unproductive tasks is 50% in every unit. Under the candidate incentive scheme, unit managers reveal all their unproductive tasks, i.e. $\alpha_i = \frac{1}{2}$ $\forall i$, if ε is larger than the lower bound $\frac{1}{n-1}$. Hence, the incentive coefficients in equilibrium are given by:

$$\beta_{ii} = 1 - \varepsilon(n-1)\tfrac{1}{2}, \beta_{ij} = \tfrac{1}{2}\varepsilon$$

Since β_{ii}, β_{ij} must be in $[0,1]$ it follows immediately that $\varepsilon \leq \min\left\{2, \frac{2}{n-1}\right\} = \frac{2}{n-1}$ for $n > 1$. (q.e.d.)

Proof of proposition 5:

In order to prevent collusion the principal must provide at least some positive utility, i.e. $0 < \beta_{ik} < 1$, for the unit managers. Since $A_i \in [0, \frac{1}{2}]$ by assumption, $0 < \varepsilon < 2$ must hold. For example, suppose that $\varepsilon = 0$. Then, each unit manager $j = 2, \ldots, n$, has utility $U_j = 0$

under the proposed incentive scheme if unit managers act noncooperatively. In this scenario, unit managers can realize an allocation via the side contract $C(0,\dots,0)$ that is weakly better for all of the unit managers. While utility remains the same for unit managers $j = 2,\dots,n$, unit manager 1 will gain through this side contract. Similar considerations hold for the upper bound of ε.

Since all unit managers receive a positive fraction of tasks in the noncooperative equilibrium, only collusion contracts characterized by the same announcements from all unit managers need to be considered. Otherwise, unit managers would receive a utility of 0. Hence, under the discriminatory tournament scheme, a possible collusion contract would be $C(\alpha_1,\dots,\alpha_n)$ with $\alpha_i \le A_i$ and $\alpha_j < A_j$ for at least one *j*. Under such a side contract utilities are given by:

$$U_1^{Co} = n + \sum_{i=1}^{n} \alpha_i (\varepsilon\alpha_i - \varepsilon - 1)$$
$$U_j^{Co} = \frac{\varepsilon}{n-1}\sum_{i=1}^{n} \alpha_i (1-\alpha_i) < \frac{\varepsilon}{n-1}\sum_{i=1}^{n} A_i (1-A_i) = U_j^{NC}$$

Hence, the utility of unit managers $j = 2,\dots,n$ would be lower with such a collusion contract and therefore side contracts do not occur. (q.e.d.)

Proof of proposition 6:

We denote by $f(\alpha_i,\alpha_j)$ the absolute share of tasks unit manager *i* receives under a given incentive scheme when the announcements are α_i and α_j. We formulate the incentive constraints IC_i and IC_j and the conditions that managers *i* and *j* do not benefit from collusion (NC_i, NC_j).

$$
\begin{aligned}
&IC_i: && f(\alpha_i,\alpha_j) \ge f(\alpha_i-\Delta_i,\alpha_j)\\
&IC_j: && 2-\alpha_i-\alpha_j-f(\alpha_i,\alpha_j) \ge 2-\alpha_i-\alpha_j+\Delta_j-f(\alpha_i,\alpha_j-\Delta_j)\\
& && or \quad f(\alpha_i,\alpha_j-\Delta_j)-\Delta_j \ge f(\alpha_i,\alpha_j)\\
&NC_i: && f(\alpha_i,\alpha_j) \ge f(\alpha_i-\Delta_i,\alpha_j-\Delta_j)\\
&NC_j: && 2-\alpha_i-\alpha_j-f(\alpha_i,\alpha_j) \ge 2-\alpha_i+\Delta_i-\alpha_j+\Delta_j-f(\alpha_i-\Delta_i,\alpha_j-\Delta_j)\\
& && or \quad f(\alpha_i-\Delta_i,\alpha_j-\Delta_j)-\Delta_i-\Delta_j \ge f(\alpha_i,\alpha_j)
\end{aligned}
$$

The proof proceeds as follows.

The IC_j condition implies that $f(\alpha_i, \alpha_j)$ is strictly monotonically decreasing in the second argument. Since $0 \le f(\alpha_i, \alpha_j) \le 2$ for $\alpha_i, \alpha_j \in [0, \frac{1}{2}]$, according to a theorem of Lebesgue (see e.g. Riesz and Sz.-Nagy (1956)) $f(\alpha_i, \alpha_j)$ must be continuous almost everywhere.

Consider $f(\frac{1}{2}, \alpha_j)$ at some α_j where the function is continuous in the second argument. Let's choose some arbitrary small $\Delta_j > 0$. The IC_j implies:

$$f(\tfrac{1}{2}, \alpha_j - \Delta_j) \ge f(\tfrac{1}{2}, \alpha_j) + \Delta_j$$

Case I:

Suppose there exists some Δ_i^* $(0 < \Delta_i^* \le \frac{1}{2})$ such that

$$f(\tfrac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j) = f(\tfrac{1}{2}, \alpha_j) + \Delta_j$$

Then, both agents will benefit from collusion through the reduction of (Δ_i^*, Δ_j) since

$$f(\tfrac{1}{2}, \alpha_j) < f(\tfrac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j) \text{ and}$$

$$f(\tfrac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j) - \Delta_i^* - \Delta_j < f(\tfrac{1}{2}, \alpha_j)$$

Case II:

Suppose that there exists no $\Delta_i^* > 0$ such that

$$f(\tfrac{1}{2} - \Delta_i^*, \alpha_j - \Delta_j) = f(\tfrac{1}{2}, \alpha_j) + \Delta_j .$$

As the IC_i requires that

$$f(\tfrac{1}{2}, \alpha_j - \Delta_j) \ge f(\tfrac{1}{2} - \Delta_i, \alpha_j - \Delta_j)$$

we must have

$$f(\tfrac{1}{2}, \alpha_j - \Delta_j) \ge f(\tfrac{1}{2} - \Delta_i, \alpha_j - \Delta_j) > f(\tfrac{1}{2}, \alpha_j) + \Delta_j \text{ for all } \Delta_i,\ \tfrac{1}{2} \ge \Delta_i > 0 .$$

Let's choose $\Delta_i = \frac{1}{2}$. Hence, agent *i* benefits from collusion since

$$f\left(\tfrac{1}{2}, \alpha_j\right) < f\left(0, \alpha_j - \Delta_j\right)$$

Suppose agent *j* does not benefit from collusion. Hence

$$f\left(0, \alpha_j - \Delta_j\right) - \tfrac{1}{2} - \Delta_j \geq f\left(\tfrac{1}{2}, \alpha_j\right)$$

From IC_i $\left(f(\alpha_i, \alpha_j) \geq f(\alpha_i - \Delta_i, \alpha_j) \right)$ follows

$$f\left(\tfrac{1}{2}, \alpha_j - \Delta_j\right) - \tfrac{1}{2} - \Delta_j \geq f\left(\tfrac{1}{2}, \alpha_j\right)$$

Since $f(\frac{1}{2}, \alpha_j)$ is continuous at α_j this is a contradiction when Δ_j is sufficiently small. Hence, agent *j* also benefits from collusion which completes the proof. (q.e.d.)

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