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## ABSTRACT

### Economic Integration and Agglomeration in a Middle Product Economy\*

The Paper examines the interactions between economic integration and population agglomeration in a middle product economy displaying neoclassical growth. There are two vertically-integrated economies. Each consists of a large number of final good competitive firms operating plants in both regions, and a large number of intermediate goods monopolistically competitive firms operating each in only one region. While immobile workers are employed with intermediate goods to produce the final good, mobile workers are used to design the line of differentiated intermediate good inputs. Capital is immobile and the final good is non-traded, whereas the intermediate goods are traded. We find that employment agglomeration and output growth need not be positively related. Furthermore, trade is not necessarily beneficial to regional growth, whereas trade between the two regions need not be associated with a widened skilled-unskilled wage gap.

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# 1 Introduction

Contemporary research focussing on the relationships between growth, trade and the location of economic activity looks like a patchwork of results that cannot be generally reconciled with data. For example, whereas there is a large agreement among economists about the positive role of international trade in fostering long-run economic growth, empirical studies fail to identify a robust and significant relation between the two processes (Levin and Renelt, 1992; Frankel and Romer, 1999). Likewise, much concern has been raised regarding the impact of trade on between-group income disparities (Leamer, 1993; Wood, 1994; Lawrence and Slaughter, 1993), while recent empirical evidence shows that economic integration has an ambiguous impact on regional income gaps (Magrini, 2004). Last, a common finding in modern regional growth is that population agglomeration and output growth are positively related when economies are sufficiently integrated (Baldwin and Martin, 2004). Here also, such a positive relation cannot be identified empirically in a robust fashion (Berliant and Wang, 2004). All of these suggest the existence of a strong tension between diverging economic forces that have not yet been captured within a unified framework. Our purpose is to contribute to the building of such a framework by tackling the problem from a very different angle.

In this respect, we want to stress the fact that most of the literature dealing with the implications of economic integration has been conducted by focussing on the final product market. However, since the seminal work of Sanyal and Jones (1982), it has been increasingly recognized that “the bulk of international trade consists of the exchange of intermediate products, raw materials, and goods which require further local processing before reaching the final consumer” (page 16). More precisely, almost all contemporary final commodities make use of inputs bought on the world markets together with inputs available in national markets. This state of affairs has triggered more and more interest in what is called the *middle*

*product* market.<sup>1</sup> In such a context, economic integration takes the special form of *vertically integrated regions* that trade a growing number as well as larger quantities of intermediate inputs from each other. The empirical relevance of this form of trade in international business is well documented and explains why we focus on it. For example, Yi (2003, page 55) observes that “vertical specialization [integration] has grown about 30 percent and accounts for about one-third of the growth in trade in the last 20-30 years”.

This paper examines the interactions between trade and population agglomeration in a neoclassical growth model with two vertically integrated economies in the presence of intermediate goods.<sup>2</sup> Its primary goal is to shed light on three still-open issues: (i) whether employment agglomeration and output growth are necessary positively related in a vertically integrated economy, (ii) whether trade in intermediate goods is always beneficial to economic growth, and (iii) whether intermediate goods trade widen skilled-unskilled wage differential when skilled workers are mobile.

Specifically, our economy involves two regions (countries or regional blocks). Each consists of a large number of final good competitive firms operating plants in both regions, and a large number of intermediate goods monopolistically competitive firms operating each in only one region. In addition, each region has a large number of unskilled-immobile and of skilled-mobile workers. The main features of our framework are as follows: (i) whereas immobile-unskilled workers are employed with intermediate goods to produce the final good, mobile-skilled workers are used to design the production line that captures the diversity of differentiated intermediate-good inputs in producing the final good; (ii) capital is immobile, as in the Hecksher-Ohlin model of international trade; (iii) the final good is a nontradeable whereas the intermediate goods are traded, as in the middle product model;

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<sup>1</sup>See the survey paper by Jones and Neary (1984, Section 3.1) and the papers cited therein.

<sup>2</sup>While we focus on a stationary equilibrium with transitional output growth, the model can be easily extended to allow for exogenous technical progress in the final good sector and hence exogenous long-run growth.

(iv) but, unlike the middle product model, we allow for imperfect competition with costly shipping as well as endogenous capital accumulation within an intertemporal optimizing setting.

The assumptions of immobile capital and of a non-traded final product are made because our main focus is on intermediate goods trade; they do not affect the nature of our analysis but vastly simplify the analysis. Given this proviso, we will see that our model enable us to study a number of important issues mentioned above concerning economic integration and economic development in the globalization process. This is accomplished by studying the intermediate goods demand and supply as well as their pricing and interregional allocation. We then characterize the steady-state equilibrium by studying how employment agglomeration, capital accumulation and output growth respond to changes in the unit transport cost and the designing efficiency of the production process.

The most distinctive feature of our model lies in the dynamic analysis of the middle product market, which allows us to shed new light on the issues mentioned above. In this respect, our main findings may be summarized as follows. Regarding the three questions raised in the foregoing, we first show that *employment agglomeration and output growth need not be positively related*, thus explaining why a positive and robust relation cannot be identified empirically. As for the last two questions, our model suggests that *trade in intermediate goods does not always benefit growth*, whereas it need not widen skilled-unskilled wage differential when skilled workers are mobile. More precisely, consider region 1 as a large economy and region 2 as a small economy. Under a more efficient design in region 1's final good production process, this region experiences more employment agglomeration, higher capital accumulation and larger output growth. However, by opening economies to trade via a decrease in trade cost, employment agglomeration declines in region 1, while its capital accumulation and output growth may be higher.

From the methodological point of view, our paper first shows that, contrary

to general beliefs, the impact of trade liberalization may differ as to the final and intermediate goods markets. Indeed, whereas lower trade costs in conventional setups generally triggers more agglomeration of the final good sector (Fujita, Krugman and Venables 1999), we will see that the opposite holds for the middle product market. Second, our modeling strategy differs from that used by Dixit and Stiglitz (1977) and Ethier (1982) in a way that will be made clear in section 3.2 below.

### **Related Literature**

Three related papers are Sanyal and Jones (1982), Ventura (1997) and Ottaviano, Tabuchi and Thisse (2002).<sup>3</sup> On the one hand, our assumption of traded intermediate goods in conjunction with non-traded final goods resembles the setups by Sanyal and Jones and by Ventura, although the structure of our model and the purpose of our study are very different. In particular, whereas Sanyal and Jones develop the first theory of trade in two middle products in a static framework, Ventura focuses primarily on how trade in two intermediate goods may support permanent growth by preventing an economy from diminishing returns. Our setting differs from theirs in several respects: (i) we have a large number of intermediate tradeables provided under imperfect competition, (ii) we allow for two types of labor (mobile and immobile), thus permitting us to endogenize the international distribution of the intermediate sector, and (iii) we deal with the impact of regional agglomeration of this sector on growth and trade.

On the other, our assumption of variety substitution with non-constant markups resembles the consumption variety setup in Ottaviano *et al.* Yet, we consider intermediate goods trade and capital accumulation, which differ sharply from their framework. An interesting finding in their paper is that with final goods trade, a

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<sup>3</sup>There is a vast theoretical literature concerning trade and growth that is remotely related to our paper. To name but a few, this includes: (i) international specialization models (Stokey 1991; Bond, Trask and Wang 2003), (ii) product variety models (Grossman and Helpman 1992; Xie 1998), (iii) reverse engineering models (Rivera-Bartiz and Romer 1991; Wan 2002), and (iv) technology transfer/adoption models (Chen and Shimomura 1998).

strong variety bias and a low transport cost make regional agglomeration sustainable. With intermediate goods trade, our result concerning variety bias corroborates with theirs, but that regarding the transport cost contrasts with their conclusion.

## 2 The Model

The global economy consists of two regions, indexed by  $i = 1, 2$ , and two sectors, the intermediate and final sectors. The final good, produced by multinational or multiregional enterprises, is homogenous and non-traded. It can be used for consumption and investment. Further, we assume that, in each region, the final sector is competitive. In what follows, we will show that the final good may be chosen as the numéraire in each region. By contrast, the intermediate sector supplies differentiated varieties and shipping one unit of any variety between the two regions requires  $\tau > 0$  units of the numéraire, whereas intraregional shipping costs are zero. When both regions import nontrivial amount of middle products of some varieties from each other, the two regions are said to be *vertically integrated*.

There are two types of labor employed in the final sector, the skilled and the unskilled workers. The skilled are mobile and can move instantaneously at zero cost from one region to the other; by contrast, the unskilled are immobile. The mass of unskilled available in each region is normalized to 1. The total mass of skilled is given and denoted by  $L$ .

There are three theaters of activities in our model: (i) the intermediate goods production, (ii) the final good production and (iii) the intertemporal consumption choice. We describe each one in order.

### 2.1 The intermediate sector

Let  $N_i$  denotes the mass of intermediate goods produced in region  $i$ . For notational convenience, we rank the intermediate goods in such a way that variety  $v \in D_1 \equiv$

$[0, N_1]$  is produced in region 1, whereas variety  $v \in D_2 \equiv [N_1, N]$  is produced in region 2.<sup>4</sup>

Even though the production of intermediate goods is decentralized, *skilled workers are hired by the final sector to design the intermediate product line*. This is done by assuming that one new variety of the intermediate sector needs  $\phi > 0$  units of skilled labor. Hence, we have

$$N_i = \frac{1}{\phi} \lambda_i L \quad i = 1, 2 \quad (1)$$

where  $\lambda_i \in [0, 1]$  is the endogenous share of skilled labor in region  $i$  ( $\lambda_1 + \lambda_2 = 1$ ). In particular, for the same mass of skilled workers, a decrease in  $\phi$  amounts to increasing the number of varieties. Furthermore, each variety is supplied by a single firm. Accordingly, the intermediate sector involves a continuum  $N$  of monopolistically competitive firms with

$$N = N_1 + N_2 \quad (2)$$

Firms operating in the intermediate sector follow a mill pricing policy. This amounts to saying that the delivered price of variety  $v$  produced in region  $i$  and transported to region  $j$  ( $p_{ij}(v)$ ) is defined by the sum of its mill price  $p_i(v)$  and transport cost. We thus have:

$$p_{ii}(v) = p_i(v) \quad \text{and} \quad p_{ij}(v) = p_i(v) + \tau \quad (j \neq i). \quad (3)$$

We also assume that each unit of variety  $v$  requires  $\eta > 0$  units of the numéraire. Let  $x(v, t)$  be the output of firm  $v$  at time  $t$ . The profit of this firm, located in region  $i$  when  $v \in D_i$ , at time  $t$  is therefore

$$\pi_i(v, t) = \max_{x(v)} [p_i(v, t) - r(t)\eta] x(v, t) \quad (4)$$

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<sup>4</sup>In each region, any single variety is inessential as it has zero measure in our continuum setup. Accordingly, we can always re-order the varieties as described in the foregoing.

where  $r(t)$  is the interest rate prevailing at time  $t$ . Thus, its discounted value from  $t$  to  $\infty$  is given by

$$\Pi(v, t) = \int_t^\infty \pi(v, \mu) e^{-\int_t^\mu [r(s) - 1] ds} d\mu. \quad (5)$$

We now describe how the demand of a variety is split between the two regions. Let  $\delta_i(v)$  be the endogenous fraction of the quantity of variety  $v \in D_i$  used to produce the final good in region  $i$ . The basic structure of the middle product economy is delineated in Figure 1. The endogenous allocation of intermediate goods (varieties) may then be summarized as follows:

Variety Region	$v \in D_1$	$v \in D_2$	Variety Demand
1	$\delta_1(v)x(v)$	$(1 - \delta_2(v))x(v)$	$x_1^d(v)$
2	$(1 - \delta_1(v))x(v)$	$\delta_2(v)x(v)$	$x_2^d(v)$
Variety Supply	$x(v)$	$x(v)$	

This enables us to write the regional demands for variety  $v$  as follows:

$$x_1^d(v) = \begin{cases} \delta_1(v)x(v) & \text{if } v \in D_1 \\ (1 - \delta_2(v))x(v) & \text{if } v \in D_2 \end{cases} \quad (6)$$

$$x_2^d(v) = \begin{cases} (1 - \delta_1(v))x(v) & \text{if } v \in D_1 \\ \delta_2(v)x(v) & \text{if } v \in D_2 \end{cases}$$

Production of intermediation goods incurs costs in terms of final goods. Setting such costs by

$$K_i \equiv \eta \left[ \int_{D_i} x(v) dv \right] \quad i = 1, 2 \quad (7)$$

we follow Romer (1990) and interpret the total quantity  $K_i$  of the numéraire used in region  $i$  for producing all the varieties of this region as its *capital good*. The capital good is rented from consumers at the market gross rental rate  $r$ .

## 2.2 The final sector

Firms producing the final good are identical and perfectly competitive. The total mass of firms is one and firms are represented by an index that is uniformly distributed over the unit interval. Because their output cannot be traded, it is optimal for each firm belonging to the final sector to operate two plants, one in each region. In other words, a final good firm can be regarded as a multinational or multiregional enterprise.

Because our main focus is on the skilled workers, we will use a framework in which unskilled workers are passive. Formally, this means that we assume that the production function of a final producer located in region  $i$  is given by

$$\text{output} = \begin{cases} Y_i & \text{if one unit of unskilled labor is used} \\ 0 & \text{otherwise} \end{cases}$$

where  $Y_i$  displays strictly decreasing returns, taking the following form:

$$\begin{aligned} Y_i &= \alpha \int_0^N x_i^d(v) dv - \frac{\beta - \gamma}{2} \int_0^N [x_i^d(v)]^2 dv - \frac{\gamma}{2} \left[ \int_0^N x_i^d(v) dv \right]^2 \\ &= \int_0^N \left[ \alpha - \frac{\beta - \gamma}{2} x_i^d(v) \right] x_i^d(v) dv - \frac{\gamma}{2} \left[ \int_0^N x_i^d(v) dv \right]^2. \end{aligned} \quad (8)$$

The parameters in (8) are such that  $\alpha > 0$  and  $\beta > \gamma$ . In this expression,  $\alpha$  expresses the intensity of production for the intermediate goods, whereas  $\beta > \gamma$  means that the level of production is higher when the production process is more sophisticated. Accordingly, we will refer to  $\beta - \gamma > 0$  as the *variety bias* in the production process. For a given value of  $\beta$ , the parameter  $\gamma > (\text{resp.}, <) 0$  implies that intermediate good inputs are Pareto substitutes (resp., complements).

Note that our technology is such that each firm in the final sector uses a fixed requirement of unskilled labor, regardless of the size of the range of intermediate goods. As a result, the wage level of the unskilled is generally undetermined. Since unskilled workers are not mobile, their market wages may be different. Nonetheless, the final good, even though it is non-traded, can still be chosen as the numéraire in

each region by adjusting the wages of the unskilled labor such that the Law of One Price applies to the final good. As there is only one final good, this normalization is inconsequential to our analysis of the effect of a reduction in trade barriers on the skilled-unskilled wage gap (see (29) below).

Because firms are identical and represented by an index uniformly distributed over the unit interval, there is no need to differentiate between the individual and aggregate output. After substituting in (6), we obtain the output of regions 1 and 2, respectively

$$\begin{aligned}
Y_1 = & \int_{D_1} \left[ \alpha - \frac{\beta - \gamma}{2} \delta_1(v)x(v) \right] \delta_1(v)x(v) dv \\
& + \int_{D_2} \left[ \alpha - \frac{\beta - \gamma}{2} (1 - \delta_2(v))x(v) \right] [1 - \delta_2(v)]x(v) dv \\
& - \frac{\gamma}{2} \left\{ \left[ \int_{D_1} \delta_1(v)x(v) dv \right]^2 + \left[ \int_{D_2} (1 - \delta_2(v))x(v) dv \right]^2 \right\},
\end{aligned}$$

and

$$\begin{aligned}
Y_2 = & \int_{D_1} \left[ \alpha - \frac{\beta - \gamma}{2} (1 - \delta_1(v))x(v) \right] (1 - \delta_1(v))x(v) dv \\
& + \int_{D_2} \left[ \alpha - \frac{\beta - \gamma}{2} \delta_2(v)x(v) \right] \delta_2(v)x(v) dv \\
& - \frac{\gamma}{2} \left\{ \left[ \int_{D_1} (1 - \delta_1(v))x(v) dv \right]^2 + \left[ \int_{D_2} \delta_2(v)x(v) dv \right]^2 \right\},
\end{aligned}$$

so that the aggregate output of the final sector is  $Y = Y_1 + Y_2$ . Thus, our setting allows for the substitution between capital and skilled labor within and between regions, via the use of *an endogenous number of intermediate goods*. However, there is no substitution between skilled labor and capital, on the one hand, and unskilled labor, on the other.

The final sector firms optimize over the two regions. Profits earned in regions 1 and 2 are given, respectively, by

$$\begin{aligned}
P_1 = & Y_1 - \int_{D_1} p_1(w)\delta_1(w)x(w)dw \\
& - \int_{D_2} (p_2(w) + \tau)(1 - \delta_2(w))x(w)dw - W_{U_1} - W_S\lambda_1L
\end{aligned}$$

and

$$\begin{aligned}
P_2 &= Y_2 - \int_{D_1} (p_1(w) + \tau)(1 - \delta_1(w))x(w)dw \\
&\quad - \int_{D_2} p_2(w)\delta_2(w)x(w)dw - W_{U_2} - W_S\lambda_2L.
\end{aligned}$$

where  $W_{U_i}$  denotes the wage rate of the unskilled workers in region  $i$  and  $W_S$  the common wage rate of the skilled workers. Hence, the global profits of the final sector are:

$$\begin{aligned}
P[x(i), \delta_1(i), \delta_2(i)] &= P_1 + P_2 \\
&= Y - \int_{D_1} [p_1(v) + \tau(1 - \delta_1(v))]x(v)dv \\
&\quad - \int_{D_2} [p_2(v) + \tau(1 - \delta_2(v))]x(v)dv - 2\bar{W}_U - W_S L.
\end{aligned}$$

where  $\bar{W}_U = \frac{1}{2} \sum_{i=1}^2 W_{U_i}$  is the average market wage for the unskilled. The flow value in time  $t$  of the final sector is therefore given by

$$v(t) = \max_{\{x(i), \delta_1(i), \delta_2(i)\}} P[x(i), \delta_1(i), \delta_2(i)] \quad (9)$$

whereas its discounted value from  $t$  to  $\infty$  is as follows:

$$V(t) = \int_t^\infty v(\mu) e^{-\int_t^\mu [r(s)-1]ds} d\mu. \quad (10)$$

## 2.3 Consumers

The total mass of consumers residing in region  $i$  is given by

$$M_i = 1 + \lambda_i L = 1 + \phi N_i. \quad (11)$$

Totally differentiating (11) with respect to time yields

$$m_i \equiv \frac{\dot{M}_i}{M_i} = \left( \frac{\lambda_i L}{M_i} \right) \frac{\dot{\lambda}_i}{\lambda_i}. \quad (12)$$

Denoting the capital depreciation rate by  $d_i$  and the amount of final good in region  $i$  used for consumption by  $Z_i$ , the dynamics of regional capital accumulation is thus governed by the equation of motion:

$$\dot{K}_i(t) = Y_i(t) - Z_i(t) - d_i K_i - \int_{D_i} [(1-r)\eta + \tau(1-\delta_i(v))] x(v) dv \quad (13)$$

where the time index  $t$  will be suppressed whenever it does not generate any confusion. In (13), the last term may be explained as follows. First,  $-\int_{D_i} (1-r)\eta x(v) dv > 0$  measures the rental revenue from capital. Second, as the transport rate is expressed in terms of the numéraire, the term  $\tau(1-\delta_i(v))x(v) > 0$  stands for the transport costs of variety  $v$  exported from region  $i$ .<sup>5</sup> In order to avoid double counting, the transport costs of region  $i$ 's imports do not appear in (13).

Because the main focus of the paper is on the agglomeration of skilled workers, we consider the simple case in which *all consumers* (regardless of their differences in skill) *equally share the capital of the region in which they reside*. As a consequence, we can denote each worker's consumption, capital and output as follows:

$$z_i \equiv \frac{Z_i}{M_i} \quad k_i \equiv \frac{K_i}{M_i} \quad y_i \equiv \frac{Y_i}{M_i} \quad i = 1, 2$$

Dividing throughout by the correspondent population, one can rewrite (13) to obtain

$$\dot{k}_i = y_i - z_i - (m_i + d_i)k_i - \frac{1}{M_i} \int_{D_i} [(1-r)\eta + \tau(1-\delta_i(v))] x(v) dv. \quad (14)$$

In this setup, any consumer living in region  $i$  chooses the consumption plan  $\{z_i(t)\}$  that maximizes her lifetime utility  $U_i$  subject to the capital accumulation equation (14).

As in the endogenous or neoclassical growth literature, we assume that the lifetime utility is time separable with a constant time preference rate ( $\rho > 0$ ) and the instantaneous utility function exhibits constant elasticity of intertemporal substitution ( $\sigma > 0$ ). Under perfect foresights, a constant parameter profile and the

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<sup>5</sup>Recall that shipping the intermediate goods requires real resources.

assumption that no one would move without a strictly positive valuation gain, the outcome of the optimization problem must have a time-invariant location solution. That is, if a skilled worker optimally chooses a particular location at time 0, it is always optimal for her to stay at that location thereafter. Since our focus is on characterizing the long-run equilibrium, we may then assume without loss of generality that skilled workers determine their residential location at the beginning of their lifetime planning. A skilled worker's optimization can, therefore, be specified as follows: at time 0,

$$\max_{i=1,2} U_i^0$$

where

$$U_i^0 = \max_{\{z_i(t)\}} \int_0^\infty \frac{[z_i(t)]^{1-\sigma^{-1}} - 1}{1 - \sigma^{-1}} e^{-\rho t} dt \quad \text{s.t. (14)} \quad (15)$$

In other words, at time 0 each skilled worker chooses a region where to reside and work; she then receives the current returns from the capital invested in the corresponding region.<sup>6</sup> The skilled worker's problem may thus be solved in two stages where the standard backward solving technique applies (i.e., second stage solved first). In the second stage, given the locational choice, the Maximum Principle applies to the intertemporal optimization problem. In the first stage, by comparing the values of  $U_1^0$  and  $U_2^0$ , the skilled worker determines the residential location that yields the higher valuation at time 0. Hence, the assumption of mobility of the skilled workers allows us to determine the way in which the intermediate sector is distributed between the two regions. By contrast, the optimization problem of unskilled workers (who are immobile) involves the second stage only.

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<sup>6</sup>In the absence of a perpetually growing force, the lifetime utility must always be bounded under the forgoing specification.

### 3 The Equilibrium

We begin by solving each agent's optimization problem by imposing *ex post* symmetry. We then study the properties of the steady-state equilibrium.

#### 3.1 The final sector

Using the expressions for  $Y_1$  and  $Y_2$ , we can differentiate  $Y$  with respect to  $x(v)$  for  $v \in D_i$  ( $i = 1, 2$ ) to obtain:

$$\frac{dY}{dx(v)} = \alpha - (\beta - \gamma)\Delta_i(v)x(v) - \gamma \int_{D_i} \Psi_i(v, w)\Delta_i(w)x(w)dw \quad (16)$$

where  $\gamma$  in the last term in (16) reflects the degree of *intraregional substitution* among varieties locally available, whereas, because of interregional transport cost, the degree of *interregional substitution* is captured by  $\gamma$  multiplied by the following two terms:

$$\begin{aligned} \Delta_i(v) &\equiv (\delta_i(v))^2 + (1 - \delta_i(v))^2 \\ \Psi_i(v, w) &\equiv [(1 - \delta_i(w)) + \delta_i(v)(2\delta_i(w) - 1)] / \Delta_i(w) \end{aligned}$$

(note that  $\Psi_i(v, v) = 1$ ). This in turn can be used with (9) to derive a final producer's first-order condition with respect to  $x(v)$  (i.e.,  $dv(t)/dx(v) = 0$ ) as follows:

$$\alpha - (\beta - \gamma)\Delta_i(v)x(v) - \gamma \int_{D_i} \Psi_i(v, w)\Delta_i(w)x(w)dw = \tilde{p}_i(v) \quad (17)$$

where, for  $v \in D_i$ ,

$$\tilde{p}_i(v) \equiv [\Gamma(i)p_1(v) + (1 - \Gamma(i))p_2(v)] + \tau \{1 - [\Gamma(i)\delta_1(v) + (1 - \Gamma(i))\delta_2(v)]\} \quad (18)$$

may be interpreted as the *average trading price* of variety  $v \in D_i$ , given that  $\Gamma(v) = 1$  for  $v \in D_1$  and  $\Gamma(v) = 0$  for  $v \in D_2$ . This price reflects the fact that variety  $v$ , produced in region  $i$ , is bought in both regions at the same mill price, a fact that

ties together the two regional markets.<sup>7</sup>

Some tedious manipulations of (17) lead to:

$$\Delta_i(w)x(w) - \Delta_i(v)x(v) = -\frac{1}{\beta - \gamma} [\tilde{p}_i(w) - \tilde{p}_i(v)] \quad (19)$$

and

$$\begin{aligned} \tilde{p}_i(v) = & \alpha - \left[ \beta - \gamma + \gamma \int_{D_i} \Psi_i(v, w) dw \right] \Delta_i(v)x(v) \\ & - \gamma \int_{D_i} \Psi_i(v, w) [\Delta_i(w)x(w) - \Delta_i(v)x(v)] dw. \end{aligned} \quad (20)$$

Hence, the difference between the demands for any two particular varieties produced in region  $i$  and weighted by  $\Delta_i$  is a linear function of the corresponding price difference. We will see that this property will enable us to solve analytically for the equilibrium price and quantity of each variety.

Combining (19) and (20), we get

$$\begin{aligned} \tilde{p}_i(v) = & \alpha - \left[ \beta - \gamma + \gamma \int_{D_i} \Psi_i(v, w) dw \right] \Delta_i(v)x(v) \\ & + \frac{\gamma}{\beta - \gamma} \int_{D_i} \Psi_i(v, w) [\tilde{p}_i(w) - \tilde{p}_i(v)] dw \end{aligned} \quad (21)$$

which yields the demand function for variety  $v \in D_i$  ( $i = 1, 2$ ):

$$x(v) = a_i(v) - b_i(v)\tilde{p}_i(v) + c_i(v) \int_{D_i} \Psi_i(v, w) [\tilde{p}_i(w) - \tilde{p}_i(v)] dw \quad (22)$$

where the three coefficients are defined as

$$b_i(v) \equiv \left\{ \Delta_i(v) \left[ \beta + \gamma \left( \int_{D_i} \Psi_i(v, w) dw - 1 \right) \right] \right\}^{-1}$$

as well as  $a_i(v) \equiv \alpha b_i(v)$  and  $c_i(v) \equiv [\gamma/(\beta - \gamma)] b_i(v)$ . In (22), the last term stands for the *competition effect*: when firm  $v$  charges a price higher (resp., lower) than

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<sup>7</sup>An alternative setting is to assume segmented markets in which firms choose one price for each regional market. The current framework, however, fits better the literature on trade and vertical integration.

competitors, the term will be negative (resp., positive), thus shifting down (resp., up) the demand curve for variety  $v$ .

Similarly, differentiating  $Y$  with respect to  $\delta_i(v)$  for  $v \in D_i$  ( $i = 1, 2$ ) yields

$$\frac{dY}{d\delta_i(v)} = - \left[ (\beta - \gamma) (2\delta_i(v) - 1) x(v) + \gamma \int_{D_i} (2\delta_i(w) - 1) x(w) dw \right] x(v) \quad (23)$$

Hence, the first-order condition for (9) with respect to  $\delta_i(v)$  becomes:

$$(\beta - \gamma) [2\delta_i(v) - 1] x(v) + \gamma \int_{D_i} [2\delta_i(w) - 1] x(w) dw = \tau x(v). \quad (24)$$

Equations (22) and (24) jointly determine the total demand for variety  $v$  ( $x(v)$ ) as well as its allocation between the two regions ( $\delta_i(v)$ ) as functions of its average trading price ( $\tilde{p}_i(v)$ ) and the range of varieties ( $D_i$ ) produced in the same region.

As in standard theory on product differentiation, we consider the case of symmetry. In this case, (24) can be simplified as follows:

$$[\beta + \gamma(N_i - 1)] (2\delta_i - 1) = \tau$$

or

$$\delta_i^* = \frac{1}{2} \left[ 1 + \frac{\tau}{\beta + \gamma(N_i - 1)} \right] \equiv \delta_i(N_i). \quad (25)$$

Thus, we have:

**Proposition 1** (Intermediate Good Allocation) *The lower the interregional transport cost and/or the higher the variety bias is, the more each region is vertically integrated. Furthermore, when varieties are substitutes (resp., complements), the larger the mass of local varieties, the more (resp., less) each region is vertically integrated.*

In words, a smaller proportion of the local varieties used by the final producers established in the corresponding region implies more vertical integration and a lower degree of regional agglomeration of skilled workers. In particular, in the extreme

case in which  $\delta_i^* = 1/2$ , the two regions are completely integrated; in the other extreme case in which  $\delta_i^* = 1$ , all skilled workers are agglomerated in region  $i$ .

The share of an intermediate good used by the final sector in the region in which it is produced increases with the interregional transport cost because the foreign varieties are less attractive. This contrasts with Ottaviano *et al.* (2002) in which lower transport costs of final goods between regions make regional agglomeration sustainable, thus showing that *results holding for the final good trade model do not necessarily carry over to the middle product economy.*

Furthermore, the share of an intermediate good used by the final sector decreases (resp., increases) with the endogenous number of local substitutable (resp., complementary) varieties because final sector firms care more (resp., less) about finding local opportunities. The main message is the positive relationship between the mass of local varieties ( $N_i$ ) and the extent of vertical integration, as captured by  $1 - \delta_i^*$ .

We now turn to the level of demand of variety  $v$ . When  $v, w \in D_i$ , the symmetry assumption implies that  $\tilde{p}_i(w) - \tilde{p}_i(v) = 0$ . Likewise, for all  $v, w \in D_i$ ,  $\Delta_i(v) = \delta_i^2 + (1 - \delta_i)^2$ ,  $\Psi_i(v, w) = 1$ ,  $b_i(v) = \{[\beta + \gamma(N_i - 1)][\delta_i^2 + (1 - \delta_i)^2]\}^{-1} \equiv b_i$ , so that the average trading price becomes:

$$\tilde{p}_i(v) = p_i + \tau(1 - \delta_i) \quad (26)$$

which is given by the sum of the mill price and the export-adjusted unit transport cost. Using (26), we can then derive the demand for variety  $v$  under symmetry as:

$$x_i = \frac{\alpha - p_i - \tau(1 - \delta_i)}{[\beta + \gamma(N_i - 1)][\delta_i^2 + (1 - \delta_i)^2]} \quad (27)$$

or,  $x_i = a_i - b_i[p_i + \tau(1 - \delta_i)]$ , which is decreasing in its own price, in the unit transport cost, and, for a given value of  $\delta_i$ , in the mass of local varieties. The effect of  $\delta_i$  on variety  $v$ 's demand is, however, ambiguous:

$$\frac{\partial x_i}{\partial \delta_i} = \tau b_i(1 - 2x_i) \begin{matrix} \geq \\ < \end{matrix} 0 \quad \text{iff} \quad x_i \begin{matrix} \leq \\ > \end{matrix} \frac{1}{2}.$$

That is, when the demand level of a particular intermediate good is low, retaining a larger share of this variety for the local final producers raises its demand. Substituting (25) into (27) leads to:

$$x_i = \frac{[\beta + \gamma(N_i - 1)][2(\alpha - p_i) - \tau] + \tau^2}{[\beta + \gamma(N_i - 1)]^2 + \tau^2}. \quad (28)$$

Some tedious calculations reveal that

$$\frac{\partial x_i}{\partial \tau} = -\frac{2\tau^2 \{2\tau(1 - \delta_i)^2 + (2\delta_i - 1)^2 [2(\alpha - p_i) - \tau]\}}{\{[\beta + \gamma(N_i - 1)]^2 + \tau^2\}^2 (2\delta_i - 1)^3} < 0.$$

Thus, the economy-wide demand for an intermediate good produced in region  $i$  ( $x_i$ ) is unambiguously decreasing in the unit transport cost ( $\tau$ ), because from (18) the gross price (inclusive of the transport cost) is higher. Summarizing, we have:

**Proposition 2** (Intermediate Good Demand) *The demand of any variety decreases in the interregional transport cost; it increases (resp., decreases) with its share in the intermediate good consumption by local plants when its input is smaller (resp., larger) than 1/2.*

In equilibrium, the wages of the skilled in the two regions must be equal because the skilled are mobile and because the final good is the numéraire in each region. In addition, free entry and exit in the intermediate sector makes the skilled workers the residual claimers, thus implying that profits are zero in the final sector. Consequently, it follows from (9) that, for every period, the equilibrium wage of the skilled workers is given by:

$$W_S = \frac{1}{L} \left\{ Y - \sum_{i=1}^2 N_i \tilde{p}_i x_i - 2\bar{W}_U \right\} \quad (29)$$

Hence, once the wage of the unskilled is set by the market, the wage of the skilled is adjusted in a way such that the entire profit made by the final good producers is absorbed by these workers.<sup>8</sup> Observe that the relationship between  $W_S$  and the wages

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<sup>8</sup>We may allow for the existence of a fixed setup cost  $V_0 > 0$  to absorb normal profits. In this case, the zero profit condition must be modified to have the term  $(r - 1)V_0$  subtracted from the righthand side.

of the unskilled workers is linear, thus confirming that our normalization rules have no impact on the wage gap.

### 3.2 The intermediate sector

Using (17) and the first equality of (26), we get

$$\frac{dp_i(v)}{dx(v)} = -\beta\Delta_i(v)$$

which can be used with (4) to derive intermediate good producers' first-order conditions ( $v \in D_i$ ,  $i = 1, 2$ ):

$$\begin{aligned} \frac{d\pi_i(v)}{dx(v)} &= p_i(v) - r\eta + x(v)\frac{dp_i(v)}{dx(v)} \\ &= p_i(v) - r\eta - \beta\Delta_i(v)x(v) = 0. \end{aligned} \quad (30)$$

This determines the supply of variety  $v \in D_i$  ( $i = 1, 2$ ):

$$x(v) = \frac{p_i(v) - r\eta}{\beta\Delta_i(v)}. \quad (31)$$

Under symmetry, the production of an intermediate good in region  $i$  becomes:

$$x_i = \frac{p_i - r\eta}{\beta[\delta_i^2 + (1 - \delta_i)^2]} \equiv x_i(N_i, p_i) \quad (32)$$

or, more intuitively,

$$x_i = b_i [\beta + \gamma (N_i - 1)] (p_i - r\eta) / \beta$$

which is increasing in its own price  $p_i$  for any given value of  $\delta_i$ . Since  $\delta_i$  is decreasing in  $N_i$  and  $x_i$  is decreasing in  $\delta_i$ , it is clear that  $x_i$  is increasing in  $N_i$ . Thus, we have:

**Proposition 3** (Intermediate Good Supply) *The higher the common price of local varieties is, the larger the supply of each of them is. Furthermore, when varieties are substitutes (resp., complements), the greater the number of local varieties is, the larger (resp., smaller) the supply of each of them is.*

Observe that, in the Dixit-Stiglitz-Ethier model, the elasticity of substitution and the mark-up are constant. This implies that a larger number of local varieties leads to a lower quantity provided by each intermediate firm. By contrast, in our setting where the mark-up is variable, the relationship between the number of local varieties and its consumption by the final sector depends on whether varieties are substitutes or complements. When they are substitutes, Proposition 3 tells us that a larger number of local varieties leads to a *higher* quantity provided by each intermediate firm.

Moreover, combining (27) and (32) enables us to solve for the common equilibrium price of intermediate goods produced in region  $i$ :

$$p_i^* = \frac{r\eta[\beta + \gamma(N_i - 1)] + \beta[\alpha - \tau(1 - \delta_i)]}{2\beta + \gamma(N_i - 1)}. \quad (33)$$

so that the average trading price can be derived as follows:

$$\tilde{p}_i^* = p_i^* + \tau(1 - \delta_i) = \frac{\alpha\beta + [\beta + \gamma(N_i - 1)][r\eta + \tau(1 - \delta_i)]}{2\beta + \gamma(N_i - 1)}. \quad (34)$$

Hence, the equilibrium mill price ( $p_i^*$ ) decreases with transport, cost whereas the average trading price ( $\tilde{p}_i^*$ ), which accounts for the trade pattern, increases.

Expressions (33) and (25) imply:

$$p_i^* = \frac{r\eta\tau + \beta(2\delta_i(N_i) - 1)[\alpha - \tau(1 - \delta_i(N_i))]}{\tau + \beta(2\delta_i(N_i) - 1)} \equiv p_i(N_i). \quad (35)$$

This shows that an increase in the mass of intermediate goods reduces the demand of each variety, hence its monopoly power, thus leading the corresponding firm to lower its price. However, because intermediate firms incur a cost associated with its spending on capital ( $r\eta$ ), they cannot afford to charge very low prices. The assumption below is imposed for these firms' profits to be positive.

**Assumption 1:**  $\alpha + \tau/2 > r\eta$ .

**Proposition 4** (Intermediate Goods Prices) *Under Assumption 1, the larger the mass of local varieties is, the lower the equilibrium mill price of these varieties*

is. Furthermore, for any given allocation of varieties, lower transport costs lead to higher equilibrium mill prices but lower average trading prices.

Substituting (35) into (32), we obtain the equilibrium output of an intermediate producer:

$$x_i^* = \frac{(2\delta_i(N_i) - 1)[\alpha - \tau(1 - \delta_i(N_i)) - r\eta]}{[\delta_i^2 + (1 - \delta_i)^2][\tau + \beta(2\delta_i(N_i) - 1)]} \equiv x_i(N_i). \quad (36)$$

By increasing the mass of local varieties, we induce a negative price effect for each intermediate producer, whereas the interregional redistribution gives rise to a positive effect. Thus, the net effect of changing the mass of local varieties on the equilibrium quantity of middle products is generally ambiguous. The equilibrium of the middle product market is depicted in Figure 2. As one can see, due to the ambiguity in demand shifts in response to a high mass of local varieties, the equilibrium quantity of each middle product may rise or fall, though its equilibrium average trading price must be lower.

### 3.3 Capital accumulation and locational choice

By symmetry and using (11), we can rewrite the capital evolution equation as follows:

$$\dot{k}_i = y_i - z_i - (m_i + d_i)k_i - \frac{N_i}{1 + \phi N_i} \{(1 - r)\eta + \tau[1 - \delta_i(N_i)]\} x_i(N_i) \quad (37)$$

The corresponding Hamiltonian is then given by:

$$\mathcal{H} = \frac{z_i^{1-\sigma^{-1}} - 1}{1 - \sigma^{-1}} + \xi_i \{y_i - z_i - (m_i + d_i)k_i - \frac{N_i}{1 + \phi N_i} [(1 - r)\eta + \tau(1 - \delta_i)] x_i\}$$

where  $\xi_i$  is the shadow price of the capital  $k_i$ . The first-order condition with respect to  $z_i$  is then

$$z_i^{-\sigma^{-1}} = \xi_i. \quad (38)$$

The Euler equation that governs the dynamics of the shadow price is given by

$$\frac{\dot{\xi}_i}{\xi_i} = \rho - \left[ \frac{\partial y_i}{\partial k_i} - (m_i + d_i) \right] \quad (39)$$

Totally differentiating (38) yields:

$$\frac{\dot{z}_i}{z_i} = -\sigma \frac{\dot{\xi}_i}{\xi_i}$$

which can be combined with (39) to derive the following Keynes-Ramsey equation:

$$\frac{\dot{z}_i}{z_i} = \sigma \left[ \frac{\partial y_i}{\partial k_i} - (\rho + m_i + d_i) \right] \quad (40)$$

Note that there is no final good trade and the production of region  $i$ 's intermediate goods only requires the final good as given by (7). Therefore, in making capital investment decision, region  $i$ 's consumers must take the intermediate goods produced in the other region  $j \neq i$  as given. Using (7), (11), the definition of  $Y_i$  as well as  $y_i \equiv Y_i/M_i$ , we obtain region  $i$ 's per capita output under symmetry as follows ( $j \neq i$ ,  $i, j = 1, 2$ ):

$$y_i(N_1, N_2, k_i) = \frac{\delta_i(N_i)k_i}{\eta} \left\{ \alpha - \frac{1 + \phi N_i}{2N_i} \frac{\tau \delta_i(N_i)k_i}{\eta[2\delta_i(N_i) - 1]} \right\} + \frac{N_j[1 - \delta_j(N_j)]x_j(N_j)}{1 + \phi N_i} \left\{ \alpha - \frac{\tau[1 - \delta_j(N_j)]x_j(N_j)}{2[2\delta_i(N_i) - 1]} \right\} \quad (41)$$

Straightforward differentiation yields

$$\frac{\partial y_i}{\partial k_i} = \frac{\delta_i(N_i)}{\eta} \left\{ \alpha - \frac{1 + \phi N_i}{N_i} \frac{\tau \delta_i(N_i)k_i}{\eta[2\delta_i(N_i) - 1]} \right\} \quad (42)$$

In what follows, we assume that the variety bias in the production process is sufficiently large for the lemma below to be established.

**Assumption 2:**  $\beta - \gamma > \phi\gamma N^2$ .

**Lemma 1** *Under Assumptions 1 and 2,  $\bar{N} > 0$  and  $\bar{k} > 0$  exist such that for all  $N_i \in (0, \bar{N})$  and  $k_i \in (0, \bar{k})$ , we have:*

$$\frac{\partial y_i}{\partial N_i} > 0 \quad \text{and} \quad \frac{\partial y_i}{\partial k_i} > 0$$

Substituting (42) into (40) leads to

$$\theta_i \equiv \frac{\dot{z}_i}{z_i} = \sigma \left[ \frac{\delta_i(N_i)}{\eta} \left\{ \alpha - \frac{1 + \phi N_i}{N_i} \frac{\tau \delta_i(N_i) k_i}{\eta [2\delta_i(N_i) - 1]} \right\} - (\rho + m_i + d_i) \right] \quad (43)$$

where, for any given value of  $N_i$ , (43) says that the rate of growth of final good consumption ( $\theta_i$ ) depends only upon the value of  $k_i$ . Combining (37) and (41), the rate of growth of capital ( $k_i$ ) depends on both  $z_i$  and  $k_i$ , for a given value of  $N_i$  and a given consumption  $\delta_i(N_i)$  of intermediate goods supplied by the foreign region. Thus, for consumers residing in each region, these two evolution equations (governing  $\dot{z}_i/z_i$  and  $\dot{k}_i/k_i$ ) jointly solve the dynamic paths of consumption and capital.

Upon solving for these optimizing paths and applying (1) and (25), each consumer's lifetime utility can be expressed as a function of  $N_i$ :  $U_i^0 = U_i^0(N_i)$ . In equilibrium, skilled workers must reach the same lifetime utility level in each region:

$$U_1^0(N_1) = U_2^0(N - N_1) \quad (44)$$

where the population identity (2) has been used and  $U_i^0$  is defined as in (15). This equilibrium condition determines the interregional distribution of skilled workers.

**Definition 1** A *dynamic market equilibrium* (DME) is a collection of quantity paths  $\{z_i, k_i, y_i, x_i, \delta_i, N_i, M_i\}$  and a collection of price paths  $\{p_i, W_S\}$  ( $i = 1, 2$ ) such that:

- (i) each consumer maximizes her lifetime utility subject to the capital evolution equation, i.e., (37) and (43) are met;
- (ii) each final good producer and each intermediate producer maximize its profit under the specified production technologies, i.e., (25) and (36) are met;
- (iii) the zero-profit condition for the final good market prevails, i.e., (29) is met;
- (iv) both the intermediate goods and the final good markets clear, i.e., (35) and (41) are met;

(v) the spatial equilibrium condition (44) and the population identities, (2) and (11), are met.

The Walras law implies that the zero-profit condition for each intermediate good firm is satisfied.

## 4 The Steady-State Equilibrium

In this section, we focus on the steady-state equilibrium of the dynamic economy described in the foregoing.

**Definition 2** A *steady-state equilibrium* (SSE) is a dynamic market equilibrium  $\{z_i, k_i, y_i, x_i, \delta_i, N_i, M_i, P_i, W_S\}$  such that, for  $i = 1, 2$ , all quantities have zero growth and skilled workers do not move ( $m_i = 0$ ).

Because the production function in the final sector is strictly concave, output growth must asymptotically come to an end. Hence, in our paper, economic growth and population agglomeration both refer to the transitional output growth process ( $y_i$ ), as in neoclassical growth theory, and the transitional agglomeration process ( $N_i$ ). Specifically, economic growth triggers a higher level of output per capita in the steady state, whereas agglomeration implies growing clustering of skilled workers in region  $i$ .

Using (2), (25), (37), (43) and setting

$$\frac{\dot{k}_i}{k_i} = \frac{\dot{z}_i}{z_i} = m_i = 0$$

we get:

$$k_i(N_i) = \frac{\eta}{\tau} \frac{2\delta_i(N_i) - 1}{\delta_i(N_i)} \frac{N_i}{1 + \phi N_i} \left[ \alpha - \frac{\eta}{\delta_i(N_i)} (\rho + d_i) \right] \quad (45)$$

$$z_i(N_i) = y_i(N_1, N_2, k_i) - d_i k_i - \frac{N_i \{(1 - r)\eta + \tau[1 - \delta_i(N_i)]\} x_i(N_i)}{1 + \phi N_i} \quad (46)$$

By differentiating (45) with respect to  $N_i$ , we obtain after some straightforward, but tedious, calculations:

**Lemma 2** *Under Assumption 1, there exists an  $\tilde{N} > 0$  such that for all  $N_i \in (0, \tilde{N})$ ,  $\frac{\partial k_i}{\partial N_i} > 0$  and for all  $N_i \in (\tilde{N}, N)$ ,  $\frac{\partial k_i}{\partial N_i} < 0$ .*

Combining Lemmas 1 and 2, we further establish:

**Lemma 3** *Under Assumptions 1 and 2, there exists an  $\hat{N} > 0$  such that for all  $N_i \in (0, \hat{N})$ ,  $\frac{\partial z_i}{\partial N_i} > 0$  and for all  $N_i \in (\hat{N}, N)$ ,  $\frac{\partial z_i}{\partial N_i} < 0$ .*

One can integrate the lifetime utility to obtain (aside from a constant)

$$U_i^0 = \frac{[z_i(N_i)]^{1-\sigma^{-1}}}{(\sigma^{-1} - 1)\rho} \quad (47)$$

Thus, because the migration of skilled workers is costless, the spatial equilibrium condition (44) together with (47) implies the following equilibrium condition:

$$z_1(N_1) = z_2(N_2) \quad (48)$$

Equations (2), (45), (46) and (48) jointly determine the steady-state equilibrium values of  $\{z_i, k_i, N_i\}$ .

By locational symmetry, we focus only upon the steady-state equilibrium outcomes in region 1. Moreover, we consider primarily a benchmark case satisfying a stronger version of dynamic efficiency in the sense that not only consumption, output and capital are positively related, but these magnitudes are also positively related to the number of local varieties.

Specifically, we restrict ourselves to the ranges  $k_i \in (0, \bar{k})$  and  $0 < N_1 < \min\{\bar{N}, \tilde{N}, \hat{N}\}$ . In this case, we can use Figure 3 to determine graphically the steady-state equilibrium. To begin with, we plot region 1's consumption function  $z_1(N_1)$  according to (46) in the upper right panel and region 2's consumption function  $z_2(N_2)$  in the upper left panel. The lower left panel is a 45° auxiliary line,

whereas the lower right panel gives the population identity (2). Combining the relationships in the upper left, lower left and lower right panels, we obtain the  $z_2(N_1)$  locus. The intersection of the  $z_1(N_1)$  and  $z_2(N_1)$  loci gives the steady-state equilibrium value of  $(N_1^*, z_1^*)$ , under which (48) is satisfied. Obviously, our benchmark solution is the dynamically efficient outcome represented by point  $E$  (while point  $A$  is dynamically inefficient). In Figure 4, we then use the final upper right panel together with (41) and the  $k_1(N_1)$  relationship in (45) to determine the steady-state equilibrium values of  $(y_1^*, k_1^*)$ .

Let us first examine the consequences of an increase in the unit transport cost ( $\tau$ ). We impose the following sufficient condition to ensure that the direct effect of transport cost dominates the indirect disincentive effect through interregional intermediate good reallocation  $(1 - \delta_1)$ , i.e.

$$\frac{d\tau[1 - \delta_1(N_1)]}{d\tau} > 0$$

which holds under

**Assumption 3:**  $\beta - \gamma > 1 + \tau$ .

For a given  $N_1$ , we see from (46) that Assumption 3 is sufficient to guarantee both  $z_1$  and  $z_2$  to decrease if the import effect through  $1 - \delta_2$  is sufficiently small. Hence, both the  $z_1(N_1)$  and  $z_2(N_1)$  loci (as well as the  $y_1(N_1)$  and  $k_1(N_1)$  loci) in Figure 4 shift towards the horizontal axis, so that the net effect on  $N_1^*$  becomes ambiguous. However, if region 1 is a “large economy” and region 2 is a “small economy” in the sense that the import effect is small in region 1 but large in region 2, then the downward shift in the  $z_1(N_1)$  locus is greater than that in the  $z_2(N_1)$  locus in magnitude. As a consequence, the range of varieties produced in region 1 in the steady-state equilibrium ( $N_1^*$ ) rises and there is employment agglomeration in the large economy (i.e., region 1), because the interregional transactions become more costly. Yet, as both  $y_1(N_1)$  and  $k_1(N_1)$  loci also shift towards the horizontal axis, capital accumulation and output growth in region 1 need not increase.

Because  $k_i$  and  $z_i$  depend only upon the size of the skilled population in region  $i$  (see (45) and (46)), we can study the effects of a more efficient design of the production process that occurs only in region 1 (i.e.,  $\phi$  in region 1 decreases). In this case, the  $z_1(N_1)$  locus shifts downwards, the  $y_1(N_1)$  and  $k_1(N_1)$  loci shift upwards, while the  $z_2(N_1)$  locus remains unchanged. The steady-state equilibrium  $N_1^*$  is unambiguously higher, as are  $y_1^*$  and  $k_1^*$ . That is, a more efficient design of the production process that occurs only in region 1 leads to employment agglomeration together with greater capital accumulation and higher regional output growth.<sup>9</sup>

In summary, we have:

**Proposition 5** (Comparative Statics) *Under Assumptions 1-3, the steady-state equilibrium possesses the following properties:*

- (i) *when region 1 is a large economy and region 2 is a small economy, a decrease in the interregional transport cost induces skilled labor mobility, promotes vertical integration, and discourages employment agglomeration in the large economy, but has ambiguous effect on capital accumulation and regional output growth;*
- (ii) *a more efficient design of the final good production process that occurs only in region 1 results in higher employment agglomeration, capital accumulation and regional output growth.*

An important message is that *employment agglomeration and output growth need not be positively related in response to changes in the underlying economic parameters*, thereby lending theoretical support to empirical observations (see Berliant and Wang (2004) and papers cited therein).

Another interesting finding of our analysis is that *trade need not be associated with a widened skilled-unskilled wage gap*. Indeed, examining (29) reveals that several effects are at work in our setting. Trade liberalization can be viewed as a

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<sup>9</sup>By examining (45) and (46), we can easily see that given identical preferences across regions, consumers' preference parameters ( $\rho$  and  $\sigma$ ) have no effect on equilibrium agglomeration outcomes.

reduction in  $\tau$ , which in turn has differentiated impacts on the determinants of the wage gap, as measured by  $W_S - \bar{W}_U$ . First, as seen in Proposition 1, a decrease in  $\tau$  leads to more vertical integration between the two regions. However, Proposition 2 tells us that the final sector uses less of each variety as more varieties are made available through the opening to trade. In addition, Proposition 4 says that the mill price charged by firms goes up when transport costs decrease, whereas the corresponding average trading price decreases. Finally, we know from Proposition 5 that, when  $\tau$  decreases, the number of varieties produced in the large region decreases but the impact on the final sector output is ambiguous. To sum-up, the decrease of  $\delta_1$  narrows the wage gap, whereas the decrease of  $x_1$ ,  $\tilde{p}_1$  and  $N_1$  all widen the wage gap. These opposite effects, together with the ambiguity of the output effect makes it very difficult to predict the total effect of trade liberalization on wage inequalities, which provides a plausible way to reconcile the heated debates in the literature.

## 5 Concluding Remarks

We have developed a two-region dynamic general equilibrium model with mobile and immobile workers and monopolistically competitive intermediate goods firms to examine economic integration and development in the process of globalization. We find that (i) employment agglomeration and output growth need not be positively related, (ii) trade is not necessarily beneficial to regional growth, and (iii) trade between the two regions need not be associated with a widened skilled-unskilled wage gap. These results may explain why it is so hard to reach a consensus in empirical studies devoted to the impact of trade.

Several extensions are worth studying. First, mobile and immobile workers could have different capital shares as well as different preferences. Second, in a more complete analysis, the final good should be considered as being tradable. Third, as our setting exhibits monopolistic competition and pecuniary agglomeration externali-

ties, it could be used to figure out how interregional transfer, tax and investment subsidy policies can alleviate the resulting social inefficiency. Last, our model could be used to reexamine the issue of optimal tariff in a vertically integrated world economy.

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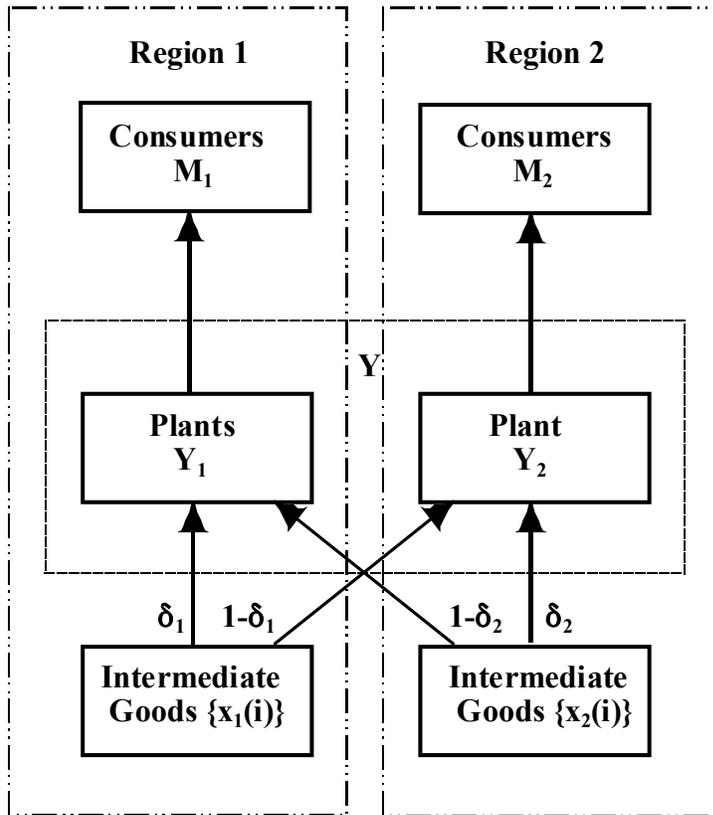


Figure 1: The Basic Environment

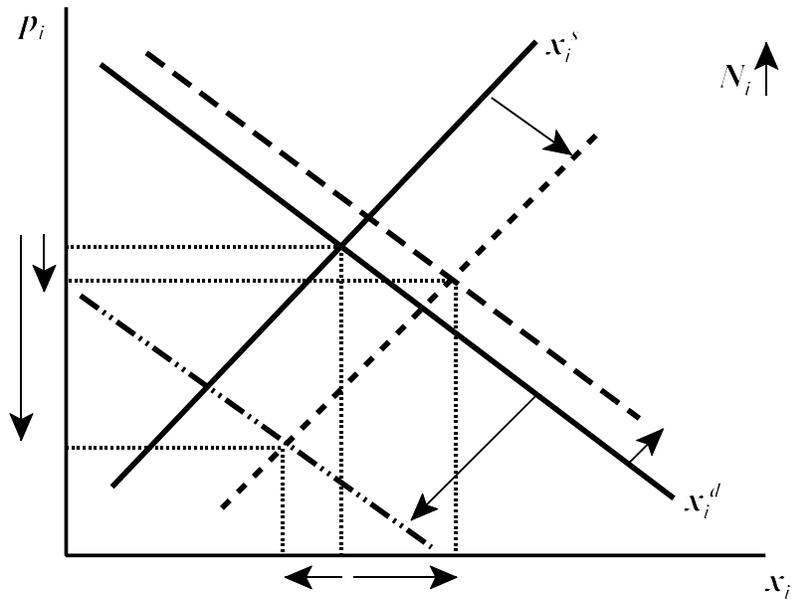


Figure 2: Middle Product Market Equilibrium

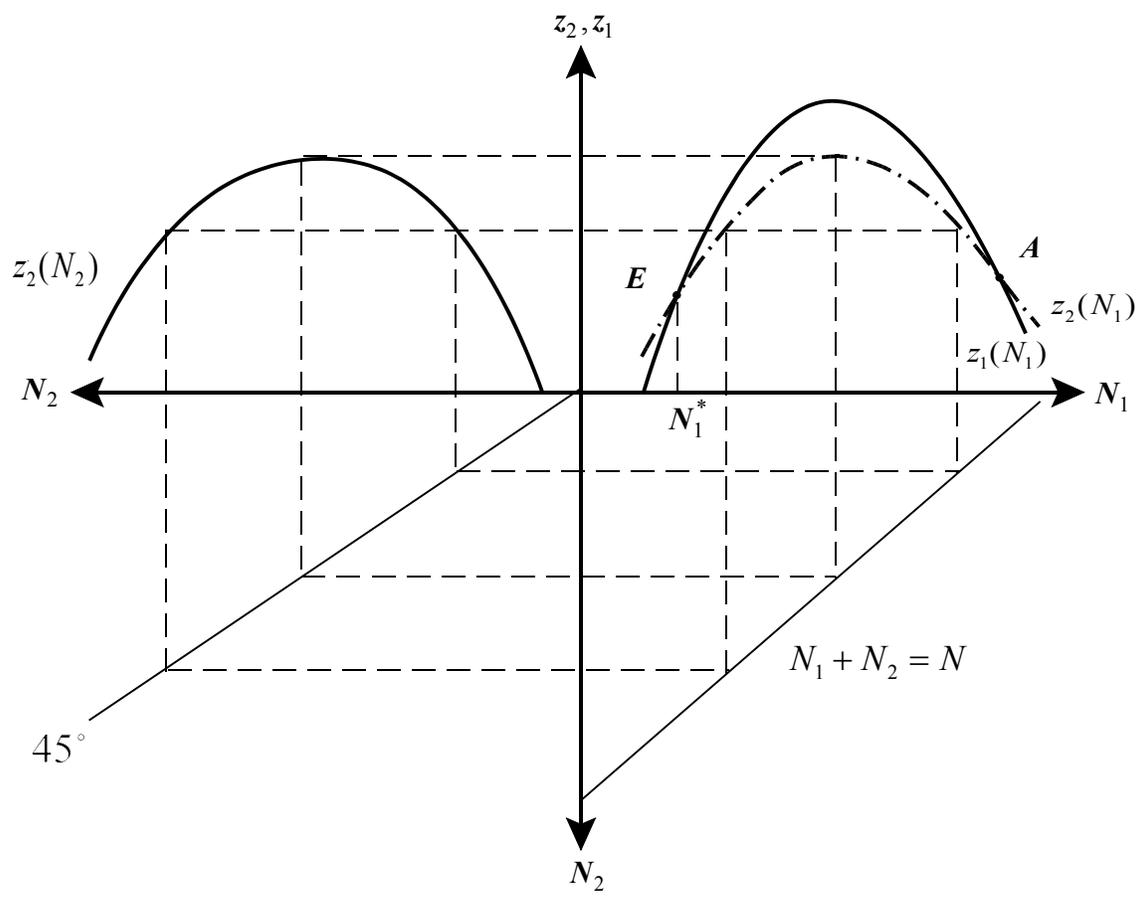


Figure 3: Steady-State No-Arbitrage Relationship

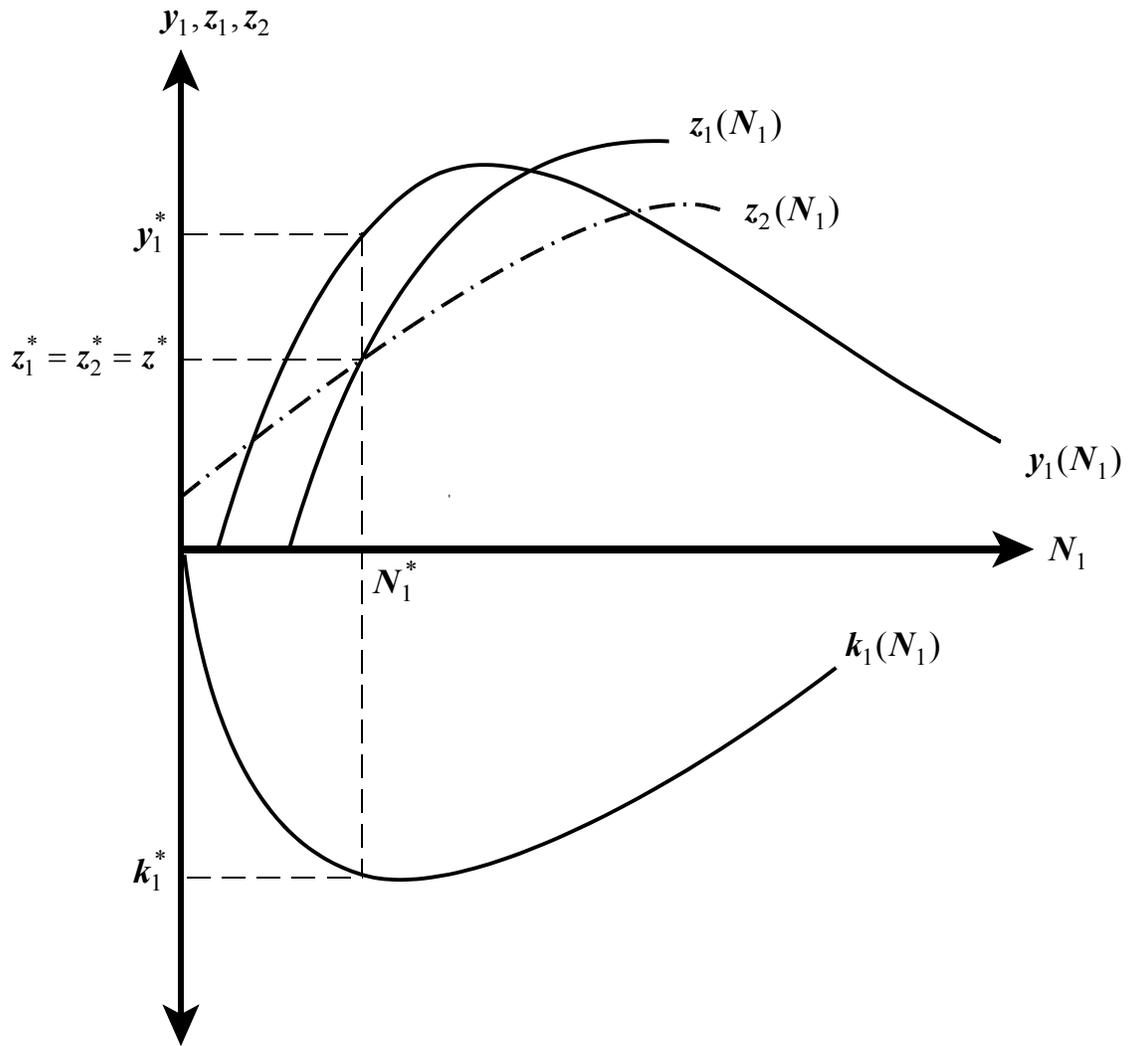


Figure 4: Determination of Steady-State Equilibrium