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## ABSTRACT

### Monetary Union: Fiscal Stabilization In The Face of Asymmetric Shocks\*

This Paper investigates the importance of fiscal policy in providing macroeconomic stabilization in a monetary union. We use a microfounded New Keynesian model of a monetary union that incorporates persistence in inflation, and examine non-cooperative interactions of fiscal and monetary authorities. We find that particularly when inflation is persistent, the use of fiscal policy for stabilization can significantly improve welfare over and above that which arises through the working of automatic stabilisers. We conclude that a regulatory framework for fiscal policy in a monetary union should allow a role for active fiscal stabilization.

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# 1 Introduction

How important is fiscal policy for macroeconomic stabilisation in a monetary union? This is an important question for members of European Monetary Union, and will become an important question for the UK should it join EMU. The present paper aims to provide an answer to this question using a microfounded New Keynesian analysis.

In a single economy, like the UK, there is now a consensus that fiscal policy should not be used for short run economic stabilisation, apart from letting the automatic stabilisers operate, and that it should be designed primarily to prevent growing public debt, so as to ensure long term fiscal solvency.<sup>1</sup> This consensus does not exist in the case of monetary union. Within the union all member countries have a common interest rate, and as a result monetary policy cannot be used to smooth asymmetric shocks. If there is no fiscal stabilisation then such shocks need to be absorbed by competitiveness changes. These changes need to come about through the relative inflation rate being lower in a country suffering a negative shock, leading to an improvement in competitiveness in that country and a resulting increase in its net exports. This adjustment mechanism may be very slow. This line of reasoning has led to the current strong criticism of the Stability and Growth Pact. Fiscal policy, which can be applied asymmetrically across the union, might aid the adjustment process. Westaway (2003) shows in an empirical model of E.M.U. that ‘active’ fiscal policy rules for stabilisation can significantly improve adjustment to equilibrium in the face of such asymmetric shocks, over and above the ‘passive’ improvements provided by allowing automatic stabilisers to operate. However, a satisfactory assessment of the welfare gains of such improvements in stabilisation requires a microfounded theoretical framework, something which we provide here.

Clearly the gains provided by fiscal stabilisation can only be large if macroeconomic volatility has significant effects on the welfare of individual agents. In the microfounded New Keynesian analysis we undertake here, we find that in a monetary union, both the total costs of macroeconomic volatility *and* the proportion of them that can be eliminated by active fiscal stabilisation increase when there is persistence in the inflation process. With realistic levels of persistence, we show that active fiscal stabilisation provides large welfare gains compared to the benchmark scenario of only using automatic stabilisers. We show that the majority of costs of macroeconomic volatility come from asymmetric supply shocks, and that it is the ability of fiscal policy to control these shocks that lead to the vast majority of the gains from fiscal stabilisation.

Our work builds on existing investigations of monetary union using microfounded New Keynesian models, developed by Benigno (2003), and extended by Beetsma and Jensen (2002), Beetsma and Jensen (2003) to include fiscal policy. We extend this framework in three ways.

Firstly, we introduce realistic changes in the structure of the model that cause a greater role for macroeconomic stabilisation. The most important modification is the introduction of persistence in inflation due to a fraction of price-setters who are backward-looking and use a rule of thumb to set prices, following Steinsson (2003). The quicker prices can adjust, the smaller will be the welfare gains from entirely eliminating macroeconomic volatility and smaller still the gains in using fiscal policy for stabilisation. Persistence

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<sup>1</sup>An overview for the U.K. is given by Treasury (2003)

in inflation slows down this adjustment and so increases the role for fiscal stabilisation. We can increase the lags with which rule of thumb of price setters respond to changes in demand. We find that a relatively small lengthening in this lag structure can cause a very large increase in the possible gains from fiscal stabilisation. Additionally, we introduce habit persistence in consumer behaviour as supported empirically (see Fuhrer (2000)). While habit persistence does affect the response of the system to shocks, we find that it does not greatly affect the case for fiscal stabilisation.

Secondly, we change the common assumption that fiscal and monetary authorities cooperate in pursuit of shared objectives<sup>2</sup>. Instead of this we model a *non-cooperative* game with differing objectives.<sup>3</sup> Apart from providing a more realistic picture of monetary and fiscal interactions, allowing fiscal and monetary authorities to have separate objectives permits us to vary the level of fiscal stabilisation (which is done by altering the objectives of fiscal authorities) without altering the objectives of monetary policy.

Thirdly, we extend the Beetsma and Jensen (2003) framework so as to study a monetary union open to the rest of the world, rather than assuming a ‘global’ monetary union. In our model, the rest of the world is a third large country (large enough not to be affected by the behaviour of the union). The relationship between the monetary union and the rest of the world is analogous to the relationship between a small open economy and the rest of the world described in Gali and Monacelli (2002). The purpose of this extension is to further explore the conventional wisdom, appealed to above, that fiscal policy is only useful in stabilising asymmetric shocks. In a ‘global’ monetary union following a symmetric shock, both fiscal and monetary policy will act on demand and thus monetary policy can fully replace any role for fiscal policy. In an open economy setting, the fact that interest rates also act on the exchange rate between the union and the rest of the world allows a second channel for stabilisation, which in turn might create a role for fiscal policy (a second instrument) in dealing with symmetric shocks. Nonetheless, we find that fiscal policy provides no significant advantage in stabilising symmetric shocks, and may in fact be (slightly) detrimental.

We evaluate welfare in our model using a microfounded measure of welfare. The extensions described above cause this measure to be quite complex. Because of this, we give both monetary and fiscal authorities simple and standard objective functions which are much simpler than the true measure of welfare. We do this because a simple objective function is more likely to capture the actual behaviour of the authority. For example a standard loss function which penalises deviations of inflation from its target and of output from its natural rate captures the essential features of a remit likely to be given to the central bank. This is what we assume for the monetary authority. We assume that the loss function of fiscal authorities, as well as penalising deviations in output and inflation (so fiscal policy supports monetary stabilisation), also penalises deviations of the fiscal deficit from its equilibrium level. Again we argue that this is more likely to

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<sup>2</sup>See, Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2003) and Beetsma and Jensen (2003) among others.

<sup>3</sup>Engwerda, van Arle, and Plasmans (2001) analyse macroeconomic adjustment in a non-cooperative setup for a dynamic stylized model of the monetary union, but they are focused on differences between cooperative and non-cooperative outcomes and only consider a Nash open-loop game between the monetary and fiscal authorities.

capture the essential features of a remit which could be given to the fiscal authorities. We then rank the equilibrium game outcomes produced under these objectives for fiscal and monetary authorities according to the theoretically correct measure of social welfare. We find that the gains of fiscal stabilisation can be large when fiscal authorities are guided by such a simple loss function; these gains would then be likely to increase further if fiscal authorities were to use an objective function more closely related to true social welfare.

We vary the amount of fiscal stabilisation by varying the weight placed on fiscal deficits in the fiscal authorities' loss functions. The larger is this weight, the less the fiscal authority will use policy to stabilise output and inflation. We compare two extreme scenarios. The first of these has the weight on the fiscal deficit which produces the best outcome in terms of social welfare.<sup>4</sup> The second has a very large weight, so high as to ensure that there is never any change in the budget deficit – this embodies extremely strong restrictions to fiscal policy of the kind which might bind when countries' fiscal policies are completely constrained. In between these extremes we consider the outcome when there is constant government spending and so automatic stabilisers are allowed to operate (henceforth referred to as the automatic stabilisers case).

We solve the policy game under monetary leadership.<sup>5</sup> We note that alternative leadership assumptions will primarily affect the way that fiscal policy contributes to the stabilisation of *symmetric* shocks, since monetary policy has no effect on outcomes caused by asymmetric shocks, while we find here that by far the most important influence of fiscal policy is in stabilising asymmetric shocks. Nonetheless, it is clearly important to see whether alternative leadership assumptions might produce different results, but this topic is left for future research.

We also follow the papers cited above in not extending the analysis to study the effects on government behaviour of a government solvency constraint; the focus here is on the short-run stabilisation of fiscal policy, assuming that this will not be restricted by solvency constraints. An analysis of solvency is left for future research.

The paper is laid out as follows. Section 2 describes the theoretical structure of model. Calibration of the model is discussed in Section 3. The main results are presented in Section 4.

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<sup>4</sup>In fact this is the best outcome in terms of social welfare *that we can calculate*. As  $\nu$  becomes small, it is increasingly difficult to achieve convergence in the iterative procedure which solves the Nash policy game of fiscal authorities. When we are constrained by this convergence problem, we use the best value of  $\nu$  in the range where we can solve the game. This gives a *lower bound* for the benefits of fiscal policy, and so does not weaken the results obtained.

<sup>5</sup>The monetary authority will be 'large,' because it sets the interest rate for the whole union but the fiscal authorities of the countries will be 'smaller' since they set policy separately in each country. This has been advanced as a reason as to why the monetary leadership assumption may be more appropriate in a monetary union. This choice about leadership may make relatively little difference as suggested above. Studying level bias, rather than stabilisation bias as we do here, Dixit and Lambertini (2003) and Lambertini and Rovelli (2003) find that fiscal leadership is preferable to monetary leadership.

## 2 The Model

### 2.1 Overall Structure of the Model and Notation

Our model of EMU is as follows. We model a monetary union of two identical small open economies, which is open to the rest of the world. In each country there are the following features:

(i) There is a Phillips curve in which domestic inflation in each country is influenced by aggregate demand and expected and past inflation. The persistence in inflation comes through a fraction of price-setters using a rule of thumb. Empirical evidence also suggests a lag in the influence of demand on inflation, and we can model a lengthened lag structure by altering the rule of thumb.

(ii) There is an intertemporal IS curve treating all private expenditure as consumption, and including the possibility of habit persistence.

(iii) There is a linearised GDP identity in each country which links aggregate output, aggregate private and public consumption, and the terms of trade.

For the union as a whole there are the following features.

(iv) Linkages between countries in the union: higher expenditure in one country causes an increase in exports from the other country, and higher prices in one country impact on inflation in another country by increasing the price of its imports, but there is no possibility of currency movements between the countries. Financial markets are complete and there is perfect risk sharing.

(v) There is a floating exchange rate between the union and the rest of the world: the exchange rate is determined by UIP. This means that monetary policy in the union can influence union-wide inflation by influencing the level of this exchange rate.

Through the text we will use the following notation:

$C_a$  – consumption basket in country  $a$

$P_a$  – price of consumption basket in country  $a$  (in country  $a$ 's currency)

$P_{Ha}$  – price of domestically produced goods in country  $a$  (in country  $a$ 's currency)

$P_{Ha}^{*b}$  – price of domestically produced goods in country  $a$ , exported to  $b$  and measured in country  $b$ 's currency

$P_{Ha}^{*w}$  – price of domestically produced goods in country  $a$ , exported to  $w$ , and measured in country  $w$ 's currency

$P_{Hb}$  – price of goods produced in country  $b$ , imported from  $b$ , and measured in country  $a$ 's currency

$P_{Hw}$  – price of goods produced in country  $w$ , imported from  $w$ , and measured in country  $a$ 's currency

$P_b^*$  – price of consumption basket in country  $b$ , measured in country  $b$ 's currency

$P_w^*$  – price of consumption basket in country  $w$ , measured in country  $w$ 's currency

Foreign currency (for country  $a$ ) denomination is denoted with asterisk.

We derive all equations for country  $a$  and then use the fact that country  $b$  is identical to  $a$  in order to write down equations for country  $b$ .

## 2.2 Behaviour of the Private Sector

### 2.2.1 Maximisation problem

Each of our small economies is inhabited by a large number of individuals. Each representative individual is a yeoman-farmer, who specialises in the production of one differentiated good, denoted by  $z$ , and spends  $h(z)$  of effort on its production. He consumes a consumption basket  $C$ , and also derives utility from per capita government consumption  $G$ . Private and public consumption are not perfect substitutes. Preferences are assumed to be:

$$\max_{\{C_s, h_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} [u(C_s/C_{s-1}^{\rho}, \xi_s) + f(G_s, \xi_s) - v(h_s(z), \xi_s)] \quad (1)$$

In each of the two economies the consumption basket consists of three composite goods, the domestic composite good (produced in the home country, subscripts  $Ha, Hb$ ), the foreign composite good from the other small open economy (produced in the foreign country, subscripts  $Hb, Ha$ ) and the good produced in the rest of the world, subscript  $Hw$ . Each composite good consists of a continuum of produced goods  $z \in [0, 1]$ . The utility function  $u(\cdot)$  incorporates habit persistence in a standard way by incorporating dependence on consumption in the period before. The parameter  $\rho$  then determines the extent of habit persistence. We also allow for taste/technology shocks  $\xi$ .

An individual chooses optimal consumption and work effort to maximise the criterion (1) subject to the demand system and the intertemporal budget constraint:

$$\sum_{s=t}^{\infty} \mathcal{E}_t(R_{t,s} P_{as} C_{as}) \leq B_{at} + \sum_{s=t}^{\infty} \mathcal{E}_t(R_{t,s} (1 - \tau) w_{as}(z) h_{as}(z))$$

where:

$$P_{at} C_{at} = \int_0^1 (p_{Ha}(z) c_{Ha}(z) + p_{Hb}(z) c_{Hb}(z) + p_W(z) c_W(z)) dz \quad (2)$$

and  $\mathcal{E}_t(R_{t,s}) = \prod_{k=t}^{s-1} \frac{1}{1+i_k}$ , and  $i_t$  is short-term interest rate. Here we assume that the labour income  $w$  is taxed at the rate  $\tau$ .

### 2.2.2 Demand system and Price Indexes

Domestically produced goods may be consumed either at home or abroad:

$$y_{at}(z) = c_{Ha,t}(z) + c_{Ha,t}^{*b}(z) + c_{Ha,t}^{*w}(z) + g_{Ha}(z) \quad (3)$$

where the asterisk denotes consumption of foreign goods, whose price is denominated in foreign currency. Namely, good  $z$ , produced at home  $Ha$ , is consumed either at home,  $c_{Ha,t}(z)$ , or abroad,  $c_{Ha,t}^{*b}(z)$ , and ‘abroad’ includes the other small open economy and the rest of the world:  $c_{Ha,t}^{*b}(z)$  and  $c_{Ha,t}^{*w}(z)$ .  $g_a(z)$  is government consumption. We assume

that the government in each country consumes the domestically produced good only, so  $g_{Ha} = g_a$ .

All goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair of goods given by  $\epsilon > 1$  (the time index  $t$  is suppressed for notational convenience):

$$C_{Ha} = \left[ \int_0^1 c_{Ha}^{\frac{\epsilon-1}{\epsilon}}(z) dz \right]^{\frac{\epsilon}{\epsilon-1}}, \quad C_{Hb} = \left[ \int_0^1 c_{Hb}^{\frac{\epsilon-1}{\epsilon}}(z) dz \right]^{\frac{\epsilon}{\epsilon-1}}, \quad C_{Hw} = \left[ \int_0^1 c_{Hw}^{\frac{\epsilon-1}{\epsilon}}(z) dz \right]^{\frac{\epsilon}{\epsilon-1}} \quad (4)$$

Every household consumes both domestic and foreign goods with the elasticity of substitution between them given by  $\eta > 0$ . Therefore, the consumption basket in country  $a$  is:

$$C_a = \left[ (\alpha_d^a)^{\frac{1}{\eta}} C_{Ha}^{\frac{\eta-1}{\eta}} + (\alpha_n^a)^{\frac{1}{\eta}} C_{Hb}^{\frac{\eta-1}{\eta}} + (1 - \alpha_d^a - \alpha_n^a)^{\frac{1}{\eta}} C_{Hw}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (5)$$

where  $\alpha_d^a$  is the share of consumption of domestic goods,  $\alpha_n^a$  is the share of consumption of goods imported from the neighbour country (the other small open economy). The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$c_{Ha}(z) = \left( \frac{p_{Ha}(z)}{P_{Ha}} \right)^{-\epsilon} C_{Ha}, \quad c_{Hb}(z) = \left( \frac{p_{Hb}(z)}{P_{Hb}} \right)^{-\epsilon} C_{Hb}, \quad c_{Hw}(z) = \left( \frac{p_{Hw}(z)}{P_{Hw}} \right)^{-\epsilon} C_{Hw} \quad (6)$$

where:

$$P_{Ha} = \left[ \int_0^1 p_{Ha}^{1-\epsilon}(z) dz \right]^{\frac{1}{1-\epsilon}}, \quad P_{Hb} = \left[ \int_0^1 p_{Hb}^{1-\epsilon}(z) dz \right]^{\frac{1}{1-\epsilon}}, \quad P_{Hw} = \left[ \int_0^1 p_{Hw}^{1-\epsilon}(z) dz \right]^{\frac{1}{1-\epsilon}} \quad (7)$$

The optimal allocation of expenditures between domestic and foreign goods implies:

$$C_{Ha} = \alpha_d^a \left( \frac{P_{Ha}}{P_a} \right)^{-\eta} C_a, \quad C_{Hb} = \alpha_n^a \left( \frac{P_{Hb}}{P_a} \right)^{-\eta} C_a, \quad C_{Hw} = (1 - \alpha_d^a - \alpha_n^a) \left( \frac{P_{Hw}}{P_a} \right)^{-\eta} C_a \quad (8)$$

where the consumer price indexes for all countries are:

$$P_a = (\alpha_d^a P_{Ha}^{1-\eta} + \alpha_n^a P_{Hb}^{1-\eta} + (1 - \alpha_d^a - \alpha_n^a) P_{Hw}^{1-\eta})^{\frac{1}{1-\eta}} \quad (9)$$

$$P_b^* = (\alpha_d^b P_{Hb}^{*b1-\eta} + \alpha_n^b P_{Ha}^{*b1-\eta} + (1 - \alpha_d^b - \alpha_n^b) P_{Hw}^{*b1-\eta})^{\frac{1}{1-\eta}} \quad (10)$$

$$P_w^* = (\alpha_a^w P_{Ha}^{*w1-\eta} + \alpha_b^w P_{Hb}^{*w1-\eta} + (1 - \alpha_a^w - \alpha_b^w) P_{Hw}^{*w1-\eta})^{\frac{1}{1-\eta}} \quad (11)$$

In introducing the international dimension, we closely follow Gali and Monacelli (2002). Since the world economy is large it can be considered as an essentially closed economy with the properties at the limit:

$$P_w^{*w} = P_{Hw}^*, \quad Y_w^* = C_w^*. \quad (12)$$

We also assume that the private and public consumption in the rest of the world are perfect substitutes.

We define terms of trade as

$$\mathcal{S}_{ab} = \frac{P_{Hb}}{P_{Ha}}, \mathcal{S}_{aw} = \frac{P_{Hw}}{P_{Ha}}, \mathcal{S}_{bw} = \frac{P_{Hw}^{*b}}{P_{Hb}^{*b}} \quad (13)$$

and the nominal exchange rates as

$$E_{ab} = \frac{P_{Hb}}{P_{Hb}^{*b}}, E_{aw} = \frac{P_W}{P_W^{*w}} \quad (14)$$

which are the relative prices of foreign goods in country  $a$ 's currency and in the foreign currency. The real exchange rates then can be defined as:

$$Q_{ab} = \frac{E_b P_b^*}{P_a}, \quad Q_{aw} = \frac{E_w P_w^*}{P_a}$$

### 2.2.3 Consumption decisions

The household optimisation problem is standard (Fuhrer (2000)) and, after linearisation, it leads to the following first order conditions (see the Additional Appendix, available from authors upon request, for a derivation):

$$c_s = \beta \phi \rho c_{s+1} + \phi \rho c_{s-1} + \frac{(1 - \beta \rho) \phi}{(1 - \sigma)} \lambda_s + \varepsilon_s \quad (15)$$

where

$$\lambda_s = \lambda_{s+1} - \sigma(i_s - \pi_{s+1}) \quad (16)$$

and parameters are defined as:

$$\sigma = -\frac{u_C(C/C^\rho, 1)}{u_{CC}(C/C^\rho, 1)C^{1-\rho}}, \quad \phi = \frac{(1 - \sigma)}{1 - \sigma\beta\rho + \beta\rho^2(1 - \sigma)}.$$

The parameter  $\rho$  determines how much current consumption depends on past: when it is zero we have a standard forward-looking Euler equation for consumption, and when it is strictly positive we get habit persistence. The shock  $\varepsilon$  is directly related to the taste/technology shock  $\xi$  in the utility function. The coefficient  $1/\sigma$  represents the coefficient of relative risk aversion associated with private consumption (and  $\sigma$  is the rate of intertemporal substitution). These equations are written in terms of 'gap' variables, i.e. in terms of the log-deviation from the efficient (given distortionary taxation) flexible-price equilibrium.

### 2.2.4 Pricing decisions

In order to describe price setting decisions we, following Steinsson (2003), split individuals into two groups according to their pricing behaviour. A proportion of agents  $1 - \omega$  are forward-looking and set prices optimally given Calvo-type constraints on price setting, while a fraction  $\omega$  are backward-looking and set their prices according to a rule of thumb.

In each period, each agent is able to reset her price with probability  $1 - \gamma$ , and otherwise, with probability  $\gamma$ , her price will rise at the steady state rate of domestic inflation. The rule of thumb used by a backward-looking agent to set her price  $P_{Ha,t}^b$  is

$$P_{Ha,t}^b = P_{Ha,t-1}^\times \Pi_{Ha,t-1} \prod_{k=1}^N \left( \frac{Y_{at-k}}{Y_{at-k}^n} \right)^{\delta_k} \quad (17)$$

where  $P_{Ha,t-1}^\times$  is the average domestic price in the previous period,  $\Pi_{Ha,t-1} = P_{Ha,t-1}/P_{Ha,t-2}$  is past period growth rate of prices and  $Y_{at-k}/Y_{at-k}^n$  is output relative to the flexible-price equilibrium. We assume that there can be more than one lag in output affecting price-setting. We present the algebra for  $N = 2$ , as this is amply sufficient to demonstrate the effects of a lengthened lag structure.

For the whole economy the resulting Phillips curve will take the form (see Steinsson (2003) and Appendix A for a detailed derivation):

$$\begin{aligned} \pi_{Hat} = & \chi\beta\pi_{Hat+1} + (1 - \chi)\pi_{Hat-1} + \kappa_c c_{at} + \kappa_{x0} x_{at} + \kappa_{x1} x_{at-1} \\ & + \kappa_{x2} x_{at-2} + \kappa_{sd} s_{as} + \kappa_{sn} s_{bs} \end{aligned} \quad (18)$$

and all coefficients are given in Appendix A. The distortionary wage income tax  $\tau$  alters the equilibrium allocation of consumption and labour, but has no effect on the dynamic equations for log-deviations from the flexible price equilibrium (see Appendix A for derivation).

The Phillips curve (18) has a familiar structure where both current and past output have an effect on inflation. Its final specification was discussed in Steinsson (2003) and we briefly repeat it in Appendix A, but explaining our modifications in detail. In the case when all consumers are forward-looking, i.e.  $\omega = 0$ , this Phillips curve collapses to the standard forward-looking Phillips curve (see Rotemberg and Woodford (1997)). If all consumers use the rule of thumb in price-setting decisions, i.e.  $\omega = 1$ , it can be brought into the form of ‘accelerationist’ Phillips curve (see Appendix A.2).

The presence of the terms of trade for both countries is due to the wedge between the consumption of basket of goods and production of a single domestic good.

The system (15) and (18) is formally equivalent to the optimising behaviour of a representative agent who maximises (1) subject to an aggregate ‘law of motion’ of the economy (the demand system, the intertemporal budget constraint and pricing decisions) when the policymaker’s behaviour is taken to be an exogenous process that is independent of the individual’s actions.

### 2.2.5 Aggregate Demand

We derive an aggregate demand for country  $a$ , following Gali and Monacelli (2002), which is given by a linearised GDP identity:

$$\begin{aligned} x_{at} = & \theta\alpha_d^a c_{at} + \theta\alpha_n^a c_{bt}^* + \theta(1 - \alpha_d^a - \alpha_n^a) c_{wt}^* + (1 - \theta)g_{at} \\ & + \eta\theta \left( 1 - (\alpha_d^a)^2 - (\alpha_n^a)^2 \right) s_{at} - 2\eta\theta\alpha_n^a\alpha_d^a s_{bt} \end{aligned} \quad (19)$$

where  $s_{at}$  and  $s_{bt}$  denote log-deviations of the terms of trade for countries  $a$  and  $b$  with respect to the rest of the world, country  $w$ . The parameter  $\theta$  denotes the share of private consumption in output, so  $1 - \theta$  is the share of the government sector in the economy.

### 2.2.6 Putting things together

We now write down the final system of equations. We simplify the analysis and notation by assuming that countries  $a$  and  $b$  are identical in all their parameters. We also substitute out for consumer inflation in terms of domestic inflation and exchange rates. The system can be written as:

$$c_{at} = \beta\phi\rho c_{at+1} + \phi\rho c_{at-1} + \frac{(1 - \beta\rho)\phi}{(1 - \sigma)}\lambda_{at} + \varepsilon_{at} \quad (20)$$

$$c_{bt} = \beta\phi\rho c_{bt+1} + \phi\rho c_{bt-1} + \frac{(1 - \beta\rho)\phi}{(1 - \sigma)}\lambda_{bt} + \varepsilon_{bt} \quad (21)$$

$$\lambda_{at} = \lambda_{at+1} - \sigma(i_t - (1 - \alpha_n)\pi_{Hat+1} - \alpha_n\pi_{Hbt+1} - (1 - \alpha_d - \alpha_n)\Delta s_{at+1}) \quad (22)$$

$$\lambda_{bt} = \lambda_{bt+1} - \sigma(i_t - (1 - \alpha_d)\pi_{Hat+1} - \alpha_d\pi_{Hbt+1} - (1 - \alpha_d - \alpha_n)\Delta s_{at+1}) \quad (23)$$

$$\begin{aligned} \pi_{Ha,t} &= \chi\beta\pi_{Ha,t+1} + (1 - \chi)\pi_{Ha,t-1} + \kappa_c\lambda_{at} + \kappa_{x0}x_{at} + \kappa_{x1}x_{at-1} + \kappa_{x2}x_{at-2} \\ &\quad + \kappa_{sd}s_{at} + \kappa_{sn}s_{bt} + \eta_{at} \end{aligned} \quad (24)$$

$$\begin{aligned} \pi_{Hb,t} &= \chi\beta\pi_{Hb,t+1} + (1 - \chi)\pi_{Hb,t-1} + \kappa_c\lambda_{bt} + \kappa_{x0}x_{bt} + \kappa_{x1}x_{bt-1} + \kappa_{x2}x_{bt-2} \\ &\quad + \kappa_{sd}s_{bt} + \kappa_{sn}s_{at} + \eta_{bt} \end{aligned} \quad (25)$$

$$x_{at} = (1 - \theta)g_{at} + \theta\alpha_d c_{at} + \theta\alpha_n c_{bt} + \eta\theta((1 - \alpha_d^2 - \alpha_n^2))s_{at} - 2\eta\theta\alpha_n\alpha_d s_{bt} + \zeta_{wt} \quad (26)$$

$$x_{bt} = (1 - \theta)g_{bt} + \theta\alpha_d c_{bt} + \theta\alpha_n c_{at} + \eta\theta((1 - \alpha_d^2 - \alpha_n^2))s_{bt} - 2\eta\theta\alpha_n\alpha_d s_{at} + \zeta_{wt} \quad (27)$$

$$i_t = s_{at+1} - s_{at} - \pi_{wt+1} + \pi_{Hat+1} + \zeta_{it} \quad (28)$$

$$s_{bt} = s_{at} - s_{at-1} - \pi_{Hbt} + \pi_{Hat} + s_{bwt-1} \quad (29)$$

Equations (20) - (23) are the consumption equations (15) and (16) written in terms of domestic inflation. Equations (26) and (27) are aggregate demand equations from (19). Equation (28) is the familiar uncovered interest rate parity condition written using terms of trade and inflation, and equation (29) follows from the requirement of the fixed nominal exchange rate between countries  $a$  and  $b$ , so the terms of trade  $s_{at}$  and  $s_{bt}$  are not independent. Shocks  $\varepsilon$  are directly related to technology/taste shocks  $\xi$  in (1). In the final specification we add extra ‘cost-push’ shocks to the inflation equations (24)-(25). Shocks  $\zeta_{wt}$  are shocks to the rest of the world’s consumption and shocks  $\zeta_{it}$  are shocks to the rest-of-the-world interest rate, or a UIP shock.

## 2.3 Behaviour of the Monetary Authorities

The union central bank uses the short-term interest rate as a policy instrument. We assume it seeks to minimise the following *traditional* loss function:

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} [(\pi_{as} + \pi_{bs})^2 + 0.5(x_{as} + x_{bs})^2]. \quad (30)$$

In other words, the central bank targets union-wide *consumer* price inflation and output. Although the microfounded social welfare function will include domestic inflation and a more complicated structure of terms concerned with output variability and inflation persistence (see Rotemberg and Woodford (1997), Beetsma and Jensen (2003), Steinsson (2003), and Appendix B to this paper), in this paper we study the implications of conventional policymaking as discussed in the introduction. We take the value of 0.5 for the weight on output variability as a conventional value in the literature.

## 2.4 Behaviour of the Fiscal Authorities

In this paper, we postulate that the fiscal authorities do not have any solvency concerns: we only study the stabilisation properties of fiscal policy. We assume that they target output and inflation (as monetary authorities do), and also target the fiscal deficit. This could reflect either a desire to run a balanced budget, or the impact of a regulatory framework such as the Stability and Growth Pact. So their loss functions are respectively:

$$\min_{\{g_{as}\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} [\pi_{as}^2 + 0.5x_{as}^2 + \nu d_{as}^2] \quad (31)$$

$$\min_{\{g_{bs}\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} [\pi_{bs}^2 + 0.5x_{bs}^2 + \nu d_{bs}^2] \quad (32)$$

where  $d_t$  denotes the *primary deficit* which can be written as

$$\delta_d d_t = (1 - \theta)g_{at} - \tau x_t$$

where  $\tau$  is wage income tax rate and  $\delta_d$  is equilibrium level of the real primary deficit.  $\nu$  denotes the weight the fiscal authority places on minimising the deviation of the deficit from the equilibrium primary deficit.

## 2.5 Social Loss Function

The three policymakers solve their optimisation problems each period, given initial conditions and time preferences. The resulting optimal policy reactions lead to stochastic equilibria that should be compared across a suitable metric, independent of initial conditions. The obvious choice of this metric is the microfounded union-wide social loss, which on the convenient assumption that social planner does not discount the future, is a sum of unconditional variances with microfounded weights.

The union-wide social loss function takes the form (see Appendix B for a discussion of the derivation):

$$\begin{aligned} \mathcal{W} = & \lambda_{\pi}(var(\pi_{Ha}) + var(\pi_{Hb})) + \lambda_{\pi}\lambda_c(var(c_a - \rho c_{a,-1}) + var(c_b - \rho c_{b,-1})) \quad (33) \\ & + \lambda_{\pi}(\lambda_x + \mu_x)(var(x_{at}) + var(x_{bt})) + \lambda_{\pi}\lambda_g(var(g_a) + var(g_b)) \\ & + \lambda_{\pi}\mu_{\Delta\pi}(var(\Delta\pi_{Ha}) + var(\Delta\pi_{Hb})) \\ & + 2\lambda_{\pi}\mu_{xx}(cov(x_a, x_{a,-1}) + cov(x_b, x_{b,-1})) \\ & + 2\lambda_{\pi}\mu_{x\Delta\pi}(cov(x_{a,-1}, \Delta\pi_{Ha}) + cov(x_{b,-1}, \Delta\pi_{Hb})) \end{aligned}$$

There are several unconventional features of this loss function. First of all, terms with  $\mu$ -coefficients are present only because of rule of thumb price setters. The presence of these terms requires that inflation and output be brought back to the equilibrium smoothly. Steinsson (2003) has shown that when the private sector is predominantly backward-looking, terms with weights denoted by  $\mu$  dominate the loss function, and the effect of other terms is negligible. For a moderate proportion of the rule of thumb price-setters, the weight of  $\mu$ -terms is limited, but not negligible.

To interpret the values of the social loss, we can express them in terms of compensating consumption – the permanent fall in the steady state consumption level that would balance the welfare gain from eliminating the volatility of consumption, government spending and leisure (Lucas (1987)). As explained in Appendix C, the percentage change in consumption level,  $\Omega$ , that is needed to compensate differences in welfare of two regimes with social loss of  $\mathcal{W}_1$  and  $\mathcal{W}_2$  is given by (33):

$$\Omega = \frac{\sigma}{(\rho\sigma + (1 - \rho))} \left( 1 - \sqrt{1 + 2\frac{(\rho\sigma + (1 - \rho))}{(1 - \rho)\sigma}(\mathcal{W}_2 - \mathcal{W}_1)} \right) \quad (34)$$

## 2.6 Policy Game

We assume that monetary and fiscal authorities are engaged in a policy game with monetary leadership. The monetary authorities move first, taking into account the reaction of the two following fiscal authorities and the private sector in both countries. The fiscal authorities play a Nash game with each other, but they follow the monetary authority and lead the private sector. Thus, the private sector is an ultimate follower, and its optimisation problem is solved out and represented by the two reaction equations, the IS curve and the Phillips curve (20)-(25). All authorities act under discretion.

The solution of a leadership game with two optimising policymakers and the private sector as an ultimate follower is discussed in Blake and Kirsanova (2003). Here, we extend their analysis, since instead of one follower we have two fiscal authorities, who each follow the monetary authority, but are engaged in a Nash game with each other. The Nash game is studied in detail in the literature on macroeconomic policy coordination, see for example Currie and Levine (1993), and its solution procedure consists essentially of two Oudiz and Sachs (1985) algorithms (one for each fiscal authority) which we need to iterate between. The Additional Appendix gives a more detailed explanation on how this is combined with the leadership framework of Blake and Kirsanova (2003).

## 3 Calibration

Because of the microfounded nature of the model, there are relatively few parameters to calibrate, given in Table 1. One period is taken as equal to one quarter of a year. We set the discount factor of the private sector (and policy makers) to  $\beta = 0.99$ .

Perhaps the most important parameter in our model is the proportion of rule of thumb price setters,  $\omega$ . A consensus figure for the forward-looking component in an empirical Phillips curve is often taken as 0.3 (see Rudebusch (2002); Batini and Nelson (2001)

also suggest a very backwards-looking Phillips curve in the UK). Thus a forward-looking coefficient of  $\chi = 0.3$  in the Phillips curve (24)-(25) can be obtained with  $\omega = 0.5$  and  $\gamma = 0.75$ . This implies that 50 percent of population reset their wages and prices not optimally, but using the rule of thumb (17).  $\gamma = 0.75$  implies that, on average, prices last for one year (as agents are yeomen-farmers, this could also be thought of as nominal wages being fixed for one year). Table 2 shows how the weights of the Phillips curve vary with  $\omega$  given  $\gamma = 0.75$ .

Calibrating the IS relationship we use  $\rho = 0.8$  for the habit persistence parameter as originally estimated by Fuhrer (2000). Subsequent research (Boldrin, Christiano, and Fisher (2000), Edge (2000), Christiano, Eichenbaum, and Evans (2001)) suggests a range of 0.5–0.9, but these estimates depend on the exact specification used in the estimation, so are not necessarily directly comparable. For the parameters related to fiscal policy, we calibrate the ratio of private consumption to output as 65 percent, according to data for the nineties for the UK; and we assume that the equilibrium ratio of domestic debt to output is 60 percent. Then the debt accumulation equation gives us the equilibrium level of the primary surplus and the tax rate.

We specify the parameters  $\delta_1$  and  $\delta_2$  for rule of thumb price-setting in (17) which determine the sensitivity of rule of thumb price setters to (lags in) demand in the following way. We first calibrate a value for their sum  $\delta = \delta_1 + \delta_2$ . This is calibrated using different assumptions from Steinsson's. Steinsson assumes that the two extreme versions of the Phillips curve – forward-looking and accelerationist – should give the same coefficient on demand pressure in the Phillips curve  $\kappa = \kappa_c + \kappa_{x0} + \kappa_{x1} + \kappa_{x2}$  i.e. he makes the constraint that with a constant  $\delta$  the sensitivity to demand in the Phillips curve should be invariant with the level of inflation persistence. This constraint determines both  $\delta$  and  $\kappa$ . However, a problem with taking this approach is that it can result in a coefficient  $\kappa$  that is unrealistically small. We instead take an empirical estimate of  $\kappa$ . Estimates of  $\kappa$  differ widely, and estimates vary from 0.05 to 0.6. We choose a value of  $\kappa = 0.3$  within this interval. Given any choice of the the key parameters in Table 1, we choose a value of  $\delta$  that will give  $\kappa = 0.3$ . To determine the lag structure, we then set  $\delta_1 = \varphi\delta$  and  $\delta_2 = (1 - \varphi)\delta$ . Setting  $\varphi = 1$  corresponds to the Steinsson specification, where only one lag in demand affects rule of thumb price-setting. Small changes in  $\varphi$ , however, can have large impacts on our results.

We calibrate the standard deviations of shocks hitting the economies as follows. There is relatively little evidence in the literature about the standard deviations of supply and demand shocks. Theoretical literature typically assumes that all standard deviations are the same and a consensus number is 0.5%, see, e.g. Jensen and McCallum (2002), Bean, Nikolov, and Larsen (2002). Empirical literature tends to suggest that demand shocks have a bigger standard deviation, although the evidence is scarce with results typically depending on goodness of fit of empirical demand and supply equations (see results from two-equation model in Bean (1998) and from a large macromodel in Barrell and Hurst (2003)). In this paper we nevertheless assume that the standard deviation of supply and demand shocks are equal, and all shocks are independent. We do this primarily for simplicity and clarity, and so that the relative welfare effects of supply and demand shocks presented in Section 4 reflects properties of the model, rather than underlying assumptions about the shocks. In fact, we can show that very little change occurs to the

results of the paper if we assume, say, that the standard deviation of demand shocks is twice that of supply shocks.

It remains to specify the common standard deviation of these shocks. We do this so that in a single country outside the monetary union (proxied here by a monetary union under symmetric shocks) the standard deviation of CPI inflation would be 0.5% were government spending kept constant (as did Jensen and McCallum (2002)).<sup>6</sup> This corresponds to a standard deviation of shocks of around 0.5% for the base-line model parametrised as above, which is widely used in the literature. Again, given any choice of the key parameters in Table 1, we then re-calibrate the standard deviation of shocks to fit with this volatility of CPI inflation.

We assume that each economy consumes 30% of imported goods, 20% of which are imported from the neighbour country in the monetary union.

## 4 Results

### 4.1 Responses to shocks

The solution of this model is a set of optimal reaction rules for monetary and fiscal authorities, given the model structure and the assumption that each country is subject to idiosyncratic supply and demand shocks. We display the responses of the economy under these rules to purely symmetric or asymmetric shocks, but it is important to note that the rules are not derived under the assumption that shocks are symmetric or asymmetric, but that they are uncorrelated across countries.

Figure 1 shows the responses of the economy to symmetric impulse supply and demand shocks, and Figure 2 displays asymmetric shocks. Supply shocks are shocks to (24) and (25), the shock terms in the Phillips curves for each country, and demand shocks are shocks to (20) and (21), shocks to the consumption components of the IS curve for each country. The figures show the responses for three cases: with stabilising fiscal policy (solid line), with automatic stabilisers in operation (dotted line) and with constant fiscal deficits (dashed line). Outcomes with automatic stabilisers are obtained simply by removing fiscal authorities from the game (so that government spending is kept constant) and calculating outcomes when only monetary policy is active. Outcomes with constant deficits are obtained by introducing fiscal authorities into the game, but placing a very large weight  $\nu \gg 1$  on the volatility of deficits in the fiscal objectives (31)-(32). Stabilising fiscal policy similarly corresponds to a small value for  $\nu \ll 1$ .

Figure 1 shows that the model produces similar responses of inflation and output to symmetric shocks in all three cases, that is whether there is stabilising fiscal policy, or the operation of the automatic stabilisers, or constant fiscal deficits. As might be expected, monetary policy can deal effectively with symmetric shocks, and fiscal stabilisation can produce no obvious improvement in the responses of inflation and output; as explained later it actually makes things slightly worse. The differences in the trajectories when fiscal policy is active are revealing. In the case of a demand shock, active fiscal policy takes the

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<sup>6</sup>In the UK, over the period of 1991 – 2003, the standard deviation of the CPI inflation is 0.54% and the standard deviation of the PPI inflation is 0.41%.

Key Parameters	Mnemonics	Value
Discount factor	$\beta$	0.99
Share of rule-of-thumb price-setters	$\omega$	0.5
Proportion of agents who able to reset their price within a period	$1 - \gamma$	0.25
Weight on demand pressure in the Phillips curve	$\kappa$	0.3
Share of the government sector in the economy	$1 - \theta$	0.65
Steady state ratio of domestic debt to output	$B/Y$	0.6
Intertemporal substitution rate	$\sigma$	0.5
Elasticity of substitution between domestic and foreign goods	$\eta$	2.0
Elasticity of substitution between two domestic goods	$\epsilon$	5.0
Production risk aversion	$1/\psi$	0.5
Share of domestic goods in consumption basket	$\alpha_d$	0.7
Openness with respect to the other small open economy	$\alpha_n$	0.2
Implied Parameters in system (20)-(29)	Mnemonics	Value
Tax Rate	$\tau$	0.356
Steady state ratio of primary real surplus to output	$\delta_d$	0.006
Weight on forward inflation in PC	$\chi$	0.3
Weight on the country's term of trade vs. the rest of the world in AD	$\theta\rho_d$	0.75
Weight on the neighbour country's term of trade vs. the rest of the world in AD	$\theta\rho_n$	-0.45
Implied Parameters in the Social Loss Function (33)	Mnemonics	Value
Consumption variability	$\lambda_c$	0.03
Output variability	$\lambda_x + \mu_x$	0.22
Variability of government consumption	$\lambda_g$	0.001
Variability of inflation growth rate	$\mu_{\Delta\pi}$	1.25
Covariance of two output lags	$\mu_{xx}$	0.00003
Covariance of the first lag in output and inflation growth rate	$\mu_{x\Delta\pi}^1$	1.03
Sum of demand-variability related terms	$\lambda$	0.25

Table 1: Parameter values

Proportion of rule of thumb price setters, $\omega$ with $\gamma = 0.75$ , $\beta = 0.99$											
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\chi^f$	1.00	0.80	0.63	0.50	0.39	0.30	0.22	0.16	0.10	0.05	0.00
$\chi^b$	0.0	0.20	0.37	0.50	0.61	0.70	0.78	0.84	0.90	0.95	1.0

Table 2: Coefficients of Phillips Curve

place of higher interest rates. In the case of a supply shock fiscal expansion in the face of output loss requires an even higher rise in the interest rate, which depresses consumption. Interestingly we will see in the next section that active fiscal policy in the face of such shocks reduces welfare; it is better that the shocks be taken care of by monetary policy alone.

Figure 2 reports the response with asymmetric shocks. Such shocks cannot be dealt with at all by monetary policy; Figure 2 confirms that they produce a zero response in interest rates. Without any fiscal stabilisation at all the economy experiences significant oscillations. This is because with a fixed exchange rate and no stabilisation by monetary policy of asymmetric shocks, the effects on the price level of any changes in inflation need to be removed. These changes in the price level need to come about through the relative inflation rate being lower in the country in which the price level is high, leading to an improvement in competitiveness in that country and a resulting increase in net exports. This adjustment mechanism is cyclical because the price level tends to overshoot: if prices are high this causes low demand and disinflation; when the price level, and demand, have returned to zero prices are still falling, leading to high demand in the future, which causes a return of inflation. As suggested by Westaway (2003), a destabilising feedback in this process is that described by the “Walters critique”. A downturn in a country will cause inflation in that country to fall. But since the nominal interest rate for the union does not fall, the real interest rate in the negatively affected country will actually rise. This increases rather than reduces the downturn prolonging the cycle.

Figure 2 shows that active fiscal policy is able to stabilise the economy in the face of such shocks. Figure 2 also suggests that the effects of the automatic stabilisers may not produce enough stabilisation to avoid cyclical outcomes, and that active fiscal policy may be necessary for this. In the case of a demand shock, active fiscal policy damps the effects on demand and prevents any effect on inflation. That stops competitiveness and the real exchange rate oscillating in the way described above. In the case of a supply shock fiscal policy can make a very great difference. In the absence of fiscal policy, inflation causes such a large output fall that inflation then overshoots downwards, and the resulting fall in the price level makes output overshoot upwards again. Active fiscal policy makes output recover much faster after its initial fall and prevents the downward overshoot of prices. It is clear that such active fiscal policy will increase welfare.

The period of the cycles shown in Figure 2 in the absence of effective fiscal policy is very short (less than ten quarters), and one would believe that in reality any cycle caused by asymmetric shocks in a monetary union is likely to have a longer period than this. However, to capture this theoretically might require a more complex model than that developed here (perhaps including the effects of capital accumulation). In our simple model fiscal policy can remove the cycle effectively. In the next section it is shown that this can cause large welfare gains, which is striking given the simple description of the frictions in the model.

## 4.2 Welfare

Overall Figures 1 and 2 suggest a possible substantial gain in welfare from fiscal stabilisation. We now explore this.

The upper panel of Figure 3 shows how social loss varies as we vary the level of fiscal activity i.e. as we vary the weight  $\nu$  on the volatility of deficits in the fiscal objectives (31)-(32). We plot the level of social loss for a particular value of  $\nu$  against a measure of the level of fiscal activity for that value of  $\nu$ . The measure we choose is the standard deviation of the fiscal deficit measured as a percentage of GDP. Values of the social loss are plotted as a percentage of the value of social welfare when only automatic stabilisers operate. So as we see in Figure 3, a policy of constant deficits increases the social loss by 4% in comparison to a policy of using automatic stabilisers, whereas a level of fiscal activity that produces a standard deviation of 1.4% for the fiscal deficit provides a decrease in the social loss of almost 30%. The lowest panel of Figure 3 shows the same graph as the upper panel, except with a standard quadratic loss function in output and inflation variances rather than the microfounded social loss. Under this measure fiscal stabilisation produces a more than 40% decrease in the loss.

The middle panel of Figure 3 shows what these changes in the social loss correspond to in terms of the change of the steady state level of consumption that would be needed to compensate for them. So a policy of constant fiscal deficits produces a level of volatility which would need a 0.25 % increase in steady state consumption to compensate for it relative to the case when only automatic stabilisers operate. A policy with a degree of fiscal activity which results in a standard deviation of 1.4% for fiscal deficits (again with deficits measured as a percentage of GDP) corresponds to a non-trivial steady state gain in consumption of over 1.5%. The lowest line indicates the consumption gain if shocks in the economy could be entirely avoided, relative to the case with automatic stabilisers and shocks. Thus active fiscal policy delivers approximately 30% of the maximum possible consumption gain.

Figure 4 shows the breakdown of the loss as the level of fiscal activity varies. The upper panel in this figure shows, for each level of fiscal activity, the proportion of the social loss that is due to its major components (e.g. inflation volatility, output volatility etc.). As can be seen, in addition to the volatility of output and inflation, large components of the social loss come from the volatility of the *change* in inflation and the covariance between inflation and output. The lower panel shows, again for each level of fiscal activity, what proportions of the social loss that can be ascribed to the symmetric and asymmetric components of supply and demand shocks respectively.

As can be seen, even with habit persistence, the vast majority of costs of volatility are due to supply shocks, with asymmetric supply shocks being significantly the largest component.

This is emphasised in Figure 5, which plots the losses due to the different shocks in *levels* rather than in percentages: we normalise all values of losses by dividing them by the value of total loss in the case when automatic stabilisers operate and the economy is affected by all shocks. The left panel shows the welfare losses measured by the microfounded loss function, the right panel by a traditional loss metric. We can see that fiscal activity generally increases the social loss when shocks are symmetric, but that these effects are relatively small. A possible reason for this loss in welfare is that fiscal policy is being pulled in unhelpful directions because it responds to the shocks in ways which are dominated by its need to deal well with asymmetric shocks. It could also be that fiscal authorities are pursuing the ‘wrong objectives’ in minimising a simple loss function (31)-

(32) rather than the true social loss; we can see that in the right panel using a traditional welfare metric in output and inflation volatility fiscal policy produces a small improvement in the response to symmetric shocks. In both cases however, the small positive or negative effects on symmetric shocks, are outweighed by a much larger decrease in losses when shocks are asymmetric, which are particularly large for asymmetric supply shocks. For asymmetric shocks, we can see that the ‘rough and ready’ stabilisation provided by a fiscal authority minimising a simple loss function produces large gains in (the complex measure of) welfare.

We would expect the stabilisation of asymmetric demand shocks to improve welfare and Figure 5 shows that it does so. However the largest amount of the welfare improvement comes from the stabilisation of asymmetric supply shocks. The first reason for this is that, even with habit persistence, the consumption equation is ‘more forward looking’ than the inflation process, and so demand disturbances are removed more quickly. Second, with a monetary union, because of the fixed exchange rate, the relative price level between the two countries must return to unity (or zero in logs) following a shock. This stabilisation of the price-level is costly because the integral of relative inflation rather than its level has to come to zero following a shock. This makes inflation shocks more costly in their effects than shocks to demand. In effect the monetary union is forcing the economy to operate like it would with price level targeting; inflationary bygones cannot be bygones. Fiscal policy can help to prevent excessive costs of this. It cannot circumvent the constraint - but it prevents overshoot in response to it<sup>7</sup>.

In summary, we find that automatic stabilisers produce relatively little benefits in stabilisation compared to the policy of constant deficits. Active fiscal policy can generate non-trivial gains in welfare, corresponding to a 1.58% increase in steady state consumption. Asymmetric supply shocks cause the majority of the costs of volatility and it is the ability of fiscal policy to deal with these that is responsible for the vast majority of the gains from fiscal stabilisation. In contrast, fiscal activity slightly increases losses when shocks are symmetric.

We now explore how these results vary as we change the parameters of the system.

### 4.3 Sensitivity of the Results: the Importance of Shocks, Persistence and Lags

Above, we discussed the result that fiscal policy does not make a large quantitative difference to the response of the economy to symmetric shocks. We find that this result holds for all reasonable parametrisations we have tested. However the improvement in responses to asymmetric shocks, and so the improvement in welfare, brought about by active fiscal policy depends strongly on key parameters in the model: the level of habit persistence  $\rho$ , the proportion of rule of thumb price setters  $\omega$ , and parameter  $\varphi$  which determines the lag structure of the Phillips curve.

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<sup>7</sup>Another contributing factor to the relative importance of supply shocks is that the (microfounded) social loss places much more weight on inflation volatility, and on volatility in the change of inflation, rather than on the volatility of output (see Table 1). Figure 6 shows the individual impact of fiscal activity on the volatility of inflation, output, consumption and government spending.

Table 3 shows the welfare changes for different fiscal regimes and parameterisations of the model in terms of the equivalent gain or loss in consumption measured as a percentage of GDP. Column (1) shows the change in steady state consumption which is equivalent to the change in welfare in moving from a policy of using automatic stabilisers to a policy of constant deficits. Column (2) shows the gain from moving from a policy of using automatic stabilisers to active fiscal policy. In Column (3), we show the consumption gain that could be obtained by entirely eliminating macroeconomic volatility (i.e. no shocks) relative to the same policy. Comparing columns (2) to column (3) then shows the proportion of the total costs of macroeconomic volatility that can be eliminated by using active fiscal policy. Column (4) is explained below.

The first row shows the base-line case described above. The base-line case parameters are denoted with asterisks. The second row of Table 3 shows results for the base-line parametrisation but with a zero level of habit persistence. The results are similar to those in the first row: varying the level of habit persistence  $\rho$  has relatively small effects on the potential gains from fiscal stabilisation compared to the base-line case. The third row of Table 3 however shows that when the proportion of forward-looking individuals,  $1 - \omega$ , is large, the potential gains from fiscal stabilisation are small. The large fraction of forward-looking price-setters implies that relative prices in the countries of the union adjust quickly, and hence asymmetric shocks die out quickly, without any policy intervention. The fourth row shows the results for the base-line levels of habit persistence and proportion of backward-looking price setters but with a lengthened lag structure on the demand terms in the Phillips curve, given by changing the parameter  $\varphi$  from 1 to 0.5 as described in Section 3. In this case, the welfare gain from fiscal stabilisation is substantially higher than in any other cases studied.

In order to understand these numbers we present Figures 7, 8 and 9. Since the differences in the welfare are a result of how much fiscal policy improves stabilisation in the face of *asymmetric* shocks, we present impulse responses to asymmetric supply and demand shocks only. In Figure 7 (which corresponds to the second row of Table 3), we see that with no habit persistence the responses to asymmetric supply shocks are similar to the base-line case in Figure 2, but the impact of asymmetric demand shocks is significantly reduced (note the reduction in the vertical scale). Consumption will immediately damp the shocks, largely preventing the response of other variables. In Figure 8 (which corresponds to the third row of Table 3), we see the responses with no habit persistence and a large proportion of forward-looking price-setters. Here the forward-lookingness of the Phillips curve means that asymmetric shocks are stabilised relatively quickly, and as a result fiscal policy makes a smaller impact in terms of welfare.

Finally, Figure 9 shows the responses to asymmetric shocks of the economy where the Phillips curve has a longer output lag structure due to changing  $\varphi$  from 1.0 to 0.5. The effect of this is to redistribute the lagged output terms in the Phillips curve (due to backward-looking price-setters) from all being at a lag of one quarter to half being at a lag of one quarter and half at a lag of two quarters. Empirically, this is an entirely plausible and seemingly relatively small change but it causes fiscal policy to have a much larger welfare impact.<sup>8</sup> Without fiscal policy, the lengthening of the lag structure causes

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<sup>8</sup>Figure 10 shows the welfare losses in this case for a varying degrees of fiscal activity.

	Key parameter values	Constant deficit, %	Active fis- cal policy, %	No shocks %	Naive rule %
		(1)	(2)	(3)	(4)
Base case	$\omega^* = 0.5, \rho^* = 0.8$ $\varphi^* = 1$	-0.25	1.58	5.34	0.96
No habits, but high inflation persistence	$\omega^*, \rho = 0, \varphi^*$	-0.45	2.44	8.37	1.45
No habits and low inflation persistence	$\omega = 0.1, \varphi^*,$ $\rho = 0$	-0.15	0.24	1.60	0.10
Increased output lags in Phillips curve	$\omega^*, \rho^*,$ $\varphi = \frac{1}{2}\varphi^*$	-38	14.03	17.70	12.25

Table 3: Welfare relative to that with Constant Government Spending: Consumption gains

the economy to have very prolonged cyclical responses to asymmetric shocks even in a model as simple as this. When the system is close to being unstable without fiscal policy, the fiscal stabilisation then produces very large welfare gains<sup>9</sup>.

Of course, the large numerical value of the welfare gain in Table 3 in the latter case is predominantly because the automatic stabilisers do not stabilise the economy well and we measure all gains relative to them. One can argue that any naive rule which feeds back on output should stabilise the economy well. The last column in Table 3 reports the percentage change in consumption level corresponding to welfare gains using a simple feedback rule,  $g_t = -\{0.5/(1 - \theta)\}x_{t-1}$ , similar to the one used by Westaway (2003). In this rule the size of the feedback coefficient of the linearised rule ensures a fall in government spending of 0.5% of GDP if GDP rises by 1%. As is apparent, in the first two rows the gain in consumption is still a non-trivial number around 1%. In all cases, a fiscal authority with a simple rule provides a large of proportion of the welfare gains obtained by our base case of a fiscal authority targetting output, inflation and the deficit as given in (31) and (32). However the base case active fiscal policy does provide a non-trivial welfare improvement over the simple rule.

## 5 Conclusion

Our work shows that the active use of fiscal policy in a monetary union can result in a significant increase in welfare. This welfare gain is predominantly brought about by the

<sup>9</sup>The ability of lags to cause a system to become more and more volatile has a long history in economic literature. In the language of Phillips (1957), fiscal policy provides ‘derivative control’ which greatly improves the behaviour of the lagged system.

ability of the fiscal policy to mitigate the effects of asymmetric shocks, and in particular asymmetric supply shocks. Monetary policy can satisfactorily stabilise symmetric shocks and fiscal policy can not greatly improve outcomes in response to such shocks. By contrast, fiscal policy greatly improves the stabilisation of asymmetric shocks and substantially increase overall welfare.

We find that the importance of fiscal policy for stabilisation depends to a large extent on the structure of the Phillips curve. When there is persistence in inflation, outcomes with fiscal stabilisation produce significantly lower social losses than non-strategic fiscal policy (of constant deficits or automatic stabilisers). This occurs with levels of inflation persistence which are well within the bounds of those postulated in the empirical literature. We also find that fiscal stabilisation is more important when there are lags in the effect of demand in the Phillips curve.

This research does not take into account many of the reasons why tight restrictions on fiscal policy might be desirable. For instance we do not take into account the possibility of fiscal insolvency, or political economy factors which may be important. Were fiscal policy to have small welfare advantages in terms of macroeconomic stabilisation, one could argue for tight restrictions to address these problems. However, given that the fiscal policy can play a very important role in macroeconomic stabilisation in a monetary union we argue that a regulatory framework for fiscal policy should allow it to play a substantial role in stabilising the economy.

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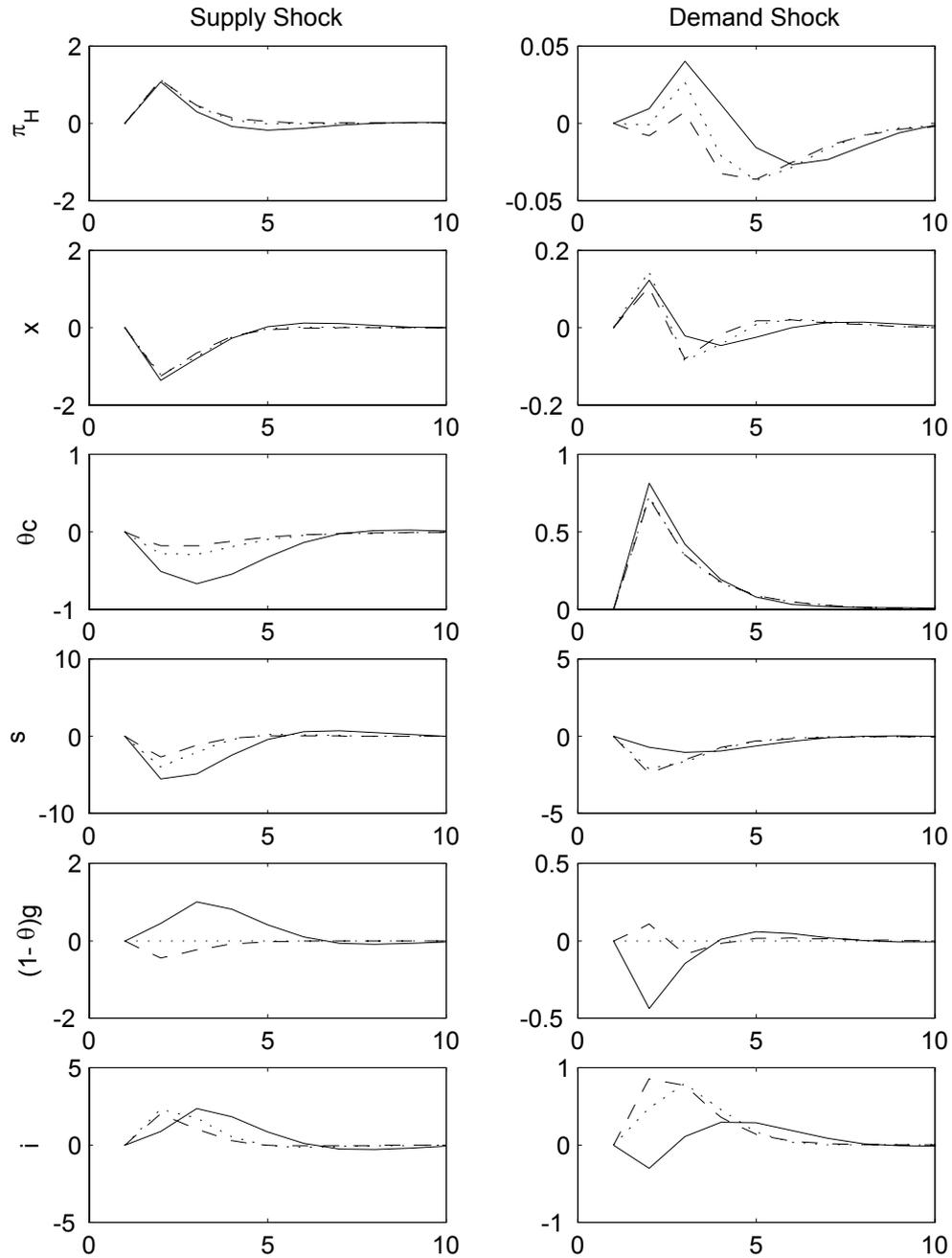


Figure 1: Symmetric shocks. Habit persistence. Solid line denotes unconstrained optimal policy, dashed line denotes policy when the budget is close to balanced every period and dotted line denotes policy with constant government spendings.

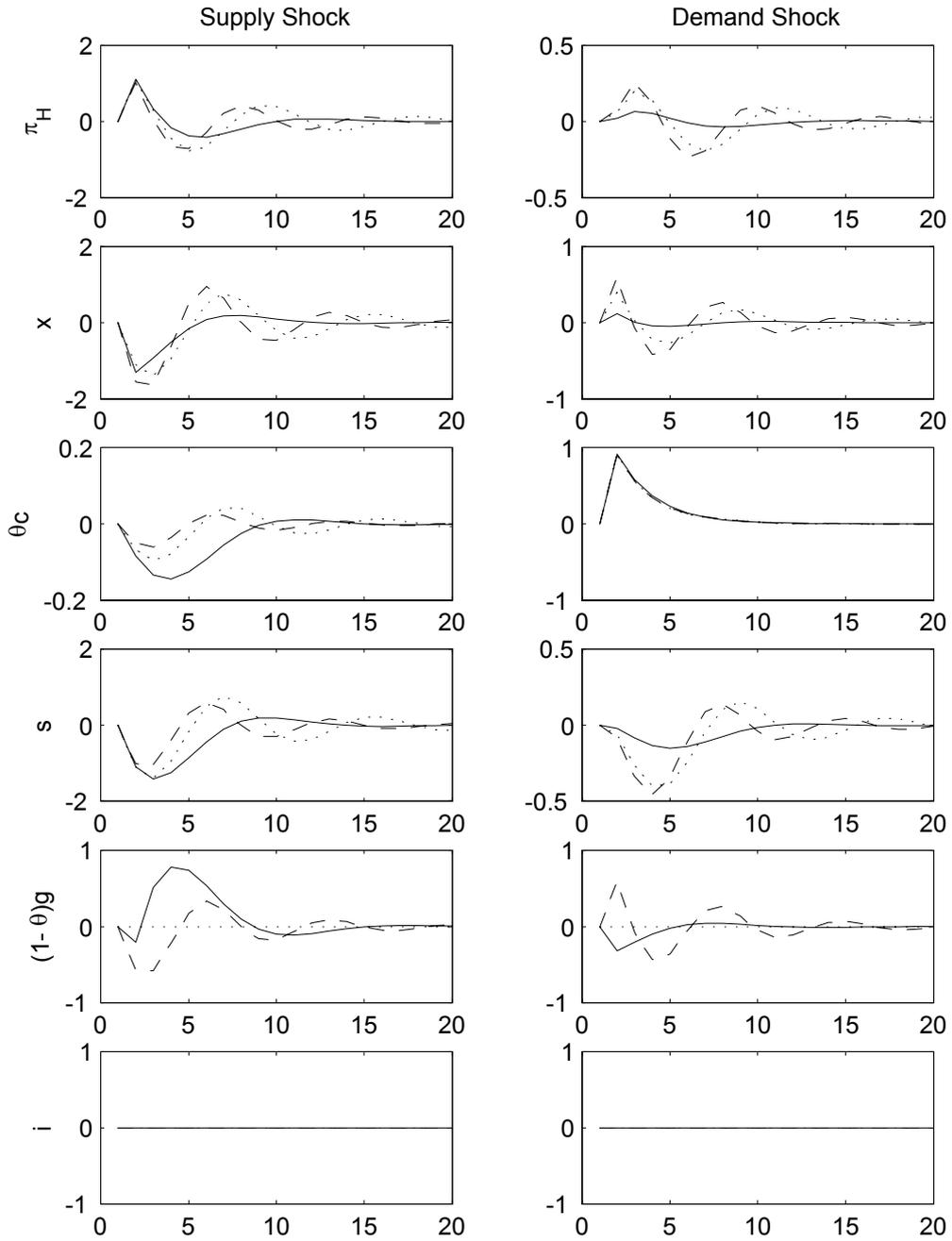


Figure 2: Asymmetric shocks. Habit persistence. Solid line denotes unconstrained optimal policy, dashed line denotes policy when the budget is close to balanced every period and dotted line denotes policy with constant government spendings.

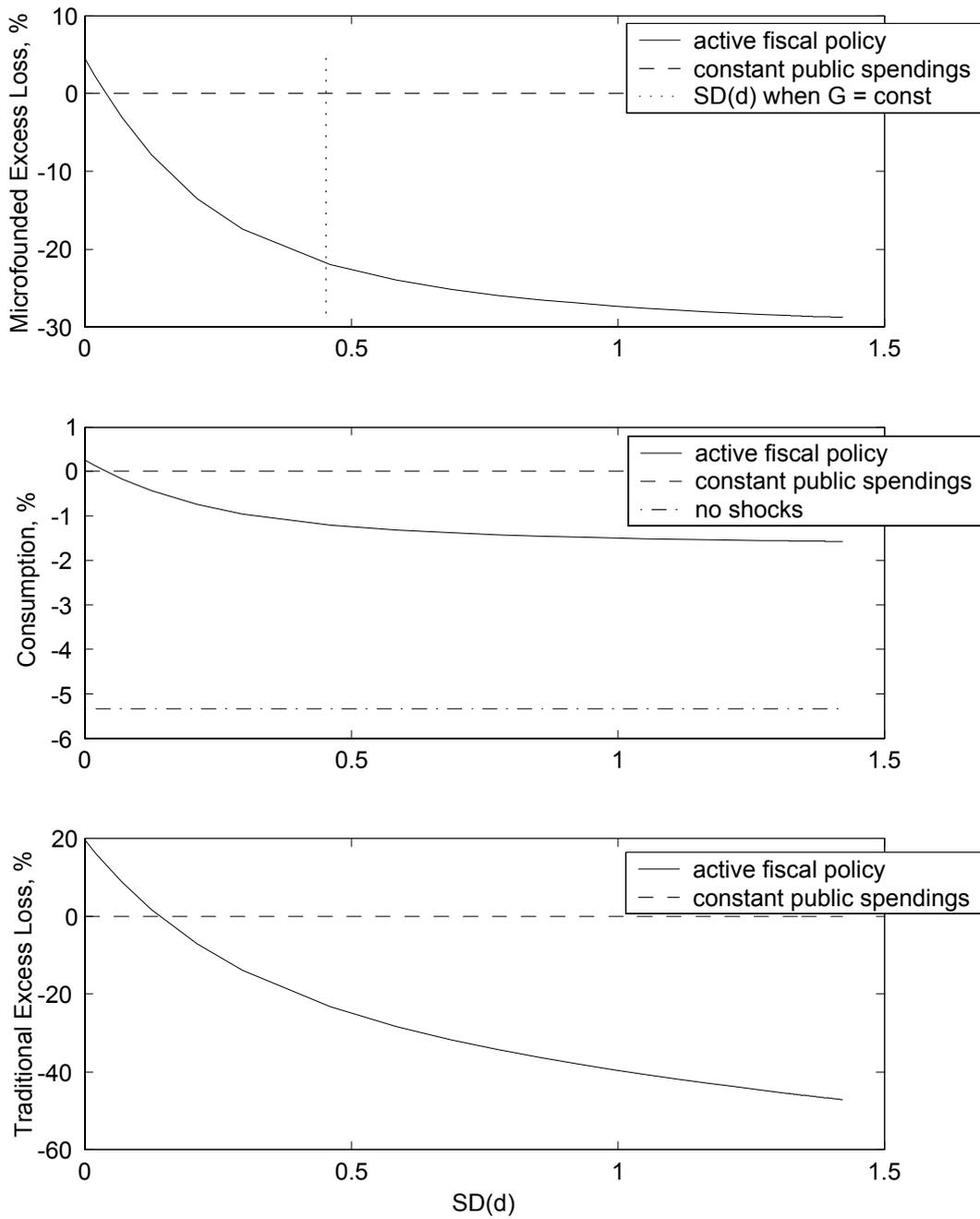


Figure 3: Social loss and consumption loss as function of fiscal activity. Habit persistence and inflation persistence.

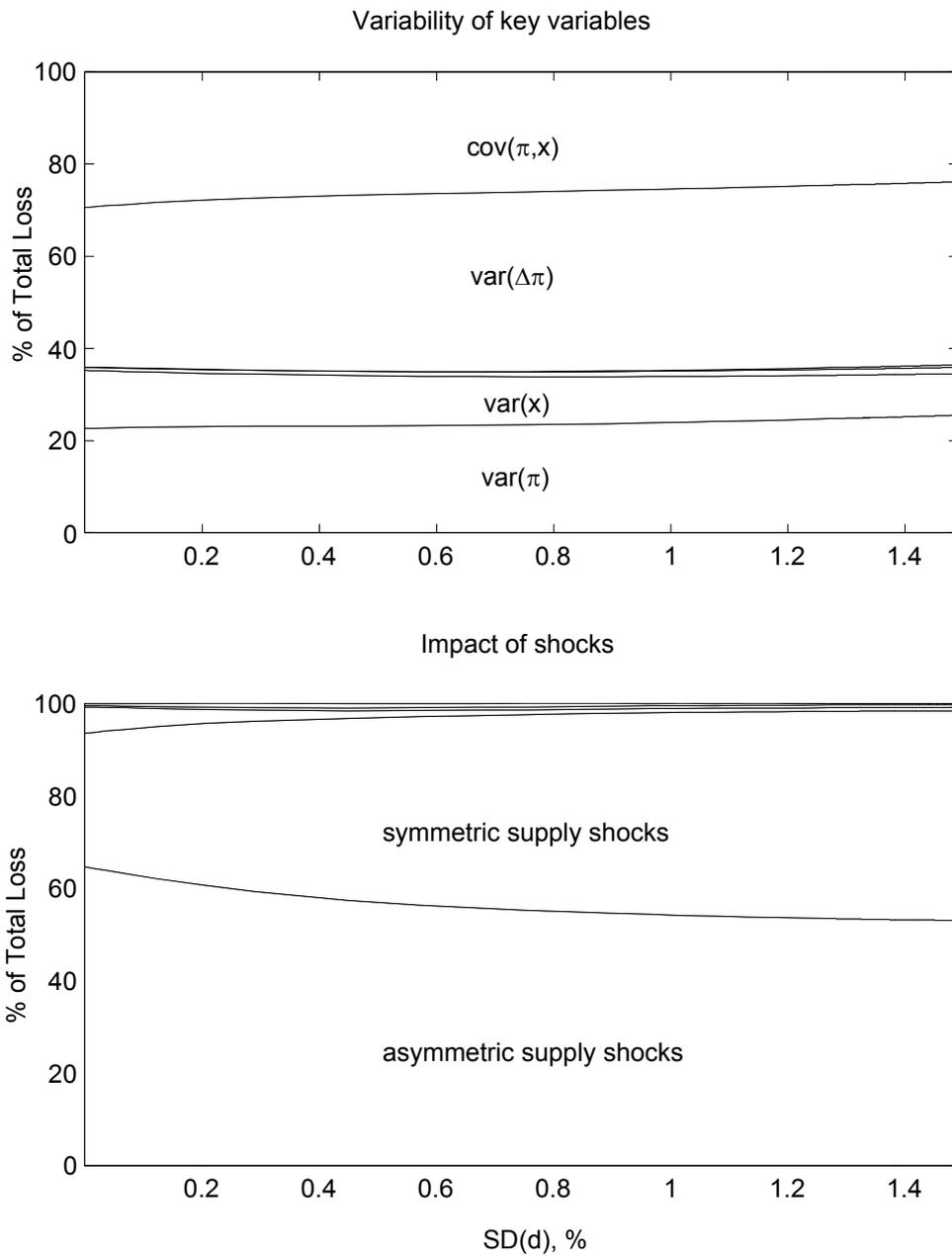


Figure 4: Components of social loss. Habit persistence and inflation persistence.

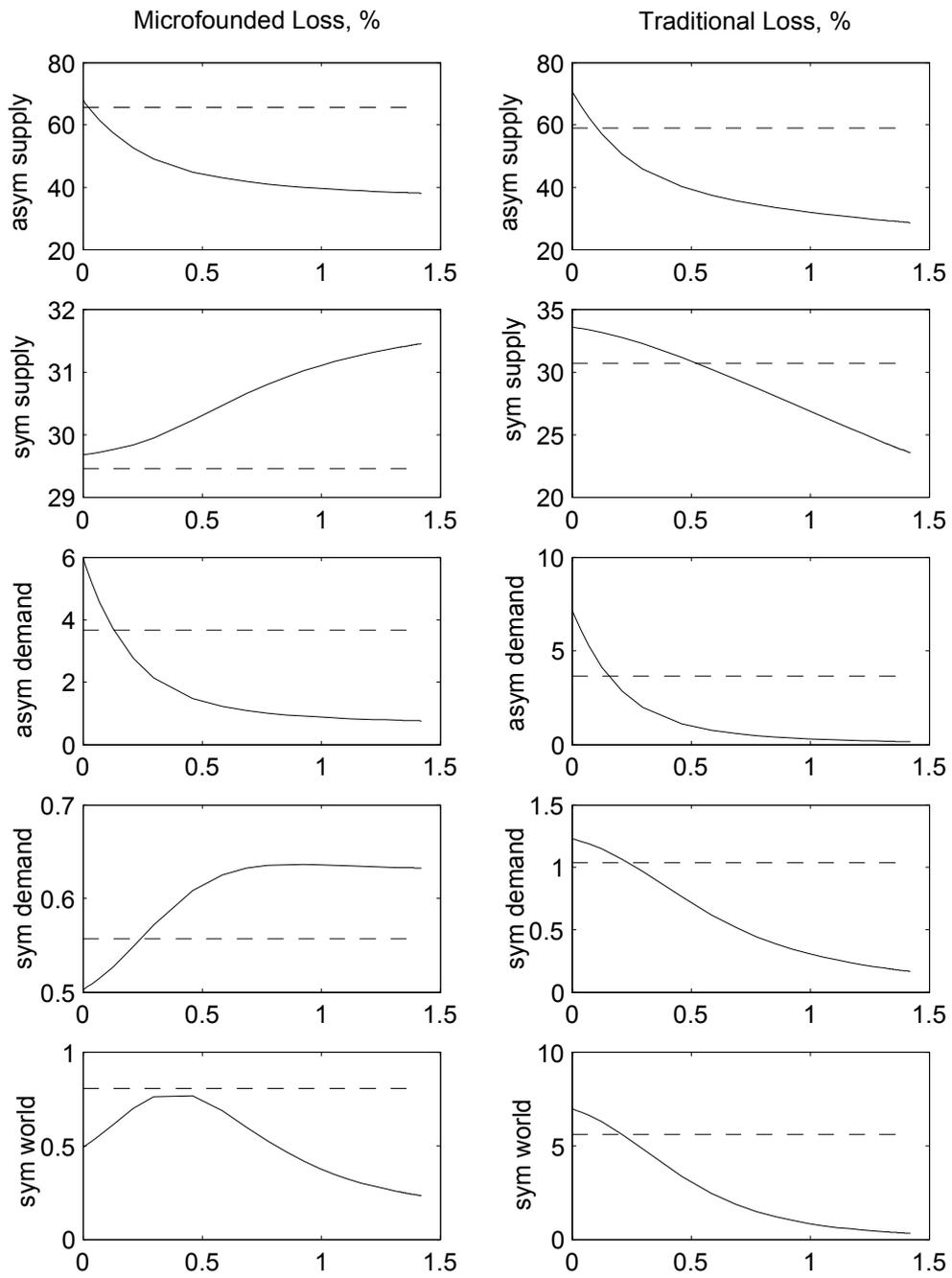


Figure 5: Social Loss due to symmetric and asymmetric shocks.

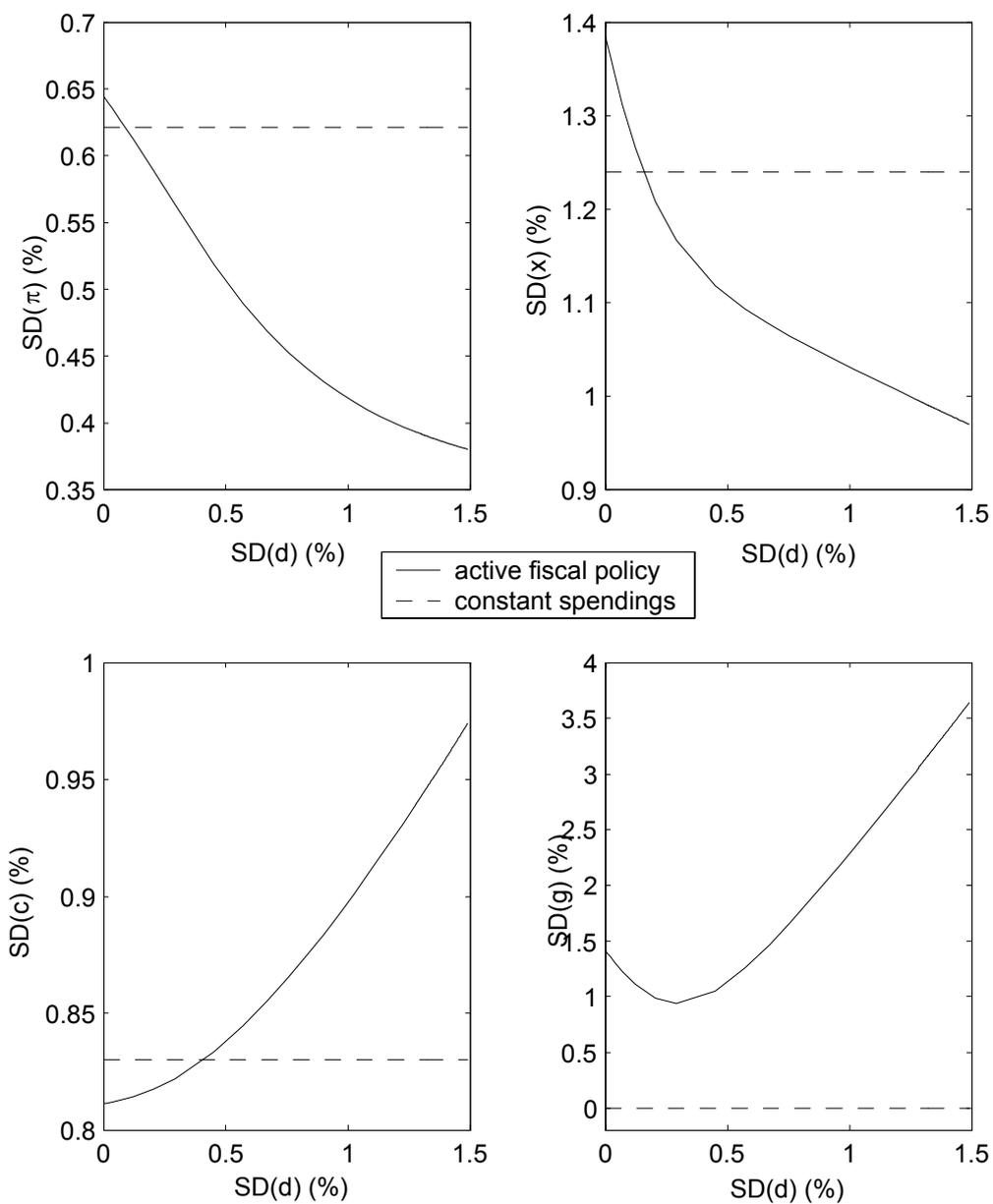


Figure 6: Standard deviations of key variables as a function of fiscal activity. Habit persistence and inflation persistence. All shocks.

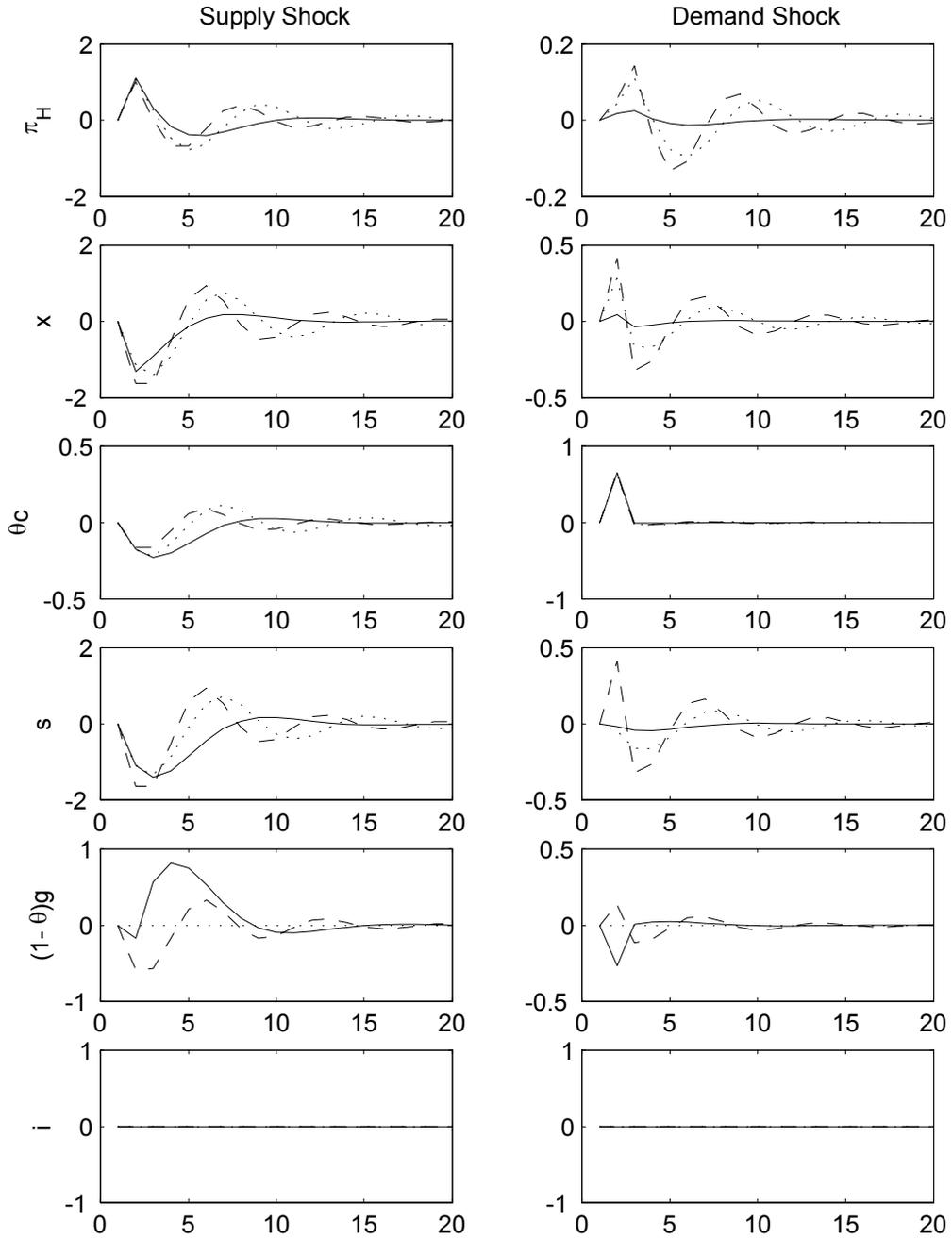


Figure 7: Asymmetric shocks. No habit persistence, but with inflation persistence. Solid line denotes unconstrained optimal policy, dashed line denotes policy when the budget is close to balanced every period and dotted line denotes policy with constant public spendings.

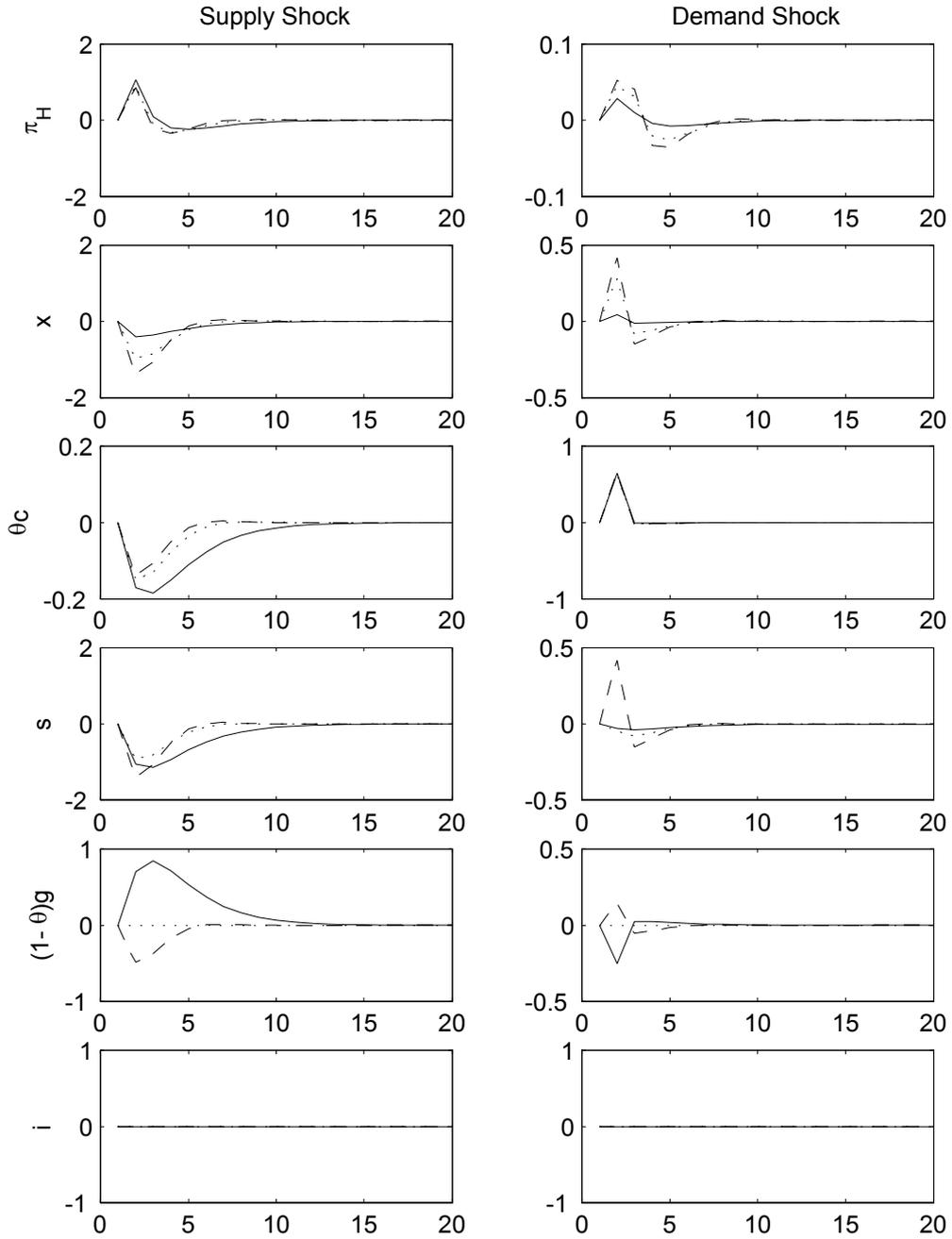


Figure 8: Asymmetric shocks. No habit persistence, high proportion of forward-looking price-setters. Solid line denotes unconstrained optimal policy, dashed line denotes policy when the budget is close to balanced every period and dotted line denotes policy with constant public spendings.

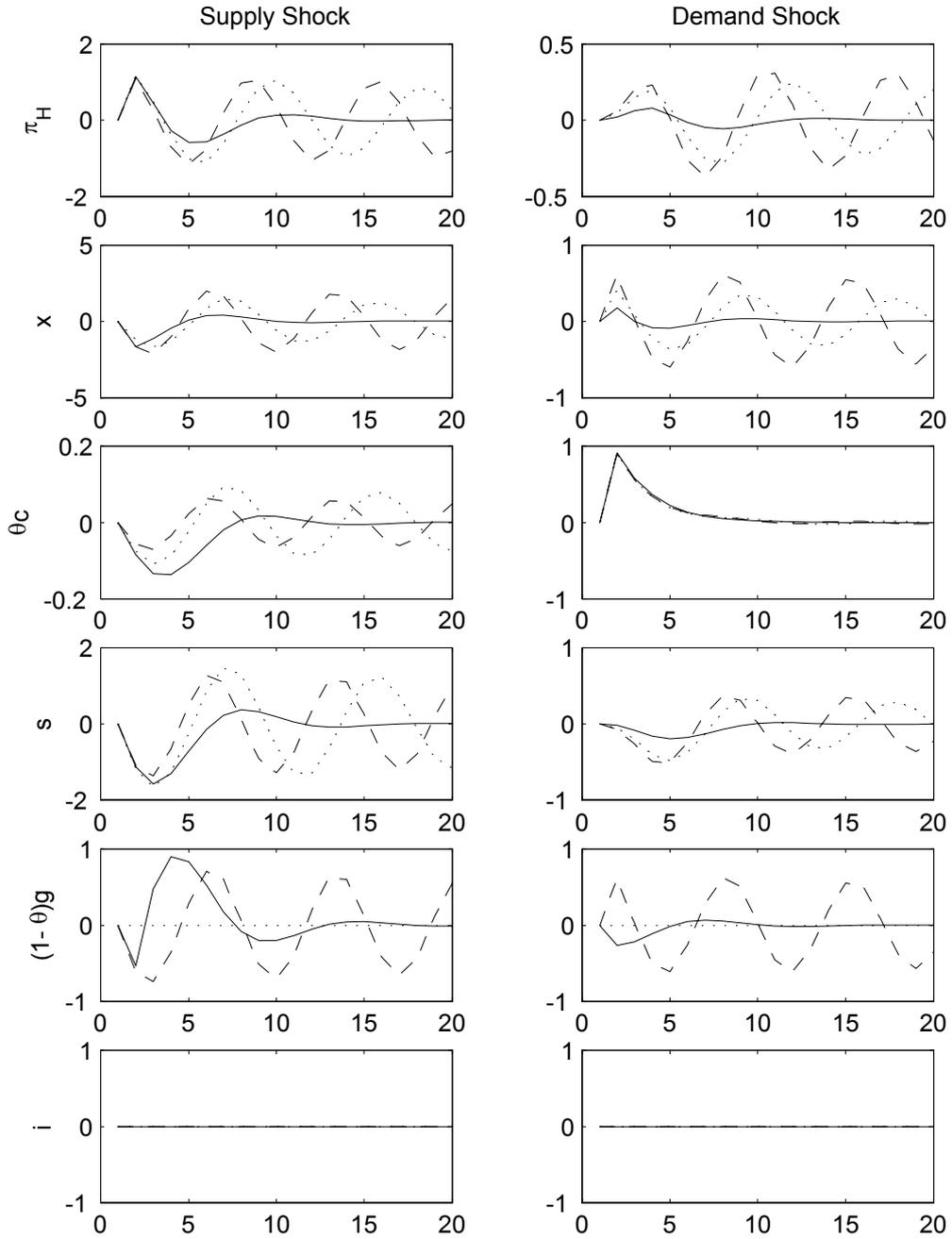


Figure 9: Asymmetric shocks. Habit persistence, inflation persistence. One extra output lag in the Phillips curve. Solid line denotes unconstrained optimal policy, dashed line denotes policy when the budget is close to balanced every period and dotted line denotes policy with constant public spendings.

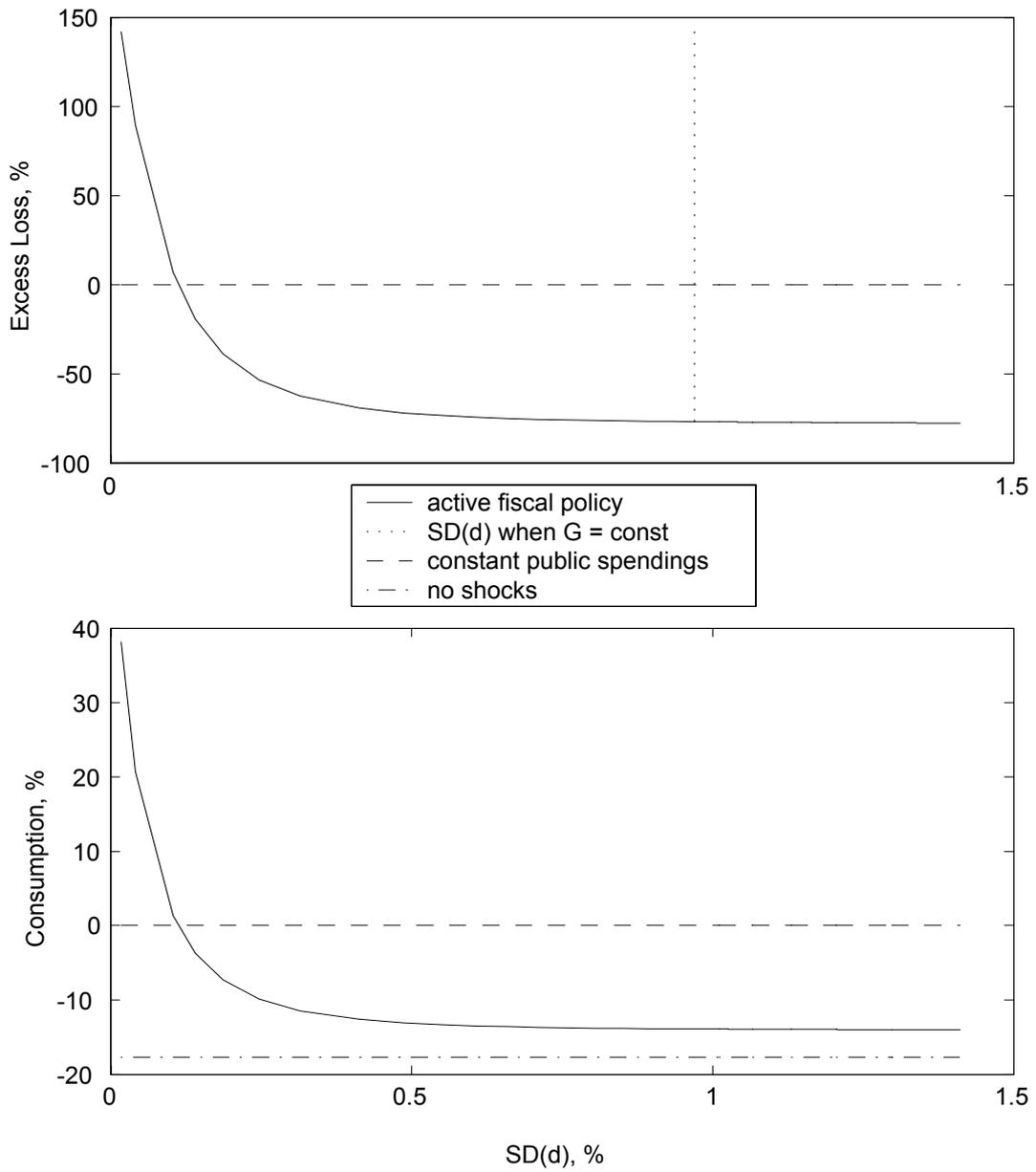


Figure 10: Social loss and consumption loss as function of fiscal activity. Habit persistence and inflation persistence. One extra output lag in the Phillips curve

## A Price-setting decisions

Pricing behaviour is taken as in Rotemberg and Woodford (1997) and Steinsson (2003). Households are able to reset their price in each period with probability  $1 - \gamma$  in which case they re-contract a new price  $P_H^n$ . For the rest of the household sector the price will rise at the steady state rate of domestic inflation  $\bar{\Pi}_H$  with probability  $\gamma$ :

$$P_{Ha,t} = \bar{\Pi}_{Ha} P_{Ha,t-1}$$

Those who recontract a new price (with probability  $1 - \gamma$ ), are split into backward-looking individuals and forward-looking individuals, in proportion  $\omega$ , such that the aggregate index of prices set by them is

$$P_{Ha,t}^\times = (P_{Ha,t}^f)^{1-\omega} (P_{Ha,t}^b)^\omega \quad (35)$$

Backward-looking individuals set their prices according to the rule of thumb:

$$P_{Ha,t}^b = P_{Ha,t-1}^\times \Pi_{Ha,t-1} \left( \frac{Y_{at-1}}{Y_{at-1}^n} \right)^\delta \quad (36)$$

where

$$\Pi_{Ha,t} = \frac{P_{Ha,t}}{P_{Ha,t-1}}$$

and  $Y_t^n$  is the efficient level of output.

We define log deviations from the steady state domestic price levels for both types of price-setters as:

$$\hat{P}_{Ha,t}^b = \ln \frac{P_{Ha,t}^b}{P_{Ha,t}}, \hat{P}_{Ha,t}^f = \ln \frac{P_{Ha,t}^f}{P_{Ha,t}}$$

### A.1 Forward-looking price-setters

Each forward-looking producer understands that sales depend on demand, which is a function of price:

$$y_{as}(z) = \left( \frac{p_{Ha}(z)}{P_{Ha}} \right)^{-\epsilon} Y_a.$$

Maximisation of expected profit requires the solution of:

$$\max_{P_{H,t}(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} [p_{Ha,t}(z) y_{as}(z) - w_{as}(z) y_{as}(z)]$$

which implies the following first order condition:

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} Y_{as} \left( \frac{p_{Ha,t}(z)}{P_{Ha,s}} \right)^{-\epsilon} [p_{Ha,t}(z) - \mu S_{a,t}(z)]$$

where  $\mu = -\frac{\epsilon}{1-\epsilon}$ ,  $S_{a,t}(z)$  is marginal cost and  $R_{t,s}$  is discount factor. This condition holds for both flexible and fixed price equilibria. However, for the fixed price equilibrium the nominal marginal cost is a function of price, set at the period  $t$ . Substituting for the nominal marginal cost, we get a final equation for the optimal  $p_{Ha,t}(z) = p_{Ha,t}^f(z)$

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} Y_{as} \left( \frac{p_{Ha,t}^f(z)}{P_{Ha,s}} \right)^{-\epsilon} (p_{Ha,t}^f(z)) \quad (37)$$

$$- \frac{\frac{\mu P_{as}}{(1-\tau)} v_y \left( \left( \frac{p_{Ha,t}^f(z)}{P_{Ha,s}} \right)^{-\epsilon} Y_{as}, \xi_s \right)}{u_C \left( \frac{C_s}{C_{s-1}^\rho}, \xi_s \right) C_{s-1}^{-\rho} - \beta \rho u_C \left( \frac{C_{s+1}}{C_s^\rho}, \xi_s \right) C_{s+1} C_s^{-\rho-1}}.$$

where  $\tau$  is wage income tax. The linearisation of the equation (37) can be found in Rotemberg and Woodford (1997) for the closed economy case. We briefly repeat it here for the open economy.

First of all, each term in the price-setting first order conditions (37) is the product of two terms, the term in curly brackets and the term in square brackets. The term in the square brackets vanishes in the equilibrium so its deviations from the equilibrium are of first order. Therefore, all products of it with the first term will be higher than of first order, unless the first term is taken at its equilibrium level, which is  $(\gamma\beta)^{s-t}$ , up to some constant multiplier.

Linearising the term in square brackets yields:

$$\frac{p_{Ha,t}^f(z)}{P_{Ha,s}} - \frac{\frac{\mu}{(1-\tau)} v_y \left( \left( \frac{p_{Ha,t}^f(z)}{P_{Ha,s}} \right)^{-\epsilon} Y_{as}, \xi_s \right)}{u_C \left( \frac{C_s}{C_{s-1}^\rho}, \xi_s \right) C_{s-1}^{-\rho} - \beta \rho u_C \left( \frac{C_{s+1}}{C_s^\rho}, \xi_s \right) C_{s+1} C_s^{-\rho-1}}$$

$$= \widehat{p}_{Ha,t}^f - \left[ \sum_{k=1}^{s-t} \pi_{Ha,t+k} + \frac{1}{\psi} \widehat{Y}_{as} + \frac{1}{\sigma} \widehat{\lambda}_s + \alpha_n^a \widehat{\mathcal{S}}_{abs} + (1 - \alpha_d^a - \alpha_n^a) \widehat{\mathcal{S}}_{aws} \right.$$

$$\left. - \frac{\epsilon}{\psi} \left\{ \widehat{p}_{Ha,t}^f - \sum_{k=1}^{s-t} \pi_{Ha,t+k} \right\} + \frac{v_y \xi(\bar{Y}, 1)}{v_y(\bar{Y}, 1)} \widehat{\xi}_s \right]$$

where  $\mathcal{S}_{ab} = \frac{P_{Hb}}{P_{Ha}}$ ,  $\mathcal{S}_{aw} = \frac{P_{Hw}}{P_{Ha}}$  are two terms of trade and

$$\sigma = -\frac{u_C(C/C^\rho, 1)}{u_{CC}(C/C^\rho, 1)C}, \quad \psi = \frac{v_y(\bar{Y}, 1)}{v_{yy}(\bar{Y}, 1)\bar{Y}}$$

We solve out this equation for prices and, using the fact that  $\sum_{s=t}^{\infty} (\gamma\beta)^{s-t} \sum_{k=1}^{s-t} \pi_{Ha,t+k} = \frac{1}{1-\gamma\beta} \sum_{k=1}^{\infty} (\gamma\beta)^k \pi_{Ha,t+k}$  we obtain the following formula for the forward-looking individuals:

$$\begin{aligned}
\widehat{P}_{Ha,t}^f &= \sum_{k=1}^{\infty} (\gamma\beta)^k \pi_{Ha,t+k} \\
&+ \frac{(1-\gamma\beta)}{1+\frac{\epsilon}{\psi}} \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} [\alpha_n^a \widehat{\mathcal{S}}_{abs} + (1-\alpha_d^a - \alpha_n^a) \widehat{\mathcal{S}}_{aws} \\
&+ \frac{1}{\psi} (\widehat{Y}_{as} - \widehat{Y}_{as}^n) + \frac{1}{\sigma} (\widehat{\lambda}_{as} - \widehat{\lambda}_{as}^n)]
\end{aligned} \tag{38}$$

Here we also used the fact that the linearisation of the similar equation for the flexible price equilibrium helps to get rid of shocks and write down the optimisation equation in terms of gaps with natural levels for output and consumption. Here  $(\alpha_n^a \widehat{\mathcal{S}}_{ab} + (1-\alpha_d^a - \alpha_n^a) \widehat{\mathcal{S}}_{aw})$  comes in as the result of the wedge between consumption of the CPI basket and the production of domestic goods and different prices set on them. The constant tax rate,  $\tau$ , does not enter the final formula when written in log-deviations from equilibrium (see Benigno and Benigno (2000) for similar derivation).

This can be rewritten in a quasi-differenced form as:

$$\begin{aligned}
\widehat{p}_{Ha,t}^f &= \gamma\beta \widehat{p}_{Ha,t+1}^f + \gamma\beta \pi_{Ha,t+1} \\
&+ \frac{1-\gamma\beta}{1+\frac{\epsilon}{\psi}} \left( \alpha_n^a \widehat{\mathcal{S}}_{abt} + (1-\alpha_d^a - \alpha_n^a) \widehat{\mathcal{S}}_{awt} + \frac{1}{\psi} (\widehat{Y}_{at} - \widehat{Y}_{at}^n) + \frac{1}{\sigma} (\widehat{\lambda}_{at} - \widehat{\lambda}_{at}^n) \right)
\end{aligned} \tag{39}$$

## A.2 Rule-of-thumb price-setters and Phillips curve

The rule-of-thumb price-setters use formula (36) to set the new price. The linearisation of this equation (using (35)) straightforwardly yields:

$$\widehat{P}_{H,t}^b = (1-\omega) \ln \frac{P_{H,t-1}^f}{P_{H,t-1}} + \omega \ln \frac{P_{H,t-1}^b}{P_{H,t-1}} - \ln \Pi_{H,t} + \ln \Pi_{H,t-1} + \delta \ln \left( \frac{Y_{t-1}}{Y_{t-1}^n} \right)$$

so we have the following equations (we use the standard notation and use  $\widehat{\mathcal{S}}_{abt} = \widehat{\mathcal{S}}_{awt} - \widehat{\mathcal{S}}_{bwt}$ )

$$\begin{aligned}
\widehat{P}_{Ha,t}^b &= (1-\omega) \widehat{P}_{Ha,t-1}^f + \omega \widehat{P}_{Ha,t-1}^b - \pi_{Ha,t} + \pi_{Ha,t-1} + \delta x_{at-1} \\
\pi_{Ha,t} &= \frac{(1-\gamma)}{\gamma} ((1-\omega) \widehat{P}_{Ha,t}^f + \omega \widehat{P}_{Ha,t}^b) \\
\widehat{P}_{Ha,t}^f &= \gamma\beta \widehat{P}_{Ha,t+1}^f + \gamma\beta \pi_{Ha,t+1} + \frac{1-\gamma\beta}{1+\frac{\epsilon}{\psi}} \left( (1-\alpha_d^a) s_{awt} - \alpha_n^a s_{bwt} + \frac{1}{\psi} x_{at} + \frac{1}{\sigma} \lambda_{at} \right)
\end{aligned}$$

Doing manipulations similar to Steinsson (2003) (A.1)-(A.6) we eliminate  $\widehat{P}_{Ha,t}^b$  and  $\widehat{p}_{Ha,t}^f$  and obtain the following specification of the Phillips curve

$$\begin{aligned} \pi_{Ha,t} &= \frac{\gamma}{\gamma + \omega(1 - \gamma + \gamma\beta)} \beta \pi_{Ha,t+1} + \frac{\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)} \pi_{Ha,t-1} \\ &+ \frac{(1 - \gamma)\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)} \delta x_{at-1} - \frac{(1 - \gamma)\gamma\beta\omega}{\gamma + \omega(1 - \gamma + \gamma\beta)} \delta x_{at} \\ &+ \frac{(1 - \gamma\beta)(1 - \gamma)(1 - \omega)}{(\gamma + \omega(1 - \gamma + \gamma\beta))} \frac{\psi}{\psi + \epsilon} \left( (1 - \alpha_d^a) s_{awt} - \alpha_n^a s_{bwt} + \frac{1}{\psi} x_{at} + \frac{1}{\sigma} \lambda_{at} \right) \end{aligned} \quad (40)$$

Note that when  $\omega = 0$  then the Phillips curve collapses to the standard forward-looking specification:

$$\begin{aligned} \pi_{Ha,t} &= \beta \pi_{Ha,t+1} + \frac{(1 - \gamma\beta)(1 - \gamma)}{\gamma} z_t \\ z_t &= \frac{\psi}{\psi + \epsilon} \left( (1 - \alpha_d^a) s_{awt} - \alpha_n^a s_{bwt} + \frac{1}{\psi} x_{at} + \frac{1}{\sigma} \lambda_{at} \right) \end{aligned}$$

When  $\omega = 1$  then the Phillips curve takes the specification

$$\pi_{Ha,t} = \frac{\gamma\beta}{(1 + \gamma\beta)} \pi_{Ha,t+1} + \frac{1}{1 + \gamma\beta} \pi_{Ha,t-1} - \frac{(1 - \gamma)}{1 + \gamma\beta} (\gamma\beta \delta x_{at} - \delta x_{at-1}). \quad (41)$$

This equation was obtained by integrating and can contain extra solutions. We are looking for solution without forward looking components, as suggested by initial formula (36). Such a solution exists and can be written in the form of accelerationist Phillips curve:

$$\pi_{Ha,t} = \pi_{Ha,t-1} + (1 - \gamma) \delta x_{at-1}. \quad (42)$$

Finally, (40) is a linear combination of forward-looking specification and rule-of-thumb specification:

$$\begin{aligned} \pi_{Ha,t} &= \frac{\omega(1 + \gamma\beta)}{\gamma(1 - \omega) + \omega(1 + \gamma\beta)} \left( \frac{\gamma\beta}{(1 + \gamma\beta)} \pi_{Ha,t+1} \right. \\ &+ \frac{1}{1 + \gamma\beta} \pi_{Ha,t-1} - \frac{(1 - \gamma)}{1 + \gamma\beta} (\gamma\beta \delta x_{at} - \delta x_{at-1}) \\ &+ \left. \frac{\gamma(1 - \omega)}{\gamma(1 - \omega) + \omega(1 + \gamma\beta)} (\beta \pi_{Ha,t+1} + \frac{(1 - \gamma\beta)(1 - \gamma)}{\gamma} z_t) \right). \end{aligned} \quad (43)$$

where we need to substitute (41) with (42) before doing numerical simulations:

$$\begin{aligned} \pi_{Ha,t} &= \frac{\omega(1 + \gamma\beta)}{\gamma(1 - \omega) + \omega(1 + \gamma\beta)} (\pi_{Ha,t-1} + (1 - \gamma) \delta x_{at-1}) \\ &+ \frac{\gamma(1 - \omega)}{\gamma(1 - \omega) + \omega(1 + \gamma\beta)} (\beta \pi_{Ha,t+1} + \frac{(1 - \gamma\beta)(1 - \gamma)}{\gamma} z_t). \end{aligned} \quad (44)$$

Finally, (44) can be rewritten as

$$\pi_t = \chi^f \beta \pi_{t+1} + \chi^b \pi_{t-1} + \kappa_c \lambda_t + \kappa_{x0} x_t + \kappa_{x1} x_{t-1} + \kappa_{sd} s_{as} + \kappa_{sn} s_{bs}$$

where

$$\begin{aligned}
\chi^f &= \frac{\gamma(1-\omega)}{\gamma(1-\omega) + \omega(1+\gamma\beta)} \\
\chi^b &= \frac{\omega(1+\gamma\beta)}{\gamma(1-\omega) + \omega(1+\gamma\beta)} \\
\kappa_c &= \frac{(1-\omega)(1-\gamma\beta)(1-\gamma)\psi}{(\gamma(1-\omega) + \omega(1+\gamma\beta))(\psi + \epsilon)\sigma} \\
\kappa_{x0} &= \frac{(1-\omega)(1-\gamma\beta)(1-\gamma)}{(\gamma(1-\omega) + \omega(1+\gamma\beta))(\psi + \epsilon)} \\
\kappa_{x1} &= \frac{\omega(1+\gamma\beta)(1-\gamma)}{\gamma(1-\omega) + \omega(1+\gamma\beta)} \delta \\
\kappa_{sd} &= \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi(1-\alpha_d^a)}{(\gamma + \omega(1-\gamma + \gamma\beta))(\psi + \epsilon)} \\
\kappa_{sn} &= -\frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi\alpha_n^a}{(\gamma + \omega(1-\gamma + \gamma\beta))(\psi + \epsilon)}
\end{aligned}$$

If we assume that rule-of-thumb individuals take more than one lag on output when setting prices, namely if equation (17) is modified to

$$P_{Ha,t}^b = P_{Ha,t-1}^\times \Pi_{Ha,t-1} \left( \frac{Y_{at-1}}{Y_{at-1}^n} \right)^{\delta_1} \left( \frac{Y_{at-2}}{Y_{at-2}^n} \right)^{\delta_2} \quad (45)$$

we obtain the modified Phillips curve:

$$\pi_t = \chi^f \beta \pi_{t+1} + \chi^b \pi_{t-1} + \kappa_c c_t + \kappa_{x0} x_t + \kappa_{x1} x_{t-1} + \kappa_{x2} x_{t-2} + \kappa_{sa} s_{as} + \kappa_{sb} s_{bs} \quad (46)$$

with

$$\begin{aligned}
\kappa_{x0} &= \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)}{(\gamma + \omega(1-\gamma + \gamma\beta))(\psi + \epsilon)} - \frac{(1-\gamma)\gamma\beta\omega\delta_1}{\gamma + \omega(1-\gamma + \gamma\beta)} \\
\kappa_{x1} &= \frac{(1-\gamma)\omega}{\gamma + \omega(1-\gamma + \gamma\beta)} (\delta_1 - \gamma\beta\delta_2) \\
\kappa_{x2} &= \frac{(1-\gamma)\omega}{\gamma + \omega(1-\gamma + \gamma\beta)} \delta_2.
\end{aligned}$$

## B Social loss function

The one-period utility function (for country  $a$ ) can be obtained by linearisation of one-period utility function in (1) up to the second-order terms (we assume symmetry):

$$\begin{aligned}
W_{sa} + W_{sb} = & u_C(C/C^\rho, 1)C^{1-\rho}[\widehat{C}_{as} - \rho\widehat{C}_{as-1} + \frac{1}{2}(1 - \frac{1}{\sigma})(\widehat{C}_{as} - \rho\widehat{C}_{as-1})^2] \\
& + \frac{u_{C\xi}(C/C^\rho, 1)}{u_C(C/C^\rho, 1)}(\widehat{C}_{as} - \rho\widehat{C}_{as-1})\widehat{\xi}_{as}] + u_C(C/C^\rho, 1)C^{1-\rho}[\widehat{C}_{bs} - \rho\widehat{C}_{bs-1} \\
& + \frac{1}{2}(1 - \frac{1}{\sigma})(\widehat{C}_{bs} - \rho\widehat{C}_{bs-1})^2 + \frac{u_{C\xi}(C/C^\rho, 1)}{u_C(C/C^\rho, 1)}(\widehat{C}_{bs} - \rho\widehat{C}_{bs-1})\widehat{\xi}_{bs}] \\
& + Gf_G(G)[\widehat{G}_a + \frac{1}{2}(1 - \frac{1}{\sigma_g})\widehat{G}_a^2 + \frac{f_{G\xi}(G, 1)}{f_G(G, 1)}\widehat{G}_{as}\widehat{\xi}_{as}] \\
& + Gf_G(G)[\widehat{G}_b + \frac{1}{2}(1 - \frac{1}{\sigma_g})\widehat{G}_b^2 + \frac{f_{G\xi}(G, 1)}{f_G(G, 1)}\widehat{G}_{bs}\widehat{\xi}_{bs}] \\
& - Yv_y(Y, 1)[\widehat{Y}_a + \frac{1}{2}(1 + \frac{1}{\psi})\widehat{Y}_a^2 + \frac{v_{y\xi}(y, 1)}{v_y(y, 1)}\widehat{Y}_{as}\widehat{\xi}_{as} + \frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})var_z\widehat{y}_a(z)] \\
& - Yv_y(Y, 1)[\widehat{Y}_b + \frac{1}{2}(1 + \frac{1}{\psi})\widehat{Y}_b^2 + \frac{v_{y\xi}(y, 1)}{v_y(y, 1)}\widehat{Y}_{bs}\widehat{\xi}_{bs} + \frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})var_z\widehat{y}_b(z)] + tip
\end{aligned} \tag{47}$$

where

$$\sigma_g = -\frac{f_G(G, 1)}{f_{GG}(G, 1)G}, \quad \psi = \frac{v_Y(Y, 1)}{v_{YY}(Y, 1)Y}.$$

We need to find an expression for  $\frac{v_y(Y)}{u_C(C/C^\rho, 1)C^{-\rho}}$  and  $\frac{f_G(G)}{u_C(C/C^\rho, 1)C^{-\rho}}$ . The first condition follows from the steady state condition

$$\frac{v_h(h_s, \xi_s)}{u_C(C/C^\rho, 1)C^{-\rho}(1 - \beta\rho)} = \frac{1 - \tau}{\mu},$$

and in order to derive the second expression we closely follow Beetsma and Jensen (2003). The second steady state relationship is

$$\frac{f_G(G)}{u_C(C/C^\rho, 1)C^{-\rho}} = (1 - \beta\rho) \frac{\frac{1-\tau}{\mu} + \frac{\theta\sigma\phi}{\psi}(\alpha_d + \alpha_n)}{(1 + (\alpha_d + \alpha_n)^2 \frac{\theta\sigma\phi}{\psi} + (1 - (\alpha_d + \alpha_n)^2) \frac{\theta\eta}{\psi}}$$

where

$$\phi = \frac{(1 - \beta\rho)}{(1 + \beta\rho^2 - \beta\sigma\rho(\rho + 1))}$$

We now need to derive a formula for  $var_z\widehat{y}(z)$ , along the lines in Rotemberg and Woodford (1997) and Steinsson (2003). This leads to the formula (for country  $a$ ):

$$\begin{aligned} \text{var}_z \widehat{y}_{as}(z) &= \frac{\epsilon^2}{(1-\gamma\beta)} \left( \frac{\gamma}{1-\gamma} \pi_{Ha,t}^2 + \frac{\omega}{(1-\omega)} \frac{1}{(1-\gamma)} (\Delta\pi_{Ha,t})^2 \right) \\ &+ \frac{\omega}{(1-\omega)} (1-\gamma) \delta^2 x_{at-1}^2 + \frac{2\omega}{(1-\omega)} \delta x_{at-1} \Delta\pi_{Ha,t} \end{aligned}$$

We substitute it into (47) and take unconditional expectation of undiscounted infinite sum of intra-period welfare losses (47). The cross terms of shocks with economic variables will disappear if we assume that shocks are uncorrelated with these variables. The linear terms will disappear as their means are zero. The welfare loss becomes:

$$\begin{aligned} \mathcal{W} &= \lambda_\pi (\text{var}(\pi_a) + \text{var}(\pi_b)) + \lambda_\pi \mu_{\Delta\pi} (\text{var}(\Delta\pi_a) + \text{var}(\Delta\pi_b)) \quad (48) \\ &+ \lambda_\pi \lambda_c (\text{var}(c_a - \rho c_{a,-1}) + \text{var}(c_b - \rho c_{b,-1})) + \lambda_\pi (\lambda_x + \mu_x) (\text{var}(x_a) + \text{var}(x_b)) \\ &+ \lambda_\pi \lambda_g (\text{var}(g_a) + \text{var}(g_b)) + \lambda_\pi \mu_{xx} (\text{cov}(x_a, x_{a,-1}) + \text{cov}(x_b, x_{b,-1})) \\ &+ \lambda_\pi \mu_{x\Delta\pi} (\text{cov}(x_{a,-1}, \Delta\pi_{Ha}) + \text{cov}(x_{b,-1}, \Delta\pi_{Hb})) \end{aligned}$$

where

$$\lambda_\pi = \frac{1}{2} \frac{\epsilon(\epsilon + \psi)(1 - \beta\rho)\gamma}{\theta\psi(1 - \gamma\beta)(1 - \gamma)} \frac{(1 - \tau)}{\mu} \quad (49)$$

$$\lambda_c = \frac{(1 - \gamma)\psi\theta(1 - \gamma\beta)(1 - \sigma)}{\epsilon(\epsilon + \psi)(1 - \beta\rho)\gamma\sigma} \frac{\mu}{(1 - \tau)} \quad (50)$$

$$\lambda_x = \frac{(1 - \gamma\beta)(1 - \gamma)(1 + \psi)}{\epsilon(\epsilon + \psi)\gamma}$$

$$\lambda_g = \frac{\psi(1 - \gamma\beta)(1 - \theta)(1 - \gamma)(1 - \sigma_g)\mu \left( \frac{1-\tau}{\mu} \psi + \theta\sigma\phi(\alpha_d + \alpha_n) \right)}{\epsilon(\epsilon + \psi)\gamma\sigma_g(1 - \tau)(\psi + (\alpha_d + \alpha_n)^2\theta\sigma\phi + (1 - (\alpha_d + \alpha_n)^2)\theta\eta)}$$

$$\mu_x = \frac{\omega(1 - \gamma)^2}{(1 - \omega)\gamma} \delta^2$$

$$\mu_{\Delta\pi} = \frac{\omega}{\gamma(1 - \omega)}$$

$$\mu_{x\Delta\pi} = 2 \frac{(1 - \gamma)\omega}{\gamma(1 - \omega)} \delta$$

Again, in the case where rule-of-thumb individuals take more than one lag on output when setting prices, the welfare function (48) should be modified to the following specification (see Additional Appendix):

$$\begin{aligned} \mathcal{W} &= \lambda_\pi (\text{var}(\pi_a) + \text{var}(\pi_b)) + \mu_{\Delta\pi} (\text{var}(\Delta\pi_a) + \text{var}(\Delta\pi_b)) \quad (51) \\ &+ \lambda_\pi \lambda_c (\text{var}(c_a) + \text{var}(c_b)) + \lambda_\pi (\lambda_x + \mu_x) (\text{var}(x_{at}) + \text{var}(x_{bt})) \\ &+ \lambda_\pi \lambda_g (\text{var}(g_a) + \text{var}(g_b)) + \lambda_\pi \mu_{xx} (\text{cov}(x_a, x_{a-1}) + \text{cov}(x_b, x_{b-1})) \\ &+ \lambda_\pi \mu_{x\Delta\pi}^1 (\text{cov}(x_{at-1}, \Delta\pi_{Ha,t}) + \text{cov}(x_{bt-1}, \Delta\pi_{Hb,t})) \\ &+ \lambda_\pi \mu_{x\Delta\pi}^2 (\text{cov}(x_{at-2}, \Delta\pi_{Ha,t}) + \text{cov}(x_{bt-2}, \Delta\pi_{Hb,t})) \end{aligned}$$

with

$$\begin{aligned}\mu_x &= \frac{\omega(1-\gamma)^2}{(1-\omega)\gamma}(\delta_1^2 + \delta_2^2), & \mu_{xx} &= 2\frac{\omega(1-\gamma)^2}{(1-\omega)\gamma}\delta_1\delta_2 \\ \mu_{x\Delta\pi}^1 &= 2\frac{(1-\gamma)\omega}{\gamma(1-\omega)}\delta_1, & \mu_{x\Delta\pi}^2 &= 2\frac{(1-\gamma)\omega}{\gamma(1-\omega)}\delta_2\end{aligned}$$

When calibrating parameters  $\delta_1$  and  $\delta_2$  we first calibrate  $\delta = \delta_1 + \delta_2$  as explained in Section 3 and then define  $\delta_1 = \varphi(\delta_1 + \delta_2)$  and  $\delta_2 = (1 - \varphi)(\delta_1 + \delta_2)$ . We vary  $\varphi$ , to explore demand effects on inflation.

## C Compensating Consumption

Having computed the social loss in stochastic equilibrium for a regime, we can give an interpretation of losses in terms of ‘real world’ variables. Suppose the two different regimes of control give us different values of losses due to volatility. We can find the level of consumption, which compensates the social planner for the difference in volatility between regimes. Indeed, suppose two regimes produce the same (dis)utility due to public consumption and work efforts, and in the first regime consumption is volatile with volatility  $\mathcal{W}_2$  and mean  $C$ , while in the second regime it is constant with mean of consumption  $C + \Omega C$ . We determine the percentage change in consumption  $\Omega$  such that we have the same utility in both regimes. We can see from the Taylor expansion derived in Appendix B that (to a second order approximation) a measure of the utility of the non-discounting social planner is given by

$$L = u(C^{1-\rho}) + f(G) - v(Y) - u_C(C^{1-\rho}, 1)C^{1-\rho}\mathcal{W}$$

where  $\mathcal{W}$ , the value of the social welfare function, is given by equation (48) and  $C$ ,  $G$  and  $Y$  refer to steady state levels of consumption, government spending and output. We compute utility level in the first regime. Since there is no volatility  $\mathcal{W} = 0$  so,

$$\begin{aligned}L_1 &= u((C + \Omega C)^{1-\rho}) + f(G) - v(Y) \\ &= u(C^{1-\rho}) + (1-\rho)C^{1-\rho}\Omega u_C(C^{1-\rho}) - \frac{C(1-\rho)}{2C^{2\rho}}\Omega^2(u_C(C^{1-\rho})\rho C^\rho \\ &\quad - C(1-\rho)u_{CC}(C^{1-\rho}) + f(G_t) - v(Y_t) + o(\Omega C)^3) \\ &= u_C(C^{1-\rho})C^{1-\rho}(1-\rho)\Omega\left(1 - \frac{\Omega}{2}\left(\rho + (1-\rho)\frac{1}{\sigma}\right)\right) \\ &\quad + u(C^{1-\rho}) + f(G) - v(Y) + o(\Omega C)^3\end{aligned}$$

For the second regime,  $\mathcal{W} = \mathcal{W}_2$  and so

$$\begin{aligned}L_2 &= u(C^{1-\rho}) + f(G) - v(Y) \\ &\quad - u_C(C^{1-\rho}, 1)C^{1-\rho}\mathcal{W}_2\end{aligned}$$

Therefore, an individual will be indifferent between these two regimes when

$$(1-\rho)\Omega\left(1 - \frac{\Omega}{2}\left(\rho + (1-\rho)\frac{1}{\sigma}\right)\right) + \mathcal{W}_2 = 0$$

which is an equation for  $\Omega$ . We can then find compensating consumption level between two regimes with losses due to volatility  $\mathcal{W}_1$  and  $\mathcal{W}_2$ :

$$(1 - \rho)\Omega\left(1 - \frac{\Omega}{2}\left(\rho + (1 - \rho)\frac{1}{\sigma}\right)\right) + (\mathcal{W}_2 - \mathcal{W}_1) = 0. \quad (52)$$

from which the relevant solution is:

$$\Omega = \frac{\sigma}{(\rho\sigma + (1 - \rho))} \left(1 - \sqrt{1 + 2\frac{(\rho\sigma + (1 - \rho))}{(1 - \rho)\sigma}(\mathcal{W}_2 - \mathcal{W}_1)}\right) \quad (53)$$