DISCUSSION PAPER SERIES

No. 4430

HETEROGENOUS WAGE FORMATION UNDER A COMMON MONETARY POLICY

Torben M Andersen

INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

ISSN 0265-8003

HETEROGENOUS WAGE FORMATION UNDER A COMMON MONETARY POLICY

Torben M Andersen, University of Aarhus and CEPR

Discussion Paper No. 4430
June 2004

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 4430

June 2004

ABSTRACT

Heterogenous Wage Formation Under A Common Monetary Policy*

How does a monetary union work when labour markets are heterogeneous? Since shocks are transmitted via both trade links and the common monetary policy and propagated via labour market responses, it follows that labour market institutions may have not only national but also union-wide implications. These issues are analysed in an intertemporal general equilibrium model for a currency union in which labour markets are heterogenous and where the monetary policy targets expected inflation. More flexibility in adjustment means more stable aggregate output, but inflation control becomes more difficult. Heterogeneity in adjustment plays a large role, in particular if country sizes are also asymmetric. This also holds in the case of aggregate shocks both for the variability of aggregate output and inflation. Considering the effects on country specific output variability it is seen that there are important spillover effects between labour market structures, and that it is not necessarily beneficial to make a unilateral move to make labour markets more flexible.

JEL Classification: E30, E52 and F41
Keywords: business cycles, monetary policy, monetary union, shocks and wage formation

Torben M Andersen
Institute of Economics
Universitet Aarhus
Building 350
DK-8000 Aarhus C
DENMARK
Tel: (45 8) 942 1609
Fax: (45 8) 613 6334
Email: tandersen@econ.au.dk

*Comments and suggestions from seminar participants at the University Paris X (Nanterre) and the Danish Central Bank are gratefully acknowledged.

Submitted 16 April 2004

Heterogenous wage formation under a common monetary policy*

Torben M. Andersen
Department of Economics, University of Aarhus
CEPR, EPRU and IZA

April 2004

Abstract

How does a monetary union work when labour markets are heterogeneous? Since shocks are transmitted via both trade links and the common monetary policy and propagated via labour market responses, it follows that labour market institutions may have not only national but also union-wide implications. These issues are analysed in an intertemporal general equilibrium model for a currency union in which labour markets are heterogenousand where the monetary policy targets expected inflation. More flexibility in adjustment means more stable aggregate output, but inflation control becomes more difficult. Heterogeneity in adjustment plays a large role, in particular if country sizes are also asymmetric.This also holds in the case of aggregate shocks both for the variability of aggregate output and inflation. Considering the effects on country specific output variability it is seen that there are important spill-over effects between labour market structures, and that it is not necessarily beneficial to make a unilateral move to make labour markets more flexible.

JEL classification: E30, E52, F41

Key words: wage formation, monetary policy, monetary union, business cycles, shocks

1 Introduction

Across European countries there are substantial differences in labour market performance as well as in labour market institutions (see e.g. OECD (2002)). This is a concern in its own right, but the creation of the Economic and Monetary Union has added to the importance of these differences since member countries share the same monetary policy. According to the traditional literature on optimal currency areas necesssary condition for a currency area is that labour

*Comments and suggestions from seminar participants at the University Paris X (Nanterre) and the Danish Central Bank are gratefully acknowledged.

markets should be flexible either in terms of high labour mobility or wage flexibility (see e.g. De Grauwe (2003)). However, none of these assumptions seem to hold for the EMU-area (see below). This raises the important question of how a monetary union works in the presence of heterogenous labour markets as well as the issue of the incentives member countries have to undertake structural reforms. Surprisingly little research has considered how a currency union with heterogenous wage setting institutions works[1].

The aim of this paper is positive in the sense of analysing the consequences of labour market heterogeneity in a monetary union in which the monetary authority targets inflation. The focus is on responses to shocks addressing the role of heterogeneity in both nominal and real wage flexibility. When monetary policy is credible and the objective is overall price stability, the interesting question becomes how differences in labour market institutions affect the stability of inflation (around the level targeted by the central bank) as well as its implications for the adjustment of activity and employment to various types of shocks. It is well-known that rigidities in adjustment of wages (nominal and real) tend to imply that employment and activity become more volatile, that is, when wages take a lower burden of adjustment, a larger burden comes to rest at employment and activity. However, the interdependencies are more complicated in a currency union due to the sharing of a common monetary policy. Labour market heterogeneity may therefore have real consequences both at the national and the union-wide level, since shocks are transmitted via both trade links and the common monetary policy, and propagated differently in national labour markets. Accordingly, there may be important spill-over effects from differences in labour market institutions. The issue of how labour market structures in one country affect other countries in a currency union is in particular important for the incentive to undertake policy reforms changing labour market structures. To address this issue it is necessary first to chart the externalities arising from different labour market institutions. In particular, do countries with inflexible labour markets exert a negative externality on countries with more flexible labour markets? Are there any gains to be made by having the most flexible labour market? To what extent do countries with more flexible labour markets perform better than countries with less flexible labour markets?

This paper presents an intertemporal general equilibrium model of a currency union in which the monetary authority credibly targets inflation and in which there may be asymmetries across unequal sized member countries in respect to both real and nominal wage flexibility. The response of inflation, aggregate and country-specific output is analysed in the presence of supply shocks for both the case of aggregate and country-specific shocks. The model structure builds on recent open economy macromodels (see e.g. Obstfeld and Rogoff (1996)) and has imperfectly competitive labour markets to model wage setting and account for rigidities. The model is cast in such a way that it is possible to find an analytical solution to the intertemporal general equilibrium model

[1] There is an important literature addressing strategic issues in wage formation when entering a monetary union, see e.g. Calmfors (2001) for a survey and references.

for a currency union with labour markets which are heterogeneous in size and structure.

Despite the frequent reference to asymmetries and heterogeneity across European labour markets there are very few studies addressing their implications. Benigno (2002) analyses from a normative point of view how monetary policy should be designed when member countries have different degrees of nominal rigidities, and it is shown that the central bank should put more weight to inflation in countries characterized by more nominal inertia. Beetsma and Jensen (2002) also allow for labour market asymmetries in analysing the interactions between monetary and fiscal policy in a monetary union. Dellas and Tavlas (2002) present a three-country model allowing for asymmetries in nominal wage flexibility, and find that countries with a high degree of nominal wage rigidity are better off in a monetary union. The incentive to undertake structural reforms is addressed by Spange (2002) in a two-country model with real shocks, and he shows that a country with a rigid labour market may not have an incentive to reform its labour market to match the more flexible country.

Several authors have addressed the issue of whether membership of EMU would lead to endogenous structural changes or reforms changing rigidities in the labour market. This line of research has taken its outset in the observation that loss of a national monetary policy will increase the need for nominal flexibility to cope with asymmetric shocks (see e.g. Calmfors (2001) for a survey and references). The present paper takes a step in addressing the need for and direction in which such reforms should go by considering the implications of various labour market rigidities (real and nominal) for business cycle fluctuations.

This paper is organized as follows: Section 2 offers some key indicators on the labour market heterogeneities prevailing across EU countries. The theoretical model is set up in section 3, and the equilibrium processes for inflation and output are presented in section 4. The implications of labour market heterogeneities are worked out in section 5 by considering both the implications for inflation and output volatility but also the spill-over effects generated by differences in labour market structures. Section 6 offers a few concluding remarks.

2 EU labour market heterogeneities

Even though there often is a common reference to the rigidity of European labour markets (cf. e.g. the debate on Eurosclerosis), it is equally clear that labour markets across Europe display substantial heterogeneities. These differences arise due to differences in institutions, policies and economic development. Table 1 offers a selective overview of some dimensions of labour market flexibility and their heterogeneity across all EU countries. The table reports measures on the structural unemployment rate and indicators of wage flexibility (real and nominal). These indicators are of importance for the wage responsiveness to shocks and the level of unemployment consistent with low and stable inflation. Note that the table can only meaningfully be read vertically, since differences in methods and data imply that the measures cannot readily be compared across the different studies used here. Reading vertically in the table clearly documents differences in labour market characteristics along all the three dimensions included.

ECB (2002) documents heterogeneity from a different perspective by considering various measures related to mis-match problems across labour markets in euro-countries and find that there are substantial differences. However, Turner and Seghezza (1999) find that a number of countries share similar long-run implications of wage adjustment in terms of e.g. the sacrifice ratio (output to be sacrificed to reduce inflation), but they also find differences in the short-run adjustment mechanism including in particular different responses to foreign prices. European Commission (2003) finds considerable inertia in nominal and particular real wage, and with considerable variation across Euro-area countries.

Table 1: Indicators of labour market flexibility for EU-countries

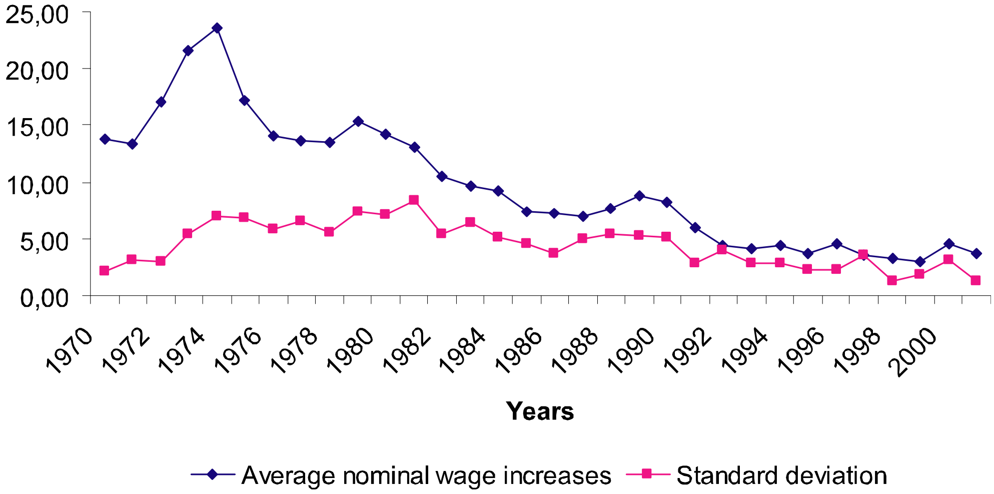
	Structural unemployment rate		Real wage flexibility		Nominal flexibility	
	OECD[1]	EU[2]	Berthold[3]	Layard.et al[4]	EU[5]	Paloviita[5]
Austria	4.9	3.8	2.5	0.11	-0.62	65.7
Belgium	8.2	7.3	2.5	0.25	-0.55	56.8
Denmark	9.3	4.3	1.7	0.58	-0.52	
Finland	9.0	8.2	1.3	0.29		13.5
France	9.5	8.8	2.1	0.23	-0.29	11.3
Germany	6.9	8.3	1.7	0.63	-0.26	18.5
Greece	9.5	10.6	1.9		-0.29	
Ireland	7.1	6.0	2.3	0.27	-0.07	20.6
Italy	10.4	9.6	2.6	0.06	-0.14	157.9
Netherlands	4.0	3.1	1.6	0.25	-0.46	59.5
Portugal	3.9	5.0	1.3		-0.16	22.3
Spain	15.1	12.8	2.4	0.52	-0.41	31.5
Sweden	5.8	5.7	1.0	0.08		
UK	7.0	5.3	1.5	0.77	-0.29	
Coefficient of variation	0.37	0.38	0.26	0.49	-0.49	0.96

Notes: 1) Source: Turner et al. (2001), 2) Source: Denis et al. (2002), 3) Responsiveness of real wages to the unemployment rate, Source Berthold et al. (1999), 4) same as 3, source Layard et al.(1991), 5) Responsiveness of inflation to unemployment rate, source, Denis et al. (2002), 6) responsiveness of inflation to activity, source: Paloviita (2002).

The important conclusion documented by the evidence reported here is that there are substantial differences across European labour markets. Adding that labour mobility across European labour markets is also small (see e.g. Descressin and Fatás (1995)) it follows that the common monetary policy is operating within a setting of heterogenous labour markets[2].

[2]In Riboud et al. (2002) it is shown the candidate countries also have labour market

Figure 1: Average wage increases and volatility, EU countries 1970-2002



Note: Wages given by hourly compensation costs in manufacturing
Source: OECD

From recent accounts of labour market policies and reforms by (see e.g. OECD (1998), HM Treasury (2003)) it is also clear that these differences are not about to be eliminated in the near future.

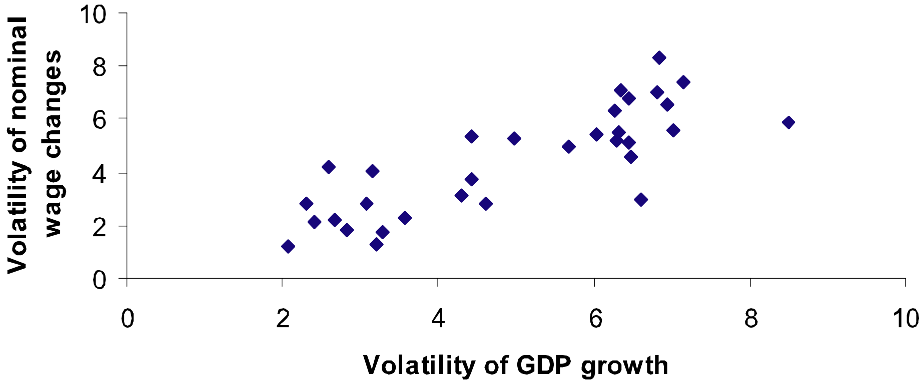
EU countries have in recent years experienced some convergence in wage developments, cf. figure 1. However, dispersion in wage increases across European countries has not fallen to the same extent, which suggests either that there are strong asymmetries or the business cycle developments have been quite different.

If difference in business cycle developments are the main reason for dispersion in wage increases one should expect to find a positive relation between the dispersion of e.g. GDP growth rates and the dispersion in nominal wage changes. Figure 2 indicates that such a positive relation is present. However, the correlation between nominal wage changes and GDP growth is falling, since the correlation was 0.74 over the period 1971-80, 0.62 over the period 1981-1990 and 0.56 over the period 1991-2002. This suggests that nominal wage changes to a lesser extent than previously reflect differences in business cycle developments.

There is thus strong empirical evidence that there are heterogeneities across European labour markets, and the next section develops a model of a currency union to analyse the importance of such asymmetries when monetary policy targets inflation.

heterogenities although they on average are not widely different from the current EU countries.

Figure 2: Volatility of nominal wage changes and GDP growth: 1971-2002



Note: Wage data as in figure 1, GDP measured in fixed prices
Source: GDP data: OECD.

3 A Monetary Union with heterogeneous labour markets

Consider a monetary union in which there is a centralized monetary authority pursuing an inflation target, and a number of separate regions/countries. Countries produce differentiated commodities (exogenous specialized production structure) and all goods are traded[3]. The countries can be of different size and they may have different labour market structures, and there is no mobility of labour between them. Labour markets are assumed to be imperfectly competitive and characterized by nominal rigidities (nominal contracts), whereas product markets for simplicity are assumed competitive. The model structure follows recent work in Open Macroeconomics (see e.g. Obstfeld and Rogoff (1996)).

Let countries/regions be indexed by $n = 1.....N$ and assume that the representative household in region i has a utility function given by

$$U_{it} = E_t \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\frac{\sigma}{\sigma - 1} C_{it+\tau}^{\frac{\sigma-1}{\sigma}} + \frac{1}{1-\varepsilon} \left(\frac{M_{it+\tau}}{P_{it+\tau}} \right)^{1-\varepsilon} - \frac{1}{1+\mu} L_{it+\tau}^{1+\mu} \right],$$

$$\sigma > 0, \quad \varepsilon > 0, \ \mu > 0, \quad 0 < \delta \leq 1.$$

[3] This leaves out possible Belassa-Samuelson effects as a cause of inflation differential among EMU-countries. For an analysis of this channel in a setting with symmetric labour markets see e.g. Duarte and Wolman (2002).

E_t is the expectations operator conditional on period t information (see below), δ the subjective discount factor, C is a real consumption index , M denotes nominal balances, P is the consumer price index, and L is the amount of labour worked. The consumption index is defined over the differentiated commodities produced in the different regions. Specifically, assume that[4]

$$C_{it+\tau} = \left[\sum_{n=1}^{N} v_n C_{int+\tau}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where v_n is the relative size of region n ($\sum_{n=1}^{N} v_n = 1$) and the corresponding consumer price index is defined

$$P_{t+\tau} = \left[\sum_{n=1}^{N} v_n \left(\frac{P_{nt+\tau}}{v_n} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Hence

$$C_{int+\tau} = \left(\frac{P_{nt}}{v_n P_t} \right)^{-\theta} C_{it+\tau}$$

The consumer's period t budget constraint is given by

$$P_t B_{it} + M_{it} + P_t C_{it} = (1 + R_{t-1}) P_t B_{it-1} + M_{it-1} + W_t N_{it} + \Pi_{it} + P_t \tau_{it}.$$

The right-hand side gives available resources as the sum of the gross return on bondholdings $(1 + R_{t-1}) P_t B_{it-1}$with R denoting the nominal interest rate (see below), initial money holdings M_{it-1}, labour income $W_t N_{it}$, nominal profit income Π_{it} and transfers from the government $P_t \tau_{it}$. Resources are allocated to consumption $P_t C_{it}$, nominal money holdings M_{it} and bondholdings $P_t B_{it}$.

The consumer maximizes expected utility subject to the budget constraint and the first-order conditions determining the optimal choice of C_{it}, B_{it}, and M_{it} are readily found (see appendix). Note that labour market variables are not individual choice variables, since employment is demand determined given the wage contracts (see below). Moreover since all agents/regions/countries share the same preferences, their choice problem is identical and the i index for consumers can therefore be dropped in the following characterization of demand.

In order to solve the model analytically, it is convenient to work with the model written in logs. Later it will be shown that the variables of the model are lognormally distributed under the assumed stochastic processes for the exogenous shock variables. When the monetary authority uses the interest rate as its instrument it follows (for details see appendix A) that the money market can be considered recursively and the key condition is the Euler equation for consumption which in logs reads (expected inflation equals zero, cf. below)

$$E_t c_{t+1} = c_t + \sigma r_t, \tag{1}$$

[4] Note that it is assumed that all commodities are tradeables, and that consumer preferences are symmetric.

where $r_t \equiv \log(1 + R_t)$, and lower-case letters denote the log-deviations from a symmetric steady state of the corresponding upper-case variables, and c denotes aggregate consumption in the currency union (see below). All constants – including conditional variance terms which are time invariant – are suppressed since the focus of this paper is on the adjustment process to shocks.

Note that it is an implication of (1) that monetary policy affects aggregate demand in all countries symmetrically.[5]

Demand for commodity i (equal to the demand for product produced in country i) is obtained by aggregating all demand function and reads

$$d_{it} = -\theta(p_{it} - p_t) + c_t$$

where $c_t = \sum_{n=1}^{N} v_n c_{nt}$.

It is well-known from the literature (see e.g. Obstfeld and Rogoff (1996), and Corsetti and Pesenti (2002)) that shocks in general will induce cross-country reallocation of wealth across countries, and therefore the steady-state equilibrium becomes path dependent. The latter poses a potential technical problem in solving the model since the net-wealth position in steady-state is not well-determined (path dependence). There are two ways to solve this problem. One approach is to assume that the numerical price elasticity of demand is unity ($\theta = 1$) since this would imply that real income and therefore wealth is unaffected by shocks. As it turns out very few of the qualitative results depend on the particular value of θ, and therefore the model is presented without any restriction on θ. However, it is straightforward to impose the constraint $\theta = 1$. Alternatively, it can be assumed that there are risk sharing arrangements such that real income (and thus consumption) is insulated from country-specific shocks, i.e. no real wealth reallocation. As the present model is specified the distribution of income does not matter for the demand side, and in the aggregate there is no wealth reallocation (closed currency union). Hence, invoking this risk sharing assumption does not have any consequences for the determination of aggregate demand. It is thus possible to interpret the general version of the model ($\theta \neq 1$) as implicitly relying on such a risk sharing arrangement.

3.1 Firms

Product markets are assumed to be perfectly competitive. The representative firm is a price and wage taker and produces subject to a decreasing returns technology linking output Y_{it} and labour input N_{it}[6]

$$Y_{it} = \frac{1}{\gamma} N_{it}^{\gamma} U_{it}^{1-\gamma}, \qquad 0 < \gamma < 1.$$

[5] Obviously this need not be the case, and there is evidence supporting asymmetries in the monetary transmission mechanism, see e.g. Suardi (2001). To focus on asymmetries originating in the labour market, this source of asymmetry is disregarded.

[6] Real capital is disregarded to simplify. Decreasing returns can be interpreted as arising from a second factor of production in fixed supply.

where U_{it} is the period t productivity in firm/region i (assumed stochastic, see below). Maximizing profits yields the following labour demand and output supply relations for the representative firm in country/region i

$$N_{it} = \left(\frac{W_{it}}{P_{it}}\right)^{\frac{-1}{1-\gamma}} U_{it} \tag{2}$$

$$Y_{it} = \frac{1}{\gamma}\left(\frac{W_{it}}{P_{it}}\right)^{\frac{\gamma}{\gamma-1}} U_{it} \tag{3}$$

Profits are distributed to households. Note that there are no nominal price rigidities, i.e. product prices are determined by the market clearing condition.

In logs the aggregate supply relation reads (neglecting constants)

$$y_{it} = \beta\left(p_{it} - w_{it}\right) + u_{it} \tag{4}$$

where $\beta \equiv \frac{\gamma}{1-\gamma}$ and the supply shock is assumed to be generated by the process

$$u_{it} = \rho u_{it-1} + \varepsilon_{it} \qquad\qquad -1 < \rho < 1$$

In the following the analysis centers on two different cases, namely, i) all innovations across countries are identical, i.e. $\varepsilon_{it} = \varepsilon_t \quad \forall i$ (implying a correlation coefficient $\rho_{ij} = 1 \; \forall i, j$) corresponding to an aggregate shock, or ii) innovations which are uncorrelated across countries ($\rho_{ij} = 0 \; \forall i, j, \; i \neq j$) corresponding to country-specific shocks. In all cases the innovations are iid $N(0, \sigma^2)$.

This way of specifying the shock makes it possible to consider the role of labour market heterogeneity in the case of both aggregate and country-specific shocks. It is often asserted that in particular country-specific shocks pose a problem in a monetary union in the presence of rigidities in wage adjustment. Related is the issue of whether a monetary union would lead to more symmetric or asymmetric business cycle fluctuations, i.e. would monetary unification lead to more similar (demand effect) or dissimilar (specialization effect) business cycles (Frankel and Rose, 1998). The present framework can be used to evaluate the consequence of different types of shocks for monetary policy and business cycle aspects when labour market differences are persistent.

3.2 Wage setting

The key issue is to capture that different countries/regions may have different degrees of flexibility both in respect to nominal and real wage flexibility, cf. section 2. A specific modeling of the labour market is suggested which in a straightforward way makes it possible to capture these two dimensions of rigidity. The specific modeling adopted here is serving the purpose of motivating the aggregate wage equation used below and the fact that key parameters depend on structural features of the labour market which differ across European countries. The model is thus suggestive, and other mechanisms could possibly lead to some of the same qualitative properties and effects of differences in institutional settings. Note that the contract structure is taken as exogenous to address the positive question of how differences in labour market structure affect a currency union, whereas the issue of explaining these differences in labour market structures is beyond the scope of this paper.

Assume that workers are hired under either long-term or short-term contracts. Specifically in country i a fraction κ_i ($\in [0,1]$) is hired under a long-term contract. For the long-term contract nominal wages are preset at the level expected to be the optimal (in the long run) to achieve a given real wage target which here (neglecting all constants) implies that the nominal wage rate for all long-term contracts is given as

$$w_{it}^L = p_t^e$$

where p_t^e denotes expectation conditional on period $t-1$ information I_{t-1}, i.e. $p_t^e = E(p_t \mid I_{t-1})$.

The short-term contract allows wages to flexibly adjust to market conditions (i.e. no nominal rigidity) and they are determined according to[7]

$$w_{it}^K = p_t + s_i y_{it}$$

where $s_i \geq 0$ is the responsiveness of wages to the level of activity (employment). Accordingly, aggregate wages in country i can be written

$$\begin{aligned} w_{it} &= \kappa_i w_{it}^L + (1-\kappa_i) w_{it}^K \\ &= \kappa_i p_t^e + (1-\kappa_i)(p_t + s_i y_{it}) \end{aligned}$$

and therefore the real wage in country i is determined as

$$w_{it} - p_{it} = p_t - p_{it} + \kappa_i (p_t^e - p_t) + (1-\kappa_i) s_i y_{it}$$

that is, changes in the product real wage can be driven by either i) changes in the terms of trade ($p_t - p_{it}$), ii) unanticipated price changes ($p_t^e - p_t$) or iii) changes in country-specific activity (y_{it}) affecting real wage demands. The larger the fraction of long-term contracts, the larger the role of unanticipated shocks. The sensitivity to activity – measured by s_i – plays a larger role, the more short-term contracts there are.

In the following nominal rigidities are captured by κ (the larger κ, the larger the nominal rigidity in wage determination), and real flexibility is captured by s (the larger s, the more flexible are real wages)

3.3 Monetary policy

The monetary authority is assumed to use the interest rate (r) as the instrument to target the rate of inflation. Specifically, the expected (one period ahead) rate of inflation is targeted, and the target is for simplicity set to zero, i.e.

$$\pi_t^e = 0$$

[7] A wage setting rule of this form is derived in Andersen(2004) as the outcome of a union wage setting procedure.

Note that this implies that the money supply is recursively determined to accommodate the money demand prevailing given the level of activity, the interest rate, and prices (see appendix). Moreover, with this inflation target it follows that control of the nominal interest rate is tantamount to control over the expected real rate of interest (presuming the inflation target to be credible).

3.4 Equilibrium conditions

For all commodities $i = 1, , , , , , N$ the equilibrium condition reads

$$d_{it} = y_{it}$$

whereas employment is demand determined in all labour markets, cf. labour demand given by (2). Note that in the aggregate for the currency union we have (see appendix)[8]

$$c_t = y_t$$

that is, no net wealth is accumulated or decumulated at the aggregate level. This reflects that the currency union is modelled as a closed area without any real capital.

4 Equilibrium inflation and output

Union wide variables like inflation and activity are defined as the weighted average of the country-specific variables, i.e.

$$\pi_t = \sum_{n=1}^{N} v_j \pi_{jt} \qquad ; \qquad y_t = \sum_{n=1}^{N} v_j y_{jt}$$

where v_j gives the relative weight (size) of country j in the currency union, $\sum_{n=1}^{N} v_j = 1$.

It can be shown that the model has a rational expectations equilibrium[9] (see appendix) where the rate of inflation and aggregate output is given as, respectively

$$\begin{aligned} \pi_t &= \sum_{n=1}^{N} \nu_n \phi_n \varepsilon_{nt} \\ y_t &= \sum_{n=1}^{N} \nu_n \varphi_n u_{nt-1} \end{aligned}$$

[8] Accordingly, the aggregate stock of bonds is zero in equilibrium.

[9] Following the standard practice of solving for the minimal state solution.

and the country-specific output can be written

$$y_{it} = \omega_{iy} y_t + \omega_{i\pi} \pi_t + \omega_{iu} u_{it}$$

All the coefficients given above are defined in appendix B. It follows that more rigidities – nominal (higher κ_i) or real (lower s_i) – imply that the sensitivity of domestic activity to aggregate activity, inflation and the country-specific shocks all increase, i.e.

$$\begin{array}{ll} \frac{\partial \omega_{iy}}{\partial \kappa_i} > 0 & \frac{\partial \omega_{iy}}{\partial s_i} < 0 \\ \frac{\partial \omega_{i\pi}}{\partial \kappa_i} > 0 & \frac{\partial \omega_{i\pi}}{\partial s_i} < 0 \\ \frac{\partial \omega_{iu}}{\partial \kappa_i} > 0 & \frac{\partial \omega_{iu}}{\partial s_i} < 0 \end{array}$$

Note that this is only a partial result, since the equilibrium distribution of aggregate activity and inflation also depends on the structural parameters determining wage adjustment, cf. below.

Considering the monetary reaction function we find that it can be written

$$r_t = \Omega_\pi \pi_t + \sum_{n=1}^{N} v_n \Omega_{un} u_{nt}$$

where $\Omega_\pi < 0$ and $\Omega_{un} < 0$. An increase in the current rate of inflation leads – other things being equal – to a decrease in the rate of interest. The intuition is to be found in the particular process assumed for the supply shock. A positive innovation to the supply shock would create a deflation pressure today, but via the autoregressive form of the shock an inflationary presure in the next period. The latter is expected and therefore the monetary authority reacts to eliminate this effect. By lowering the current interest rate the central bank induces consumers to engage in intertemporal substitution, increasing current consumption (counteracting the immediate deflationary response) and decreasing future consumption and this lowering future expected inflation. Note that[10]

$$\frac{\partial \Omega_{un}}{\partial \kappa_n} > 0; \frac{\partial \Omega_{un}}{\partial s_n} < 0$$

Other things being equal the monetary authority reacts less strongly to a shock originating in a country with much nominal wage rigidity, and more srongly if it originates in a country with much real wage flexibility.

As shown in Appendix B aggregate output is not affected by current innovations to productivity in equilibrium. This implies that innovations to the shock have no immediate effect on output but only on inflation. To see the intuition for this result note that equilibrium aggregate output can be written

$$y_t = -\sigma r_t + \sum_{n=1}^{N} \nu_n \varphi_n u_{nt}$$

[10] Proof follows by noting that $\Omega_{un} = \frac{\rho - 1}{\sigma} \frac{1}{\rho} \varphi_n$.

that is, there is a direct impact effect of the state variables (u_{nt}) and the interest rate (r_t) set by the monetary authorities. An implication of strict inflation targeting is that the actual rate of inflation will have to be white noise, otherwise expected inflation would differ from zero violating the inflation target. This means that all systematic parts in the inflation process will have to be eliminated via the way interest rates are set[11]. Since the determination of demand involves consumption smoothing (see (1)) it follows that unanticipated changes in income would lead to persistent changes in demand, i.e. a period t innovation would affect period $t+1$ demand and therefore inflation. Accordingly, it is implied by the optimal determination of the interest rate under strict inflation targeting that the period t income or demand effect of shocks is neutralized, cf. appendix. However, output adjusts to the expected value of productivity at period t, which comes as no surprise.

5 Volatility of inflation and activity

This section considers the implications of labour market heterogeneity for the adjustment to aggregate and country-specific shocks within the currency area. First, the overall macroeconomic stability is considered by analysing volatility of aggregate inflation and output in the currency area. Next the implications for country-specific output fluctuations are considered.

To avoid unnecessary technical complications the results are presented for the case $N = 2$. Below analytical results are supplemented by numerical illustrations[12]. The latter serve an illustrative purpose, which in particular is useful when the analytical results are ambiguous or very complicated.

5.1 Symmetric labour markets

As a benchmark case for evaluating the consequence of labour market heterogeneities consider first the case where labour markets are homogeneous or symmetric, i.e. all labour markets are characterised by the same degree of nominal and real rigidity ($\kappa_1 = \kappa_2 = \kappa$ and $s_1 = s_2 = s$). In this case we have (see appendix C) $\phi_1 = \phi_2 = \phi$, and $\varphi_1 = \varphi_2 = \varphi$ and therefore equilibrium inflation and output can be written

$$\begin{aligned} \pi_t &= \phi \left[v_1 \varepsilon_{1t} + v_2 \varepsilon_{2t} \right] \\ y_t &= \varphi \left[v_1 u_{1t-1} + v_2 u_{2t-1} \right] \end{aligned}$$

[11] In the appendix it is shown how the interest rate is set so as to eliminate the effects of the past rate of inflation, expected shocks etc. on the current rate of inflation.

[12] The baseline parameter values are: $\nu_1 = \nu_2 = 0.5$, $\beta = 0.66/0.34, \theta = 1, \kappa = 0.5, s = 0.2$, $\rho = 0.9$, and $\sigma_\varepsilon^2 = 1$.

where

$$\begin{aligned}\phi &< 0; \frac{\partial \phi}{\partial \kappa} > 0; \frac{\partial \phi}{\partial s} = 0 \\ \varphi &> 0; \frac{\partial \varphi}{\partial \kappa} > 0; \frac{\partial \varphi}{\partial s} < 0\end{aligned}$$

Positive realizations of the shocks lower inflation and increase output (with a one period lag) and these signs are as should be expected given the nature of the shock (supply). More nominal inertia (higher κ) means more inflation variability, whereas less real rigidity (higher s) does not affect inflation volatility in the case of symmetric labour markets. More nominal flexibility (lower κ) and more real flexibility (higher s) both lead to less sensitivity of output to the shocks.

In the case of aggregate/common shocks we have that aggregate inflation and output variability is given as

$$\begin{aligned}Var(\pi_t) &= \phi^2 \sigma_\varepsilon^2 \\ Var(y_t) &= \varphi^2 \sigma_u^2\end{aligned}$$

Note that in the case of symmetric labour markets and common shocks there are no relative price changes, i.e. the terms-of-trade effect does not arise (i.e. $p_{nt} = p_t \ \forall n$). It is therefore a useful benchmark case since the terms-of-trade effect may then arise either from country-specific shocks or asymmetric labour markets, both of which are analysed in the following.

In the case of country-specific shocks we have

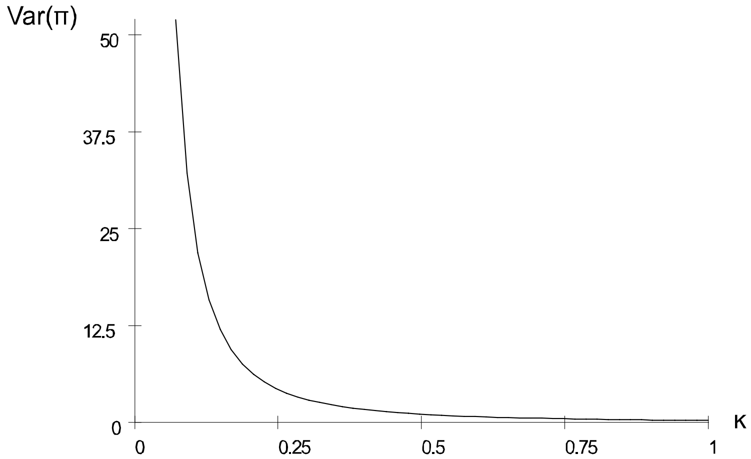
$$\begin{aligned}Var(\pi_t) &= \phi^2 \left[(\nu_1)^2 + (\nu_2)^2\right] \sigma_\varepsilon^2 \\ Var(y_t) &= \varphi^2 \left[(\nu_1)^2 + (\nu_2)^2\right] \sigma_u^2\end{aligned}$$

The latter brings out the straightforward point that asymmetries in size can have important implications for aggregate variability in the currency area in the case of country-specific shocks since $(\nu_1)^2 + (\nu_2)^2$ varies between 1 (for $\nu_1 = 1$, and $\nu_2 = 0$, or vice versa) and 0.5 (for $\nu_1 = \nu_2 = \frac{1}{2}$). This, of course, captures the standard smoothing property arising when aggregating independent stochastic variables. In words it implies that aggregate variability is always higher when country sizes are asymmetric. Note that since $(\nu_1)^2 + (\nu_2)^2 \leq 1$ it follows that variability of both inflation and output are never larger in the case of country-specific shocks as compared to the case of common shocks.

Figure 3 plots the variability of aggregate inflation[13] as a function of the parameter κ characterizing nominal rigidities in the case of symmetric (homogeneous) labour markets. The important point is that inflation variability is

[13] Note the level of variability is very high for κ going to zero – this is in contrast to figures reported below - the difference arises because this figure has $\kappa_1 = \kappa_2$, while the other figures only allow one of the κ´s to change at the time.

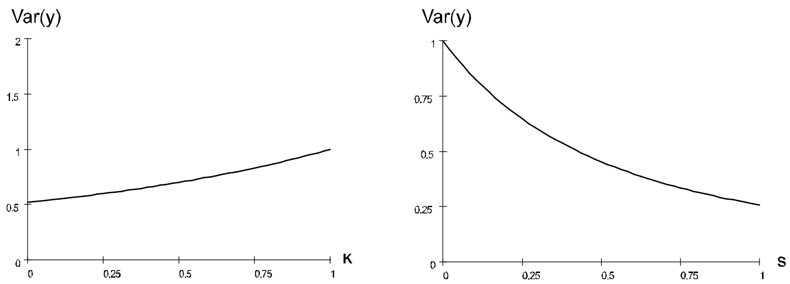
Figure 3: Aggregate inflation variability, homogenous labour market – nominal rigidities



very sensitive to nominal rigidities. Note that real wage flexibility is inconsequential for inflation variability in the case of homogenous labour markets (see above). Figure 4 below displays how aggregate output variability depends on real and nominal rigidities, respectively.

Figure 4 shows that aggregate output volatility is fairly sensitive to both nominal and real flexibility when countries have symmetric labour market structures.

Figure 4: Aggregate output variability, homogeneous labour markets - real and nominal rigidity



5.2 Heterogenous labour markets - aggregate output and inflation

It is now possible to turn to the case where there are asymmetries in labour market structures across the member countries of the currency union. The heterogeneity applies both to real and nominal rigidities.

Note that in the case of labour market heterogeneities even common shocks may affect relative prices among member countries of the currency union, and this may change the transmissions of shocks critically. Adding country-specific shocks would then compound with the effects of labour market heterogeneities. This section considers first the aggregate consequences and then turns to country-specific output fluctuations.

Aggregate output and inflation
In equilibrium the aggregate rate of inflation and aggregate output can be written (see appendix B)

$$\begin{aligned}\pi_t &= \nu_1\phi_1\varepsilon_{1t} + \nu_2\phi_2\varepsilon_{2t} \\ y_t &= \nu_1\varphi_1 u_{1t-1} + \nu_2\varphi_2 u_{2t-1}\end{aligned}$$

where

$$\begin{aligned}\frac{\partial\phi_1}{\partial\kappa_1} &\gtreqless 0, \frac{\partial\phi_1}{\partial s_1} > 0, \frac{\partial\phi_1}{\partial\kappa_2} \gtreqless 0, \frac{\partial\phi_1}{\partial s_2} < 0 \\ \frac{\partial\varphi_1}{\partial\kappa_1} &> 0, \frac{\partial\varphi_1}{\partial s_1} < 0, \frac{\partial\varphi_1}{\partial\kappa_2} > 0, \frac{\partial\varphi_1}{\partial s_2} < 0\end{aligned}$$

The effect of changes in nominal rigidities is in general ambiguous, whereas more real wage flexibility at home makes aggregate inflation more sensitive to domestic shocks and less to foreign shocks. More flexibility (nominal or real) unambiguously makes output less sensitive to both domestic and foreign shocks.

The variability of inflation is in the case of common shocks given as

$$Var(\pi_t) = [\nu_1\phi_1 + \nu_2\phi_2]^2 \sigma_\varepsilon^2$$

where

$$\begin{aligned}sign\frac{\partial[\nu_1\phi_1 + \nu_2\phi_2]}{\partial s_1} &= sign(\kappa_2 - \kappa_1) \\ sign\frac{\partial[\nu_1\phi_1 + \nu_2\phi_2]}{\partial s_2} &= sign(\kappa_1 - \kappa_2)\end{aligned}$$

whereas $\frac{\partial[\nu_1\phi_1+\nu_2\phi_2]}{\partial\kappa_1} \gtreqless 0, \frac{\partial[\nu_1\phi_1+\nu_2\phi_2]}{\partial\kappa_2} \gtreqless 0$, cf. the numerical illustrations below. It is an implication that more real flexibility leads to more inflation variability if it happens in the least nominal rigid country and vice versa. Notice that in the case where nominal flexibility is the same across the two countries ($\kappa_2 = \kappa_1$)

variations in real flexibility do not affect inflation variability, cf. also numerical illustration below.

In the case of country-specific shocks inflation variability is given by

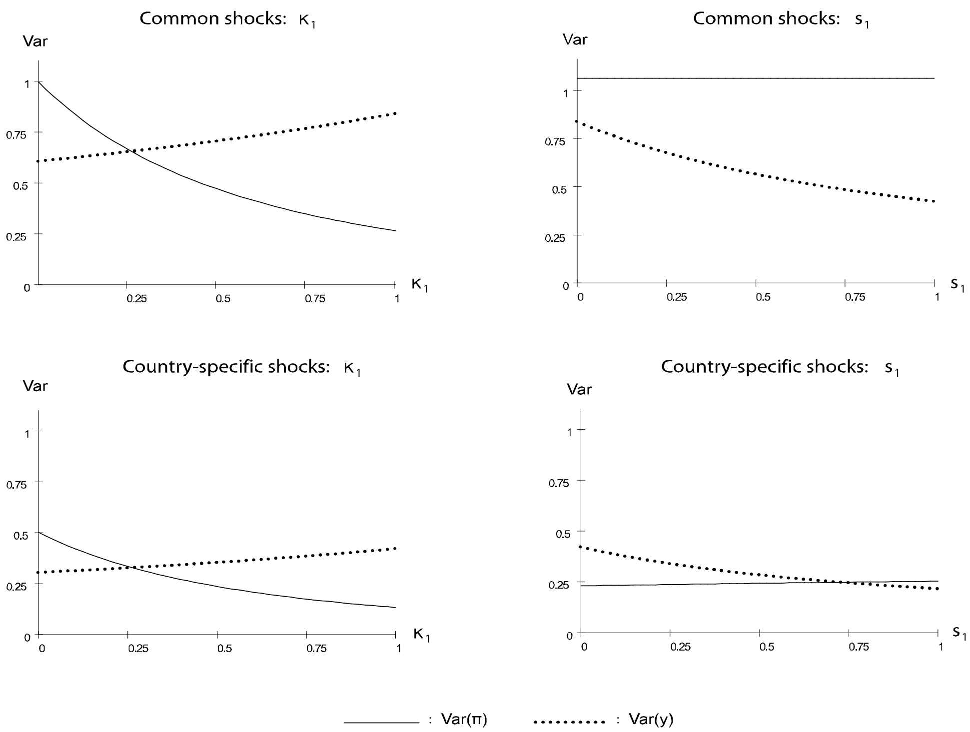
$$Var\left(\pi_t\right) = \left[\left(\nu_1\phi_1\right)^2 + \left(\nu_2\phi_2\right)^2\right]\sigma_\varepsilon^2$$

It follows that if $\kappa_1 \geq \kappa_2$, and $s_1 \leq s_2$ then

$$|\phi_1| \leq |\phi_2|$$

Accordingly, inflation has the lowest variance if the largest country has the most rigid wages (high κ, small s) i.e. if ν_1 high and $|\phi_1|$small), and vice versa.

Figure 5: Inflation and output variability and flexibility: Common and Country specific shocks



The variability of aggregate output is in the case of common shocks given as

$$Var\left(y_t\right) = \left[\nu_1\varphi_1 + \nu_2\varphi_2\right]^2\sigma_u^2$$

where

$$\frac{\partial\left[\nu_1\varphi_1+\nu_2\varphi_2\right]}{\partial s_1} < 0; \frac{\partial\left[\nu_1\varphi_1+\nu_2\varphi_2\right]}{\partial s_2} < 0$$
$$\frac{\partial\left[\nu_1\varphi_1+\nu_2\varphi_2\right]}{\partial \kappa_1} < 0; \frac{\partial\left[\nu_1\varphi_1+\nu_2\varphi_2\right]}{\partial \kappa_2} < 0$$

and for country-specific shocks as

$$Var\left(y_t\right)=\left[\left(\nu_1\varphi_1\right)^2+\left(\nu_2\varphi_2\right)^2\right]\sigma_u^2$$

Note in the latter case output variability is lowest if the largest country has the most flexible labour market (i.e. if ν_1 high, φ_1 low).

Figure 5 illustrates how the volatility of aggregate inflation (bold line) and output (dotted line) depend on the degree of nominal and real flexibility in one country (here the home country). Observe that there is symmetry between the two countries in all other respects. Nominal flexibility matters most for inflation variability, whereas real flexibility matters most for output flexibility.

The interesting issue here is how heterogeneity affects the overall performance of inflation and output in the currency union. Since heterogeneity in adjustment interacts with country size it is useful to consider how aggregate performance is affected by different asymmetries in adjustment across the two countries, but for a given aggregate flexibility. To address this issue consider variations in nominal flexibility for a given aggregate or average flexibility, i.e.

$$v_1\kappa_1+v_2\kappa_2=\overline{\kappa}$$

where $\overline{\kappa}$ is some given value. Similarly for real flexibility consider variations across countries for a given aggregate or average flexibility, i.e.

$$v_1 s_1+v_2 s_2=\overline{s}$$

where $\overline{s}$ is some given value. Figure 4 and 5 show inflation and output variability, respectively, under such experiments with heterogeneity. The charts include two lines – the thin line is for the case where country sizes are equal ($v_1 = v_2 = 0.5$) and the thick line is for the case where there is asymmetric country size ($v_1 = 0.25, v_2 = 0.75$). The parameter on the x-axis is the flexibility in country 2 (which in the case of asymmetric size is the large country).

The figure brings out the interesting finding that heterogeneity in flexibility matters little (thin line) if countries are of symmetric size (reflecting that for symmetric country sizes the variance terms are almost linear in the adjustment parameters). However, when country sizes are asymmetric heterogeneity in flexibility for a given average flexibility may have a substantial effect on overall performance (bold line). Especially for aggregate output volatility in the case of aggregate shocks the effects may be large. Interestingly, this is more so for common/aggregate than country-specific shocks, which points to the fact that

Figure 6: Output variability - heterogeneity for given average flexibility

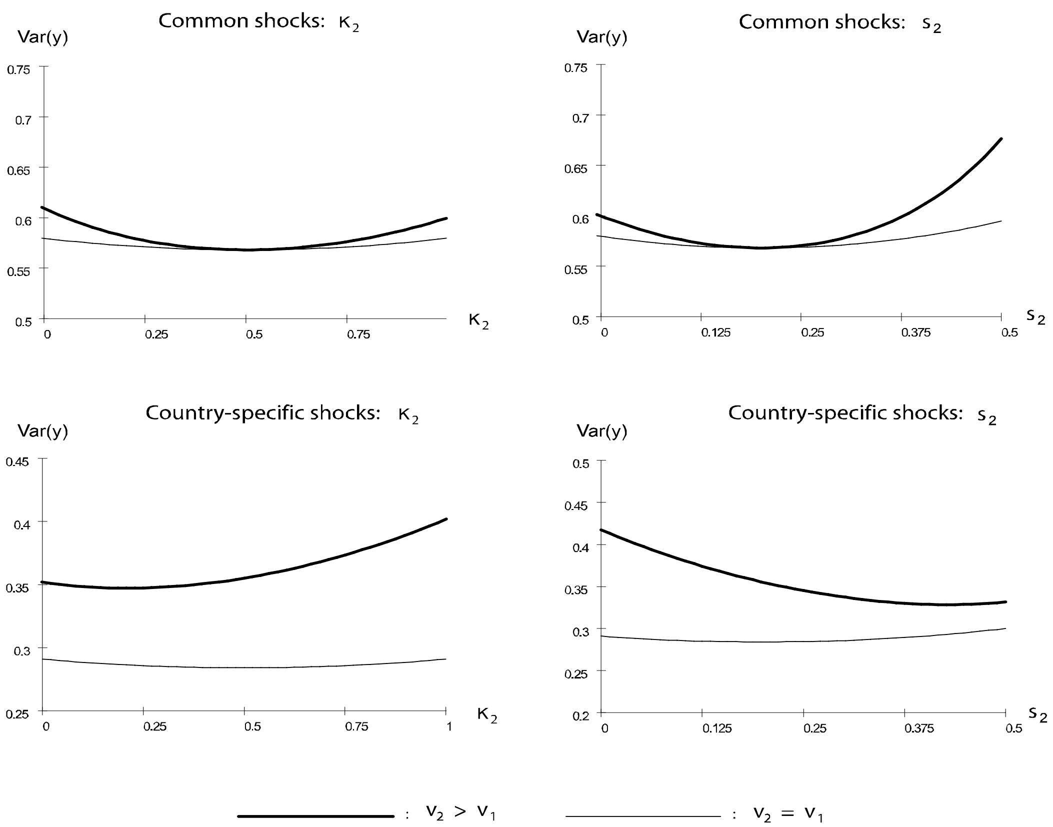
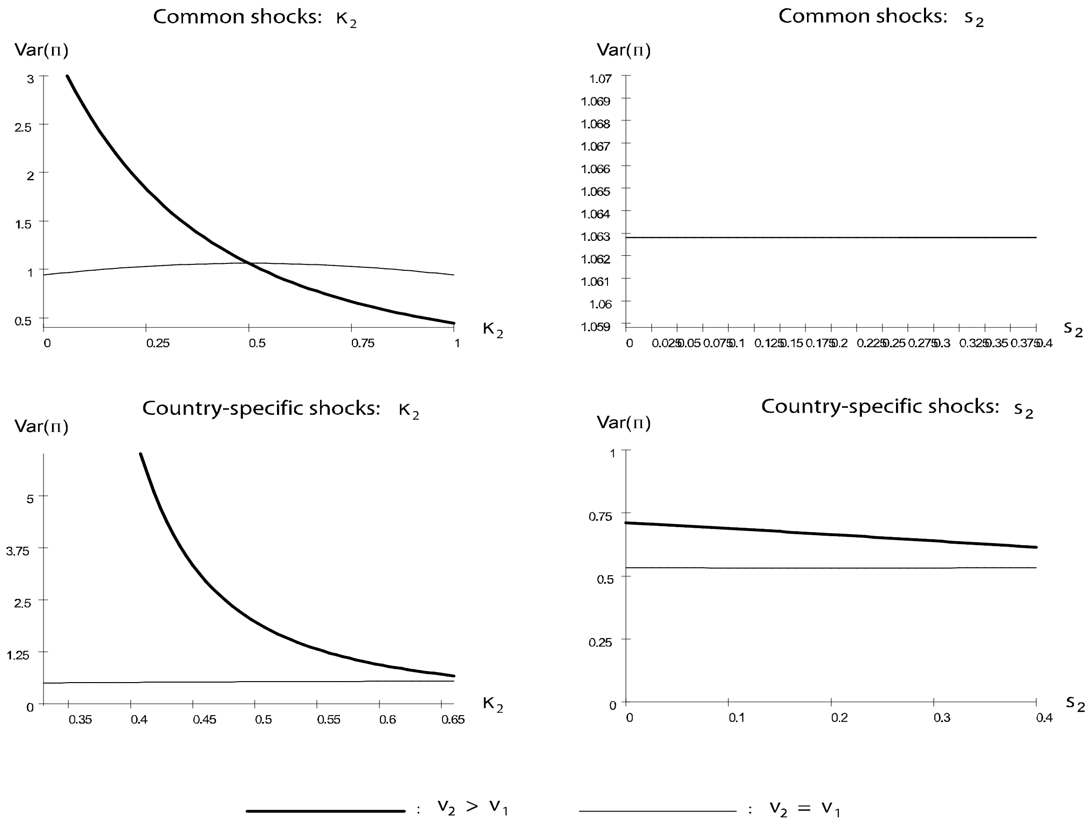


Figure 7: Inflation variability - heterogeneity for given average flexibility



the traditional viewpoint on the importance of flexibility in respect to country-specific shocks, may overlook the important and strong interdependencies arising in the case of aggregate or common shocks. Heterogeneity is also seen to make inflation control more difficult.

Notice in the case of country-specific shocks the level of variability is unambiguously higher with asymmetric country sizes. The reason is the effect of asymmetric country size outlined in the case of symmetric labour markets, cf. section 5.1.

To sum up: the analysis has shown that for the performance of the monetary union more labour market flexibility (higher s, or lower κ) leads to less variability in aggregate output and more variability of inflation. Nominal rigidities have a substantial effect on inflation variability. The effects on output variability are small but not trivial. More real flexibility seems in particular to have large effects on output variability if the initial flexibility is low. Heterogeneity is potentially of large importance for aggregate volatility when countries are of asymmetric size.

5.3 Heterogeneous labour markets: Country-specific output

Having considered the role of labour market heterogeneities for the overall performance of the currency union, the next question is the implication for country-specific output. This issue is considered by analysing common and country specific shocks in turn.

Common shocks
Country-specific output is first considered in the case of common shocks, where we have

$$y_{1t} = \Gamma_{1u} u_{t-1} + \Gamma_{1\varepsilon} \varepsilon_t \quad (5)$$

$$y_{2t} = \Gamma_{2u} u_{t-1} + \Gamma_{2\varepsilon} \varepsilon_t \quad (6)$$

with coefficients given in the appendix C, where

$$\Gamma_{1u} > 0; sign\ \Gamma_{1\varepsilon} = sign\ (\kappa_2 - \kappa_1)$$

$$\Gamma_{2u} > 0; sign\ \Gamma_{2\varepsilon} = sign\ (\kappa_1 - \kappa_2)$$

Differences in nominal rigidities have an influence on how shocks affect output across countries. It is seen that the country with the most nominal rigid wages ($e.g.\ \kappa_1 > \kappa_2$) would experience an output reduction, while the less rigid country would experience an increase. The reason being that the terms of change shifts in the favour of the more flexible country. A productivity innovation calls for a price reduction and since the largest reduction takes place in the most flexible country it follows that the most rigid country experiences a relative price increase which has a contractionary effect. [14]

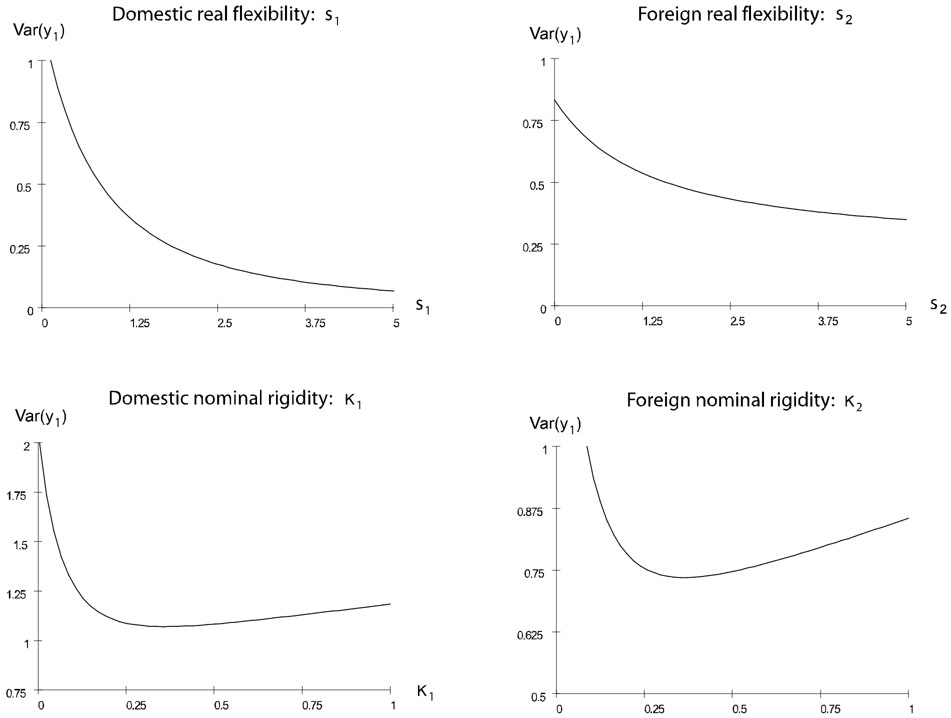
Table 2 summarizes how the parameters in (5) and (6) depend on the underlying parameters. The ambiguity in the results indicates that at the country level the interdependencies between countries are strong, and it does not necessarily follow that more flexibility at home or abroad leads to less output variability. This is clearly seen from the numerical results reported in figure 5.

Table 2: Γ-coefficients

	Γ_{1u}	Γ_{2u}	$\Gamma_{1\varepsilon}$	$\Gamma_{2\varepsilon}$
κ_1	>0	>0	$\lesseqgtr 0$	$\lesseqgtr 0$
κ_2	>0	>0	$\lesseqgtr 0$	$\lesseqgtr 0$
s_1	<0	<0	$sign(\kappa_1 - \kappa_2)$	$sign(\kappa_2 - \kappa_1)$
s_2	<0	<0	$sign(\kappa_1 - \kappa_2)$	$sign(\kappa_2 - \kappa_1)$

[14] Note that consistent with the findings reported above on the aggregate output effects we have that $\nu_1 \Gamma_{1\varepsilon} + \nu_2 \Gamma_{2\varepsilon} = 0$, i.e. the current shock innovations do not have any aggregate output effects within the currency union.

Figure 8: Domestic output variability, common shocks: domestic and foreign labour market flexibility



To see more clearly the importance of labour market heterogeneities, let country 1 be the country with the most rigid labour market in the sense that it has more nominal rigidity and less flexible real wages relative to country 2 ($\kappa_1 \geq \kappa_2, s_1 \leq s_2$). Considering inflation it turns out that the rigid country contributes more to inflation variability than the flexible, since

$$|\phi_1| \leq |\phi_2|$$

The intuition is straightforward since there is less wage adjustment in the rigid country and therefore the price response to the shock is also smaller.[15] At the same time the most rigid country has the highest output variability (provided country size is not too unequal)[16], i.e.

$$Var(y_{1t}) \geq Var(y_{2t})$$

[15] Obviously, if there are price rigidities rather than wage rigidities, this result would turn around.

[16] The "perverse" result that $Var(y_{1t}) > Var(y_{2t})$ even if $\Lambda_1 > \Lambda_2$ requires that the difference between $\Lambda_1$and Λ_2 is not too large, and that countries are very asymmetrically sized.

Figure 8 shows that more real wage flexibility – at home and foreign – is always reducing domestic output variability. However, the effects of both domestic and foreign nominal rigidity is ambiguous – up to a certain threshold level more nominal inflexibility – both at home and foreign may contribute to less output variability. The reason is that too large differences in nominal flexibility induce volatility in the terms of trade.

Country specific output - country-specific shocks
Turning to country-specific shocks we have

$$\begin{aligned} y_{1t} &= \Upsilon_{1u1}u_{1t-1} + \Upsilon_{1u2}u_{2t-1} + \Upsilon_{1\varepsilon1}\varepsilon_{1t} + \Upsilon_{1\varepsilon1}\varepsilon_{2t} \\ y_{2t} &= \Upsilon_{2u1}u_{1t-1} + \Upsilon_{2u2}u_{2t-1} + \Upsilon_{2\varepsilon1}\varepsilon_{1t} + \Upsilon_{2\varepsilon1}\varepsilon_{2t} \end{aligned}$$

with coefficients given in the appendix. This case is very complicated and few unambiguous theoretical results can be found. Figure 6 reports on the importance of domestic and foreign flexibility for domestic output variability. More domestic flexibility - nominal or real - unambiguously contributes to less domestic output variability. More foreign real wage flexibility is also reducing domestic output variability, whereas more foreign nominal rigidity at low levels of κ would lower domestic output variability and at high levels increase it (the curve is U-shaped).

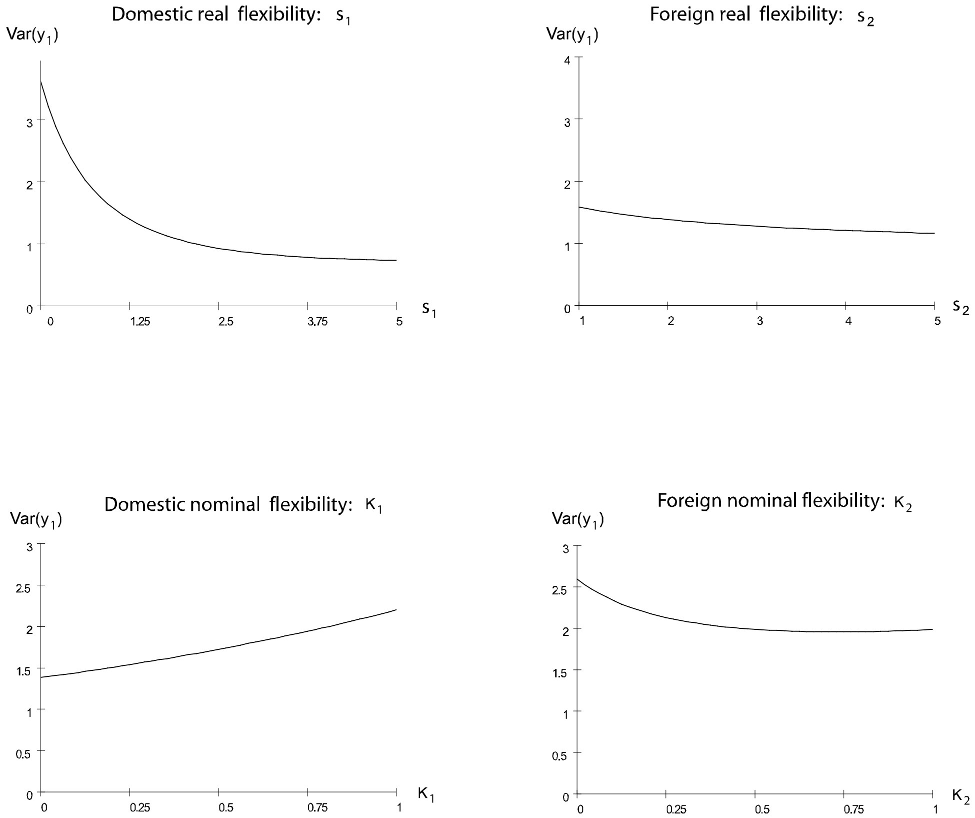
5.4 Spill-overs

A key question is the spill-overs induced by labour market heterogeneities. This question is obviously of importance for evaluating the overall performance of the currency union. Is it the case that more rigid countries contribute to more inflation and output variability in the currency union? Equally important are the implications at the country level. In what way is a country affected if other countries have more rigid labour markets? The latter is important for the need for policy coordination in relation to structural policies affecting labour market flexibility. The following summarizes the results on the interdependencies.

For real flexibility it was found that more real wage flexibility is good for aggregate output variability, but bad for inflation variability. Moreover, it is good for both domestic and foreign output variability. If there are costs associated with structural reforms affecting real wage flexibility and if domestic decisions on the benefit side include output stability it follows that the non-cooperative choice of structural policies may leave too little real wage flexibility. The reason is that countries acting non-cooperatively do not take into account how domestic real wage flexibility affects foreign output variability.

For nominal flexibility the issue is more complex. More nominal flexibility leads to less aggregate output volatility and more inflation variability, whereas the effect on domestic output variability is in general ambiguous. It is possible that more nominal rigidity may lower output variability. This is due to the terms-of-trade effect which is larger the more different the flexibility is at home

Figure 9: Domestic output variability, country-specific shocks: domestic and foreign labour market flexibility



and foreign. This suggests a strong strategic complementarity in nominal flexibility in the sense that there can be large costs in terms of output variability of having a degree of flexibility too much different to the degree of flexibility prevailing in other countries. The effect of nominal flexibility on foreign output variability is also ambiguous, which implies that for a given level of foreign nominal rigidity it is not necessarily optimal to choose full nominal flexibility even if that can be done at no costs. Note also that when more foreign nominal rigidity leads to more domestic output volatility we have a situation where noncooperative decisions on structural reforms may lead to excessive flexibility.

An interesting observation is that the spill-over effects are much stronger in the case of aggregate than country specific shocks. This brings a new perspective on the traditional view on the role of wage flexibility in a monetary union. It is usually argued that wage flexibility in particular is important for the ability to deal with country-specific shocks. The numerical illustrations confirm that the degree of flexibility (nominal or real) in the domestic labour market is important for domestic output volatility in respect to both aggregate and country-specific shocks. However, for country-specific shocks the spill-over effect is relatively invariant to the degree of foreign labour market flexibility. The important spill-over effects which give reason for concern about coordination of structural policies directed at making labour markets more flexible seem to arise in relation to aggregate shocks.

6 Concluding remarks

This paper has considered the implications of labour market heterogeneities in a monetary union in which the monetary authority targets inflation. It was found that such heterogeneities may play an important role both for the overall performance of the currency union, and the country-specific performance. In particular, heterogeneity is associated with potentially strong nonlinearity and interdependencies in structural factors.

An important caveat is that structural issues have here only be considered in the perspective of adjustment to shocks. Accordingly, the underlying rationale for various institutional structures in the labour market have not been addressed. Issues which it is obviously important to integrate when considered the implications of structural reforms.

While the model captures some basic mechanisms it is in some respects very stylized. In future work it would be interesting to allow for richer dynamics in the structure (including e.g. capital accumulation and other dynamic aspects) as well in the adjustment (sluggishness in adjustment of wages and prices over time). On the empirical side it would be interesting to considered more carefully the implications of the existing heterogeneities across member countries in the Euro-area and assess the implications of various reform options.

References

Andersen, T.M., 2004, Real and Nominal Propagation of Nominal Shocks, Economic Journal, 114, 174-195.

Beetsma, R. and H. Jensen, 2002, Monetary and Fiscal Policy Interactions in a Micro-founded model of a Monetary Union, Working paper

Berthold, N., R. Fehn and E. Thode, 1999, Real Wage Rigidities, Fiscal Policies, and the Stability of EMU in the transition phase, IMF Working paper WP/99/83.

Benigno, P., 2002, Optimal Monetary Policy in a Currency Area,Working Paper.

Calmfors, L., 2001, Wages and Wage-Bargaining Institutions in the EMU - A Survey of the Issues, Empirica, 28, 325-351.

Corsetti, G. and P. Pesenti, 2002, Welfare and Macroeconomics, Quarterly Journal of Economics,

Dellas, H., and G. Tavlas, 2002, Wage Rigidity and Monetary Union, CEPR Discussion paper 3679.

Denis, C., K. Morrow and W. Röger, 2002, Production function approach to calculating potential growth and output gaps - Estimates for the EU Member States and the US, European Commission Economic Papers, 176.

Decressin, J., and A. Fatás, 1995, Regional Labour Market Dynamics in Europe, European Economic Review, 39, 1627-1655.

De Grauwe, P. 2003, Economics of Monetary Union, Oxford University Press, 5th edition.

Duarte, M. and A.L. Wolman, 2002, Regional Inflation in a Currency Union: Fiscal Policy vs. Fundamentals, ECB Working paper 180.

ECB, 2002, Labour market mismatches in euro area countries, March 2002.

European Commission, 2003, The EU Economy 2003 Review, Ch. 4: Wage Flexibility and Wage Interdependencies in EMU.

Erlandsson, M., 2003, Nominal wage flexibility in a monetary union, Working paper, University of Gothenburg.

Frankel, J.A. and A. K. Rose, 1998, The Endogeneity of the Optimal Currency Area Criteria, Economic Journal 108, 1008-1025.

H.M. Treasury, 2003, Meeting the Challenge: Economic Reform in Europe

Iversen, T., and D. Soskice, 2000, The Non-neutrality of monetary policy with large price and wage setters, Quarterly Journal of Economics, CXV, 265-284.

Layard.R., S. Nickell, and R.Jackman,1991, Unemployment: Macroeconomic Performance and the Labour Market, Oxford University Press, Oxford.

Obstfeld, M., and K. Rogoff, 1995, Exchange Rate Economics Redux, Journal of Political Economy,103,624-660

Obstfeld, M., and K. Rogoff, 1996, Foundations of International Macroeconomics, MIT Press.

OECD, 1998, The OECD Jobs Strategy: Progress Report on Implementation of Country-Specific Recommendations, Economics Department Working Papers no 196.

OECD, 2002, Employment Outlook

Paloviita, M., 2002, Inflation dynamics in the euro area and the role of expectations, Bank of Finland Discussion paper 2002-20.

Riboud et al., 2002, Does Eurosclerosis Matter? Institutional Reform and Labour Market Performance in Central and Eastern European Countries in the 1990s, Social Protection Discussion Paper Series No. 0202, The World Bank, www.worldbank.org/sp

Spange, M., 2002, International Spill-over Effects of Labour Market Rigidities, Working Paper, University of Aarhus

Suardi, M., 2001, EMU and asymmetries in the monetary transmission mechanism, European Commission, Economic Papers 154.

Turner, D., L. Boone, C. Giorno, M. Meacci, D. Rae, and P. Richardson, 2001, Estimating the Structural Rate of Unemployment for the OECD countries, OECD Economic Studies, 33, 171-216.

Turner, D., and E. Seghezza, 1999, Testing for a common OECD Phillips curve, OECD Economics Department, Working Paper 219.

A Steady state and log-linearization

First-order conditions
The first-order conditions to the household decision problem are

$$C_{it}^{-\frac{1}{\sigma}} = \delta\left(1+R_t\right)E_t\left(C_{it+1}^{-\frac{1}{\sigma}}\right) \quad (7)$$

$$C_{it}^{-\frac{1}{\sigma}} = \lambda\left(\frac{M_{it}}{P_{it}}\right)^{-\varepsilon} + E_t\left(\delta C_{it+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right) \quad (8)$$

Note that given the exogenous wage contracts, labour supply is determined by demand (provided the wage exceeds the reservation wage, which is assumed always to be the case).

Steady state

The steady-state version of the model is similar to that analyzed in, for example, Obstfeld and Rogoff (1995). We focus on a symmetric non-stochastic steady state where $B_i = 0$, $C_i = C = Y_i = Y$, $R = \delta^{-1} - 1$, $P_i = P$, $W_i = W$ $\forall i$. Real incomes are $Y_i = \frac{P_i Y_i}{P}$. Steady-state values are indicated by omission of time subscripts.

Note that it is an implication of the equilibrium condition that $C_t = Y_t$, i.e. in the aggregate there is no net-wealth accumulation or decumulation. This is an obvious implication of the fact that there is no real capital in the model. For each single country, however, the situation is different, and a country-specific shock would in general induce a wealth reallocation between countries. More specifically following a non-common shock the economy moves away from the initial steady state in the distribution of consumption possibilities for countries and does not return for $\theta \neq 1$ (see section 3 for a discussion).

Log-linear representation
Next step is to write the model in a log-linear way, where the following formula for normally distributed variables is used

$$\log E\left(X^f\right) = fE\left[\log\left(X\right)\right] + \frac{f^2}{2}Var\left[\log\left(X\right)\right],$$

where f is a scalar and X is lognormally distributed.

The log-version of the Euler equation (1) follows straightforward by noting that all constants are left out. The money demand warrants a comment. Since the monetary authority is assumed to be setting the interest rate it follows that money supply is endogenously determined by the money demand prevailing at the given interest rate, output and prices, cf. (8). Hence, the money stock is determined recursively, and since the money stock is of no specific interest for the analysis undertaken here it follows that the money market can be disregarded.

B Stochastic Equilibrium

The appendix first solves for the inflation rate to determine monetary policy reactions. Next the level of output is solved for

B.1 Inflation rate

From the output relation (4) we have

$$y_{it} = \beta \left(p_{it} - w_{it}\right) + u_{it}$$

where the wage rate is determined by

$$w_{it} = \kappa_i p_t^e + (1 - \kappa_i)(p_t + s_i y_{it})$$

implying that output supply can be written

$$\begin{aligned} y_{it} &= \left[1 + \beta(1 - \kappa_i)s_i\right]^{-1} \left[\beta(p_{it} - p_t + \kappa_i(p_t - p_t^e)) + u_{it}\right] \\ &= \Lambda_i^{-1} \left[\beta(p_{it} - p_t + \kappa_i(p_t - p_t^e)) + u_{it}\right] \end{aligned}$$

where

$$\Lambda_i \equiv 1 + \beta(1 - \kappa_i)s_i$$

It follows that

$$y_{it} - y_{it-1} = \frac{\beta}{\Lambda_i} \left[(\pi_{it} - \pi_t) - \kappa_i \left(\pi_t^e + \pi_{t-1} - \pi_{t-1}^e - \pi_t\right) + \beta^{-1} \left(u_{it} - u_{it-1}\right)\right]$$

Aggregate demand for country i commodities can be written

$$y_{it} - y_{it-1} = -\theta \left(\pi_{it} - \pi_t\right) + c_t - c_{t-1}$$

From the Euler equation we have

$$E_t c_{t+1} = c_t + \sigma r_t$$

Using that in equilibrium $c_t = y_t$ and that

$$y_{t+1} = E_t y_{t+1} + \varepsilon_{t+1}$$

where the residual ε is yet undetermined (see below) but has the property that $E_t \varepsilon_{t+1} = 0$. It follows that

$$\begin{aligned} c_t - c_{t-1} &= E_{t-1} c_t + \varepsilon_t - c_{t-1} \\ &= \sigma r_{t-1} + \varepsilon_t \end{aligned}$$

and therefore

$$y_{it} - y_{it-1} = -\theta \left(\pi_{it} - \pi_t\right) + \sigma r_{t-1} + \varepsilon_t$$

Country-specific inflation
Combining demand and supply we have that the equilibrium condition requires

$$\frac{\beta}{\Lambda_i}\left[(\pi_{it}-\pi_t)-\kappa_i\left(\pi_t^e+\pi_{t-1}-\pi_{t-1}^e-\pi_t\right)+\beta^{-1}\left(u_{it}-u_{it-1}\right)\right]=-\theta\left(\pi_{it}-\pi_t\right)+\sigma r_{t-1}+\varepsilon_t$$

or

$$\pi_{it}=\lambda_{1i}\left(-\pi_{t-1}^e+\pi_t^e+\pi_{t-1}-\pi_t\right)+\lambda_{2i}\pi_t+\lambda_{3i}(\sigma r_{t-1}+\varepsilon_t)-\lambda_{4i}\left(u_{it}-u_{it-1}\right)$$

where

$$\begin{array}{ll} \lambda_{1i}=\frac{\beta\kappa_i}{\beta+\theta\Lambda_i}<1 & \lambda_{2i}=1 \\ \lambda_{3i}=\frac{\Lambda_i}{\beta+\theta\Lambda_i} & \lambda_{4i}=\frac{1}{\beta+\theta\Lambda_i} \end{array}$$

Aggregate inflation
Aggregating over country-specific inflation we get

$$\begin{aligned}\pi_t &= \left(\pi_t^e+\pi_{t-1}-\pi_{t-1}^e-\pi_t\right)\left[\sum_{n=1}^N v_n\lambda_{1n}\right]+\pi_t\left[\sum_{n=1}^N v_n\lambda_{2n}\right]+\sum_{n=1}^N v_n\lambda_{3n}\left(\sigma r_{t-1}+\varepsilon_t\right) \\ &\quad -\sum_{n=1}^N v_n\lambda_{4n}\left(u_{nt}-u_{nt-1}\right) \end{aligned} \tag{9}$$

For the supply shock it is assumed that it follows an AR(1) process , i.e.

$$u_{nt}=\rho u_{nt-1}+\varepsilon_{nt} \qquad ;0<\rho<1$$

Taking expectations we find

$$\begin{aligned}\pi_t^e &= \left(\pi_t^e+\pi_{t-1}-\pi_{t-1}^e-\pi_t^e\right)\left[\sum_{n=1}^N v_n\lambda_{1n}\right]+\pi_t^e\left[\sum_{n=1}^N v_n\lambda_{2n}\right] \\ &\quad +\sum_{n=1}^N v_n\lambda_{3n}\left(\sigma r_{t-1}\right)-(\rho-1)\sum_{n=1}^N v_n\lambda_{4n}u_{nt-1}\end{aligned}$$

Monetary policy - inflation targeting
Monetary policy instrument is set so as to meet inflation target - for simplicity assumed to be a zero inflation rate, i.e.

$$\pi_t^e=E_{t-1}\pi_t=0$$

this implies that the interest rate is set at the level which ensures that the inflation target is reached, which requires

$$0=\pi_{t-1}\left[\sum_{n=1}^N v_n\lambda_{1n}\right]+\sigma r_{t-1}\left[\sum_{n=1}^N v_n\lambda_{3n}\right]-(\rho-1)\sum_{n=1}^N v_n\lambda_{4n}u_{nt-1} \tag{10}$$

which is accomplished by the following interest rate rule

$$r_{t-1} = \frac{-1}{\sigma} \left[\frac{\sum_{n=1}^{N} v_n \lambda_{1n}}{\sum_{n=1}^{N} v_n \lambda_{3n}} \right] \pi_{t-1} + \frac{\rho - 1}{\sigma} \left[\frac{\sum_{n=1}^{N} v_n \lambda_{4n} u_{nt-1}}{\sum_{n=1}^{N} v_n \lambda_{3n}} \right]$$

Combining (10) and (9) and using that $E_{t-1}\pi_t = 0$ for all t we find that aggregate inflation can be written

$$\pi_t = \left[\sum_{n=1}^{N} v_n \lambda_{1n} \right]^{-1} \left[\sum_{n=1}^{N} v_n \lambda_{3n} \varepsilon_t - \sum_{n=1}^{N} v_n \lambda_{4n} \varepsilon_{nt} \right]$$

and the country-specific inflation rate can be written

$$\pi_{it} = \lambda_{1i} \left(\pi_{t-1} - \pi_t \right) + \pi_t + \lambda_{3i} (\sigma r_{t-1} + \varepsilon_t) - \lambda_{4i} \left(\left(\rho - 1 \right) u_{nt-1} + \varepsilon_{it} \right)$$

Implying that inflation differentials are given as

$$\pi_{it} - \pi_t = \lambda_{1i} \left(\pi_{t-1} - \pi_t \right) + \lambda_{3i} (\sigma r_{t-1} + \varepsilon_t) - \lambda_{4i} \left(\left(\rho - 1 \right) u_{nt-1} + \varepsilon_{it} \right)$$

Note that $\frac{\partial \pi_i}{\partial \pi} = 1 - \lambda_{1i} > 0$, $\frac{\partial \pi_i}{\partial \varepsilon_i} < 0$.

The monetary reaction function can be written

$$r_{t-1} = \Omega_\pi \pi_{t-1} + \sum_{n=1}^{N} v_n \Omega_{4n} u_{nt-1}$$

where

$$\begin{aligned} \Omega_\pi &= \frac{-1}{\sigma} \left[\frac{\sum_{n=1}^{N} v_n \lambda_{1n}}{\sum_{n=1}^{N} v_n \lambda_{3n}} \right] < 0 \\ \Omega_{un} &= \frac{\rho - 1}{\sigma} \left[\frac{\lambda_{4n}}{\sum_{n=1}^{N} v_n \lambda_{3n}} \right] < 0 \end{aligned}$$

B.2 Equilibrium output

We have that output can be written

$$y_{it} = \Lambda_i^{-1} \left[\beta (p_{it} - p_t + \kappa_i (p_t - p_t^e)) + u_{it} \right]$$

Using that demand for country i output reads

$$d_{it} = -\theta (p_{it} - p_t) + y_t$$

it follows from the equilibrium condition $y_{it} = d_{it}$ that the relative price (terms of trade) is determined by

$$\Lambda_i^{-1} \left[\beta (p_{it} - p_t + \kappa_i (p_t - p_t^e)) + u_{it} \right] = -\theta (p_{it} - p_t) + y_t$$

Hence

$$(p_{it}-p_t) = [\Lambda_i\theta+\beta]^{-1}\left[\Lambda_i y_t - [\beta\kappa_i(\pi_t-\pi_t^e)+u_{it}]\right]$$

Inserting this in the demand relation yields

$$\begin{aligned} y_{it} &= -\theta(p_{it}-p_t)+y_t \\ &= \frac{1}{\Lambda_i\theta+\beta}\left[\theta\beta\kappa_i(\pi_t-\pi_t^e)+\theta u_{it}+\beta y_t\right] \\ &= \omega_{iy}y_t+\omega_{i\pi}\pi_t+\omega_{iu}u_{it} \end{aligned}$$

where it has been used that $\pi_t^e = 0$, and with coefficients defined as

$$\begin{aligned} \omega_{iy} &= \frac{\beta}{\Lambda_i\theta+\beta} \\ \omega_{i\pi} &= \frac{\theta\beta\kappa_i}{\Lambda_i\theta+\beta} \\ \omega_{iu} &= \frac{\theta}{\Lambda_i\theta+\beta} \end{aligned}$$

Aggregating over country-specific output we get

$$\begin{aligned} y_t &= \sum_{n=1}^{N} v_n y_{nt} \\ &= \frac{\sum_{n=1}^{N} v_n\omega_{n\pi}}{1-\sum_{n=1}^{N} v_n\omega_{ny}}\pi_t+\frac{\sum_{n=1}^{N} v_n\omega_{nu}u_{nt}}{1-\sum_{n=1}^{N} v_n\omega_{ny}} \end{aligned} \tag{11}$$

Next conjecture a solution of the following form

$$y_t = \sum_{n=1}^{N}\nu_n\varphi_n u_{nt-1}+\sum_{n=1}^{N}\nu_n\chi_n\varepsilon_{nt} \tag{12}$$

where the φ and χ- coefficients are undetermined coefficients to be determined below. Given the conjecture we have

$$E_t y_{t+1} = \sum_{n=1}^{N}\nu_n\varphi_n u_{nt}$$

Using the Euler equation it follows that

$$\begin{aligned} E_t y_{t+1} &= y_t+\sigma r_t \\ y_{t+1} &= E_t y_{t+1}+\varepsilon_{t+1} \end{aligned}$$

For this to be consistent with (12) requires

$$\varepsilon_{t+1} = \sum_{n=1}^{N}\nu_n\chi_n\varepsilon_{nt+1}$$

Note that these relations imply

$$y_t = -\sigma r_t + \sum_{n=1}^{N} \nu_n \varphi_n u_{nt}$$

The next step is to prove that the conjecture (12) actually constitutes an equilibrium. To this end use (11) and the expression for inflation (9) to yield

$$\begin{aligned} y_t &= \frac{\sum_{n=1}^N v_n \omega_{n\pi}}{1-\sum_{n=1}^N v_n \omega_{ny}} \frac{1}{\left[\sum_{n=1}^N v_n \lambda_{1n}\right]} \left[\left(\sum_{n=1}^N v_n \lambda_{3n}\right)\varepsilon_t - \sum_{n=1}^N v_n \lambda_{4n}\varepsilon_{nt}\right] + \frac{\sum_{n=1}^N v_n \omega_{nu} u_{nt}}{1-\sum_{n=1}^N v_n \omega_{ny}} \\ &= \frac{\sum_{n=1}^N v_n \omega_{n\pi}}{1-\sum_{n=1}^N v_n \omega_{ny}} \frac{1}{\left[\sum_{n=1}^N v_n \lambda_{1n}\right]} \left[\left(\sum_{n=1}^N v_n \lambda_{3n}\right)\varepsilon_t - \sum_{n=1}^N v_n \lambda_{4n}\varepsilon_{nt}\right] \\ &\quad + \frac{\sum_{n=1}^N v_n \omega_{nu}\left(\rho u_{nt-1} + \varepsilon_{nt}\right)}{1-\sum_{n=1}^N v_n \omega_{ny}} \end{aligned} \tag{13}$$

Consistency between (12) and (13) requires:

$$\sum_{n=1}^N v_n \varphi_n u_{nt-1} = \rho \frac{\sum_{n=1}^N v_n \omega_{nu} u_{nt-1}}{1-\sum_{n=1}^N v_n \omega_{ny}}$$

which is ensured if

$$\varphi_i = \rho \frac{\omega_{iu}}{1-\sum_{n=1}^N v_n \omega_{ny}} \forall i$$

and

$$\begin{aligned} \sum_{n=1}^N v_n \chi_n \varepsilon_{nt} &= \frac{\sum_{n=1}^N v_n \omega_{n\pi}}{1-\sum_{n=1}^N v_n \omega_{ny}} \frac{1}{\left[\sum_{n=1}^N v_n \lambda_{1n}\right]} \left[\sum_{n=1}^N v_n \lambda_{3n}\varepsilon_t - \sum_{n=1}^N v_n \lambda_{4n}\varepsilon_{nt}\right] \\ &\quad + \frac{\sum_{n=1}^N v_n \omega_{nu}\varepsilon_{nt}}{1-\sum_{n=1}^N v_n \omega_{ny}} \\ &= \frac{\sum_{n=1}^N v_n \omega_{n\pi}}{1-\sum_{n=1}^N v_n \omega_{ny}} \frac{1}{\left[\sum_{n=1}^N v_n \lambda_{1n}\right]} \left[\sum_{n=1}^N v_n \chi_n \varepsilon_{nt} \sum_{n=1}^N v_n \lambda_{3n} - \sum_{n=1}^N v_n \lambda_{4n}\varepsilon_{nt}\right] \\ &\quad + \frac{\sum_{n=1}^N v_n \omega_{nu}\varepsilon_{nt}}{1-\sum_{n=1}^N v_n \omega_{ny}} \end{aligned}$$

which is ensured if

$$\begin{aligned} \chi_i &= \frac{\sum_{n=1}^N v_n \omega_{n\pi}}{1-\sum_{n=1}^N v_n \omega_{ny}} \frac{1}{\left[\sum_{n=1}^N v_n \lambda_{1n}\right]} \left[\chi_i \sum_{n=1}^N v_n \lambda_{3n} - \lambda_{4n}\right] + \frac{\omega_{iu}}{1-\sum_{n=1}^N v_n \omega_{ny}} \\ &= \frac{-\lambda_{4i}\sum_{n=1}^N v_n \omega_{n\pi} + \omega_{iu}\left(\sum_{n=1}^N v_n \lambda_{1n}\right)}{\left(1-\sum_{n=1}^N v_n \omega_{ny}\right)\left(\sum_{n=1}^N v_n \lambda_{1n}\right) - \left(\sum_{n=1}^N v_n \omega_{n\pi}\right)\left(\sum_{n=1}^N v_n \lambda_{3n}\right)} \end{aligned}$$

Note that

$$-\lambda_{4i}\sum_{n=1}^{N} v_n\omega_{n\pi} + \omega_{iu}\left(\sum_{n=1}^{N} v_n\lambda_{1n}\right) = 0 \qquad \forall i$$

which is seen by using the definition of the λ- and ω-coefficients which is equivalent to

$$\frac{-1}{\beta+\theta\Lambda_i}\left(\sum_{n=1}^{N} v_n\frac{\theta\beta\kappa_n}{\beta+\theta\Lambda_n}\right) + \frac{\theta}{\beta+\theta\Lambda_i}\left(\sum_{n=1}^{N} v_n\frac{\beta\kappa_n}{\beta+\theta\Lambda_n}\right) = 0$$

it follows that $\chi_i = 0$ for all i.

Output in country n can now be written

$$\begin{aligned} y_{it} &= \omega_{iy}y_t + \omega_{i\pi}\pi_t + \omega_{iu}u_{it} \\ &= \left[\omega_{iy}\frac{\sum_{n=1}^{N} v_n\omega_{n\pi}}{1-\sum_{n=1}^{N} v_n\omega_{ny}} + \omega_{i\pi}\right]\pi_t + \omega_{iy}\frac{\sum_{n=1}^{N} v_n\omega_{nu}u_{nt}}{1-\sum_{n=1}^{N} v_n\omega_{ny}} + \omega_{iu}u_{it} \end{aligned}$$

Finally note, that inflation can be written

$$\begin{aligned} \pi_t &= \left[\sum_{n=1}^{N} v_n\lambda_{1n}\right]^{-1}\left[\sum_{n=1}^{N} v_n\lambda_{3n}\varepsilon_t - \sum_{n=1}^{N} v_n\lambda_{4n}\varepsilon_{nt}\right] \\ &= \sum_{n=1}^{N}\nu_n\phi_{\varepsilon n}\varepsilon_{nt} \end{aligned}$$

where

$$\begin{aligned} \phi_{\varepsilon i} &= \left[\sum_{n=1}^{N} v_n\lambda_{1n}\right]^{-1}\left[\chi_i\sum_{n=1}^{N} v_n\lambda_{3n} - \lambda_{4n}\right] \\ &= \frac{-\lambda_{4i}}{\sum_{n=1}^{N} v_n\lambda_{1n}} \end{aligned}$$

C Labour market structures

C1 Symmetric labour markets

If $\kappa_i = \kappa$ and $s_i = s$ for all i, we have

$$\begin{aligned}
\pi_t &= \phi \sum_{n=1}^{N} \nu_n \varepsilon_{nt} \\
y_t &= \varphi \sum_{n=1}^{N} \nu_n u_{nt-1}
\end{aligned}$$

where

$$\phi = -\frac{\lambda_4}{\lambda_1} = -\frac{1}{\beta\kappa}$$

Hence ϕ is negative and increasing in κ

$$\varphi = \rho \frac{\omega_u}{1-\omega_y} = \frac{\rho}{\Lambda}$$

Hence, φ is positive and increasing in κ and decreasing in s. It follows that more nominal rigidity leads to more variability in the rate of inflation, while increases in nominal and real rigidity both contribute to higher output variability.

Note for aggregate shocks we have

$$\begin{aligned}
Var(\pi) &= \phi^2 \sigma_\varepsilon^2 \\
Var(y) &= \varphi^2 \sigma_u^2
\end{aligned}$$

whereas in the case of country-specific shocks we have

$$\begin{aligned}
Var(\pi) &= \phi^2 \left[(\nu_1)^2 + (\nu_2)^2 \right] \sigma_\varepsilon^2 \\
Var(y) &= \varphi^2 \left[(\nu_1)^2 + (\nu_2)^2 \right] \sigma_u^2
\end{aligned}$$

ie. asymmetry in size is in itself a source of increased variance.

C2: Asymmetric labour markets $N = 2$

Inflation

$$\pi_t = \nu_1 \phi_1 \varepsilon_{1t} + \nu_2 \phi_2 \varepsilon_{2t}$$

where

$$\begin{aligned}
\phi_1 &= \frac{-\lambda_{41}}{v_1\lambda_{11} + v_2\lambda_{12}} \\
&= -\frac{\beta + \theta\Lambda_2}{\nu_1\beta\kappa_1(\beta+\theta\Lambda_2) + \nu_2\beta\kappa_2(\beta+\theta\Lambda_1)} < 0 \\
\phi_2 &= \frac{-\lambda_{42}}{v_1\lambda_{11} + v_2\lambda_{12}} \\
&= -\frac{\beta + \theta\Lambda_1}{\nu_1\beta\kappa_1(\beta+\theta\Lambda_2) + \nu_2\beta\kappa_2(\beta+\theta\Lambda_1)} < 0
\end{aligned}$$

For later reference note

$$\nu_1\phi_1 + \nu_2\phi_2 = -\frac{\nu_1(\beta+\theta\Lambda_2) + \nu_2(\beta+\theta\Lambda_1)}{\nu_1\beta\kappa_1(\beta+\theta\Lambda_2) + \nu_2\beta\kappa_2(\beta+\theta\Lambda_1)} < 0$$

where

$$\begin{array}{rclcl}
\frac{\partial(\nu_1\phi_1+\nu_2\phi_2)}{\partial s_1} & = & sign(\kappa_2-\kappa_1) & ; & \frac{\partial(\nu_1\phi_1+\nu_2\phi_2)}{\partial \kappa_1} \gtreqless 0 \\
\frac{\partial(\nu_1\phi_1+\nu_2\phi_2)}{\partial s_2} & = & sign(\kappa_2-\kappa_1) & ; & \frac{\partial(\nu_1\phi_1+\nu_2\phi_2)}{\partial \kappa_2} \gtreqless 0
\end{array}$$

Hence, if $\kappa_1 \geq \kappa_2$ and $s_1 \leq s_2$ then

$$|\phi_1| \leq |\phi_2|$$

We have

$$\begin{array}{c}
\frac{\partial\phi_1}{\partial\kappa_1} \gtreqless 0, \frac{\partial\phi_1}{\partial s_1} > 0, \frac{\partial\phi_1}{\partial\kappa_2} \gtreqless 0, \frac{\partial\phi_1}{\partial s_2} < 0 \\
\frac{\partial\phi_2}{\partial\kappa_1} \gtreqless 0, \frac{\partial\phi_2}{\partial s_1} < 0, \frac{\partial\phi_2}{\partial\kappa_2} \gtreqless 0, \frac{\partial\phi_2}{\partial s_2} > 0
\end{array}$$

The variability of inflation is in the case of aggregate shocks given as

$$Var(\pi_t) = [\nu_1\phi_1 + \nu_2\phi_2]^2 \sigma_\varepsilon^2$$

For the country-specific shocks we have

$$Var(\pi_t) = \left[(\nu_1\phi_1)^2 + (\nu_2\phi_2)^2\right] \sigma_\varepsilon^2$$

note that variability has the lowest variance if the largest country has the most nominal rigid wages (i.e. if ν_1 high, $|\Theta_1|$small).

Aggregate output
Turning to aggregate output we have

$$y_t = \nu_1\varphi_1 u_{1t} + \nu_2\varphi_2 u_{2t}$$

where

$$\begin{array}{rcl}
\varphi_1 & = & \rho\frac{\omega_{1u}}{1-\sum_{n=1}^{2} v_n\omega_{ny}} = \rho\frac{\frac{\theta}{\beta+\theta\Lambda_1}}{1-\nu_1\frac{\beta}{\beta+\theta\Lambda_1}-\nu_2\frac{\beta}{\beta+\theta\Lambda_2}} \\
 & = & \rho\frac{\frac{\theta}{\beta+\theta\Lambda_1}}{\nu_1\frac{\theta\Lambda_1}{\beta+\theta\Lambda_1}+\nu_2\frac{\theta\Lambda_2}{\beta+\theta\Lambda_2}} = \rho\frac{\theta(\beta+\theta\Lambda_2)}{\nu_1\theta\Lambda_1(\beta+\theta\Lambda_2)+\nu_2\theta\Lambda_2(\beta+\theta\Lambda_1)} \\
\varphi_2 & = & \rho\frac{\frac{\theta}{\beta+\theta\Lambda_2}}{\nu_1\frac{\theta\Lambda_1}{\beta+\theta\Lambda_1}+\nu_2\frac{\theta\Lambda_2}{\beta+\theta\Lambda_2}} = \rho\frac{\theta(\beta+\theta\Lambda_1)}{\nu_1\theta\Lambda_1(\beta+\theta\Lambda_2)+\nu_2\theta\Lambda_2(\beta+\theta\Lambda_1)}
\end{array}$$

$$\frac{\partial \varphi_1}{\partial \Lambda_1} = -\rho \frac{\theta (\beta + \theta\Lambda_2) [\nu_1 \theta (\beta + \theta\Lambda_2) + \nu_2 \theta \Lambda_2 \theta \Lambda_1]}{[\nu_1 \theta \Lambda_1 (\beta + \theta\Lambda_2) + \nu_2 \theta \Lambda_2 (\beta + \theta\Lambda_1)]^2} < 0$$

$$\begin{aligned} \frac{\partial \varphi_1}{\partial \Lambda_2} &= \rho \frac{\theta\theta [\nu_1 \theta \Lambda_1 (\beta + \theta\Lambda_2) + \nu_2 \theta \Lambda_2 (\beta + \theta\Lambda_1)] - \theta (\beta + \theta\Lambda_2) [\nu_1 \theta \Lambda_1 \theta + \nu_2 \theta (\beta + \theta\Lambda_1)]}{[\nu_1 \theta \Lambda_1 (\beta + \theta\Lambda_2) + \nu_2 \theta \Lambda_2 (\beta + \theta\Lambda_1)]^2} \\ &= \rho \frac{[\nu_1 \Lambda_1 - \nu_1 \Lambda_1] \theta^3 (\beta + \theta\Lambda_2) + [\theta \Lambda_2 (\Lambda_2 - 1) - \beta] \theta \nu_2 (\beta + \theta\Lambda_1)}{[\nu_1 \theta \Lambda_1 (\beta + \theta\Lambda_2) + \nu_2 \theta \Lambda_2 (\beta + \theta\Lambda_1)]^2} \\ &= \rho \frac{-\theta^2 \nu_2 (\beta + \theta\Lambda_1) \beta}{[\nu_1 \theta \Lambda_1 (\beta + \theta\Lambda_2) + \nu_2 \theta \Lambda_2 (\beta + \theta\Lambda_1)]^2} < 0 \end{aligned}$$

For later reference also note that

$$\nu_1 \varphi_1 + \nu_2 \varphi_2 = \rho \frac{\nu_1 \theta (\beta + \theta\Lambda_2) + \nu_2 \theta (\beta + \theta\Lambda_1)}{\nu_1 \theta \Lambda_1 (\beta + \theta\Lambda_2) + \nu_2 \theta \Lambda_2 (\beta + \theta\Lambda_1)}$$

and

$$\begin{aligned} \frac{\partial (\nu_1 \varphi_1 + \nu_2 \varphi_2)}{\partial \Lambda_1} &< 0 \\ \frac{\partial (\nu_1 \varphi_1 + \nu_2 \varphi_2)}{\partial \Lambda_2} &< 0 \end{aligned}$$

It is easily verified that if $\kappa_1 \geq \kappa_2$ and $s_1 \leq s_2$ then

$$\varphi_1 \geq \varphi_2$$

The variability of aggregate output is in the case of common shocks given as

$$Var(y_t) = [\nu_1 \varphi_1 + \nu_2 \varphi_2]^2 \sigma^2$$

and for country-specific shocks as

$$Var(y_t) = \left[(\nu_1 \varphi_1)^2 + (\nu_2 \varphi_2)^2\right] \sigma^2$$

Note in the latter case output variability is lowest if the largest country has the most flexible labour market (i.e. if ν_1 high, φ_1 low).

Country-specific output
Country-specific output reads

$$\begin{aligned} y_{1t} &= \omega_{1y} y_t + \omega_{1\pi} \pi_t + \omega_{1u} u_{1t} \\ &= \omega_{1y} [\nu_1 \varphi_1 u_{1t-1} + \nu_2 \varphi_2 u_{2t-1}] + \omega_{1\pi} [\nu_1 \phi_1 \varepsilon_{1t} + \nu_2 \phi_2 \varepsilon_{2t}] + \omega_{1u} u_{1t} \end{aligned}$$

in the case of common shocks we have

$$\begin{aligned} y_{1t} &= \omega_{1y} [\nu_1 \varphi_1 + \nu_2 \varphi_2] u_{t-1} + \omega_{1\pi} [\nu_1 \phi_1 + \nu_2 \phi_2] \varepsilon_t + \omega_{1u} u_t \\ &= \Gamma_{u1} u_{t-1} + \Gamma_{\varepsilon 1} \varepsilon_t \end{aligned}$$

where

$$\begin{aligned}\Gamma_{u1} &= \omega_{1y}\left[\nu_1\varphi_1+\nu_2\varphi_2\right]+\rho\omega_{1u}\\ \Gamma_{\varepsilon 1} &= \omega_{1\pi}\left[\nu_1\phi_1+\nu_2\phi_2\right]+\omega_{1u}\end{aligned}$$

Hence

$$Var(y_{1t})=\left(\Gamma_{u1}\right)^2\sigma_u^2+\left(\Gamma_{\varepsilon 1}\right)^2\sigma_\varepsilon^2$$

Inserting the respective coefficients we find

$$\begin{aligned}\Gamma_{u1} &= \left[\omega_{1y}\left[\nu_1\varphi_1+\nu_2\varphi_2\right]+\rho\omega_{1u}\right]\\ &= \frac{\rho\theta}{\beta+\theta\Lambda_1}\left[\beta\left[\frac{\nu_1\left(\beta+\theta\Lambda_2\right)+\nu_2\left(\beta+\theta\Lambda_1\right)}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}\right]+1\right]\\ &= \frac{\rho\theta}{\beta+\theta\Lambda_1}\left[\frac{\beta\nu_1\left(\beta+\theta\Lambda_2\right)+\beta\nu_2\left(\beta+\theta\Lambda_1\right)+\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}\right]\\ &= \rho\theta\left[\frac{\beta+\theta\Lambda_2}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}\right]>0\\ \Gamma_{u2} &= \frac{\rho\theta}{\beta+\theta\Lambda_2}\left[\beta\left[\frac{\nu_1\left(\beta+\theta\Lambda_2\right)+\nu_2\left(\beta+\theta\Lambda_1\right)}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}\right]+1\right]\\ &= \rho\theta\left[\frac{\beta+\theta\Lambda_1}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}\right]>0\end{aligned}$$

Note that $\Lambda_1<\Lambda_2$ implies $\Gamma_{u1}>\Gamma_{u2}$.Furthermore it follows that

$$\begin{aligned}\frac{\partial\Gamma_{u1}}{\partial\Lambda_1} &= \frac{\partial}{\partial\Lambda_1}\left[\rho\theta\left[\frac{\beta+\theta\Lambda_2}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}\right]\right]\\ &= -\rho\theta\frac{\left(\beta+\theta\Lambda_2\right)\left[\nu_1\theta\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\theta\right]}{\left[\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)\right]^2}<0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\Gamma_{u1}}{\partial\Lambda_2} &= \frac{\partial}{\partial\Lambda_2}\left[\rho\theta\left[\frac{\beta+\theta\Lambda_2}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}\right]\right]\\ &= \rho\theta\frac{\theta\left[\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)\right]-\left(\beta+\theta\Lambda_2\right)\left[\nu_1\theta\Lambda_1\theta+\nu_2\theta\left(\beta+\theta\Lambda_1\right)\right]}{\left[\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)\right]^2}\\ &= \rho\theta\frac{-\beta\nu_2\theta}{\left[\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)\right]^2}\left(\beta+\theta\Lambda_1\right)<0\end{aligned}$$

Turning to the Γ_ε-coefficients, we have

$$\begin{aligned}
\Gamma_{\varepsilon_1} &= \left[\omega_{1\pi}\left[\nu_1\phi_1+\nu_2\phi_2\right]+\omega_{1u}\right] \\
&= \frac{\theta}{\beta+\theta\Lambda_1}\left[1-\beta\kappa_1\left[\frac{\nu_1(\beta+\theta\Lambda_2)+v_2(\beta+\theta\Lambda_1)}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}\right]\right] \\
&= \theta v_2\beta\left[\frac{\kappa_2-\kappa_1}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}\right] \\
\Gamma_{\varepsilon 2} &= \left[\omega_{2\pi}\left[\nu_1\phi_1+\nu_2\phi_2\right]+\omega_{2u}\right] \\
&= -\frac{\theta\beta\kappa_2}{\beta+\theta\Lambda_2}\left[\frac{\nu_1\left(\beta+\theta\Lambda_2\right)+\nu_2\left(\beta+\theta\Lambda_1\right)}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}\right]+\frac{\theta}{\beta+\theta\Lambda_2} \\
&= \theta v_1\beta\left[\frac{\kappa_1-\kappa_2}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}\right]
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial\Gamma_{\varepsilon 1}}{\partial\kappa_1} &= \theta v_2\beta\left[\frac{-\left[\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)\right]-\left(\kappa_2-\kappa_1\right)\left[\nu_1\beta\left(\beta+\theta\Lambda_2\right)-\nu_2\beta\kappa_2\theta\beta s_1\right]}{\left[\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)\right]^2}\right] \\
&= \theta v_2\beta\left[\frac{-\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)-\kappa_2\nu_1\beta\left(\beta+\theta\Lambda_2\right)+(\kappa_2-\kappa_1)\nu_2\beta^2\kappa_2\theta s_1}{\left[\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)\right]^2}\right]
\end{aligned}$$

A sufficient condition that this is negative is $\kappa_2-\kappa_1<0$. Note that for $\kappa_2-\kappa_1<0$ we have $\Gamma_{\varepsilon 1}<0$ and hence the numerical value of $\Gamma_{\varepsilon 1}$ is increasing in κ_1.

$$\begin{aligned}
\frac{\partial\Gamma_{\varepsilon 1}}{\partial\kappa_2} &= \theta v_2\beta\left[\frac{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)-\left(\kappa_2-\kappa_1\right)\left[-\nu_1\beta\kappa_1\theta\beta s_2+\nu_2\beta\left(\beta+\theta\Lambda_1\right)\right]}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}\right] \\
&= \theta v_2\beta\left[\frac{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+(\kappa_2-\kappa_1)\nu_1\beta\kappa_1\theta\beta s_2+\kappa_1\nu_2\beta\left(\beta+\theta\Lambda_1\right)}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}\right]
\end{aligned}$$

A sufficient condition that this is positive is $\kappa_2-\kappa_1>0$. Note that for $\kappa_2-\kappa_1>0$ we have $\Gamma_{\varepsilon 1}>0$ and hence the numerical value of $\Gamma_{\varepsilon 1}$ is increasing in κ_1.

It is easily verified that

$$\nu_1\left[\omega_{1\pi}\left[\nu_1\phi_1+\nu_2\phi_2\right]+\omega_{1u}\right]+\nu_2\left[\omega_{2\pi}\left[\nu_1\phi_1+\nu_2\phi_2\right]+\omega_{2u}\right]=0$$

which implies that current innovations to the shock do not have any aggregate consequences.

Comparing variances of output we have that

$$Var(y_{1t})=\left(\Gamma_{u1}\right)^2\sigma_u^2+\left(\Gamma_{\varepsilon 1}\right)^2\sigma_\varepsilon^2>Var(y_{2t})=\left(\Gamma_{u2}\right)^2\sigma_u^2+\left(\Gamma_{\varepsilon 2}\right)^2\sigma_\varepsilon^2$$

holds for $\Lambda_1<\Lambda_2$ under the sufficient condition that $\nu_1\leq\nu_2$.

We have that $sign\frac{\partial\Gamma_{\varepsilon 1}}{\partial s_1}=sign(\kappa_1-\kappa_2)$, $sign\frac{\partial\Gamma_{\varepsilon 1}}{\partial s_2}=sign(\kappa_1-\kappa_2)$, since

$$\begin{aligned}
&\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right) \\
=\;&\nu_1\beta\kappa_1\left(\beta+\theta\left(1+\beta(1-\kappa_2)s_2\right)\right)+\nu_2\beta\kappa_2\left(\beta+\theta\left(1+\beta(1-\kappa_1)s_1\right)\right)
\end{aligned}$$

is increasing in s_1, and s_2.

Country specific output - country specific shocks

Country-specific output reads

$$\begin{aligned} y_{1t} &= \omega_{1y}y_t + \omega_{1\pi}\pi_t + \omega_{1u}u_{1t} \\ &= \omega_{1y}\left[\nu_1\varphi_1 u_{1t-1} + \nu_2\varphi_2 u_{2t-1}\right] + \omega_{1\pi}\left[\nu_1\phi_1\varepsilon_{1t} + \nu_2\phi_2\varepsilon_{2t}\right] + \omega_{1u}u_{1t} \\ &= \Upsilon_{u1}u_{1t-1} + \Upsilon_{u2}u_{2t-1} + \Upsilon_1\varepsilon_{1t} + \Upsilon_2\varepsilon_{2t} \end{aligned}$$

where

$$\begin{aligned} \Upsilon_{1u1} &= \omega_{1y}\nu_1\varphi_1 + \rho\omega_{1u} = \nu_1\frac{\beta}{\beta+\theta\Lambda_1}\rho\frac{\theta\left(\beta+\theta\Lambda_2\right)}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)} + \rho\frac{\theta}{\beta+\theta\Lambda_1} \\ &= \left[\frac{\theta\left(\beta+\theta\Lambda_2\right)\nu_1\beta}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)}+1\right]\frac{\rho\theta}{\beta+\theta\Lambda_1} \\ \Upsilon_{1u2} &= \omega_{1y}\nu_2\varphi_2 = \nu_2\frac{\beta}{\beta+\theta\Lambda_1}\rho\frac{\theta\left(\beta+\theta\Lambda_1\right)}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)} \\ &= \frac{\theta\nu_2\beta\rho}{\nu_1\theta\Lambda_1\left(\beta+\theta\Lambda_2\right)+\nu_2\theta\Lambda_2\left(\beta+\theta\Lambda_1\right)} \\ \Upsilon_{1\varepsilon1} &= \omega_{1\pi}\left[\nu_1\phi_1+\omega_{1u}\right] = \frac{\theta\beta\kappa_1}{\beta+\theta\Lambda_1}\left[-\frac{v_1\left(\beta+\theta\Lambda_2\right)}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}+\rho\frac{\theta}{\beta+\theta\Lambda_1}\right] \\ &= \frac{\theta\beta\kappa_1}{\beta+\theta\Lambda_1}\left[\frac{-\nu_1\left(\beta+\theta\Lambda_2\right)\left(\beta+\theta\Lambda_1\right)+\rho\theta\left[\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)\right]}{\left(\beta+\theta\Lambda_1\right)\left[\nu_1\left(\beta+\theta\Lambda_2\right)\beta\kappa_1+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)\right]}\right] \\ &= \frac{\theta\beta\kappa_1}{\beta+\theta\Lambda_1}\left[\frac{\left[-\nu_1\left(\beta+\theta\Lambda_1\right)+\rho\theta\nu_1\beta\kappa_1\right]\left(\beta+\theta\Lambda_2\right)+\rho\theta\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)}{\left(\beta+\theta\Lambda_1\right)\left[\nu_1\left(\beta+\theta\Lambda_2\right)\beta\kappa_1+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)\right]}\right] \\ \Upsilon_{1\varepsilon2} &= \nu_2\phi_2 = -\frac{v_2\left(\beta+\theta\Lambda_1\right)}{\nu_1\beta\kappa_1\left(\beta+\theta\Lambda_2\right)+\nu_2\beta\kappa_2\left(\beta+\theta\Lambda_1\right)} \end{aligned}$$

Hence

$$Var(y_{1t}) = \left[\left(\Upsilon_{u1}\right)^2+\left(\Upsilon_{u2}\right)^2\right]\sigma_u^2 + \left[\left(\Upsilon_1\right)^2+\left(\Upsilon_2\right)^2\right]\sigma_\varepsilon^2$$