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**DOES THE LENGTH OF THE  
PERIOD REALLY MATTER FOR  
THE IDENTIFICATION AND THE  
MODELLING OF MONETARY  
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***INTERNATIONAL MACROECONOMICS***



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# DOES THE LENGTH OF THE PERIOD REALLY MATTER FOR THE IDENTIFICATION AND THE MODELLING OF MONETARY POLICY SHOCKS?

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## **ABSTRACT**

### **Does the Length of the Period Really Matter for the Identification and the Modelling of Monetary Policy Shocks?**

In this Paper, we ask whether our empirical and theoretical knowledge about the effect of monetary policy shocks is robust to the choice of the period length. We think that such a question is particularly relevant in the monetary literature, as frictions are often introduced under the form of a one-period lag in agents' reaction. We first show that it is possible to use more efficiently the available information when identifying monetary policy shocks. Using together quarterly series for GDP and monthly series for monetary aggregates and interest rates, it is possible to identify monetary shocks with the assumption that they do not have any impact on GDP within a month, by restricting ourselves to the identification of third-month-of-a-quarter shocks. With this new method, we obtain very similar estimated IRFs, as compared with the results obtained with quarterly data, although the price puzzle appears to be more pronounced in our estimates. Such a similarity is a new fact that quantitative models need to match. In the second part of the Paper, we propose a model-based explanation for this result, by computing a limited participation model predictions, when the time period is reduced from one quarter to one month, and when the model predictions are time-aggregated at the quarterly frequency. We show that the introduction of adjustment costs to portfolio reallocation into the model is not only improving its fit, but is necessary for obtaining qualitatively realistic predictions, when the length of the period is thought to be the month and not the quarter.

JEL Classification: E40 and E50

Keywords: limited participation, monetary policy shocks and time aggregation

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# Does the Length of the Period Really Matter for the Identification and the Modelling of Monetary Policy Shocks?

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April 2004

## Abstract

In this paper, we ask whether our empirical and theoretical knowledge about the effect of monetary policy shocks is robust to the choice of the period length. We think that such a question is particularly relevant in the monetary literature, as frictions are often introduced under the form of a one-period lag in agents' reaction. We first show that it is possible to use more efficiently the available information when identifying monetary policy shocks. Using together quarterly series for GDP and monthly series for monetary aggregates and interest rates, it is possible to identify monetary shocks with the assumption that they do not have any impact on GDP within a month, by restricting ourselves to the identification of third-month-of-a-quarter shocks. With this new method, we obtain very similar estimated IRFs, as compared with the results obtained with quarterly data, although the price puzzle appears to be more pronounced in our estimates. Such a similarity is a new fact that quantitative models need to match. In the second part of the paper, we propose a model-based explanation for this result, by computing a limited participation model predictions, when the time period is reduced from one quarter to one month, and when the model predictions are time-aggregated at the quarterly frequency. We show that the introduction of adjustment costs to portfolio reallocation into the model is not only improving its fit, but is necessary for obtaining qualitatively realistic predictions, when the length of the period is thought to be the month and not the quarter.

**Key Words :** Time Aggregation – Monetary Policy Shocks – Limited Participation Model

**JEL Classification :** E4, E5

## 1 Introduction

A minimal level of consensus has been reached among economists on how to identify and model monetary policy. As of identification, Bernanke and Blinder [1992], Christiano and Eichenbaum [1992b], Strongin [1995], Christiano, Eichenbaum, and Evans [1996] and Bernanke and Mihov [1999] have extended the initial Sims [1972] and Sims [1980] Vectorial Auto Regressive approach to monetary shocks identification. As shown by Christiano, Eichenbaum, and Evans [1999] in the Handbook of Macroeconomics, identification is broadly speaking relying on some particular recursiveness assumption in the

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orthogonalization of the VAR innovations. In words, by assuming that some variables do not respond to some shocks *within a period*, one identifies monetary policy shocks. We shall come back later to the italicized words<sup>1</sup>. Relatively robust results are obtained from those studies: a contractionary monetary policy shock increases the Federal Funds rate, decreases Non Borrowed Reserves, Total Reserves and M1, and have a small but negative initial effect on the price level and on output.

As of modelling, macroeconomists are taking those monetary “facts” as a benchmark to be confronted with the predictions of their artificial Dynamic General Equilibrium economies. First generation of “cash-in-advance” monetary flex-price models (Cooley and Hansen [1989]) were not successful in reproducing the recessionary effect of a monetary contraction, while first sticky price models (Hairault and Portier [1993] Yun [1996], Cho and Cooley [1994]) failed at reproducing the so-called liquidity effect, *i.e.* the increase in nominal interest rate following a monetary contraction. A second generation of models that put together nominal frictions on financial markets (limited participation) and/or stickiness in prices and wages adjustment (Rotemberg and Woodford [1998], Christiano, Eichenbaum, and Evans [2001], Ireland [1997]) is more successful in modelling the dynamic effect of monetary policy shocks. In those models, the fact that prices (wages) are set *for n periods* or that portfolio allocation is decided *one-period in advance* is crucial for the quantitative properties of the model. Again, the italics call for some justification. This is what we discuss now.

The fact that the length of the period is an important choice has been recognized for a long time. It matters both for the empirical analysis and for the theory. Let us first consider the empirical analysis. Economists do not choose the sampling frequency of the data, and this is a constraint they have to deal with. GDP is only available on a quarterly basis. Therefore, it seems unavoidable that any empirical work that imposes a zero impact effect of monetary shock on GDP is *de facto* imposing a three month lag for money to affect output. This is clearly a strong assumption, that is well acknowledged for in the literature. For example, Bernanke and Mihov [1999] mentioned the

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<sup>1</sup>We should also mention some recent work of Canova and de Nicolò [2000] and Uhlig [1999] that use sign restrictions rather than imposing zeros. Again, *for how many periods* the sign restriction has to be imposed is an important choice.

fact that “because of our identifying assumption is that there is no feedback from policy variables to the economy within the period, the length of “the period” is potentially important”. Many authors have therefore checked the robustness of their results using monthly (or even weekly) proxies for GDP (Industrial Production Index, interpolated GDP, Business Week production index, etc...). Results seem to be robust, although we do not know really what are we comparing, given that the information set is not the same. Even though we share the common wisdom that “it should not change that much the results if we could do the exercise with monthly GDP”, we would prefer a formal check. The first contribution of this paper is show that we can do this exercise by simply using more efficiently the available information, namely by mixing frequencies. The intuition is the following: observing GDP at the quarter does not impose using only quarterly averages for some other variables that are available at a higher frequency (interest rates, monetary aggregates,..). Using a simple mixed frequency VAR that does not add any other assumptions to the ones of the literature, we can test the consequences of reducing the length of the zero effect constraint *ceteris paribus*, that is to say without interpolating GDP or switching to some proxy of aggregate activity. The simple idea we implement is to identify only some monthly monetary shocks, namely third-month-of-a-quarter ones in the monthly case. Such a shock is only restricted to have a zero impact on current quarterly GDP, but not on next quarter GDP, meaning that it can affect GDP after one month. We insist on the fact that such a method does not put more constraints on the data than a quarterly VAR does, but simply fully exploits the fact that some variables are sampled at a higher frequency than GDP, and that such an information should not be discarded. The result of this exercise is that mixed-frequency estimated IRFs are very similar to the quarterly estimated ones, , although the price puzzle appears to be more pronounced in the mixed-frequency exercise. This similarity is providing us information on which model we should use in quantitative work. If the true model were a monthly one, the quarterly estimated VAR would mix impulse and propagation mechanisms, and should be *a priori* different from the results of a well identified procedure.

Can this result be understood with a structural monetary model? This is the modelling question we ask in the second part of the paper. Again, it is well-known that sampling frequency needs not to be the frequency at which agents take decisions in reality and in models. The drawbacks of such an assumption have been emphasized by Christiano, Eichenbaum, and Marshall [1991], when considering a Permanent Income model. More recently, Aadland [2001] has shown that labor market properties of a Real Business Cycle model were dependant on the period length. It is nevertheless the case that almost all the literature on monetary DGE assume that agents take decisions at the quarter (Chari, Christiano, and Eichenbaum [1995] is a notable exception, but no time aggregated predictions are derived). Therefore, the same caveat that the one underlined by Bernanke and Mihov [1999] for VARs applies. The nominal frictions that are introduced in monetary DGE models are imposing some lags/adustment costs in some decision taking (price or wage setting, portfolio allocation, available money balances, *etc...*). One should not impose a lower bound on those frictions by setting *a priori* the length of the period as this choice has non trivial implications. If one agrees that a month is a more reasonable assumption for pre-determination of prices or portfolios, do we still get reasonable predictions when looking at the quarterly implications of the model? Contrarily to the VARs, the common wisdom is that the period length matters for the model predictions once aggregated at the quarterly frequency. Considering as a benchmark economy a simple limited participation model (in the lines of Christiano and Eichenbaum [1992b] and Christiano, Eichenbaum, and Evans [1998]), we show that the model's predictions are robust to time aggregation only when the model has strong propagation mechanisms. In such a case, it is possible to explain the similarity of IRFs obtained in the VAR.

The paper is organized as follows. Section 2 presents our new estimates of monetary shocks impulse responses using mixed frequency VARs and compares them with quarterly VARs ones. Section 3 shows how the VAR results should be interpreted, as read through the lens of a simple limited participation model. Section 4 concludes.



## 2 Monetary Policy Shocks Identification Using Monthly Data

The VAR methodology is a very popular one to identify the effect of monetary policy (see Christiano, Eichenbaum, and Evans [1999] (hereafter CEE) for a survey). In this section we first briefly recall this methodology, taking as a benchmark the CEE's specification. We then expose how we can relax the zero-impact-during-one-quarter identifying assumption for the monetary shock if we use the data available at higher frequency. Our method is simple to implement, use efficiently the available information and results are broadly in line with the literature, although we do observe a more pronounced price puzzle.

### 2.1 The State of the Art

This paragraph is directly inspired from CEE. We start from the assumption that the Central Bank implements its monetary policy using a reaction function. At each period  $t$ , the policymaker sets its instrument,  $S_t$ , in a systematic way, in relation with the information set of the period,  $\Omega_t$ . The monetary policy rule can be written:

$$S_t = f(\Omega_t) + \sigma_s \epsilon_s^t \quad (1)$$

where  $S_t$  is the instrument of the Central Bank and  $f(\cdot)$  is a linear function that relates the instrument to the information set of the central bank  $\Omega_t$ .  $\epsilon_s^t$  defines the monetary policy shocks, with standard deviation  $\sigma_s$ .

To estimate both the monetary policy shocks and their effects on other macroeconomics variables, one assumes that the variables of interest - including the instruments of the monetary policy -  $\{Z_t, t = 1, \dots, T\}$  follow a VAR of order  $q$ :

$$A_0 Z_t = A_1 Z_{t-1} + \dots A_q Z_{t-q} + \epsilon_t \quad (2)$$

This is the structural representation of the VAR. The structural shocks  $\epsilon_t$ , which include the monetary policy shocks, are by construction orthogonal one to each others.  $A_0$ , the matrix of contemporaneous impact, is assumed to be invertible. Pre-multiplying (2) by  $A_0^{-1}$ , one obtains the reduced VAR

representation:

$$Z_t = B_1 Z_{t-1} + \dots B_q Z_{t-q} + u_t \quad (3)$$

where  $B(L) = A_0^{-1}A(L)$  and  $u_t = A_0^{-1}\epsilon_t$  has covariance matrix  $V$ .  $u_t$  are uncorrelated with lagged values of  $Z_t$ , and the parameters  $\{B_1, \dots, B_q, V\}$  can be estimated consistently using Ordinary Least Squares method.

To identify fundamental shocks in the economy and particularly the monetary shock, a so called recursive approach is followed. Assume that the matrix of contemporaneous impacts  $A_0$  is lower triangular and recall that structural shocks are orthogonal. One can recover  $A_0^{-1}$  by the Cholevsky decomposition of  $V$ . Therefore, one has a recursive system which depends on the order of the variables in  $Z_t$ . Let's decompose  $Z_t$  in three groups, according to the location of the instrument  $S_t$ :

$$Z_t = \begin{pmatrix} X_t^1 \\ S_t \\ X_t^2 \end{pmatrix}$$

The recursiveness assumption implies that when the central bank sets its instrument  $S_t$ , it does not observe the contemporaneous values  $X_t^1$ , but does observe  $X_t^2$ . Another implication is that the variables in  $X_t^1$  react to a monetary policy shock with a delay of one period. CEE estimate such a specification for the US economy. In their benchmark VAR, the instrument used by the central bank is the three months federal fund rate;  $X_t^1$  is composed of GDP, GDP deflator and an Index of Crude Good price; while  $X_t^2$  regroups the variables actually used by the Federal Reserve bank to control the level of the Federal Fund rate: Non Borrowed Reserve, Total Reserve and M1. After the estimation of the parameters of the structural VAR, one can compute impulse response functions to a monetary policy shock.

Because GDP and its deflator are only available each quarter, CEE estimate the VAR using all variables at this frequency. When one considers the reaction function (1), this implies that the central bank reacts only at quarterly frequency. And the identification scheme imposes that variables  $X_t^1$  react to the monetary policy with a delay of one quarter. This assumption is unlikely to hold in the real world.

Assume that in the real world, such a delay is one month long. If we use quarterly data, we will make two types of errors. First, a misspecification one as we will omit a MA component in the quarterly aggregated VAR. If the equation representation of the reaction function is supposed to be true at the monthly frequency, we add autocorrelation in the error term when estimating it in quarterly frequency. And we will estimate a combination of past, present and future monetary policy shocks, instead of the monetary policy shock alone. Second, we will make an identification error. The recursiveness assumption assumes that the set of variables  $X_t^1$  reacts to the monetary policy with a delay of one quarter, while it does react within the quarter if the true model is a monthly one. Therefore, the estimated impulse of  $X_t^1$  will include part of the propagation mechanism.

Given that the frequency choice is an important one, and that the data we use to estimate monetary policy and its effects are available at different sampling frequencies, why don't we use all the information available? And if we do, how should we do it?

There are three ways to estimate such a VAR when data are not all available at the highest frequency: aggregation, interpolation and mixed frequency estimation. The first possibility is to take all variables at the lowest frequency. That implies a loss of information and requires the estimation of an ARMA. The second possibility is to take all the variables at the highest frequency. This method needs to add some extra identifying assumptions to be able to interpolate variables. The third is the one we promote: use all the information available but do not add any extra identifying assumptions. Zadrozny and Baolue [1998] propose a way to estimate a model with mixed frequency data. The reference frequency is the highest frequency. The variables sampled at the lowest frequency are considered as series with missing observations, and a Kalman filter is used to estimate such a VAR with missing observations. This method is complex, and requires much more technical skills than the one needed to estimate a VAR. We will show that in the case of monetary policy shocks identification, we can keep the tractability of the VAR approach (that has been part of its success in economic administrations and central banks), and still use more efficiently the available information.

## 2.2 An Alternative Methodology: VAR in mixed frequency

The approach we present now is simple. We simply impose the fact that monetary policy does not affect real variables during a month, instead of one quarter.

To illustrate the methodology, let us consider a simple VAR with only three variables: real GDP  $Y$ , Federal Fund Rate  $FF$  and monetary base  $M1$  (the empirical implementation will be done with a larger VAR). Suppose first that those three variables are available monthly, and that they are ordered in the following way:

$$Z_t = \begin{pmatrix} Y_t \\ FF_t \\ M1_t \end{pmatrix}$$

If we use the recursiveness assumption of CEE, we obtain the following decomposition of the contemporaneous parameters:

$$A_0 = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

The zeros on the first row mean that  $Y_t$  does not react contemporaneously to  $FF_t$  and  $M1_t$  shocks.

The zero of the middle row means that the policy maker does not observe  $Y_t$  when setting  $FF_t$ .

Suppose now the  $Y$  is observable only every two months, while  $FF$  and  $M1$  are observable every month. Let us write a mixed frequency vector  $Z_\tau$  that preserves the ordering,

$$Z_\tau = \begin{pmatrix} Y_\tau = Y_t + Y_{t+1} \\ FF_t \\ M1_t \\ FF_{t+1} \\ M1_{t+1} \end{pmatrix} \quad t = 1, 3, 5, \dots, T-1, \quad \tau = \frac{t+1}{2} \quad \text{and } T \text{ even}$$

The subscript  $\tau$  is used for bi-monthly observations,  $t$  for first-month-of-the-two-months-period observations and  $t+1$  for second-month-of-the-two-months-period ones. To solve the identification problem, let us use the same recursiveness hypothesis as in the monthly case:

$$A_0 = \begin{pmatrix} a_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} & 0^1 & 0^1 \\ a_{21} & a_{22} & 0^2 & 0^3 & 0^3 \\ a_{31} & a_{32} & a_{33} & 0^3 & 0^3 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0^2 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix}$$

The two zeros ( $0^1$ ) of the first line mean that  $Y$  does not react contemporaneously to shocks to  $FF$  or  $M1$ . It implies that  $Y_t + Y_{t+1}$  does not react to shocks to  $FF_{t+1}$  and to  $M1_{t+1}$ . The zeros ( $0^2$ )

correspond to the assumption that the  $FF$  does not react to  $M1$  contemporaneously. Finally the zeros ( $0^3$ ) correspond to the assumption that time is irreversible: contemporaneous values does not respond to future innovations. In other words,  $FF_t$  and  $M1_t$  do not react to shocks to  $FF_{t+1}$  and  $M1_{t+1}$ .

Of course, we do not have enough restrictions to identify  $\mathbf{a}_{12}$  and  $\mathbf{a}_{13}$ . Those two parameters drive the response of  $Y_{t+1}$  to  $FF_t$  and  $M1_t$ . This point is straightforward: we have less information than if everything were observable in monthly data, and we therefore need more identifying assumptions if we want to recover all the structural innovations. But we do not need those parameters to be able to identify *some* monetary policy shocks and their impact on all the variables. We can compute the impulse response functions for all variables to a shock on  $FF_{t+1}$ , for  $t = 1, 3, 5, \dots$ . In words, we can estimate the response function to a monetary policy shock, *restricting ourselves to shocks that occur the last month of every reference period* (bi-monthly in this example). We can then calculate the impact on all the variables of the system.

This specification allows us to use all the information that we have, without adding more assumptions than the ones needed when estimating the VAR with time aggregated data. In the next subsection, we apply this method to the same data than CEE, therefore mixing monthly and quarterly variables.

### 2.3 Implementation on US data

We extend the method presented above to a bigger mixed frequency model, with monthly and quarterly variables, to identify US monetary policy shocks. We take CEE's monetary policy VAR as benchmark. The variables we use are the real GDP, the GDP deflator, a PPI component (crude material for further processing), the Federal Fund Rate (FF), Non Borrowed Reserve, Total Reserve and M1. All variables are in log, except the Federal Fund Rate. We consider FF as the monetary policy instrument, and then defined the monetary policy shocks as the disturbances on FF, using the recursiveness identification procedure. The GDP and its deflator are available only each quarter. The others variables exist in

monthly frequency.<sup>2</sup> The period of estimation goes from 1960:Q1 to 1999:Q2.

Figures (2 and 1 report the estimated IRFs for all the variables after a contractionary monetary policy shock — that is a positive shock on the federal fund rate. In both cases, the monetary policy shock has been calibrated such that Non Borrowed Reserves drop by one percent at the impact. The solid line reports the point estimates of the various dynamic response functions. The dashed lines correspond to the 95 per cent confidence interval obtained through Monte-Carlo simulations<sup>3</sup>.

The first column is the benchmark case (estimated with only quarterly data), while the second column is the mixed frequency case, with a shock to  $FF$  in the third month of a quarter. The IRFs reported are measured in quarters (monthly responses for the mixed frequency case have been time-aggregated).

Figure 1 shows that our procedure does identify the same type of shock than in the quarterly VAR<sup>4</sup>: the Federal Fund Rate increases on impact and stays above its non shocked level for at least 10 quarters, Non Borrowed Reserves are reduced by 1% on impact (by normalization), and stay below their non shocked level for at least 3 quarters. It is hard to find any significant movements of Total Reserves and M1 in both the quarterly and the mixed frequency VARs. Let us turn now to the impact of a monetary contraction on prices and quantities, as shown on Figure 2. We do observe an output contraction of the same order of magnitude in both VARs, although the response is not significantly different from zero after two quarters in the mixed-frequency VAR. The responses of prices is more problematic, as the price puzzle is extremely pronounced in the mixed frequency VAR. In the quarterly VAR, a crude material price that controls for monetary authorities reactions to price increases is included, which allows for the elimination of the price puzzle originally found by Sims [1992]. It seems that this variable does not do such a good job in the mixed-frequency case. If we

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<sup>2</sup>We use Federal Reserve economic data series, as obtained from <http://www.stls.frb.org/fred/>.

<sup>3</sup>From the VAR model, we obtain an estimate of the Data Generating Process (DGP) of the  $Z_t$  vector — that is the estimations of the parameters  $\hat{A}$ . This DGP is used to simulate  $I=1000$  realizations,  $\{Z_t(i)\}_{i=1}^I$ , of the initial vector of time series  $Z_t$ , by sampling from the estimated residuals. Using these sets of simulated time series, we re-estimate the model, and obtain a set of new parameters  $\{\hat{A}(i)\}_{i=1}^I$ , from which we can derive a simulated distribution of the IRFs. The confidence intervals are obtained by taken for all lags the 24<sup>th</sup> and the 974<sup>th</sup> simulated IRF.

<sup>4</sup>It should be noted that the responses are less smooth in the mixed-frequency VAR, as we allow for a larger number of monthly lags.

use a smoothed index of commodity prices (*PCOM*), as in Christiano, Eichenbaum, and Evans [1999] (which is unfortunately only available on a shorter sample (1960:1-1995:3), we eliminate most of the price puzzle, even in the mixed frequency VAR (see figure 3). All in all, our conclusion is that results from the mixed frequency VAR are very similar to the quarterly VAR ones.

## 2.4 Discussion

Are our results surprising? A first answer would be no. Among others, Bernanke and Mihov [1999] have shown that very similar IRFs are obtained when using a monthly specification with monthly proxies for the variables that are not available at a monthly frequency (for example using an industrial production index instead of GDP). The conclusion is that, even if the quarterly VAR is likely to be misspecified, the identification scheme is robust<sup>5</sup>. We want to go further and explore the models implication of such a result. In a world in which monetary shocks occur every month, a quarterly VAR provides us with responses that mix impulse and propagation mechanisms. What does the result tell us is that it is not important to confuse impulse and propagation mechanisms because they are essentially going in the same way. This is a useful information, as many monetary models could be rejected along this dimension. This is what we show in the next section in the case of a simple limited participation model with or without adjustment costs to portfolio reallocations.

## 3 Monetary Policy Shocks Using Monthly Data Models

In this section, we briefly present a canonical limited participation model of monetary policy, and show that portfolio reallocations adjustment costs are necessary to understand the similarity between quarterly and monthly-quarterly estimated IRFs to a monetary policy impulse.

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<sup>5</sup>We have tried to go further in the mixed frequency VAR by using weekly data for interest rates and some monetary aggregates. Unfortunately, multicollinearity problems occur as imposing 4 lags for quarterly GDG amounts to imposing 16 lags for a weekly interest rate. The solution would then be to estimate SURE systems with different lags for different variables. As we were aiming at comparison with the literature, we did not pursue this route any further in this paper, and leave it for future research.

### 3.1 A Limited Participation Model

We consider here a simple version of a Limited Participation model, in the lines of Christiano and Eichenbaum [1992a] and Christiano, Eichenbaum, and Evans [1998]. Let us briefly present the model. There are three agents in the economy: a representative household, a representative firm and a representative financial intermediary. The only source of shocks is monetary policy. The important source of friction that allows monetary policy shocks to generate a liquidity effect is the fact that the household has to allocate her portfolio before the realization of the current shock. The explanation of this friction given by Christiano and Eichenbaum [1992a] is that a fraction of the disposable money stock owned by the households "is held by firms and financial intermediaries in the form of retained earnings or pension funds and cannot readily be allocated by households". The other explanation is fixed costs that prevent households to reallocate their portfolio continuously.

**Household :** At the beginning of the period the household holds predetermined stocks of money  $M_t$  and capital  $K_t$ . The allocation of money is also predetermined, and cannot react to shocks within the period:  $Q_t$  has been allocated to buy goods (consumption and investment) and  $M_t - Q_t$  has been put on a bank account, that serves a gross interest rate  $R_t$ . Consumption and investment should be paid with money that comes from current labor income  $W_t N_t$  and from  $Q_t$ . Changing the composition of the portfolio from one period to another is costly, and it is assumed that the cost incurred,  $H_t$ , is measured in unit of time.

The household program can be written in the following way :

$$\max V(K_t, M_t, Q_t) = \log \left\{ C_t - \psi_0 \frac{(N_t + H_t)^{(1+\psi)}}{1 + \psi} \right\} + \beta E_t [V(K_{t+1}, M_{t+1}, Q_{t+1})] \quad (4)$$

subject to

$$K_{t+1} \leq (1 - \delta)K_t + I_t \quad (5)$$

$$M_{t+1} \leq R_t(M_t - Q_t) + \Pi_t + Q_t + W_t L_t + z_t K_t - P_t(I_t + C_t) \quad (6)$$

$$P_t(C_t + I_t) \leq Q_t + W_t N_t \quad (7)$$



$$H_t = d \times \left( e^{c\left(\frac{Q_t}{Q_{t-1}} - 1 - x\right)} + e^{-c\left(\frac{Q_t}{Q_{t-1}} - 1 - x\right)} \right) \quad (8)$$

$\Pi_t$  represents the profit of firms and banks that is redistributed to the household,  $z_t$  is the rental rate of capital and  $P_t$  the monetary price of consumption and investment. The household is assumed to hold capital and rent it to the firm at a rate  $z_t$ . Equation (6) describes the evolution of the household's stock of money, (7) is the cash in advance constraint and (8) gives the parametric form chosen for adjustment costs, with  $x$  being the growth rate of  $Q$  along a deterministic steady growth path.

**Firm :** Given a production function (9), the firm equalizes factor prices with marginal products in order to maximize profits (equations (10) and (11)). The supplementary constraint faced by the firm is that it must pay workers in advance of production. This is done by borrowing the wage bill from the bank, at the gross interest rate  $R_t$ . The effective marginal cost of labor is therefore not  $W_t$  but  $R_t W_t$ .

$$Y_t = K_t^\alpha L_t^{(1-\alpha)} \quad (9)$$

$$z_t = \alpha Y_t / K_t \quad (10)$$

$$R_t W_t = (1 - \alpha) Y_t / L_t \quad (11)$$

**Banks :** The financial intermediary has two sources of funds: lump sum money injection  $X_t$  and household's deposit  $M_t - Q_t$ . Those funds are used to finance the firm's wage bill, according to a constant return to scale technology (equation (12)). It is assumed that  $X_t$  is such that money growth rate follows an  $AR(1)$  process with persistence  $\rho$  and mean  $\bar{g}$ , as shown in equations (13) and (14).

$$W_t L_t + Q_t = M_t + X_t = g_t M_t \quad (12)$$

$$M_{t+1} = g_t M_t \quad (13)$$

$$g_t = \bar{g}^{1-\rho} g_{t-1}^\rho \varepsilon_t \quad (14)$$

**Competitive Equilibrium :** As the model is very standard, we skip the extensive definition of the competitive equilibrium . In the next subsection, we study the response of the economy to a monetary growth shock. To do so, we perform a simulation of a log linear approximation of the model.

### 3.2 Calibration of the Quarterly and Monthly Models

We start by assuming that agents take decisions at the quarterly frequency, as it is done in the literature. In particular, this amounts to assume that portfolio allocations are fixed for three months. The parameter calibration we chose is the one of Christiano, Eichenbaum, and Evans [1998], and is given on the first line of table 1. Much of the parameters are standard.  $\psi_0$  is chosen so that employment is equal to unity at the non stochastic steady state.

In the monthly case, we first assume that all functional forms (utility function, production function, adjustment costs, capital law of motion) are unchanged. We made this assumption to keep working with “standard” specifications <sup>6</sup>. Therefore, we only modified the parameters that have a time dimension. The discount factor  $\beta$ , depreciation rate  $\delta$ , money growth average  $\bar{g}$  and persistence  $\rho$  are simply expressed on a monthly basis, while capital elasticity of output  $\alpha$  is maintained constant. The labor disutility scale parameter  $\psi_0$  is chosen so that employment is now equal to one third at the steady state of the monthly model

As adjustment cost parameters have also a time dimension, we need to carefully choose them in order to compare the monthly and quarterly model. For the quarterly model, we use as a benchmark the calibration of Christiano, Eichenbaum, and Evans [1998]. For the monthly case, we kept constant the convexity parameter  $c$ , but adjust the scale parameter  $d$  in order to keep roughly constant the “size” of the adjustment costs. This “size” is measured in the following way: we computed the response of the economy to a one percent shock to money growth rate in a quarterly economy without adjustment costs. From the sequence of portfolio allocation  $\{Q_t\}$ , we compute a sequence of adjustment cost  $\{H_t\}$

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<sup>6</sup>The alternative would have been to assume that the true model is the monthly one, and to find the exact time-aggregated counterpart of the model for the quarterly frequency. This would have implied a change in the functional forms for preferences, technology, stochastic processes.

that would have had to be paid if there were some adjustment costs. We do the same in the case of the monthly model for a “typical shock” (to be defined later), and adjust the value of  $d$  to obtain similar pattern for the two sequences of relative adjustment costs  $\{H_t/L_t\}$ . Those sequences are plotted on figure 4.

We also need to be specific in the description of the shocks we consider. At the quarterly frequency, the typical monetary policy shock is a -1 % shock on money growth. At the monthly frequency, the discipline we follow is to study shocks that are observationally equivalent to the quarterly shock once aggregated over the first three months. We consider two type of shocks. The first one is a third month of a quarter shock of 1%. In such a case, the impact response of the quarterly-aggregated monthly model should be quite similar to the one of the quarterly model, as the former does not mix impulse and propagation mechanisms. The second type of shock we consider is what we refer to as the typical monthly shock, and corresponds to what happens on average in the monthly economy. Every month of the first quarter, the agents are surprised by a innovation  $\varepsilon$ . The size of the innovation is chosen such that the resulting aggregated sequence of money growth displays a 1% deviation the first quarter. In this case, temporal aggregation can create a wedge between the quarterly-aggregated monthly model and the quarterly one, as the former does mix impulse and propagation in the first quarter. This shock is the one that is identified when using a quarterly VAR, if decisions are taken at the monthly frequency in the real world. The responses to this shock will be the theoretical counterparts to the estimated IRFs.

The precise values of those shocks are given in table 2. The pattern of money growth following different shocks are plotted on figure 5.

### **3.3 Impulse Response Comparison**

Here, we aim at comparing the responses of output and nominal interest rate to a typical shock, both in the quarterly and monthly model, the responses of the monthly model being time-aggregated at the quarter, in order to understand the empirical results we obtained in the first section.

Let us first consider the two upper panels of figure 6, that display the response of output and nominal interest rate to a third month of a quarter monetary impulse, in monthly and quarterly models without adjustment costs. The quarterly and quarterly-aggregated monthly model are qualitatively similar on impact and onwards, as the quarterly-aggregated monthly model does not mix impulse and propagation effect in the first quarter. With adjustment costs (lower panels of figure 6), we also obtain very similar responses in the two models.

Let us now turn to a typical monthly shock. It is clear from the two upper panels of figure 7 that the prediction of the limited participation model are not robust to time aggregation: a monetary contraction has almost no liquidity effect nor contractionary effect in the monthly model, while it does in the quarterly one. This result comes from the lack of internal persistence of the model as the impact response of the (quarterly aggregated) monthly model can be thought as the sum of the first three periods of response of the quarterly model <sup>7</sup>. After one period, the model is clearly at odds with the data, as we observe an output expansion and a drop in the nominal interest rate. At the quarterly frequency, the weakness of the limited participation model was not its response on impact, but its lack of propagation, and more precisely its anti-persistence. Once written at the monthly frequency and quarterly-aggregated, the weakness is transmitted to the first quarter response, as it is now already a mix of impulse and propagation mechanisms.

It is then easy to understand that the results we have obtained in our empirical exercise can be obtained in the model by introducing adjustment costs (the two lower panels of figure 7). But it should be noted that adjustment costs are not just “improving” the predictions of the model, but have qualitative consequences on the monthly model, once aggregated at the quarterly frequency. It is only when the underlying model of the economy displays a lot of persistence that one can expect a correct identification of monetary shocks with a quarterly VAR. In the case of a limited participation model, adjustment costs to portfolio reallocations is a way of obtaining this persistence.

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<sup>7</sup>This is not exactly the case, as some deep parameters have different values in the monthly and quarterly model, but it is a useful way to get the intuition of the results

## 4 Conclusion

In this paper, we have asked whether our empirical and theoretical knowledge about the effect of monetary policy shocks was robust to the length of the period. We think that such a question is particularly relevant in this literature in which frictions are often introduced, as one-period lags in agents' reaction. Typically, monetary policy shocks are identified in VARs by assuming that monetary policy does not react within the period to a given set of innovations, and limited participation models assume that portfolio reallocations are decided one period before the monetary shock occurs. Clearly, whether one period is a week or a year is important in assessing the pertinence of such empirical and theoretical exercises.

We first show that it was possible to use more efficiently the available information when identifying monetary policy shocks. Using together quarterly series for GDP and monthly series for monetary aggregates and interest rates, it is possible to identify monetary shocks with the assumption that they do not have any impact on GDP within a month, by restricting ourselves to third month of a quarter shocks. This new piece of evidence is pretty much in line with the results obtained with quarterly data, which is a puzzling result. We then explain this result using a limited participation model, in which the time period is reduced from one quarter to one month, and when the model predictions are time-aggregated at the quarterly frequency. We show that the introduction of adjustment costs to portfolio reallocation into the model is necessary to account for our empirical results.

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# Appendix

## A Tables

Table 1: Parameters Values

	$\beta$	$\delta$	$\psi_0$	$\psi$	$\alpha$	$d$	$c$	$\rho$	$\bar{g}$
Quarterly model	.9926	.02	2.652	.40	.36	0 or 85	2	.5	1.01
Monthly model	.9975	.0067	7.7	.40	.36	0 or 110	2	.794	1.0033

*This table presents the parameter values that we use for the simulation of the monthly and quarterly model. For each frequency, two cases are considered, depending whether the adjustment cost parameter  $d$  is null or not.*

Table 2: Shocks

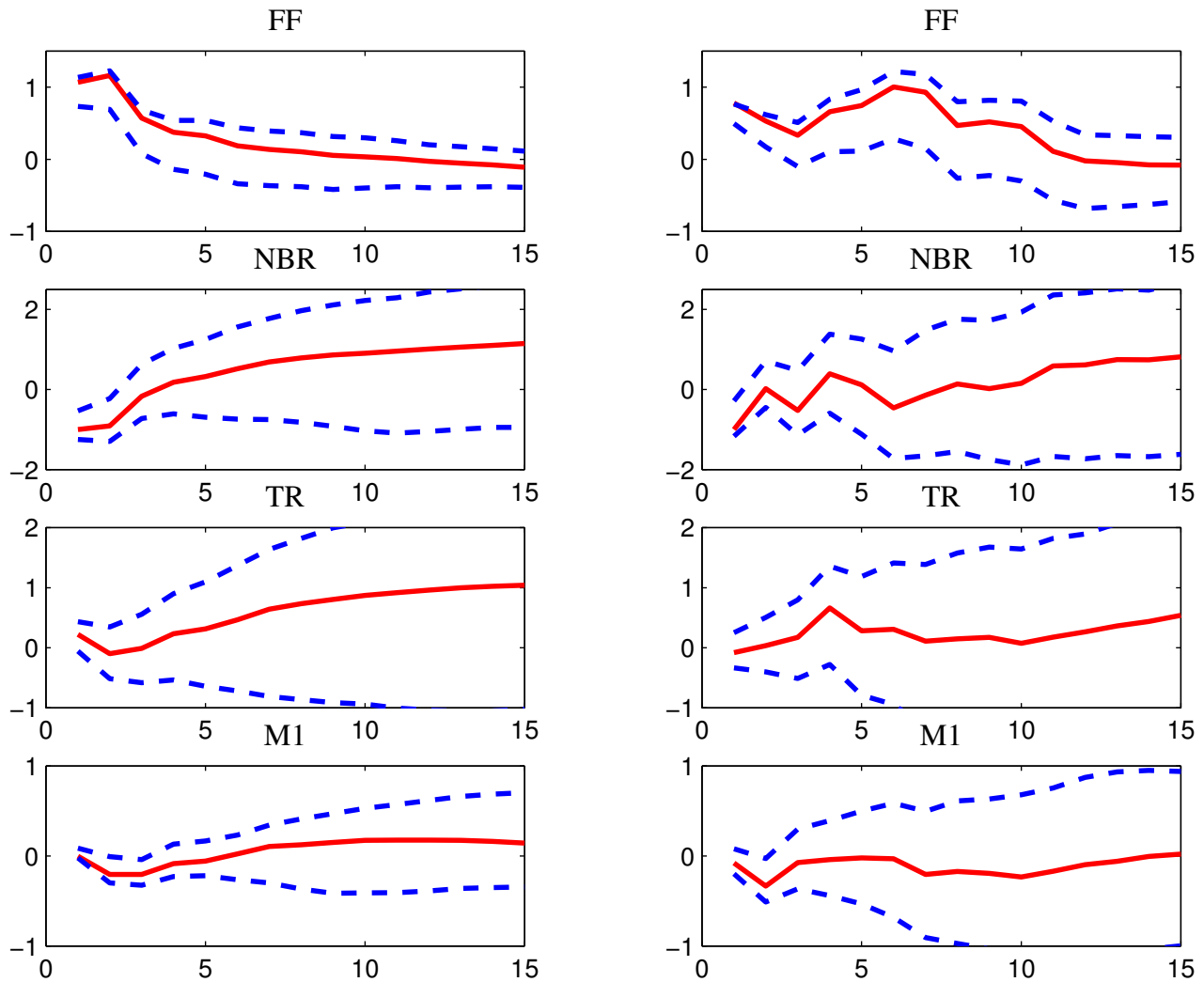
Typical Quarterly shock	: .99 the first quarter
Typical Monthly Shock	: .9981 each first three month
Third Month Shock	: .99 the third month of the first quarter

*This table presents the value of the multiplicative innovation to money growth that we impose for the computation of the models' IRFs. The discipline is that, in each case, the impact response of money growth rate, aggregated at the quarter, is 1% below its steady state level.*

## B Figures

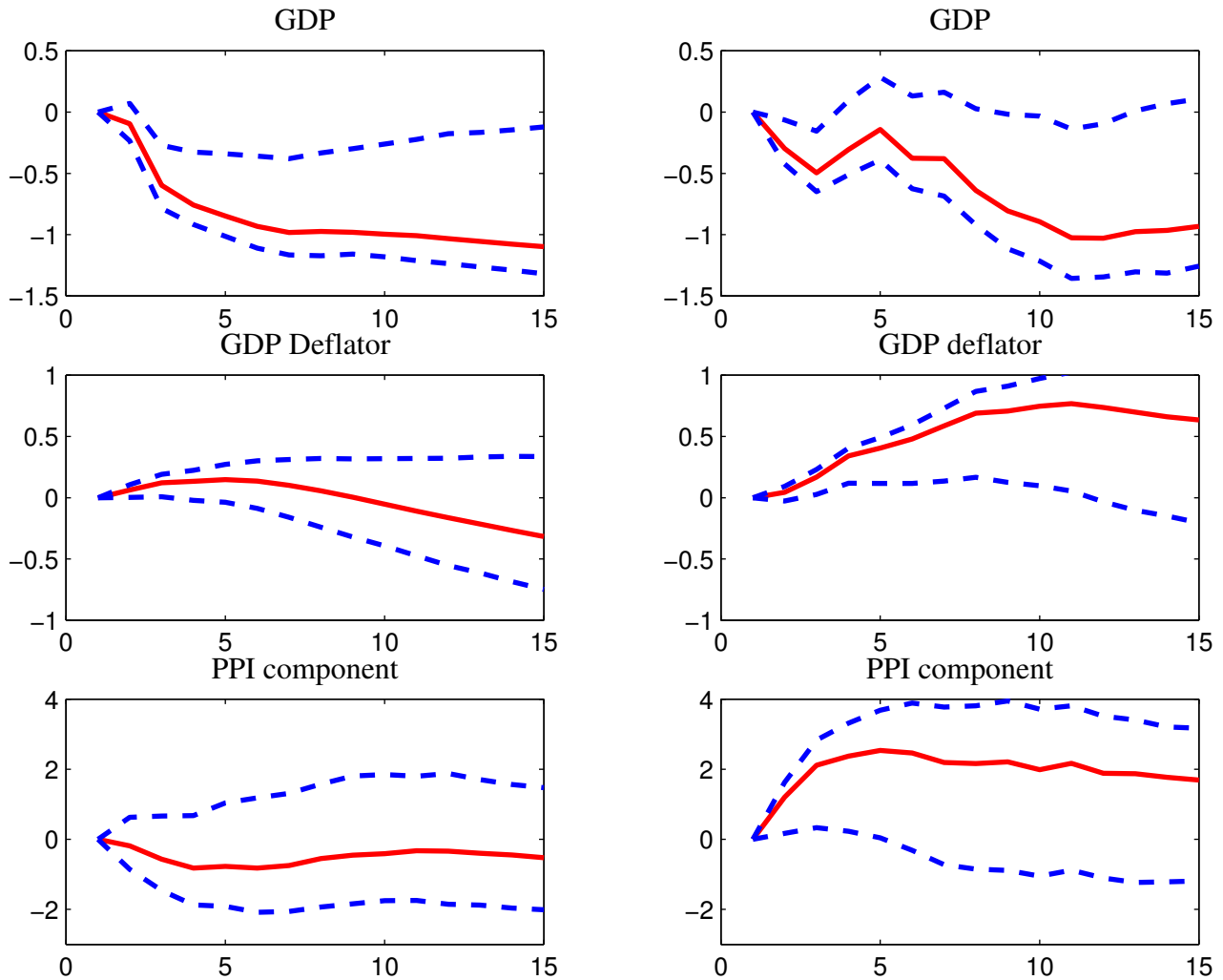


Figure 1: Response to a temporary contractionary monetary shock. Quarterly VAR (left column) and Mixed-Frequency VAR (right column)



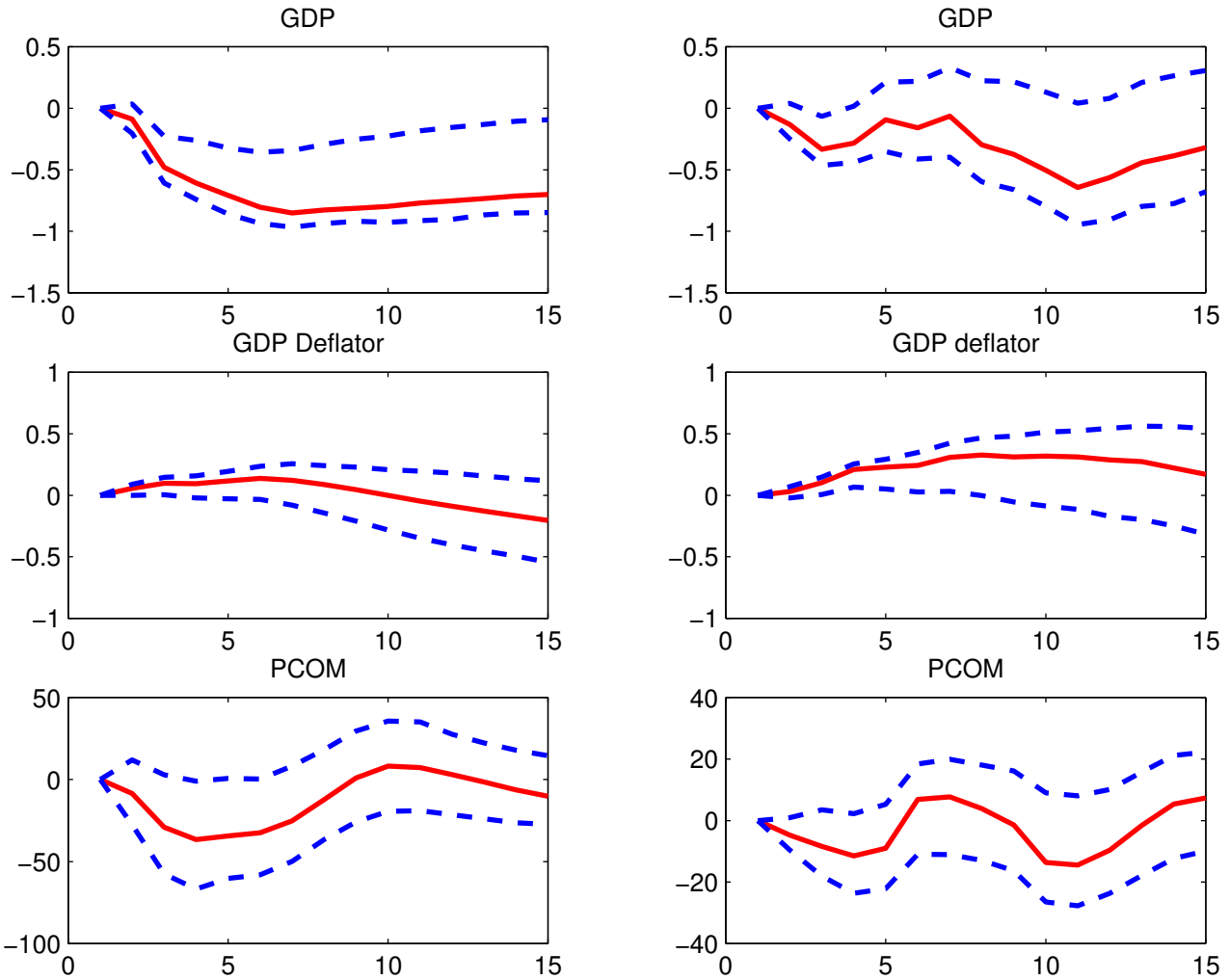
*This figure compares the response of the Federal Fund Rate (FF), Non Borrowed Reserves (NBR), Total Reserves (TR) and the monetary aggregate M1 to a contractionary monetary shock. The left column corresponds to a quarterly VAR while the right one corresponds to our mixed-frequency VAR. In both cases, the monetary policy shock is normalized so that Non Borrowed Reserves drop by one percent at the impact.*

Figure 2: Response to a temporary contractionary monetary shock. Quarterly VAR (left column) and Mixed-Frequency VAR (right column)



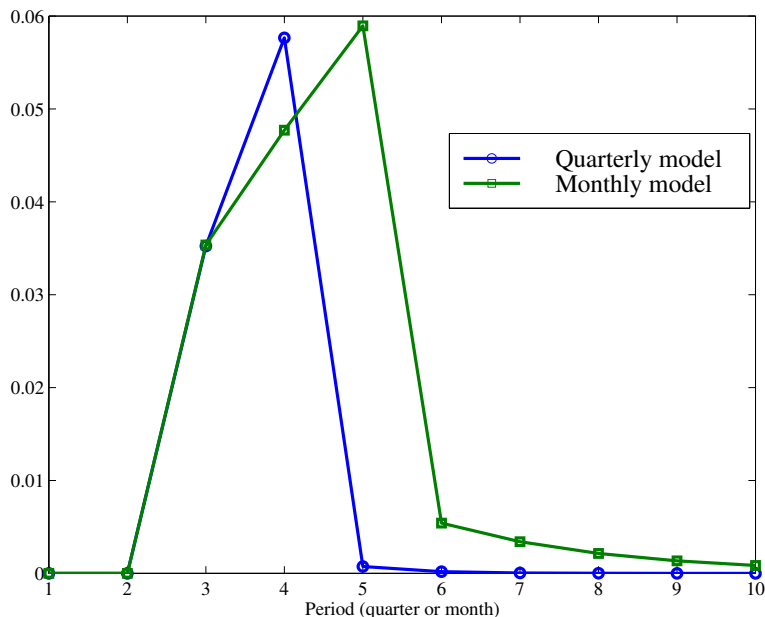
*This figure compares the response of GDP, Price Deflator and a Producer Price Index (PPI) component (Price Index of crude material for further processing) to a contractionary monetary shock. The left column corresponds to a quarterly VAR while the right one corresponds to our mixed-frequency VAR. In both cases, the monetary policy shock is normalized so that Non Borrowed Reserves drop by one percent at the impact.*

Figure 3: Response to a temporary contractionary monetary shock. Quarterly VAR (left column) and Mixed-Frequency VAR (right column), using PCOM



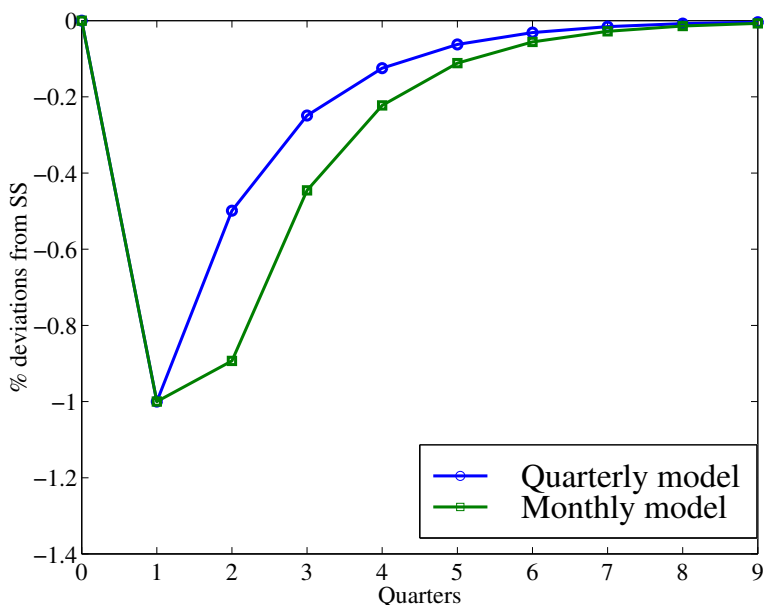
*This figure compares the response of GDP, Price Deflator and a Price of Commodity Index (PCOM) to a contractionary monetary shock. The left column corresponds to a quarterly VAR while the right one corresponds to our mixed-frequency VAR. In both cases, the monetary policy shock is normalized so that Non Borrowed Reserves drop by one percent at the impact.*

Figure 4: Relative size of the adjustment costs in the monthly and quarterly model



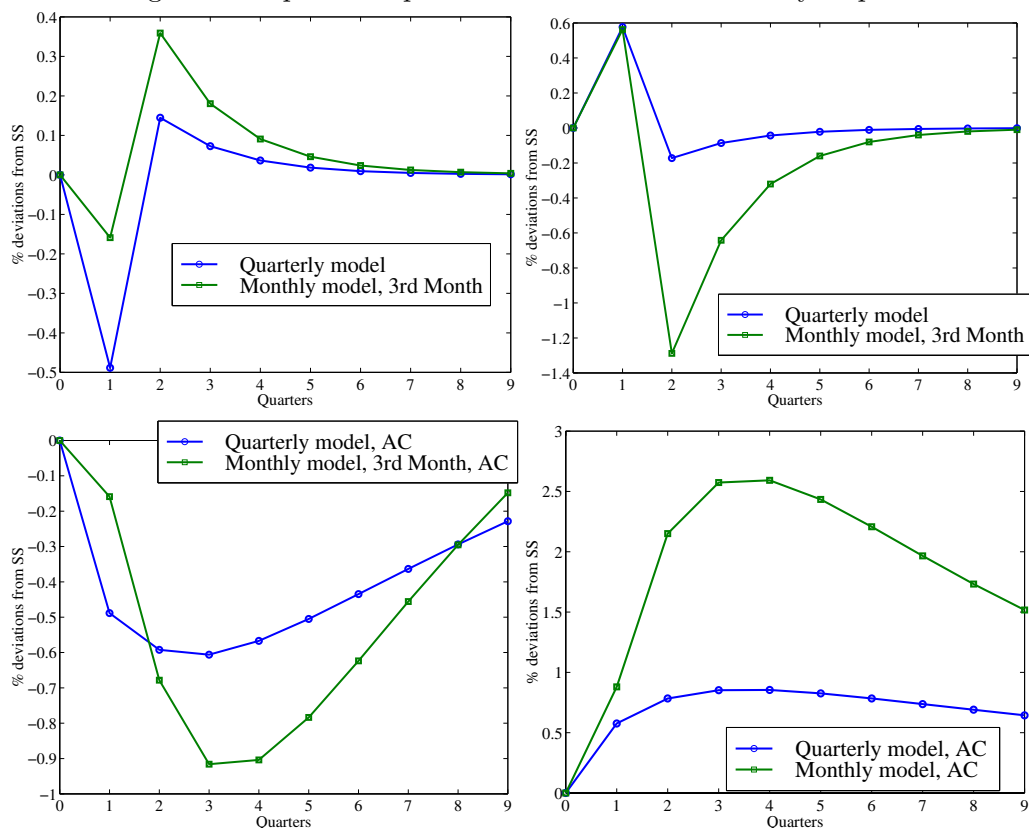
*This figure compares the size of the adjustment costs in the monthly and quarterly models. The ratio of time spent to reallocate portfolio over total time worked is displayed for the two economies, following a typical contractionary monetary policy shock. The ratio is constructed as follows. We first compute the sequence of portfolio reallocations in an economy without adjustment costs, and then compute what it would have cost in term of time spent had the agent realized ex post that adjustment was costly.*

Figure 5: Typical Monetary Impulse



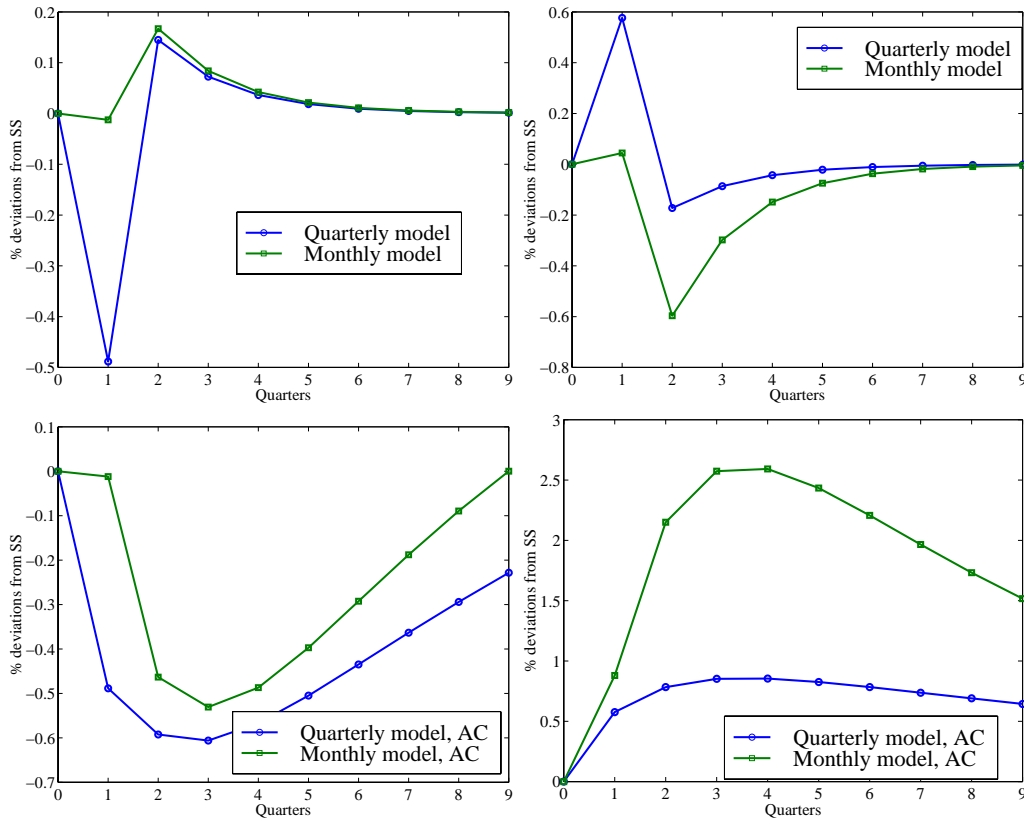
*This figure compares the response of money growth rate to a typical contractionary monetary policy shock in the quarterly and the quarterly-aggregated monthly economy. The shock is a -1% shock to the growth rate at quarter 1 for the quarterly economy. In the monthly economy, the shock is a sequence of three unexpected monthly shocks of -0.191% at month 1, 2 and 3 of quarter 1. This shock is such that, once aggregated at the quarter, the impact response of the money growth rate is exactly -1%.*

Figure 6: Impulse Response to a 3rd Month Monetary Impulse



This figure displays the response of output (right panels) and nominal interest rate (left panels) to a third month contractionary monetary policy shock, both in models without (upper panels) and with (lower panels) adjustment costs to portfolio reallocation. A typical monetary shock is -1% shock to money growth in the quarterly model, and a sequence of three unexpected -1.91% monthly shocks in the monthly model.

Figure 7: Impulse Response to a Typical Monetary Impulse



This figure displays the response of output (right panels) and nominal interest rate (left panels) to a typical contractionary monetary policy shock, both in models without (upper panels) and with (lower panels) adjustment costs to portfolio reallocation. A typical monetary shock is -1% shock to money growth in the quarterly model, and a sequence of three unexpected -1.91% monthly shocks in the monthly model.