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Hans Gersbach and Bernhard Pachl

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Hans Gersbach, Universität Heidelberg and CEPR
Bernhard Pachl, Universität Heidelberg

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Flexible Majority Rules for Central Banks*

We propose a flexible majority rule for central bank councils where the size of the majority depends monotonically on the change in interest rate within a particular time frame. Small changes in interest rate require a small share of supporting votes, even less than 50%. We show that flexible majority rules are superior to simple majority rules and can implement the optimal monetary policy under a variety of circumstances.

JEL Classification: D72, E52, E58 and F33
Keywords: central bank, flexible majority rules, majority rule and voting

Hans Gersbach
Alfred-Weber-Institut
Universität Heidelberg
Grabengasse 14
69117 Heidelberg
GERMANY
Tel: (49 62) 2154 3173
Fax: (49 62) 2154 3 578
Email: gersbach@uni-hd.de

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Bernhard Pachl
Alfred-Weber-Institut
Universität Heidelberg
Grabengasse 14
69117 Heidelberg
GERMANY
Tel: (49 62) 2154 2958
Fax: (49 62) 2154 3578
Email: bpachl@ix.urz.uni-heidelberg.de

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=160906

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1 Introduction

This paper proposes a flexible majority decision rule for central banks. The flexible majority rule works as follows. Within a prespecified time frame, the size of the majority necessary for adopting a change in the interest rate depends on the change in the interest rate itself. For small changes in interest rate, only a small share of the votes is required, possibly even less than 50%. For large interest rate changes, a larger majority is necessary, tending to total unanimity.

We consider a model where N central bankers, representing countries, regions or different constituencies within a country, decide on monetary policy. The central bank loss function is composed of the weighted losses function of countries, regions or constituencies. This is the typical case for the European Central Bank (ECB) which we take as our leading example. When the monetary union is hit by a regional shock, some countries desire a change in monetary policy while the other countries want to retain the status quo. For instance, some countries may be affected negatively by a negative supply or demand shock and concern for their own country's welfare makes them want to ease monetary policy through interest-rate cuts. Other countries not affected by the shock will prefer no change in the interest rate. Under simple majority rule, a change in interest rate will occur, if and only if, a simple majority of votes desires a change. Under flexible majority rule, small changes in interest rate will only require a small share of supporting votes and hence a small number of countries to join in, whereas large changes in interest rate require large majorities.

The key advantage of the flexible rule is that a number of countries hit by negative shocks can partially ease the consequences by a small interest-rate cut. Larger changes in interest rate, however, require larger majorities, which can only be achieved if a larger number of countries are affected by the shock. The flexible majority rule aligns the severity of shocks and the socially desirable change in the interest rate. The drawbacks of simple majority rules and unanimity rules (possible exploitation by minorities, unanimity rules creating extreme veto power) can be overcome by flexible majority rules.

We distinguish two cases. First, the vote of every central banker has the same weight; second, the vote of a central banker is weighted to the same degree as his country is weighted in the central bank loss function. Our main results are as follows. First, in both cases the flexible majority rule always leads to smaller central bank losses than the simple majority rule. Second, if every vote is weighted as described above, flexible majority rules can implement the socially optimal solution. Third, it is socially optimal for small interest-rate changes within a particular time frame to be brought about by minorities - either one large country or a set of small countries. Similarly, it is socially desirable for large interest-rate changes to require large majorities. The main intuition for our results is that flexible majority rules of the kind described above can mimic aggregate social loss minimisation, which calls for small interest rate changes, when shocks are small and affect only a few countries and large interest-rate changes, when shocks are larger and affect many countries.

The paper is organised as follows: In section 2 we explain the flexible majority rule and relate our work to the literature. Section 3 presents our model, with the specific properties of the shock function, the assumed central bank loss function and the constitutional process that determines how a change in interest rate is implemented. In section 4 we describe the different decision rules and their outcomes. In section 5 we discuss the results and section 6 concludes the article. Most of the proofs can be found in the appendix A and we give a simple example with three countries for all decision rules in appendix B.

2 Relation to the Literature

2.1 Regional Bias in Central Bank Decisions

A socially desirable procedure for making decisions in central bank councils has been the focus of a substantial recent literature, mostly centered around the ECB.

The ECB's Governing Council makes decisions about interest rates. The council consists of the Executive Board of the ECB (president, vice-president and four other members) and the central-bank govenours of the 12 euro countries). The one person one vote principle prevails. Two main issues have been investigated. First, before the (virtual euro) was introduced in 1999 the optimal institutional design of the ECB had focused on the degree of centralization. Von Hagen and Süppel (1994), Lohmann (1997) and Bindseil (2001) have highlighted the advantages of a stronger role for the centrally nominated ECB[1] As the current decision-making procedure relies strongly on the national representatives which have a political weight of about $\frac{2}{3}$ of all votes, flexible majority rules might partially act as a substitute for a lack of centralization.

Second, national and regional considerations appear to play a substantional role in

[1]The advantages of centralization have gained renewed interest in the current process of EU enlargement (Baldwin, Berglof, Giavazzi, and Widgren (2001) and Berger, de Haan, and Inklaar (2003)).

ECB's decision-making as has been suggested by (Heinemann and Hüfner (2002)). In such circumstances, matching economic size and voting power by vote-weighting improves welfare as we demonstrate in this paper and as suggested for instance by Berger and de Haan (2002). Under such schemes votes of national representatives are weighted by the member countries' share in GDP of the euro area. We show that weighting and flexible majority rules can yield the first-best monetary policy. Our suggestion is potentially applicable to any central bank where different members of the decision-making body represent different groups or regions. For instance, recent research has highlighted that heterogeneity of preferences and even a regional bias play a significant role at the Federal Reserve. For instance, governors tend to vote against the majority when there is a significant gap between the unemployment rate in their *region* and the national rate (Meade and Sheets (2002)). Therefore, flexible majority rules might also be appropriate for the Fed.

2.2 Efficient Collective Decision-making

On a broad conceptual level, our paper addresses the optimal design of majority rules, which has along tradition in economic and political science.

In every collective decision problem, the question arises which decision rule should be used in order to achieve socially desirable outcomes. One of the most widely employed decision rule is the simple majority rule where a proposal is accepted if it obtains more than 50% of the votes. For example, in countries with a democratic constitution, most of the processes in which politicians are elected and parliamentary decisions are taken follow the simple majority rule. An early discussion when this rule is optimal can be found in Rae (1969) and Taylor (1969). Nevertheless, the simple majority rule is not optimal in all cases. The classic work of Buchanan and Tullock (1962) showes that a majority other than 50% might be optimal. Other majorities are realized, for example, in the veto or the unanimity rule in the United Nations Security Council or the $\frac{2}{3}$ majority needed for an amendment of the constitution in the Federal Republic of Germany. The problem, however, is that these fixed majorities very often lead to inefficiencies. Consider, for example, a collective decision problem where two groups have preferences located at two extremes. If one group is at least as big as the fixed majority needed in this decision problem,[2] it can always overrule the other group, which may lead to serious dissatisfaction on the part of the minority (see for instance Guinier (1994)), and which is not optimal from a utilitarian perspective. In this paper,

[2]In this case the majority has to be greater than 50%.

we design flexible majority rules that can imitate a first-best solution in an utilitarian sense. Furthermore, in the recent past there has been a renewed interest in new decision rules. In a recent paper, Casella (2000) suggests a system of storable votes, where the voters can choose between the possibility of voting now, or storing the vote and having an additional vote in the future. Another paper by Grüner and Kiel (2001) compares the mean and median of all proposals by individuals.[3] We suggest flexible majority rules for monetary policy. Flexible majority rules, where the size of the majority depends on the number of people taxed, have been introduced by Erlenmeier and Gersbach (2001) for public good provision.[4] In this paper we design flexible majority rules for monetary policy.

3 The model

3.1 Central Bank Council

We consider a monetary union consisting of $N \in \mathbb{N}$ countries, which jointly makes decisions about monetary policy in a single central bank such as the ECB. The social loss function[5] for every single country is given by:

$$L_t^k = (i_t - i_t^k)^2 \tag{1}$$

The variable i_t denotes the interest rate adopted by the central bank in period t and i_t^k denotes the interest rate of the $k-th$ ($k \in \mathcal{N} = \{1, ..., N\}$) central banker, who represents country k, wants to implement in period t. i_t^k is the bliss point of country k and depends on a shock ϵ that occurs at the end of period $t-1$. The aggregated loss function for the whole union is assumed to be given by the weighted sum of the single loss functions:

$$\mathcal{L}_t = \sum_{k=1}^{N} g_k L_t^k \tag{2}$$

[3]Dixit and Jensen (2000) model the way in which governments could influence the central bank by offering incentive contracts.

[4]There are real world examples of flexible majority rules as has been pointed out by Amihai Glazer. For instance, when a person buys property in Irvine in Southern California, he signs a contract making him a member of a homeowner association which provides local public goods, and which has the right to levy annual fees. The required share of votes to implement an increase of the fees depends on the proposed fee change.

[5]Gersbach and Hahn (2001) show that one can obtain such a functional form of losses when considering standard demand and supply equations without time lags, social losses that are quadratic in inflation and output, and supply shocks that are normally distributed.

with $g_k \in (0,1)$ and $g_k \leq g_l, \forall k < l, k, l \in \mathcal{N}$ and $\sum_{k=1}^{N} g_k = 1$, where g_k are the weights of the countries representing, for example, differences[6] in GDP or population.

We assume that the monetary union is affected by a supply or demand shock which affects countries in a different way. We divide the union into two parts: countries affected by the shock and countries not affected by it. The subset of countries affected is denoted by the set $\mathcal{K}$, with $\mathcal{K} \subseteq \mathcal{N}$.

We consider shocks that can affect any subset of countries. It is natural to assume a shock function $\epsilon(.)$, which is a strictly monotonically increasing function of $(-1)^{\nu} G_{\mathcal{K}}^{+}$, with $G_{\mathcal{K}}^{+} = \sum_{k \in \mathcal{K}} g_k$ and $\nu = 0$, if the shock is positive and $\nu = 1$, if the shock is negative. Furthermore, we assume $\epsilon(0) = 0$.

Assuming that i_{t-1} represents the optimal interest rate of countries where no shock occurs, we can write i_t^k as a function of the shock:

$$i_t^k(\epsilon((-1)^{\nu} G_{\mathcal{K}}^{+})) = \gamma_k \Delta i_t(\epsilon((-1)^{\nu} G_{\mathcal{K}}^{+})) + i_{t-1} \tag{3}$$

where $\Delta i_t(\epsilon)$ is the desired change in the interest rate and $\Delta i_t(.)$ is therefore a strictly increasing function with $\Delta i_t(0) = 0$. γ_k is a geographical indicator variable describing whether a country is affected by the shock or not. γ_k is then given by:

$$\gamma_k = \begin{cases} 1 & \text{for } k \in \mathcal{K} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Altogether we have $2^{N+1} - 1$ possible different shock scenarios in the union represented by $(\mathcal{K}, \nu)$.[7] We assume that these shocks are distributed according to an arbitrary probability distribution and we define $p_{\mathcal{K}}^{\nu}$ as the probability of a particular shock, where all countries labeled with numbers element of $\mathcal{K}$ are positively $(\nu = 0)$ or negatively $(\nu = 1)$ affected. Note that $-p_{\{\emptyset\}}^{1} + \sum_{\nu=0}^{1} \sum_{n=1}^{N} \sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}}^{\nu} = 1$ when $|\mathcal{K}|$ is the number of elements in $\mathcal{K}$ and $\sum_{\sigma(|\mathcal{K}|=n)}$ is the sum over all subsets $\mathcal{K}$ of $\mathcal{N}$ containing n elements.[8]

If we consider that i_t can be written as

$$i_t = i_{t-1} + \Delta \hat{i}_t \tag{5}$$

[6]In this paper we do not focus on the determination of g_k, but take it as given.

[7]Note that degeneracies are possible if the g_i's are specified. For example $g_1 = 0.05$, $g_2 = 0.1$, $g_3 = 0.2$, $g_4 = 0.3$, $g_5 = 0.35$. Although $\epsilon(g_1 + g_5) = \epsilon(g_2 + g_4)$, these are considered to be two different shocks, because the shock does not affect the same countries.

[8]We have to subtract $p_{\{\emptyset\}}^{1}$ when summing up the probabilities, because $p_{\{\emptyset\}}^{0}$ and $p_{\{\emptyset\}}^{1}$ both describe the shock scenario when the union faces no shock at all.

where $\Delta\hat{i}_t$ is the actual change in interest rate from period $t-1$ to period t and with (3) we can write

$$L_t^k = (\Delta\hat{i}_t - \gamma_k \Delta i_t)^2 \quad (6)$$

In the following, we drop the time index t, since we are focussing on a specific period. Now we can write the social loss function of the union in any specific shock scenario, denoted by $\mathcal{L}_{\mathcal{K}}^{\nu}$, as:

$$\begin{aligned} \mathcal{L}_{\mathcal{K}}^{\nu} &= G_{\mathcal{K}}^{+}\left(\Delta\hat{i}\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right) - \Delta i\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right)\right)^2 + G_{\mathcal{K}}^{-}\left(\Delta\hat{i}\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right) - 0\right)^2 && (7)\\ &= \left(\Delta\hat{i}\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right) - G_{\mathcal{K}}^{+}\Delta i\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right)\right)^2 + G_{\mathcal{K}}^{+}G_{\mathcal{K}}^{-}\left(\Delta i\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right)\right)^2 && (8)\end{aligned}$$

with $G_{\mathcal{K}}^{-} = 1 - G_{\mathcal{K}}^{+}$.[9] For simplicity of exposition, we write in the following:

$$\Delta i\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right) = \Delta i_{\mathcal{K}}^{\nu} \quad \text{and} \quad \Delta\hat{i}\left((-1)^{\nu}G_{\mathcal{K}}^{+}\right) = \Delta\hat{i}_{\mathcal{K}}^{\nu} \quad (9)$$

The expected social loss function is then given by:

$$E[\mathcal{L}] = -p_{\{\emptyset\}}^{1}\mathcal{L}_{\{\emptyset\}}^{1} + \sum_{\nu=0}^{1}\sum_{n=1}^{N}\sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}}^{\nu}\mathcal{L}_{\mathcal{K}}^{\nu} \quad (10)$$

In the following, we will calculate $E[\mathcal{L}]$ applying different decision rules determining $\Delta\hat{i}_{\mathcal{K}}^{\nu}$.

3.2 Constitution

To examine optimal decision rules for central banks, we consider a constitutional design problem where governments of the monetary union decide on the decision rule the central bank of the union will use. The decision about the decision rule is governed by the unanimity rule and occurs under a veil of ignorance, i.e. at a time when shocks are not yet known and no conflicts of interest are present. The stages of the constitutional design process are as follows:

Stage 1: The governments of the monetary union decide by unanimity on the decision rule.

Stage 2: Central bankers in the council observe whether their countries and other countries are affected by the shock or not.

Stage 3: The council decides on the change in the interest rate in accordance with the decision rule.

[9] Note that we leave out ϵ and write Δi directly as a function of $(-1)^{\nu}G_{\mathcal{K}}^{+}$, because Δi is a strictly increasing function of ϵ.

We will restrict rules to democratic decision processes where each central bank has one vote, which may or may not be weighted by the size of the country.

4 Decision Rules

We distinguish between simple majority (SM) and flexible majority (FM) decision rules:

SM: i_{t-1} will be changed in t if and only if a change receives a majority of more than 50% of the votes. The central bank implements the maximal interest rate change that receives a majority of 50% of the votes when the interest rate is varied starting from i_{t-1}. Equivalently, the central bank implements the preferred interest rate change of the median voter.[10]

FM: i_{t-1} will be changed in t if the proposed $\Delta\hat{i}^{\nu}_{\mathcal{K}}$ obtains a majority of $\alpha(\Delta\hat{i}^{\nu}_{\mathcal{K}})$ with $\alpha(.)$ decreasing for $\Delta\hat{i}^{\nu}_{\mathcal{K}} \leq 0$ and increasing for $\Delta\hat{i}^{\nu}_{\mathcal{K}} \geq 0$ and $0 \leq \alpha(.) \leq 1$. The central bank implements the maximum interest rate change $\Delta\hat{i}^{\nu}_{\mathcal{K}}$ that receives a share of $\alpha(\Delta\hat{i}^{\nu}_{\mathcal{K}})$ votes, when the interest rate is varied starting from i_{t-1}.

The simple majority rule represents the standard median voter outcome. The important feature of flexible majority rules is that the size of the majority α depends on the proposed interest rate change $\Delta\hat{i}^{\nu}_{\mathcal{K}}$. We will see that it is optimal for small interest rate changes to require a small share of votes, while large interest rate changes require a large share of supporting votes. We now proceed as follows. We examine each decision rule separately and provide the comparison afterwards. The maximum interest rate change for which a supporting majority exists will be chosen. We analyze both the case where every country has only one vote and the case where the vote of every country is weighted with its importance for overall welfare g_k. We describe in the following four different decision rules:

1. A flexible majority rule with weighted votes that is indexed by FM_w.

2. A simple majority rule with weighted votes that is indexed by SM_w.

[10] When preferences are one-dimensional and single-peaked as in this paper, starting from any status quo, the median voters most preferred outcome is the maximal change of the status quo that receives a simple majority of votes.

3. A simple majority rule without weighted votes that is indexed by SM_{nw}.

4. A flexible majority rule without weighted votes that is indexed by FM_{nw}.

4.1 FM_w: Flexible Majority Rule with Weighted Votes

In this case, we construct a FM_w rule, that minimises $E[\mathcal{L}]$. Here, the vote of every country is weighted by its g_k. We are looking for an optimal voting function rule $\alpha(\Delta\hat{i}^{\nu}_{\mathcal{K}})$. According to (7) and (9), social losses of the union for every possible shock scenario (i.e. for fixed subset $\mathcal{K}$ and fixed ν) are given by:

$$\mathcal{L}^{\nu}_{\mathcal{K}} = G^{+}_{\mathcal{K}}\left(\Delta\hat{i}^{\nu}_{\mathcal{K}} - \Delta i^{\nu}_{\mathcal{K}}\right)^2 + G^{-}_{\mathcal{K}}\left(\Delta\hat{i}^{\nu}_{\mathcal{K}}\right)^2 \tag{11}$$

and from (8) we see at once that this is minimised with respect to $\Delta\hat{i}^{\nu}_{\mathcal{K}}$ by $G^{+}_{\mathcal{K}}\Delta i^{\nu}_{\mathcal{K}}$. This optimal value for $\Delta\hat{i}^{\nu}_{\mathcal{K}}$ we define by

$$[\Delta\hat{i}^{\nu}_{\mathcal{K}}]^{FM_w} = G^{+}_{\mathcal{K}}\Delta i^{\nu}_{\mathcal{K}} \tag{12}$$

In the next proposition we provide existence of an optimal flexible majority rule.

Proposition 1
There exists a function $\alpha^{FM_w}(\Delta\hat{i}^{\nu}_{\mathcal{K}})$ which determines the share of votes in such a way that under the flexible majority rule $\alpha^{FM_w}(\Delta\hat{i}^{\nu}_{\mathcal{K}})$, occurs the central bank council will always implement a change of interest rate that minimises the loss function (11) *for every shock scenario. $\alpha^{FM_w}(\Delta\hat{i}^{\nu}_{\mathcal{K}})$ is given by:*

$$\alpha^{FM_w}(\Delta\hat{i}^{\nu}_{\mathcal{K}}) = \begin{cases} \left([\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}\right)^{-1}(\Delta\hat{i}^{0}_{\mathcal{K}}) & \text{for} \quad \nu = 0 \\ \left([\Delta\hat{i}^{1}_{\mathcal{K}}]^{FM_w}\right)^{-1}(\Delta\hat{i}^{1}_{\mathcal{K}}) & \text{for} \quad \nu = 1 \end{cases} \tag{13}$$

The proof is given in the appendix. Note that the share of votes required for support of a proposal to change the interest rate is monotonically increasing in the absolute value of the interest rate change.

Implementing the decision rule α^{FM_w} and inserting $\Delta\hat{i}^{\nu}_{\mathcal{K}} = [\Delta\hat{i}^{\nu}_{\mathcal{K}}]^{FM_w}$ in (11), we obtain

$$[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_w} = G^{+}_{\mathcal{K}}G^{-}_{\mathcal{K}}\left(\Delta i^{\nu}_{\mathcal{K}}\right)^2 \tag{14}$$

and the expected loss function (10) is then given by:

$$E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_w}\right] = \sum_{\nu=1}^{1}\sum_{n=1}^{N}\sum_{\sigma(|\mathcal{K}|=n)} p^{\nu}_{\mathcal{K}}G^{+}_{\mathcal{K}}G^{-}_{\mathcal{K}}\left(\Delta i^{\nu}_{\mathcal{K}}\right)^2 \tag{15}$$

An immediate consequence of proposition 1 is the following corollary:

Corollary 1
Applying α^{FM_w} leads to the first-best solution.

Corollary 1 follows from the observation that a first-best solution means implementing the interest rate change, which minimises $\mathcal{L}_{\mathcal{K}}^{\nu}$ for every single shock scenario. Therefore corollary 1 follows directly from (11), (12), and proposition 1.

As a very simple example, we consider a case with two countries $g_1 = 0.3$, $g_2 = 0.7$ and $\Delta i_{\{\emptyset\}}^{\nu} = (-1)^{\nu} \cdot 0$, $\Delta i_{\{1\}}^{\nu} = (-1)^{\nu} \cdot 1$, $\Delta i_{\{2\}}^{\nu} = (-1)^{\nu} \cdot 2$ and $\Delta i_{\{1,2\}}^{\nu} = (-1)^{\nu} \cdot 3$. The optimal function $\alpha(\Delta \hat{i}_{\mathcal{K}}^{\nu})$ is then given by:

$$\alpha^{FM_w}(\Delta \hat{i}_{\mathcal{K}}^{\nu}) = \begin{cases} 0 & \text{if} & & |\Delta \hat{i}_{\mathcal{K}}^{\nu}| & = & 0 \\ 0.3 & \text{if} & 0 & < & |\Delta \hat{i}_{\mathcal{K}}^{\nu}| & \leq & 0.3 \\ 0.7 & \text{if} & 0.3 & < & |\Delta \hat{i}_{\mathcal{K}}^{\nu}| & \leq & 1.4 \\ 1 & \text{if} & 1.4 & < & |\Delta \hat{i}_{\mathcal{K}}^{\nu}| & \leq & 3 \end{cases} \tag{16}$$

Applying this flexible majority rule, we see that no change in interest rate occurs when no country is affected, a change of 3 occurs when all countries are positively affected, and a change of -3 when all countries are negatively affected, because in this case every country wants an exact change of 3 or -3 respectively. If only the smaller country is affected, it desires a change of 1 or -1. But with its share of 0.3 of the total votes, it can only implement a change up to 0.3 or down to -0.3 respectively. Since its private losses are descending in $[0, 0.3]$ for a positive shock and ascending in $[-0.3, 0]$ for a negative shock, the central bank will adopt a change in the interest rate of 0.3 when it is positively affected and of -0.3 when it is negatively affected. By the same argumentation, the change in interest rate will be 1.4 or -1.4 when only the large country is affected. For the seven different shock scenarios together, the implemented changes in the interest rate will be $\Delta \hat{i}_{\{\emptyset\}}^{\nu} = (-1)^{\nu} \cdot 0$, $\Delta \hat{i}_{\{1\}}^{\nu} = (-1)^{\nu} \cdot 0.3$, $\Delta \hat{i}_{\{2\}}^{\nu} = (-1)^{\nu} \cdot 1.4$ and $\Delta \hat{i}_{\{1,2\}}^{\nu} = (-1)^{\nu} \cdot 3$ and the social loss function of the union will also be minimised in every associated shock scenario.

4.2 SM_w: Simple Majority Rule with Weighted Votes

In the next step we examine the simple majority rule with weighted votes. In this case the change in interest rate $[\Delta \hat{i}_{\mathcal{K}}^{\nu}]^{SM_w}$ is given by:

$$[\Delta \hat{i}_{\mathcal{K}}^{\nu}]^{SM_w} = \begin{cases} \Delta i_{\mathcal{K}}^{\nu} & \text{if} \quad G_{\mathcal{K}}^{+} > \frac{1}{2} \\ \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

and thus $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w}$ is given by:

$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} = \begin{cases} G_{\mathcal{K}}^{+} (\Delta i_{\mathcal{K}}^{\nu})^2 & \text{if} \quad G_{\mathcal{K}}^{+} \leq \frac{1}{2} \\ \\ G_{\mathcal{K}}^{-} (\Delta i_{\mathcal{K}}^{\nu})^2 & \text{if} \quad G_{\mathcal{K}}^{+} > \frac{1}{2} \end{cases} \tag{18}$$

and the expected social loss can be calculated as:

$$E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w}\right] = \sum_{\nu=0}^{1} \left[\sum_{j=1}^{\bar{j}} p_{\mathcal{K}_j}^{\nu} G_{\mathcal{K}_j}^{+} \left(\Delta i_{\mathcal{K}_j}^{\nu}\right)^2 + \sum_{j=\bar{j}+1}^{2^N-1} p_{\mathcal{K}_j}^{\nu} G_{\mathcal{K}_j}^{-} \left(\Delta i_{\mathcal{K}_j}^{\nu}\right)^2 \right] \tag{19}$$

where we have ordered all 2^N $G_{\mathcal{K}}^{+}$ in the ascending row

$$0 = G_{\mathcal{K}_0}^{+} < G_{\mathcal{K}_1}^{+} \leq G_{\mathcal{K}_2}^{+} \leq ... \leq G_{\mathcal{K}_{2^N-2}}^{+} < G_{\mathcal{K}_{2^N-1}}^{+} = 1 \tag{20}$$

and $\bar{j}$ is determined by the fact that there exists a unique $\bar{j} \in \{0, 1..., 2^N - 1\}$ with $G_{\mathcal{K}_{\bar{j}}}^{+} \leq \frac{1}{2}$ and $G_{\mathcal{K}_{\bar{j}+1}}^{+} > \frac{1}{2}$.[11] The expected loss under a simple majority rule includes both kinds of inefficiencies associated with collective decisions. First, interest rate changes are not implemented and are also too small, since a share of countries smaller than 50% are affected by shocks. Second, adopted interest rate changes are too large since more than 50% of the weighted countries are affected by shocks and the minority has no impact on monetary policy.

We next turn to the simple majority rule with unweighted votes.

4.3 SM_{nw}: Simple Majority Decision Rule without Weighted Votes

In this case, the change in interest rate $[\Delta i_{\mathcal{K}}^{\nu}]^{SM_{nw}}$ is determined by:

$$[\Delta i_{\mathcal{K}}^{\nu}]^{SM_{nw}} = \begin{cases} \Delta i_{\mathcal{K}}^{\nu} & \text{if} \quad |\mathcal{K}| > \frac{N}{2} \\ \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

thus $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$ is given by:

$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}} = \begin{cases} G_{\mathcal{K}}^{+} (\Delta i_{\mathcal{K}}^{\nu})^2 & \text{if} \quad |\mathcal{K}| \leq \frac{N}{2} \\ \\ G_{\mathcal{K}}^{-} (\Delta i_{\mathcal{K}}^{\nu})^2 & \text{if} \quad |\mathcal{K}| > \frac{N}{2} \end{cases} \tag{22}$$

[11] The $p_{\mathcal{K}_j}^{\nu}$'s are the corresponding probabilities for the specific shock scenario and $\Delta i_{\mathcal{K}_j}^{\nu}$ is similarly defined as $\Delta i_{\mathcal{K}}^{\nu}$ in (9).

and the expected social loss can be calculated to

$$E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}\right] = \sum_{\nu=0}^{1}\left[\sum_{n=1}^{n\leq\frac{N}{2}}\sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}}^{\nu} G_{\mathcal{K}}^{+}\left(\Delta i_{\mathcal{K}}^{\nu}\right)^{2} + \sum_{n>\frac{N}{2}}^{N}\sum_{\sigma(|\mathcal{K}|=n)} p_{\mathcal{K}}^{\nu} G_{\mathcal{K}}^{-}\left(\Delta i_{\mathcal{K}}^{\nu}\right)^{2}\right] \tag{23}$$

The simple majority rule with unweighted votes exhibits the same sources of inefficiency as the simple majority rule with weighted votes, although inefficiencies here are more pronounced, since now more than half of the countries can change the interest, even if together they only have a small weight in the union.

4.4 FM_{nw}: Flexible Majority Decision Rule without Weighted Votes

Finally we analyse flexible majority rules with unweighted votes. Since at the voting stage we can no longer distinguish between the countries on the basis for their weights, we minimise overall shock possibilities, where the number n of affected countries and ν is fixed. The minimisation is bounded by the fact that the change in the interest rate possible for n countries must not exceed the change in interest rate possible for $n+1$ countries for $\nu = 0$ and must not fall under the possible change in interest rate for $n+1$ countries for $\nu = 1$. In order to formulate this analytically, we rearrange the order of summation in (10) and restrict ourselves to the case of $\nu = 0$, since the argumentation for $\nu = 1$ is similar.

First note that for every fixed n we can arrange the $G_{\mathcal{K}}^{+}|_{|\mathcal{K}|=n}$ in a similar way to that of all $G_{\mathcal{K}}^{+}$ in (20). For n fixed, we have $\binom{N}{n}$ different $G_{\mathcal{K}}^{+}|_{|\mathcal{K}|=n}$. We order them in the ascending row

$$G_{\mathcal{K}_1^n}^{+} \leq G_{\mathcal{K}_2^n}^{+} ... \leq G_{\mathcal{K}_{j_n}^n}^{+} \leq ... \leq G_{\mathcal{K}_{\binom{N}{n}}^n}^{+} \tag{24}$$

where n is the number of affected countries and $j_n \in \{1, ... \binom{N}{n}\}$ represents the position of this shock scenario in the ordering given by (24). Now with (12) we can write for[12] (8):

$$\mathcal{L}_{\mathcal{K}_{j_n}^n}^{0} = \left(\Delta \hat{i}_{\mathcal{K}_{j_n}^n}^{0} - [\Delta \hat{i}_{\mathcal{K}_{j_n}^n}^{0}]^{FM_w}\right)^2 + G_{\mathcal{K}_{j_n}^n}^{-} \Delta i_{\mathcal{K}_{j_n}^n}^{0} [\Delta \hat{i}_{\mathcal{K}_{j_n}^n}^{0}]^{FM_w} \tag{25}$$

Assume now that we have an arbitrary FM_{nw} rule. Since in this case, every central banker's vote has the same weight, this rule has to be increasing in the number n of affected countries. Thus the rule is fully determined if for any n, we can give a maximum possible interest rate change. We denote this interest rate change which only depends

[12] Again $\Delta i_{\mathcal{K}_{j_n}^n}^{\nu}$ is defined in a way similar to $\Delta i_{\mathcal{K}}^{\nu}$ in (9).

on n with $\Delta\hat{\hat{i}}(n)$, where $\Delta\hat{\hat{i}}(n)$ has to be an increasing function of n, since we cannot distinguish between large and small countries during the voting stage.$[\mathcal{L}^0_{\mathcal{K}^n_{j_n}}]^{FM_{nw}}$ is then given by:

$$[\mathcal{L}^0_{\mathcal{K}^n_{j_n}}]^{FM_{nw}} = \begin{cases} \left(\Delta\hat{\hat{i}}(n) - [\Delta\hat{i}^0_{\mathcal{K}^n_{j_n}}]^{FM_w}\right)^2 + G^-_{\mathcal{K}^n_{j_n}}\,\Delta i^0_{\mathcal{K}^n_{j_n}}\,[\Delta\hat{i}^0_{\mathcal{K}^n_{j_n}}]^{FM_w} & \text{if} \quad \Delta\hat{\hat{i}}(n) \leq \Delta i^0_{\mathcal{K}^n_{j_n}} \\ G^-_{\mathcal{K}^n_{j_n}}\left(\Delta i^0_{\mathcal{K}^n_{j_n}}\right)^2 & \text{if} \quad \Delta\hat{\hat{i}}(n) > \Delta i^0_{\mathcal{K}^n_{j_n}} \end{cases} \tag{26}$$

because if $\Delta\hat{\hat{i}}(n) > \Delta i^0_{\mathcal{K}^n_{j_n}}$, the n affected countries will simply minimise their own social loss function by voting for $\Delta i^0_{\mathcal{K}^n_{j_n}}$ but not for a higher change in interest rate.

Now we can formulate the minimisation problem.

Proposition 2
There exists an optimal FM_{nw} rule that solves the following problem:[13]

$$\min_{\Delta\hat{\hat{i}}(0)\ldots\Delta\hat{\hat{i}}(N)} \sum_{n=0}^{N}\sum_{j_n=1}^{\binom{N}{n}} p^0_{\mathcal{K}^n_{j_n}}[\mathcal{L}_{\mathcal{K}^n_{j_n}}]^{FM_{nw}} \quad s.t. \quad \Delta\hat{\hat{i}}(n) \leq \Delta\hat{\hat{i}}(n+1) \tag{27}$$

Since the actual calculation of a best FM_{nw} rule is very technical[14] and depends strongly on the actual values of g_k, $p^0_{\mathcal{K}^n_{j_n}}$ and the functional form of $\Delta i(.)$, we provide a simple possible FM_{nw} rule, which need not solve (27), but which has some nice properties that can be exploited when we compare FM_{nw} with SM_{nw}. Consider, therefore, the following FM_{nw} rule:

$$\Delta\hat{i}^{FM_{nw}}(n)|_{\nu=0} = \begin{cases} [\Delta\hat{i}^0_{\mathcal{K}^n_1}]^{FM_w} & \text{if} \quad n \leq \frac{N}{2} \\ [\Delta\hat{i}^0_{\mathcal{K}^n_{\binom{N}{n}}}]^{FM_w} & \text{if} \quad n > \frac{N}{2} \end{cases} \tag{28}$$

This is an increasing function in n, which follows from the fact that $G^+_{\mathcal{K}^n_1}$ and $G^+_{\mathcal{K}^n_{\binom{N}{n}}}$ are both increasing in n and $G^+_{\mathcal{K}^n_1} \leq G^+_{\mathcal{K}^n_{\binom{N}{n}}}$. Considering also $\nu = 1$, the complete FM_{nw} rule is given by:

$$\Delta\hat{i}^{FM_{nw}}((-1)^\nu n) = \begin{cases} [\Delta\hat{i}^\nu_{\mathcal{K}^n_1}]^{FM_w} & \text{if} \quad |(-1)^\nu n| \leq \frac{N}{2} \\ [\Delta\hat{i}^\nu_{\mathcal{K}^n_{\binom{N}{n}}}]^{FM_w} & \text{if} \quad |(-1)^\nu n| > \frac{N}{2} \end{cases} \tag{29}$$

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[13]The $p^\nu_{\mathcal{K}^n_{j_n}}$'s are again the corresponding probabilities for the specific shock scenario.
[14]This is done for a specific example in section 8.

Applying this rule means that for $\nu = 0$ and fixed n, $[\Delta\hat{i}^{\nu}_{\mathcal{K}^n_1}]^{FM_w}$ for $n \leq \frac{N}{2}$ will be implemented after the voting stage, while $\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}$ for $n > \frac{N}{2}$ will be implemented if $|\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}| \leq |[\Delta\hat{i}^{\nu}_{\mathcal{K}^n_{\binom{N}{n}}}]^{FM_w}|$ and $[\Delta\hat{i}^{\nu}_{\mathcal{K}^n_{\binom{N}{n}}}]^{FM_w}$ if $|\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}| > |[\Delta\hat{i}^{\nu}_{\mathcal{K}^n_{\binom{N}{n}}}]^{FM_w}|$. Altogether $[\mathcal{L}^{\nu}_{\mathcal{K}^n_{j_n}}]^{FM_{nw}}$ is then given by:

$$[\mathcal{L}^{\nu}_{\mathcal{K}^n_{j_n}}]^{FM_{nw}} = \begin{cases} G^+_{\mathcal{K}^n_{j_n}} G^-_{\mathcal{K}^n_{j_n}} \left(\Delta i^{\nu}_{\mathcal{K}^n_1}\right)^2 \\ \text{if} \quad \left(G^+_{\mathcal{K}^n_{j_n}} = G^+_{\mathcal{K}^n_1} \wedge |(-1)^{\nu} n| \leq \frac{N}{2}\right) \vee \left(G^+_{\mathcal{K}^n_{j_n}} = G^+_{\mathcal{K}^n_{\binom{N}{n}}} \wedge |(-1)^{\nu} n| > \frac{N}{2}\right) \\ \\ \left([\Delta\hat{i}^{\nu}_{\mathcal{K}^n_1}]^{FM_w} - [\Delta\hat{i}^{\nu}_{\mathcal{K}^n_{j_n}}]^{FM_w}\right)^2 + G^+_{\mathcal{K}^n_{j_n}} G^-_{\mathcal{K}^n_{j_n}} \left(\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}\right)^2 \\ \text{if} \quad |\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}| > |[\Delta\hat{i}^{\nu}_{\mathcal{K}_1{}^n}]^{FM_w}| \wedge |(-1)^{\nu} n| \leq \frac{N}{2} \\ \\ \left(\Delta\hat{i}^{\nu}_{\mathcal{K}^n_{\binom{N}{n}}} - [\Delta\hat{i}^{\nu}_{\mathcal{K}^n_{j_n}}]^{FM_w}\right)^2 + G^+_{\mathcal{K}^n_{j_n}} G^-_{\mathcal{K}^n_{j_n}} \left(\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}\right)^2 \\ \text{if} \quad |\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}| \geq |[\Delta\hat{i}^{\nu}_{\mathcal{K}^n_{\binom{N}{n}}}]^{FM_w}| \wedge |(-1)^{\nu} n| > \frac{N}{2} \\ \\ G^-_{\mathcal{K}^n_{j_n}} \left(\Delta i^{\nu}_{\mathcal{K}^n_{j_n}}\right)^2 \\ \text{otherwise} \end{cases} \tag{30}$$

Finally we insert (30) in (10) and end up with

$$E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_{nw}}\right] = \sum_{\nu=0}^{1} \sum_{n=1}^{N} \sum_{j_n=1}^{\binom{N}{n}} p^{\nu}_{\mathcal{K}^n_{j_n}} [\mathcal{L}^{\nu}_{\mathcal{K}^n_{j_n}}]^{FM_{nw}} \tag{31}$$

The intuition for the FM_{nw} rule given in (29) is that in every shock scenario the outcome will never be worse than under the SM_{nw} rule without weighted votes. This will now be demonstrated in section 4.5.

4.5 SM versus FM

Here we compare the different decision rules and obtain:

Proposition 3

$$\begin{array}{llllll} (i) & E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_w}\right] & < & E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{SM_w}\right] & \leq & E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{SM_{nw}}\right] \\ (ii) & E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_w}\right] & \leq & E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_{nw}}\right] & < & E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{SM_{nw}}\right] \end{array} \tag{32}$$

An important corollary follows directly from the proof of proposition 3:

Corollary 2
In any given shock scenario we have

$$(i) \quad [\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w} \;\leq\; [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} \;\leq\; [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}} \tag{33}$$

and there exists a FM_{nw} rule with

$$(ii) \quad [\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w} \;\leq\; [\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_{nw}} \;\leq\; [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}} \tag{34}$$

Note that the second part of corollary 2 is only shown for the FM_{nw} rule given in (29). This is because it need not be true for the optimal FM_{nw} rule given by the minimisation problem in (27). Nevertheless, the second part of proposition 3 holds for both FM_{nw} rules without weighted votes, since in the proof we use $E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_{nw}}\right]$ calculated by (30) and the value $E\left[[\tilde{\mathcal{L}}_{\mathcal{K}}^{\nu}]^{FM_{nw}}\right]$ calculated from (27) can never be greater than $E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_{nw}}\right]$ calculated from (30), where $\tilde{\mathcal{L}}_{\mathcal{K}}^{\nu}$ represents the social loss function derived from proposition 2.

Since the second part of corollary 2 means that there exists a FM_{nw} rule which is *ex post* never worse than the SM_{nw} rule, another plausible minimisation procedure than (27) can be considered for the flexible majority rule without weighted votes:

$$\begin{aligned}
&\min_{\Delta\hat{\hat{i}}(0)\ldots\Delta\hat{\hat{i}}(N)} \sum_{n=0}^{N}\sum_{j_n=1}^{\binom{N}{n}} p^0_{\mathcal{K}^n_{j_n}}\,[\mathcal{L}^0_{\mathcal{K}^n_{j_n}}]^{FM_{nw}} \qquad && \text{s.t. } \ \Delta\hat{\hat{i}}(n) \leq \Delta\hat{\hat{i}}(n+1) \\
& && \text{s.t. } \ [\mathcal{L}^0_{\mathcal{K}}]^{FM_{nw}} \leq [\mathcal{L}^0_{\mathcal{K}}]^{SM_{nw}} \\
&\min_{\Delta\hat{\hat{i}}(0)\ldots\Delta\hat{\hat{i}}(-N)} \sum_{n=0}^{N}\sum_{j_n=1}^{\binom{N}{n}} p^1_{\mathcal{K}^n_{j_n}}\left[\mathcal{L}^1_{\mathcal{K}^n_{j_n}}\right]^{FM_{nw}} && \text{s.t. } \ \Delta\hat{\hat{i}}(-n) \geq \Delta\hat{\hat{i}}(-(n+1)) \\
& && \text{s.t. } \ [\mathcal{L}^1_{\mathcal{K}}]^{FM_{nw}} \leq [\mathcal{L}^1_{\mathcal{K}}]^{SM_{nw}}
\end{aligned} \tag{35}$$

The problem given in (35) has at least one solution, which follows from the same argument as the proof of proposition 2 and from the fact that (29) is a FM_{nw} rule that satisfies $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_{nw}} \leq [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$.

From corollary 2 we observe that, given the weights of the votes (every vote is equally weighted or they are weighted according to the weights of the countries in the social loss function in the union), flexible majority rules are better than simple majority rules.[15] The intuition underlying the advantages of flexible majority rules runs as follows. It is socially desirable for small interest rate changes to be possible if only a small part of the union is affected by a shock. This is not possible under simple majority rules, because

[15] Comparing the FM_{nw} rule and the SM_w rule, we observe that without specifying $p_{\mathcal{K}}^{\nu}$, g_k and $\Delta i_{\mathcal{K}}^{\nu}(.)$ the relationship is ambiguous (as indicated by the example in section 8).

the 50% majority always fully determines the monetary policy. In contrast, applying flexible majority rules means that minorities can also change the interest rate to a small degree. Additionally, for the social optimum large interest rate changes should only be possible if a large part of the union is really affected by a shock. But again, simple majority rules already provide the possibility for large interest rate changes if only a percentage less more than 50% of the union is affected. Under flexible majority rules, it is the case that the larger the interest rate change , the larger is the required share of votes is. This means that large interest rate changes can require a share of votes larger than 50%.

5 Discussion and Robustness

Our investigation suggests that flexible majority rules might be a useful tool for central banks. In Appendix B we provide a detailed example how the various decision rules can be computed.

In this section we address a variety of conceptual and practical issues which need to be dealt with. First, allowing minorities to initiate a change of the interest rates may invite cycling in a dynamic setting, since interest rate changes might be revised immediately. Such undesirable cycling can be avoided by restricting flexible majority rules to majorities or a revision rule. A revision rule stipulates that interest rate change reversals within a particular time frame, say a year, require a share of supporting votes larger than 50% or opposing votes for the initial interest rate change. One still has to worry and eliminate strategic voting under such reversal rules.[16]

Second, the shock scenarios can be more complicated. For instance, the sign of shocks can be different across countries. In such cases, the direction of the interest rate change must be determined by the relative size of votes supporting one direction. The increment change is the determined by the flexible majority rule.

Third, we have assumed that if the members of the council have registered the overall shock and know whether they are affected, they will want to implement the same change in the interest rate. But in a given shock scenario preferences may be different and different members may have different opinions about the appropriate change in interest rate. Such a scenario makes the optimal rule more complicated. While it is possible to calculate flexible majority rules for such cases, in practice one might want to opt for

[16]Details and design of such rules are available upon request.

a simple step function, to implement flexible majority rules, i.e. only stipulating the size of the required majority for a sequence of normalised interest rate changes 0.25%, 0.50%, 0.75%, etc.

6 Conclusion

Our discussion suggests that majority rules can be improved by making the size of the majority dependent on the proposal. This improvement will apply not only to the expected social losses but also to every single shock scenario, if the flexible majority rule is chosen properly. We have also shown that there is a first-best flexible majority rule with weighted votes.

Our investigation of flexible majority for central banks, however, can only be the beginning of the underlying research agenda. There are a variety of conceptual and practical issues that await further research effort. Nevertheless, the present paper suggests that flexible majority rules might improve simple majority rules by a considerable margin.

7 Appendix A

Proof of Proposition 1:

Since $[\Delta\hat{i}^{\nu}_{\mathcal{K}}]^{FM_w} = G^{+}_{\mathcal{K}}\Delta i^0_{\mathcal{K}}$ is strictly increasing in $G^{+}_{\mathcal{K}}$ and $[\Delta\hat{i}^{\nu}_{\mathcal{K}}]^{FM_w} = G^{+}_{\mathcal{K}}\Delta i^1_{\mathcal{K}}$ is strictly decreasing in $G^{+}_{\mathcal{K}}$, we can invert $[\Delta\hat{i}^{\nu}_{\mathcal{K}}]^{FM_w}$ separately for $\nu = 0$ and $\nu = 1$. Together, we can define

$$\alpha^{FM_w}(\Delta\hat{i}^{\nu}_{\mathcal{K}}) = \begin{cases} \left([\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}\right)^{-1}(\Delta\hat{i}^{0}_{\mathcal{K}}) & \text{for} \quad \nu = 0 \\ \left([\Delta\hat{i}^{1}_{\mathcal{K}}]^{FM_w}\right)^{-1}(\Delta\hat{i}^{1}_{\mathcal{K}}) & \text{for} \quad \nu = 1 \end{cases} \tag{36}$$

which is strictly decreasing for $\nu = 0$, strictly increasing for $\nu = 1$ and $0 \leq \alpha^{FM_w} \leq 1$. Consider $\nu = 0$. By construction of α^{FM_w}, in every shock scenario $[\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}$ will be implemented, as the affected countries want to change the interest rate up to $\Delta\hat{i}^{0}_{\mathcal{K}} = \Delta i^{0}_{\mathcal{K}} \geq [\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}$. However the FM_w rule restricts the possible change of the interest rate with a majority of $G^{+}_{\mathcal{K}}$ to the upper bound $[\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}$. We next observe that every proposal $\Delta\hat{i}^{0}_{\mathcal{K}}$ with $\Delta\hat{i}^{0}_{\mathcal{K}} < [\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}$ would lose against a proposal $\Delta\tilde{i} \in (\Delta\hat{i}^{0}_{\mathcal{K}}, [\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}]$ in a pairwise decision, because all countries where the shock has occurred, strictly prefer to vote for $\Delta\tilde{i}$ and, due to the construction of α^{FM_w}, $\Delta\tilde{i}$ would again be adopted, because $\Delta\tilde{i}$ needs a majority lower than or equal to $G^{+}_{\mathcal{K}}$ to be accepted. Therefore, there will be an ascending pairwise ballot until $[\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}$ is reached. If $\nu = 1$, reversing the inequalities and changing $[\Delta\hat{i}^{0}_{\mathcal{K}}]^{FM_w}$ in $[\Delta\hat{i}^{1}_{\mathcal{K}}]^{FM_w}$, the argument remains the same. Altogether, α^{FM_w} minimises $E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_w}\right]$ because $E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_w}\right]$ is a $2^{N+1}-1$ dimensional paraboloid in $\vec{\Delta}\hat{i} = (\Delta\hat{i}^{0}_{\mathcal{N}}, ..., \Delta\hat{i}^{0}_{\{\emptyset\}}, ..., \Delta\hat{i}^{1}_{\mathcal{N}})$.

■

Proof of Proposition 2:

This problem has at least one solution because $\tilde{\mathcal{L}}(\vec{\Delta}\hat{\hat{i}}) = \sum_{n=0}^{N}\sum_{j_n=1}^{\binom{N}{n}} p^0_{\mathcal{K}^n_{j_n}}[\mathcal{L}^0_{\mathcal{K}^n_{j_n}}]^{FM_{nw}}$, with $\vec{\Delta}\hat{\hat{i}} = (\Delta\hat{\hat{i}}(0), ..., \Delta\hat{\hat{i}}(N))$ is continuous in $[\mathrm{IR}^{+}_{0}]^{N+1}$ and for $\tilde{\mathcal{L}}$ there exists a $\vec{\delta} \in [\mathrm{IR}^{+}]^{N+1}$ with $\tilde{\mathcal{L}}(\vec{0}) > \tilde{\mathcal{L}}(\vec{\delta})$, $\tilde{\mathcal{L}}(\vec{\gamma})$ =const. for $\gamma_r \geq \Delta i(1)$ $\forall r \in \{0, 1..., N\}$, $\vec{\gamma} = (\gamma_0, ...\gamma_N)$ and $\delta_r < \gamma_r$.

■

Proof of Proposition 3:

Assume that $\mathcal{K}$ and ν are fixed.

(i) If we compare (14) and (18), we see that for all $G_{\mathcal{K}}^{+} \in (0,1)$ we have $G_{\mathcal{K}}^{+}G_{\mathcal{K}}^{-} < G_{\mathcal{K}}^{+}$ and $G_{\mathcal{K}}^{+}G_{\mathcal{K}}^{-} < G_{\mathcal{K}}^{-}$; since $\mathcal{L}^{FM_w} = \mathcal{L}^{SM_w}$ for $G_{\mathcal{K}}^{+} \in \{0,1\}$, we can conclude that $E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w}\right]$ is strictly lower than $E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w}\right]$.

If we compare (18) and (22), we see that $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} = [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$ if $(G_{\mathcal{K}}^{+} \leq \frac{1}{2} \wedge |\mathcal{K}| \leq \frac{1}{2})$ and if $(G_{\mathcal{K}}^{+} > \frac{1}{2} \wedge |\mathcal{K}| > \frac{N}{2})$. It is also possible that for some $G_{\mathcal{K}}^{+}$ we have first $(G_{\mathcal{K}}^{+} \leq \frac{1}{2} \wedge |\mathcal{K}| > \frac{N}{2})$ and second $(G_{\mathcal{K}}^{+} > \frac{1}{2} \wedge |\mathcal{K}| \leq \frac{N}{2})$ (for example $N = 3$ and $g_1 = 0.1$, $g_2 = 0.2$, $g_3 = 0.7$). The comparison for these two cases gives:

1. $G_{\mathcal{K}}^{+} \leq \frac{1}{2} \wedge |\mathcal{K}| > \frac{N}{2}$

$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} = G_{\mathcal{K}}^{+}\left(\Delta i_{\mathcal{K}}^{\nu}\right)^2 \tag{37}$$
$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}} = G_{\mathcal{K}}^{-}\left(\Delta i_{\mathcal{K}}^{\nu}\right)^2 \tag{38}$$
$$\Longleftrightarrow$$
$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} - [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}} = \left(2G_{\mathcal{K}}^{+} - 1\right)\left(\Delta i_{\mathcal{K}}^{\nu}\right)^2 \leq 0 \tag{39}$$

2. $G_{\mathcal{K}}^{+} > \frac{1}{2} \wedge |\mathcal{K}| \leq \frac{N}{2}$

$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} = G_{\mathcal{K}}^{-}\left(\Delta i_{\mathcal{K}}^{\nu}\right)^2 \tag{40}$$
$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}} = G_{\mathcal{K}}^{+}\left(\Delta i_{\mathcal{K}}^{\nu}\right)^2 \tag{41}$$
$$\Longleftrightarrow$$
$$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} - [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}} = \left(1 - 2G_{\mathcal{K}}^{+}\right)\left(\Delta i_{\mathcal{K}}^{\nu}\right)^2 < 0 \tag{42}$$

Altogether, we can conclude that $E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w}\right]$ is always lower or equal to $E\left[[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}\right]$.

(ii) If we compare (14) and (30) we see again that for any specific shock scenario, the FM_{nw} rule can never be strictly better than the FM_w rule.

In order to compare FM_{nw} and SM_{nw}, we look at the change in interest rate implemented in these cases in a specific shock scenario. We begin with n fixed and $n \leq \frac{N}{2}$. Here in the SM_{nw} case the change in interest rate is 0, while in the FM_{nw} case the change is $[\Delta \hat{i}_{\mathcal{K}_1^n}^{\nu}]^{FM_w}$. Since 0 and $[\Delta \hat{i}_{\mathcal{K}_1^n}^{\nu}]^{FM_w}$ are both in the downward part of the parabola for $\nu = 0$ and the upward part for $\nu = 1$, the FM_{nw} rule is strictly better than SM_{nw} for every $\mathcal{K}$, with $|\mathcal{K}| = n$, $(n > 0)$. If $n > \frac{N}{2}$ SM_{nw} is as good as FM_{nw}, if $\Delta i_{\mathcal{K}_{j_n}^n}^{\nu} \leq [\Delta \hat{i}_{\mathcal{K}_{\binom{N}{n}}^n}^{\nu}]^{FM_w}$. Otherwise FM_{nw} is strictly better than SM_{nw}, because now $\Delta i^{\nu}_{\mathcal{K}^n_{jn}}$ and $[\Delta \hat{i}^{\nu}_{\mathcal{K}^n_{\binom{N}{n}}}]^{FM_w}$ are in the upward part of the parabola for $\nu = 0$ and the downward part for $\nu = 1$. Accordingly, we have shown that $E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_{nw}}\right]$ is always strictly lower than $E\left[[\mathcal{L}^{\nu}_{\mathcal{K}}]^{SM_{nw}}\right]$.

■

8 Appendix B

To illustrate our results we give a simple example, involving three countries. We assume that the weights are given by $g_1 = 0.1$, $g_2 = 0.2$ and $g_3 = 0.7$. Ordering the $G^+_{\mathcal{K}}$ according to (20), we get $G^+_{\mathcal{K}_0} = 0$, $G^+_{\mathcal{K}_1} = 0.1$, $G^+_{\mathcal{K}_2} = 0.2$, $G^+_{\mathcal{K}_3} = 0.3$, $G^+_{\mathcal{K}_4} = 0.7$, $G^+_{\mathcal{K}_5} = 0.8$, $G^+_{\mathcal{K}_6} = 0.9$ and $G^+_{\mathcal{K}_7} = 1$. Furthermore, we assume that $\Delta i\big((-1)^{\nu} G^+_{\mathcal{K}_j}\big) = (-1)^{\nu} \cdot j$ and that all shocks are uniformly distributed, which implies that $p^{\nu}_{\mathcal{K}} = \frac{1}{15}\ \forall\, \mathcal{K}, \nu$. $[\mathcal{L}^{\nu}_{\mathcal{K}}]^{SM_{nw}}$, $[\mathcal{L}^{\nu}_{\mathcal{K}}]^{SM_w}$ and $[\mathcal{L}^{\nu}_{\mathcal{K}}]^{FM_w}$ can then be directly be calculated from (22), (18) and (14). Now we calculate the FM_{nw} rules according to (27) and (35). We use the indication FM_{nw_1} for (27) and FM_{nw_2} for (35). First, we consider (26) and calculate the social loss functions for any specific shock scenario (as we have formulated a symmetric problem, we can restrict ourselves to the case of $\nu = 0$):

$$[\mathcal{L}^0_{\{1\}}]^{FM_{nw}} = \begin{cases} \left(\Delta\hat{\hat{i}}(1) - 0.1\right)^2 + 0.09 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 1 \\ 0.9 & \text{otherwise} \end{cases} \tag{43}$$

$$[\mathcal{L}^0_{\{2\}}]^{FM_{nw}} = \begin{cases} \left(\Delta\hat{\hat{i}}(1) - 0.4\right)^2 + 0.64 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 2 \\ 3.2 & \text{otherwise} \end{cases} \tag{44}$$

$$[\mathcal{L}^0_{\{3\}}]^{FM_{nw}} = \begin{cases} \left(\Delta\hat{\hat{i}}(1) - 2.8\right)^2 + 3.36 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 4 \\ 4.8 & \text{otherwise} \end{cases} \tag{45}$$

$$[\mathcal{L}^0_{\{1,2\}}]^{FM_{nw}} = \begin{cases} \left(\Delta\hat{\hat{i}}(2) - 0.9\right)^2 + 1.89 & \text{if} \quad \Delta\hat{\hat{i}}(2) < 3 \\ 6.3 & \text{otherwise} \end{cases} \tag{46}$$

$$[\mathcal{L}^0_{\{1,3\}}]^{FM_{nw}} = \begin{cases} \left(\Delta\hat{\hat{i}}(2) - 4.0\right)^2 + 4 & \text{if} \quad \Delta\hat{\hat{i}}(2) < 5 \\ 5 & \text{otherwise} \end{cases} \tag{47}$$

$$[\mathcal{L}^0_{\{2,3\}}]^{FM_{nw}} = \begin{cases} \left(\Delta\hat{\hat{i}}(2) - 5.4\right)^2 + 3.24 & \text{if} \quad \Delta\hat{\hat{i}}(2) < 6 \\ 3.6 & \text{otherwise} \end{cases} \tag{48}$$

Obviously, in the case of $\mathcal{K} = \{\emptyset\}$, the best choice of $\Delta\hat{\hat{i}}(0)$ is $\Delta\hat{\hat{i}}(0) = 0$ and in the case of $\mathcal{K} = \{1, 2, 3\}$ $\Delta\hat{\hat{i}}(3) = 7$ is the best choice. Summing up the loss functions for $n = 1$ and $n = 2$ respectively, we get:

$$\mathcal{L}^0_{|\mathcal{K}|=1} = \left[\mathcal{L}^0_{\{1\}}\right]^{FM_{nw}} + \left[\mathcal{L}^0_{\{2\}}\right]^{FM_{nw}} + \left[\mathcal{L}^0_{\{3\}}\right]^{FM_{nw}} =$$

$$= \begin{cases} 3(\Delta\hat{\hat{i}}(1) - 1.1)^2 + 8.47 & \text{if} \quad \Delta\hat{\hat{i}}(1) < 1 \\ 2(\Delta\hat{\hat{i}}(1) - 1.6)^2 + 7.78 & \text{if} \quad 1 \leq \Delta\hat{\hat{i}}(1) < 2 \\ (\Delta\hat{\hat{i}}(1) - 2.8)^2 + 7.46 & \text{if} \quad 2 \leq \Delta\hat{\hat{i}}(1) < 4 \\ 8.9 & \text{if} \quad \Delta\hat{\hat{i}}(1) \geq 4 \end{cases} \tag{49}$$

and

$$\mathcal{L}^0_{|\mathcal{K}|=2} = \left[\mathcal{L}^0_{\{1,2\}}\right]^{FM_{nw}} + \left[\mathcal{L}^0_{\{1,3\}}\right]^{FM_{nw}} + \left[\mathcal{L}^0_{\{2,3\}}\right]^{FM_{nw}} =$$

$$= \begin{cases} 3(\Delta\hat{\hat{i}}(2) - 3.4\overline{3})^2 + 19.73\overline{6} & \text{if} \quad \Delta\hat{\hat{i}}(2) < 3 \\ 2(\Delta\hat{\hat{i}}(2) - 4.7)^2 + 14.52 & \text{if} \quad 3 \leq \Delta\hat{\hat{i}}(2) < 5 \\ (\Delta\hat{\hat{i}}(2) - 5.4)^2 + 14.54 & \text{if} \quad 5 \leq \Delta\hat{\hat{i}}(2) < 6 \\ 14.9 & \text{if} \quad \Delta\hat{\hat{i}}(2) \geq 6 \end{cases} \tag{50}$$

We see at once that $\mathcal{L}^0_{|\mathcal{K}|=1}$ is minimised at $\Delta\hat{\hat{i}}(1) = 2.8$ and $\mathcal{L}^0_{|\mathcal{K}|=2}$ is minimised at $\Delta\hat{\hat{i}}(2) = 4.7$. Thus the FM_{nw_1} rule is given by:

$$[\Delta\hat{i}]^{FM_{nw_1}}(0) = 0 \quad [\Delta\hat{i}]^{FM_{nw_1}}(1) = 2.8 \tag{51}$$

$$[\Delta\hat{i}]^{FM_{nw_1}}(2) = 4.7 \quad [\Delta\hat{i}]^{FM_{nw_1}}(3) = 7 \tag{52}$$

For FM_{nw_2} the solution is calculated numerically. This is feasible, as we know that the solution has to be between 0 and the beginning of the constant part of $\mathcal{L}^0_{|\mathcal{K}|=1}$ for $n = 1$ and between 0 and the beginning of the constant part of $\mathcal{L}^0_{\mathcal{K}_{|\mathcal{K}|=2}}$ for $n = 2$. We obtain $[\Delta\hat{i}]^{FM_{nw_2}}(1) = 0.2$ and a degenerated solution for $[\Delta\hat{i}]^{FM_{nw_2}}(2)$, with $[\Delta\hat{i}]^{FM_{nw_2}}(2) = 4.8$ or $[\Delta\hat{i}]^{FM_{nw_2}}(2) = 5.4$. From this we get two FM rules indicating the first as $FM_{nw_{21}}$ and the second as $FM_{nw_{22}}$. They are given by:

$$[\Delta\hat{i}]^{FM_{nw_{21}}}(0) = 0 \quad [\Delta\hat{i}]^{FM_{nw_{21}}}(1) = 0.2 \tag{53}$$

$$[\Delta\hat{i}]^{FM_{nw_{21}}}(2) = 4.8 \quad [\Delta\hat{i}]^{FM_{nw_{21}}}(3) = 7 \tag{54}$$

and

$$[\Delta\hat{i}]^{FM_{nw_{22}}}(0) = 0 \quad [\Delta\hat{i}]^{FM_{nw_{22}}}(1) = 0.2 \tag{55}$$

$$[\Delta\hat{i}]^{FM_{nw_{22}}}(2) = 5.4 \quad [\Delta\hat{i}]^{FM_{nw_{22}}}(3) = 7 \tag{56}$$

For completeness, we also give the shares of votes necessary for the FM_w rule:

$$\alpha^{FM_w}(\Delta\hat{i}^\nu_\mathcal{K}) = \begin{cases} 0 & \text{if} & & & |\Delta\hat{i}^\nu_\mathcal{K}| & = & 0 \\ 0.1 & \text{if} & 0 & < & |\Delta\hat{i}^\nu_\mathcal{K}| & \leq & 0.1 \\ 0.2 & \text{if} & 0.1 & < & |\Delta\hat{i}^\nu_\mathcal{K}| & \leq & 0.4 \\ 0.3 & \text{if} & 0.4 & < & |\Delta\hat{i}^\nu_\mathcal{K}| & \leq & 0.9 \\ 0.7 & \text{if} & 0.9 & < & |\Delta\hat{i}^\nu_\mathcal{K}| & \leq & 2.8 \\ 0.8 & \text{if} & 2.8 & < & |\Delta\hat{i}^\nu_\mathcal{K}| & \leq & 4.0 \\ 0.9 & \text{if} & 4.0 & < & |\Delta\hat{i}^\nu_\mathcal{K}| & \leq & 5.4 \\ 1.0 & \text{if} & 5.4 & < & |\Delta\hat{i}^\nu_\mathcal{K}| & \leq & 7 \end{cases} \tag{57}$$

Now we can compare all the different decision rules. The numbers are given in table 1 and table 2. Comparing the columns for FM_{nw_1}, $FM_{nw_{21}}$, $FM_{nw_{22}}$ and SM_w, we see that we do indeed have cases where SM_w is better than FM_{nw} and vice versa. For example FM_{nw_1} is worse than SM_w for $G^+_\mathcal{K} = 0.1$ and better for $G^+_\mathcal{K} = 0.7$. $FM_{nw_{2j}}$ is worse than SM_w for $G^+_\mathcal{K} = 0.3$ and better for $G^+_\mathcal{K} = 0.2$ ($j = 1, 2$). It can also be seen that in a specific shock scenario FM_{nw_1} can be worse than SM_{nw}. Take for example $G^+_\mathcal{K} = 0.2$. If we calculate the expected social losses for all decision rules, we get:

$$\begin{array}{llllll} E\left[[\mathcal{L}^\nu_\mathcal{K}]^{SM_{nw}}\right] = 54 & & E\left[[\mathcal{L}^\nu_\mathcal{K}]^{FM_{nw_1}}\right] = 43.96 & & E\left[[\mathcal{L}^\nu_\mathcal{K}]^{FM_{nw_{21}}}\right] = 50.88 \\ E\left[[\mathcal{L}^\nu_\mathcal{K}]^{FM_{nw_{22}}}\right] = 50.88 & & E\left[[\mathcal{L}^\nu_\mathcal{K}]^{SM_w}\right] = 34 & & E\left[[\mathcal{L}^\nu_\mathcal{K}]^{FM_w}\right] = 26.44 \end{array}$$

Referring only to the expected loss function, we see that as generally shown in proposition 3 SM_{nw} is worse than FM_{nw}. Furthermore, in this specific example SM_w is better than FM_{nw}. But the opposite is also possible, as can be seen if we take another example where $g_j = g_k \; \forall j, k \in \mathcal{N}$. Here, the FM_{nw} rule coincides with FM_w and therefore $E\left[[\mathcal{L}^\nu_\mathcal{K}]^{FM_{nw}}\right]$ must be less than $E\left[[\mathcal{L}^\nu_\mathcal{K}]^{SM_w}\right]$. A graphical illustration of corollary 2 and the comparison of the different decision rules for a specific shock scenario is given in figures 1 and 2. We have plotted the social losses of the different decision rules versus every single shock scenario.

$G^+_{\mathcal{K}_j}$	$\Delta i\left(G^+_{\mathcal{K}_j}\right)$	n	$[\mathcal{L}^0_{\mathcal{K}}]^{SM_{nw}}$	$[\mathcal{L}^0_{\mathcal{K}}]^{FM_{nw_{21}}}$	$[\mathcal{L}^0_{\mathcal{K}}]^{FM_{nw_{22}}}$	$[\mathcal{L}^0_{\mathcal{K}}]^{FM_{nw_1}}$	$[\mathcal{L}^0_{\mathcal{K}}]^{SM_w}$	$[\mathcal{L}^0_{\mathcal{K}}]^{FM_w}$
0	0	0	0	0	0	0	0	0
0.1	1	1	0.1	0.1	0.1	0.9	0.1	0.09
0.2	2	1	0.8	0.68	0.68	3.2	0.8	0.64
0.3	3	2	6.3	6.3	6.3	6.3	2.7	1.89
0.7	4	1	11.2	10.12	10.12	3.36	4.8	3.36
0.8	5	2	5	4.64	5	4.49	5	4
0.09	6	2	3.6	3.6	3.24	3.73	3.6	3.24
1	7	3	0	0	0	0	0	0

Table 1:
This table gives the different values for the different decision rules and all single shock scenarios where $\nu = 0$.

$G^+_{\mathcal{K}_j}$	$\Delta i\left(-G^+_{\mathcal{K}_j}\right)$	n	$[\mathcal{L}^1_{\mathcal{K}}]^{SM_{nw}}$	$[\mathcal{L}^1_{\mathcal{K}}]^{FM_{nw_{21}}}$	$[\mathcal{L}^1_{\mathcal{K}}]^{FM_{nw_{22}}}$	$[\mathcal{L}^1_{\mathcal{K}}]^{FM_{nw_1}}$	$[\mathcal{L}^1_{\mathcal{K}}]^{SM_w}$	$[\mathcal{L}^1_{\mathcal{K}}]^{FM_w}$
0	0	0	0	0	0	0	0	0
0.1	-1	1	0.1	0.1	0.1	0.9	0.1	0.9
0.2	-2	1	0.8	0.68	0.68	3.2	0.8	0.64
0.3	-3	2	6.3	6.3	6.3	6.3	2.7	1.89
0.7	-4	1	11.2	10.12	10.12	3.36	4.8	3.36
0.8	-5	2	5	4.64	5	4.49	5	4
0.9	-6	2	3.6	3.6	3.24	3.73	3.6	3.24
1	-7	3	0	0	0	0	0	0

Table 2:
This table gives the different values for the different decision rules and all single shock scenarios where $\nu = 1$.

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$[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w} \leq [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w} \leq [\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$

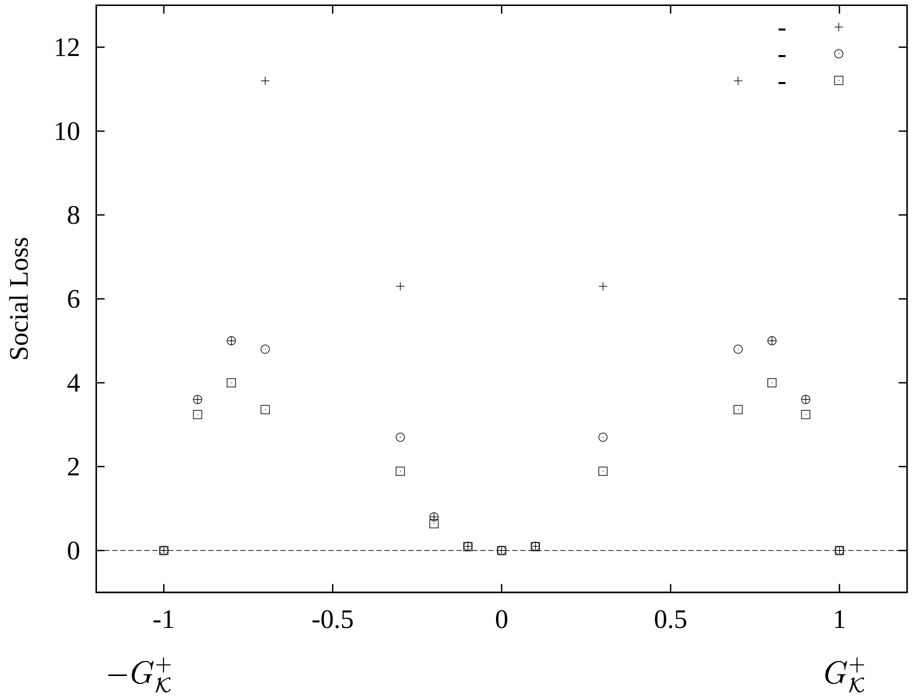

Figure 1:
"+" corresponds to $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$,"$\odot$" to $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w}$ and "$\square$" to $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w}$. As we can see $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_w}$ is always between $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w}$ and $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$.

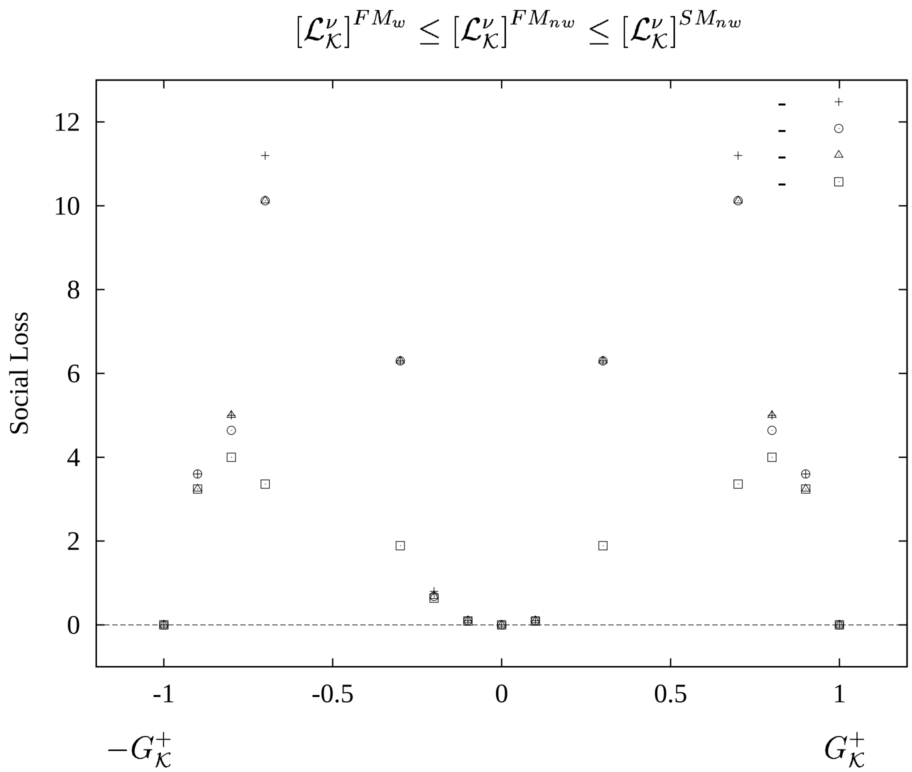

Figure 2:
"+" corresponds with $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$, "$\odot$" to $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_{nw_{21}}}$, "$\triangle$" to $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_{nw_{22}}}$ and "$\square$" to $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w}$. As we can see $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{Fm_{21}}$ and $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_{nw_{22}}}$ is always between $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{FM_w}$ and $[\mathcal{L}_{\mathcal{K}}^{\nu}]^{SM_{nw}}$.