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## INNOVATION AND COMPETITIVE PRESSURE

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## **ABSTRACT**

### **Innovation and Competitive Pressure\***

I analyse the effects of competition on R&D effort (in a non-tournament context) and obtain robust results that hold for a variety of market structures, including markets with and without barriers to entry and markets characterized by either price or quantity competition. The approach encompasses models of direct investment to reduce costs as well as models where cost reduction arises because the agency problem between managers and owners in an asymmetric information context (X-inefficiency) is better resolved. It is found that increasing the number of firms tends to reduce R&D effort, whereas increasing the degree of product substitutability, with or without free entry, increases R&D effort – provided that the total market for varieties does not shrink. Increasing the total market size increases both R&D effort and (weakly) the number of varieties.

JEL Classification: L13 and O30

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# 1 Introduction

Competition is known to have an ambiguous impact on innovation incentives. There is by now a large body of work, going back at least to Schumpeter and continuing with Arrow (1962) and many other scholars, that obtains conflicting results with regard to the effect of competitive pressure on innovation incentives. Schumpeter himself oscillated between thinking that monopoly profit or competitive pressure were the drivers of innovation. Porter (1990), Geroski (1990, 1994), Baily and Gersbach (1995), Nickell (1996), Blundell, Griffith, and Van Reenen (1999), Symeonidis (2002a,b), and Galdón-Sánchez and Schmitz (2002), among others, have collected favorable empirical evidence of the effect of competition on innovation. There is a host of models displaying a variety of results in particular cases. Leading nontournament models like Dasgupta and Stiglitz (1980) and Spence (1984) end up relying on constant elasticity functional forms to obtain results. They both find in such examples that increasing the number of firms reduces innovation effort. Bester and Petrakis (1993) and Qiu (1997) compare innovation incentives in Cournot and Bertrand markets with a linear-quadratic specification. Similarly, models of X-inefficiency in which there is an agency problem between owners and managers rely on very simple and parameterized specifications of market competition. This is the case, for instance, in the linear model of Martin (1993), the examples in Schmidt (1997), and the linear-quadratic model of Raith (2003).

Most of the models display a trade-off between fixed and variable costs. Those include R&D and cost-reduction models as well as agency models. In the latter, the innovation incentive of owners typically translates monotonically, via the incentive scheme of the manager, into the managers' incentives. The owner must pay the manager his reservation utility, the cost of effort, and an information rent (owing to asymmetric information) in order to reduce costs. In this way, for example, more competition may induce a higher cost-reduction effort through an incentive scheme that is more sensitive to performance.<sup>1</sup>

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<sup>1</sup>Hubbard and Palia (1995) and Cuñat and Guadalupe (2002) provide evidence of how competition increases the performance-pay sensitivity, respectively, of CEOs in the U.S. banking industry after deregulation and of CEO, executives, and workers in a panel of U.K. firms after the pound's appreciation in 1996. Competition may also provide information (e.g., on the cost structure of firms) and enlarged opportunities for

In this paper I consider a benchmark symmetric reduced-form non-tournament model, with no spillovers, that displays the trade-off between fixed and variable costs and where in R&D investment has no strategic commitment value. This central scenario is plausible on empirical grounds. Indeed, patents (inducing a patent race or tournament) do not seem to be the major source of returns to innovative activity (Schankerman (1991))<sup>2</sup> and, according to Cohen, “The empirical findings to date do not establish whether the net effect of appropriability on R&D incentives is positive or negative” (1995, p. 230). At the same time, it is possible that strategic effects have been overplayed in the literature. For example, the same source states: “Despite the considerable theoretical attention devoted to strategic interaction, we know surprisingly little about its empirical relevance” (Cohen (1995, p. 234; see also Griliches (1995)). Geroski (1991) hints that strategic effects may be of second-order importance in determining innovation incentives. It is worth remarking that even though R&D investment typically precedes market interaction, this does not mean that it can be used strategically. That is, it does not follow that R&D investment, or contracts with managers that reward effort, are observable and that firms can commit to it.<sup>3</sup> Despite this, I do check the robustness of the results to the strategic commitment effect of R&D and the effect of spillovers. No claim is made about the realism of the symmetry assumption.<sup>4</sup>

Firms compete à la Cournot (with homogenous product) or à la Bertrand (with differentiated products), and innovation is investment in cost reduction (although I look also at product introduction when goods are differentiated). I consider four (classical) different possible measures of enhanced competitive pressure:<sup>5</sup>

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comparison, and therefore stronger incentives. The informational role of competition in enhancing efficiency has been highlighted in a series of models. I will not pursue this line of inquiry in this paper but see Hart (1983), Scharfstein (1988), Hermalin (1992, 1994), and Meyer and Vickers (1985).

<sup>2</sup>Recent empirical analysis does not seem to favor the patent race model (with its first-mover advantage). See Tellis and Golder (1997) and Lieberman and Montgomery (1998).

<sup>3</sup>The evidence on the strategic commitment value of R&D is scant.

<sup>4</sup>See Boone (2000) for an analysis of innovation incentives with asymmetric industry structures.

<sup>5</sup>Sometimes a change from Cournot to Bertrand behavior is interpreted as an increase in competitive pressure. This may be so, since Bertrand equilibria tend to be more competitive than Cournot, but this interpretation need not make sense within a given industry. Indeed, the mode of competition is typically dictated by the structural conditions in the industry (see Vives (1999, Chap.7)).

- with *barriers to entry*, as an increase
  - (1) in the number of competitors or
  - (2) in the degree of product substitutability;
- with *free entry*, as an increase
  - (3) in the total size of the market or
  - (4) in the degree of product substitutability.

In the central scenario considered, individual firms' cost-reduction incentives depend on the output per firm because the value of a reduction in unit costs will increase with the output produced by the firm. Output per firm depends in turn on demand and price-pressure effects. For a given total market size, competition affects the effective market of a firm, its residual demand (a level or size effect), and the elasticity of the residual demand faced by the firm (an elasticity effect). Typically an increase in competition for a given total market size will decrease the residual demand for the firm and will increase the demand elasticity. The first effect will tend to decrease R&D effort because a unit cost reduction will benefit a diminished output, whereas the second will tend to increase R&D effort, because a unit reduction in costs will allow the firm to decrease price with a higher output impact.<sup>6</sup>

I obtain the following results. In a market with barriers to entry:

- Increasing the number of firms tends to reduce R&D effort (either in Cournot or Bertrand) because the residual demand (size) effect dominates the price pressure (elasticity) effect. Exceptions are difficult to find: in Cournot the result holds in the usual case of outputs being strategic substitutes; in Bertrand the result holds for all leading examples (including linear, constant elasticity, constant expenditure, and logit demand systems).
- With Bertrand and product differentiation, increasing the degree of product substitutability increases R&D effort provided the total market for varieties does not shrink.

The reason is that the demand effects and the price-pressure effects both work in the

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<sup>6</sup>See Kamien and Schwartz (1970) and Willig (1987) for related analyses.

same direction. This holds for leading examples such as linear (Shapley–Shubik specification), constant elasticity, and constant expenditure demand systems. With logit there is neither demand effect nor price–pressure effect.

The results in a market with barriers to entry generalize those obtained by (among others) Dasgupta and Stiglitz (1980), Spence (1984), Tandon (1984), and Martin (1993) in the context of a Cournot model with constant elasticity or linear specifications, and by Raith (2003) with price competition and product differentiation à la Salop (1979). Bertrand and Kramarz (2002) and Ebell and Haefke (2003) provide evidence on the output expansion effect of competition.

In a market with free entry:

- In a Cournot homogenous product market, increasing the total market size increases R&D effort (and per–firm output). Increasing the market size has a direct positive impact on R&D effort and output per firm, but at the same time it may increase the free–entry number of firms. However, the latter increases less than proportionately, owing to the reduction in margins, and the direct effect prevails. In fact, the free–entry number of firms may even decrease with market size. In a constant elasticity example with no entry cost, the free–entry number of firms is independent of market size.
- In a Bertrand market with product differentiation:
  - Increasing the total market size increases per–firm output, R&D effort, and (weakly) the number of varieties (entering firms). Both process and product innovation are enhanced in larger markets.
  - Increasing the degree of product substitutability increases R&D effort (and per–firm output) provided the total market does not shrink. The number of varieties introduced may diminish.

The results with free entry formalize Adam Smith’s intuition that the division of labor is limited by the extent of the market: in larger markets, more products are introduced and they are produced at lower cost. Schmookler (1959, 1962) emphasized the role of demand and market size in the innovation incentive. The empirical literature tends to confirm the

role of market size in explaining the incentives to innovate (see Scherer and Ross (1990) and Cohen (1995) for surveys as well as Symeonides (2002a, chap. 6)).<sup>7</sup> Syverson (2003) provides evidence that industries' median productivity levels are increasing in the degree of product substitutability of the industry products. The results here also generalize those obtained by Raith (2003). The result that increasing product substitutability increases innovation effort but may decrease the number of varieties introduced is consistent with the findings in Boone (2000) for symmetric market structures.

The results with free entry suggest also that market integration and opening of markets may yield unambiguous benefits in terms of innovation effort. Indeed, an increase in market size can result from international market integration or the dismantling of barriers to trade. We would thus have a connection between globalization, understood as the general lowering of transport costs and barriers to trade, and innovation effort.<sup>8</sup> Our results in particular are consistent with the findings in Baily and Gersbach (1995) that competition in the global marketplace is what gives companies a productivity advantage.

The plan of the paper is as follows. Section 2 considers the case of markets with barriers to entry and studies price-pressure and demand effects of increasing competition in Cournot and Bertrand markets. Section 3 deals with the free-entry case and performs a comparative statics exercise with market size, the level of entry costs, and the degree of product substitutability. Section 4 explores extensions of the results. Concluding remarks close the paper, and the Appendix collects several proofs and the details of the examples.

## **2 Barriers to entry: Price pressure and demand effects**

In this section I consider Cournot and Bertrand markets with barriers to entry and perform a comparative statics exercise on the number of firms (Cournot and Bertrand) and on the degree of product substitutability (Bertrand). I consider first a Cournot market with homogenous product and then a Bertrand market with differentiated products. In both cases, a robustness exercise to the strategic commitment effect of R&D is performed. The

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<sup>7</sup>Blundell, Griffith, and Van Reenen (1999) provide evidence on the positive impact of market share on innovation output for a panel of British manufacturing firms.

<sup>8</sup>See e.g. Krugman (1995).

section concludes by establishing some connections with the empirical literature.

## 2.1 Cournot competition with homogenous product

Consider an  $n$ -firm Cournot market for a homogenous product with smooth inverse demand  $P(\cdot)$ ,  $P' < 0$ . Firm  $i$  can invest  $z_i$  to reduce its constant marginal cost of production  $c_i$  according to a smooth function  $c_i = c(z_i)$  with  $c(z) > 0$ ,  $c'(z) < 0$ , and  $c''(z) > 0$  for all  $z > 0$ . The profit to firm  $i$  is given by

$$\pi_i = P(X) x_i - c(z_i) x_i - z_i,$$

where  $x_i$  is the output of the firm and  $X$  is total output. For convenience we will think of  $n$  as a continuous variable but all results hold with  $n$  discrete.

Consider a simultaneous-move game where firm  $i$ , for each  $i$ , chooses  $(z_i, x_i)$ . This can be interpreted also as an open-loop strategy in a two-stage investment-quantity game. Consider an (interior) symmetric equilibrium  $(x, z)$  of the game. From the first-order conditions (FOCs) we have  $P(xn) + xP'(xn) - c(z) = 0$ , which yields a Cournot equilibrium  $x(z, n)$ , and  $-xc'(z) - 1 = 0$ . Let

$$\phi(z, n) \equiv -x(z, n)c'(z) - 1.$$

If  $(n+1)P' + nP''x < 0$  (which implies in turn uniqueness and local stability of the Cournot equilibrium  $x(z, n)$ ; see Vives (1999, Sec. 4.3.1)) then  $\frac{\partial x}{\partial z} = -\frac{-c'}{(n+1)P' + xnP''}$ . We have thus  $\frac{\partial \phi}{\partial z} = -xc'' - c'\frac{\partial x}{\partial z} < 0$  if and only if

$$D \equiv ((n+1)P' + nP''x)c''x + (c')^2 < 0.$$

Note that  $D < 0$  implies  $(n+1)P' + nP''x < 0$ . We thus conclude that the symmetric equilibrium is unique if  $D < 0$ . From  $\frac{dz}{dn} = -\left(-c'\frac{\partial x(z, n)}{\partial n}\right) / \frac{\partial \phi}{\partial z}$  and  $\frac{\partial \phi}{\partial z} < 0$  we obtain that

$$\text{sign} \left\{ \frac{dz}{dn} \right\} = \text{sign} \left\{ \frac{\partial x(z, n)}{\partial n} \right\}.$$

This is the output effect, there is more incentive to invest in reducing costs (with  $c' < 0$ ) when the output is larger. Furthermore,  $\frac{\partial x}{\partial n} = -x\frac{P' + xP''}{(n+1)P' + xnP''}$  and therefore

$$\text{sign} \left\{ \frac{\partial x(z, n)}{\partial n} \right\} = \text{sign} \{P' + P''x\}.$$

In conclusion, the innovation effort  $z$  increases or decreases with the number of firms according to the impact on output, and this in turn depends on whether Cournot best responses are increasing or decreasing (i.e., on whether  $P' + P''x$  is positive or negative at the equilibrium).<sup>9</sup> The following proposition summarizes the result.

**Proposition 1** *Let  $P' < 0$  and let  $c' < 0$ , and  $c'' > 0$ . Consider a symmetric interior equilibrium  $(x^*, z^*)$ . If  $D \equiv ((n + 1)P' + nP''x)c''x + (c')^2 < 0$  (at the candidate equilibrium) then the symmetric equilibrium is unique and*

$$\text{sign} \left\{ \frac{dz^*}{dn} \right\} = \text{sign} \left\{ \frac{dx^*}{dn} \right\} = \text{sign} \{ P'(nx^*) + P''(nx^*)x^* \}.$$

The normal case is that best responses are decreasing. Indeed, the conditions for upward sloping best replies in Cournot oligopoly are quite stringent. Letting

$$E \equiv -XP''(X)/P'$$

we have upward sloping best responses (with constant marginal costs) if  $n + 1 > E > n$ . The first inequality yields uniqueness (and stability) of the symmetric Cournot equilibrium ( $(n + 1)P' + nP''x < 0$  is equivalent to  $n + 1 > E$ ); the second yields upward sloping best responses (see Seade (1980) and Vives (1999, Sec. 4.3.1)). In practice this means that upward sloping best responses will hold, if at all, for a single change in the number of firms  $n$ . This is clearly the case if  $E$  is constant. Then demands are of the form  $P(X) = a - bX^{1-E}$  if  $E \neq 1$  or  $P(X) = a - b \log X$  if  $E = 1$ , with  $a \geq 0$  and  $b > 0$  if  $E \leq 1$  and  $b < 0$  if  $E > 1$ , and they include linear and constant elasticity. If  $E$  is constant and we require  $n + 1 > E$  for all  $n \geq 1$ , then  $2 > E$  and only  $2 > E > 1$  is possible.

A sufficient condition to have upward sloping best replies is that  $P$  is log-convex and costs zero. Indeed, we need to have  $P' + x_i P'' > 0$  only along best responses (and  $P$  log-convex with zero costs is sufficient to ensure that). Then the game is log-supermodular, and extremal individual Cournot equilibrium outputs are increasing in  $n$ . However, for  $c > 0$ ,  $P(X) - c$  cannot be log-convex (Amir (1996); see also Vives (1999, Sec. 4.1)). Positive costs bias best responses decisively toward being downward sloping.

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<sup>9</sup>Note that  $\text{sign} \left\{ \frac{dz^*}{dn} \right\} = \text{sign} \left\{ \frac{\partial z}{\partial n} \right\}$  because  $\frac{dz^*}{dn} = \frac{\partial z}{\partial z} \frac{dz}{dn} + \frac{\partial z}{\partial n}$  and  $\frac{\partial z}{\partial z} > 0$ . (Alternatively, differentiating the FOCs yields  $\frac{dz}{dn} = -\frac{x^2(P' + xP'')c''}{D} < 0$  so  $\frac{dz}{dn} = \frac{xc'(P' + xP'')}{D} < 0$ , and  $\text{sign} \left\{ \frac{dz^*}{dn} \right\} = \text{sign} \left\{ \frac{dx^*}{dn} \right\} = \text{sign} \{ P'(nx^*) + P''(nx^*)x^* \}$ .)

## Remarks

- Sufficient conditions for  $D < 0$  when  $c'' > 0$  are that  $P' + x_i P'' < 0$  and  $(2P' + xP'')c''x_i + (c')^2 < 0$ . These conditions imply that  $\pi_i = P(X)x_i - c_i x_i - z_i$  is strictly concave in  $(x_i, z_i)$ .<sup>10</sup>
- Under the assumptions of Proposition 1, it is easily checked that equilibrium profits  $\pi_n^*$  are decreasing in  $n$ . Indeed, with  $D < 0$  we have that

$$\text{sign}\{d\pi_n^*/dn\} = \text{sign}\{(2P' + xP'')c''x_i + (c')^2\},$$

and the second-order necessary condition yields  $(2P' + xP'')c''x_i + (c')^2 \leq 0$ . Profits are strictly decreasing in  $n$  if  $\pi_i = P(X)x_i - c_i x_i - z_i$  is strictly concave in  $(x_i, z_i)$ .

**Examples** The models of Dasgupta and Stiglitz (1980) and Tandon (1984) are particular cases of Proposition 1.

Constant elasticity (Dasgupta and Stiglitz (1980)). Let  $P(X) = bX^{-\varepsilon}$  ( $a = 0$ ,  $E - 1 = \varepsilon > 0$ ) and let  $c(z) = \alpha z^{-\gamma}$ . The parameter  $\alpha$  can be interpreted as the underlying scientific base in the industry, while the elasticity  $\gamma$  of  $c(\cdot)$  would indicate innovation opportunities in the industry (with a higher  $\gamma$  increasing opportunities). The condition  $n + 1 > E > n$  becomes in this case  $n > \varepsilon > n - 1$ . Assume that  $\varepsilon(1 + \gamma)/\gamma \geq n > \varepsilon$  (this implies that  $D < 0$ ); then there is a unique symmetric equilibrium with

$$z^* = [b(\gamma/n)^\varepsilon \alpha^{\varepsilon-1} (1 - \varepsilon/n)]^{1/(\varepsilon - \gamma(1-\varepsilon))}$$

and

$$x^* = (1/\gamma\alpha) [b(\gamma/n)^\varepsilon \alpha^{\varepsilon-1} (1 - \varepsilon/n)]^{(1+\gamma)/(\varepsilon - \gamma(1-\varepsilon))}.$$

If we require that  $n > \varepsilon$  for all  $n \geq 1$ , then  $z^*$  and  $x^*$  increase with  $n$  only when going from monopoly to duopoly. Total output  $nx^*$  and industry R&D expenditure  $nz^*$  both

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<sup>10</sup>Profits  $\pi_i$  are strictly concave in  $(x_i, z_i)$  if  $c'' > 0$ ,  $2P' + x_i P'' < 0$ , and  $(2P' + xP'')c''x_i + (c')^2 < 0$ . If  $P' + x_i P'' < 0$  then a sufficient condition to have that  $(2P' + xP'')c''x_i + (c')^2 < 0$  is that  $c(\cdot)$  is sufficiently convex, that is,  $-c''x/c' > c'/P' > 0$ . Strict concavity plus a mild boundary condition implies the existence of an interior equilibrium.

increase with  $n$ . R&D intensity  $\frac{z^*}{p^*x^*} = \gamma \left(1 - \frac{\varepsilon}{n}\right) \left(\frac{\alpha}{\beta}\right)^{\varepsilon-1}$  increases with  $n$  and with  $\gamma$ . It is immediate also that  $z^*$  and profit  $\pi^*$  increase (decrease) with  $\alpha$  if  $\varepsilon > 1$  ( $\varepsilon < 1$ ).

Linear demand (Tandon (1984)). Consider a market with linear demand  $p = a - bX$  and  $c(z) = a - \beta z^\delta$ . We need  $\delta < \frac{1}{2}$  to guarantee strict concavity of profits of firm  $i$  with respect to  $x_i$  and  $z_i$  (if  $\delta < 1$  then  $c(\cdot)$  is strictly convex). Then  $z^* = \left(\frac{\delta\beta^2}{b(n+1)}\right)^{1/(1-2\delta)}$  and  $x^* = \left(\frac{\beta}{(n+1)b}\right) \left(\frac{\delta\beta^2}{b(n+1)}\right)^{\delta/(1-2\delta)}$  are both decreasing in  $n$  for  $\delta < \frac{1}{2}$ , while R&D intensity  $z^*/p^*x^*$  may decrease or increase with  $n$  (it decreases for  $\delta \in (\frac{1}{5}, \frac{1}{2})$ ).

**Multiple equilibria** We may use lattice-theoretic methods to extend Proposition 1 in order to encompass multiple equilibria and remove the regularity conditions, as long as we restrict attention to extremal equilibria. All that is needed is downward sloping demand and a decreasing innovation function plus some mild boundary conditions. The following proposition states the result and the proof is in the Appendix.

**Proposition 2** *Let  $P' < 0$  and  $c' < 0$ , and let the following boundary conditions hold: There exist  $\bar{c} > \underline{c} > 0$  and  $\bar{X} > 0$  such that  $\bar{c} > c(z) > \underline{c} > 0$ ,  $c'(0+) = -\infty$ ,  $c'(z) \rightarrow 0$  as  $z \rightarrow \infty$ ,  $P(xn) \leq \underline{c}$  if  $xn \geq \bar{X}$ , and  $\lim_{x \rightarrow 0} \{P(xn) + xP'(xn)\} \geq \bar{c}$ . Consider an extremal symmetric interior equilibrium  $(x^*, z^*)$ . Then  $x^*$  and  $z^*$  are strictly decreasing (increasing) in  $n$  if Cournot best replies are strictly decreasing (increasing).*

Downward sloping demand ensures that for a given symmetric investment profile  $z$  there exist extremal symmetric Cournot equilibria  $\underline{x}(z, n)$  and  $\bar{x}(z, n)$  that are increasing in  $z$  (Amir and Lambson (2000), Vives (1999, pp. 106–107)). This means that there exist extremal symmetric equilibria in the game. It can be shown, under the boundary assumptions, that the required regularity condition characterizing the equilibrium (i.e.,  $\phi(z, n)$  decreasing in  $z$ ) is fulfilled at a symmetric interior extremal equilibrium. It follows then as before that  $z$  is strictly increasing (decreasing) in  $n$  if and only if the extremal  $x(z, n)$  is strictly increasing (decreasing) in  $n$ , and this, in turn, depends on the slopes of Cournot best replies.

### 2.1.1 Strategic commitment effects

It may be asked if the results are robust with respect to strategic effects. Toward this end we analyze the subgame-perfect equilibria (SPE) of the two-stage game where firms first invest in cost reduction and then compete in quantities. Denote by  $x^*(z_i, z_{-i})$ ,  $i = 1, \dots, n$ , a second-stage Cournot equilibrium for a given investment profile and let

$$V(z_i, z_{-i}) \equiv P(X^*(z))x_i^*(z) - c(z_i)x_i^*(z) - z_i$$

be the associated profit for firm  $i$ . The following proposition strengthens the requirements on demand to ensure that investments in the first stage are strategic substitutes ( $\frac{\partial^2 V}{\partial z_i \partial z_j} < 0$ ,  $j \neq i$ ) and that increasing  $n$  reduces both output and innovation effort. When investments are strategic substitutes, increasing the number of firms will tend to decrease innovation effort of any firm because the aggregate investment of rivals increases. The following proposition states the result formally (with proof in the Appendix).

**Proposition 3** *Consider a symmetric interior SPE of the two-stage game:  $(z^*, \{x^*(z_i, z_{-i})\}_{i=1}^n)$ . Suppose that  $P'' \leq 0$  and that  $-P'$  is log-concave (i.e.,  $P'P''' - (P'')^2 \leq 0$ ). Then investments are strategic substitutes at the first stage, and we have  $\frac{dz^*}{dn} < 0$  and  $\frac{dx^*}{dn} < 0$ .*

Assume for the rest of this section that  $E$  is constant,  $E < 1 + n$ ,  $n > 1$ , and  $c(\cdot)$  is sufficiently convex ( $-c''x/c' > c'/((1 + \min(n - E, 0))P') > 0$ ). Then<sup>11</sup>

$$\text{sign} \left\{ \frac{dz^*}{dn} \right\} = \text{sign} \{ E - 2(n - E)^2 \}.$$

Therefore,  $\frac{dz^*}{dn} < 0$  for  $E \leq 0$  (or  $P'' \leq 0$ )<sup>12</sup> and  $\frac{dz^*}{dn} > 0$  for  $1 + n > E > n$  (strategic complementarity at the output stage). Note that we could have  $\frac{dz^*}{dn} > 0$  for  $E$  close to  $n$  and  $0 < E < n$ , i.e., with strategic substitutes at the output stage.<sup>13</sup> This is the case in the

<sup>11</sup>It can be checked after some tedious computations (see Suzumura (1995)) that with these assumptions  $\frac{dz^*}{dn} = -\frac{\partial \varphi / \partial n}{\partial \varphi / \partial z}$  with  $\partial \varphi / \partial z = -(1 + G(x, n))(c''x + (c')^2 / (1 + n - E)P') < 0$  and  $\partial \varphi / \partial n = xc'(z)(n - 1)(2(n - E)^2 - E) / (1 + n - E)^2 n^2$ .

<sup>12</sup>This actually follows from Proposition 2 because  $-P'$  is log-concave if  $E$  is constant and  $E \leq 0$  (i.e.,  $P'' \leq 0$ ).

<sup>13</sup>With  $E$  constant and  $E < n$  (strategic substitutes at the Cournot stage), investments are also strategic substitutes at the investment stage.

constant elasticity demand model considered by Spence (1984). Then  $E = 1 - \varepsilon$  and, with an exponential innovation function (as in the following example),  $z^*$  increases from  $n = 1$  to  $n = 2$  for  $\varepsilon = 1/2$ ; otherwise,  $z^*$  is decreasing with  $n$ . Note that for  $\varepsilon < 1$  and  $1 \leq n \leq 2$ ,  $E - 2(n - E)^2 > 0$  (whereas for  $n \geq 3$  it is negative).

**An agency model with linear demand (Martin (1993))** Here every firm has an owner and a manager and the manager's unobservable effort reduces cost. The constant marginal cost of firm  $i$  is given by

$$c(\theta_i) = m + \theta_i e^{-l_i}$$

for  $m > 0$ ,  $\theta_i$  a random variable (IID across firms) with compact positive support  $[\underline{\theta}, \bar{\theta}]$ , and  $l_i$  the labor input (effort) of the firm's manager. The manager observes  $\theta_i$  and knows  $l_i$  but the owner does not. The latter sets up an incentive scheme with a cost target  $c(\theta_i)$  and a payment schedule  $\varphi(\theta_i)$ . The interpretation is that, given a reported efficiency  $\theta_i$ , the manager must achieve the cost target  $c(\theta_i)$  in order to obtain the compensation  $\varphi(\theta_i)$ . The utility of the manager equals the compensation minus the disutility of effort  $\lambda l_i$ , where  $\lambda > 0$ . It is easy to check that an incentive-feasible compensation schedule must satisfy  $\varphi(\theta_i) = \lambda \log \frac{\bar{\theta}}{c(\theta_i) - m}$ . Market competition is à la Cournot with linear demand, and in the first stage owners compete by setting cost targets. It is then immediate that the optimal cost target and the compensation are constant. We are thus in the frame of our model with an innovation function (or reduced-form cost function)

$$c(z) = m + \bar{\theta} \exp\{-z/\lambda\}, \lambda > 0.$$

Note that  $c' < 0$  and  $c'' > 0$ . Given that demand is linear ( $E = 0$ ) we have that  $\frac{dz}{dn} < 0$  or that increasing the number of firms reduces cost-reduction effort and increases costs. Indeed, this is the result obtained by Martin (1993).

## 2.2 Bertrand competition with product differentiation

Consider a differentiated product market with  $n$  firms, where each firm produces a different variety. The demand system for the varieties is symmetric and is given by the smooth (whenever demand is positive) and exchangeable functions  $x_i = D_i(p)$ ,  $p = (p_1, \dots, p_n)$ ,

$i = 1, \dots, n$ .<sup>14</sup> Demand is downward sloping  $\frac{\partial D_i}{\partial p_i} < 0$ , products are gross substitutes  $\frac{\partial D_i}{\partial p_j} > 0$ ,  $j \neq i$ , and the Jacobian of the demand system is negative definite. The cost function for firm  $i$  is  $C(x_i; z_i) = c(z_i)x_i$ , with  $c' < 0$  and  $c'' > 0$ . The profits for firm  $i$  are therefore

$$\pi_i = (p_i - c(z_i)) D_i(p) - z_i.$$

Consider the simultaneous-move game in which each firm chooses an investment-price pair. Let  $H(p; \alpha) \equiv D_i(p, \dots, p; \alpha)$  be the demand for a variety when all firms set the same price (the Chamberlinian DD function) where  $\alpha$  is a parameter that affects demand. I will consider  $\alpha = n$ , the number of firms, and  $\alpha = \sigma$ , a measure of product substitutability (typically, the elasticity of substitution between any two products, either the Allen-Hicks or the direct elasticity of substitution). It follows from our assumptions that  $\frac{\partial H}{\partial p}(p; \alpha) \equiv \frac{\partial D_i}{\partial p_i}(p, \dots, p; \alpha) + \sum_{j \neq i} \frac{\partial D_i}{\partial p_j}(p, \dots, p; \alpha) < 0$ .<sup>15</sup> Let  $h(p; \alpha) \equiv \frac{\partial D_i}{\partial p_i}(p, \dots, p; \alpha)$  and note that  $h(p; \alpha) < 0$ . The parameter  $\alpha$  will be suppressed to ease notation in functions when no confusion is possible. A very wide range of demand systems fulfill the assumptions.

Fix a symmetric profile of investment  $z_i = z$  and consider an associated (interior) symmetric Bertrand equilibrium  $p(z, \alpha)$  satisfying

$$L \equiv \frac{p - c}{p} = \frac{1}{\eta},$$

where  $L$  is the Lerner index and

$$\eta \equiv -\frac{p}{H(p)} h(p)$$

is the elasticity of demand for an individual firm. The first-order condition for a symmetric interior equilibrium is  $(p - c) \frac{\partial D_i}{\partial p_i} + D_i = 0$ , or

$$\phi(p; \alpha) \equiv (p - c) h(p; \alpha) + H(p; \alpha) = 0.$$

Therefore, this symmetric equilibrium will be unique if  $\frac{\partial \phi}{\partial p} = (p - c) \frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p} < 0$ . It is then immediate that

$$\text{sign} \left\{ \frac{\partial p(z, \alpha)}{\partial \alpha} \right\} = \text{sign} \left\{ (p - c) \frac{\partial h}{\partial \alpha} + \frac{\partial H}{\partial \alpha} \right\} = \text{sign} \left\{ -\frac{\partial \eta}{\partial \alpha} \right\}.$$

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<sup>14</sup>That is, interchanging the prices of rival goods does not affect the demand for any good (as a function of its own price) and any two goods that sell at the same price have the same demand. Formally, the demand system can be described by a unique demand function for any good depending on its own price and the prices of rivals,  $D_i(p_i; p_{-i}) = D(p_i; p_{-i})$  for all  $i$ .

<sup>15</sup>See Vives (1999, Sec. 6.3).

If  $\alpha = n$  then typically  $\frac{\partial \eta}{\partial n} > 0$ , and increasing the number of firms increases the elasticity of demand and decreases prices. If  $\alpha = \sigma$  then typically  $\frac{\partial \eta}{\partial \sigma} > 0$ , and increasing product substitutability decreases prices.<sup>16</sup> Table 1 provides properties of examples of several commonly used demand systems: linear (Shapley–Shubik (1969) and Bowley (1924) specifications), location (Salop (1979)), constant elasticity, constant expenditure demand systems (with exponential and constant elasticity specifications) and logit. The term  $S$  parameterizes total market size.

A symmetric (interior) equilibrium of the full investment–price game<sup>17</sup> will satisfy the first-order condition for investment:  $-xc'(z) - 1 = 0$  or

$$\Psi(z, \alpha) \equiv -H(p(z, \alpha); \alpha) c'(z) - 1 = 0.$$

It follows that if  $\partial \Psi / \partial z < 0$ , for which a sufficient condition is

$$B \equiv \left( (p - c) \frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p} \right) c'' H + (c')^2 h \frac{\partial H}{\partial p} < 0,$$

then the symmetric equilibrium will be unique (note that  $B < 0$  implies that  $(p - c) \frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p} < 0$ ).

We are now ready to assess the impact of the parameter  $\alpha$  on the equilibrium  $z$ . From  $\frac{dz}{d\alpha} = -\frac{\partial \Psi / \partial \alpha}{\partial \Psi / \partial z}$  we have that

$$\text{sign} \frac{dz}{d\alpha} = \text{sign} \frac{\partial \Psi}{\partial \alpha} = \text{sign} \left\{ -c' \frac{\partial x(z; \alpha)}{\partial \alpha} \right\},$$

where  $x(z; \alpha) \equiv H(p(z, \alpha); \alpha)$  is the equilibrium output per firm in the Bertrand equilibrium for a given  $z$ . This means that, as before, investment increases if and only if output per

<sup>16</sup>Suppose that demands come from a representative consumer with (strictly quasiconcave) utility function  $U(x_0, x)$ , where  $x$  is the vector of differentiated commodities and  $x_0$  is the numéraire (this a generalization of the quasilinear case, for which  $W(x_0, x) = x_0 + U(x)$ ). For a symmetric allocation, denote by  $\sigma$  the (Allen–Hicks) elasticity of substitution between any pair of differentiated goods, by  $\sigma_0$  the elasticity of substitution between the numéraire and a differentiated good, and by  $\eta^I$  the income elasticity of the demand for a differentiated good. Assuming that the latter is bounded, at a symmetric Bertrand equilibrium we have  $\eta = \mu \sigma_0 + (1 - \mu) \sigma (n - 1) n^{-1} + (1 - \mu) \eta^I n^{-1}$ , where  $\mu$  is the expenditure share of the numéraire good. It is clear that  $\eta$  increases with  $\sigma$ . Increasing  $n$  has a more complex effect in the formula, but typically it will (among other effects) increase  $\eta$  by weakly increasing  $\sigma$ . See Benassy (1989) and Vives (1999, Sec. 6.4).

<sup>17</sup>If  $\pi_i = (p_i - c(z_i)) D_i(p) - z_i$  is strictly concave in  $(p_i, z_i)$ , then some mild boundary conditions ensure the existence of an interior equilibrium.

firm increases. The decomposition

$$\frac{\partial x}{\partial \alpha} = \frac{\partial H}{\partial p} \frac{\partial p}{\partial \alpha} + \frac{\partial H}{\partial \alpha}$$

is instructive. The term

$$\frac{\partial H}{\partial p} \frac{\partial p}{\partial \alpha}$$

is the price pressure effect: increasing  $\alpha$  decreases  $p$  (in the leading examples considered with either  $\alpha = n$  or  $\alpha = \sigma$ ), which in turn increases demand. The term

$$\frac{\partial H}{\partial \alpha}$$

is the demand effect: the direct impact of  $\alpha$  on demand. We will see how, when  $\alpha = n$ , the price pressure and the demand effects have different signs— provided that there is a limited market for the differentiated varieties ( $\frac{\partial H}{\partial n} < 0$ ) and the latter typically dominates. On the other hand, if  $\alpha = \sigma$  then typically both the price–pressure effect and the demand effect (weakly) work in favor of more R&D effort. Indeed, there is no presumption that increasing the elasticity of substitution will decrease the symmetric demand for varieties. Proposition 4 summarizes the results so far.

**Proposition 4** *Let the demand system fulfill  $\frac{\partial D_i}{\partial p_i} < 0$  and  $\frac{\partial D_i}{\partial p_j} > 0$  for  $j \neq i$  with negative definite Jacobian, and let  $c' < 0$  and  $c'' > 0$ . Consider a symmetric and interior equilibrium  $(p^*, z^*)$ . Then the following statements hold.*

(i) *If  $B \equiv \left( (p^* - c) \frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p} \right) c'' H + (c')^2 h \frac{\partial H}{\partial p} < 0$  (evaluated at the candidate equilibrium), then the equilibrium is the unique symmetric one and*

$$\text{sign} \left\{ \frac{dz^*}{d\alpha} \right\} = \text{sign} \left\{ \frac{\partial x(z; \alpha)}{\partial \alpha} \right\} = \text{sign} \left\{ \frac{\partial H}{\partial p} \frac{\partial p(z, \alpha)}{\partial \alpha} + \frac{\partial H}{\partial \alpha} \right\},$$

where  $(p(z, \alpha), x(z; \alpha))$  is the symmetric Bertrand equilibrium for given  $\alpha$  and  $z$ .

(ii) *When changing the number of firms  $n$  for linear, constant elasticity, logit, and constant expenditure demand systems, we have  $\frac{\partial H}{\partial n} < 0$  and  $\frac{\partial \eta}{\partial n} > 0$ ; the demand effect dominates the price–pressure effect, and  $\frac{\partial x(z; n)}{\partial n} < 0$ .<sup>18</sup>*

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<sup>18</sup>When  $B < 0$  we have that  $\text{sign} \frac{dz^*}{dn} = \text{sign} \left\{ \frac{\partial H}{\partial p} (p - c) \frac{\partial h}{\partial n} - \frac{\partial H}{\partial n} \left( (p - c) \frac{\partial h}{\partial p} + h \right) \right\}$  and  $\text{sign} \frac{dp^*}{dn} = \text{sign} \left\{ \left( (p - c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n} \right) c'' H + (c')^2 h \frac{\partial H}{\partial n} \right\}$ . A sufficient condition for  $\frac{dz^*}{dn} < 0$  is that  $\frac{\partial h}{\partial n} > 0$  and  $\frac{\partial h}{\partial p} < 0$ .

(iii) When varying product substitutability  $\sigma$  in all cases considered,  $\frac{\partial \eta}{\partial \sigma} > 0$ . For linear (Shapley–Shubik specification), logit, and constant expenditure demand systems,  $\frac{\partial H}{\partial \sigma} = 0$ ; for constant elasticity,  $\frac{\partial H}{\partial \sigma} > 0$ . The logit system (like classical location models) is a boundary case with neither price–pressure nor demand effects so  $\frac{\partial x(z;\sigma)}{\partial \sigma} = 0$ . For the other cases, price–pressure and demand effects work in the same direction and so  $\frac{\partial x(z;\sigma)}{\partial \sigma} > 0$ . For the linear demand specification of Bowley,  $\frac{\partial H}{\partial \sigma} < 0$  and  $\frac{\partial x(z;\sigma)}{\partial \sigma} < 0$ .

### Remarks

- Under the assumptions of Proposition 4 we have that  $\text{sign} \left\{ \frac{dx}{d\alpha} \right\} = \text{sign} \left\{ \frac{\partial x(z;\alpha)}{\partial \alpha} \right\}$  for  $\alpha = n$  and  $\alpha = \sigma$ , because  $\frac{dx}{d\alpha} = \frac{\partial x(z;\alpha)}{\partial z} \frac{dz}{d\alpha} + \frac{\partial x(z;\alpha)}{\partial \alpha}$  and  $\text{sign} \frac{\partial x(z;\alpha)}{\partial z} = \text{sign} \{-c'h\} > 0$ .

It follows, indeed, that innovation effort and individual output move in the same direction.

- Strengthening the condition  $B \equiv \left( (p^* - c) \frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p} \right) c''H + (c')^2 h \frac{\partial H}{\partial p} < 0$  to

$$\widehat{B} \equiv \left( (p^* - c) \frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p} \right) c''H + (c')^2 h^2 \leq 0$$

(note that  $|\partial H/\partial p| < |h|$ ), and assuming  $\frac{\partial H}{\partial n} < 0$  and  $\frac{\partial \eta}{\partial n} > 0$ , yields that profits at the symmetric equilibrium,  $\pi_n^*$ , are strictly decreasing in  $n$ . This follows because

$$\frac{d\pi_n^*}{dn} = (p^* - c) \left( \frac{\partial H}{\partial n} + \frac{dp^*}{dn} \left( \frac{\partial H}{\partial p} - h \right) \right)$$

and  $\frac{d\pi_n^*}{dn} < 0$  if and only if  $-\frac{\partial H}{\partial n} \widehat{B} + \left( \frac{\partial H}{\partial p} - h \right) \left( (p^* - c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n} \right) c''H < 0$  (since  $\frac{\partial H}{\partial p} - h > 0$ ,  $c''H > 0$ , and  $\text{sign} \left\{ (p^* - c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n} \right\} = \text{sign} \left\{ -\frac{\partial \eta}{\partial n} \right\}$ ). Alternatively, with  $B < 0$ , a sufficient condition for  $d\pi_n^*/dn < 0$  is that  $dp^*/dn < 0$ .

- The parameter  $\alpha$  could also be interpreted as “regulatory pressure”. It is then akin to our product substitutability measure with  $\frac{\partial \eta}{\partial \alpha} > 0$  and  $\frac{\partial H}{\partial \alpha} = 0$ . Increasing  $\alpha$  would exert price pressure, increasing output and R&D effort.
- Similarly to the Cournot case, we can extend the characterization in Proposition 4(i) to multiple equilibria situations as long as we restrict attention to extremal symmetric interior equilibria.

Table 1 provides the properties of the examples claimed in Proposition 4, as well as computed equilibrium solutions for  $c(z) = \alpha z^{-\gamma}$  with  $\alpha > 0$  and  $\gamma > 0$  and for the demand systems of constant elasticity, constant expenditure (constant elasticity specification), and logit. For those computed examples we find also that R&D intensity (R&D expenditures over sales)  $\frac{z^*}{p^*x^*}$  is increasing in  $n$  and  $\sigma$ . The Appendix provides details for each example.

Table 1: Exogenous Market Structure

Demand system	Linear (Shapley-Shubik)	Linear (Bowley)	Location	CES	Constant Exp. (Exponential)	Constant Exp. (CES)	Logit
$D_i(p) =$	$S\left(\frac{\alpha - \beta p_i}{n} - \frac{\beta \gamma p_i}{n} + \frac{\beta \gamma \sum_j p_j}{n^2}\right)$	$S(a_n - b_n p_i + c_i \sum_{j \neq i} p_j)$	$\frac{St + n(p - p_i)}{nt}$	$\frac{S(\beta \theta)^{\frac{1}{1-\beta \theta}} p_i^{\frac{1}{1-\beta}}}{[\sum_j p_j^{\frac{\beta}{\beta-1}}]^{\frac{1-\theta}{1-\beta \theta}}}$	$\frac{S \exp -\beta p_i}{p_i \sum_j \exp -\beta p_j}$	$\frac{S p_i^{-r}}{p_i \sum_j p_j^{-r}}$	$\frac{S \exp\left[-\frac{p_i}{\mu}\right]}{\sum_i \exp\left[-\frac{p_i}{\mu}\right]}$
$\sigma \equiv$	$\gamma$	$\gamma$	$\frac{1}{t}$	$\beta$	$\beta$	$r$	$\frac{1}{\mu}$
$H =$	$\frac{S(\alpha - \beta p)}{n}$	$\frac{S(\alpha - p)}{\beta + (n-1)\gamma}$	$\frac{S}{n}$	$\frac{S(\beta \theta)^{\frac{1}{1-\beta \theta}} p^{\frac{1}{\beta \theta - 1}}}{n^{\frac{1-\theta}{1-\beta \theta}}}$	$\frac{S}{np}$	$\frac{S}{np}$	$\frac{S}{n}$
$\eta =$	$\frac{(n+n\gamma-\gamma)p}{(\alpha-\beta p)n}$	$\frac{[\beta+(n-1)\gamma]p}{(\beta-\gamma)(\alpha-p)}$	$\frac{np}{t}$	$\frac{1 - \frac{\beta}{n} \frac{1-\theta}{1-\beta \theta}}{1-\beta}$	$\frac{n+\beta(n-1)p}{n}$	$\frac{n(r+1)-r}{n}$	$\frac{p(n-1)}{\mu n}$
<i>sign :</i>							
$\partial H / \partial n$	-	-	-	-	-	-	-
$\partial \eta / \partial n$	+	+	+	+	+	+	+
$\partial H / \partial \sigma$	0	-	0	+	0	0	0
$\partial \eta / \partial \sigma$	+	+	+	+	+	+	+
$\partial x / \partial n$	-	-	-	-	-	-	-
$\partial x / \partial \sigma$	+	-	0	+	+	+	0
Innovation Function $c(z) = \alpha z^{-\gamma}$							
$z^* =$				$\left[\frac{(S\alpha\gamma)^{\beta\theta} n\eta}{n^\theta S\beta\theta\gamma(\eta-1)}\right]^{\frac{1}{\beta\theta(\gamma+1)-1}}$		$\frac{S\gamma r(n-1)}{n(n(r+1)-r)}$	$\left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}}$
$p^* =$				$\frac{\eta}{\eta-1} \alpha (z^*)^{-\gamma}$		$\frac{\alpha(z^*)^{-\gamma}[n(r+1)-r]}{r(n-1)}$	$\frac{n\mu}{n-1} + \alpha[z^*]^\gamma$
$\pi^* =$				$z^* \left[\frac{1-\gamma(\eta-1)}{\gamma(\eta-1)}\right]$		$\frac{S[n-\gamma r(n-1)]}{n(n(r+1)-r)}$	$\frac{S\mu}{n-1} - z^*$
$\frac{z^*}{p^*x^*} =$				$\frac{\gamma(\eta-1)}{\eta}$		$\frac{\gamma r(n-1)}{n(r+1)-r}$	$\frac{1}{\frac{1}{S\gamma} + \frac{\mu}{(n-1)z^*}}$

### 2.2.1 Strategic commitment effects

I analyze the subgame-perfect equilibria of the two-stage game in which firms first invest in cost-reducing R&D and then compete in prices. Denote by  $p^*(z_i, z_{-i})$ ,  $i = 1, \dots, n$ , a second-stage Bertrand equilibrium for a given investment profile  $z$ , and let

$$(p_i^*(z_i, z_{-i}) - c(z_i)) S D_i(p^*(z_i, z_{-i})) - z_i$$

be the corresponding profit of firm  $i$  in the reduced-form game at the first stage.

It is not difficult to see that, at a symmetric interior SPE of the two-stage game  $(p, z)$ , we have  $H(p) + (p - c(z)) h(p) = 0$  and  $-c'(z) S H - 1 - (p - c(z)) S (n - 1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i} = 0$ .

The term

$$-(p - c(z)) S (n - 1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i}$$

is the strategic commitment effect and it does not appear in the characterization of the equilibrium in the simultaneous move game. With strategic complementarity in prices

and the condition for uniqueness of a symmetric Bertrand equilibrium at the second stage

$\left(\frac{\partial H}{\partial p} + h + (p - c) \frac{\partial h}{\partial p} < 0\right)$ , it follows that  $\frac{\partial p_j^*}{\partial z_i} \leq 0$  and therefore  $-(p - c(z)) S (n - 1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i} \geq$

0. Increasing the innovation effort of firm  $i$  reduces the equilibrium prices of rivals, because firm  $i$  is more aggressive and best responses are upward sloping. In order to perform comparative statics with respect to  $n$  note that the SPE  $z$  will be characterized by

$$F(z; n) \equiv -c'(z) S x(z; n) - 1 - (p^*(z, n) - c) S (n - 1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i} = 0$$

with  $x(z; n) \equiv H_n(p^*(z, n))$  and, provided  $\frac{\partial F}{\partial z} < 0$ , we have  $\text{sign} \frac{dz}{dn} = \text{sign} \frac{\partial F}{\partial n}$ .

We know that  $\text{sign} \frac{\partial}{\partial n} (-c'(z) S H - 1) = \text{sign} \left(-c' S \frac{\partial x(z; n)}{\partial n}\right) = \text{sign} \frac{\partial x(z; n)}{\partial n}$ . This confirms the result in the simultaneous-move game, with  $dz/dn < 0$  when  $\partial x(z; n)/\partial n < 0$ .

The derivative of the strategic effect has an ambiguous sign. The reason is that increasing the number of firms may induce the firms in the first stage to distort their investment more (because there will be more competition at the second stage) or to distort it less (because, with more firms, the possibilities of manipulating the second-stage price equilibrium diminish).

However, tedious algebra shows that, in the case of constant expenditure demand system (with constant elasticity specification for demand and constant elasticity innovation costs)

as well as in the Shapley–Shubik linear demand system and the logit case (both for a general innovation cost function) the result of the simultaneous game holds and  $dz/dn < 0$ . In all these examples investments at the first stage are strategic substitutes. Furthermore, in these examples the same comparative statics with respect to  $\sigma$  hold:  $dz/d\sigma > 0$  for the first and second cases and  $dz/d\sigma = 0$  for the logit.<sup>19</sup> Using the Bowley linear demand system, Qiu (1997) finds that  $\text{sign } \frac{dz}{d\sigma} = \text{sign } \frac{\partial x}{\partial \sigma} < 0$  in the strategic two-stage game. This is the same result as in the simultaneous game according to Proposition 4.

### 2.2.2 Bertrand and Cournot

We can think of still another way to change competitive pressure in the market: by switching from Bertrand to Cournot. It is well known that Bertrand equilibria tend to be more competitive than Cournot equilibria (see Vives (1985), Singh and Vives (1984), and Vives (1999, Chap. 6) for a precise statement of the needed conditions). Typically we would then have, at symmetric equilibria and for the same level of costs, that the Bertrand output will be larger than the Cournot output and hence the incentive for cost reduction is greater in the former. However, this conclusion need not be robust to strategic commitment effects. Indeed, in Cournot (with competition between strategic substitutes) it pays a firm to overinvest in order to gain an advantage, whereas in Bertrand (with strategic complements) it pays to underinvest in order to gain an advantage (Fudenberg and Tirole (1984)). This means that Cournot competition may induce more cost–reduction effort owing to this strategic effect even though the output in Bertrand may be higher (see the linear-quadratic models of Bester and Petrakis (1993), Qiu (1997), and Symeonidis (2003); in the latter, R&D increases product quality in a quality-augmented version of the Bowley demand system). However, it should be noted that, in general, if we want to know how an increase of competitive pressure in a particular industry affects innovation effort, then a comparison between Bertrand and Cournot equilibria will not be appropriate. Indeed, institutional features of the market typically determine the mode of competition.<sup>20</sup>

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<sup>19</sup>The (lengthy) computations of the examples are available upon request.

<sup>20</sup>See Vives (1999, Chap.7).

## 2.3 Empirical results

The result obtained—that in markets with barriers to entry, the innovation effort per firm decreases in the number of firms—should be contrasted with some results in the empirical literature where an inverted U-shaped relationship is found between market concentration and R&D effort or output (see e.g. Scherer and Ross (1990), Caves and Barton (1990), and Aghion et al. (2002))<sup>21</sup>. For highly concentrated markets, a decrease in concentration seems to benefit innovation, although the effect is reversed for lower concentration levels. Aghion et al. (2002) relate a measure of innovative output (the count of successful patent applications) to a measure of competition (the Lerner index<sup>22</sup>) as a proxy for competitive pressure measured by  $\sigma$  in a market with a fixed number of firms. In their step-by-step innovation model there are two forces: competition may increase the incremental profit from innovating (i.e., escape the competition effect for firms that are neck-and-neck) but also may reduce innovation incentives for laggards when it is intense enough (by reducing rents to innovation). When competition is low the first force dominates, yet when competition is intense the second does owing to a composition effect in the steady-state distribution of technology gaps.

These empirical results can be reconciled with the analysis in this paper provided that competition involves also a liquidation effect that induces cost-reduction effort.<sup>23</sup> By reducing profits, competition may put in danger the survival of the company and/or its management and so induce more effort whenever there are liquidation costs (be they bankruptcy costs or termination costs for the manager; see e.g. Schmidt (1997)). This means, for example, that increasing the number of firms increases the probability of liquidation and thus tends to increase innovation effort. This effect is then dominated by the reduction in profit (or demand) effect when the number of firms grows large.

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<sup>21</sup>See also Ceccagnoli (2003) for a nonmonotonic effect in an increase in the number of non-innovating firms.

<sup>22</sup>In fact, they use average cost instead of marginal cost and hence their measure of competition (in terms of our model) is instead  $\widehat{L} \equiv \frac{p-c-(z/x)}{p} = L - \frac{z}{px}$ .

<sup>23</sup>We might also try to explain the inverted U-shaped relationship between an average Lerner index and average innovation output (or effort) in an industry with asymmetric firms and composition effects.

### 3 Free entry

In this section I analyze markets with free entry and perform a comparative statics analysis with the size of the market, the size of the entry cost, and the degree of product substitutability. As before, I consider first a Cournot market with homogenous product and then the Bertrand market with differentiated products.

#### 3.1 Cournot competition with homogenous product

Parameterize the demand by the size of the market  $S > 0$ . Inverse demand is now given by  $p = P(X/S)$ , and we look for a free-entry equilibrium in which entering firms incur a fixed cost  $F \geq 0$ . Firms choose whether to enter or not at a first stage and then choose simultaneously investment and output. Suppose that for any  $n$  there is a unique, symmetric equilibrium with associated profits per firm of  $\pi_n$ . At a free-entry equilibrium with  $n$  firms in the market, each firm makes nonnegative profits,  $\pi_n \geq F$ , and further entry would result in negative profits,  $\pi_{n+1} < F$ . If  $\pi_n$  is strictly decreasing in  $n$  then there can be at most one free-entry equilibrium, and there will be one if  $\pi_n$  tends to zero as  $n$  grows. I will finesse the game form positing a free-entry zero-profit condition. If  $n^e$  is such that  $\pi_n = F$ , then the free-entry number of firms is  $[n^e]$ .<sup>24</sup> Obviously, if we have a result, say, that  $\frac{dn^e}{dS} > 0$ , this means that  $\frac{d[n^e]}{dS} \geq 0$ .

Alternative game forms involve firms choosing simultaneously whether to enter, their investment in cost reduction, and level of output (Dasgupta and Stiglitz (1980)); or entry and investment at a first stage followed by market competition (Boone (2000)); or a sequential three-stage entry–investment–market competition (Sutton (1991), Suzumura (1995)).<sup>25</sup>

**Proposition 5** *Suppose that the assumptions of Proposition 1 hold and let  $(x^e, z^e, n^e)$  be a symmetric (interior) free-entry equilibrium. Let profits of firm  $i$  be strictly concave with respect to  $x_i$  and  $z_i$ , and let  $D \equiv xc'' \left( \frac{n+1}{S} P' + \frac{nx}{S^2} P'' \right) + (c')^2 < 0$ . Then the equilibrium is*

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<sup>24</sup>The brackets  $[x]$  denote the largest integer less than  $x$ .

<sup>25</sup>See Mas-Colell, Whinston, and Green (1995, Sec.12E) for a careful discussion of the differences in the game forms. Novshek (1980) and Kihlstrom (1999) consider simultaneous entry and output or price decisions.

unique and

$$\text{sign} \left\{ \frac{dz^e}{dS} \right\} = \text{sign} \left\{ \frac{dx^e}{dS} \right\} > 0.$$

Furthermore,

$$\text{sign} \left\{ \frac{dz^e}{dF} \right\} = \text{sign} \left\{ \frac{dx^e}{dF} \right\} = \text{sign} \left\{ - (P' + (x/S) P'') \right\} \text{ and } \frac{dn^e}{dF} < 0.$$

Under the assumptions we know that equilibrium profits  $\pi_n^*$  for a given  $n$  are strictly decreasing in  $n$ . We will thus have a unique  $(x, z, n)$  fulfilling the FOCs for output and innovation effort as well as the zero profit condition  $(P(xn/S) - c(z))x - z - F = 0$ . The results follow by differentiating totally the equilibrium conditions under the assumptions (see the Appendix).

Some intuition for the market size  $S$  comparative statics result in the proposition can be gained as follows. Increasing  $S$  will have a positive direct impact on  $x$  and  $z$  and an indirect effect because of the changes in  $n$ . However, the indirect effect is always dominated because  $n$  increases (if at all) less than proportionately than  $S$ . The reason is that, with constant marginal costs, increasing the market size increases also the toughness of competition and puts pressure on margins, moderating the rate of entry.<sup>26</sup> In fact,  $n$  may even decline as a result of the intensity of the R&D competition. A condition for this not to be the case is strategic substitutability in outputs (i.e.,  $P' + (x/S)P'' < 0$ ) and  $c(\cdot)$  sufficiently convex (i.e.,  $-c''x/Sc' > nc'/P' > 0$ ). Then  $\frac{dn^e}{dS} > 0$ .

The comparative statics results on  $F$  are very intuitive. Increasing the entry cost decreases the free-entry number of firms, and it increases (decreases) output and R&D effort whenever outputs are strategic substitutes (complements).

## Remarks

- It is easy to check that in equilibrium

$$L \equiv \frac{p - c}{p} = \frac{1 + \frac{F}{z}}{1 + \frac{1}{\gamma(z)} + \frac{F}{z}} = \frac{\varepsilon(nx)}{n},$$

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<sup>26</sup>For example, if  $P(X/S) = (X/S)^{-1}$  then, letting  $n(S)$  denote the free-entry number of firms for a given symmetric investment profile  $z$  and Cournot competition, we have  $n(S)/S = (F/S)^{1/2}$ .

where  $\gamma(z) \equiv -zc'(z)/c(z)$  and  $\varepsilon(X) \equiv -XP'(X)/P(X)$ .<sup>27</sup> It is immediate also that  $L = (z + F)/px$ .

- If  $F = 0$  then  $L = z/px$  (R&D intensity) and

$$n^e = \varepsilon(n^e x^e) \frac{1 + \gamma(z^e)}{\gamma(z^e)}.$$

If  $\gamma$  is increasing in  $z$ , then increasing  $S$  increases  $z$  and R&D intensity. Note that, for a given inverse elasticity  $\varepsilon$ , increasing the technological opportunities  $\gamma$  will tend to increase concentration. This is consistent with the empirical findings that industries with more technological opportunities are more concentrated (see, e.g. Scherer and Ross (1990)).

- With constant elasticity innovation and demand functions and  $F > 0$ , we have that  $L$  decreases (strictly) with  $z$  and hence increasing  $S$  increases  $z$ , decreases  $L$ , and increases  $n$ . However, if  $F = 0$ , then  $L = \gamma/(1 + \gamma)$  and  $n^e = \varepsilon(1 + \gamma)/\gamma$  are independent of  $S$ .

### Constant elasticity examples

- Let  $p = (X/S)^{-\varepsilon}$  ( $E - 1 = \varepsilon > 0$ ),  $c(z) = \alpha z^{-\gamma}$  and  $F = 0$ . Then, indeed, both

$$z^e = \left( S^\varepsilon \gamma^{2\varepsilon} \alpha^{\varepsilon-1} \varepsilon^{-\varepsilon} (1 + \gamma)^{-(1+\varepsilon)} \right)^{1/(\varepsilon-\gamma(1-\varepsilon))}$$

and

$$x^e = \frac{1}{\gamma\alpha} \left( S^\varepsilon \gamma^{2\varepsilon} \alpha^{\varepsilon-1} \varepsilon^{-\varepsilon} (1 + \gamma)^{-(1+\varepsilon)} \right)^{(1+\gamma)/(\varepsilon-\gamma(1-\varepsilon))}$$

increase with  $S$  and the free-entry number of firms is  $[\varepsilon(1 + \gamma)/\gamma]$  (Dasgupta and Stiglitz (1980)).

- Tandon (1984) considers a linear demand example  $p = a - bX$  with  $c(z) = a - \beta z^\delta$  and  $F = 0$ . Strict concavity of profits of firm  $i$  with respect to  $x_i$  and  $z_i$  requires  $\delta < \frac{1}{2}$ . Then  $n^e = \frac{1-\delta}{\delta}$ . Both  $z^e = \left( \frac{\delta^2 \beta^2}{b} \right)^{\frac{1}{1-2\delta}}$  and  $x^e = \left( \frac{\delta\beta}{b} \right) \left( \frac{\delta^2 \beta^2}{b} \right)^{\frac{\delta}{1-2\delta}}$ , as well as R&D intensity  $z^e/p^e x^e$ , increase in  $S$  (decrease in  $b$ ) since  $\delta < \frac{1}{2}$ , and  $n^e z^e$  increases in  $\delta$ .

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<sup>27</sup>Note that  $\text{sign } \varepsilon' = \text{sign } \{1 - \varepsilon - E\}$ .

- Sutton (1991) considers a three-stage game featuring (i) an entry decision, (ii) investment in cost reduction, and (iii) quantity competition. Demand is given by  $P(X/S) = (X/S)^{-1}$  and the innovation curve by  $c(z) = (z\gamma a^{-1} + 1)^{-1/\gamma}$ , where  $\gamma > \max\{1, 2a/3F\}$  and  $F$  is the sunk cost of entry. Then, for  $S$  small,  $z^e = 0$ ; for larger  $S$ ,  $z^e$  is increasing in  $S$  while  $n^e$  decreases (increases) in  $S$  if  $F < a/\gamma$  ( $F > a/\gamma$ ). This model can also be given a quality investment or advertising interpretation. In this example, investment has a strategic commitment effect.

## 3.2 Bertrand with free entry

### 3.2.1 Comparative statics with market size and entry cost

Consider again the Bertrand market of Section 2.2 and let  $S$  denote the size of the market (number of consumers, say). Then  $x_i = SD_i(p)$  and the profit of firm  $i$  is

$$\pi_i = (p_i - c(z_i))SD_i(p) - z_i - F,$$

where  $S$  is total market size and  $F \geq 0$  is the sunk cost of entry.

**Proposition 6** *Consider a symmetric interior free-entry equilibrium  $(p^e, z^e, n^e)$ . Under the assumptions of Proposition 4, if at the equilibrium  $\widehat{B} \leq 0$  then the equilibrium is unique. Suppose that  $\frac{\partial H}{\partial n} < 0$  and  $\frac{\partial \eta}{\partial n} > 0$ ; then*

$$\text{sign} \left\{ \frac{dz^e}{dS} \right\} = \text{sign} \left\{ \frac{dx^e}{dS} \right\} = \text{sign} \left\{ -\frac{dp^e}{dS} \right\} > 0$$

and

$$\text{sign} \frac{dn^e}{dS} \geq 0 \quad \left( \frac{dn^e}{dS} = 0 \text{ if } \widehat{B} = 0 \right).$$

Furthermore,

$$\frac{dn^e}{dF} < 0, \quad \text{sign} \left\{ \frac{dz^e}{dF} \right\} = \text{sign} \left\{ -\frac{dz_n}{dn} \right\},$$

and

$$\text{sign} \left\{ \frac{dp^e}{dF} \right\} = \text{sign} \left\{ -\frac{dp_n}{dn} \right\},$$

where  $(p_n, z_n)$  is the equilibrium with exogenous  $n$  evaluated at  $n = n^e$ .

Under the assumptions we know that equilibrium profits  $\pi_n^*$  for a given  $n$  are strictly decreasing in  $n$ . We will thus have a unique  $(p^e, z^e, n^e)$  fulfilling the FOCs for price and innovation effort as well as the zero profit condition  $(p - c(z))SH(p; n) - z - F = 0$ . The results follow by differentiating totally the equilibrium conditions under the assumptions (see the Appendix).

Increasing the size of the market increases variety (product innovation) and also reduces cost (process innovation); and, indeed, total R&D effort  $n^e z^e$  increases with  $S$ . As in the Cournot case, increasing market size increases the number of firms less than proportionately, if at all, and thus increases individual firm output and innovation effort. The potential downward pressure exerted on innovation effort by an increase in the number of firms is overwhelmed by the expanded market. An interesting difference with the Cournot case is that the number of firms cannot decrease when the market size expands. Here Adam Smith's dictum that the division of labor is limited by the extent of the market comes true both in the sense of increasing the number of varieties and in the cost of producing each one.

Increasing the entry cost reduces the number of products introduced and firms (indeed, under our assumptions profits are decreasing in  $n$ ), and it affects price and R&D effort depending on the impact of a decreased number of firms. Typically (see examples in Table 1) we have that decreasing  $n$  increases  $z$  and  $p$ , and increasing  $F$  will therefore decrease  $n$  and increase  $p$  and  $z$ . Increasing the entry cost then has the (perhaps paradoxical) effect of increasing innovation effort. The reason is that it decreases the number of entrants, and each entrant produces more and has more incentive to reduce costs.

All the demand systems considered (linear, constant elasticity, constant expenditure, and logit demand systems) fulfill  $\frac{\partial H}{\partial n} < 0$  and  $\frac{\partial \eta}{\partial n} > 0$  (see Table 1).

### 3.2.2 Comparative statics with product substitutability

As in Section 2.2, we parameterize the demand function by the degree of product substitutability  $\sigma$ , yielding  $H(p) \equiv D_i(p, \dots, p; \sigma)$ , and assume that  $\frac{\partial \eta}{\partial \sigma} > 0$  and  $\frac{\partial H}{\partial \sigma} \geq 0$ .

**Proposition 7** *Consider a symmetric interior free-entry equilibrium  $(p^e, z^e, n^e)$ . If  $\hat{B} \leq$*

0,  $\frac{\partial H_n}{\partial n} < 0$ ,  $\frac{\partial \eta}{\partial n} > 0$ ,  $\frac{\partial H}{\partial \sigma} \geq 0$ , and  $\frac{\partial \eta}{\partial \sigma} > 0$ , then at the equilibrium

$$\text{sign} \left\{ \frac{dz^e}{d\sigma} \right\} = \text{sign} \left\{ \frac{dx^e}{d\sigma} \right\} = \text{sign} \left\{ -\frac{dp^e}{d\sigma} \right\} > 0$$

and  $\text{sign} \left\{ \frac{dn^e}{d\sigma} \right\}$  is ambiguous but

$$\frac{dn^e}{d\sigma} < 0 \text{ if } \frac{\partial H}{\partial \sigma} = 0.$$

The proof of the proposition follows along similar lines than that of Proposition 6 and can be found in the Appendix. Increasing the degree of product substitutability increases output per firm and R&D effort, provided the total market does not shrink. The assumptions on demands are fulfilled for all the examples (except the Bowley variation of linear demands). In the linear (Shapley–Shubik), constant expenditure, and logit demand systems,  $\frac{\partial H}{\partial \sigma} = 0$  and therefore  $\frac{dn^e}{d\sigma} < 0$ . It should be clear why this is so. When changes in  $\sigma$  are demand-neutral, increasing  $\sigma$  decreases profits and so the zero-profit entry condition is restored, decreasing the number of entrants.

## Remarks

- The parameter  $\sigma$  could also be interpreted as “regulatory pressure”, with  $\frac{\partial \eta}{\partial \sigma} > 0$  and  $\frac{\partial H}{\partial \sigma} = 0$ . An increase in regulatory pressure would then decrease price while increasing R&D effort and concentration.
- As before, in a free-entry equilibrium

$$L \equiv \frac{p - c}{p} = \frac{1 + \frac{F}{z}}{1 + \frac{1}{\gamma(z)} + \frac{F}{z}} = \frac{1}{\eta(p, n, \alpha)},$$

where  $\gamma(z) \equiv -zc'(z)/c(z)$  and  $\alpha$  may represent  $S$  or  $\sigma$ . We also have that  $L = (z + F)/px$  (here this is the R&D intensity including expenditure  $F$  on product introduction).

- The relationship between market power and innovation effort is ambiguous:

$$\text{sign} \frac{\partial L}{\partial z} = \text{sign} \left\{ -\frac{F}{z^2} \gamma^{-1} + \left( 1 + \frac{F}{z} \right) \frac{\gamma'}{\gamma^2} \right\}.$$

If  $\gamma' \leq 0$  and  $F > 0$ , then  $L = \frac{1+F/z}{1+(\gamma(z))^{-1}+F/z}$  is strictly decreasing in  $z$ . If  $F = 0$  then  $\text{sign } \frac{\partial L}{\partial z} = \text{sign } \{\gamma'\}$  and thus, if  $\gamma' > 0$ ,  $L$  is strictly increasing in  $z$ . It follows that, if  $\sigma$  or  $S$  increase (and hence  $z$  also increases) then  $L$  decreases when  $\gamma' \leq 0$  and  $F > 0$  or if  $\gamma' < 0$  and  $F = 0$ . This is the case in particular if  $\gamma$  is constant with  $F > 0$ . If  $\gamma$  is increasing in  $z$  and  $F = 0$ , then increasing  $S$  increases  $z$ ,  $L$ , and R&D intensity. If  $F = 0$  and  $\gamma$  is constant,  $n^e$  increases with  $S$  if  $\eta$  is increasing in  $p$ . This is so because  $\eta$  is independent of  $S$  and strictly increasing in  $n$ , and also increasing  $S$  decreases  $p$ . In this case the degree of monopoly power  $L$  is determined by technological considerations (the elasticity of the innovation function).

- Increases in product substitutability  $\sigma$  need not go together with decreases in the Lerner index  $L$ . In particular, it could be that an increase in  $\sigma$  increases market power ( $L$ ) and innovation effort  $z$ . This will happen, for example, with  $F = 0$  and  $\gamma' > 0$ .<sup>28</sup> This situation would be at odds with work (e.g. Aghion et al. (2002)) in which the Lerner index, or an approximation to it, is taken as a proxy for competitive pressure measured by  $\sigma$ .
- The Lerner index and the level of concentration may move in opposite directions. If  $F > 0$  and  $\gamma$  is constant, then the Lerner index is strictly decreasing with  $\sigma$ . It follows that increasing  $\sigma$  increases  $z$ , decreases  $L$  (and R&D intensity) and also decreases  $n^e$  if  $\frac{\partial H}{\partial \sigma} = 0$ .

**Incentives in the Salop (1979) model (Raith (2003))** The incentive model by Raith (2003) provides a nice illustration of our results. The author considers the model of Salop (1979) with a mass of consumer  $S$  uniformly distributed within a circle of circumference 1 and with quadratic transportation costs having parameter  $t$ . Each of the  $n$  firms has a cost

$$c_i = \bar{c} - e_i - u_i,$$

where  $e_i$  is the unobservable effort exerted by the manager of the firm and  $u_i$  is normally distributed idiosyncratic noise with mean 0 and variance  $v$ . Owner  $i$  makes decisions and offers a linear contract to his manager, with compensation  $w_i = s_i + b_i(\bar{c} - c_i)$ , to reduce

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<sup>28</sup>Note that  $\text{sign } \gamma' = \text{sign } \left\{ 1 + \gamma + \frac{c''z}{c'} \right\}$ . Then  $-\frac{c'z}{c} + \frac{c''z}{c'} < 0$  if and only if  $c$  is log-convex.

costs. After all managers have chosen their effort levels, costs are realized (and are private information to the firms), firms compete in prices, and a (Bayesian) Bertrand equilibrium obtains. Managers have constant absolute risk aversion  $\rho$ , quadratic cost of effort  $\frac{k}{2}(e_i)^2$ , and a reservation utility of 0. Given that the manager of  $i$  will choose  $e_i = b_i/k$ , firm  $i$  will set  $s_i = -\frac{1}{2k}(1 - k\rho v)(b_i)^2$  so that the manager will obtain a zero expected utility. The expected compensation of the manager will be  $\bar{w}_i = s_i + b_i e_i = \frac{1+k\rho v}{2k}(b_i)^2$ ; the expected cost,  $\bar{c}_i = \bar{c} - b_i/k = \bar{c} - \sqrt{\frac{2\bar{w}_i}{k(1+k\rho v)}}$ . In terms of our model, then,

$$c(z) = \bar{c} - \sqrt{\frac{2z}{k(1+k\rho v)}}.$$

Under some parameter restrictions, and for a fixed number of firms  $n$ , Raith shows that there is a symmetric equilibrium for the overall game and that cost reduction effort is independent of  $\sigma \equiv 1/t$ . Furthermore, with free entry and with firms paying an entry cost  $F$ , cost-reduction effort is increasing in  $\sigma$ ,  $S$ , and  $F$ . All these results follow from Propositions 4, 7, and 8 —noting that in the Salop model  $\frac{\partial H}{\partial p} = \frac{\partial H}{\partial \sigma} = 0$ .

Table 2 provides the endogenous market structure counterpart of Table 1 with computed examples when  $c(z) = \alpha z^{-\gamma}$  with  $\alpha >$  and  $\gamma > 0$ . The Appendix provides computational details of the results reported in Table 2. We see that the Lerner index is increasing in  $F$  and decreasing in  $\sigma$  in the constant expenditure (constant elasticity)<sup>29</sup> and logit cases. In the constant elasticity (CES) case we have that  $n^e$  is strictly decreasing in  $\sigma$  when  $F = 0$  even though  $\partial H/\partial \sigma > 0$ .

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<sup>29</sup>The fact that  $L$  is decreasing in  $\sigma$  validates the conjecture of Aghion et al. (2002, p. 13, fn.9).

Table 2: Endogenous Market Structure (with innovation function  $c(z) = \alpha z^{-\gamma}$ )

Demand system	CES $F = 0$	CES $F > 0$	Constant Expenditure (CES)	Logit
$D_i(p) =$		$\frac{S(\beta\theta)^{\frac{1}{1-\beta\theta}} p_i^{\frac{1}{1-\beta}}}{\left(\sum_j p_j^{\frac{\beta}{\beta-1}}\right)^{\frac{1}{1-\beta\theta}}}$	$\frac{S p_i^{-r-1}}{\sum_j p_j^{-r}}$	$\frac{S \exp\left(-\frac{p_i}{\mu}\right)}{\sum_j \exp\left(-\frac{p_j}{\mu}\right)}$
$n^e$	$= \frac{\beta\gamma(1-\theta)}{(1-\beta\theta)(\beta\gamma+\beta-1)}$	$\left[\frac{(S\alpha\gamma)^{\beta\theta} (n^e)^{1-\theta} \eta^e}{S\beta\theta\gamma(\eta^e-1)}\right]^{\frac{1}{\gamma\beta\theta+\beta\theta-1}} =$ $= \frac{F\gamma(\eta^e-1)}{1-\gamma(\eta^e-1)}$	$= \frac{(F-S\gamma)r+S}{2F(r+1)} +$ $\frac{[(Fr+S-\gamma rS)^2+4\gamma rFS(r+1)]^{1/2}}{2F(r+1)}$	$\frac{S\mu}{n^e-1} -$ $\left[\frac{S\alpha\gamma}{n^e}\right]^{\frac{1}{\gamma+1}} = F$
$z^e =$		$\left[\frac{(S\alpha\gamma)^{\beta\theta} (n^e)^{1-\theta} \eta^e}{S\beta\theta\gamma(\eta^e-1)}\right]^{\frac{1}{\gamma\beta\theta+\beta\theta-1}}$	$\frac{S\gamma r(n^e-1)}{n^e(n^e(r+1)-r)}$	$\left[\frac{S\alpha\gamma}{n^e}\right]^{\frac{1}{\gamma+1}}$
$p^e =$		$\frac{n^e}{\eta^e-1} \alpha (z^e)^{-\gamma}$	$\alpha \left[\frac{S\gamma}{n^e} \left(\frac{r(n^e-1)}{n^e(r+1)-r}\right)^{\frac{\gamma+1}{\gamma}}\right]^{-\gamma}$	$\frac{n^e \mu}{n^e-1} +$ $\alpha \left[\frac{S\alpha\gamma}{n^e}\right]^{-\frac{\gamma}{\gamma+1}}$
$L^e \left(= \frac{1}{\eta^e}\right) =$		$\frac{\beta n^e (1-\beta\theta)(1-\beta)}{n^e (1-\beta\theta) + \beta(\theta-1)}$	$\frac{n^e}{n^e(r+1)-r}$	$\frac{1}{1 + \frac{n^e-1}{S\mu\gamma} \left(\frac{S\alpha\gamma}{n^e}\right)^{\frac{1}{\gamma+1}}}$
$sign \partial n^e / \partial \sigma$			–	–
$sign \partial L^e / \partial \sigma$			–	–
$sign \partial L^e / \partial F$		+	+	+
$sign \partial z^e / \partial F$		+	+	+
$sign \partial p^e / \partial F$		+	+	ambiguous

**Cournot** The results could be easily extended to Cournot competition. In fact, Spence (1984) has shown how a certain class of cost-reduction Cournot models with homogenous product can be reinterpreted in a product differentiation environment. In the constant elasticity case, for example, it is possible to check that, under quantity competition, the same comparative statics with respect to  $S$  hold as in the Bertrand case. That is,  $\frac{dn^e}{dS} > 0$  for  $F > 0$  and  $\frac{dn^e}{dS} = 0$  for  $F = 0$ .<sup>30</sup>

<sup>30</sup>Inverse demand is given by  $p_i = \frac{S^{1-\beta\theta} \alpha^\theta \beta\theta x_i^{\beta-1}}{\left(\sum_j x_j^\beta\right)^{\frac{1}{1-\beta\theta}}}$  for  $i = 1, \dots, n$  (Koenker and Perry (1981)).

## 4 Extensions

### 4.1 An alternative measure of competitive pressure

Competitive pressure could be measured also by the extent that each firm internalizes the profits of other firms. This could arise, for example, when firms in the industry have cross-shareholdings. Suppose that firm  $i$  maximizes

$$\pi_i + \lambda \sum_{j \neq i} \pi_j,$$

where  $\lambda$  ranges from  $\lambda = 0$  (no internalization as before) to  $\lambda = 1$  (full internalization or collusion), and consider the simultaneous-move game. An increase in  $\lambda$  will then mean a decrease in competitive pressure. It is possible to check (proofs available on request) that, under Cournot and under Bertrand competition, an increase in competitive pressure  $1/\lambda$  will:

- increase output and innovation effort with barriers to entry; and
- increase innovation effort and decrease the number of entrants with free entry.

The intuition is straightforward. With barriers to entry, if firms are more aggressive (a lower  $\lambda$ ) then output per firm and the incentive to innovate will increase. With free entry, a firm (when deciding whether to enter) considers only its own profits but knows that, once in the market, competition will be softer if  $\lambda$  is higher. Tougher competition thus means that fewer firms will enter and that output per firm will be larger, inducing a larger innovation effort. The results with  $\lambda$  parallel those obtained in the Bertrand case with degree of substitutability  $\sigma$  whenever changes in  $\sigma$  are demand-neutral ( $\frac{\partial H}{\partial \sigma} = 0$ ).

### 4.2 Investment in quality

The cost-reduction model can be interpreted as investment in quality (product innovation) in the context of the Cournot model (see e.g. Spence (1984) or Sutton (1991)). This is consistent with the result that, with free entry, increasing the market size need not increase the number of entrants under Cournot competition owing to the pressure in margins. Indeed, the increased intensity of competition as the market grows large —when a better

quality requires a higher fixed-cost investment— may prevent further entry and so market concentration need not decline (Sutton (1991)). However, the same reinterpretation does not hold with price competition and, under regularity conditions, a larger market is less concentrated.<sup>31</sup> Despite this, results by Symeonidis (2000) are in line with those obtained in this paper. Symeonidis (2000) considers a (strategic) three-stage game of entry, investment in product quality, and quantity competition within a model in which horizontal and vertical product differentiation coexist. Demand functions are linear (a quality-augmented version of the Bowley demand system) and the innovation function is of the power variety. The author finds that increasing the degree of horizontal product substitutability increases concentration and R&D effort and that increasing the market size increases R&D effort.

### 4.3 Spillovers

When the effort of one firm affects (favorably) the cost reduction of other firms, we say that there are (positive) spillovers.<sup>32</sup> With high enough (positive) spillovers, the R&D cost reduction investments of rivals may be strategic complements in a two-stage game with investment at the first stage and Cournot competition in the second. This is what happens in the linear-quadratic specifications of d’Aspremont and Jacquemin (1988, 1990) and Ceccagnoli (2003).<sup>33</sup> In principle this suggests that, with high enough spillovers and with Cournot competition, it could be that increasing the number of firms increases individual firm innovation effort. However, it can be checked that this does not happen in the linear-quadratic specification where increasing the number of firms always lowers innovation effort.<sup>34</sup>

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<sup>31</sup>Interestingly, Berry and Waldfogel (2003) show that in the restaurant industry (where quality is produced mostly with *variable* costs) the range of qualities increases with market size, whereas in daily newspapers (where quality is produced mostly with *fixed* costs) the average quality increases with market size and there is no fragmentation as the market grows large.

<sup>32</sup>See Spence (1984), d’Aspremont and Jacquemin (1988, 1990) and Amir (2000).

<sup>33</sup>Ceccagnoli (2003) also shows that with fringe firms that do not invest in R&D and do not benefit from the spillover, strategic complementarity among the investing firms increases with the number of fringe firms.

<sup>34</sup>The setting is as follows:  $P(X) = a - bX$  and  $c(z) = \bar{c} - z_i - \beta \sum_{j \neq i} z_j$ . If a firm invests  $\gamma z_i^2/2$ , then its marginal cost will be reduced by  $z_i + \beta \sum_{j \neq i} z_j$ , where  $\beta > 0$  is the spillover rate.

## 5 Concluding remarks

The testable empirical implications of our results may be summarized as follows.

- *In markets with barriers to entry:* More competitive pressure in terms of more firms means less R&D effort per firm, whereas more competitive pressure in terms of a greater product substitutability (that does not shrink the total market for varieties) means more R&D effort per firm.
- *With free entry:* Increasing the market size increases innovation effort, per firm output, and the number of varieties introduced (with price competition and product differentiation); and higher product substitutability (that does not shrink the total market for varieties) means more R&D effort and output per firm. The effect of market size on the number of varieties will hold only for cost-reduction innovations; with quality innovations, the number of varieties introduced may diminish.

A potential application of these results is to deregulated markets. For example, in banking both the deregulation process in Europe and the removal of restrictions on U.S. intrastate and interstate branching, interpretable as increases in market size and/or product substitutability, have been claimed (by Gual and Neven (1993) and Jayaratne and Strahan (1998), respectively) to deliver cost efficiencies.

Many extensions of the analysis could be envisioned. I have already commented on alternative ways of parameterizing competitive pressure, investments to enhance quality with price competition, and spillovers. An immediate extension would be to consider investment that affects the slope of (increasing) marginal costs. More substantial extensions would include asymmetric market structures and performing a welfare analysis with a view toward competition and industrial policy. Leahy and Neary (1997) have developed part of this program.

## 6 Appendix:

### 6.1 Proofs

**Proof of Proposition 2:** Given a symmetric investment profile  $z$  and given that  $P' < 0$ , there exist extremal symmetric Cournot equilibria  $\underline{x}(z)$  and  $\bar{x}(z)$  that are increasing in  $z$  (Amir and Lambson (2000), Vives (1999, pp. 106–107)). This means that there exist extremal symmetric equilibria in the game. Indeed, just consider  $\underline{x}(z)$ , where  $\underline{z}$  is the smallest equilibrium associated to  $\underline{x}(\cdot)$  and  $\bar{z}$  is the greatest equilibrium associated to  $\bar{x}(\cdot)$ . At an extremal interior equilibrium  $(x^*, z^*)$ , we have  $P(xn) + xP'(xn) - c(z) = 0$  and  $-xc'(z) - 1 = 0$ . Therefore,  $\phi(z, n) \equiv -x(z, n)c'(z) - 1 = 0$ , where  $x(z, n)$  is an extremal Cournot equilibrium given  $z$ . We know that  $\phi(\cdot, n)$  cannot jump down, since  $x(z, n)$  is increasing in  $z$ ;  $\phi(0+, n) > 0$ , since  $c'(0+) = -\infty$ ; and  $\phi(z, n) < 0$  for  $z$  large, since  $c'(z) \rightarrow 0$  as  $z \rightarrow \infty$ . It follows that, for extremal  $z$ ,  $\phi(z, n)$  is decreasing in  $z$  (indeed, it could not otherwise be an extremal equilibrium) and therefore, if  $\phi(z, n)$  is strictly increasing (decreasing) in  $n$  then so will  $z$  be. We have that  $\phi(z, n)$  is strictly increasing (decreasing) in  $n$  if and only if  $x(z, n)$  is strictly increasing (decreasing) in  $n$ . Given that  $x(z, n)$  fulfills  $\varphi(x, z, n) \equiv P(xn) + xP'(xn) - c(z) = 0$  and that, at extremal equilibria,  $\varphi$  is decreasing in  $x$ —because (a)  $\varphi(x, z, n) < 0$  for  $x$  large (for  $xn \geq \bar{X}$  we have  $p \leq \underline{c}$ ) and (b)  $\varphi(0+, z, n) > 0$  (since  $\lim_{x \rightarrow 0} \{P(xn) + xP'(xn)\} > \bar{c}$ ) we conclude that  $x(z, n)$  is strictly increasing (decreasing) in  $n$  if and only if  $\varphi(x, z, n)$  is, and this happens if  $P'(xn) + xP''(xn)$  is positive (negative). ■

**Proof of Proposition 3:** At the symmetric SPE we have that

$$\begin{aligned} \varphi(z) &\equiv \frac{\partial V(z_i, z_{-i})}{\partial z_i} \Big|_{z_i=z} = -xc'(z) \left[ 1 + (n-1) \frac{P' + xP''}{(n+1)P' + nxP''} \right] - 1 \\ &= -x(z, n)c'(z)(1 + G(x, n)) - 1 = 0, \end{aligned}$$

where  $G(x, n) = ((n-1)/n)(n-E)/(1+n-E)$ .<sup>35</sup> Note that  $E(X) \leq 0$  because  $P'' \leq 0$  and therefore  $E(X) < 1+n$  (so that, for a given symmetric profile of investments,

<sup>35</sup>With  $n+1 > E$ , we have that  $\text{sign } G = \text{sign}\{n-E\}$ . That is, innovation effort is larger (smaller) in the two-stage (simultaneous) game depending on whether best responses in the Cournot game are downward (upward) sloping.

there is a unique and stable symmetric Cournot equilibrium). Hence,  $\frac{dz^*}{dn} = -\frac{\partial\varphi/\partial n}{\partial\varphi/\partial z}$ . We have  $\frac{\partial\varphi}{\partial z} = \frac{\partial^2 V}{\partial z_i^2} + (n-1)\frac{\partial^2 V}{\partial z_i\partial z_j}$  evaluated at a symmetric solution. Tedious algebra shows that  $\frac{\partial^2 V}{\partial z_i\partial z_j} < 0$  when  $P' < 0$ ,  $P'' \leq 0$ , and  $P'P''' - (P'')^2 \leq 0$ ; therefore, investments are strategic substitutes at the first stage. Moreover, the second order necessary condition at the equilibrium is  $\frac{\partial^2 V}{\partial z_i^2} \leq 0$  and so  $\frac{\partial\varphi}{\partial z} < 0$ . Under the assumptions it is possible to check also that  $\partial\varphi/\partial n < 0$ . ■

**Proof of Proposition 5:** We know that equilibrium profits  $\pi_n^*$  for a given  $n$  are strictly decreasing in  $n$ . We will thus have a unique  $(x, z, n)$  fulfilling:

$$\begin{aligned} P(xn/S) + (x/S)P'(xn/S) - c(z) &= 0 \\ -xc'(z) - 1 &= 0 \\ (P(xn/S) - c(z))x - z - F &= 0 \end{aligned}$$

Differentiating totally the equilibrium conditions and evaluating at the equilibrium, we find that

$$\frac{dx^e}{dS} = \frac{(xc'')(x/S^2)P'}{(2P' + (x/S)P'')c''(x/S) + (c')^2}$$

and

$$\frac{dz^e}{dS} = -\frac{c'(x/S^2)P'}{(2P' + (x/S)P'')c''(x/S) + (c')^2}.$$

We have that  $\text{sign}\left\{\frac{dz^e}{dS}\right\} = \text{sign}\left\{\frac{dx^e}{dS}\right\} > 0$  because the denominator is negative (strict concavity of profits of firm  $i$  with respect to  $x_i$  and  $z_i$  implies  $xc''(((n+1)P'/S) + ((xn/S)(P''/S))) + (c')^2 < 0$  for any  $n$ , which in turn implies the result). Furthermore,

$$\frac{dn^e}{dS} = \frac{((n+1)P' + (x/S)nP'')c''(x/S^2) + (n/S)(c')^2}{(2P' + (x/S)P'')c''(x/S) + (c')^2}.$$

Sufficient conditions for  $dn^e/dS > 0$  are that  $P' + (x/S)P'' < 0$  and  $-c''x/S > nc'/P' > 0$ .

We obtain also

$$\begin{aligned} \frac{dx^e}{dF} &= \frac{c''\left(P' + \frac{x}{S}P''\right)}{-xP'\left(\frac{xc''}{S}\left(2P' + \frac{xP''}{S}\right) + (c')^2\right)}, \\ \frac{dz^e}{dF} &= \frac{-c'\left(P' + \frac{x}{S}P''\right)}{-xP'\left(\frac{xc''}{S}\left(2P' + \frac{xP''}{S}\right) + (c')^2\right)}, \end{aligned}$$

and

$$\frac{dn^e}{dF} = \frac{\frac{xc''}{S} \left( (n+1)P' + \frac{nxP''}{S} \right) + (c')^2}{\frac{x^2}{S}P' \left( \frac{xc''}{S} (2P' + \frac{xP''}{S}) + (c')^2 \right)} < 0.$$

As before, we have that  $\frac{xc''}{S} \left( 2P' + \frac{xP''}{S} \right) + (c')^2 < 0$ , and the inequality follows because  $D = \frac{xc''}{S} \left( (n+1)P' + \frac{nxP''}{S} \right) + (c')^2 < 0$ . ■

**Proof of Proposition 6:** Under the assumptions ( $\widehat{B} < 0$ ), equilibrium profits for a given  $n$ ,  $\pi_n^*$ , are strictly decreasing in  $n$ . We will therefore have a unique symmetric equilibrium  $(p^e, z^e, n^e)$  fulfilling:

$$\begin{aligned} (p - c(z))h(p; n) + H(p; n) &= 0 \\ -SH(p; n)c'(z) - 1 &= 0 \\ (p - c(z))SH(p; n) - z - F &= 0 \end{aligned}$$

It can be checked that the Jacobian of the system is negative definite under the assumptions. Differentiating totally the equilibrium conditions with respect to  $S$  and evaluating at the equilibrium, we find that

$$\begin{aligned} \text{sign} \frac{dp^e}{dF} &= \text{sign} \left\{ H^2 S c'' (p - c) \left[ (p - c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n} \right] \right\} < 0, \\ \text{sign} \frac{dz^e}{dF} &= -H \left[ (p - c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n} \right] > 0, \end{aligned}$$

and

$$\text{sign} \frac{dn^e}{dS} = \text{sign} \left\{ H^2 S h^{-1} \widehat{B} \right\} = \text{sign} \left\{ -\widehat{B} \right\} \geq 0.$$

Differentiating totally the equilibrium conditions with respect to  $F$  and evaluating at the equilibrium, we find that

$$\text{sign} \frac{dn^e}{dF} = \text{sign} \left\{ c'' \left( h + \frac{\partial H}{\partial p} + (p - c) \frac{\partial h}{\partial p} \right) + (c')^2 \frac{\partial H}{\partial p} \right\} < 0,$$

because  $h + \frac{\partial H}{\partial p} + (p - c) \frac{\partial h}{\partial p} < 0$  (the assumption  $\widehat{B} < 0$  implies this) and  $Hc''S > 0$ ; and  $\frac{\partial H}{\partial p} < 0$  and  $(c')^2 HS > 0$ . Furthermore,

$$\text{sign} \frac{dp^e}{dF} = -\text{sign} \left[ (c')^2 h \frac{\partial H}{\partial n} + Hc'' \left( (p - c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n} \right) \right] = -\text{sign} \frac{dp_n}{dn}$$

and

$$\text{sign} \frac{dz^e}{dF} = -\text{sign} \left[ \frac{\partial H}{\partial p} \frac{\partial h}{\partial n} (p-c) - \frac{\partial H}{\partial n} \left( h + (p-c) \frac{\partial h}{\partial p} \right) \right] = -\text{sign} \frac{dz_n}{dn},$$

where  $(p_n, z_n)$  is the equilibrium with exogenous  $n$  evaluated at  $n = n^e$ . ■

**Proof of Proposition 7:** Similarly as in the proof of Proposition 6, differentiating totally the equilibrium conditions and evaluating at the equilibrium yields

$$\text{sign} \frac{dp^e}{d\sigma} = \text{sign} \{ Hc'' (p-c) \Omega \},$$

and

$$\text{sign} \frac{dz^e}{d\sigma} = \text{sign} \{ Hc' \Omega \},$$

where

$$\Omega = \left[ \frac{\partial H}{\partial \sigma} \left[ (p-c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n} \right] - \frac{\partial H}{\partial n} \left[ (p-c) \frac{\partial h}{\partial \sigma} + \frac{\partial H}{\partial \sigma} \right] \right].$$

We obtain that  $\frac{dp^e}{d\sigma} < 0$  and  $\frac{dz^e}{d\sigma} > 0$  because  $\Omega < 0$  ( $\frac{\partial H}{\partial \sigma} \geq 0$ ,  $\text{sign} \{ -[(p-c) \frac{\partial h}{\partial n} + \frac{\partial H}{\partial n}] \} = \text{sign} \frac{\partial \eta}{\partial n} > 0$ ,  $\frac{\partial H}{\partial n} < 0$ , and  $\text{sign} -[(p-c) \frac{\partial h}{\partial \sigma} + \frac{\partial H}{\partial \sigma}] = \text{sign} \frac{\partial \eta}{\partial \sigma} > 0$ ); and

$\text{sign} \frac{dn^e}{d\sigma} = \text{sign} \{ -H\Gamma \}$ , where

$$\begin{aligned} \Gamma &\equiv \frac{\partial H}{\partial \sigma} \left[ c'' (p-c) \left( h + \frac{\partial H}{\partial p} + (p-c) \frac{\partial h}{\partial p} \right) - (c')^2 H \right] \\ &\quad - \left( H + (p-c) \frac{\partial H}{\partial p} \right) c'' \left( (p-c) \frac{\partial h}{\partial \sigma} + \frac{\partial H}{\partial \sigma} \right). \end{aligned}$$

■

## 6.2 Examples

### 6.2.1 Exogenous market structure (barriers to entry)

Denote by  $x$  and  $p$  the symmetric Bertrand equilibria for a given  $z$ , and let  $S$  parameterize total market size.

Linear demand (Shapley and Shubik (1969)).<sup>36</sup> Let  $S = 1$  and  $D_i(p) = \frac{S}{n} (\alpha - \beta [p_i + \gamma (p_i - \frac{1}{n} \sum_i p_i)])$  for  $i = 1, \dots, n$ , where  $\alpha, \beta, \gamma$  are positive constants. We

<sup>36</sup>This linear demand system can be derived from a quadratic utility function (with preferences linear in the numéraire) in which the number of firms  $n$  enters as a parameter. See Vives (1999, Chap. 6).

have  $H = (\alpha - \beta p)/n$ . At a symmetric solution, the direct elasticity of substitution is given by  $\sigma = (1 + \gamma)(\alpha - nx)/nx$ , and it increases with the substitutability parameter  $\gamma$ .<sup>37</sup> We have that  $\frac{\partial H}{\partial n} < 0$  and that  $\frac{\partial h}{\partial n} > 0$ ,  $\frac{\partial \eta}{\partial n} > 0$ ,  $\frac{\partial \eta}{\partial \gamma} > 0$ , and  $\frac{\partial H}{\partial \gamma} = 0$ . For a given symmetric profile  $z$ , there is a unique and symmetric Bertrand equilibrium with price  $p$  and output per firm  $x$ . We have that  $\frac{\partial p}{\partial n} < 0$ ,  $\frac{\partial x}{\partial n} < 0$ ,  $\frac{\partial x}{\partial \gamma} > 0$ , and  $\frac{\partial p}{\partial \gamma} < 0$ . In summary,  $\frac{\partial x}{\partial n} < 0$  and  $\frac{\partial x}{\partial \sigma} > 0$ .

Linear demand (Bowley (1924)). Let  $D_i(p) = S \left( a_n - b_n p_i + c_n \sum_{j \neq i} p_j \right)$  for  $i = 1, \dots, n$ , where  $a_n = \alpha / (\beta + (n - 1)\gamma)$ ,

$b_n = (\beta + (n - 2)\gamma) / ((\beta + (n - 1)\gamma)(\beta - \gamma))$ , and

$c_n = \gamma / ((\beta + (n - 1)\gamma)(\beta - \gamma))$  and where  $\alpha > 0$  and  $\beta > \gamma > 0$  are utility parameters.<sup>38</sup>

At a symmetric solution, the direct elasticity of substitution  $\sigma = p / (\beta - \gamma)x$  increases with  $\gamma$ . The Chamberlinian DD demand function is given by  $H = (\alpha - p) / (\beta + (n - 1)\gamma)$ , where  $\frac{\partial H}{\partial n} < 0$ ,  $\frac{\partial h}{\partial n} < 0$ ,  $\frac{\partial \eta}{\partial n} > 0$ ,  $\frac{\partial \eta}{\partial \gamma} > 0$ , and  $\frac{\partial H}{\partial \gamma} < 0$ . For a given symmetric profile  $z$ , there is a unique and symmetric Bertrand equilibrium with price  $p$  and output per firm  $x$ :  $p = (a_n + b_n c(z)) / (2b_n - (n - 1)c_n)$ . We have that  $\frac{\partial p}{\partial n} < 0$ ,  $\frac{\partial x}{\partial n} < 0$ , and  $\frac{\partial p}{\partial \gamma} < 0$  but  $\frac{\partial x}{\partial \gamma} < 0$ . Hence, in this case, increasing competitive pressure by increasing the elasticity of substitution decreases output. With this particular demand system we have the unusual feature that  $\frac{\partial H}{\partial \sigma} < 0$ . In summary,  $\frac{\partial x}{\partial n} < 0$  and  $\frac{\partial x}{\partial \sigma} < 0$ .

Location models (Salop (1979)). Although formally in models with localized competition the demand system is not exchangeable for  $n > 2$ , the analysis is easily adapted. A uniform mass of customers  $S$  is distributed within a circle in which  $n$  firms have located symmetrically and each produces at constant marginal cost  $c$ . Consumers have a linear transportation cost  $t > 0$ . Then the demand of firm  $i$  setting price  $p_i$  (with neighbors setting a price equal to  $p$ ) is  $\frac{S}{n} + \frac{p - p_i}{t}$  when there is direct competition among firms. We can take  $\sigma \equiv 1/t$ . Therefore  $H = S/n$  and  $H$  is independent of  $p$  and  $\sigma$ . There is neither price–pressure effect nor a demand effect coming from  $\sigma$ . The unique Bertrand equilibrium

<sup>37</sup>For symmetric solutions (with demands arising from the maximization of a quasilinear utility function), the (direct) elasticity of substitution is given by  $\sigma = (\varepsilon_{ij} + \varepsilon_i)^{-1}$ , where  $\varepsilon_{ij}$  is the cross-elasticity of inverse demand,  $\varepsilon_{ij} = \frac{q_j}{p_i} \frac{\partial p_i}{\partial q_j}$ . Note also that  $\varepsilon_{ij} \leq 0$  and  $\varepsilon_i \geq 0$ .

<sup>38</sup>This linear demand system can be derived also from a quadratic utility function (with preferences linear in the numéraire). See Vives (1999, Chap. 6).

is  $p = c + t/n$  and  $\eta = 1 + nc/t$ , which for given  $c$  is increasing in  $n$  and in  $\sigma$ . If the transportation cost is quadratic with parameter  $t$ , then the Bertrand equilibrium is given by  $p = c + t/n^2$ .

Constant elasticity. Let

$$D_i(p) = S(\beta\theta)^{\frac{1}{1-\beta\theta}} \frac{p_i^{\frac{1}{1-\beta}}}{\left(\sum_{j=1}^n p_j^{\frac{\beta}{\beta-1}}\right)^{\frac{1-\theta}{1-\beta\theta}}} \text{ for } i = 1, \dots, n, \text{ with } 0 \leq \beta < 1 \text{ and also } 0 \leq \beta\theta < 1.$$

The (direct) elasticity of substitution is  $\sigma = 1/(1-\beta)$ ; for  $\beta = 0$  goods are independent, and for  $\beta = 1$  they are perfect substitutes. We have that  $H = S(\beta\theta)^{1/(1-\beta\theta)} p^{-1/(1-\beta\theta)} n^{-(1-\theta)/(1-\beta\theta)}$  and that  $\frac{\partial H}{\partial p} < 0$ ,  $\text{sign } \frac{\partial H}{\partial n} = \text{sign}(\theta - 1)$ , and  $\frac{\partial H}{\partial \beta} > 0$ . Restrict attention to the case  $\theta - 1 < 0$  in order to ensure a limited market for the differentiated varieties:  $\frac{\partial H}{\partial n} < 0$ . We have that  $\eta = \frac{1}{1-\beta} \left(1 - \frac{\beta}{n} \frac{1-\theta}{1-\beta\theta}\right)$ , which is strictly increasing in  $n$  and  $\beta$  (and therefore the Lerner index  $L$  will be decreasing in  $n$  and  $\beta$ ). For a given symmetric profile  $z$ , there is a unique and symmetric Bertrand equilibrium with price  $p$  and output per firm  $x$  (the price game is log-supermodular and there is a unique symmetric equilibrium, hence the symmetric equilibrium is the unique one). In equilibrium,  $\frac{\partial h}{\partial n} > 0$ ,  $\frac{\partial \eta}{\partial n} > 0$  (for  $n > 1$ ), and  $p = (n(1-\beta\theta) + \beta(\theta-1))c / (\beta n(1-\beta\theta) + \beta(\theta-1))$ , and it is easily checked that  $\text{sign } \frac{\partial p}{\partial n} = \text{sign}(\theta - 1) < 0$  and  $\frac{\partial x}{\partial n} < 0$ . Furthermore,  $\frac{\partial p}{\partial \beta} < 0$  and  $\frac{\partial x}{\partial \beta} > 0$  because  $\frac{\partial x}{\partial \beta} = \frac{\partial H}{\partial \beta} + \frac{\partial H}{\partial p} \frac{\partial p}{\partial \beta}$ ,  $\frac{\partial H}{\partial \beta} > 0$ ,  $\frac{\partial H}{\partial p} < 0$ , and  $\frac{\partial p}{\partial \beta} < 0$ . In summary,  $\frac{\partial x}{\partial n} < 0$  and  $\frac{\partial x}{\partial \sigma} > 0$ .

Assuming that  $c(z) = \alpha z^{-\gamma}$  with  $\alpha > 0$  and  $\gamma > 0$ , we can obtain a closed-form solution. It can be shown that, evaluating at a symmetric equilibrium,  $B < 0$  if and only if  $\beta\theta < \frac{1}{\gamma+1}$ . Some computations then yield

$$z^* = \left( \alpha^{\beta\theta} (\gamma S)^{\beta\theta-1} n^{1-\theta} (\beta\theta)^{-1} \frac{n(1-\beta\theta) + \beta(\theta-1)}{\beta n(1-\beta\theta) + \beta(\theta-1)} \right)^{\frac{1}{\gamma\beta\theta + \beta\theta - 1}},$$

and

$$p^* = \left[ (S\alpha\gamma)^{\beta\theta-1} (\beta\theta)^{-1} n^{1-\theta} \left( \alpha \frac{n(1-\beta\theta) + \beta(\theta-1)}{\beta n(1-\beta\theta) + \beta(\theta-1)} \right)^{\frac{(\gamma+1)(1-\beta\theta)}{\gamma}} \right]^{\frac{\gamma}{1-\beta\theta - \beta\theta\gamma}}.$$

By Proposition 4 it follows that if  $\beta\theta < \frac{1}{\gamma+1}$  then  $\frac{dz}{dn} < 0$  and  $\frac{dz}{d\beta} > 0$ . Indeed, for  $\beta\theta < \frac{1}{\gamma+1}$  we have  $\text{sign } \frac{dz}{dn} = \text{sign}(\gamma\beta\theta + \beta\theta - 1) < 0$  and  $\frac{dp}{dn} < 0$  because  $\text{sign } \frac{dp}{dn} =$

$-sign \frac{\gamma\beta\theta + \beta\theta - 1}{\gamma\beta\theta + \beta\theta - 1 - \gamma} < 0$ . Furthermore,  $\pi_n^* = z^* \frac{1 - \beta - \beta\gamma \left(1 - \frac{1}{n} \frac{1 - \theta}{1 - \beta\theta}\right)}{\gamma\beta \left(1 - \frac{1}{n} \frac{1 - \theta}{1 - \beta\theta}\right)} = z^* \left( \frac{1}{(\eta - 1)\gamma} - 1 \right)$  and  $sign \widehat{B} = -sign \frac{1}{1 - \beta} \left\{ 1 - \beta - \beta\gamma \left(1 - \frac{1}{n} \frac{1 - \theta}{1 - \beta\theta}\right) \right\}$ . As a result,  $\pi_n^* > 0$  if and only if  $\widehat{B} < 0$ . This means that  $\pi_n^*$  is strictly decreasing in  $n$  whenever positive. Note also that  $\beta \leq \frac{1}{\gamma + 1}$  guarantees that  $\widehat{B} < 0$  for all  $n$ .

Constant expenditure model. Let  $D_i(p) = \frac{S}{p_i} \frac{g(p_i)}{\sum_{j=1}^n g(p_j)}$ ,  $i = 1, \dots, n$ , with  $g > 0$ ,  $g' < 0$ , and  $S > 0$ . We have that  $H = S/np$  and therefore  $\frac{\partial H}{\partial n} < 0$  and  $\frac{\partial H}{\partial \sigma} = 0$ . We have also that  $\frac{d\eta}{dn} > 0$  because  $\frac{\partial H}{\partial n} + (p - c) \frac{\partial h}{\partial n} = -\frac{S}{pn^2} \left[ \frac{c}{p} - \frac{g'(p)}{g(p)} \right] < 0$ .

Let  $g(p) \equiv e^{-\beta p}$  with  $\beta > 0$ . Observe that goods are independent for  $\beta = 0$  yet are perfect substitutes for  $\beta \rightarrow \infty$ . Let  $S = 1$ . For a given symmetric profile  $z$ , there is a unique and symmetric Bertrand equilibrium with price  $p$  and output per firm  $x$  (the price game is log-supermodular and symmetric and there is a unique symmetric equilibrium, so the symmetric equilibrium is the unique one). We have  $p = \left( c + (c^2 + (4cn / (\beta(n - 1))))^{1/2} \right) / 2$ ,  $x = S/np$ ,  $\frac{\partial p}{\partial n} < 0$ ,  $\frac{\partial x}{\partial n} < 0$ ,  $\frac{\partial p}{\partial \beta} < 0$ ,  $\frac{\partial H}{\partial \beta} = 0$ , and  $\frac{\partial x}{\partial \beta} > 0$ .

Another example is of the constant elasticity variety:  $g(p) \equiv p^{-r}$  where  $r > 0$  (see Anderson, de Palma and Thisse (1992, Chap. 7)).<sup>39</sup> Goods are perfect substitutes when  $r \rightarrow \infty$  but are independent when  $r \rightarrow 0$ . We can take  $\sigma = 1 + r$ . (The demand system may arise from  $W(x_0, x) = \left( \sum_i x_i^{\frac{r}{1+r}} \right)^{\frac{1+r}{r}} x_0^\alpha$  for  $\alpha > 0$ , yielding  $S = I/(1 + \alpha)$ , where  $I$  is the income of the representative consumer.) We have that  $h = -S \frac{n(r+1)-r}{(np)^2}$ ,  $\frac{\partial h}{\partial p} = -S \frac{2(n(r+1)-r)}{n^2 p^3} < 0$ , and  $\eta = \frac{n(r+1)-r}{n}$  (which increases with  $n$  and  $r$ ). For a given symmetric profile  $z$ , there is a unique and symmetric Bertrand equilibrium with price  $p$  and output per firm  $x$  (the price game is log-supermodular and symmetric and there is a unique symmetric equilibrium, hence the symmetric equilibrium is the unique one). We have that  $p = c \frac{n(r+1)-r}{r(n-1)}$ ,  $\frac{\partial p}{\partial n} = -\frac{c}{r(n-1)^2} < 0$ ,  $\frac{\partial x}{\partial n} < 0$ ,  $\frac{\partial p}{\partial r} = -\frac{cn}{r^2(n-1)} < 0$ , and  $\frac{\partial x}{\partial r} > 0$  because  $\frac{\partial H}{\partial p} < 0$  and  $\frac{\partial H}{\partial r} = 0$ .

Assuming that  $c(z) = \alpha z^{-\gamma}$  with  $\alpha > 0$  and  $\gamma > 0$ , we can obtain a closed-form solution. It can be shown that  $B < 0$  if and only if  $\frac{\gamma+1}{\gamma} > \frac{r(n-1)}{r(n-1)+4n}$ . This is always true. The equilibrium solution is  $z^* = \frac{S\gamma r(n-1)}{n(n(r+1)-r)}$  and  $p^* = \alpha \left[ \frac{S\gamma}{n} \left( \frac{r(n-1)}{n(r+1)-r} \right)^{\frac{\gamma+1}{\gamma}} \right]^{-\gamma}$ . Indeed, we have that  $sign \frac{dz}{dn} =$

<sup>39</sup>This is also the specification in Aghion et al. (2002).

$sign \left\{ \frac{d}{dn} \left( \frac{r(n-1)}{n(r+1)-r} \right) < 0 \right\}$  and  $sign \frac{dp}{dn} = -sign \left[ -\frac{1}{n^2} \left( \frac{r(n-1)}{n(r+1)-r} \right)^{1/\gamma} \frac{r}{n(r+1)-r} \left( 1 - \frac{\gamma+1}{\gamma} \frac{n}{n(r+1)-r} \right) \right]$ . We have also that  $\frac{dx}{dn} < 0$ .

Profits are given by  $\pi_n = S \left[ \frac{n-\gamma r(n-1)}{n(r(n-1)+n)} \right]$ ; they are strictly decreasing in  $n$ , and  $\pi_n > 0$  if and only if  $n > \gamma r(n-1)$ . This holds for all  $n$  if  $\gamma r < 1$ . Positive profits imply that  $\widehat{B} < 0$  ( $\widehat{B} < 0$  if and only if  $\frac{\gamma+1}{\gamma} \frac{n}{r(n-1)+n} > \frac{r(n-1)}{r(n-1)+4n}$ ) and  $\frac{dp}{dn} < 0$ .

We have that  $L = \frac{n}{n(r+1)-r}$ , which is decreasing in  $n$  and  $r$ . The R&D expenditure/sales ratio  $\frac{z^*n}{p^*x^*n} = \frac{z^*n}{S} = \frac{\gamma r(n-1)}{n(r+1)-r}$  is increasing in  $r$  and  $n$ .

Logit. Let  $D_i(p) = \frac{e^{-p_i/\mu}}{\sum_j e^{-p_j/\mu}} S$ ,  $i = 1, \dots, n$ ,  $\mu > 0$ . We have that goods are perfect substitutes for  $\mu = 0$  and are independent for  $\mu = \infty$ , and the elasticity of substitution is  $pn/\mu$ . Furthermore,  $H(p) = S/n$  and  $h(p) = -(S/n)(1 - 1/n)/\mu$ ,  $\frac{\partial H}{\partial n} < 0$ , and  $\frac{\partial h}{\partial n} > 0$ . We have that  $\eta = \frac{p(n-1)}{\mu n}$ , which is increasing in  $n$  and  $\sigma \equiv 1/\mu$ . For a given symmetric profile  $z$ , there is a unique and symmetric Bertrand equilibrium with price  $p$  and output per firm  $x$  (the price game is log-supermodular and symmetric and there is a unique symmetric equilibrium, so the symmetric equilibrium is the unique one). We have that  $p = c + n\mu/(n-1)$ ,  $\frac{\partial p}{\partial n} < 0$ , and  $\frac{\partial x}{\partial n} < 0$ . There is no price-pressure effect because  $\frac{\partial H}{\partial p} = 0$ , but there is a demand effect  $\frac{\partial H_n}{\partial n} < 0$ . Furthermore,  $\frac{\partial H}{\partial \sigma} = 0$  and therefore there is no demand effect. Neither there is a price-pressure effect, (because  $\frac{\partial H}{\partial p} = 0$ ) and hence, despite that  $\frac{\partial p^*}{\partial \sigma} < 0$ , we have that  $\frac{\partial x^*}{\partial \sigma} = 0$ . (In this case  $B < 0$  always because  $\frac{\partial H}{\partial p} = 0$ .)

As before, assuming that  $c(z) = \alpha z^{-\gamma}$  with  $\alpha > 0$  and  $\gamma > 0$  yields a closed-form solution:  $p = \frac{n\mu}{n-1} + \alpha \left[ \frac{S\alpha\gamma}{n} \right]^{-\frac{\gamma}{\gamma+1}}$  and  $z = \left[ \frac{S\alpha\gamma}{n} \right]^{\frac{1}{\gamma+1}}$ . We have that  $L = \left[ 1 + \frac{n-1}{\mu\gamma S} \left( \frac{S\alpha\gamma}{n} \right)^{\frac{1}{\gamma+1}} \right]^{-1}$ , which is decreasing in  $n$  and  $\sigma \equiv 1/\mu$  and that  $\frac{z^*}{p^*x^*} = \left[ \frac{1}{S\gamma} + \frac{\mu}{n-1} \left( \frac{n}{S\alpha\gamma} \right)^{\frac{1}{\gamma+1}} \right]^{-1}$ , which is increasing in  $n$  and  $\sigma$ .

## 6.2.2 Endogenous market structure (free entry)

Constant elasticity. It can be shown that, evaluating at a symmetric equilibrium,  $B < 0$  if and only if  $\beta\theta < \frac{1}{\gamma+1}$  and  $sign \widehat{B} = -sign \left\{ \frac{1}{1-\beta} - \beta - \beta\gamma \left( 1 - \frac{1-\theta}{n} \frac{1-\theta}{1-\beta\theta} \right) \right\}$ . We have, that for given  $n$ ,

$$z_n = \left( \alpha^{\beta\theta} (\gamma S)^{\beta\theta-1} n^{1-\theta} \frac{n(1-\beta\theta) + \beta(\theta-1)}{\beta n(1-\beta\theta) + \beta(\theta-1)} \right)^{\frac{1}{\gamma\beta\theta + \beta\theta - 1}}$$

and

$$p_n = \left[ (S\alpha\gamma)^{\beta\theta-1} (\beta\theta)^{-1} n^{1-\theta} \left( \frac{\alpha(n(1-\beta\theta)) + \beta(\theta-1)}{\beta[n(1-\beta\theta) + (\theta-1)]} \right)^{\frac{(\gamma+1)(1-\beta\theta)}{\gamma}} \right]^{\frac{\gamma}{1-\beta\theta-\beta\theta\gamma}}.$$

The free-entry number of firms is  $[n^e]$ , where  $n^e$  is the solution to

$$\pi_n = z_n \frac{1 - \beta - \beta\gamma \left( 1 - \frac{1}{n} \frac{1-\theta}{1-\beta\theta} \right)}{\gamma\beta \left( 1 - \frac{1}{n} \frac{1-\theta}{1-\beta\theta} \right)} - F = 0$$

given that variable profits (whenever positive) are strictly decreasing with  $n$ . It is straightforward to check that profits are strictly increasing in  $S$  because  $\partial z / \partial S > 0$ . It follows then that  $\frac{dn^e}{dS} > 0$ .

The following expression implicitly defines  $n^e$ :

$$\begin{aligned} & \left( (\gamma S \alpha)^{\beta\theta-1} n^{1-\theta} (\beta\theta)^{-1} \alpha \frac{n(1-\beta\theta) + \beta(\theta-1)}{\beta n(1-\beta\theta) + \beta(\theta-1)} \right)^{\frac{1}{\gamma\beta\theta+\beta\theta-1}} \\ &= \frac{F\gamma\beta(n(1-\beta\theta) - (1-\theta))}{(1-\beta-\beta\gamma)n(1-\beta\theta) + (1-\theta)\beta\gamma}. \end{aligned}$$

In equilibrium it should hold that  $z^e = \frac{F\gamma\beta([n^e](1-\beta\theta) - (1-\theta))}{(1-\beta-\beta\gamma)[n^e](1-\beta\theta) + (1-\theta)\beta\gamma}$  or  $z^e = \frac{F\gamma(\eta-1)}{1-\gamma(\eta-1)}$ , where  $\eta = \frac{1}{1-\beta} \left( 1 - \frac{\beta}{n} \frac{1-\theta}{1-\beta\theta} \right)$ . From this expression, knowing that  $\frac{dn^e}{dS} > 0$  it follows that  $\frac{dz^e}{dS} > 0$ . (This holds even if  $[n^e]$  stays constant for increasing  $S$ ; in this case, the direct impact of  $S$  increases  $z$ .)

With constant elasticity demand and  $\gamma$  constant, the Lerner index is decreasing in  $z$ . Therefore, increasing  $S$  increases  $z$ , decreases  $L$ , and increases  $\eta$ . The result is that  $n$  must increase.

We know also that increasing  $F$  increases  $z$  (because  $\text{sign} \frac{dz^e}{dF} = -\text{sign} \frac{dz_n}{dn} > 0$ ) and increases  $p$  (because  $\text{sign} \frac{dp^e}{dF} = \text{sign} \frac{dp_n}{dn} > 0$ ). If  $F = 0$  and  $\beta \leq \frac{1}{\gamma+1}$ , then profits are strictly positive for all  $n$  and  $n^e = \infty$ .

If  $F = 0$ ,  $\beta > \frac{1}{\gamma+1}$ , and  $\beta\theta < \frac{1}{\gamma+1}$ , then we still know that profits (whenever positive) are strictly decreasing with  $n$ . Then the free-entry number of firms is  $\left[ \frac{\beta\gamma(1-\theta)}{(1-\beta\theta)(\beta\gamma+\beta-1)} \right]$  because, at this  $n$ , adding one more firm would result in negative profits. In this case the free-entry number of firms is independent of  $S$ , and  $n^e \geq 1$  as long as  $\beta > \frac{1}{\gamma+1}$ ; as before,

under our assumptions  $\left(\beta\theta < \frac{1}{\gamma+1}\right)$ ,  $\frac{dz}{dS} > 0$  and  $\frac{dp}{dS} < 0$ . (Note that for  $n = \frac{\beta\gamma(1-\theta)}{(1-\beta\theta)(\beta\gamma+\beta-1)}$  we have  $\widehat{B} = 0$ .) Furthermore,  $\frac{dn^e}{d\beta} < 0$  (using the assumption  $\beta\theta < \frac{1}{\gamma+1}$ ).

Constant expenditure (and constant elasticity). Given  $n$ ,  $z_n = \frac{S\gamma r(n-1)}{n(n(r+1)-r)}$  and profits are given by  $\pi_n = S \left[ \frac{n-\gamma r(n-1)}{n(r(n-1)+n)} \right]$ . They are strictly decreasing in  $n$ , and  $\pi_n > 0$  if and only if  $n > \gamma r(n-1)$ . This holds for all  $n$  if  $\gamma r < 1$ . Positive profits imply that  $\widehat{B} < 0$  ( $\widehat{B} < 0$  if and only if  $\frac{\gamma+1}{\gamma} \frac{n}{r(n-1)+n} > \frac{r(n-1)}{r(n-1)+4n}$ ).

Using the zero profit-entry condition we obtain

$$n^e = \frac{(F - S\gamma)r + S + \sqrt{(Fr + S - S\gamma r)^2 + 4\gamma r S F (r + 1)}}{2F(r + 1)},$$

which is strictly increasing in  $S$  provided that  $\gamma r < 1$ . Furthermore, as expected,  $\frac{dz^e}{dS} > 0$ ,  $\frac{dz^e}{dr} > 0$ , and  $\text{sign} \frac{dn}{dr} = \text{sign} \frac{d\pi}{dr} < 0$  (recall that  $\partial H / \partial r = 0$ ). We know also that increasing  $F$  increases  $z$  (because  $\text{sign} \frac{dz^e}{dF} = -\text{sign} \frac{dz_n}{dn} > 0$ ) and increases  $p$  (because  $\text{sign} \frac{dp^e}{dF} = -\text{sign} \frac{dp_n}{dn} > 0$ ). The Lerner index is given by  $L = \frac{n}{r(n-1)+n}$ , and it can be checked that (a)  $\frac{dL}{dr} < 0$  whenever  $n > r\gamma(n-1)$  and (b)  $L$  is increasing in  $F$  because  $n$  is decreasing in  $F$ .

Logit. Given  $n$ , we have  $p_n = c(z) + \frac{\mu n}{n-1}$  and  $z_n = \left[ \frac{S\alpha\gamma}{n} \right]^{\frac{1}{\gamma+1}}$ . Profits (gross of fixed costs) are given by  $\pi_n = \frac{S\mu}{n-1} - \left[ \frac{S\alpha\gamma}{n} \right]^{\frac{1}{\gamma+1}}$ . For profits to be decreasing in  $n$  we need  $\frac{S\mu(\gamma+1)}{n-1} - \frac{n-1}{n} \left[ \frac{S\alpha\gamma}{n} \right]^{\frac{1}{\gamma+1}} > 0$  (which is equivalent to  $\widehat{B} < 0$  and is implied by positive profits  $\pi_n > 0$ ).<sup>40</sup> We conclude that if  $\pi_n$  is positive then it is strictly decreasing in  $n$ . The free-entry number of firms is implicitly defined by  $[n^e]$  where  $n^e$  solves  $\frac{S\mu}{n-1} - \left[ \frac{S\alpha\gamma}{n} \right]^{\frac{1}{\gamma+1}} = F$ . Consistent with our other results, we have  $\frac{dn^e}{dS} > 0$  and  $\frac{dz^e}{dS} > 0$ ;  $\frac{dx}{d\mu} < 0$  or  $\frac{dx}{d\sigma} > 0$ ;  $\frac{dz}{d\mu} < 0$  or  $\frac{dz}{d\sigma} > 0$ ; and  $\frac{dn^e}{d\mu} > 0$  or  $\frac{dn^e}{d\sigma} < 0$ . Increasing  $F$  increases  $z$  (because  $\text{sign} \frac{dz^e}{dF} = -\text{sign} \frac{dz_n}{dn} > 0$ ) and the impact on  $p$  is ambiguous (because  $\text{sign} \frac{dp^e}{dF} = -\text{sign} \frac{dp_n}{dn}$ ). We have that  $L = \left[ 1 + \left( \frac{\mu}{n-1} \right)^{-1} \left( \frac{S\alpha\gamma}{n} \right)^{\frac{1}{\gamma+1}} \frac{1}{S\gamma} \right]^{-1}$  and  $dL/d\sigma < 0$  (taking into account the impact of  $\sigma$  on  $L$ ), and  $dL/dF > 0$  because  $L$  decreases in  $n$  and  $n$  decreases with  $F$ .

<sup>40</sup>It can be checked that  $B^* < 0$  at  $n = n^*$ : observe that  $B^* < 0$  if and only if  $\gamma + 1 + \frac{1-n}{\mu n} \gamma \alpha z^{-\gamma} > 0$ , and this holds at equilibrium if and only if  $\frac{S\mu}{n-1} - F = \left[ \frac{S\alpha\gamma}{n} \right]^{\frac{1}{\gamma+1}} < \frac{\mu S(\gamma+1)}{n-1}$ .

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