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## FUNDAMENTALS AND JOINT CURRENCY CRISES

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## ABSTRACT

### Fundamentals and Joint Currency Crises\*

In this note we demonstrate that in affine models for bilateral exchange rates, the nature of return interdependence during crises depends on the tail properties of the fundamentals' distribution. We denote crisis linkages as either strong or weak, in the sense that the dependence remains or vanishes asymptotically. We show that if one currency return reaches crisis levels, the probability that the other currency breaks down as well vanishes asymptotically if the fundamentals' distributions exhibit light tails (like, for example, the normal). If, however, the marginal distributions exhibit heavy tails, the probability that the other currency breaks down as well remains strictly positive even in the limit. This result implies that linearity and heavy tails are sufficient conditions for joint or contagious currency crises to happen systematically through fundamentals.

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# FUNDAMENTALS AND JOINT CURRENCY CRISES

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## 1. INTRODUCTION

Financial crises are usually described as failures of financial institutions or sharp falls in asset prices. Since long there is an active debate about the origins and nature of such crises. For example, one view holds that they are the expression of an occasional inherent malfunctioning of financial institutions or markets. Another view rather sees crises as caused by bad outcomes in underlying economic variables (fundamentals). Representative of the first view is the literature modelling univariate crises as self-fulfilling events in the presence of multiple equilibria (sunspots). For example, Diamond and Dybvig (1983) show that bank depositor runs can occur as a self-fulfilling prophecy, which would imply that they happen more or less randomly. Obstfeld (1986) argues that also currency crises can occur as a consequence of multiple equilibria. This is in contrast with the literature pointing to the fact that many such crises seem to have occurred in relation to unfavourable macroeconomic conditions, sometimes caused by bad policies. For example, Gorton (1988) makes forcefully the point that most episodes of banking instability in US history seem to have been related to business cycle downturns rather than occurring randomly. Krugman (1979) shows how unsustainably large budget deficits can lead to currency attacks.

The primary concern with financial crises is that they reach a large breadth, in the sense that banks fail or markets crash together. The reason for this concern is that these widespread (or systemic) crises have the strongest real effects, in that aggregate consumption, investment and growth are adversely affected. A more recent literature on financial contagion and systemic risk has therefore started to pay attention to the breadth of crises. For example, Allen and Gale (2000) model the spreading of bank failures through interbank exposures. Masson (1999) illustrates various forms of joint currency crises in

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a macroeconomic two-country model, covering both self-fulfilling and fundamentals-based crises. First empirical tests of joint currency crises have already been provided by Eichengreen, Rose and Wyplosz (1996) or Kaminsky and Reinhart (2000).<sup>1</sup> As understanding the spreading of financial crises is very important (for the reason given above), the present paper provides a new perspective on this issue. More specifically, by combining exchange rate theory with extreme value analysis, we derive conditions under which widespread crises occur systematically.

We develop our point in the context of exchange rates and currency crises, but – as we will explain below – the argument is more general, applying to crises in many financial markets. Within the context of a simple affine exchange rate model, we show that the magnitude of the cross-currency interdependence during crisis periods hinges upon the tail properties of the marginal distributions of the variables determining exchange rates. More specifically, suppose that the logarithmic exchange rate returns are an affine function of the domestic and foreign fundamentals. This implies that different exchange rate returns against the same base currency are correlated. Nevertheless, we show that if one currency return reaches crisis levels, the probability that the other currency breaks down as well (increasing the threshold at which one speaks of a crisis without bound) vanishes asymptotically, if the forex fundamentals are thin tailed (e.g. normally distributed). In plain English, joint currency crises are neither very frequent nor do they exhibit vehemence under this condition. Alternatively, if the fundamentals exhibit heavy tails, the probability that the other currency breaks down as well remains strictly positive in the limit. In this case severe joint crises do happen relatively frequently.

Our main result is as simple as it may surprise. Two basic conditions are sufficient for systemic (widespread) currency market crises to occur frequently and with vehemence. First, the univariate distributions describing the behaviour of economic variables are heavy tailed. Loosely speaking, the heavy tail feature means that the probability of univariate currency collapses is much higher than what one would expect if the underlying fundamentals were normally distributed. Second, nominal bilateral exchange rates, expressed against the same currency, are linear expressions of the domestic and base currency fundamentals. The interesting and first novel element of this result is that the degree of cross-sectional dependence between exchange rate returns

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<sup>1</sup>For a broader survey of the contagion literature, see De Bandt and Hartmann (2000).

during crisis periods (so called asymptotic dependence), and thereby the breadth of currency crises, seems to be related to the univariate frequency of extreme realizations in macroeconomic fundamentals. We derive this result by combining standard exchange rate economics with multivariate statistical extreme value analysis.<sup>2</sup> While fat tails and tail dependence of asset returns have by now been extensively documented in the empirical literature,<sup>3</sup> how the marginal tail thickness relates theoretically to the bivariate tail dependence of returns in standard asset pricing models has – to the best of our knowledge – not been dealt with before. The combination of asset pricing (here exchange rate) theory with extreme value analysis is the second, more methodological novel element of our paper.

Based on this analysis one may classify currency linkages during times of market stress into a weak and a strong type, depending on whether the conditional crash probability respectively vanishes or persists asymptotically. Correspondingly, the international monetary and financial system may be characterized as being relatively stable in the former case, while it is more fragile in the latter case. Our two conditions, linearity and univariate heavy tails, are sufficient for having a more fragile system.

Our result has also some relevance for policy, as public authorities can influence the distribution of underlying economic variables. If policy makers abstained from any action that may cause or accommodate extreme movements in economic fundamentals, then they would help avoiding widespread currency crises. In normal times this may mean

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<sup>2</sup>Recently, a number of studies contributed to the financial contagion literature by employing multivariate extreme value analysis to estimate extreme asset return linkages (tail dependence). A first generation of papers provides bivariate analyses in the same asset class; see e.g. Straetmans (2000), Longin and Solnik (2001) and Poon et al. (2001) for stock markets and Starica (1999) and Hartmann et al. (2003) for foreign exchange markets. A second generation of papers either offers bivariate analyses across different asset classes, such as stock-bond linkages and the flight-to-quality phenomenon in G-5 economies analyzed in Hartmann et al. (forthcoming), or higher order multivariate linkages.

<sup>3</sup>Since the seminal work by Mandelbrot (1963), numerous studies have estimated the tail thickness of univariate asset return distributions, generally finding more frequent crashes than would be predicted by the normal distribution. The relative occurrence of stock market extremes has by far received most of the attention; see e.g. Blattberg and Gonedes (1974), Jansen and de Vries (1991) or Jondeau and Rockinger (2003). Bond market extremes have been considered in Hartmann et al. (forthcoming) for example. Boothe and Glassman (1987), Hols and de Vries (1991) or Mittnik et al. (2000) e.g. investigate the tail fatness of foreign exchange rate returns.

e.g. to conduct monetary and fiscal policies with a ‘steady hand’, avoiding drastic changes in money supply or government expenditures. In very volatile times it may mean to counteract fluctuations in fundamentals through decisive action. In this fashion the tails of the distributions of underlying economic variables would be made thinner.

The remainder of this note proceeds as follows. In section 2 we introduce the canonical affine exchange rate model in which we study the relationship between marginal tail thickness and bivariate tail dependence. A discussion and comparison of different measures to characterize currency linkages during periods of market stress is provided in section 3. The central result of the note on the relationship between the univariate properties of economic fundamentals and the frequency and severity of exchange rate linkages during crises we derive in section 4. The two cases of thin tailed and fat tailed marginals are treated in two separate subsections. Finally, section 5 contains a summary and conclusions.

## 2. AFFINE EXCHANGE RATE MODELS

Consider the standard monetary model of the log price of currency  $j$  in terms of currency 0

$$\begin{aligned} s_{0j} &= (m_0 - \phi y_0 + \lambda R_0) - (m_j - \phi y_j + \lambda R_j) \\ &= g_0 - g_j, \quad j = 1, \dots, n \quad . \end{aligned}$$

$g_0$  and  $g_j$  are composite fundamentals consisting of the logarithmic money measure  $m$ , the negative of the income elasticity times log real income  $-\phi y$  and the semi interest rate elasticity times the interest rate  $\lambda R$  (see e.g. Frenkel, 1976, or Obstfeld and Rogoff, 1996, ch. 8). In first differences the monetary model can be concisely summarized as

$$(2.1) \quad \Delta s_{0j} = \Delta g_0 - \Delta g_j.$$

The linear in first difference specification reveals two properties that will prove crucial in the following sections. First, the set of multiple exchange rates  $\Delta s_{0j}$  ( $j = 1, \dots, n$ ) all have the fundamental  $\Delta g_0$  in common. This exposure to shocks in the numéraire currency may be important, as illustrated e.g. in Aghion, Bachetta and Banerjee (2001). For a set of emerging market currencies, they plot the ratio of dollar denominated liabilities to claims with respect to foreign banks in 1997 right before the start of the Asian crisis. Given the high content of dollar denominated debt, most of the emerging market currencies were therefore highly exposed to the same US interest rate fluctuations. Second, (2.1) is linear in the first differences of the composite fundamental

$g$  and hence the individual fundamentals as well. The linear specification conforms e.g. to the linear factor model used in Forbes and Chinn (2003), who show that trade linkages are important transmitters of shocks between countries.<sup>4</sup>

The use of linear models is by no means limited to the monetary model or the exchange rate literature, cf. the popular Arbitrage Pricing Theory for explaining equilibrium equity returns (Ross, 1976). Thus our results pertain to linkages between other classes of assets as well. Investment banks, for example, often hold sizable portfolios of commercial company equity. Sharpe fluctuations in the companies' equity portfolios in turn influence the banks' own shareholder value. As long as different investment banks hold stakes in the same companies with heavy tailed distributed returns, bank stocks are necessarily interdependent (see e.g. Acharya and Yorulmazer, 2003).

### 3. MEASURES OF DEPENDENCE

**3.1. The correlation measure.** A standard measure of dependence is the coefficient of correlation  $\rho$ . As is well known the means, variances and the correlation coefficient of a pair of random variables completely characterize the bivariate normal distribution. One must ask, however, how well  $\rho$  captures the dependence if it is unknown whether the data are normally distributed or not. Specifically, one wonders whether  $\rho$  adequately captures the interdependence at crisis levels. Boyer, Gibson, and Loretan (1997) have noticed that even if the normal model applies, verifying the market speak of increased correlation during times of crisis by calculating conditional correlation coefficients can be illusory. Forbes and Rigobon (2002) show that, indeed, if one corrects  $\rho$  not much correlation change can be identified around crisis times.

One of the problems associated with the concept of correlation is that the data may be dependent, while the correlation coefficient is zero. Consider e.g. the discrete uniform distribution on the 8 points  $(x, y) \in \{(1, 1), (1, -1), (-1, 1), (-1, -1), (2, 2), (2, -2), (-2, 2), (-2, -2)\}$ . Due to the symmetry of this distribution it is immediate that  $\rho = 0$ , though the data are not independent. If  $x = -1$ ,  $y$  cannot be equal to 2, and  $P\{Y > 1|X > 1\} = 1/2$ , while unconditionally  $P\{Y > 1\} = 1/4$  only. Hence,  $X$  and  $Y$  are dependent and  $\rho$  does not capture this dependence.<sup>5</sup>

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<sup>4</sup>Note that the monetary model captures the mirror image of the trade account through movements in the capital account.

<sup>5</sup>The bivariate Student-t distribution constitutes another popular example. Even if  $\rho = 0$ , the model still exhibits dependence because the joint distribution cannot

Lastly, economists evaluating investments within expected utility theory frameworks are not so much interested in the correlation measure itself; they rather have an interest in the trade-offs between risk measured as a probability and the gains or losses, which are the quantiles of the return distribution. As such the correlation is only an intermediate step in the calculation of this trade-off between quantile and probability. Therefore we like to turn to a measure which is not conditioned on a particular multivariate distribution and which directly reflects the probabilities and associated crash levels.

**3.2. Co-crash probabilities.** What is worrying for supervisors and industry representatives is that a heavy loss in one market goes hand in hand with a heavy loss in another market, destroying the real value of a diversified investment portfolio. More specifically, one asks given that  $Y > s$ , what is the probability that  $X > s$ , where  $X$  and  $Y$  stand for currency returns and  $s$  is the common high loss level.<sup>6</sup> Since we are interested in the extreme linkage probabilities, we will try to directly evaluate these probabilities, bypassing the correlation concept.

If two random variables  $X$  and  $Y$  are not independent, having some information about one variable, say  $X$ , implies that one has also information about the other variable,  $Y$ . This can be readily expressed as a conditional probability  $P\{Y > s | X > s\}$ . We will, however, adopt the related probability measure that conditions on any market crash, without indicating the specific market. This is the linkage measure

$$\frac{P\{X > s\} + P\{Y > s\}}{1 - P\{X \leq s, Y \leq s\}}$$

proposed in Hartmann et al. (forthcoming). The linkage measure, even though it is the sum of two conditional probabilities, reflects the expected number of currency crashes given that least one currency has collapsed. To see this, let  $\kappa$  denote the number of simultaneously crashing currencies, i.e., returns exceeding  $s$ , and write the conditionally expected number of currency crashes given a collapse of at least one currency as  $E\{\kappa | \kappa \geq 1\}$ .

From probability theory we have that

$$E\{\kappa | \kappa \geq 1\} = 1 \frac{P\{X > s, Y \leq s\} + P\{X \leq s, Y > s\}}{1 - P\{X \leq s, Y \leq s\}} + 2 \frac{P\{X > s, Y > s\}}{1 - P\{X \leq s, Y \leq s\}} =$$

be factorized into the marginal dfs. In general, statistical or stochastic independence is sufficient for a zero correlation coefficient but not vice versa, see e.g. Feller (1971).

<sup>6</sup>Without loss of generality we can take the two quantiles on which we condition equal to  $s$ .

$$(3.1) \quad \frac{P\{X > s\} + P\{Y > s\}}{1 - P\{X \leq s, Y \leq s\}}.$$

The conditional expectation measure  $E\{\kappa|\kappa \geq 1\}$  has also the advantages that it can be easily extended beyond the bivariate setting and that one does not need to specify the crashing, conditioning asset whereby one would look only into one direction in the plane.

To develop some intuition for this measure as a device for measuring dependence during times of market stress, consider two polar cases.

**Case 1.** *If  $X$  and  $Y$  are independent and identically distributed (i.i.d.) and writing  $p = P\{X > s\}$ , then*

$$E\{\kappa|\kappa \geq 1\} = \frac{2p}{1 - (1 - p)^2} = \frac{2}{2 - p}.$$

*In the limit  $p \rightarrow 0$  as  $s \rightarrow \infty$ , and hence  $E\{\kappa|\kappa \geq 1\} \rightarrow 1$ .*

**Case 2.** *If  $X = Y$  and writing  $p = P\{X > s\}$ , then*

$$E\{\kappa|\kappa \geq 1\} = \frac{2p}{1 - (1 - p)} = 2.$$

*Clearly, even as  $p \rightarrow 0$ , still  $E\{\kappa|\kappa \geq 1\} = 2$ .*

These two cases show that  $1 \leq E\{\kappa|\kappa \geq 1\} \leq 2$ . In case the return pair is completely independent (Case 1),  $E\{\kappa|\kappa \geq 1\}$  reaches its lower bound for very large quantiles  $s$ , which implies that the data are also asymptotically independent. On the other hand, if the data are completely dependent, then in the limit ( $s \rightarrow \infty$ ),  $E\{\kappa|\kappa \geq 1\}$  will still equal 2 (complete asymptotic dependence). Also notice that even though in the first case the data are independent, the dependence measure  $E\{\kappa|\kappa \geq 1\}$  is higher than 1 at all finite levels of  $p$  since even with independent returns, there is a nonzero probability that ‘two markets will crash, given that at least one market crashes’.

As for the intermediate case of imperfectly correlated returns ( $\rho \neq 0$ ,  $|\rho| < 1$ ), either  $E\{\kappa|\kappa \geq 1\} = 1$  (asymptotic independence) or  $1 < E\{\kappa|\kappa \geq 1\} \leq 2$  (asymptotic dependence), if the quantile  $s$  gets large. In particular, one cannot rule out that currency returns are asymptotically independent in the presence of a nonzero correlation.

#### 4. WEAK AND STRONG CURRENCY CRISIS LINKAGES

Within the affine currency model framework from section 2, we are now ready to prove that the limiting value of (3.1) critically depends on the tail properties of the marginal distributions of the currency fundamentals. We dub the crisis linkage as weak (asymptotic independence)

whenever  $E\{\kappa|\kappa \geq 1\} = 1$  in the limit, and strong (asymptotic dependence) otherwise. If the former case applies, the international monetary and financial system is more stable as severe crises in one currency are not associated with crises in other currencies, whereas in the latter case it is subject to systemic risk and therefore more fragile. For example, the existence of only weak crisis linkages implies the absence of the statistically significant occurrence of currency contagion.

Assume that each of the countries' composite fundamentals  $\Delta g$  in (2.1) is independent from all the other countries' composite fundamentals. Regarding the distribution of  $\Delta g$ , we either assume normality or that the distribution exhibits heavy tails in the sense that tail probabilities are declining as a power function of the quantile (to be made precise below). Notice that tails of the normal distribution are governed by the exponential function whereas a heavy tailed model like the Student-t exhibits a Pareto distribution-type decline. It is more or less a stylized fact that many asset returns are heavy tailed. We show that this necessarily leads to asymptotic dependence. Conversely, we also show that if the fundamentals exhibit light tails, such as the normal distribution, then the forex returns are asymptotically independent

In order to derive our main result it is sufficient to consider a three currency system with composite fundamentals  $\Delta g_0 = X$ ,  $\Delta g_1 = -Y$ , and  $\Delta g_2 = -Z$  such that  $\Delta s_{01} = X + Y$  and  $\Delta s_{02} = X + Z$ . We may assume that  $X$ ,  $Y$ , and  $Z$  are i.i.d.<sup>7</sup>

**4.1. Fundamentals with light tails.** In this subsection we assume that  $X, Y$  and  $Z$  are standard normally distributed random variables. As normality is preserved under summation the pair of random variables  $(\Delta s_{01}, \Delta s_{02})$  exhibits a bivariate normal distribution with correlation coefficient  $\rho = 1/2$ .

**Proposition 1.** *If  $\Delta s_{01}$  and  $\Delta s_{02}$  follow a bivariate normal distribution with  $\rho = 1/2$ , then  $\lim_{s \rightarrow \infty} E\{\kappa|\kappa \geq 1\} = 1$ , so that the crisis linkage is weak.*

In order to prove this claim we use Sibuya's (1960) approach and the following asymptotic expansion for the tail probability of a normally

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<sup>7</sup>In practice, basic fundamentals like money supplies, national income levels and interest rates cannot be considered as being independent across countries. However, it can be easily shown that the relationship we derive between marginal tail heaviness and bivariate tail dependence still holds for pairwise dependent  $X, Y$  and  $Z$ . The dependence – if present – actually even strengthens our results. By assuming independence we isolate the most difficult case to prove.

distributed random variable:

$$(4.1) \quad \Pr\{\theta X > s\} \sim \frac{1}{\sqrt{2\pi}} \frac{\theta}{s} \exp\left(-\frac{1}{2}\left(\frac{s}{\theta}\right)^2\right), \quad s \text{ large}$$

(see e.g. Abramowitz and Stegun, 1972, p. 932). To indicate equality in distribution we use the double arrow symbol " $\Rightarrow$ ".

*Proof.* We start by noticing that the expectational linkage measure (3.1) can be transformed as follows:

$$(4.2) \quad \begin{aligned} E\{\kappa|\kappa \geq 1\} &= \frac{\Pr\{\Delta s_{01} > s\} + P\{\Delta s_{02} > s\}}{1 - \Pr\{\Delta s_{01} \leq s, \Delta s_{02} \leq s\}} \\ &= \frac{1}{1 - \frac{\Pr\{\Delta s_{01} > s, \Delta s_{02} > s\}}{\Pr\{\Delta s_{01} > s\} + P\{\Delta s_{02} > s\}}}. \end{aligned}$$

Thus, we are left with proving that

$$\lim_{s \rightarrow \infty} \frac{\Pr\{\Delta s_{01} > s, \Delta s_{02} > s\}}{\Pr\{\Delta s_{01} > s\} + P\{\Delta s_{02} > s\}} = 0.$$

Evidently, the marginal tail probabilities in (4.2) are governed by the asymptotic expansion (4.1), e.g. for  $\Delta s_{01}$ :

$$\begin{aligned} \Pr\{\Delta s_{01} > s\} &= \Pr\{X + Y > s\} \\ &\Rightarrow \Pr\{\sqrt{2}X > s\} \\ &\sim \frac{1}{\sqrt{\pi}} \frac{1}{s} e^{-s^2/4} \end{aligned}$$

for large  $s$ . As for the joint exceedance probability in (4.2) an upward bound exists:

$$\begin{aligned} \Pr\{\Delta s_{01} > s, \Delta s_{02} > s\} &\leq \Pr\{\Delta s_{01} + \Delta s_{02} > 2s\} \\ &= \Pr\{2X + Y + Z > 2s\} \\ &\Rightarrow \Pr\left\{\frac{1}{2}\sqrt{6}X > s\right\} \\ &\sim \sqrt{\frac{3}{\pi}} \frac{1}{2s} e^{-s^2/3}. \end{aligned}$$

Thus, upon combining the last expressions and under the stated normality assumptions

$$\begin{aligned} \frac{\Pr\{\Delta s_{01} > s, \Delta s_{02} > s\}}{\Pr\{\Delta s_{01} > s\} + P\{\Delta s_{02} > s\}} &= \frac{\Pr\{\Delta s_{01} > s, \Delta s_{02} > s\}}{2 \Pr\{\Delta s_{01} > s\}} \\ &\leq \frac{\sqrt{3}}{4} \exp\left(-\frac{s^2}{3} + \frac{s^2}{4}\right) \rightarrow 0 \text{ as } s \rightarrow \infty. \end{aligned}$$

Hence,

$$\lim_{s \rightarrow \infty} E\{\kappa | \kappa \geq 1\} = 1$$

□

This asymptotic independence results is by no means limited to the class of normal distributions. A similar procedure can be used to verify the asymptotic independence for many other types of joint distributions. But the normal distribution appears most interesting, since it is so often assumed in theoretical and empirical work on exchange rate returns and in other asset pricing applications. Note that we have just shown that this assumption implies that currency (or other financial market) contagion cannot occur systematically.

**4.2. Fundamentals with heavy tails.** Prior to relating the tail fatness of exchange rate fundamentals to their degree of asymptotic dependence, we need a formal definition of what ‘fat tails’ exactly means. A random variable exhibits heavy tails if its distribution function  $F(s)$  far into the tails has a first order term identical to the Pareto distribution, i.e.

$$(4.3) \quad F(s) = 1 - s^{-\alpha}L(s) \quad \text{as } s \rightarrow \infty,$$

where  $L(s)$  is a slowly varying function such that

$$(4.4) \quad \lim_{t \rightarrow \infty} \frac{L(ts)}{L(t)} = 1, \quad s > 0.$$

It can be easily shown that conditions (4.3)-(4.4) are equivalent to

$$(4.5) \quad \lim_{t \rightarrow \infty} \frac{1 - F(ts)}{1 - F(t)} = s^{-\alpha}, \quad \alpha > 0, \quad s > 0,$$

i.e., the distribution varies regularly at infinity. The tail index  $\alpha$  can be interpreted as the number of bounded distributional moments. And as not all moments are bounded, we speak of heavy tails. Distributions like the Student-t, F-distribution, Burr distribution, sum-stable distributions with unbounded variance all fall into this class. It can be shown that the unconditional distribution of the ARCH and GARCH

processes belongs to this class as well. Note that Student-t distributions are often used in the empirical modelling of the unconditional return of exchange rates, see e.g. Boothe and Glassmann (1987), while GARCH process are extremely popular conditional models.

To derive our result, we need to use Feller's convolution theorem (Feller, 1971, VIII.8).

**Theorem 1.** *Let  $X_i$  be i.i.d. random variables with regularly varying symmetric tails, i.e. as  $s \rightarrow \infty$*

$$\Pr\{X_i \leq -s\} = \Pr\{X_i > s\} = s^{-\alpha}L(s).$$

*Then for the tail of the distribution of the sum of  $X_i$  ( $i = 1, \dots, n$ ) ( $n$ -fold convolution) as  $s \rightarrow \infty$*

$$\Pr\left\{\sum_{i=1}^n X_i \leq s\right\} = 1 - ns^{-\alpha}L(s).$$

In three dimensions this theorem implies by the independence of the  $X_i$  that for large  $s$

$$\begin{aligned} \Pr\left\{\sum_{i=1}^3 X_i \leq s\right\} &\sim 1 - \sum_{i=1}^3 \Pr\{X_i > s\} \\ &\sim \Pr\{X_1 \leq s\} \Pr\{X_2 \leq s\} \Pr\{X_3 \leq s\} \\ &= \Pr\{X_1 \leq s, X_2 \leq s, X_3 \leq s\}. \end{aligned}$$

In other words, the probability on the area below the plane  $\sum_{i=1}^3 X_i = s$  equals the probability on the lower bar  $\{X_1 \leq s, X_2 \leq s, X_3 \leq s\}$ . The first step is the Theorem 1. The second step is a consequence of the independence, which implies that the joint probability

$$\begin{aligned} \Pr\{X_1 \leq s, X_2 \leq s, X_3 \leq s\} &= \Pr\{X_1 \leq s\} \Pr\{X_2 \leq s\} \Pr\{X_3 \leq s\} \\ &= [1 - s^{-\alpha}L(s)]^3 \\ &= 1 - 3s^{-\alpha}L(s) + o(s^{-\alpha}). \end{aligned}$$

Thus for large quantiles  $s$  all mass concentrates along the axes, so that hyperplanes and bars that cut the three axes at the same points separate the same probability mass. This implies the following:

**Proposition 2.** *Let  $X, Y$  and  $Z$  be i.i.d. random variables with regularly varying tails, i.e. as  $s \rightarrow \infty$*

$$\begin{aligned} \Pr\{X \leq -s\} &= \Pr\{Y \leq -s\} = \Pr\{Z \leq -s\} = s^{-\alpha}L(s), \\ \Pr\{X > s\} &= \Pr\{Y > s\} = \Pr\{Z > s\} = s^{-\alpha}L(s). \end{aligned}$$

Then

$$\lim_{s \rightarrow \infty} E \{ \kappa | \kappa \geq 1 \} = \frac{4}{3}.$$

*Proof.* By definition

$$\begin{aligned} \lim_{s \rightarrow \infty} E \{ \kappa | \kappa \geq 1 \} &= \lim_{s \rightarrow \infty} \frac{\Pr\{\Delta s_{01} > s\} + P\{\Delta s_{02} > s\}}{1 - \Pr\{\Delta s_{01} \leq s, \Delta s_{02} \leq s\}} \\ (4.6) \qquad &= \lim_{s \rightarrow \infty} \frac{\Pr\{X + Y > s\} + \Pr\{X + Z > s\}}{1 - \Pr\{X + Y \leq s, X + Z \leq s\}}. \end{aligned}$$

By Feller's convolution theorem 1 we directly have for the numerator in (4.6) that

$$\Pr\{X + Y > s\} + \Pr\{X + Z > s\} \sim 2s^{-\alpha}L(s) + 2s^{-\alpha}L(s).$$

For the denominator

$$1 - \Pr\{X + Y \leq s, X + Z \leq s\}$$

note that the lines  $X + Y = s$  and  $X + Z = s$  are two of the three edges of the triangular plane  $\sum_{i=1}^3 X_i = s$  in the positive quadrant. We noted above that Feller's theorem implies that for large  $s$  all mass is along the three axes. Hence, if we are interested in the joint probability of being below any two of the three edges of the triangular plane, this is necessarily equal to the probability of being below the triangular plane, since the set of two edges cuts the three axes at the same points (as the triangular plane). Hence,

$$\begin{aligned} 1 - \Pr\{X + Y \leq s, X + Z \leq s\} &\sim 1 - \Pr\{X + Y + Z \leq s\} \\ &\sim 3s^{-\alpha}L(s). \end{aligned}$$

Thus

$$\begin{aligned} \lim_{s \rightarrow \infty} \frac{\Pr\{X + Y > s\} + \Pr\{X + Z > s\}}{1 - \Pr\{X + Y \leq s, X + Z \leq s\}} &= \lim_{s \rightarrow \infty} \frac{2s^{-\alpha}L(s) + 2s^{-\alpha}L(s)}{3s^{-\alpha}L(s)} \\ &= \frac{4}{3}. \end{aligned}$$

□

The two exchange rates returns  $\Delta s_{01}$  and  $\Delta s_{02}$  are asymptotically dependent, since  $\lim_{s \rightarrow \infty} E \{ \kappa | \kappa \geq 1 \} = 4/3 > 1$ . Thus the crisis linkage for this class of distributions is strong and the international monetary and financial system appears relatively fragile, exhibiting systemic risk.

Note, however, that proposition 2 does not imply that there are no joint distributions that have heavy tailed marginals, positive correlation

and asymptotic independence. In fact one can easily verify that for e.g. the bivariate Gumbel-Pareto distribution

$$F(x, y) = (1 - x^{-\alpha})(1 - y^{-\alpha})(1 + \beta x^{-\alpha} y^{-\alpha}), \quad \alpha > 0, 0 < \beta < 1,$$

(constructed from the Farlie-Gumbel-Morgenstern copula) the marginals exhibit Pareto shapes, i.e.,  $F_x(s) = F_y(s) = 1 - s^{-\alpha}$  and that the two variates are not independent. Nevertheless, the distribution exhibits asymptotic independence. In this sense is the assumption about the linearity of asset returns in the fundamentals in proposition 2 crucial. One can also construct joint distributions, where the marginals have exponential type thin tails, but which nevertheless exhibit asymptotic dependence. A systematic analysis of crisis linkages implied by non-linear exchange rate (or more general asset pricing) models is beyond the scope of this note and left to future research. The above result, however, implies that if the dependence arises from the linear properties of the problem, the marginals necessarily have to exhibit fat tails to obtain asymptotic dependence.

Finally, it has recently become popular to model dependence structures by choosing specific copulas. Proposition 2 shows that if economic theory implies that the dependence arises from a linear problem, then one should limit oneself to the subclass of copulas that are consistent with linearly induced asymptotic dependence.

## 5. CONCLUSION

It is by now well known that financial returns exhibit heavy tails and are thus nonnormally distributed. This implies that extreme market conditions tend to happen more frequently than expected on the basis of the normal distribution, which is used so often in standard asset pricing approaches. From the point of view of international financial stability and portfolio diversification, the strength of asset linkages during crisis periods matters even more, as the linkages determine the stability of the system as a whole. Several papers talk about increased correlation between financial assets or markets during crisis periods. As has been argued before, the use of correlation analysis is not without problems though. Since the correlation concept is just an intermediary step in calculating probabilities, we prefer to define market linkages in terms of conditional probabilities and the expected number of market crashes.

In the present paper we try to make two contributions. First, we make a first step to combine foreign exchange theory with extreme value analysis, so as to better understand the nature of market linkages in crisis periods. Second, we examine the role of the univariate

properties of economic fundamentals for the strength and severity of extreme market spillovers. Choosing the case of currency markets we show that the fragility of the international monetary and financial system or its systemic stability hinges critically on the type of marginal distribution that applies to the country fundamentals. More precisely, we demonstrate that in linear exchange rate models the nature of interdependence between different currencies in times of crisis is fundamentally related to the univariate frequency with which large movements in underlying economic variables occur.

Suppose that logarithmic exchange rate returns are a linear function of the domestic and foreign fundamentals. This implies that different exchange rate returns against the same base currency are correlated, because they have partly common fundamentals. Nevertheless, if one currency crashes, the probability that the other currency breaks down as well vanishes asymptotically if the forex fundamentals exhibit thin tails, as the case for the normal distribution. Alternatively, if the marginal distributions exhibit heavier tails than the normal, e.g. are Student-t distributed, the probability that the other currency breaks down as well remains strictly positive even in the limit. We therefore speak of, respectively, weak and strong crisis linkages between different currencies. Correspondingly, the international monetary and financial system may be characterized as relatively robust in the former case, where destabilising phenomena like contagion do not occur systematically, while it is relatively fragile in the latter case.

Two simple conditions are sufficient for the spreading of financial instability to be directly related to the distribution of the economic fundamentals, fat tails and linearity. (We focus on exchange rates here, but the results apply to any other asset, as long as its pricing is linear and the marginals are heavy tailed.) While the latter condition is more intrinsic to the structure of the economy, the former condition is related to economic policy. In regular circumstances, by pursuing their policies with a ‘steady hand’ instead of orchestrating drastic changes in variables like money supply, interest rates or public expenditure, public authorities can diminish the scope for fat tails in fundamentals. In specific circumstances of large market-driven fluctuations the same result can be attained through strong counteracting measures. In the light of our argument, policy institutions may in this way contribute to the stability of the international exchange rate system.

Two directions for future research emerge from the note. On the side of theory, it appears interesting to extend our analysis to non-linear exchange rate (or asset pricing) models. Non-linear relationships between exchange rates and fundamentals could emerge from target

zones (see Krugman, 1991) or from various forms of transaction costs (see e.g. Dumas, 1992). On the empirical side, the numerous studies of the tail behaviour of asset prices should be extended by systematic studies of the tail behaviour of the main macroeconomic fundamentals. This will indicate how frequent and severe spillovers of exchange market crises can be. Both directions are beyond the ambition of the present note.

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