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No. 4288

## **CALVO CONTRACTS: A CRITIQUE**

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***INTERNATIONAL MACROECONOMICS***



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Discussion Paper No. 4288  
March 2004

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CEPR Discussion Paper No. 4288

March 2004

## **ABSTRACT**

### **Calvo Contracts: A Critique**

The Calvo contract Phillips Curve is widely indexed for general inflation, using either core inflation or other backward-looking formulae. Such a Phillips Curve implies a high and persistent degree of nominal rigidity. It is argued here that optimal indexation would by contrast use the rational expectation of inflation. If this scheme is implemented, the relationship defaults to a familiar 'surprise' Phillips Curve, removing all except temporary monetary rigidity.

JEL Classification: E31 and E32

Keywords: indexing, new-Keynesian synthesis, Phillips curve, price stickiness and rational expectations

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Submitted 16 January 2004

# CALVO CONTRACTS — A CRITIQUE\*

by Patrick Minford and David Peel

## Abstract

The Calvo contract Phillips Curve is widely indexed for general inflation, using either core inflation or other backward-looking formulae. Such a Phillips Curve implies a high and persistent degree of nominal rigidity. It is argued here that optimal indexation would by contrast use the rational expectation of inflation. If this scheme is implemented, the relationship defaults to a familiar ‘surprise’ Phillips Curve, removing all except temporary monetary rigidity.

Keywords: Price Stickiness, Indexing, Rational Expectations, Phillips Curve, New-Keynesian Synthesis (JEL E31, E32)

The theoretical basis for nominal rigidity set out by Calvo (1983) has been widely adopted in recent work of the so-called New Keynesian type — also known as the New NeoKeynesian Synthesis — see Clarida, Gali and Gertler (1999). In the Calvo contract nominal rigidity can last indefinitely in the sense that there is a limited chance for wage- or price-setters to change their setting in any period. Hence once a price or wage is ‘out of line’ with its equilibrium there is a chance it will continue for ever. This has seemed an attractive set-up for modellers who wanted a basis for nominal rigidity with substantial potential real effects.

Nevertheless recent work has exposed a variety of puzzles arising from this set-up. On the one hand there are the apparently counter-factual implications of the theory noted for example by Mankiw (2001), Mankiw and Reis (2002), Ball (1994), Fuhrer and Moore (1995), Bakhshi et al (2003), Rudd and Whelan (2003) and Eichenbaum and Fisher (2003). On the other hand, a number of articles have pointed to the time-inconsistency problems posed for policy. We give some examples of the latter below. The essence of these problems lies in that indefiniteness of duration for rigidity; once prices or wages have got out of line there is a strong incentive to take some action that might push them back, even if commitments have been made to stabilising prices or inflation along a particular initially-optimal path. A partial list of work that has wrestled with these issues would include: Goodfriend and King (2001), Khan, King and

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\*We are grateful for comments and discussion from Harris Dellas, Huw Dixon and Bennett McCallum.

Wolman (2002), Svensson and Woodford (2003), McCallum (2003), Collard and Dellas (2003), and Woodford (2000).

More recently, it has been recognised (Erceg, Henderson and Levin, 2000; Christiano, Eichenbaum and Evans, 2002 for example) that the uncompromising nominal rigidity in Calvo (1983) ought to be modified to allow for some indexing process whereby general inflation is passed through by wage/price-setters. The argument has been that the chances of changing price identified in the Calvo model relate to the changing of a relative price, for example on the grounds that some micro menu cost threshold is stochastically disturbed by some micro event. If then there is some general ongoing inflation this would be passed on by all including those who would not on micro grounds wish to change their relative price. Thus the ‘menus’ outside the restaurants are all uprated for general inflation; some individual menus are then raised more or less than that according to micro shocks. Two specific ways have been widely pursued for doing this: indexing to ‘core’ inflation and alternatively to lagged inflation.<sup>1</sup>

It is not our intention in this paper to add to the literature of optimal policy under Calvo contracts. Rather our contention is that the indexing modification is correct but has been incorrectly implemented: the Calvo contract should instead be indexed to expected inflation. We argue the case for this in what follows and show that should this be adopted the puzzles we have referred to for optimal policy are eliminated, as the Calvo contract Phillips Curve defaults to the familiar ‘surprise’ Phillips Curve of Lucas (1972) and Sargent and Wallace (1975).

## I The Calvo contract reworked to allow for indexing to expected inflation

Let us rework the Calvo set-up allowing for an environment where inflation is expected. We will assume that all price-setters ‘index’ their contracts for this expected inflation; and that this indexing is like a relabelling and carries no menu cost. The information set of price-setters at  $t$  consists of  $t - 1$  macro information (on the general price level, for example) but  $t$  micro information; thus each agent observes his own output and real costs, so that on average relative prices are accurately set for those able to do so.

We begin with the same steps as Calvo; in these price-setters set their relative prices in terms of their expected real costs. Thus the variables that follow are all to be read as real variables, relative prices. Price-setters (or wage-setters, analysed analogously) operate under imperfect competition where if prices were

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<sup>1</sup> Further examples are Casares (2002), Ascari (2003); additionally Calvo, Celasun and Kumhof (2003, 2001) and Cespedes, Kumhof and Parrado (2003) have recently suggested further forms of indexing based on rules of thumb based on learning. All these schemes violate the strict natural rate hypothesis (that no monetary policy should be capable of permanently changing output and employment) whose absence from the original Calvo set-up was noted by McCallum (1998) - see also Minford and Peel (2002a) for examples of how monetary policy can ‘manipulate’ real outcomes.

flexible they would be continuously set as a mark-up on marginal cost. They are assumed to face a menu cost of changing their price: this takes the form of a lump sum cost which acts as a threshold. If some unexpected shock to costs exceeds this threshold, they will change their price and set it to the newly expected marginal cost. It is assumed that there is a constant probability,  $1 - \xi$ , of such a shock for each (identical) price-setter. The expected losses at  $t = 0$  of the  $h$ th price-setter can be written:

$$(1) \quad \sum_{t=0}^{\infty} E_0^h \beta^t [p_t^h - (1+m)c_t^h]^2$$

where  $m$  is the mark-up,  $c$  is the (real) marginal cost and  $p_t^h$  is the relative price of the  $h$ th agent.  $E_t^h$  is the rational expectation of the  $h$ th agent conditional on micro information at  $t$  and macro information at  $t-1$  (for simplicity we assume no signal extraction about macro data at  $t$  from micro data then).

The first-order condition with respect to the decision to set his price at  $\tilde{p}_0^h$  then implies:

$$(2) \quad \tilde{p}_0^h = (1+m)(1-\beta\xi) \sum_{t=0}^{\infty} (\beta\xi)^t E_0^h c_t^h$$

In other words the reset price is equal to a weighted average of all future expected marginal costs plus the mark-up. This expression is justified as follows. Consider the losses associated with the decision to set  $\tilde{p}_0^h$ . For  $t = 0$ ,  $\tilde{p}_0^h$  can be freely set and so the loss is  $[\tilde{p}_0^h - (1+m)c_0^h]^2$ . At  $t = 1$  there is a  $\xi$  chance of being unable to change his prices from  $\tilde{p}_0^h$  and a  $1-\xi$  chance of being able to reset it, in which case any loss is not to do with today's decision; hence the expected loss at  $t = 1$ , due to today's decision, is  $\beta\xi[\tilde{p}_0^h - (1+m)E_0^h c_1^h]^2$ . Similarly at  $t = 2$ , there is a  $\xi^2$  chance of being unable to change it from  $\tilde{p}_0^h$  in either  $t = 1$  or  $t = 2$ ; there is a  $(1-\xi)^2$  of being able to change it in both periods, a  $\xi(1-\xi)$  chance of changing it in  $t = 1$  but not in  $t = 2$ , and similarly of not changing it in  $t = 1$  but doing so in  $t = 2$ . In all these last three cases nothing decided for  $t = 0$  affects the losses which are due to decisions taken in later periods. So the expected loss at  $t = 2$  due to the decision at  $t = 0$  is  $\beta^2\xi^2[\tilde{p}_0^h - (1+m)E_0^h c_2^h]^2$ . Analogously at  $t = i$  it is  $\beta^i\xi^i[\tilde{p}_0^h - (1+m)E_0^h c_i^h]^2$ . Equation (1) will only contain these terms in  $\tilde{p}_0^h$  in other words. So differentiating it with respect to  $\tilde{p}_0^h$  yields the first-order condition above. In general at time  $t$  the  $h$ th's agent's decision is therefore:

$$(3) \quad \tilde{p}_t^h = (1+m)(1-\beta\xi) \sum_{i=0}^{\infty} (\beta\xi)^i E_t^h c_{t+i}^h$$

As noted above, all these costs and prices are in real terms. Thus equation (3) expresses the optimum on the assumption that prices can be set in real terms — e.g. under perfect contemporaneous indexing or in a world of complete fixity in the general price level.

Plainly we are dealing with a world in which prices cannot be set in real terms, because the general price level is expected to have moved and perfect contemporaneous indexing is not available; nor do agents know the current general price level but are assumed to know last period's (for convenience we assume no signal extraction from the micro data that they do have contemporaneously). In such a world the optimal course for agents is well-known to be to set their prices at the optimal expected real level, that is the planned real level for  $t$  adjusted for the rationally-expected price level for  $t$ ; the reason is that they so build into today's setting of prices the best estimate of what the general price level will be this period — see Appendix for a proof. This estimate incorporates all available information about the existing price level and so is superior merely to adding last period's price level ('indexing to the lagged level') which is known but will not in general be expected to remain the same in the current period; as it is also superior to adding 'core' inflation which in general will not be equal to expected inflation.

Thus all will raise their (nominal) prices in  $t$  by the expected rate of inflation from  $t - 1$  to  $t$  while those who are able to change their relative price at  $t$  will additionally raise their prices in line with equation (3). We will at this stage assume that the actual price index is sufficiently close to unity at  $t - 1$  for its change to be approximately equal to the change in the logarithm of prices, inflation. Hence those who can change their relative price set the current price at the relative cost plus the expected general price level, while those who cannot change their relative price set their current price at the expected general price level. Hence inflation is

$$(4) \quad \pi_t = P_t - P_{t-1} = (1 - \xi)[\tilde{p}_t + E_{t-1}P_t] + \xi[E_{t-1}P_t] - P_{t-1}$$

$$(5) \quad = (1 - \xi)\tilde{p}_t + E_{t-1}\pi_t$$

where  $\tilde{p}_t$ , the aggregated equilibrium price, is:

$$\tilde{p}_t = \int \left\{ (1 + m)(1 - \beta\xi) \sum_{i=0}^{\infty} (\beta\xi)^i E_t^h c_{t+i} \right\} dh$$

and  $E_{t-1}\pi_t$  is the rational expectation of inflation formed using the macro data available (i.e. at  $t - 1$ ).

Now let  $(1 + m)c_{t+i} = \delta(y_{t+i} - y^*) + u_{t+i}$ , that is marginal costs are a rising function of output (and approximately linear in the region of  $y^*$ , the natural rate of output, which we will for simplicity assume constant); we allow for a supply shock here which we will treat as random for simplicity. In aggregating we will assume that the  $h$ -specific shocks at  $t$  observed by each agent are i.i.d. and so of no help in predicting future costs; therefore  $E_t^h y_{t+i} = E_{t-1}y_{t+i}$  ( $i \geq 1$ ), where all agents are using the common macro data available (from  $t - 1$ ) to project the economy's future state. Hence

$$(6) \quad \tilde{p}_t = (1 - \beta\xi)[\delta(y_t - y^*) + u_t] + \sum_{i=1}^{\infty} (\beta\xi)^i E_{t-1}[\delta(y_{t+i} - y^*) + u_{t+i}]$$

By construction  $E_{t-1}u_{t+i} = 0$  ( $i \geq 1$ ) we can rewrite (6) as

$$(7) \quad \tilde{p}_t = (1 - \beta\xi) \left\{ \delta \frac{(y_t - y^*)}{1 - \beta\xi E_{t-1}B^{-1}} + u_t \right\} = \\ \beta\xi E_{t-1}\tilde{p}_{t+1} + (1 - \beta\xi) \{ \delta(y_t - y^*) + u_t \}$$

where  $E_{t-1}B^{-1}$  is the lead operator preserving the  $t - 1$  date of expectations.

$E_{t-1}\tilde{p}_{t+1} = 0$  ( $i \geq 1$ ) by equation (5) led  $i$  periods and taking expectations at  $t - 1$ . Imposing this on (5), leading it  $i$  periods and taking expectations of these at  $t$  yields:

$$E_{t-1}y_{t+i} = y^* \text{ (all } i \geq 1)$$

and

$$(8) \quad \tilde{p}_t = (1 - \beta\xi) \{ \delta(y_t - y^*) + u_t \}$$

whence the Phillips Curve is:

$$(9) \quad \pi_t = (1 - \xi)(1 - \beta\xi) \{ \delta(y_t - y^*) + u_t \} + E_{t-1}\pi_t$$

Note therefore that  $E_{t-1}[(1 - \xi)(1 - \beta\xi) \{ \delta(y_t - y^*) + u_t \}] = 0$ . It follows that monetary policy in the form of interest rates operating through the IS curve must be expected to set output at its natural rate. The Calvo Phillips Curve equation does not determine expected inflation, any more than a Sargent-Wallace Phillips Curve does. What it does determine is surprise inflation, which depends on the surprise in current output and  $u_t$ .

To find the expected rate of inflation we add, in the usual manner of New Keynesian models, an IS curve (following the McCallum and Nelson, 1999, approximation within a micro-founded model) and a Taylor rule (in the real interest rate,  $r_t$ ) following Henderson and McKibbin (1993) and Taylor (1993):

$$(10) \quad y_t = \theta_0 - \theta_1 r_t + \theta_2 E_t y_{t+1}$$

$$(11) \quad r_t = r^* + \alpha_y(y_t - y^*) + \alpha_\pi(\pi_t - \pi^*)$$

where  $r^* = \frac{\theta_0 - (1 - \theta_2)y^*}{\theta_1}$  is the equilibrium real interest rate. Here we assume in the usual way of this literature that somehow the authorities can make interest rates react contemporaneously to current output and inflation.

Taking expectations and solving we find that:

$$(12) \quad E_{t-1}\pi_t = \pi^*$$

Hence the Taylor Rule calibrated appropriately ensures that inflation will be expected to hit the inflation target. Output and inflation will therefore differ from their targets only through surprises. The New Keynesian model when agents (optimally) use expected inflation as their indexing instrument therefore replicates the behaviour of the New Classical model. We now briefly pursue the implications of using the Calvo model, both unadjusted and adjusted for this optimal indexing procedure, for the problem of optimising monetary policy within a standard set-up. This will illustrate the significance of the adjustment.

## II Optimisation in the Calvo contract

We begin by going over the optimisation in the unindexed Calvo set-up.

For example take the following simple set-up. The maximand (derivable as a second-order Taylor expansion from the model; Rotemberg and Woodford, 1997) is

$$(13) \quad V_t = -0.5E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2)$$

and the Calvo Phillips Curve is

$$(14) \quad \pi_t = \gamma y_t + u_t + \mu E_t \pi_{t+1} = E_t \frac{\gamma y_t + u_t}{1 - \mu B^{-1}}$$

where the error term,  $u_t$ , as before is i.i.d.. We have set target and steady state inflation and output at zero. We assume, as is usual in this literature, that current macro data is observed and that the authorities can set  $\pi_t$  and  $y_t$  at  $t$ . Thus  $E_t$  is the rational expectation conditional on  $t$ -period macro information.

For this problem it turns out that the value function approach is not convenient as the reaction function changes over time, implying that the value function will change also. Instead we follow McCallum (2003) and set up the Lagrangean:

$$(15) \quad L_t = -0.5E_t \sum_{i=0}^{\infty} \{ \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2) + \delta_i (\pi_{t+i} - \gamma y_{t+i} - u_{t+i} - \mu E_t \pi_{t+i+1}) \}$$

The first-order conditions yield for  $i = 0$ :

$$(16) \quad \pi_t = \frac{-\lambda}{\gamma} y_t$$

Using the Calvo Phillips Curve we then obtain:

$$(17) \quad \frac{-\lambda}{\gamma} y_t = E_t \left\{ \frac{\gamma y_t + u_t}{1 - \mu B^{-1}} \right\}$$

Multiplying through by the lead operator expression and collecting terms yields  $\{\gamma^2 + \lambda - \lambda\mu B^{-1}\} E_t y_t = -\gamma E_t u_t$ ; Now since expectations of future  $u_{t+i}$  are by construction zero seen from time  $t$ , the current plan is  $E_t y_t = y_t = \frac{-\gamma u_t}{\lambda + \gamma^2}$  and by implication  $\pi_t = \frac{\lambda}{\lambda + \gamma^2} u_t$ .

Repeating this operation for  $i \geq 1$  yields:

$$(18) \quad E_t \pi_{t+i} = \frac{-\lambda}{\gamma} E_t (y_{t+i} - \frac{\mu}{\beta} y_{t+i-1}) = \frac{-\lambda}{\gamma} E_t (1 - \frac{\mu}{\beta} B) y_{t+i} \quad (6)$$

This yields an equation for the path of output:

$$(19) \quad E_t \left\{ \left( -\lambda\mu B^{-1} + (\gamma^2 + \lambda + \frac{\lambda\mu^2}{\beta}) - \frac{\lambda\mu}{\beta} B \right) y_{t+i} \right\} = -\gamma u_{t+i} \quad (7)$$

The left-hand side expression of (19) in  $B$  can be factorised into

$$(20) \quad k_0(1 - k_1B^{-1})(1 - k_2B)$$

where  $k_1, k_2$  are chosen as the stable forward and backward roots of the difference equation in  $B$  (see Minford and Peel, 2002b, pp. 70–71). In this particular case we find that

$$(21) \quad k_2 = \beta^{-1}k_1$$

and the roots of  $k_1$  are given by:

$$(22) \quad k_1^2 - \left( \frac{\beta(\gamma^2 + \lambda) + \lambda\mu}{\lambda\mu} \right) k_1 + \beta = 0$$

which for normal values of the parameters will yield a stable and unstable value for  $k_1$ ; we choose the stable one (the unstable is plainly suboptimal). The value of  $k_2$  is given as  $\beta^{-1}$  times that value; again for normal parameter values this should also be stable.

Using these solutions we can rewrite (18) as:

$$(23) \quad E_t \left\{ \left( \frac{\lambda\mu}{k_2} (1 - \beta k_2 B^{-1}) (1 - k_2 B) \right) y_{t+i} = -\gamma u_{t+i} \right\} (i \geq 1)$$

Note that  $E_t \frac{-\gamma u_{t+i}}{(1 - \beta k_2 B^{-1})} = 0$  because seen from period  $t$  the expected values of the errors at  $t+1$  onwards are all zero. Hence we are left with

$$(24) \quad E_t \{(1 - k_2 B) y_{t+i} = 0\} (i \geq 1)$$

It follows that the solution for  $E_t y_{t+i} = k_2 E_t y_{t+i-1}$ . where the initial value is  $y_t$ . Thus  $E_t y_{t+i} = k_2^i y_t (i \geq 1)$  and  $E_t \pi_{t+i} = -\zeta k^{i-1} y_t (i \geq 1)$  where  $\zeta = \frac{-\lambda}{\gamma} (k - \frac{\mu}{\beta})$ . As has been noted (e.g. McCallum, 2003) this path is time-inconsistent since at  $t+1$  it will be optimal to make

$$(25) \quad \pi_{t+1} = \frac{-\lambda}{\gamma} y_{t+1} \neq \frac{-\lambda}{\gamma} E_t \left( 1 - \frac{\mu}{\beta} B \right) y_{t+1}$$

This problem can be dealt with either by assuming that there is a penalty for reoptimising so that the original path from the policy's enactment must be kept (however this has to be enforced in perpetuity as in every future period there will be an incentive to renege); thus forcing the original path to be ‘timeless’ (i.e. having the form of (18) so that the monetary authority is not allowed to neglect lagged output when it opens the policy even though at this point bygones are bygones). It is not the purpose however of this paper to contribute to this issue,<sup>2</sup> because it does not arise when the Calvo contract is appropriately indexed.

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<sup>2</sup>If we consider the Calvo contract with a lagged inflation term to proxy inertia in price-setting as in Clark, Goodhart and Huang (1999), essentially the same problems occur though

### III Optimisation in the Calvo contract augmented for indexing to expected inflation

However, when we turn to the Calvo contract once indexed to expected prices the optimum problem is rather different. The first point to note is that the Taylor-series approximation to the welfare function will now not go through in terms of inflation since it is only the surprise in output and so in inflation that causes relative prices to be disturbed. Thus surprise inflation, not inflation, will enter the objective function, leaving indeterminate the target for expected inflation.

If one nevertheless retains the objective function in its previous form, on some alternative — say political economy — grounds, then one can formulate the commitment strategy as follows. The Calvo Phillips Curve is now:

$$(26) \quad \pi_t = (1 - \xi)(1 - \beta\xi)\delta \frac{(y_t - y^*)}{1 - \beta\xi E_{t-1} B^{-1}} + E_{t-1}\pi_t + u_t$$

Set up the Lagrangean (letting  $\delta' = (1 - \xi)(1 - \beta\xi)\delta$ ):

$$(27) \quad L_t = -0.5E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2) + n_i ([1 - \beta\xi E_{t-1} B^{-1}] (\pi_{t+i} - E_{t+i-1} \pi_{t+i}) - \delta' y_{t+i} - u_{t+i})$$

Suppose the policy is announced and committed one-period ahead at  $t-1$ . we may take expectations at  $t-1$  of  $L_t$  and maximize it with respect to expected inflation and output at all  $t+i$  periods. Trivially we obtain that expected inflation and output are zero for all  $i$ , that is in our notation respectively at the target inflation rate and the natural rate. Imposing this commitment constraint on (27) yields a new Lagrangean to be maximised in period  $t$  with respect to

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with more complexity. The Lagrangean is now

$$L_t = -0.5E_t \sum_{i=0}^{\infty} \{ \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2) + \delta_i (\pi_{t+i} - \gamma y_{t+i} - u_{t+i} - \mu E_{t+i} \pi_{t+i+1} - \rho \pi_{t+i-1}) \}$$

here the first order conditions for  $t$  deliver

$$\pi_t = \frac{-\lambda}{\gamma} E_t \{ 1 - \rho \beta B^{-1} \} y_t$$

while

$$E_t \pi_{t+i} = \frac{\lambda}{\gamma} E_t \left\{ \beta \rho B^{-1} - 1 + \frac{\mu}{\beta} B \right\} y_{t+i}$$

The solution can be found by similar principles though in this case there are three roots for the difference equation driving the system, two forward and one backward, the forward roots will drop out for the same reason as in the problem without inertia. Plainly the time-inconsistency still applies since the lag appears in the  $t+i$  periods while it is absent in the  $t$  period.

planned output and inflation: Note that all expected future terms in surprise inflation are necessarily zero as are all expected future errors

$$(28) \quad L_t = -0.5E_t \left[ \begin{array}{l} \sum_{i=0}^{\infty} \{ \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2) \} + \\ n_0 (\pi_t - \delta' y_t - u_t) + \sum_{i=1}^{\infty} n_i (-\delta' y_{t+i}) \end{array} \right]$$

The optimising strategy is to choose all expected future output and inflation to be zero and to react solely to the shock at  $t$ :

$$(29) \quad \pi_t = \frac{-\lambda}{\delta'} y_t$$

Hence

$$(30) \quad y_t = \frac{-\delta'}{\lambda + \delta'^2} u_t$$

$$(31) \quad \pi_t = \frac{\lambda}{\lambda + \delta'^2} u_t$$

Each period the authority will stick to its plan of zero inflation and output at the natural rate, other than to stabilise current supply shocks — exactly as in the orthodox (Sargent and Wallace, 1975) Phillips Curve set-up (for example Svensson, 1997).

The discretionary optimum turns out in this particular set-up to be identical; we could make it different in the conventional way by having the objective function target a higher output than the natural rate. However this adds nothing new to the orthodox analysis under the standard sort of Phillips Curve considered e.g. by Barro and Gordon (1983).

## IV Conclusion

We conclude that the Calvo contract, which produces a variety of curious and puzzling results in its usually-used form, should be adjusted for expected inflation which is the appropriate indexing process for a rational agent to use. When this adjustment is carried out, we find we recover a Phillips Curve that is in its essentials the same as that of Sargent and Wallace. Optimisation of monetary policy under a standard welfare function with such a Phillips Curve is well-known and poses no particular problems.

## Appendix

We now repeat the steps in the text but allow an ‘indexing scheme’ whereby prices set by price-setters are automatically raised in line with the general price index,  $\tilde{P}_t$  (which we conveniently define as a natural logarithm). The expected losses at  $t = 0$  of the  $h$ th price-setter can now be written:

$$(A1) \quad \sum_{t=0}^{\infty} E_0 \beta^t [\bar{p}_t^h + \tilde{P}_t - P_t - (1+m)c_t^h]^2$$

where  $m$  is the mark-up,  $c$  is the (real) marginal cost, both as before but now  $\bar{p}_t^h$  is the relative price of the  $h$ th agent as set in real terms whereas the actual price prevailing in the market is  $\bar{p}_t^h + \tilde{P}_t$  and the actual real price is therefore  $\bar{p}_t^h + \tilde{P}_t - P_t$  (where again we conveniently define  $P$  as the ln of the general price index and  $\bar{p}^h \cdot c$  are to be thought of as indices normalised around unity).

To find the first order condition note that  $E_0 \bar{p}_t^h = \xi E_0 \bar{p}_{t-1}^h + (1-\xi) E_0 \tilde{p}_t^h$  where as before  $\tilde{p}_t^h$  is the real price actually freely set at period  $t$ . Thus for example

$$(A2) \quad E_0 \bar{p}_1^h = \xi \tilde{p}_0^h + (1-\xi) E_0 \tilde{p}_1^h$$

and

$$(A3) \quad E_0 \bar{p}_2^h = \xi^2 \tilde{p}_0^h + (1-\xi)\xi E_0 \tilde{p}_1^h + (1-\xi)^2 E_0 \tilde{p}_2^h$$

Now expanding the expected loss gives:

$$(A4) \quad L = \sum_{t=0}^{\infty} E_0 \beta^t \left( \begin{array}{c} (\bar{p}_t^h)^2 + (\tilde{P}_t - P_t)^2 - \{(1+m)c_t^h\}^2 - 2(1+m)\bar{p}_t^h c_t^h \\ - 2\bar{p}_t^h(\tilde{P}_t - P_t) + 2(1+m)(\tilde{P}_t - P_t)c_t^h \end{array} \right)$$

The first-order condition with respect to the decision to set  $\tilde{p}_0^h$  then implies:

$$(A5) \quad \tilde{p}_0^h = (1-\beta\xi) \sum_{t=0}^{\infty} (\beta\xi)^t E_0 \{(1+m)c_t^h + (\tilde{P}_t - P_t)\}$$

The difference with the expression in the text is that there is an additional term in the expected indexing errors which now enter the expected loss.

Rational-expectations indexing as in the text is written as  $\tilde{P}_t = E_{t-1} P_t$ . It follows that  $\tilde{P}_t - P_t = \epsilon_t$  where  $\epsilon_t$  is an i.i.d. error produced by macro news. Thus  $E_0^h(\tilde{P}_t - P_t) = 0$  for all  $t \geq 0$  which restores to us equation (3) in the text; under rational-expectations indexing, the derivation from then on is also the same.

We may note that under rational-expectations indexing since  $\bar{p}_t^h$  and  $c_t^h$  are both functions of real micro variables affecting agent  $h$ , current and expected, we can note that  $q_t^h = \bar{p}_t^h - (1+m)c_t^h$ , is independent of  $\epsilon_t$  and write the expected loss as:

$$(A6) \quad L = \sum_{t=0}^{\infty} E_0^h \beta^t [q_t^h + \epsilon_t]^2 = \sum_{t=0}^{\infty} E_0^h \beta^t (q_t^h)^2 + \frac{1}{1-\beta} \sigma_{\epsilon}^2$$

An alternative indexing scheme must raise the expected loss. Let this scheme be altered from the rational scheme by the amount  $z_{t-1}$  where this macro variable is anything known at  $t-1$  and is also independent of agent  $h$ 's micro shocks. Then  $\tilde{P}_t = E_{t-1}P_t + z_{t-1}$ . Let us assume for simplicity that  $z_t = \lambda z_{t-1} + u_t$  where  $u_t$  is i.i.d. so that  $E_0 z_{t+i}$  is  $\lambda^{i+1} z_{t-1}$ .<sup>3</sup>

It follows that the optimal setting of  $\bar{p}_t^h$  will now contain a term reacting to this variable so that

$$(A7) \quad \bar{p}_t^h - (1+m)c_t^h = q_t^h + kz_{t-1}$$

(where the evaluation of  $k$  requires numerical analysis).

Furthermore the error in the indexing scheme will now become:

$$(A8) \quad \tilde{P}_t - P_t = \epsilon_t + z_{t-1}$$

so that the expected loss becomes:

$$(A9) \quad \begin{aligned} L &= \sum_{t=0}^{\infty} E_0^h \beta^t [q_t^h + (1+k)z_{t-1} + \epsilon_t]^2 \\ &= \sum_{t=0}^{\infty} E_0^h \beta^t (q_t^h)^2 + \frac{1}{1-\beta} \{ \sigma_{\epsilon}^2 + (1+k)^2 \sigma_z^2 \} \end{aligned}$$

Only if  $k = -1$  would this loss be reduced to the minimum achieved by rational-expectations indexing; but note that  $k = -1$  implies that all price-setters can completely offset the inappropriate index movement by changing their prices and thus restore de facto rational-expectations indexing. However, only those able to change prices can do so and therefore such complete offsetting is impossible; thus  $k \neq -1$ . Therefore, only explicit rational-expectations indexing can achieve the minimum loss.

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<sup>3</sup>This can easily be generalised, for example to where  $z_t = a\phi_t$  where  $a, \phi$  are vectors respectively of coefficients and variables known at  $t$ ; then let  $\phi_t = \Lambda\phi_{t-1}$ , a VAR. We will then obtain  $\bar{p}_t^h - (1+m)c_t^h = q_t^h + \kappa\phi_{t-1}$  where  $\kappa$  is now a vector of coefficients.

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