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ABSTRACT

Fiscal Policy with Agents Differing in Altruism and in Ability*

This Paper explores the effects of a menu of inter-generational fiscal policies (public debt financed by taxes, PAYG social security system and inheritance taxation) in an overlapping generations model with perfect altruism. It generalizes the model by Barro (1974) by introducing intra-generational heterogeneity. In other words, households differ in productivity and altruism. Within such a model wealth is entirely held in the steady-state by the families with the highest degree of altruism. Under plausible assumptions both public debt and social security are neutral à la Ricardo, while increasing inequality. Also, estate taxation can be Pareto-worsening even though it fosters income equality.

JEL Classification: D64, E62, H20 and H55
Keywords: altruism, estate tax, public debt and social security

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1 Introduction

The literature on the effects of fiscal policy is founded on two canonical models: the Barro-Ramsey model of infinitely lived families and the Diamond-Samuelson model of overlapping generations.[1] In the first model any attempt by the government to reallocate resources among generations through public borrowing, pay-as-you-go social security or estate taxation is totally or partially neutralized by families who want to smooth their consumption over time by means of their bequests. In the second model individuals smooth consumption only over their lifetimes. As a consequence, fiscal policy cannot be neutralized and has real effects. For example, government debt is completely neutral within the Barro-Ramsey model; in the Diamond-Samuelson model, it crowds out capital and reduces steady-state utility levels.[2]

As argued by a number of people, these models are not appropriate for a realistic analysis of fiscal policy. One of the reasons is that real societies don't consist of identical individuals who are perfectly altruistic or perfect life-cyclers. There is heterogeneity, and by taking that simple fact into account we may get a better understanding of fiscal policy.

This paper generalizes the model of Barro (1974) by introducing intra-generational heterogeneity. Our economy is populated by infinite dynasties of two-period lived agents whose welfare depends on their own consumption in two periods of their life, as well as the welfare of their immediate de-

[1] See Barro (1974), Ramsey (1928), Diamond (1965), Samuelson (1958).

[2] Granted that there is underaccumulation.

scendants. There is a finite variety of dynasties which differ in terms of two parameters: the fixed level of labor endowment given to young individuals in every generation of a dynasty and the factor by which they discount welfare of their immediate descendants.

Thus we state and derive the steady-state dynamic general equilibrium. In such a setting, only the dynasties characterized by the highest degree of altruism (i.e., placing the highest weight on the descendants welfare in their utility functions) end up with "operative" bequests, namely transferring assets to their young heirs. The equilibrium optimal bequests of the agents of all other dynasties turn out to be inoperative. As a consequence all these other agents end up behaving like pure life-cyclers, regardless of their factor of altruism. As such society consists of what Mankiw (2000) calls "spenders" (life-cyclers) and "savers" (altruists with operative bequests) as well as, and what we call (Michel, Pestieau, 1998) the "non-altruists" and the "altruists".

Within this setting, we explore the effects of a menu of intergenerational fiscal policies: public debt financed by taxes, pay-as-you-go (PAYG) social security systems and inheritance taxation. Anticipating the results, we show that both public debt and PAYG social security are neutral at the macrolevel, but increase income disparity between the most altruistic agents and the others. We also show that estate taxation can be Pareto worsening even while decreasing income inequality.

We thus generalize an earlier paper (see Michel-Pestieau (1998)) in which we distinguish two types of individuals, altruists and non altruistsn and we use Cobb-Douglas production functions and loglinear utility functions. Here we consider a number of types of individuals differing in their degree of altruism and their level of productivity, and we use more general utility and production functions.

The rest of the paper is organized by sections. Section 2 presents the basic model, and gives the steady-state market solutions and optimality conditions. Section 3 introduces debt policy and also pay-as-you-go social security, studying their impact on the long run distribution of wealth. Section 4 considers the case of a progressive inheritance taxation while assuming homothetic preferences. A final section concludes.

2 Market equilibrium and social optimality

2.1 The model

2.1.1 Dynasties and generations

Society consists of m dynasties of two-period overlapping generations. The population growth rate is constant and the same for all dynasties; it is denoted by n. An individual belonging to dynasty i $(1, ..., m)$ and to generation t $(0, ..., \infty)$ is characterized by a factor of altruism γ_i and an index of productivity h_i. For further use, we posit that $\gamma_1 > \gamma_2 > \gamma_m \geqslant 0$. He works in period t, he may receive a bequest x_{it}, earns $h_i w_t$, consumes c_{it} and saves s_{it}; he retires in period $t+1$, consumes d_{it+1} and may leave x_{it+1} to his

$(1+n)$ children. His retirement income comes from the gross return on saving, $R_{t+1}s_{it}$. The variables w_t, the wage rate per efficiency unit, and R_{t+1}, one plus the rate of interest, will be shown below to be the marginal productivity of labor and capital, respectively.

Note that the two characteristics h_i and γ_i as well as their correlation, are time-invariant. We don't make any assumption on the correlation between them, even though it is tempting to think that the most altruistic individuals (and thus the wealthiest) are also the most productive.

For further use, we define p_i as the proportion of type i's agents (and dynasties) in society with $\sum p_i = 1$. The size of generation t is denoted by N_t with $N_t = N_{t-1}(1+n)$.

2.1.2 Households behavior

We can write both the utility function[3] and the budget constraints of individuals belonging to dynasty i and generation t as:

$$v_{it}^{*} = \max \quad u\left(c_{it}, d_{it+1}\right) + \gamma_i \; v_{it+1}^{*} \tag{1}$$

$$c_{it} = x_{it} + h_i\, w_t - s_{it} \tag{2}$$

$$d_{it+1} = R_{t+1}\, s_{it} - (1+n)\; x_{it+1} \tag{3}$$

[3]See Weil (1987).

where u is strictly concave, continuously differentiable, satisfying Inada conditions and excluding inferior goods. It is assumed that bequests cannot be negative:

$$x_{it+1} \geqslant 0. \tag{4}$$

Below we distinguish lifetime resources given by $x_{it} + h_i\, w_t$ and lifetime consumption spending denoted by Ω_{it} with

$$\Omega_{it} = c_{it} + d_{it+1}/R_{t+1}.$$

Let us repeat that type i's agents have consistently children of the same type; in other words, types are hereditary.

2.1.3 Dynasties choices

If $\gamma_i > 0$, the problem of dynasty i at time 0 can now be expressed as:

$$Max \sum_{t=0}^{\infty} \gamma_i^t\, u\left(c_{it}, d_{it+1}\right)$$

subject to (2), (3), (4) for $t \geqslant 0$ and given x_{i0}.

Solving the above problem one obtains the following optimality condition for s_{it} and x_{it+1} respectively:

$$-u_c'\left(c_{it}, d_{it+1}\right) + R_{t+1}\, u_d'\left(c_{it},\, d_{it+1}\right) = 0$$

and

$$-\left(1+n\right)\, u_d'\left(c_{it},\, d_{it+1}\right) + \gamma_i\, u_c'\left(c_{it+1}, d_{it+2}\right) \leqslant 0 \qquad \left(= 0 \text{ if } x_{it+1} > 0\right).$$

If $\gamma_m = 0$, all generations of dynasty m behave a life-cyclers in the line of Diamond. Their saving is denoted:

$$s_{mt} = s^D \left(w_t h_m - s, R_{t+1} s \right).$$

The above first-order conditions are satisfied with $x_{mt+1} = 0$.

2.1.4 Firms and production

We assume a CRS production function $F(K_t, H_t)$ with labor H_t in efficiency units and capital K_t as inputs. We also assume total depreciation. At equilibrium factor prices are equal to marginal productivity:

$$\left. \begin{array}{l} w_t = F'_H \left(K_t, H_t \right) = \omega \left(k_t \right) \\ R_t = F'_K \left(K_t, H_t \right) = \varrho \left(k_t \right) \end{array} \right\} \qquad (\mathrm{F}_t)$$

with $k_t = K_t / H_t$ and for further use $f(k_t) = F(k_t, 1)$.

2.2 Market equilibrium

2.2.1 Intertemporal equilibrium

At each period of time, the labor market equilibrium is given by:

$$H_t = \sum p_i \, h_i \, N_t = \bar{h} \, N_t$$

where $\bar{h}$ is average productivity. The resource constraint in period t is:

$$F(K_t, H_t) = \sum p_i \, N_t \, c_{it} + \sum p_i \, N_{t-1} \, d_{it} + K_{t+1}.$$

This constraint is shown to be equivalent to the capital accumulation equation:

$$K_{t+1} = \sum p_i \, N_t \, s_{it},$$

after use of the Euler condition $F(K_t, H_t) = R_t K_t + w_t \, H_t$ and consumer's budget constraints $x_{it} + h_i w_{it} = c_{it} + s_{it}$ and $R_t s_{it-1} = d_{it} + (1+n) \, x_{it}$.

In the steady-state one obtains:

$$(1+n) \, k \, \bar{h} = \sum p_i \, s_i \tag{5}$$

with

$$w = \omega(k) \quad \text{and} \quad R = \varrho(k) \, . \tag{6}$$

2.2.2 Steady-state market equilibrium

The characteristics of the steady-state equilibrium of such an economy are now quite well-known. As shown by Becker (1980), Altig and Davis (1980) and by Michel and Pestieau (1998), only the dynasties characterized by the highest degree of altruism end up transferring assets to their children. All the other dynasties turn out to leave no bequests. In the presentation of Becker (1980) and much earlier in that of Ramsey (1928)[4], one speaks of time preference and of impatience. In such presentation only the most patient

[4]In Ramsey (1928) there is no liquidity constraint unlike Becker (1980).

agents hold a positive amount of wealth in equilibrium; the other have no wealth.

In such an economy, all agents of types $i \geqslant 2$ are constrained by $x_i \geqslant 0$ and their saving is given by:

$$s_i = s^D (wh_i, R) = \arg\max_s \ u(wh_i - s, Rs) \tag{7}$$

One has to make sure that this economy is non degenerate, namely that $x_1 > 0$. Otherwise we fall back upon Diamond's framework. Following Nourry and Venditti (2001, Prop. 1), there exists a steady-state equilibrium with $x_1 > 0$ and with stock of capital $\hat{k}$ consistent with the modified golden rule $\gamma_1 \hat{R} = 1 + n$ iff

$$(1+n)\,\hat{k}\;\bar{h} > \sum_{i=1}^{n} p_i s^D \left(\hat{w} h_i, \hat{R}\right) \tag{8}$$

with $\hat{w} = \omega(\hat{k})$ and $\hat{R} = \varrho(\hat{k})$. In words, there are bequests if the equilibrium stock of capital is larger than the amount that could be accumulated in an economy *à la Diamond*.[5]

2.2.3 Welfare analysis

When (8) holds, the steady-state value of $\hat{k}$ depends only on γ_1 with the modified golden rule relation $\hat{R} = \dfrac{1+n}{\gamma_1}$. It is independent of productivity h_i and frequency p_i. The welfare of constrained dynasties depends on both h_i and γ_i in the steady-state but not on the frequency p_i. In contrast the welfare of unconstrained dynasties is also a function of p_1.

[5]See Thibault (2002) for a generalization of this model with endogenous labor supply.

This point was already made by Michel and Pestieau (1998) and generalized by Nourry and Venditti (2001, Prop. 3). It can be expressed for the purposes of this paper through the following proposition.

Proposition 1 *Both the optimal steady-state bequest and the steady-state welfare of the unconstrained households increase as their proportion decreases. When this proportion tends to 0, the level of bequest tends to infinity.*

Proof. Let us denote $q_i = \dfrac{p_i}{1-p_1}$, $i = 2, \dots, m$. Using $s_1 = \gamma_1 x_1 + s^D\left(\Omega_1, \hat{R}\right)$ and $\Omega_1 = \hat{w}h_1 + (1+\gamma_1)\, x_1$ and $s_i = s^D\left(\hat{w}h_i, \hat{R}\right)$ $i = 2, \dots, m$, we write (5) as:

$$\begin{aligned} & p_1 \left[\gamma_1 x_1 + s^D\left(\hat{w}h_1 + (1-\gamma)\, x_1, \hat{R}\right) - \sum_{i=2}^{m} q_i s^D\left(\hat{w}h_i, \hat{R}\right)\right] \\ = \quad & (1+n)\, \hat{k}\ \bar{h} - \sum_{i=2}^{m} q_i s^D\left(\hat{w}h_i, \hat{R}\right). \end{aligned} \tag{9}$$

Decreasing p_1 for unchanged q_i clearly increases x_1. Furthermore when $p_1 \to 0$, $x_1 \to \infty$. ■

One of the implications of this proposition is that even though the unconstrained households had a very low productivity relative to some constrained households, when p_1 becomes small they end up having the highest lifetime income. In other words as $p_1 \to 0$,

$$\hat{w}h_1 + (1-\gamma_1)\, x_1 > \hat{w}h_i$$

even if h_i is much higher than h_1.

3 Intergenerational fiscal policy

3.1 Public debt

Let us now introduce some public debt B_t, its service financed by lump-sum taxes levied on both the workers τ^y_{it} and on the retirees τ^0_{it} (τ^y and τ^0 are both non negative). If there is no other public spending, the government revenue constraint is equal to:

$$B_t = R_t \, B_{t-1} - \sum_i p_i \left(\tau^y_{it} \, (1+n) + \tau^0_{it}\right) N_{t-1}$$

In the steady-state this constraint can be rewritten as:

$$b \, (R - (1+n)) = \sum_i p_i \left(\tau^y_i \, (1+n) + \tau^0_i\right)$$

where $b_t = B_t/N_t$ is constant: $b_t = b$.

As taxes are non distortionary the optimal conditions for individual choices c, d and x are not changed. If, as assumed, $x_1 > 0$ (non degenerate equilibrium) one has as before:

$$R = \hat{R} = (1+n) \, / \gamma_1 \ ; \ k = \hat{k}$$

and

$$x_i = 0 \qquad \text{for } i \geqslant 2.$$

In other words, debt policy has no impact on prices and aggregate variables but it has some on individuals' resources. For $i \geqslant 2$ life-cycle income with debt b can be written as:

$$\Omega_i^b = h_i \, \hat{w} - \tau_i^y - \tau_i^0/\hat{R}.$$

As a consequence, if the constrained households have to pay for the interest payments on the debt, then we see an decrease in their lifetime income, their two consumptions and their utility.[6]

What about type 1's individuals? Their initial response is to look ahead to their future tax liabilities and to adjust their bequests accordingly. Let us consider the resource equilibrium constraint:

$$\left[f\left(\hat{k}\right) - (1+n)\,\hat{k}\right]\bar{h} = \sum_{i=1}^{m} p_i \left(c_i + \frac{d_i}{1+n}\right) \tag{10}$$

and observe that disposable income has not changed. Given that the consumption of type i for $i \geqslant 2$ decreases, one knows that the consumption of type 1 increases. It also implies that x_1 increases along with the life-cycle income of type 1 individuals, which leads us to our second proposition.

Proposition 2 *Compare two steady-states with and without debt and assume operative bequests. The stock of capital is the same but with debt the welfare of the unconstrained individual is higher and that of the constrained individuals is lower than without debt.*

[6]It is possible that when the debt is issued, its short run effect allows the life-cyclers to consume relatively more than they would in the absence of debt. But here we restrict the analysis to the steady-state.

This result that the Ricardian equivalence holds, even with heterogeneous agents, can be found in slightly a different setting in Smetters (1999) and Mankiw (2000).

3.2 Pay-as-you-go social security with redistribution

We now turn to a pay-as-you-go pension system that provides the same benefits θ_t to all, and finances them by a proportional payroll tax of rate τ. We thus have:

$$N_{t-1}\,\theta_t = N_t\,\tau\,w_t\,\bar{h},$$

or

$$\theta_t = (1+n)\,\tau w_t\,\bar{h}.$$

In the steady-state with $x_1 > 0, w = \hat{w}$ and $R = \hat{R} = (1+n)/\gamma_1$, one can write the life-cycle income of constrained dynasties ($i \geqslant 2$):

$$\begin{aligned} \Omega_i^\theta &= (1-\tau)\,\hat{w}\,h_i + \theta/\hat{R}, \\ &= \hat{w}\,h_i + \tau\hat{w}\left(\gamma_1\,\bar{h} - h_i\right). \end{aligned} \tag{11}$$

One sees that $\Omega_i^\theta \lesseqgtr \Omega_i$ as $h_i \lesseqgtr \gamma_1\,\bar{h}$. In other words, the introduction of social security has a positive (negative) effect on type i individuals ($i \geqslant 2$) if their productivity index is inferior (superior) to the average productivity times the altruism factor of type 1. There are two redistributive effects involved .

First there is some redistribution from all constrained individuals to the unconstrained ones. This is made clear when $h_i = \bar{h}$ for all i. Then social security, like public debt, penalizes initially the constrained individuals in the steady-state who have to finance the "free lunch" offered when the system was introduced. There is also some redistribution from high productivity individuals to low productivity workers. In the limit case when γ_1 tends to 1, any individual with below average productivity benefits from this redistributive pension system.

Let us consider the benchmark case where $h_i = \gamma_1 \bar{h}$ for $i \geqslant 2$. From (12), $\Omega_i^\theta = \Omega_i^0$. In other words, social security has no effect on the lifetime income of agents i and thus on their consumption. Redistributive pension benefits compensate for the burden a pay-as-you-go system imposes upon constrained households. Given the overall resource constraint, the welfare of the unconstrained dynastic is also unaffected. We now turn to the other cases where some agents loose and others gain.

In the extreme case where all constrained dynasties have a relatively low productivity such that $h_i < \gamma_1 \bar{h}$ for all $i \geqslant 2$, then they all gain and the welfare of the unconstrained dynasty necessarily decreases.

Consider now the case when $h_i > \gamma_1 \bar{h}$ for $i \geqslant 2$. From (12) it is clear that the welfare of constrained individuals decrease and thus that of the unconstrained one increase. This case includes the particular situation with $h_i = \bar{h}$ for $i \geqslant 2$ studied by Michel and Pestieau (1998). Thus,

$$\Omega_i^\theta < \Omega_i^0 \qquad \text{for } i \geqslant 2$$

and both levels of consumption as well as utility decrease. By the same token, the welfare of the unconstrained households increases. Naturally we could have a wide dispersion among the h_i'$s\,(i \geqslant 2)$ so that those with low $h_i\left(<\gamma_1\bar{h}\right)$ would benefit from social security, and those with higher $h_i\left(>\gamma_1\bar{h}\right)$ would be hurt by it. If we have homothetic preferences[7], $c_i + \dfrac{d_i}{1+n} = \lambda\left(\hat{R}\right)\Omega_i^\theta$ for all agents. Then, the sum $\sum_{i=1}^m p_i\left(\Omega_i^\theta - \Omega_i^0\right)$ is equal to 0 (since the total consumption is unchanged). Consequently, $\Omega_1^\theta - \Omega_1^0$ is positive if $\sum_{i=1}^m p_i\left(\Omega_i^\theta - \Omega_i^0\right) = \tau\hat{w}\left[\gamma_1\bar{h}\,(1-p_1) - \bar{h} + p_1 h_1\right]$ is strictly negative. And this will also be the case if $p_1 h_1 < \bar{h}\,(1-\gamma_1\,(1-p_1))$.

Note that we have to assume at the start that the values (h_i, p_i) verify (8) with γ_1. With this assumption the steady-state equilibrium with pay-as-you-go pensions necessarily implies operative bequests $(x_1 > 0)$. For each type, a positive tax implies a decline in savings whatever the non-negative benefits received by the retirees (normality is assumed). Proposition 3 summarizes the above findings.

Proposition 3 *Compare two steady-states with and without a PAYG redistributive pension system and assume operative bequests. PAYG system has no effect on the steady-state when $h_i = \gamma_1\bar{h}$ for $i \geqslant 2$. It improves the welfare of constrained dynasties for which $h_i < \gamma_1\bar{h}$ and worsens that of those with $h_i > \gamma_1\bar{h}$. Assuming homothetic preferences it improves the welfare of the unconstrained dynasty if and only if $h_1 < \left(\gamma_1 + \dfrac{1-\gamma_1}{p_1}\right)\bar{h}$.*

[7]See the appendix.

If thus appears that the steady-state effect of redistributive social security is to transfer resources from higher ability workers $\left(h_i > \gamma_1 \; \bar{h}\right)$ to lower ability ones $\left(h_i < \gamma_1 \; \bar{h}\right)$.

Redistributive social security is thus shown to have a positive influence on income equality. Yet it does not have such an influence on the distribution of inherited wealth. In fact, Propositions 3 and 4 imply that societies with growing public endebtment and expanding social security schemes should all things being equal experience increasing wealth inequality. This is the point made by Laitner (2000).

4 Estate taxation

One of the interesting about studying a society of heterogenous individuals with unequal wealth is to note the redistributive incidence of estate taxation. We assume that a flat tax is levied on the estate and that its proceeds are redistributed uniformly to all households. Denoting by τ the estate tax rate and by T the uniform transfer, we rewrite the budget constraints of the individuals as:

$$(1-\tau)\, x_{it} + h_i\, w_t + T_t = c_{it} + s_{it}$$

$$R_{t+1}\, s_{it} = d_{it+1} + (1+n)\, x_{it+1}$$

The revenue constraint is:

$$T_t = \tau \sum_i p_i x_{it}.$$

Clearly, the optimal condition for saving is unchanged, but that for bequest is now distorted:

$$
\begin{aligned}
-\left(1-n\right)u_d'\left(c_{it},d_{it+1}\right)+\gamma_i\left(1-\tau\right)u_c'\left(c_{it},d_{it+1}\right) &\leqslant 0 \\
&= 0 \text{ if } x_{it+1}>0.
\end{aligned}
$$

In the steady-state the sufficient conditions for an optimum are:

$$
\left.
\begin{array}{c}
\left(1-\tau\right)x_i+h_iw+T=c_i+s_i \\
Rs_i=d_i+\left(1+n\right)x_i \\
u_c'\left(c_i,d_i\right)=R\,u_d'\left(c_i,\,d_i\right) \\
\left(1-\tau\right)\gamma_i\,R\leqslant 1+n \qquad \left(=1+n \;\text{ if } x_i>0\right).
\end{array}
\right\}
\tag{SC}
$$

As seen above, for $i \geqslant 2$, $x_i = 0$ and $s_i = \sigma\left(wh_i + T, R\right)$. Assuming $x_1 > 0$, we have in the steady-state $\hat{k}_\tau$ a level of capital stock being defined by $\left(1-\tau\right)\gamma_1\,\varrho(\hat{k}_\tau) = 1+n$. We note that $\hat{k}_\tau < \hat{k}_0$, the capital stock consistent with the modified golden rule when there is no estate tax. Disposable income thus decreases: $\bar{h}(f(\hat{k}_\tau) - (1+n)\,\hat{k}_\tau)$. At the same time, revenue constraint implies that

$$T = \tau\, p_1\, x_1.$$

Then for $i \geqslant 2$ we have in the steady-state:

$$c_i + d_i/R = \Omega_i\left(\tau\right) = wh_i + \tau p_1 x_1$$

where $w = w\left(\hat{k}_\tau\right)$ and $R = \varrho\left(\hat{k}_\tau\right) = \dfrac{1+n}{\left(1-\tau\right)\gamma_1}$.

And for the most altruistic agents ($i = 1$), we have:

$$c_1 + d_1/R = \Omega_1(\tau) = wh_1 + \tau p_1 x_1 + \left(1 - \tau - \frac{1+n}{R}\right) x_1. \qquad (12)$$

4.1 Bequest with homothetic preferences

For now on we assume homothetic preferences that imply that consumption levels are proportional to life-cycle consumption expenditures Ω_i. We have for all agents i:

$$c_i + \frac{d_i}{1+n} = \lambda(R)\,\Omega_i(\tau)$$

where $\lambda(R)$ is increasing for $R > 1/(1+n)$. More precisely the equilibrium condition $u'_c = Ru'_d$ is equivalent to $c/d = \phi(R)$ where $\phi(R)$ is decreasing. Also $\lambda(R)$ is defined by:

$$\lambda(R) = \frac{\phi(R) + 1/(1+n)}{\phi(R) + 1/R}$$

as shown in the appendix.

We now calculate the value of x_1. We start with the resource constraint:

$$\bar{h}\left(f(k) - (1+n)k\right) = \sum_1^m p_i\left(c_i + d_i/(1+n)\right) = \lambda(R)\sum_1^m p_i\Omega_i(\tau).$$

Furthermore we have:

$$\sum_{i=1}^m p_i\Omega_i(\tau) = w\,\bar{h} + \tau p_1 x_1 + \left(1 - \tau - \frac{1+n}{R}\right) p_1 x_1 = w\,\bar{h} + \left(1 - \frac{1+n}{R}\right) p_1 x_1.$$

Thus

$$\begin{aligned}\left(1 - \frac{1+n}{R}\right)\frac{p_1 x_1}{\bar{h}} &= \frac{1}{\lambda(R)}\left[f(k) - (1+n)k\right] - w \\ &= \frac{1}{\lambda(R)}\left(Rk + w - (1+n)k\right) - w.\end{aligned}$$

Using the expression for $\lambda(R)$ we obtain:

$$\frac{p_1\ x_1}{\bar{h}} = \frac{1}{\phi(R) + 1/(1+n)} \left((R\phi(R) + 1)\, k - w/(1+n) \right). \qquad (13)$$

One notes that the value of x_1 does not depend on the distribution of $\gamma_i\,(i \geqslant 2)$ nor of h_i; $p_1 x_1/\bar{h}$ only depends on $(1-\tau)\,\gamma_1$. In other words x_1 can be studied as if we were in a single agent economy.

Remark: *In the following we make the assumption that* $\frac{\partial x_1}{\partial \tau} < 0$. *In other words, estate taxation does not only depress per capita accumulation but also bequests. At first sight this is what we expect. Yet one can construct examples that yield the opposite outcome. We illustrate this by considering two extreme cases. In the first* $c = 0$. *Then* $w + x = s = (1+n)\,k$ *or* $(1+n)k - f(k) + f'(k)k = x$. *Thus*

$$\frac{dx}{dk} = 1 + n + f''(k)k.$$

In the second case $d = 0$. *Then* $f'(k)k = x$ *and thus* $\frac{dx}{dk} = f'(k)\left(1 + \frac{f''(k)k}{f'(k)}\right)$. *In each of these extreme cases of preferences, there exist well behaved production functions that give the two possibilities* $\frac{\partial x_1}{\partial \tau} \gtrless 0$.

We now turn to incidence of τ on Ω_i and u_i for $i \geqslant 2$, and then on Ω_1 and u_1.

4.2 Incidence of τ on Ω_i and u_i for $i \geqslant 2$.

Life-cycle income is $\Omega_i = w\ h_i + \tau p_1 x_1$ and thus:

$$\frac{\partial \Omega_i}{\partial \tau} = h_i \frac{\partial w}{\partial \tau} + p_1 x_1 + \tau p_1 \frac{\partial x_1}{\partial \tau}.$$

We note that

$$\frac{\partial R}{\partial \tau} = f'' \frac{\partial k}{\partial \tau} = \frac{1+n}{\gamma_1 (1-\tau)^2} = \frac{R}{1-\tau}$$

and

$$\frac{\partial w}{\partial \tau} = -kf'' \frac{\partial k}{\partial \tau} = -\frac{Rk}{1-\tau}.$$

Hence,

$$\left.\frac{\partial \Omega_i}{\partial \tau}\right|_{\tau=0} = -Rkh_i + p_1 x_1 = -Rk\left(h_i - \bar{h}\right) + p_1 x_1 - Rk\bar{h}$$

and using (15) we know that for $R > 1+n$.

$$p_1 x_1 - Rk\bar{h} = \frac{\bar{h}}{\phi(R) + 1/(1+n)} \left[-k\left(\frac{R}{1+n} - 1\right) - \frac{w}{1+n}\right] < 0. \quad (14)$$

From (15) and (16) we note that for $h_i = \bar{h}$ the estate tax has a negative effect on the life-cycle income of individuals i. The marginal gain from the transfer is more than offset by the loss in earnings.[8] In fact this conclusion applies to any i with $h_i > \tilde{h}$ where $\tilde{h}$ is defined by:

$$Rk\left(\tilde{h} - \bar{h}\right) = p_1 x_1 - Rk\bar{h} < 0.$$

We now turn to the tax incidence on u_i. In the appendix we show that:

$$\frac{\partial u_i}{\partial \tau} = u_c' \left[\frac{\partial \Omega_i}{\partial \tau} + \frac{\Omega_i}{R^2\phi + R}\frac{\partial R}{\partial \tau}\right],$$

and thus:

$$\left.\frac{\partial u_i}{\partial \tau}\right|_{\tau=0} = u_c' \left[R\ k\left(\bar{h} - h_i\right) - \frac{\bar{h}}{\phi + 1/(1+n)}\left(k\frac{R}{1+n} - 1\right) + \frac{w}{1+n}\right] \quad (15)$$

[8] This result holds for $\tau > 0$ if $\frac{\partial x_1}{\partial \tau} \leqslant 0$.

given that when $\tau = 0$:

$$\frac{\Omega_i}{R^2\phi + R}\frac{\partial R}{\partial \tau} = \frac{w\ h_i + \tau\ p_1 x_1}{(R\ \phi + 1)(1-\tau)} = \frac{w\left(h_i - \bar{h}\right)}{R\phi + 1} + \frac{w\ \bar{h}}{R\phi + 1}.$$

We can rewrite (17):

$$\frac{1}{u_c'}\left.\frac{du_i}{d\tau}\right|_{\tau=0} = A\left(\bar{h} - h_i\right) - B\ \bar{h},$$

with

$$A = Rk - \frac{w}{R\ \phi + 1} > 0 \qquad \left(x_1 > 0 \text{ implies } (R\ \phi + 1)\, k > \frac{w}{1+n} > \frac{w}{R}\right)$$

and

$$B = \frac{k\left(\frac{R}{1+n} - 1\right)}{\phi + \frac{1}{1+n}} + \frac{w\ \phi\,(R - (1+n))}{((1+n)\,\phi + 1)\,(R\ \phi + 1)} > 0.$$

This leads us to our next proposition.

Proposition 4 : *Assuming homothetic preferences, if all workers have the same productivity, then the introducion of estate taxation has a negative incidence on all constrained workers. If productivity is not uniform, constrained households with sufficiently low productivity can benefit from the tax. They gain if, and only if,* $h_i < \bar{h}\,(1 - B/A)$.

4.3 Incidence of τ on Ω_1 and u_1.

$$\frac{1}{u_c'}\frac{du_1}{d\tau} = \frac{\partial \Omega_1}{\partial \tau} + \frac{\Omega_1}{R^2 + \phi R}\frac{\partial R}{\partial \tau}$$

with

$$\Omega_1 = \omega\left(\hat{k}_\tau\right) h_1 + \tau p_1 x_1 + (1-\tau)\,(1-\gamma_1)\, x_1.$$

Hence,

$$\frac{\partial \Omega_1}{\partial \tau} = h_1 \frac{\partial w}{\partial \tau} - [1-\gamma_1 - p_1]\, x_1 + [(1-\tau)(1-\gamma_1) + \tau p_1] \frac{\partial x_1}{\partial \tau}.$$

We know that $\dfrac{\partial R}{\partial \tau} = \dfrac{R}{1-\tau}$ and then $\left(\dfrac{\Omega_1}{R^2\phi + R}\right) \dfrac{\partial R}{\partial \tau} = \dfrac{\Omega_1}{(R\phi+1)(1-\tau)}$. We also have $\dfrac{\partial w}{\partial \tau} = -\dfrac{Rk}{1-\tau}$.

This gives:

$$(1-\tau)\frac{\partial \Omega_1}{\partial \tau} = -h_1 Rk - (1-\gamma_1)(1-\tau)\, x_1 + (1-\tau)\, p_1 x_1 + (1-\tau)\, D \frac{\partial x_1}{\partial \tau}$$

where

$$D = (1-\tau)(1-\gamma_1) + \tau p_1 > 0.$$

We now write:

$$\begin{aligned} \frac{(1-\tau)}{u_c'}\frac{\partial u_1}{\partial \tau} &= -h_1 Rk - (1-\gamma_1)(1-\tau)\, x_1 - \tau p_1 x_1 + p_1 x_1 + (1-\tau)\, D \frac{\partial x_1}{\partial \tau} \\ &\quad + \frac{1}{R\phi+1}\left[w h_1 + \tau p_1\ x_1 + (1-\tau)(1-\gamma_1)\, x_1\right], \end{aligned}$$

or

$$\frac{(1-\tau)}{u_c'}\frac{\partial u_1}{\partial \tau} = -h_1\left(Rk - \frac{w}{R\phi+1}\right) + p_1 x_1 - D\left(\frac{R\phi x_1}{R\phi+1} - (1-\tau)\frac{\partial x_1}{\partial \tau}\right).$$

From (14) we have:

$$\begin{aligned} p_1\ x_1 - \bar{h}\ Rk + \tfrac{w\ \bar{h}}{R\phi+1} &= \bar{h}\ k\left(\tfrac{R\phi+1}{\phi+\frac{1}{1+n}} - R\right) + \bar{h}w\left(\tfrac{1}{R\phi+1} - \tfrac{1}{(1+n)\phi+1}\right) \\ &= \tfrac{\bar{h}k}{\phi+\frac{1}{1+n}}\left(1-\tfrac{R}{1+n}\right) + \tfrac{\bar{h}w}{(R\phi+1)((1+n)\phi+1)}\,(1+n-R)\,\phi < 0, \end{aligned}$$

given that $R > 1+n$.

This implies that

$$\frac{(1-\tau)}{u'_c}\frac{\partial u_1}{\partial \tau} < \left(\bar{h}-h_1\right)\left(Rk-\frac{w}{R\phi+1}\right)-D\left(\frac{R\phi x_1}{R\phi+1}-(1-\tau)\frac{\partial x_1}{\partial \tau}\right).$$

We can thus conclude that $\frac{\partial u_1}{\partial \tau} < 0$ if $h_1 \geqslant \bar{h}$ and $\frac{\partial x_1}{\partial \tau} \leqslant 0$, which leads us to our last proposition.

Proposition 5 *: Assuming that preferences are homothetic and that estate tax discourages bequest it has a depressive effect on the utility of unconstrained households if these have a productivity not smaller than the average one.*

Combining Propositions 5 and 6 we observe that estate taxation is Pareto worsening if everyone has the same productivity $\left(h_i = \bar{h}\right)$ and $\frac{\partial x}{\partial \tau} \leqslant 0$. Both are just sufficient conditions and by no means necessary ones.

5 Conclusion

The purpose of this paper was to generalize some results developed in Michel and Pestieau (1998) and in Mankiw (2000). The setting is that of an overlapping generations economy consisting of households who differ in earnings ability and intergenerational altruism – two characteristics assumed to be hereditary. We showed that both public debt and pay-as-you-go social security are neutral *à la Ricardo,* but that there is an increase in wealth inequality between households who are the most altruistic versus all the others. We also demonstrated that in the steady-state redistributive estate taxation is likely to be Pareto worsening, even though it reduces wealth inequality.

This same paradoxical result was obtained in a different studies by Stiglitz (1978). In other words, when capital is taxed its quantity falls, and this in turn depresses real wages. The effect of such a reduction may be large enough to make any tax on wealth transfer undesirable, even from the standpoint of people who own no wealth, pay no tax and benefit from the transfer. Naturally, with different productivities we expect that life cyclers with low productivity will benefit from the tax transfer.

Finally, a word of on the robustness of the above results. As stated at the outset we focused on the steady-state.[9] Only if the steady-state is stable, the conclusions apply to the dynamic (locally) near the steady-state.

Michel and Pestieau (1998) have studied the global stability property in the simple special case with a log-linear utility function and a Cobb-Douglas production function. A study of local stability in the general case has already been done by Nourry and Venditti (2001), who show that there is no indeterminacy and give necessary and sufficient conditions for the steady-state to be a saddle-point. In this paper we assume that these conditions are satisfied, thus implying a stable steady-state.

[9]Regarding an attempt to study the dynamics of an economy with other form of altruism, see Falk and Stark (2001).

Appendix

Homothetic preferences

Continuous homothetic preferences can be represented by an homogenous utility function of degree 1. We take a utility function $u(c, d)$ homogenous of degree 1, continuously differentiable, increasing and strictly concave with respect to c and to d, and satisfying Inada's conditions:

$$\lim_{c \to 0} u'_c(c, 1) = +\infty \quad \text{and} \quad \lim_{d \to 0} u'_d(1, d) = +\infty. \tag{A.1}$$

This implies that the equation

$$u'_c(\phi, 1) = R\ u'_d(\phi, 1) \tag{A.2}$$

admits a unique solution $\phi(R)$ that is decreasing and one-to-one from R^*_+ into R^*_+ (the set of positive numbers).

Indeed $R(\phi) = u'_c(\phi, 1)/u'_d(1, 1/\phi)$ is decreasing and one-to-one. Thus the same holds for the inverse function $\phi(R)$.

The problem of the consumer is to maximize $u(c, d)$ subject to

$$c + d/R = \Omega. \tag{A.3}$$

Optimality conditions are given by (A.3) and

$$c/d = \phi(R). \tag{A.4}$$

Henceforth, we have:

$$\begin{aligned}
d(\Omega,R) &= \frac{\Omega}{\phi(R)+1/R} \quad \text{with} \quad d'_R(\Omega,R)>0\ , \\
c(\Omega,R) &= \frac{\Omega\phi(R)}{\phi(R)+1/R} \quad \text{with} \quad c'_R(\Omega,R)\gtrless 0\ , \\
\lambda(R) &= \frac{c(\Omega,R)+d(\Omega,R)/(1+n)}{\Omega} = 1+\frac{1/(1+n)-1/R}{\phi(R)+1/R}.
\end{aligned}$$

We note that $\lambda(1+n)=1$ and $\lambda'(R)>0$ for $R>1+n$.

We now derive the indirect utility function:

$$\begin{aligned}
\tilde{u}(\Omega,R) &= u(c(\Omega,R),d(\Omega,R)) = c(\Omega,R)\,u'_c + d(\Omega,R)\,u'_d \\
&= u'_c(c+d/R) = \Omega\ u'_c(\phi(R),1)
\end{aligned}$$

when we use the homogeneity of degree 1.

Function $\tilde{u}$ has the following properties:

$$\frac{\partial\tilde{u}}{\partial\Omega}=u'_c\ ;\ \frac{\partial\tilde{u}}{\partial R}=u'_c\ c'_R+u'_d\ d'_R=u'_c\left(c'_R+\frac{d'_R}{R}\right).$$

Given that $c'(R)+\dfrac{d'(R)}{R}=\dfrac{d}{R^2}$, we can write:

$$\frac{\partial\tilde{u}}{\partial R}=u'_c\frac{d}{R^2}=u'_c\frac{\Omega}{R^2\phi(R)+R}.$$

We can now give the derivative of $\tilde{u}$ with respect to τ:

$$\begin{aligned}
\tfrac{\partial\tilde{u}}{\partial\tau} &= u'_c\tfrac{\partial c}{\partial\tau}+u'_d\tfrac{\partial d}{\partial\tau}=u'_c\left(\tfrac{\partial c}{\partial\tau}+\tfrac{1}{R}\tfrac{\partial d}{\partial\tau}\right) \\
&= u'_c(\phi(R(\tau)),1)\left(\tfrac{\partial\Omega}{\partial\tau}+\frac{d}{R^2}\tfrac{\partial R}{\partial\tau}\right) \qquad \text{(A.5)} \\
&= u'_c(\phi(R(\tau)),1)\left(\tfrac{\partial\Omega}{\partial\tau}+\tfrac{\Omega}{R^2\phi(R)+R}\tfrac{\partial R}{\partial\tau}\right)
\end{aligned}$$

following

$$\frac{\partial \Omega}{\partial \tau} = \frac{\partial c}{\partial \tau} + \frac{1}{R}\frac{\partial d}{\partial \tau} - \frac{d}{R^2}\frac{\partial R}{\partial \tau}.$$

Examples:

$$\begin{aligned} u(c,d) &= c^{\alpha} d^{1-\alpha},\ \phi(R) = \frac{c}{d} = \frac{\alpha}{(1-\alpha)R} \\ u(c,d) &= \left(c^{1-1/\sigma} + \beta\ d^{1-1/\sigma}\right)^{\frac{\sigma}{\sigma-1}},\ \phi(R) = (\beta\ R)^{-\sigma}. \end{aligned}$$

References

[1] Altig, D. and S. Davis, [1992], The timing of intergenerational transfers, tax policy and aggregate savings, *American Economic Review*, 1199-1220.

[2] Barro, R.J., [1974], Are government bonds net wealth?, *Journal of Political Economy*, 82, 1095-1117.

[3] Becker, R., [1980], On the long run steady-state in a simple dynamic model of equilibrium with heterogeneous households, *Quarterly Journal of Economics*, 95, 375-382.

[4] Diamond, P., [1965], National debt in a neoclassical growth model, *American Economic Review*, 55, 1126-50.

[5] Falk, I. and O. Stark, [2001], Dynasties and destiny: on the roles of altruism and impatience in the evolution of consumption and bequests, *Economica*, 68, 505-518.

[6] Laitner, J., [2000], Secular changes in wealth inequality and inheritance, University of Michigan, unpublished.

[7] Mankiw, G., [2000], The savers-spenders theory of fiscal policy, *American Economic Review*, 90, 120-24.

[8] Michel, Ph. and P. Pestieau, [1998], Fiscal policy in a growth model with both altruistic and non altruistic agents, *Southern Economic Journal*, 64, 682-97.

[9] Nourry, C. and A. Venditti, [2001], Determinacy of equilibrium in an overlapping generations model with heterogeneous agents, *Journal of Economics Theory*, 96, 230-255.

[10] Ramsey, F.P. , [1928], A mathematical theory of saving, *Economic Journal*, 38, 543-559.

[11] Samuelson, P.A., [1958], An exact consumption loan model of interest with or without the social contrivance of money, *Journal of Political Economy*, 66, 467-82.

[12] Smetters, K., [1999], Ricardian equivalence: long-run leviathan, *Journal of Public Economic*s, 73, 395-421.

[13] Stiglitz, J.E., [1978], Notes on estate taxes, redistribution and the concept of balanced growth path incidence, *Journal of Political Economy*, 86, 137-150.

[14] Thibault, E., [2002)], Existence and specific characters of capitalists: a savers-spenders theory approach, GREMAQ Working Paper.

[15] Weil, P., [1987], Love thy children. Reflections on the Barro debt neutrality theorem, *Journal of Monetary Economics*, 19, 377-391.