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ABSTRACT

Overconfidence and Delegated Portfolio Management*

Following extensive empirical evidence about 'market anomalies' and overconfidence, the analysis of financial markets with agents overconfident about the precision of their private information has received a lot of attention. All these models consider agents trading for their own account. In this article, we analyse a standard delegated portfolio management problem between a financial institution and a money manager who may be of two types: rational or overconfident. We consider several situations. In each case, we derive the optimal contract and results on the performance of financial institution hiring overconfident managers relative to institutions hiring rational agents, and results on the price impact of overconfidence.

JEL Classification: D82 and G11
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1 Introduction

Following extensive empirical evidence about "market anomalies"[1] and overconfidence[2], the analysis of financial markets with overconfident agents has received a lot of attention (see Kyle and Wang (1997), Odean (1998), Wang (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Wang (2001)).[3] The common results of all these studies is that overconfidence always leads to overly risky investment strategy but may also provide higher expected return.[4]

This last result led to conclusions about delegated portfolio management. For example, Kyle and Wang (1997) conclude that "for some parameter values a fund facing a major rival in an efficient market should hire an overconfident manager" and Wang (2001) shows that institutions hiring overconfident portfolio managers grow faster than those hiring rational managers. However, in order to reach these conclusions, these studies implicitly assume that *(i)* overconfidence is an observable characteristic, and *(ii)* managers' incentives are aligned with those of the employing institution. What if these two assumptions do not hold? What is then the compensation contract proposed by the financial institution? What are the consequences on the investment strategy of overconfident agents? In such a situation, what is the impact of overconfidence on prices?

To answer these questions, we analyse a standard delegated portfolio management problem in which a risk neutral financial institution (the principal) hires a money manager (the agent) who may be of two types: rational or overconfident. If exerting effort, the agent acquires private information about the value of a risky asset. If the agent is rational, he updates his beliefs about the expected value of

[1]See Daniel, Hirshleifer and Subrahmanyam (1998, Appendix I) for a review.

[2]See, for example, Alpert and Raiffa (1982) and Heath and Tversky (1991). See Odean (1998, Section II) for a review of the literature.

[3]Another way of modelling irrational behavior is misinterpretation of the expected value of the asset traded (Delong, Shleifer, Summers and Waldmann (1990), Palomino (1996)).

[4]The exception is if agents are strategic (i.e., have market power) and do not trade simultaneously. In such a case, overconfidence yields lower expected returns.

the risky asset in a Bayesian fashion. However, if overconfident, the agent over-estimates the precision of his private signal. Based on his updated beliefs, the agent then makes an investment decision. We consider two different cases. First, the agent is risk-averse and price taker (Case PT, hereafter). Second, the agent is risk-neutral, has limited liability and has market power. (Case MP, hereafter).

In these situations, the principal faces moral hazard problems on both effort and risk (i.e., the amount invested in risky assets). In Case PT, if overconfidence is an observable characteristic, we show that there exist optimal contracts such that rational and overconfident agents choose the same investment strategy. Hence, overconfident and rational agents perform equally well and undertake the same amount of risk. This result implies that results about overconfidence obtained in the case of agents trading for own account may not hold in the case of delegated portfolio management since the contract offered by the principal modifies investment incentives.

If overconfidence is not observable, we derive conditions under which there exists a separating equilibrium such that the principal offers a menu of contracts, rational and overconfident agents choose different contracts, rational agents exert a low effort and overconfident agent exert a high effort. In this equilibrium, overconfident agents perform better (earn higher expected return) and undertake less risk (the variance of return is lower) than rational agents. This result is due to the fact that the contracts proposed by the principal align the risk taking incentives of the overconfident agent with his owns' while still giving incentives to overconfident agents to acquire a large amount of information (given their beliefs.)

In Case MP, we assume that trading takes place in a market similar to that described in Easley and O'Hara (1987). If overconfidence is observable, we derive conditions under which the contract offered by the principal is first-best. Hence, rational and overconfident agents choose the same investment strategy. This implies that overconfidence does not have any impact on prices if informed agents trade on the behalf of a principal, while overconfidence would have some price impact if informed agents were trading for their own account. This result shows that results about the price impact of overconfidence depend on whether one considers trading for own account or delegated portfolio management. In the latter case, contracts offered by the principal influence investment strategies, hence the impact of overconfidence on prices.

If overconfidence is not observable, we show that overconfidence has an impact on prices in situations in which rational and overconfident agents do not acquire the same amount of information. In such situations, the market maker does not know whether informed trades come from rational of overconfident agents. As a consequence, he does not know the precision of the information of the informed agent. Therefore, if the market maker operates in a competitive environment (as is usually assumed in market microstructure models), then the quotes he posts take into account the fact that an informed order may come either from a rational agent or an overconfident agent.

Our results have several implications. First, in terms of accumulation of wealth, they should be compared to those of Wang (2001). In our model, if overconfidence is observable, then a principal can choose what type of agent to hire. Given that an agent overconfident about the precision of his private information believes that he will realize a good performance with a higher probability than the correct one, he accepts "cheaper" contracts than a rational agent. This makes overconfident agents more attractive than rational agents for the principal. Hence, our results extends those of Wang (2001) to the case of delegated portfolio management: financial institutions hiring overconfident agents grow faster than those hiring rational agents. However, in our case, the result is not due to strategic market interaction between rational and overconfidence acting as a commitment to trade aggressively (as in Kyle and Wang (1997) and Wang (2001)). This is due to their acceptance of "cheap" contracts that rational agents refuse.

Second, our results have implications for the debate on the origin of the very high trading volume in financial markets. As already mentioned, the literature on overconfidence has established that overconfident agents trade too large quantities. In the case of delegated portfolio management, Dow and Gorton (1997) provide a second reason for excessive trading volume: agency problem between principals and agents who want to show that they are informed, hence are very active in the market. Our results suggest that a principal offering the appropriate contract can mitigate agents' incentives to trade due to overconfidence. Hence, excessive trading volume would have two sources: overconfidence in the case of agents trading for own account and agency problems in the case of delegated portfolio management.

The organization of the paper is as follows. Section 2 reviews the related literature. Section 3 presents the model with a risk-averse, price-taking agent and consider trading for own account as a benchmark case. Section 4 presents the results in case of delegated portfolio management. Section 5 analyses the case with a risk-neutral agent with market power and limited liability, while Section 6 concludes. All the proofs are contained in Appendix A.

2 Related literature

Our article bridges the literature on delegated portfolio management and that on overconfidence in financial markets.

Bhattacharya and Pfleiderer (1985) were the first to study delegated portfolio management in a principal-agent framework. However, their model is more one of hidden information rather than hidden action since the principal can verify the level of risk taken by the agent.[5]

Cohen and Starks(1988), Admati and Pfleiderer (1996), Diamond (1998), Palomino and Prat (2003) study delegated portfolio management with moral hazard on both effort and risk. Cohen and Stark (1988) derive conditions under which the manager exerts more effort but chooses a riskier portfolio than investors prefer. Admati and Pfleiderer (1996) look at the impact of benchmarking on behavior. They show that, in general, benchmarking is inconsistent with obtaining the optimal portfolio and tends to decrease incentives to exert effort. Diamond (1998) shows that if the control space of the agent has full dimensionality, (i.e., the principal has fewer degrees of freedom in setting the incentives than the agent has degrees of freedom in responding), then as the cost of effort shrinks, the optimal contracts converges

[5]This type of problem is also analysed in Stoughton (1993).

to a linear contract. Palomino and Prat (2003) consider the case in which the agent has limited liability. They show that there exists an optimal contract which takes the form of a bonus contract.

The influence of overconfidence on contracts has been studied by Gervais, Heaton and Odean (2002) and Hackbart (2002). Gervais, Heaton and Odean consider a capital budgeting problem faced by a risk averse manager who may be overoptimistic or overconfident. They find that a risk neutral principal may be better off hiring a moderately overconfident agent than a rational one. The main difference between their study and ours is that we study the case in which the overconfidence level of the managers is not observable and the agent has the choice between several effort levels. In such a case, we derive conditions under which the principal can use a menu of contracts to screen agents.

Hackbart (2002) studies the impact of overconfidence on capital structure and also looks at contracts based on a cash salary, a bonus, an equity stake, and executive stock options. However, the optimal contract is not derived.

The consequences of overconfidence in financial markets has been studied both in the context of perfectly and imperfectly competitive markets. Under the assumption of perfect competition, Odean (1998, Section III.A) studies a market in which all informed agents are overconfident about the precision of their information. He shows that as overconfidence increases, trading volume and price volatility increase and overconfident agents' expected utility is lower than if their beliefs were properly calibrated.

Daniel *et al.*(1998) study price reactions to public and private information. They show that overconfidence increases price volatility around private signals, and that price moves resulting from the arrival of private information are on average partially reversed in the long run.

Wang (2001, Section III) studies population dynamics in the presence of rational and overconfident agents. He shows that if overconfident agents are moderately overconfident and their initial share of the population is above some treshold, then overconfident agents as a group will dominate the economy in the long run.

Finally, Daniel *et al.* (2001) derive an asset pricing model taking into account agents' overconfidence.

In an economy in which agents are risk averse with negative exponential utility, and uncertainty is normally distributed, they show that price overreacts to private signals and true expected returns decompose additively into a risk premium and components arising from mispricing.

In imperfectly competitive markets, Odean (1998, Section III.A) shows that overconfidence can lead to market breakdowns, and that when a market equilibrium exists, expected volume, market depth, price volatility and the level of informational efficiency increase as the insider's overconfidence increases.

Kyle and Wang (1997) and Wang (2001, Section II) show that in market with two informed agents, overconfidence acts as a commitment to trade aggressively. As a consequence, an overconfident informed agent may earn a higher expected utility than a rational one and overconfident agents may dominate the economy in the long run.

Finally, Caballe and Sakovics (2003) differentiate between private self-confidence (the self-confidence of the speculators) and public self-confidence (the self-confidence they attribute the their competitors). They show that public self-confidence and private self-confidence have different effects (sometimes opposite) on trading volume, price volatility, informational efficiency and expected profits.

3 The model

We consider the following economy. There is one risky asset and a risk-free asset with return normalized to 1. The return V of the risky asset is $V_H > 1$ with probability 1/2, $V_L < 1$ with probability 1/2, and $E(V) = 1$.

If exerting effort at a cost c, the agent receives private information. The signal he receives is either s_H or s_L. Conditional on signals, the distribution of the return of the risky asset is

$$\begin{aligned} &\text{Prob}(V_i|s_i) = (1+k)/2 \quad j = H, L \\ &\text{Prob}(V_i|s_j) = (1-k)/2 \quad i = H, L,\ j = H, L,\ i \neq j \end{aligned}$$

with $k \in (0, 1)$.

We define overconfidence as in Gervais, Heaton and Odean (2002). That is, after receiving a signal s_i ($i = H, L$), an overconfident agent believes that

$$\text{Prob}(V_i|s_i) = (1+K)/2 \quad j = H, L$$

$$\text{Prob}(V_i|s_j) = (1-K)/2 \quad i = H, L,\ j = H, L,\ i \neq j$$

with $K \in (k, 1)$. Hence, overconfidence means that the agent perceives the information as more reliable than what it really is. The difference $K - k \in (0, 1-k)$ measures the degree of overconfidence of the agent.

The agent is risk averse with utility

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

with $\gamma \in (0, 1)$.

3.1 Benchmark case: Trading for own account

As a Benchmark, we consider the case in which the agent first acquires information and then trades for his own account.

Proposition 1 *Let*

$$A_H(K) = \left(\tfrac{(1+K)(V_H-1)}{(1-K)(1-V_L)}\right)^{1/\gamma} \quad A_L(K) = \left(\tfrac{(1-K)(V_H-1)}{(1+K)(1-V_L)}\right)^{1/\gamma} \tag{1}$$

$$x(K, c, s_i) = \frac{(A_i(K)-1)(1-c)}{(V_H-1) + A_i(K)(1-V_L)} \qquad i = H, L \tag{2}$$

and

$$U_o(K, s_i) = (1-c)^{1-\gamma}\left\{\frac{(1+K)}{2(1-\gamma)}\left[1 + \frac{x(K,c,s_i)}{(1-c)}(V_i - 1)\right]^{1-\gamma} + \frac{(1-K)}{2(1-\gamma)}\left[1 + \frac{x(K,c,s_i)}{(1-c)}(V_j - 1)\right]^{1-\gamma}\right\} \quad j \neq i \tag{3}$$

(i) *If*

$$c < 1 - \left(\frac{2}{U_o(K, s_H) + U_o(K, s_L)}\right)^{1/(1-\gamma)}, \tag{4}$$

an agent with overconfidence level K acquires information and trades a quantity $x(K, c, s_i)$ when receiving a signal s_i.

(ii) *The expected return of an overconfident agent is larger than that of a rational agent*

(iii) *The variance of return of an overconfident agent is larger than that of a rational agent.*

These results are similar to those of De Long, Shleifer, Summers and Waldmann (1990), Odean (1998) and Wang (2001) in the context of perfectly competitive markets. Overconfidence (or overoptimism) generates incentives to trade larger quantities than rational agents. As a consequence, overconfident agents earn a higher expected return but their investment is riskier.

4 Delegated portfolio management

The principal is risk-neutral and cannot acquire information at any cost. This implies that if he does not hire an agent, his expected revenue is zero. The amount invested in the risky asset x belongs to $[-\bar{x}, \bar{x}]$. This means that there is an upper limit to the amount the agent can borrow to invest in the risky asset or shortsell. To make the problem interesting, we assume $\bar{x} > x(K, c, s_H)$ and $-\bar{x} < x(K, c, s_L)$. That is, if trading for own account, an informed agent can choose the optimal trading strategy given his beliefs.

Finally, we assume that only non-decreasing compensation contracts are offered. If, as in Palomino and Prat (2003, Assumption 2), the agent can sabotage (i.e., given an actual return r, he can report any return $r' \leq r$), then an optimal contract must be non-decreasing in r. Here, we restrict our attention to such non-decreasing contracts directly.

We denote $\bar{U} \geq 0$ the reservation utility of the agent.

In such a situation, *if* an agent acquires information and acts in the interest of the principal, he trades a quantity $\bar{x}$ if he receives the signal $s = s_H$ and he trades a quantity $-\bar{x}$ if he receives the signal $s = s_L$.

However, we assume that the portfolio chosen by the agent is not verifiable. Therefore, it cannot be contracted upon. Given the difference in risk aversion between the principal and the agent, the moral hazard problem faced by the principal is twofold. He must provide the agent incentives to 1) exert effort and acquire information, and 2) take the appropriate level of risk.

We consider two cases. First, overconfidence is observable. That is the level of overconfidence of the agent is common knowledge to the principal and the agent. In the second case, we will assume that the principal does not know whether he is making an offer to a rational or an overconfident agent.

4.1 Overconfidence is observable

Denote $R[x(s_i), V_j]$ $(i, j = H, L)$ the realized return of the agent if he trades a quantity x after having received a signal s_i and V_j is realized, i.e.,

$$R[x(s_i), V_j] = 1 + x(s_i)(V_j - 1)$$

As is standard in contract theory, the principal has all the bargaining power and makes a take-it-or-leave-it offer to the agent (see, e.g., Salanie (1997) and Laffont and Martimort (2001) for surveys on contract theory).

Denote $E_K(.)$ and $E_k(.)$ the expectation operators using overconfident or rational beliefs, respectively. The problem of the principal is to choose a contract $h^*(R)$ which maximizes

$$\frac{1}{2}\left(E_k\{R(x^*(s_H), V) - h[R(x^*(s_H), V)]|s_H\} + E_k\{R(x^*(s_L), V) - h[R(x^*(s_L), V)]|s_L\}\right) \tag{5}$$

subject to

$$x^*(s_i) \in \text{argmax} E_K\{U[h^*(R(x, V) - c)]|s_i\} \qquad i = H, L \tag{6}$$

$$x^*(\emptyset) \in \text{argmax} E\{U[h^*(R(x, V))]|\emptyset\} \tag{7}$$

where "$\emptyset$" means that the agent has not acquired information.

$$\frac{1}{2}E_K\{U[h^*(R(x^*(s_H), V))-c]|s_H\}+\frac{1}{2}E_K\{U[h^*(R(x^*(s_L), V))-c]|s_L\} \geq E\{U[h^*(R(x^*(\emptyset), V))]|\emptyset\} \tag{8}$$

$$\frac{1}{2}E_K\{U[h^*(R(x^*(s_H),V))-c]|s_H\}+\frac{1}{2}E_K\{U[h^*(R(x^*(s_L),V))-c]|s_L\}\geq \bar{U} \tag{9}$$

Equations (6) and (7) are the incentive compatibility constraints on risk if the agent has acquired information and has not acquired information, respectively. Equation (8) represents the incentive compatibility constraint on effort while equation (9) represents the participation constraint.

Definition 2 (i) *A contract is optimal if it is a solution of the maximization problem (5) - (9).*

(ii) *A contract is first-best if 1) it is optimal, 2) the agent chooses $x(s_H) = \bar{x}$, $x(s_L) = -\bar{x}$ and 3) $E_K[U(h^*)] = \bar{U}$.*

Hence, a contract is said to be first-best if the agent chooses the same trading strategy as the principal would choose for his own account (Condition 2) and if the contract leaves no (expected) rent to the agent (Condition 3).

Now, we analyze optimal contracts and derive conditions under which a first-best contract exists.

Lemma 3 (i) *The set of optimal contracts contains contracts of the shape*

$$h(R|\alpha_0,\alpha_1)=\begin{cases}\alpha_0 & \text{if} \quad R\leq 1\\ \alpha_1+\beta R & \text{if} \quad R>1\end{cases} \tag{10}$$

with $\beta > 0$.

(ii) *For any such contract, an informed agent trades quantities $x(s_H) = \bar{x}$ and $x(s_L) = -\bar{x}$.*

Lemma 3 means that there always exist optimal contracts which solve the moral hazard problem on risk. In other words, such optimal contracts align overconfident and rational agents' investment incentives. This result highlights the differences between trading for the own account and the delegated portfolio management when dealing with the impact of overconfidence on investment strategies: if agents trade for own account, then overconfidence generates risk-taking incentives, i.e., an overconfident agent takes more risk than a rational one (Proposition 1). This is not the case with delegated portfolio management. The optimal contract proposed by the principal aligns overconfident and rational agents' risk taking incentives.

In the rest of the paper, we restrict our attention to optimal contracts of the shape of $h(.)$.

Proposition 4 (i) *Assume that the agent is overconfident. There exist* $\bar{\bar{U}}$ *and* $\bar{c}(\bar{U})$ *such that if* $\bar{U} \in (0, \bar{\bar{U}})$ *and* $c < \bar{c}(\bar{U})$*, then there exists a first-best contract*

$$h_K^*(R|\alpha_0^*(K), \alpha_1^*(K)) = \begin{cases} \alpha_0^*(K) & \text{if} \quad R \leq 1 \\ \alpha_1^*(K) + R & \text{if} \quad R > 1 \end{cases}$$

where $\alpha_1^*(K)$ *and* $\alpha_0^*(K)$ *are given by Equations (16) and (17), respectively, in Appendix.*

(ii) *If the agent is rational, there is no first-best contract.*

The Proposition states that under some conditions on parameters, the principal has the possibility to offer optimal contracts that leave no rents to overconfident agents while this possibility does not exist with rational agents. The reason is the following. For the IC constraint on effort to be satisfied, it must be the case that $\frac{\alpha_1+R-c}{\alpha_0-c} > 1$, i.e., the compensation net of information cost in case of good performance ($R > 1$) is strictly larger than in the case of bad performance ($R < 1$). Now, for a given expected compensation C paid by the principal (i.e., $\frac{(1-k)}{2}\alpha_0 + \frac{(1+k)}{2}(\alpha_1 + R) = C$), the expected utility of an agent is maximized at

$$\frac{\alpha_1 + R - c}{\alpha_0 - c} = \frac{(1+K)(1-k)}{(1-K)(1+k)} = M(k, K)$$

It is straightforward that $M(k, k) = 1$, and for all $K > k$, $M(k, K) > 1$. Now, if the principal minimizes the expected compensation subject to some participation constraint $\bar{U}$, he wants $\frac{\alpha_1+R-c}{\alpha_0-c} = M(k, K)$ (as above). As a consequence, if the agent is overconfident (and the cost of information acquisition c is small enough), it is possible for the principle to set α_0 and α_1 such that we have, simultaneously, $\frac{\alpha_1+R-c}{\alpha_0-c} = M(k, K)$ and the IC constraint on effort satisfied. If the agent is rational, this is not possible. Therefore, the principal must leave some rent to the agent is order to satisfy the IC constraint on effort.

From Proposition 4, we also derive the following result.

Corollary 5 *Assume that for a first best contract h_K^* exists. Then, the principal is better off hiring an overconfident agent than a rational one.*

The corollary states that principals will prefer to hire overconfident rather than rational agents. They do so, not because they expect overconfident agents to outperform the rational ones, but because they know that, although both types choose the same investment strategy in equilibrium, the overconfident agents are willing to do so at a lower compensation level than the rational agents.

These results should be compared to those of Wang (2001) on the comparison of performances between rational and overconfident money managers. Our results show that those of Wang (2001) hold when the compensation contract of the agent is taken into account. However, financial institutions hiring overconfident agents do not accumulate more wealth than those hiring rational agents because overconfident agents perform better than rational agents, but because overconfident agents accept cheaper contracts than rational agents.

4.2 Overconfidence is not observable

We now assume that the principal cannot observe whether the agent is rational or overconfident. The principal correctly believes that the agent is overconfident with probability θ and rational with probability $1 - \theta$.

Denote h_k^o an optimal contract of the shape of (10) proposed by the principal to a rational agent if types are observable.

When overconfidence is not observable, the principal faces an additional incentive compatibility constraint. If h_k^o is offered by the principal and chosen by an overconfident agent, then his participation constraint is not binding. In other words, the contract h_k^o leaves some rent to an overconfident agent. As a consequence, an overconfident agent prefers the contract h_k^o relative to the contract h_K^*. This implies that if overconfidence is not observable, then the principal faces a trade-off. Either he proposes contracts which are accepted by both types of agents (but leaves some rent to overconfident agents), or the principal proposes the contract h_K^* which will be rejected by the agent with probability $(1-\theta)$, i.e., if the agent happens to be rational. We derive the following result.

Proposition 6 *Assume that a first-best contract h_K^* exists. There exists $\bar{\theta}$ such that if $\theta > \bar{\theta}$, the principal only offers the contract h_K^*. The contract is accepted by an overconfident agent and rejected by a rational one.*

The Proposition states that if the probability that the agent is overconfident is large (i.e., larger than $\bar{\theta}$), then the principal screens agents and hires only overconfident agents. This implies that if θ is large, the principal only hires overconfident agents, as if overconfidence were observable.

4.3 Two levels of information.

So far, we have assumed that there is only one level of private information. Here, we extend the previous analysis by assuming that there are two levels (1 and 2) of private information. If a rational manager pays a cost c_i ($i = 1, 2$ and $c_1 < c_2$), he receives a signal $s_{i,j}$ ($j = H, L$) such that

$$\text{Prob}(V_j|s_{ij}) = (1 + k_i)/2 \quad j = H, L$$
$$\text{Prob}(V_j|s_{ij'}) = (1 - k_i)/2 \quad j = H, L,\ j' = H, L,\ j' \neq j$$

with $k_i \in (0, 1)$ and $k_1 < k_2$.

Hence, if exerting a high effort and paying a high cost (c_2), the manager receives a more precise information than if exerting a low effort and paying a low cost (c_1).

After paying a cost c_i ($i = 1, 2$) and receiving a signal s_{ij} ($j = H, L$), an overconfident agent believes that

$$\text{Prob}(V_j|s_{ij}) = (1 + K_i)/2 \quad j = H, L$$
$$\text{Prob}(V_j|s_{ij'}) = (1 - K_i)/2 \quad j = H, L,\ j' = H, L,\ j \neq j'$$

with $K_i \in (k_i, 1)$ and $K_1 < K_2$.

In such a situation, we have the following results.

Proposition 7 *There exist $\bar{c}_1 > 0$, $\bar{c}_2 > 0$, $\bar{\delta} > 0$, $\bar{\theta} > 0$ and $\bar{K} < 1$ such that if $c_1 < \bar{c}_1$, $c_2 - c_1 < \bar{c}_2$, $k_2 - k_1 < \delta$, $K_2 > \bar{K}$ and $\theta < \bar{\theta}$, then there exists a separating equilibrium such that*

(i) *the principal offers a menu of contracts (h_1, h_2),*

(ii) *a rational agent chooses the contract h_1 and exerts a low effort*

(iii) *an overconfident agent chooses h_2 and exerts a high effort.*

The proposition states that for some sets of parameters, both overconfident and rational agents are hired in equilibrium, and types are revealed. In this equilibrium, overconfident agents exert a high effort and rational agents exert a low effort. The contracts offered by the principal aligns rational and overconfident agents' risk-taking incentives but leaves overconfident agent with more incentives to acquire information given their beliefs.

To complete Proposition 7, it should also be mentioned that there is no separating equilibrium such that rational agent exerts a high effort and overconfident agent exerts low effort, the reason being that any contract which provides a rational agent with incentives to exert a high effort, also provides an overconfident agent with incentives to exert a high effort.

The separating equilibrium described in Proposition 7 has implication for the comparison of performances and risk undertaken by rational and overconfident agent.

Corollary 8 *Assume that the separating equilibrium of Proposition 7 holds. Then,*

(i) *Overconfident agents perform better than rational agents (i.e., their expected return is higher than that of rational agents).*

(ii) *Overconfident agents take less risk than rational agents (i.e., the variance of return of overconfident agents is lower than that of rational agents)*

Part *(ii)* of the corollary contrasts with previously established results showing that overconfidence leads to investment strategies riskier than those of rational agents. However, these results were obtained in the context of agents trading for their own account. Corollary 8 shows how, in the context of delegated portfolio management, the contract offered by the principal influences investment incentives and the risk taking behavior of overconfident agents relative to rational agents. The principal can provide an overconfident agent with incentives to choose an effort level higher than that chosen by a rational agent. The direct consequence is that overconfident agents obtain a higher expected return and the variance of their return is lower than that of rational agents.

4.4 Discussion

It can be argued that our model represents a corner case since, given the optimal contract, the level of risk undertaken by the agent is just set by an exogenous constraint(i.e., $\bar{x}$). First, this is not true since the level of risk chosen by the agent is set by the contract proposed by the principal. For some (sub-optimal) contracts of other shapes, the agent would choose less risky investment strategies (i.e., $|x| < \bar{x}$). Second, if the principal is risk averse, the equilibrium level of risk $|x^*|$ is set endogenously at a lower level than $\bar{x}$. In such a case, the contract offered by the principal contains a cap on the compensation of the agent: there exists $\bar{R} > 1$ such that for any $R > \bar{R}$, $h(R) = h(\bar{R})$. This puts an upper-bound on the risk-taking incentives for the agents. The case of a risk-averse principal and observable types is analyzed formally in Appendix B. In particular, we show that the equilibrium level of risk undertaken by the agent is not always increasing with the level of overconfidence. Again, this is different from results obtained in the case of agents trading for own account.

5 Extension: A risk-neutral strategic agent.

In this section, we consider a situation in which the agent is risk neutral and has market power when trading the risky asset. As a consequence, overconfidence will have an impact on prices.

As in the previous sections, we assume that there is one risky asset. Its value V is $V_H > 1$ with probability 1/2, $V_L < 1$ with probability 1/2 and $E(V) = 1$. This asset is traded in a market similar to that described in Easley and O'Hara (1987). That is, three types of agents participate in the market: a market maker, noise traders and the informed agent. With equal probabilities, noise traders buy a small quantity (Z_1), buy a large quantity (Z_2), sell a low quantity ($-Z_1$) , or sell a large quantity ($-Z_2$). The timing of the trading game is the following. The market maker posts bid and ask prices for the various quantities submitted. With probability 1/2, an order is sent to the market maker by the informed agent, and with probability 1/2, it is sent by a noise trader. The market maker operates in a competitive environment. This implies that for any order (X) that he receives, the market maker expects zero profit from trade. After trading takes place, the value V of the asset is realized.

If the agent acquires information, then the precision of his signal and his beliefs are as in the previous sections.

5.1 Benchmark case: Trading for own account.

Assume that the informed agent's beliefs are common knowledge. When trading for his own account, the informed agent acts so as to maximize his expected profit. Denote $P(X)$ the price posted by the market for a trading order X. We have the following results about strategies and market prices.

Proposition 9 *Let $\rho = Z_2/Z_1$.*
(i) *If $\rho \geq \frac{3K}{3K-2k}$, then there exists a unique separating equilibrium: an agent who observes a signal s_H (s_L) always trades a quantity Z_2 (-Z_2). For all X different from Z_2 and $-Z_2$, $P(X) = 1$.*

$$P(Z_2) = \frac{1}{3}\left((1+k)V_H + (1-k)V_L + 1\right)$$

and

$$P(-Z_2) = \frac{1}{3}\left((1-k)V_H + (1+k)V_L + 1\right)$$

(ii) *If $\rho < \frac{3K}{3K-2k} < 3$ then there exists a unique pooling equilibrium: an agent receiving a signal s_H (s_L) trades a quantity Z_2 (-Z_2) with probability μ_K^* and trades a quantity Z_1 (-Z_1) with a probability $(1-\mu_K^*)$*

where μ_K^* is the unique positive solution of Equation (40) in Appendix. For all X different from Z_1, Z_2, $-Z_1$ and $-Z_2$, $P(X) = 1$.

$$P(Z_2) = \frac{\mu_K^*[(1+k)V_H + (1-k)V_L] + 1}{2\mu_K^* + 1}$$

$$P(Z_1) = \frac{(1-\mu_K^*)[(1+k)V_H + (1-k)V_L] + 1}{2(1-\mu_K^*) + 1}$$

$$P(-Z_1) = \frac{(1-\mu_K^*)[(1+k)V_L + (1-k)V_H] + 1}{2(1-\mu_K^*) + 1}$$

$$P(-Z_2) = \frac{\mu_K^*[(1+k)V_L + (1-k)V_H] + 1}{2\mu_K^* + 1}$$

The proposition states that overconfidence has an impact on the bid-ask spread when agents trade for their own account. First, if $\rho \in [3K/(3K-2k), 3]$ and the insider is rational then there is a strictly positive bid-ask spread for both small and large quantities. Conversely, if the insider is overconfident, he always trade large quantities, hence $P(Z_1) - P(-Z_1) = 0$. Second, if $\rho < 3K/(3K-2k)$, then there are positive bid-ask spreads for both small and large quantities. However, these spreads are different if the insider is rational or overconfident. The reason is that, due to their difference in beliefs, different types of traders use different probabilities for randomization between the small and the large quantity in equilibrium, i.e., the unique positive solution of Equation (40) is different if $K = k$ (rational insider) and if $K > k$ (overconfident insider).

5.2 Delegated portfolio management

As in Section 4, we assume that the principal does not have access to private information (hence, if he does not hire an agent his expected profit from trading is zero). However, we now assume that the agent is risk neutral and has limited liability.

First, we consider the case where overconfidence is observable. Two cases have to be distinguished. If $\rho > 3$, then there is no moral hazard on risk. Provided that the agent acquires information, the principal and the agent's incentives are aligned. This implies that the principal only faces moral hazard on effort when hiring an agent. Conversely, if $\rho \in [3K/(3K-2k), 3]$, then the principal also faces moral hazard on risk: the expected trading volume of the agent is too large (i.e., if maximizing his expected return, the agent trades a quantity $|Z_2|$ with probability 1.)

Note that, with respect to the Section 4, the moral hazard problem on risk is different. Here, the objective of the principal is to reduce the trading intensity of the agent. This is due to the fact that the agent is risk neutral and has limited liability. Conversely, in Section 4, the principal was aiming at increasing trading quantities.

The following proposition derives conditions under which contracts align overconfident agents' incentives with those of rational agents.

Proposition 10 (i) *Assume that* $\rho > 3$. *If* $0 < \bar{U} < \frac{Z_2}{2(1+k)}[(1+k)V_H + (1-k)V_L - 2]$ *and* $c < \frac{K}{1+K}\bar{U}$ *then there exists a first-best contract*

$$h_K^o(\pi) = \begin{cases} c & if \quad \pi \leq 0 \\ \frac{2\bar{U}}{1+K} + c - Z_2(V_H - P(Z_2)) + \pi & if \quad \pi > 0 \end{cases} \tag{11}$$

The agent acquires information and trades a quantity Z_2 $(-Z_2)$ *when observing* s_H (s_L).

(ii) *Assume that* $\rho < 3$. *There exists* $\bar{\bar{U}} > 0$ *such that if* $0 < \bar{U} < \bar{\bar{U}}$, *and* $c < \frac{K}{1+K}\bar{U}$ *then there exists a first best contract*

$$g_K(\pi) = \begin{cases} c & if \quad \pi \leq 0 \\ \frac{2\bar{U}}{1+K} + c & if \quad \pi > 0 \end{cases} \tag{12}$$

An agent receiving a signal s_H (s_L) *trades a quantity* Z_2 *(*$-Z_2$*) with probability* μ_k^* *and trades a quantity* Z_1 *(*$-Z_1$*) with a probability* $(1-\mu_k^*)$ *where* μ_k^* *is the unique positive solution of Equation (40) with* $K = k$.

The main difference with the results of Section 4 is that (under some conditions on parameters) there exist first-best contract either if the agent is rational or overconfident. Also the shape of the first-best contract changes with the market structure (i.e., the volume of liquidity trading). If $\rho > 3$, then, there exists a first-best contract of the shape described in Lemma 3. If $\rho < 3$, the objective of the principal is to reduce the risk-taking incentives of an overconfident agent. As a consequence, the first-best contract has the shape of a bonus-contract (i.e., a fixed salary c plus a fixed bonus in case of good performance ($\pi > 0$)). This result is along the lines of Palomino and Prat (2003).

A consequence of Proposition 10 is that (for sets of parameters such that first-best contracts exist), rational and overconfident agents choose the same investment strategy. This implies that overconfidence does not influence bid-ask spreads (hence prices) in the case of delegated portfolio management, although overconfident agents have market power.

From Equations (11) and (12), we also deduce the following result.

Corollary 11 *Assume that first-best contracts h^o_K, h^o_k, g_k and g_K exist. The principal is better-off hiring an overconfident agent.*

As in the model of the previous section, the principal is better off hiring an overconfident agent. However, this is not because an overconfident agent performs better but because he accepts cheaper contracts.

5.3 Overconfidence is not observable

We now assume that overconfidence is not observable and the principal and the market maker correctly believe (ex-ante) that the agent is overconfident with probability θ and rational with probability $1 - \theta$.

The principal faces the same trade-off as in the case of a price-taking agent: if $\rho > 3$ ($\rho < 3$) either the principal proposes the contract h^o_k (g_k) which is accepted by both types of agents (but leaves some rent to an overconfident agent), or the principal proposes the contract h^o_K (g_K) which will be rejected by the agent with probability $(1 - \theta)$, i.e., if the agent happens to be rational. Hence, we have the same type of result as in the previous section.

Proposition 12 *Assume that $\rho \geq 3$ ($\rho < 3$), overconfidence is not observable and for all $K \in (k, 1)$, first-best contracts h^o_K (g_K) exist. Then, there exists $\bar{\theta}$ such that if $\theta > \bar{\theta}$ the principal only offers the* *contract h^o_K (g_K). The contract is accepted by an overconfident agent and rejected by a rational one. If $\theta \leq \bar{\theta}$, the principal only offers the contract h^o_k (g_k) and the contract is accepted by both types of agents.*

As in the previous section, if the probability that the agent is overconfident is large, the principal screens agents with contracts and only hires overconfident agents. If the probability that the agent is overconfident is small, we have a pooling equilibrium. Here, again, overconfidence does not have any impact on prices.

If there are two levels of information, the situation is different. In equilibria in which rational and overconfident agents pool on the same effort level, then overconfidence will not have an impact on the bid-ask spread since the principal will offer contracts of the shape of h^o_j and g_j $(j = k, K)$ in these equilibria. Conversely, in equilibria in which rational and overconfident agents do not pool on the same effort level, overconfidence always has an impact on bid-ask spreads. We first show that such equilibria exist.

Proposition 13 *Assume that $\rho > 3$. Let*

$$\hat{h}_1(\pi) = \begin{cases} c_1 & \text{if} \quad \pi \leq 0 \\ \frac{2\bar{U}}{1+k_1} + c_1 - Z_2(V_H - \hat{P}(Z_2)) + \pi & \text{if} \quad \pi > 0 \end{cases} \tag{13}$$

and

$$\hat{h}_2(\pi) = \begin{cases} c_2 & \text{if} \quad \pi \leq 0 \\ \frac{2\bar{U}(1+K_1)}{(1+k_1)(1+K_2)} + c_2 - Z_2(V_H - \hat{P}(Z_2)) + \pi & \text{if} \quad \pi > 0 \end{cases} \tag{14}$$

with

$$\hat{P}(Z_2) = \frac{2}{3}[\theta E_{k_2}(V|s_H) + (1-\theta)E_{k_1}(V|s_H)] + 1$$

There exists $\bar{\theta}$, $\bar{c}_1$, $\bar{c}_2$, $\bar{\delta}$, and $\bar{K}_2 < 1$ such that if $c_1 < \bar{c}_1$, $c_2 - c_1 < \bar{c}_2$, $k_2 - k_1 < \delta$, $K_2 > \bar{K}$ and $\theta < \bar{\theta}$, then there exists a separating equilibrium such that

(i) *the principal offers the menu of contracts $(\hat{h}_1, \hat{h}_2)$,*

(ii) *a rational agent chooses the contract* $\hat{h}_1$ *and exerts a low effort.*

(iii) *an overconfident agent chooses* $\hat{h}_2$ *and exerts a high effort.*

For sets of parameters such that the separating equilibrium exist, overconfidence has an impact of prices. An informed trading order comes with probability θ from an overconfident and with probability $1-\theta$ from a rational agent. As a consequence, the expected value of the asset conditional on receiving an order Z_2 is

$$E(V|Z_2) = \frac{2}{3}[\theta E_{k_2}(V|s_H) + (1-\theta)E_{k_1}(V|s_H)] + 1$$

Since market makers operate in a competitive environment, $E(V|Z_2) = \hat{P}(Z_2)$. This implies that overconfidence has an impact on price. However, this is not due to trading intensity. This is due to over-acquisition of information by overconfident agents.

6 Conclusion

We have studied models of delegated portfolio management in which a risk neutral principal hires an agent who is either rational or overconfident.

When overconfidence is observable, we have derived conditions under which the contract proposed to the agents is first-best. This implies that the optimal contract fully aligns overconfident agents' incentives to invest with those of rational agents. As a consequence, overconfident and rational agents perform equally well and overconfidence does not have any impact on prices.

When overconfidence is not observable, we have derived conditions under which there exists a separating equilibrium such that the principal offers a menu of contracts, rational and overconfident agents choose different contracts, rational agents exert a low effort while overconfident agent exert a high effort. In this situation, overconfident managers may perform better than rational managers and take less risk and, if they have market power, they will have an impact on prices

These results have consequences for the analysis of the price impact of overconfidence. If overconfidence can be detected before proposing compensation contracts (i.e., overconfidence is observable), then contracts can align overconfident agents' risk taking incentives with those of rational agents. This implies that overconfidence has no impact on price. If overconfidence is not observable, it will have an impact on price if it leads to over-acquisition of information by overconfident money managers.

Our article extends the current literature on the performance and risk taking behavior of overconfident agents in financial markets. In particular, it shows the difference between comparing the performance of agents trading for their own account and comparing performances in the context of delegated portfolio management.

Also, our results have implications for the debate on the origin of the very high trading volume in financial markets. Our results suggest that a principal offering the appropriate contract can mitigate agents' incentives to trade due to overconfidence. Hence, overconfidence is not a source of excessive trading volume in case of delegated portfolio management. This implies that the excessive trading volume observed in financial markets may have two sources: overconfidence in the case of agents trading for own account and, as suggested by Dow and Gorton (1997), agency problems in the case of delegated portfolio management.

Appendix A: Proofs

Proof of Proposition 1

Proof of (i) Assume that after paying a cost c, the agent receives a signal s_H.[6] He maximizes

$$U(W(x)) = \frac{1}{2(1-\gamma)} \left((1+K)[1-c+x(V_H-1)]^{1-\gamma} + (1-K)[1-c+x(V_L-1)]^{1-\gamma} \right)$$

The first-order condition of utility maximization yields

$$(1+K)(V_H-1)[1-c+x(V_H-1)]^{-\gamma} + (1-K)(V_L-1)[1-c+x(V_L-1)]^{-\gamma} = 0$$

[6]The proof for the case $s = s_L$ is identical.

This is equivalent to

$$x = x(K, c, s_H) = \frac{(A_H(K) - 1)(1 - c)}{(V_H - 1) + A_H(K)(1 - V_L)}$$

It follows that the expected utility of the agent before receiving his signal is

$$U = \frac{1}{2(1-\gamma)}(1-c)^{1-\gamma}[U_o(K, s_H) + U_o(K, s_L)]$$

where $U_o(K, s_H)$ and $U_o(K, s_L)$ are given by Equation (3).

The agent acquires information if $U > 1$. This is equivalent to

$$c < 1 - \left(\frac{2}{U_o(K, s_H) + U_o(K, s_L)}\right)^{1/(1-\gamma)},$$

Proof of (ii) The expected return of the agent is

$$\begin{aligned} R(K) = 1 &+ \tfrac{(1+k)}{4}[x(K, c, s_H)(V_H - 1) + x(K, c, s_L)(V_L - 1)] \\ &+ \tfrac{(1-k)}{4}[x(K, c, s_H)(V_L - 1) + x(K, c, s_L)(V_H - 1)] \end{aligned}$$

Since A_H and A_L are increasing and decreasing in K, respectively. This implies that $x(K, c, s_H)$ and $x(K, c, S_L)$ are increasing and decreasing in K, respectively. Hence, $R(K)$ is increasing in K.

Proof of (iii) Given that $(V_H + V_L)/2 = 1$, we have $1 - V_L = V_H - 1$, and $x(K, c, s_L) = -x(K, c, s_H)$. The variance of the return is then

$$Var_K = x_H^2(1 - k^2)(V_H - 1)^2$$

where x_H stands for $x(K, c, s_H)$. Given that $A_H(K)$ is increasing in K, x_H is increasing in K. Hence, Var_K in increasing in K. □

Proof of Lemma 3: Assume that a contract $f(R)$ is optimal. This contract induces an agent who has acquired information to choose an investment strategy $(x_f(s_L), x_f(s_H))$. The expected revenue of the principal is then

$$\begin{aligned} Rev(f) = &\tfrac{1-k}{4}\,(2 + x_f(s_L)(V_H - 1) + x_f(s_H)(V_L - 1) - f[1 + x_f(s_L)(V_H - 1)] - f[1 + x_f(s_H)(V_L - 1)]) \\ &+ \tfrac{1+k}{4}\,(2 + x_f(s_L)(V_L - 1) + x_f(s_H)(V_H - 1) - f[1 + x_f(s_L)(V_L - 1)] - f[1 + x_f(s_H)(V_H - 1)]) \end{aligned}$$

Step 1: If $x_f(s_L) \neq -\bar{x}$ or $x_f(s_H) \neq \bar{x}$, then f is not an optimal contract.

Proof: From the assumption that the unconditional distribution of the risk asset is V_H with probability $1/2$, V_L with probability $1/2$ and $E(V) = 1$, we deduce that $V_H - 1 = 1 - V_L$. Let $r = (V_H - 1) = (1 - V_L)$. Since $x_f(s_L) \geq -\bar{x}$, $x_f(s_H) \leq \bar{x}$, we have

$$Rev(f) \leq \frac{1-k}{4}(2 - 2\bar{x}r - f[1 + x_f(s_L)r] - f[1 - x_f(s_H)r]) + \frac{1+k}{4}(2 + 2\bar{x}r - f[1 - x_f(s_L)r] - f[1 + x_f(s_H)r])$$

Now, consider the following contract:

$$g_1(r) = \begin{cases} \text{Min}\,(f[1 + x_f(s_L)r], f[1 - x_f(s_H)r]) & \text{if} \quad R \leq 1 \\ \text{Min}\,(f[1 + x_f(s_H)r], f[1 - x_f(s_L)r]) & \text{if} \quad R > 1 \end{cases}$$

With such a contract, $x_f(s_L) = -\bar{x}$ and $x_f(s_H) = \bar{x}$ are optimal strategies for the agent, and the expected compensation paid by the principal is smaller with g_1 than with f. Furthermore, any contract which pays the same expected compensation as g_1 but induces $x_f(s_L) > -\bar{x}$ or $x_f(s_H) < \bar{x}$ is dominated by g_1. This implies that any optimal contract must satisfy $x_f(s_L) = -\bar{x}$ or $x_f(s_H) = \bar{x}$.

Step 2: Step 1 implies that $x_f(s_L)(V_H - 1) = x_f(s_H)(V_H - 1) = -\bar{x}r$ and $x_f(s_H)(V_H - 1) = x_f(s_L)(V_L - 1) = \bar{x}r$. From Step 1, we also deduce that given any optimal contract, the return realized by an agent is $1 + \bar{x}r$ with probability $(1 + k)/2$ and $1 - \bar{x}r$ with probability $(1 - k)/2$. It implies that the set of optimal contract contains a contract of the shape

$$g_2(r) \begin{cases} X_0 & \text{if} \quad R \leq 1 \\ X_1 & \text{if} \quad R > 1 \end{cases}$$

(As mentioned in Step 1, with such a contract, $x_f(s_L) = -\bar{x}$ and $x_f(s_H) = \bar{x}$ are optimal strategies for the agent.) Taking $\alpha_0 = X_0$ and $\alpha_1 + \beta R = X_1$, we deduce that the set of optimal contract contains a contract of the shape of h.

Proof of (ii) We have already proved that an optimal contract induces $x(s_L) = -\bar{x}$ and $x(s_H) = \bar{x}$.

Here is a direct proof that if the principal chose a contract of the shape of h, then the agent chooses $x(s_L) = -\bar{x}$ and $x(s_H) = \bar{x}$.

The revenue of the principal is Given that $h(R)$ is constant if $R \leq 1$ and increasing in R if $R > 1$, it is straightforward that the agent will trade a positive quantity when receiving a signal s_H and will trade a negative quantity when receiving a signal s_L. Therefore, if observing s_H, the expected utility of the agent is

$$E_K[U(h^*)|s_H] = \frac{1-K}{2}\frac{\alpha_0^{1-\gamma}}{(1-\gamma)} + \frac{1+K}{2}\frac{(\alpha_1 + \beta x(V_H - 1))^{1-\gamma}}{(1-\gamma)}$$

and if observing s_L, the expected utility of the agent is

$$E_K[U(h^*)|s_L] = \frac{1-K}{2}\frac{\alpha_0^{1-\gamma}}{(1-\gamma)} + \frac{1+K}{2}\frac{(\alpha_1 + \beta x(V_L - 1))^{1-\gamma}}{(1-\gamma)}$$

$E_K[U(h^*)|s_H]$ and $E_K[U(h^*)|s_L]$ are increasing and decreasing in x, respectively. Hence, the agent chooses $x(s_H) = \bar{x}$ and $x(s_L) = -\bar{x}$. □

Proof of Proposition 4:

Proof of (i): The proof is divided in three steps.

Step 1: Assume that the agent exerts effort, then the contract

$$h(R) = \begin{cases} \alpha_0^* & \text{if} \quad R \leq 1 \\ \alpha_1^* + R & \text{if} \quad R > 1 \end{cases}$$

with

$$M(k, K) = \left(\frac{(1-k)(1+K)}{(1+k)(1-K)}\right)^{1/\gamma} \tag{15}$$

$$\alpha_1^* = c - 1 - \bar{x}(V_H - 1) + \left(\frac{2(1-\gamma)\bar{U}M(k,K)^{1-\gamma}}{1 - K + (1+K)M(k,K)^{1-\gamma}}\right)^{1/(1-\gamma)} \tag{16}$$

$$\alpha_0^* = c + \frac{\alpha_1^* + \bar{x}(V_H - 1) + 1 - c}{M(k,K)} \tag{17}$$

maximizes the principal's expected revenue and leaves no rent to the agent.

Proof: Let $\bar{r} = \bar{x}(V_H - 1) = \bar{x}(1 - V_L)$. From Step 1, we deduce that the objective of the principal is to maximize

$$\frac{1-k}{2}[-\alpha_0+1-\bar{r}]-\frac{1+k}{2}\alpha_1 \tag{18}$$

subject to

$$\frac{1-K}{2(1-\gamma)}(\alpha_0-c)^{1-\gamma}+\frac{1+K}{291-\gamma)}(\alpha_1+1+\bar{r}-c)^{1-\gamma}\geq \bar{U} \tag{19}$$

The first order condition of revenue maximization for the principal yields

$$\frac{(1-k)(\alpha_0-c)^{\gamma}}{(1-K)}=\frac{(1+k)(\alpha_1+1+\bar{r}-c)^{\gamma}}{(1+K)} \tag{20}$$

and the constraint (19) is binding. The system of equations (20) and constraint (19) binding has a unique solution (α_0^*, α_1^*).

Step 2: Assume that the contract h^* is proposed. Then, there exists $\bar{c}>0$ such that if $c<\bar{c}$, then the agent exerts effort.

Proof: The agent exerts effort if

$$\frac{1}{2}\left[U\left(\alpha_0^*\right)+U(\alpha_1^*+1+\bar{r})\right]<\frac{(1-K)}{2}U\left(\alpha_0^*-c\right)+\frac{1+K}{2}U(\alpha_1^*+1+\bar{r}-c) \tag{21}$$

Let

$$Z=\left(\frac{2(1-\gamma)\bar{U}M(k,K)^{1-\gamma}}{1-K+(1+K)M(k,K)^{1-\gamma}}\right)^{1/(1-\gamma)} \tag{22}$$

Using Equation (22), Inequality (21) can be rewritten as

$$\frac{1}{2}\left[U\left(\frac{Z}{M(k,K)}+c\right)+U(Z+c)\right]<\frac{(1-K)}{2}U\left(\frac{Z}{M(k,K)}\right)+\frac{(1+K)}{2}U(Z) \tag{23}$$

If $\bar{U}_K>0$ then $Z>0$. Given, that $K>k>0$, $M(k,K)>1$. Therefore, given that U is concave,

$$\frac{1}{2}\left[U\left(\frac{Z}{M(k,K)}\right)+U(Z)\right]<\frac{(1-K)}{2}U\left(\frac{Z}{M(k,K)}\right)+\frac{(1+K)}{2}U(Z) \tag{24}$$

Given that Z is increasing in $\bar{U}_K$, We deduce that there exists $\bar{c}_1(\bar{U})>0$ such that if $c<\bar{c}_1(\bar{U})$, we have the desired result.

Step 3: There exists $\bar{\bar{U}}$ such that if $\bar{U} < \bar{\bar{U}}$, then, the expected revenue of the principal from hiring an agent and proposing the contract h_K^* is strictly positive.

Proof: The expected revenue of the principal is

$$Rev = \frac{(1-k)}{2}\left(-c - \frac{Z}{M(k,K)} + 1 - \bar{r}\right) + \frac{1+k}{2}\left(1 + \bar{r} - c - Z\right)$$

The principal hires an agent if $Rev \geq 0$. This is equivalent

$$c < 1 + k\bar{r} - \frac{Z}{2}\left(1 + k + \frac{1-k}{M(k,K)}\right) \tag{25}$$

If Z is small then the RHS of this inequality is strictly positive. Denote $\bar{\bar{c}}(\bar{U})$ the RHS of (25) as a function of $\bar{U}$. Given that Z is increasing in $\bar{U}$, there exists $\bar{U}^o > 0$, then the such that if $\bar{U} < \bar{U}^o$ then $\bar{\bar{c}}(\bar{U}) > 0$ and if $c < \bar{\bar{c}}(\bar{U})$ then $Rev > 0$.

Hence, if $\bar{U} < \bar{U}^o$ and taking $\bar{c}(\bar{U}) = Min(\bar{c}_1(\bar{U}), \bar{\bar{c}}(\bar{U}))$, we have the desired result.

Proof of (ii) the objective of the principal is to maximize

$$\frac{1-k}{2}[-\alpha_0 + 1 - \bar{r}] - \frac{1+k}{2}\alpha_1 \tag{26}$$

subject to

$$\frac{1-k}{2(1-\gamma)}(\alpha_0 - c)^{1-\gamma} + \frac{1+k}{2(1-\gamma)}\left(\alpha_1 + 1 + \bar{r} - c\right)^{1-\gamma} \geq \bar{U} \tag{27}$$

and

$$\frac{1-k}{2}(\alpha_0 - c)^{1-\gamma} + \frac{1+k}{2}\left(\alpha_1 + 1 + \bar{r} - c\right)^{1-\gamma} \geq \frac{1}{2}(\alpha_0)^{1-\gamma} + \frac{1}{2}\left(\alpha_1 + 1 + \bar{r}\right)^{1-\gamma} \tag{28}$$

The first constraint is the participation constraint while the second is the IC constraint on effort. Note that, generically, only one constraint will be binding at the optimum. Using the proof of *(i)* Step 2, $M(k,k) = 1$ implies that for any $c > 0$, if the agent is rational then the optimum with Constraint (27) binding does not satisfy the IC Constraint (28). As a consequence, the solution of the maximization problem is such that the IC Constraint (28) is binding. The FOC is then,

$$\frac{(1-k)(\alpha_0-c)^{-\gamma}-\alpha_0^{-\gamma}}{(1+k)(\alpha_1+1+\bar{r}-c)^{-\gamma}-(\alpha_1+1+\bar{r})^{-\gamma}}=1 \tag{29}$$

The optimal contract is such that (α_0, α_1) is the solution of the system (29) and Constraint (28) binding.

$\square$.

Proof of Corollary 5:

The expected revenue of a principal hiring an overconfident agent is

$$Rev(K)=\frac{1-k}{2}(1-\bar{r}-\alpha_0^*(K))+\frac{1+k}{2}(-\alpha_1^*(K))$$

where $\alpha_0^*(K)$ and $\alpha_1^*(K)$ are given by Equations (17) and (16), respectively.

Differentiating Rev with respect to K and rearranging, we obtain

$$\frac{dRev}{dK}=\frac{M(k,K)^{1-\gamma}}{(1-g)}\frac{1-k+(1+k)M(k,K)}{1-K+(1+K)M(k,K)}>0$$

This implies that $Rev(k)<Rev(K)$. Denote (α_0^o, α_1^o) the solution of the system of equations (29) and (28). Now, using from the proof of Proposition 4, we know that

$$Rev(k)>\frac{1-k}{2}(1-\bar{r}-\alpha_0^o)+\frac{1+k}{2}(-\alpha_1^o)$$

Therefore, we have the desired result. $\square$.

Proof of Proposition 6: Denote $P^+(h)$ and $P^-(h)$ the payoff of the of the agent if $R>1$ and $R<1$, respectively if the contract h is proposed by the principal. For a given utility level U obtained by the agent with this contract, the IC constraint on effort is satisfied if

$$\frac{1}{2(1-\gamma)}\left[(P^-+c)^{(1-\gamma)}+(P^++c)^{(1-\gamma)}\right]<U \tag{30}$$

We deduce that we must have $P^+ > [(1-\gamma)U]^{1/(1-\gamma)}$. Let

$$F(P^+, U, k) = \frac{2U(1-\gamma) - (1+k)(P^+)^{(1-\gamma)}}{1-k} \tag{31}$$

Then, for any $P^+ > U^{1/(1-\gamma)}$, $\partial F/\partial k < 0$. Therefore, $F(P^+, U, k) > F(P^+, U, K)$ if $K > k$. This implies that any optimal contract h^o offered by the principal to rational agents is also proposed to overconfident agents, h^o is preferred by overconfident agents relative to h^*_K. Let $\bar{U}^o_K = E_K[U(h^o)]$. Then, we have $\bar{U}^o_K > \bar{U}$.

Let the contract $\hat{h}$ of the shape of (10) with $\beta = 1$ be a solution of

$$\text{Max}\frac{1-k}{2}(1-\bar{r}-\alpha_0) + \frac{1+k}{2}(-\alpha_1)$$

subject to $E_K[U(\hat{h})] \geq E_K[U(h^o)]$ and $E_k[U(\hat{h})] \leq E_k[U(h^o)]$.

Denote $\hat{Rev}$ the revenue of the principal if $\hat{h}$ is chosen by overconfident agents. It is straightforward that $\hat{Rev} < Rev(K)$. Let

$$Rev^o = \frac{1-k}{2}(1-\bar{r}-\alpha^o_0) + \frac{1+k}{2}(-\alpha^o_1)$$

and

$$\bar{\theta} = \frac{Rev^o}{Rev(K) + Rev^o - \hat{Rev}}$$

Then, the principal is better off offering only the contract h^*_K rather than the pair of contracts $(h^o, \hat{h})$.

□

Proof of Proposition 7: Let $\bar{R} = 1 + \bar{r}$. A separating equilibrium $(h_1(R|\alpha_{0,1}, \alpha_{1,1}), h_2(R|\alpha_{0,2}, \alpha_{1,2}))$ must satisfy the following conditions:

1) An overconfident agent choosing the contract $h_2(R|\alpha_{0,2}, \alpha_{1,2})$ is better off exerting a high effort (i.e., paying c_2) than exerting no effort, i.e.,

$$\frac{1-K_2}{2}(\alpha_{0,2} - c_2)^{1-\gamma} + \frac{1+K_2}{2}(\alpha_{1,2} + \bar{R} - c_2)^{1-\gamma} \geq \frac{1}{2}\left(\alpha_{0,2}^{1-\gamma} + (\alpha_{1,2} + \bar{R})^{1-\gamma}\right) \tag{32}$$

2) An overconfident agent choosing the contract $h_2(R|\alpha_{0,2},\alpha_{1,2})$ is better off exerting a high effort (i.e., paying c_2) than exerting a low effort (i.e. paying a cost c_1), i.e.,

$$\frac{1-K_2}{2}(\alpha_{0,2}-c_2)^{1-\gamma}+\frac{1+K_2}{2}(\alpha_{1,2}+\bar{R}-c_2)^{1-\gamma} \geq \frac{1-K_1}{2}(\alpha_{0,2}-c_1)^{1-\gamma}+\frac{1+K_1}{2}(\alpha_{1,2}+\bar{R}-c_1)^{1-\gamma} \quad (33)$$

3) An overconfident agent choosing the contract $h_2(R|\alpha_{0,2},\alpha_{1,2})$ and exerting a high effort is better off than choosing the contract $h_1(R|\alpha_{0,1},\alpha_{1,1})$ and exerting a low effort, i.e.,

$$\frac{1-K_2}{2}(\alpha_{0,2}-c_2)^{1-\gamma}+\frac{1+K_2}{2}(\alpha_{1,2}+\bar{R}-c_2)^{1-\gamma} \geq \frac{1-K_1}{2}(\alpha_{0,1}-c_1)^{1-\gamma}+\frac{1+K_1}{2}(\alpha_{1,1}+\bar{R}-c_1)^{1-\gamma} \quad (34)$$

4) An overconfident agent choosing the contract $h_2(R|\alpha_{0,2},\alpha_{1,2})$ and exerting a high effort is better off than choosing the contract $h_1(R|\alpha_{0,1},\alpha_{1,1})$ and exerting no effort, i.e.,

$$\frac{1-K_2}{2}(\alpha_{0,2}-c_2)^{1-\gamma}+\frac{1+K_2}{2}(\alpha_{1,2}+\bar{R}-c_2)^{1-\gamma} \geq \frac{1}{2}\left(\alpha_{0,1}^{1-\gamma}+(\alpha_{1,1}+\bar{R})^{1-\gamma}\right) \quad (35)$$

5) A rational agent choosing the contract $h_1(R|\alpha_{0,1},\alpha_{1,1})$ is better off exerting a low effort than exerting no effort, i.e.,

$$\frac{1-k_1}{2}(\alpha_{0,1}-c_1)^{1-\gamma}+\frac{1+k_1}{2}(\alpha_{1,1}+\bar{R}-c_1)^{1-\gamma} \geq \frac{1}{2}\left(\alpha_{0,1}^{1-\gamma}+(\alpha_{1,1}+\bar{R})^{1-\gamma}\right) \quad (36)$$

6) A rational agent choosing the contract $h_1(R|\alpha_{0,1},\alpha_{1,1})$ and exerting low effort is better off than choosing the contract $h_2(R|\alpha_{0,2},\alpha_{1,2})$ and exerting no effort, i.e.,

$$\frac{1-k_1}{2}(\alpha_{0,1}-c_1)^{1-\gamma}+\frac{1+k_1}{2}(\alpha_{1,1}+\bar{R}-c_1)^{1-\gamma} \geq \frac{1}{2}\left(\alpha_{0,2}^{1-\gamma}+(\alpha_{1,2}+\bar{R})^{1-\gamma}\right) \quad (37)$$

7) A rational agent choosing the contract $h_1(R|\alpha_{0,1},\alpha_{1,1})$ and exerting low effort is better off than choosing the contract $h_2(R|\alpha_{0,2},\alpha_{1,2})$ and exerting a low effort, i.e.,

$$\frac{1-k_1}{2}(\alpha_{0,1}-c_1)^{1-\gamma}+\frac{1+k_1}{2}(\alpha_{1,1}+\bar{R}-c_1)^{1-\gamma} \geq \frac{1-k_1}{2}(\alpha_{0,2}-c_1)^{1-\gamma}+\frac{1+k_1}{2}(\alpha_{1,2}+\bar{R}-c_1)^{1-\gamma} \quad (38)$$

8) A rational agent choosing the contract $h_1(R|\alpha_{0,1},\alpha_{1,1})$ and exerting low effort is better off than choosing the contract $h_2(R|\alpha_{0,2},\alpha_{1,2})$ and exerting a high effort (i.e., paying c_2), i.e.,

$$\frac{1-k_1}{2}(\alpha_{0,1}-c_1)^{1-\gamma}+\frac{1+k_1}{2}(\alpha_{1,1}+\bar{R}-c_1)^{1-\gamma} \geq \frac{1-k_2}{2}(\alpha_{0,2}-c_2)^{1-\gamma}+\frac{1+k_2}{2}(\alpha_2+\bar{R}-c_2)^{1-\gamma} \quad (39)$$

A separating equilibrium is then a pair of contracts $h_i(R|\alpha_{0,i}\alpha_{1,i})$ $(i = 1, 2)$ that maximizes the revenue of the principal and satisfies constraints (32)-(39).

Step 1: Let the contract $h_i^o(.|\alpha_{0,i}^o, \alpha_{1,i}^o)$ be a solution of the system made of Equation (29) and Constraint (28) binding, with $k = k_i$. Furthermore, assume that parameters k_1, k_2, c_1, c_2 are such that $Rev(h_1^o) > Rev(h_2^o)$. For any given difference $c_2 - c_1$, this will happen if $k_2 - k_1$ is small enough. In other words, there exists $\delta(c_2 - c_1)$ such that if $k_2 - k_1 < \delta(c_2 - c_1)$, then $Rev(h_1^o) > Rev(h_2^o)$. For such parameters, if overconfidence is observable and the principal faces a rational agent, then the contract h_1^o will be proposed by the principal.

Let

$$U_1^{K_1} = \frac{1 - K_1}{2(1-\gamma)}(\alpha_{0,1}^o - c_1)^{1-\gamma} + \frac{1 + K_1}{2(1-\gamma)}(\alpha_{1,1}^o + \bar{R} - c_1)^{1-\gamma}$$

That is, $U_1^{K_1}$ represents the expected utility of an overconfident agent if he chooses h_1^o and exerts a low effort level.

Let $\hat{h}_2(.|\hat{\alpha}_{0,2}, \hat{\alpha}_{1,2})$ be a solution of the program the (5)- (9) with $K = K_2$ and $U_1^{K_1}$ substituting $\bar{U}$. From the proof of Proposition 4, we know that for any c_2, there exists $\bar{K}_2 < 1$ such that if $K_2 > \bar{K}_2$ then a solution exists such that the *(i)* IC constraint on effort is not binding and *(ii)* this solution is such that $\hat{\alpha}_{0,2} - c_2 < \alpha_0^o - c_1$ and $\hat{\alpha}_{1,2} - c_2 > \alpha_1^o - c_1$.

By construction, if the pair of contracts $(h_1^o, \hat{h}_2)$ is proposed then Constraints (32), (34), and (36) are satisfied. Furthermore, $U_1^{K_1} > U_1^{k_1}$. Since $U_1^{k_1}$ is larger than the RHS of Inequality (35) by definition of h_1^o, it follows that Constraint (35) is satisfied since its LHS is equal to $U_1^{K_1}$ by definition of $\hat{h}_2$.

Now, From the proof of Proposition 4, we know that if $K_2 > \bar{K}_2$, then $\frac{\hat{\alpha}_{0,2} - c_2)}{\hat{\alpha}_{1,2} + \bar{R} - c_2} = \frac{1}{M(k_2, K_2)}$. Hence, the ratio $\frac{\hat{\alpha}_{0,2} - c_2)}{\hat{\alpha}_{1,2} + \bar{R} - c_2}$ is decreasing in K_2.

By continuity arguments, we deduce that if c_1 and $c_2 - c_1$ are small enough, and K_2 is large enough, then Constraints (33), (37), (38), and (39) are satisfied.

Step 2: For this menu of contracts to be an equilibrium, it must also be the case that the principal does not have incentives to deviate from this equilibrium.

A first possible deviation for the principal is to offer only a first-best contract with high effort for overconfident agents. (Obviously, this contract yields a higher revenue that the contract $\hat{h}_2$.) However, using the same argument as in the proof of Corollary 5, if θ is small enough, this deviation is not profitable. Similarly, if θ is small enough, offering only the first-best contract with low effort for overconfident agents is not a profitable deviation either.

Hence, the only possibly profitable deviations for the principal are such that they induce agents to choose the same effort level.

The most profitable deviation such that both types of agents choose the high effort level is such that the contract proposed to rational agents is $h_2^o(.|\alpha_{0,2}^o, \alpha_{1,2}^o)$. Given that $Rev(h_1^o) > Rev(h_2^o)$, we deduce that if θ is small enough, then this deviation is not profitable.

Finally, assume that the principal offers a pair of contracts such that both types of agents choose the low effort level. Obviously, one of the proposed contract is $h_1^o(.|\alpha_{0,1}^o, \alpha_{1,1}^o)$. If c_1 is small enough, then, there is a contract which solves the program the (5)- (9) with $K = K_1$ and $U_1^{K_1}$ substituting $\bar{U}$ and the IC constraint on effort is not binding. Denote h_1 this contract. For the principal, the pair of contract (h_1^o, h_1) is the revenue maximizing pair of contracts such that both types of agents choose a low effort level. The pair of contracts $(h_1^o, \hat{h}_2)$ is preferred to (h_1^o, h_1) if $Rev(\hat{h}_2) > Rev(h_1)$.

We now show that if $c_2 - c_1$ is small enough then $Rev(\hat{h}_2) > Rev(h_1)$. Assume that $c_2 = c_1$. Then, from Corollary 5, we deduce that in such a case $Rev(\hat{h}_2) > Rev(h_1)$. Therefore, by continuity, if $c_2 - c_1$ is small enough then $Rev(\hat{h}_2) > Rev(h_1)$. This completes the proof. □

Proof of Corollary 8: The expected return of an agent paying an effort cost c_i $(i = 1, 2)$ is

$$E(R|k_i) = 1 + \frac{\bar{x}}{2}k_i(V_H - V_L)$$

which is an increasing function of k_i.

The variance of the return of an agent paying an effort cost c_i $(i = 1, 2)$ is

$$Var(R|k_i) = \frac{\bar{x}^2}{2}\left((1 + k_i)(V_H - 1)^2 + (1 - k_i)(V_L - 1)^2 - \frac{1}{2}k_i^2(V_H - V_L)^2\right)$$

Given that $(V_H + V_L)/2 = 1$, $\frac{Var(R|k_i)}{dk_i} < 0$ is equivalent to $k_i > 0$. Hence, $Var(R|k_2) < Var(R|k_1)$.

□

Proof of Proposition 9: Assume that the agent observes s_H. (The proof for the case s_L is similar). First, it is straightforward for any quantity different from Z_1, Z_2 the market makers knows that he is facing an insider. Hence, for any X different from Z_1, Z_2, he sets $P(X) = V_H$. Second, if the market maker anticipates the insider to always trade a quantity Z_2, then he sets $P(Z_1) = 1$ and

$$P(Z_2) = \frac{\mu_K[(1+k)V_H + (1-k)V_L] + 1}{2\mu_K + 1}$$

Therefore, for a separating equilibrium to exist, it must be the case that

$$(E_K(V|S_H) - P(Z_2))Z_2 > (E_K(V|S_H) - P(Z_1))Z_1$$

Using the assumption that $(V_H + V_L)/2 = 1$, this last inequality is equivalent to

$$\frac{Z_2}{Z_1} > \frac{3K}{3K - 2k}$$

If this inequality does not hold then the only equilibria are of pooling type. Assume that the insider when observing s_H chooses trades a quantity Z_2 with probability μ and Z_1 with probability $(1-\mu)$. A pooling equilibrium must satisfy the following three conditions:

$$P(Z_2, \mu) = \frac{\mu_K[(1+k)V_H + (1-k)V_L] + 1}{2\mu_K + 1}$$

$$P(Z_1, \mu) = \frac{(1-\mu_K)[(1+k)V_H + (1-k)V_L] + 1}{2(1-\mu_K) + 1}$$

$$Z_2\left(E_K(V|s_H) - P(Z_2, \mu)\right) = Z_1\left(E_K(V|s_H) - P(Z_1, \mu)\right)$$

This last equation is then equivalent to

$$4\mu^2(K-k)(\rho-1) + 2\mu[(K-k)(2-3\rho) + K(1+\rho)] + K(1-3\rho) = 0 \tag{40}$$

where $\rho = Z_2/Z_1$.

This completes the proof. □.

Proof of Proposition 10: Assume that $\rho \geq 3$.

Step 1: The set of first-best contracts contains a contract of the shape of (11).

Proof: Assume that a contract f is first-best. Let $\Pi^+ = Z_2(V_H - P(Z_2)) = Z_2(P(-Z_2) - V_L)$ and $\Pi^- = Z_2(V_L - P(Z_2)) = Z_2(P(-Z_2) - V_H)$. Then, we have

$$\frac{1-K}{2} f(\Pi^-) + \frac{1+K}{2} f(\Pi^+) - c = \bar{U}$$

and

$$\frac{1-K}{2} f(\Pi^-) + \frac{1+K}{2} f(\Pi^+) - c \geq \frac{1}{2}[f(\Pi^-) + f(\Pi^+)]$$

Denote $Rev(f)$ the revenue of the principal generated by f, i.e.,

$$Rev(f) = \frac{1-k}{2}(\Pi^- - f(\Pi^-)) + \frac{1+k}{2}(\Pi^+ - f(\Pi^+))$$

It is straightforward that the contract

$$h(\pi) = \begin{cases} f(\Pi^-) & \text{if} \quad \pi \leq 0 \\ f(\Pi^+) - \Pi^+ + \pi & \text{if} \quad \pi > 0 \end{cases}$$

is also first best.

Step 2: A first best contract of the shape of h has $h(\pi) = c$ if $\pi < 0$.

Proof: We have

$$\frac{1-K}{2} h(\Pi^-) + \frac{1+K}{2} h(\Pi^+) = \bar{U} + c$$

This is equivalent to

$$h(\Pi^-) = \frac{2(\bar{U} + c)}{1-K} - \frac{1+K}{1-K} h(\Pi^+)$$

Therefore, if h is proposed, then the revenue of the principal is

$$Rev(h) = \frac{1-k}{2}\left[\Pi^- - \left(\frac{2(\bar{U}+c)}{1-K} - \frac{1+K}{1-K} h(\Pi^+)\right)\right] + \frac{1+k}{2}(\Pi^+ - h(\Pi^+))$$

Given that $k < K$, $Rev(h)$ is increasing in $h(\Pi^+)$. As a consequence, the revenue of the principal is maximized for $h(\Pi^-) = c$. Hence, the optimal contract is given by (11).

Assume that $\rho < 3$. The proof is identical to the proof of the case $\rho \geq 3$ taking a contract of the shape of (12) instead of (11). With a contract of the shape of (12), the agent is indifferent between trading small and large quantities. As a consequence, he is willing to randomize and trade a large quantity with probability μ_k^* and a small quantity with probability $1 - \mu_k^*$. In such a case, prices are as given in Proposition 9 *(ii)*. This completes the proof. □

Proof of Proposition 12:

Step 1: Consider any pair of contracts (h', h_k^o) such that rational agents choose h_k^o and overconfident agents choose h' if the pair of contracts (h', h_k^o) is offered. Then the principal is better off offering only h_K^o (which is accepted by both types of agents.)

Proof: Let

$$\hat{U} = \frac{1-K}{2}c + \frac{1+K}{2}\left(\frac{2\bar{U}}{1+k} + c\right)$$

then $\hat{U}$ is the utility an overconfident agent derives from accepting h_k^o. If the pair of contracts (h', h_k^0) is offered, h' must provide the at least a utility level $\hat{U}$ to an overconfident agent. Now, from the proof of Proposition 10, we know that the revenue of the principal is maximized if $h(\Pi^+)$ is maximized (i.e., $h(\Pi^-) = c$). This implies that the contract that provides a utility level $\hat{U}$ to overconfident agents, satisfies the IC constraints and maximizes the revenue of the principal is h_k^o.

Step 2: Let $\bar{\theta} = \frac{Rev(h_k^o)}{Rev(h_K^o)}$. From the previous paragraph, we deduce that if $\theta < \bar{\theta}$, the principal only offers h_k^0 and if $\theta > \bar{\theta}$, the principal only offers h_K^o.

The proof for the case $\rho < 3$ is identical. □

Proof of Proposition 13: First, the 8 conditions of Proposition 7 must hold.

If $c_2 < \frac{1+K_1}{1+k_1}\bar{U}$, then Condition 1) holds.

If $c_2 - c_1 < \frac{K_2-K_1}{1+K_2}\frac{1+K_1}{1+k_1}\bar{U}$ then Condition 2) holds.

By construction, an overconfident agent is indifferent between $\hat{h}_2$ with a high effort and $\hat{h}_1$ with a low effort. As a consequence, Condition 3) holds.

If $c_1 < \frac{K_1}{1+k_1}\bar{U}$, then Condition 4) holds.

The contract $\hat{h}_1$ is the first best contract for rational agents when only the low level of information is available. From Proposition 10, we know that of c_1 is small enough, the rational agent will not deviate to not exert effort. Hence, Condition 5) holds.

If $c_2 < \frac{K_1-k_1}{1+k_1}\bar{U}$, then Condition 6) holds.

If $c_2 < \frac{K_1}{1+k_1}\bar{U}$, then Condition 7) holds.

Finally, if $\frac{1+K_2}{1+K_1} > \frac{1+k_2}{1+k_1}$ then Condition 8) holds.

Now, we must show that there are no profitable deviations for the principal. From the proof of Proposition 10, we know that there exists $\bar{\theta}$ such that if $\theta < \bar{\theta}$, then a pooling equilibrium is preferred to a separating one in which only overconfident agents accept the contract or a separating one in which two different contracts are accepted by rational and overconfident agents, but they exert the same effort level. Hence, we only need that there are no profitable deviation such that agents pool on the same contract and exert the same effort.

First, consider the case of pooling on the low effort level. A sufficient condition for this deviation not to be profitable is that $Rev(\hat{h}_2) > Rev(\hat{h}_1)$. Let $\Pi^+ = Z_2(V_H - P(Z_2)) = Z_2(P(-Z_2) - V_L)$ and $\Pi^- = Z_2(V_L - P(Z_2)) = Z_2(P(-Z_2) - V_H)$. Given the assumption about the distribution of uncertainty, $\Pi^+ = -\Pi^-$. Then, $Rev(\hat{h}_2) > Rev(\hat{h}_1)$ is equivalent to

$$c_2 - c_1 < (k_2 - k_1)\Pi^+ + \bar{U}\left(1 - \frac{1+k_2}{1+k_1}\frac{1+K_1}{1+K_2}\right)$$

Second, consider the case of pooling on the high effort level. It is straightforward that the most profitable of such deviation is the contract $h^0_{k_2}$ with $c = c_2$. The equilibrium pair of contracts is preferred if $\theta Rev(\hat{h}_2) + (1-\theta)Rev(\hat{h}_1) > Rev(h^o_{k_2})$. This is equivalent to

$$c_2 - c_1 > (k_2 - k_1)\Pi^+ - \bar{U}\frac{\theta}{1-\theta}\left(1 - \frac{1+k_2}{1+k_1}\frac{1+K_1}{1+K_2}\right)$$

The RHS of this last inequality becomes negative when k_2 converges to k_1. This completes the proof. □

Appendix B: A risk-averse principal

We assume that the principle is risk averse with utility

$$U(W) = \frac{W^{1-\gamma_p}}{1-\gamma_p}$$

with $\gamma_p \in (0,1)$, and that overconfidence is observable. (Assumptions concerning the agent are the same as those of Section 3).

Proposition 14 *The set of optimal contracts contains a contract of the shape*

$$h(.|\alpha_0, \alpha_1, \bar{R}) = \begin{cases} \alpha_0 & \textit{if} \quad R \leq 1 \\ \alpha_1 + R & \textit{if} \quad R \in (1, \bar{R}] \\ \alpha_1 + \bar{R} & \textit{if} \quad R > \bar{R} \end{cases} \tag{41}$$

Proof: Let f be an optimal contract and $r = (V_H - 1) = (1 - V_L)$. Denote $x^*_f(.)$, the optimal strategy of the agent given f. From the distribution of V, it is straightforward that $x^*_f(s_H) = -x^*_f(s_L) = x^*_f$. Take $\bar{R} = 1 + x^*_f r$, $\alpha_0 = f(1 - x^* r)$ and $\alpha_1 = f(1 + x^*_f r) - 1 + x^*_f r$. Proceeding as in the proof of Lemma 3, part *(ii)*, we obtain that if h is offered by the principal, then the agent choose the investment strategy x^*_f. Now, given α_0 and α_1, the expected utilities of the principal and the agents are identical with h and f. Hence, h is an optimal contract. □

Proposition 15 *Assume that the agent is overconfident (i.e., $K > k$). There exists $\bar{\bar{U}}$ and $\bar{c}$ such that if $\bar{U} < \bar{\bar{U}}$ and $c < \bar{c}$, then there exists an optimal contract of the shape of (41) with*

$$\bar{R} = \frac{(1-c)[1 - T(k,\gamma_p)] + Z[T(k,\gamma_p) - T(K,\gamma)]}{1 + T(k,\gamma_p)}$$

$$\alpha_1^* = Z\frac{1+T(K,\gamma)}{1+T(k,\gamma_p)} - \frac{2(1-c)}{1+T(k,\gamma_p)}$$

$$\alpha_0^* = c + ZT(K,\gamma_p)$$

$$T(k,\gamma_p) = \left(\frac{1-k}{1+k}\right)^{1/\gamma_p}$$

$$Z = \left(\frac{2(1-\gamma)\bar{U}}{(1-K)T(K,\gamma)^{1-\gamma}+1+K}\right)^{\frac{1}{1-\gamma}}$$

Proof : Let $r = (V_H - 1) = (1 - V_L)$. The problem of the principal is to maximize

$$\frac{(1-k)}{2(1-\gamma_p)}[1-xr-h(1-xr)]^{1-\gamma_p} + \frac{(1+k)}{2(1-\gamma_p)}[1+xr-h(1+xr)]^{1-\gamma_p} \tag{42}$$

subject to constraints (6)- (9).

Step 1: For a given utility level of the agent, a necessary condition for the maximization of the expected utility of the principal is

$$\frac{1-xr-h(1-xr)}{1+xr-h(1+xr)} = T(k,\gamma_p)$$

Proof: the FOC condition of maximization of (42) subject to a participation constraint yields the desired result.

Step 2: For a given utility level of the principal, a necessary condition for the maximization of the expected utility of the agent is

$$\frac{h(1-xr)}{h(1+xr)} = T(K,\gamma)$$

Proof: the FOC condition of maximization of the expected utility of the agent subject to a participation constraint yields the desired result.

Step 3: If a contract of the shape of (41) is proposed by the principal, the agent chooses $x(s_H) = -x(s_L) = \bar{R}/r = x^*$

Proof: Similar to the proof of Lemma 3, part *(ii).*

Step 4: α_0^*,α_1^* and x^* are the solution of the system of equations

$$\frac{1-x^*r-\alpha_0}{-\alpha_1}=T(k,\gamma_p)$$

$$\frac{\alpha_0-c}{1+x^*r+\alpha_1-c}=T(K,\gamma)$$

$$\frac{(1-K)}{2(1-\gamma)}[\alpha_0-c]^{1-\gamma}+\frac{(1+K)}{2(1-\gamma)}[1+x^*r+\alpha_1-c]^{1-\gamma}=\bar{U}$$

Given that $K>k$, if $\bar{U}$ and c are small enough, then α_0^*,α_1^* and x^* satisfy

$$\frac{(1-K)}{2(1-\gamma)}[\alpha_0^*-c]^{1-\gamma}+\frac{(1+K)}{2(1-\gamma)}[1+x^*r+\alpha_1^*-c]^{1-\gamma}\geq\frac{1}{2(1-\gamma)}\left([\alpha_0^*]^{1-\gamma}+[1+x^*r+\alpha_1^*]^{1-\gamma}\right)$$

$$\frac{1-k}{2(1-\gamma_p)}[1-x^*r-\alpha_0^*]^{1-\gamma_p}+\frac{1+k}{2(1-\gamma_p)}(-\alpha_1^*)^{1-\gamma_p}\geq 1$$

(This last condition means that the principal is better off hiring an agent who invests only in the risk-free asset. Given that there is no risk premium in absence of private information, investing all his wealth in the risk-free asset is optimal.)

□

With the contract (41), the agent invests $x^*=\bar{R}/r$ in the risky asset if observing s_H and $-x^*$ if observing s_L. We now show that x^* may be decreasing in the level of overconfidence.

Proposition 16 *For any set of parameters such that the contract (41) is optimal, there exists $\bar{K}<1$ such that if $K>\bar{K}$, $\bar{R}$ is decreasing in K. In other words, the level of risk undertaken by the agent is decreasing in the level of overconfidence.*

Proof: Let $F(K)=((1-K)T(K,\gamma)+1+K)^{-\frac{1}{1-\gamma}}$ and $G(K)=(T(k,\gamma_p)-T(K,\gamma))F(K)$. Then, $\frac{d\bar{R}}{dK}>0$ is equivalent to $\frac{dG(K)}{dK}>0$. We have

$$\frac{dG(K)}{dK}=-\frac{F(K)^{2-\gamma}}{(1-\gamma)}[T(k,\gamma_p)-T(K,\gamma)]\left(1-T(K,\gamma)^{1-\gamma}-\frac{2(1-\gamma)(1-K)}{\gamma(1+K)^2}T(K,1)T(K,\gamma)^{1-\gamma}\right)+\frac{2T(K,\gamma)^{1-\gamma}F(K)}{\gamma(1+K)^2}.$$

We deduce that

$$\text{Lim}_{K\to 1}\frac{dG(K)}{dK} = -\frac{F(K)^{2-\gamma}}{(1-\gamma)}T(k,\gamma_p) < 0$$

So, by continuity, we have the desired result. $\square$

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