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FACTOR SHADOW PRICES IN DISTORTED OPEN ECONOMIES

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ABSTRACT

In a competitive constant-returns small open economy world prices are the appropriate shadow prices for traded goods in public sector cost-benefit analysis. If there are at least as many factors of production as there are goods produced and traded, it is straightforward to derive 'foreign-exchange-equivalent' factor shadow prices. Bertrand (American Economic Review, 1979) has argued that the case of more goods being produced and traded than there are factors (referred to as 'diversification' below) is of greater empirical relevance, and that in this case it is impossible to define factor shadow prices. In this paper, I argue that this view is incorrect, in the sense that if the economic forces which bring about the diversification of production are consistently and explicitly modelled, just enough information is available to define the factor shadow prices. The cases considered are: (i) a small open economy in which the government's objective is to influence the distribution of income, in which case diversification could only result from an irrational use of the policy tools available to the government; (ii) a small open economy in which there are political constraints on production levels in some activities, in which case world prices are not the correct shadow prices for the constrained activities; and (iii) a closed economy (which can be thought of as a model in which world prices are endogenous) in which the shadow prices of goods are not world prices.

JEL classification: 021, 024, 321, 323, 400

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NON-TECHNICAL SUMMARY

The proposition, originally advanced by Ian Little and James Mirrlees, that in an open economy the social opportunity cost of a good is the 'world price' at which it can be bought from or sold to foreigners, is valid in a wide range of circumstances. Several economists have discussed the implications of the Little-Mirrlees proposition for the calculations of the value of non-traded inputs, and derived 'foreign-exchange-equivalent factor shadow prices'. However, these derivations have made the technical assumption that the number of traded goods does not exceed the number of non-traded inputs, and Trent Bertrand has argued that the cases where this assumption does not hold are particularly relevant, and that factor shadow prices cannot, in principle, be derived in these cases. I argue in this paper that Bertrand's conclusion is too pessimistic.

There are two plausible cases in which more goods will be produced than there are factors of production: a small open economy in which diversification of production is itself an objective of government policy; or a large open economy or closed economy in which there is consumer demand for a wide range of goods. In the first case, in order to derive shadow prices for inputs it is necessary to introduce an explicit description of government policy into the model and assume that government policy is applied consistently. In the second case, an explicit treatment of consumer demand is needed.

The common factor in the two cases is the need to model explicitly the factors bringing about diversification of production. In a small open economy in which there are no constraints on government policy other than a set of fixed market distortions, it is straightforward to derive foreign-exchange-equivalent factor shadow prices. As we move away from that simple case, the calculation of shadow prices for both goods and factors requires more detailed information about the structure of the economy, but it is the usefulness of shadow prices rather than their existence which is called into question.

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1.0 INTRODUCTION

Much attention in the theory of cost-benefit analysis has been given to the proposition advanced by Little and Mirrlees (1969) that in an open economy, even in the presence of distortions, world prices measure the social opportunity costs of goods and so are the appropriate shadow prices for project appraisal. The proposition is valid in a wide range of circumstances, the key requirements being that world prices be exogenous and that the distortions do not include quantitative restrictions on foreign trade. The question of what are the correct shadow prices for non-traded factors of production when traded goods are valued at world prices has been addressed by Diamond and Mirrlees (1976), Findlay and Wellisz (1976), Srinivasan and Bhagwati (1978), Bhagwati and Wan (1979), and Bertrand (1979).

For an economy with constant returns to scale in which there are two traded goods and two factors, "foreign-exchange-equivalent" factor shadow prices are derived by Findlay and Wellisz, and further discussed by Srinivasan and Bhagwati. The vector ω of factor shadow prices is defined by

$$(1) \qquad \pi = \omega A$$

where π is the vector of world prices, A is the matrix whose typical entry a_{ij} is the quantity of factor i used in the production of a unit of good j, the input coefficients being chosen by private producers who face domestic prices for goods which are different from world prices.

The justification for the prices defined by (1) being shadow prices is as follows. Consider a competitive economy in which n goods are produced using m factors of production. Let the endowment of factors of production be given by the m-dimensional vector v, and the prices faced by producers for goods be given by the n-dimensional vector p. Then the output vector x and the vector w of competitive factor prices are related to v and p through the m+n equilibrium equations

$$(2) \qquad Ax = v$$

$$(3) \qquad A'w = p$$

whose differentials are

$$(4) \qquad Adx + Sdw = dv$$

$$(5) \qquad A'dw = dp$$

where

$$S = x'\partial A/\partial w = [\sum_k x_k \partial a_{ik}/\partial w_j]$$

and where the envelope property $dA'w = 0$ is used in deriving

(5). If $m \geq n$, the equations (2) and (3) determine x and w as functions of p and v.

If $m = n$ (≥ 2) and if the input coefficient matrix A is non-singular, it is straightforward to invert (4) and (5) and thereby to establish that the supply function $x(p,v)$ has the property that $\partial x/\partial v = A$.

Suppose now that world prices are the correct measure of the social opportunity cost of goods. The social opportunity cost of factors used in the public sector is then the value of output lost in consequence from the competitive production sector, that output being, of course, evaluated at world prices. Thus the shadow prices ω of factors are related to the shadow prices (world prices) π of goods by

$$(6) \qquad \omega = \pi(\partial x/\partial v)$$

which implies (and is implied by) equation (1). Thus the Findlay-Wellisz-Srinivasan-Bhagwati shadow factor prices value at world prices the marginal contributions of factors to national product at the distorted equilibrium.

In fact, in this model, equation (1) is a reflection of a more general proposition established by Diamond and Mirrlees (1976), that at the optimal level of public production in a distorted economy with constant returns to scale in the private sector, public sector shadow prices should have the property that private production activities make zero profits when evaluated at shadow prices. Clearly, this property is satisfied by the shadow prices $\{\pi, \omega\}$ above, for (1) is a statement of the Diamond-Mirrlees property. (The above derivation, though for a more restricted class of models, is more general than that of Diamond and Mirrlees in the specific respect that it does not require that public production be at an optimal level.) Indeed, since (1) and (6) are equivalent one could say that the Diamond-Mirrlees property <u>defines</u> the relation between goods and factor shadow prices.

(It may also be observed that the factor shadow prices derived above will be "stationary"; that is, invariant with respect to changes in the vector of factor supplies available to the private sector, for competitive factor market prices $w(p,v)$ have the local factor-price-equalisation property that $\partial w/\partial v = 0$, so A, which is a function of w, is locally constant.)

When the number m of factors exceeds the number n of goods, the derivation of factor shadow prices is a little more complex. Now the equation (1) is not <u>sufficient</u> to determine ω. However, from the fact that $x(p,v)$ and $w(p,v)$ are determined by the equations (2) and (3) it is possible, following Diewert and Woodland (1977), to show that if the matrix $AA' - S$ is nonsingular with inverse Z, then the system (4)-(5) can be inverted to give

$$\partial x/\partial v = [A'ZA]^{-1}A'Z$$

so that the factor shadow prices are well defined by (6) and satisfy (1), the Diamond-Mirrlees property. (Because this economy does not satisfy a factor-price-equalisation theorem, the factor shadow prices are not "stationary". As Bhagwati and Wan (1979) have discussed, factor shadow prices will in general vary as the public sector draws factors away from the private sector, even with goods prices constant.)

It is in the case of $n>m$ that more substantial difficulties seem to arise. The function $x(p,v)$ does not exist; for many different values of x may be compatible with a particular p,v. The simplest illustrative example is the two-good one-factor Ricardian model in which the transformation curve is a straight line, showing that the production vector is indeterminate if relative goods prices happen to coincide with the slope of the transformation line. In such a case, $\partial x/\partial v$ is not well defined: and the marginal cost to the economy of an input cannot be evaluated in the absence of knowledge of what its marginal effect is on the pattern of production. This problem is extensively discussed by Bertrand (1979) who argues that this is a particularly relevant case and that the problems which arise in this case are fatal to the attempt to derive satisfactory factor shadow prices in an open economy. This paper is addressed to that and related problems, and argues that Bertrand's conclusion is too pessimistic.

The central argument is that if world prices are exogenous, that is, if we are looking at a small open economy, then one would normally expect only m goods to be produced. The number of goods produced will exceed the number of factors available only if government policy is deliberately geared towards causing such diversification. This is the case described by Bertrand as having "greater relevance ... price interferences [being] often instituted so as to permit domestic viability for industries not competitive at world standards and to tax sectors efficient by world standards" (p.909-910). In such circumstances, however, one must explicitly model the objectives and instruments of tax policy, and this will introduce enough extra information into the model to pin down the factor shadow prices. As Drèze and Stern (1983) have emphasised, we need to assume that government policy other than public production plans need to be optimally chosen (subject to whatever constraints exist on such policies) if shadow prices for public production are to be well defined.

The other type of explanation that can be offered for diversification with n>m is that goods prices are endogenous, and will adjust so that the world as a whole produces all goods demanded by consumers. The simplest model in which this can arise is one of a closed economy (which can be thought of as a world with no trade restrictions and factor price equalisation) in which there are more goods than factors and the distortions are consumer taxes. Here the solution to the apparent indeterminacy of factor shadow prices is that the determination of both goods and factor shadow prices requires explicit modelling of consumer behaviour, which is not needed in a small open economy.

The common characteristic of these two cases is that the economic forces which bring about the diversification must be modelled.

2.0 THE SMALL OPEN ECONOMY

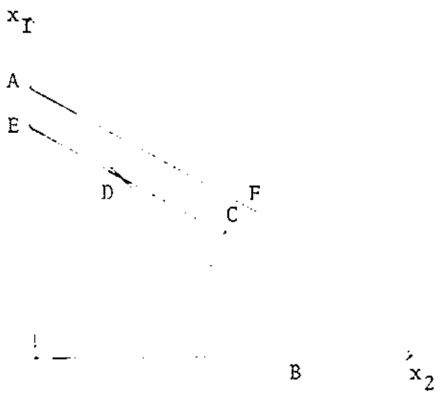

Figure 1

Consider first a very simple case, which, though of an essentially trivial nature in the present context, serves to illustrate the general principle applicable to more economically interesting cases. Figure 1 illustrates a Ricardian model with two goods and one factor, and a single consumer. If the domestic price ratio is equal to the slope of the transformation line AB, all points on the line are equally profitable. Suppose, however, that the world relative price of good 2 is lower than the domestic price, the different domestic price ratio being the consequence of a subsidy to production of good 2. Consumers face world prices. Then consumer welfare is increased the higher is production of good 1, and maximised by specialisation in good 1. When production takes place at D, consumption will be at C, where D and C have the same value at world prices, and C maximises utility on the budget line through D and C. The value of factor income in terms of good 1 is given by the point A where the transformation line intersects the x_1 axis, and AE represents the lump-sum tax which the government levies on the consumer in order to finance the subsidy to the production of x_2. There is a superior equilibrium in which production is at A, and consumption at F, so that production of the subsidised good has fallen to zero. That equilibrium is attainable simply by reducing the lump-sum tax to zero: with the consumer budget line being the line through AF, consumer demand is at F, and equilibrium requires that domestic production be at A.

This example is not very plausible economically because there is no reason why production of one good should be subsidised in a one-factor Ricardian model, but an analogous argument may be made in the more interesting case shown in

Figure 2, which essentially replicates Bertrand's Figure 2 (p.910).

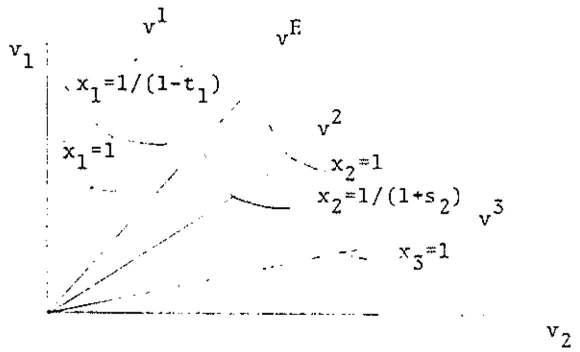

Figure 2

In a small open economy, three traded goods are produced using two factors of production. Figure 2 illustrates the unit value isoquants for the three goods. Units are chosen so that one unit of each good has unit value at world prices. Production of good 1 is taxed at the rate t_1 so that where isoquant $x_1=1$ represents the output level which has unit value at the world price, isoquant $x_1=1/(1-t_1)$ generates a unit value of output at the domestic price. Production of good 2 is subsidised at the rate s_2, and isoquants $x_2=1$ and $x_2=1/(1+s_2)$ represent unit value of output at domestic and world prices respectively of good 2. Production of good 3 takes place at the world price, and $x_3=1$ is the unit value isoquant.

In the absence of intervention, good 2 would not be produced, while the tax and subsidy rates have been chosen so that production of all three goods is competitive. The rays v^k show the proportions in which factors will be used in the respective sectors, and if the economy's endowment ratio of factors is inside the cone $v^1 0 v^3$, production of all three goods will be compatible with full employment, but the amounts of each good produced will be indeterminate. The effect of intervention on factor prices is to raise the relative price of factor 2.

In the particular case illustrated in Figure 2, in which v^E represents the factor endowment ratio, it would be compatible with full employment and competitive pricing to produce only goods 1 and 2, or only goods 1 and 3, though not to produce only goods 2 and 3. In the three-dimensional commodity space, the transformation surface has the property of being a ruled surface: to every set of domestic prices compatible with the production of all three goods (such as the set of domestic prices shown in Figure 2) there corresponds a straight line lying along the transformation surface.

Figure 3 illustrates the full three-dimensional transformation curve, and the straight line AB is the locus of points corresponding to the domestic prices of Figure 2: at A there is specialisation in goods 1 and 3, at B specialisation in goods 1 and 2, and any linear combination of the two points

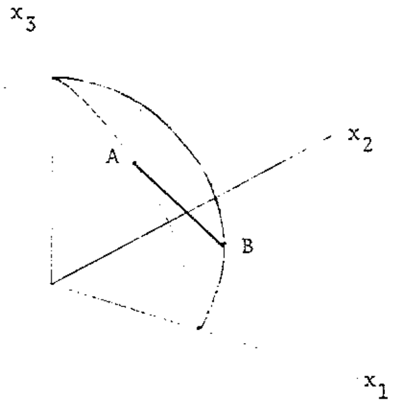

Figure 3

is compatible with full employment at the same prices. (Compare the line AB in Figure 1.) However, welfare is not constant with respect to changes in production plan, as is most easily seen by supposing, as in the Ricardian case of Figure 1, that there are no consumption distortions so that consumers prices are world prices and that there is a single representative consumer in the economy, with any surplus or deficit arising from the production taxes and subsidies being distributed in lump-sum fashion to the consumer. Then, if the plane through A representing world relative prices (not shown in the Figure) lies strictly above the rest of AB, it follows that consumer welfare is higher with production at A than with production anywhere else on AB. Optimal production of good 2 is zero.

The argument is essentially the same as in Figure 1. In the present case, however, point A is not the point at which the value of output is maximised at world prices, for there is still a tax on good 1: the value of output at world prices would be higher with more of good 1 and less of good 3 than are produced at A. But, in contrast with the Ricardian case, there is a possible rationale for intervention: the tax and subsidy shift the distribution of income in favour of the owners of factor 2. However, analogously to the earlier case, factor prices are independent of how much of each good is actually produced, and it is desirable to drive production of the subsidised good to zero.

To put the same point another way, there is no rationale for the subsidisation of good 2; for the same apparent policy objectives are better met by only taxing good 1.

Now once production has been driven to the point A in Figure 3 (or the point A in Figure 1) the same number of goods is being produced as there are factors of production, and there will be no difficulty about deriving foreign-exchange-equivalent shadow prices.

The general point is that we need to assume that existing government policies are optimally chosen (subject to whatever constraints there are on choice) in order to define shadow prices, and the general argument is as follows. Let there be n goods and m factors, where n>m. (The argument below can be extended to the case of $n \leq m$ with some modification to deal with the possibility that factor prices vary with public sector production but since the case of $n>m$ is the apparently problematic one, let us concentrate on that one.) Let $c(q,u)$ be the compensated demand function of the representative consumer, where c is an n-dimensional vector of goods consumed, q is the corresponding n-dimensional vector of domestic consumer prices for goods, and u is a measure of real income. Domestic private net outputs of goods are given by the n-dimensional vector x. The public sector may also engage in production and the net outputs of goods by the public sector are given by the n-dimensional vector g. World prices are fixed at π, and domestic consumer and producer prices are therefore fixed also if trade taxes and consumer taxes are given. Denote by p the n-dimensional vector of prices faced by domestic producers. In equilibrium, the value of consumption at world prices must be equal to the value of total production:

$$(7) \quad \pi c(q,u) = \pi(x + g)$$

The factor endowments of the economy are given by the m-dimensional vector v, and competitive factor prices are given by the m-dimensional vector w. Competitive producers faced by a constant returns technology choose cost-minimising input coefficients; and the typical entry a_{ij} of the mxn matrix A is the cost-minimising quantity of input i used in the production of a unit of good j. A will, in general, depend on w, and the competitive producer prices p will satisfy (3), which will, since $n>m$, define w as a function of p, so that w will be fixed. In competitive equilibrium, it is also required that all factors be fully employed. Let v^x be the factors used by private producers, so that

$$(8) \qquad Ax = v^x$$

and if v^g are the factors used in the public sector, full employment requires that

$$(9) \qquad v = v^x + v^g$$

Some aspects of the production side of this model have already been discussed in section 1: and (8) and (9) differ from (2) only in that we now allow for public sector factor use.

It is important to understand the connection between (7), which is a balance of payments equation, and the government's budget equation. Total consumer income m satisfies

$$\begin{aligned} m &= qc = (q - \pi)c + \pi(x + g) \\ (10) \qquad &= (q - \pi)c + (\pi - p)x + [\pi g - wv^g] + wv \end{aligned}$$

using (3) and (7)-(9). (Note how (3) and (8) give the standard result that in a constant returns economy the value of output to producers will be the same as the value of factor earnings.) The first three terms on the right of (10) are net government revenues from taxation of consumption, from taxation of private production, and from public production. Thus, consumer income is the sum of government revenue and factor incomes; so any government surplus or deficit from taxes and subsidies is distributed as a lump-sum to the consumer.

Suppose that consumer and producer taxes, and therefore prices, are fixed, and recall that factor prices also are determined by the fixed producer prices. Let public production and factor use also be constant. With more factors than goods, there will typically be multiple equilibria, since the output vector is not uniquely determined. Consider how the equilibrium changes with x. The variables which may change are u and x only. (7) and (8) require that such changes satisfy

(11) $(\pi c_u)du = \pi dx$

(12) $Adx = 0$

and the change will be welfare-improving if

(13) $du > 0$

Motzkin's theorem (see Mangasarian (1969, p.34) states that <u>either</u> there exists a solution x to the matrix inequalities $Ex >> 0$, $Fx \geq 0$, $Gx = 0$ (where the matrix E is non-vacuous) <u>or</u> there exist solutions $y^1 > 0$, $y^2 \geq 0$, y^3 to $y^1E + y^2F + y^3G = 0$, but not both. Thus there exist no du, dx satisfying (11)-(13) if and only if there exist scalars $\lambda > 0$, and μ, and an m-dimensional vector ω such that

(14) $\lambda = \mu(\pi c)$

(15) $\mu\pi = \omega A$

Equations (14) and (15) are conditions for optimal choice of u and x, and they have natural interpretations in terms of shadow prices. λ is the social value of utility, μ is the shadow price of foreign exchange, and (14) requires that the marginal benefit and cost of utility should be equated. (15) requires the equality of the social marginal benefit and cost of production of each good. (Note that if we replaced the equalities in (11) and (12) with inequalities permitting excess supply of foreign exchange and of factors, it would follow that $\mu \geqq 0$, and $\omega \geqq 0$.)

(14) implies that μ is in fact positive also, and without loss of generality μ can be set to 1 in (14) and (15). Then (15) asserts the existence of Findlay-Wellisz-Srinivasan--Bhagwati factor shadow prices, if no solutions to (11)-(13) exist, that is if the government has used its power of lump-sum distribution to ensure that the welfare-maximising equilibrium is chosen given the fixed prices, taxes, and public production.

It is now a simple matter to confirm that the shadow prices derived above are indeed appropriate for evaluating the public sector's use of factors (and that world prices are appropriate for the evaluation of goods production). Consider a perturbation of the equilibrium described by (3) and (7)-(10) by a change $\{dg, dv^g\}$ in public production. Assume that the government is optimally choosing among multiple equilibria so that (15) is satisfied (with $\mu = 1$). Differentiation of (7) gives

$$(16) \qquad \begin{aligned} (\pi c_u)du &= \pi dx + \pi dg = \omega A dx + \pi dg \\ &= \omega dv^x + \pi dg = \pi dg - \omega dv^g \end{aligned}$$

where we have used the differentials of (8) and (9) and the fact that prices are constant so that A is constant. Thus, since from (14) we know that (πc_u) is positive, it follows that a cost-benefit test in which the project $\{dg, dv^g\}$ is evaluated at the prices π for goods and ω for factors will be an appropriate test for the effect of the project on welfare.

The central point of the argument above is that the derivation of shadow prices for public production in the presence of a set of fixed distortions requires one to assume that the government is making appropriate use of those policy instruments which it does control. The importance to the theory of shadow prices of the assumptions made about the rationality of and constraints upon other public sector policies is the central theme of Drèze and Stern's recent survey.

Thus one interpretation of the problem identified by Bertrand in defining shadow prices is that the problem arises only because there is an unexplained implicit irrationality in government policy: if the objective of the government when it imposes distorting taxes is to influence factor prices and the distribution of income, then selection of the best among the continuum of equilibria which satisfy this objective will imply that shadow prices are well defined and will usually, though not necessarily, imply that production of all but m goods will be driven to zero.

However, another interpretation is that there are further constraints on government actions which have not yet been made explicit. Suppose, specifically, that the government actually wanted (for strategic or political or other reasons) to maintain a particular production level of good 2 in the three-good example above, and this was its reason for its distorting the price system so as to make production of all three goods possible. But then it is necessary to treat the output level of good 2 as explicitly constrained, and to evaluate the social cost of the public sector's use of factors by calculating the effect on output only of goods 1 and 3. This measure too will be well defined.

In general, suppose that the subset x^f of the private production vector is subject to a constraint that outputs must not fall below certain levels. The government might not be able to satisfy such a constraint simply through lump-sum distribution, and it might need direct control over output levels, although since production of all goods is equally profitable, there would be no cost to producers in complying with such a request. We have to add to (11)-(13) the inequality

$$(17) \qquad dx^f \gtrless 0$$

and Motzkin's theorem now implies that the assumption that the government is optimising with respect to its existing policy options is equivalent to the existence of a positive scalar λ, a non-negative n-dimensional vector η, a scalar μ, and an m-dimensional vector ω such that

$$(18) \qquad \lambda = \mu(\pi c_u)$$

$$(19) \qquad \eta + \mu\pi = \omega A$$

where η has the property that it has zero entries corresponding to the goods not in the constrained vector x^f, and non-negative entries corresponding to goods in x^f.

It is certainly true that factor shadow prices do not now satisfy (1); but this is for the simple reason that world prices are not goods shadow prices. The shadow prices of goods whose production levels are constrained have a component reflecting the social cost of that constraint. It remains the case, however, that factor shadow prices are well-defined if optimal use is made of the policies which are available to the government. In the example of Figure 2, if production of good 2 were constrained to be positive, the shadow prices of goods 1 and 3 would be their world prices, the factor shadow prices would be the prices at which production of goods 1 and 3 would just be profitable at world prices, given the distorting effect of the tax on good 1 on the choice of technique of production, and the shadow price of good 2 would actually exceed both the domestic price and the world price.

3.0 THE CLOSED ECONOMY

We now turn to the second broad case in which it seems plausible for there to be more goods produced than there are factors: the case where a combination of consumer demand for a wide range of goods and the endogeneity of world prices ensures that more goods are produced in this country than there are factors. The simplest case is the one identified by Bertrand where "common technologies across countries and free trade may lead to factor-price equalization, implying that every commodity can be produced in every country even without trade restrictions" (p.909). In fact this case can be modelled by treating the world as a closed economy.

The model is as in the previous section except that (7) is replaced by

$$(20) \qquad c(q,u) = x + g$$

Equations (3),(8) and (9) continue to hold.

The crucial difference is that, whereas in the previous case p, q and w were fixed by the assumption that world prices and taxes were fixed, now prices are endogenous. Let $q_i = (1+t_i)p_i$, where the consumer tax rates t_i are fixed. Changes du, dx, dp and dw are feasible for given v, g and v if

(21) $c_u du + CTdp = dx$

(22) $dp = A'dw$

(23) $Adx + Sdw = 0$

where S was defined at (5) above; and (21) and (22) combine to give

(24) $c_u du + CTA'dw + dx = 0$

By Motzkin's theorem, there exist no such changes which also satisfy

(25) $du > 0$

if and only if there exists a positive scalar λ, an n-dimensional vector k, and an m-dimensional vector ω such that

(26) $\lambda = kc_u$

(27) $k = \omega A$

(28) $kCTA' = -\omega S$

These equations like (14)-(15) are interpreted as optimality conditions. λ is the social value of utility, k are the shadow prices of goods, and ω the shadow prices of factors. (26) states that the marginal social value of utility should equal its social cost, (27) that the value of goods should equal their social cost of production, and (28) that the social costs and benefits of feasible price changes should be equal. (As in the previous case, replacing the equalities in (24) and (25) with the appropriate inequalities incorporating free disposal assumptions make k and ω non-negative, as the shadow price interpretation requires.)

As before, to suppose that (26)-(28) are not satisfied is to suppose that an equilibrium exists which is superior to the current equilibrium and which is attainable purely by a change in the government's lump-sum distribution.

Now with dg and dv non-zero, (21)-(23) become

$$c_u du + CTdp = dx + dg \quad (29)$$

$$dp = A'dw \quad (30)$$

$$Adx + Sdw = -dv^g \quad (31)$$

Using (26)-(28) we obtain

$$\begin{aligned}(kc_u)du &= kdx + kdg - kCTA'dw \\ &= \omega Adx + kdg + \omega Sdw \\ &= kdg - \omega dv^g \quad (32)\end{aligned}$$

and since kc_u is positive, it is confirmed that k and ω are the appropriate shadow prices for use in public sector cost-benefit analysis.

It follows from (27) that these shadow prices follow a Diamond-Mirrlees-Findlay-Wellisz-Srinivasan-Bhagwati rule analogous to (1) independently of the relative numbers of goods and factors. Of course, there is no question in this particular model of "world prices" being shadow prices for goods, for "world prices" do not exist, but the conclusion will generalise to any model in which prices are endogeneous. There is no difficulty about the goods prices being somehow insufficient to pin down the factor prices because once we leave the small open economy model, both sets of shadow prices are simultaneously and endogenously determined.

4.0 CONCLUSION

It remains only to repeat what was said at the end of the introduction. The apparent problems that diversification of production creates for the theory of shadow pricing cannot be sensibly discussed in isolation from the economic forces which bring about the diversification. The derivation of shadow prices requires the explicit incorporation of assumptions about the nature of and constraints upon government intervention, and also requires an explicit treatment of the determination of equilibrium prices when the determination of the market prices of goods is endogeneous.

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