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ABSTRACT

Optimal Dynamic Taxation with Indivisible Labour*

In this Paper, we take the field of optimal dynamic taxation further in two directions. Using a model with invisible labour, as in Hansen (1986) and Rogerson (1988), we first explore the short-run dynamics of the capital-income tax, particularly whether the tax, under the second-best programme, goes to zero in finite or infinite time. We derive two classes of preferences for which the optimal capital tax reaches zero in a finite time. Second, we ask what preference structures will leave labour untaxed at all times? An exponential preference structure is derived, for which labour is untaxed both in the short and long run.

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1. INTRODUCTION

The theory of optimal dynamic taxation is primarily concerned with the question: how should a government arrange taxes over time? A central result is that a capital tax should be zero in a steady state (Judd, 1985 and Chamley, 1986). This is a robust second-best result,[1] and is verified under various frameworks.[2] However, there is little analysis of the optimal taxes out of a steady state. An exception is Chamley (1986) who shows that the capital tax reaches zero in finite time for an example where the utility function is iso-elastic in consumption. Jones, Manuelli, and Rossi (1993) have computed transition paths of taxes numerically. [3] Also, little attention has been given to the labour tax. There is in principle nothing that precludes the government from taxing capital and labour in the beginning to the optimization period, accumulating assets, and levy no taxes in the steady state.[4]

In this paper we take Chamley's (1986) analysis further in mainly two directions. First we explore the dynamics of the tax paths, particularly whether capital income tax goes to zero in finite or infinite time. The answer to this question depends on the preference structure. We derive two classes of preferences for which the optimal capital tax approaches zero in a finite time (these are necessary and sufficient). Second, we explore seriously the optimal labour-tax implications of the Chamley model. What preference structures will leave labour untaxed at all times? The issue is important because one may question whether the labour income tax is also zero in the steady state like capital tax. If the government can accumulate capital, it could raise all necessary

[1] There are two ways of introducing the second-best. The first is when the government has to raise an exogenously specified amount of revenue without recourse to lump-sum taxation. The second-best tax system then minimizes the distortions. The second alternative is to introduce heterogeneous individuals. The government then resorts to distortionary taxation for redistributive reasons. Chamley (1986) takes the first view, and Judd (1985) the second.

[2] See Atkeson et. al. (1999) and Renström (1999) for a surveys.

[3] Jones, Manuelli and Rossi (1993) address the issue of optimal labour taxation including physical and human capital. However, their analysis is mostly based on simulation with specific functional forms, and does not admit a closed form solution with a fairly general preference structure.

[4] Jones, Manuelli and Rossi (1997) address the issue of optimal labour taxation including human capital in addition to physical capital, so that the labour tax has an intertemporal distortion. They show that there are certain cases when the labour tax is zero in steady state. See also Reinhorn (2003) for a clarification of those results, in particular regarding interior solutions of a model with human capital in addition to physical.

revenues by taxing capital and labour at the beginning of the optimization period, and set all taxes zero at the steady state.

Both these questions are answered in a framework with indivisible labour supply, as in Hansen (1986) and Rogerson (1988). We choose a model with invisible labor for two reasons. First reason is macroeconomic realism. An indivisible-labour economy explains the business cycle stylized facts better than a divisible-labor model. The volatility of employment is better explained in a model with indivisible labour (see, Hansen (1986)) than divisible labour.[5] One may question whether there is any scope for government interference by optimal choice of capital and labor taxes as instruments. We find that the answer depends on the preference structure.

The second reason why we use a model with indivisible labour is modeling convenience. With indivisible labour, we can establish a connection between the household's demand for unemployment insurance, and the normality of leisure.[6] When leisure is normal, in order to raise the tax base, the fiscal authority would tax labour to induce the household to work harder. When labour supply is indivisible, and the leisure is normal, the individual buys insurance to equate the utility gain from not working to the utility cost of the insurance purchase. The novelty of our approach is that we derive an exponential class of preferences with non-separable leisure for which this insurance demand is zero. This means leisure is a *neutral* good and the immediate implication is the labour should remain untaxed. In this scenario, labour should be untaxed at all times. For the same exponential preferences, capital should be taxed at 100% for a finite time period, and then not taxed at all.

The paper is organized as follows. Section 2 outlines the economy. Section 3 presents the key optimal-tax results. Section 4 concludes.

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[5] Greenwood and Huffman (1996) find additional implications for the natural rate of unemployment using an indivisible-labour model. Mulligan (1999) finds that the optimal tax implications differ between indivisible labor and divisible-labour models.

[6] In Basu and Renström (2002) we analyzed the sign of the optimal labour tax in an indivisible-labour economy in relation to the household's insurance demand, and established a connection between insurance demand and normality of leisure.

2. THE ECONOMY

As in Hansen (1985) and Rogerson (1988), we consider an economy where labour supply is indivisible, so that individuals can work either full time, h_0 or not at all. In each period the household member participates in an employment lottery, choosing the probability of working. This makes her wage income uncertain. It is assumed that households have access to an actuary fair insurance market, where they can buy unemployment insurance. This makes the economy Pareto efficient in the absence of distortionary taxation. In this way we preserve the standard second-best framework: the underlying economy is Pareto efficient in the absence of government intervention and the government has only access to distortionary instruments, which are used so as to minimise these distortions.

2.1. *Individual Economic Behavior*

In each period the household member participates in an employment lottery. With probability $\alpha(t)$, the individual works full time, h_0, and with probability $(1-\alpha(t))$ the individual is unemployed. She has access to an insurance market where she buys unemployment insurance, $y(t)$. The household's consumption ($c^s(t)$) and asset accumulation ($\dot{a}^S(t)$) are thus potentially contingent on whether the household works (s=1) or not (s=2). There is no intrinsic uncertainty, which means that preferences and technology are non-stochastic. The household thus solves the following maximization problem:

$$(1) \quad J(a_0) \equiv \max_{c,y,\alpha} \int_0^\infty e^{-\theta t}\left[\alpha(t)u\left(c^1(t),1-h_0\right)+(1-\alpha(t))u\left(c^2(t),1\right)\right]dt$$

$$(2) \quad \dot{a}^1(t) = \rho(t)a(t) + \omega(t)h_0 - p(\alpha(t))y(t) - c^1(t)$$

$$(3) \quad \dot{a}^2(t) = \rho(t)a(t) + y(t) - p(\alpha(t))y(t) - c^2(t)$$

$$(4) \quad a(0) = a_0$$

where $a(t)$ equals the sum of outstanding public debt, $b(t)$, and the capital stock, $k(t)$, that earn the after-tax interest at rate $\rho(t)= (1-\tau^K(t))r(t)$, and $\omega(t)= (1-\tau^L(t))w(t)$ is the after tax wage; $r(t)$ and $w(t)$ are the rental- and wage-rates, respectively, $\tau^K(t)$ and $\tau^L(t)$ are the proportional tax rates on capital- and labour-income, respectively, and $p(\alpha(t))$ is the competitive price of insurance. The insurance company behaves competitively and maximizes the expected profit, $p(\alpha(t))y(t)$-$(1-\alpha(t))y(t)$, which gives rise to the zero-profit condition, $p(\alpha(t))=1-\alpha(t)$.[7]

Substituting the zero-profit condition into (2) and (3), the current value Hamiltonian of the representative household can be written as:

$$(5)\quad H=\alpha(t)u(c^1(t), 1-h_0) + (1-\alpha(t))u(c^2(t),1)$$
$$+ \alpha(t)\, q^1(t)\, [\rho(t)a(t)+\omega(t)h_0-(1-\alpha(t)y(t)-c^1(t)]$$
$$+ (1-\alpha(t))\, q^2(t)\, [\rho(t)a(t)+ \alpha(t)y(t) -c^2(t)]$$

The first order conditions are (subscripts denoting partial derivatives):

$$(6)\quad \frac{\partial H}{\partial c^1(t)} = u_c(c^1(t),1-h_0) - q^1(t) = 0$$

$$(7)\quad \frac{\partial H}{\partial c^2(t)} = u_c(c^2(t),1) - q^2(t) = 0$$

$$(8)\quad \frac{\partial H}{\partial y(t)} = q^1(t) - q^2(t) = 0$$

Using (6), (7) and (8) it follows that

[7] The household randomizes the labour supply decision in this setting by choosing a probability of work $\alpha(t)$. A realistic description of this arrangement is that the representative household consists of a family of N members. In each period the household decides the proportion, $\alpha(t)$, of members working. The labour supply is then $\alpha(t)h_0N$. The household can buy insurance on a competitive market to diversify the income uncertainty arising from $(1-\alpha(t))N$ of its members not working. After choosing the probability of work, $\alpha(t)$ the household is pre-committed to it, and cannot renege. The insurance company then charges the actuarially fair premium, 1-α. This rules out adverse selection in the model. The household then realizes that, when choosing work probability, the insurance premium is a linear function of the probability of its working.

(9) $u_c(c^1(t),1-h_0)=u_c(c^2(t),1)=q(t)$

which is a result of perfect insurance. In other words, by buying insurance, the individual equalizes the marginal utilities across states. Equation (9) gives the optimal time paths of state-contingent consumption, $c^1(q(t))$ and $c^2(q(t))$, as functions of the co-state variable $q(t)$. However, this does not necessarily imply that household will equalize consumption across states. For consumption equalization, one requires an additional restriction on the preferences that the utility function is additively separable between consumption and leisure, meaning $u_{c(1-h)}=0$. It turns out that without any such restriction on the preferences, the household will not choose to have full consumption insurance as in Hansen (1985). This can be seen from (6), (7), and (8). Since $1-h_0$ is not equal to unity, c^1 cannot equal c^2 unless $u_{c(1-h)}$ is equal to 0. Next, since $q^1(t)=q^2(t)$ it follows that the optimal asset holding decisions must satisfy:

(10a) $\dot{a}^1(t)=\dot{a}^2(t)$,

(10b) $\dot{q}(t)=(\theta-\rho(t))q(t)$.

An individual's asset accumulation is thus independent of her employment history. This implies that individuals starting with the same a_0 will have the same $a(t)$ at all t, regardless of their employment history. Substituting (10a) in (2) and (3) gives

(11) $y(t)=\omega(t)h_0+c^2(q(t))-c^1(q(t))$

Notice now that the household chooses full insurance if the optimal consumption bundles are such that $c^1(q(t))=c^2(q(t))$. In the absence of any restriction on $y(t)$, the household can choose to have positive, negative or zero insurance.[8] Finally, the optimal choice of $\alpha(t)$ must be such that

(12) $$\frac{\partial H}{\partial \alpha(t)}=u(c^1(q(t)),1-h_0)-u(c^2(q(t)),1)-q(t)\left[c^1(q(t))-c^2(q(t))-\omega(t)h_0\right]=0$$

which upon the use of (11) can be rewritten as:

[8] One needs to be careful about the non-negativity constraint on consumption while thinking about negative unemployment benefit. $y(t)$ can be negative as long as $c^2(t)$ is non-negative. We assume interior solutions, meaning $c^2(t)>0$.

(12') $u(c^2(q(t)),1) - u(c^1(q(t)),1-h_0) = q(t)y(t)$.

The household chooses to buy a positive insurance, $y(t)$, if the utility gain from not working balances the utility cost of the insurance purchase.

2.2. *Production*

There is large number of competitive firms in the economy each operating under the following constant returns-to-scale technology:

(13) $f(k(t), \alpha(t)h_0) = f_1 \cdot k(t) + f_2 \cdot \alpha(t)h_0$

2.3. *The government*

The government taxes labour and capital income to finance an exogenously specified sequence of public spending, $g(t)$, the use of which, is not explicitly modeled. It adjusts two tax rates, $\tau^L(t)$ and $\tau^K(t)$, continuously. The government is assumed to borrow and lend freely at the market rate of interest, $r(t)$. The government's budget constraint is, therefore, given by:

(14) $\dot{b} = r(t)b(t) - \tau^k(t)r(t)a(t) - \tau^L(t)w(t)\alpha(t)h_0 + g(t)$

with $b(0)=b_0$.

2.4. *Equilibrium*

The equilibrium is characterized by the following conditions:

(a) Facing $w(t)$, $r(t)$, $\tau^L(t)$, $\tau^K(t)$, the household chooses optimal sequences of $c(t)$, $a(t)$, $\alpha(t)$, $y(t)$ that solves the problem stated in (1), (2) and (3).[9]

[9] We assume no-Ponzi games.

(b) Given an exogenous steam of government spending, $g(t)$, the government pre-commits to a tax sequence, $\tau^L(t)$ and $\tau^K(t)$, and a debt sequence, $b(t)$, that satisfies the government budget constraint (14).

(c) Goods, labour, rental markets clear meaning

(15) $\dot{k}(t) = f(k(t), \alpha(t)h_0) - \alpha(t)c^1(t) - [1-\alpha(t)]c^2(t) - g(t)$,

(16) $\omega(t) = (1-\tau^L(t))f_2(k(t), \alpha(t)h_0)$,

(17) $\rho(t) = (1-\tau^K(t))f_1(k(t), \alpha(t)h_0)$,

Notice that the equilibrium level of employment, $h(t)$ ($\equiv \alpha(t)h_0$) is determined by the time path of the probability of work, $\alpha(t)$. The equilibrium time path of $\alpha(t)$ can be determined in two steps. First, using (12) one determines the market clearing after tax wage, $\omega(t)$ as a function of $q(t)$. Define that equilibrium wage function as:

(18) $\omega(t) = \Omega(q(t))$

Next, using (16) and (18), one can characterize the path of $\alpha(t)$ as a function of $k(t)$, $q(t)$, and $\tau^L(t)$ as follows:

(19) $\alpha(t) = \alpha(k(t), \tau^L(t), q(t))$

3. OPTIMAL DYNAMIC TAXATION

3.1. *Government's problem*

We now solve for the optimal tax problem for the government for this economy with indivisible labour. The government solves a Ramsey problem for pre-committed tax sequences, $\tau^L(t)$ and $\tau^K(t)$, that maximize the household's utility functional (1) subject to its own budget constraint (14), the economy-wide resource constraint (15), the first-order optimality conditions (9), (10b), and (12), and a no-confiscation constraint on capital income as follows:[10]

[10] No such confiscation constraint is relevant for labour income taxation because if labour income is confiscated by the government it is optimal for the household to set $\alpha(t)$=0 which means no production. On the other hand, in principle, the capital income can be confiscated and the government can eventually own all the capital to run production.

(20) $\rho(t) \geq 0$

Using (16) and (17), and the CRS property of the production function, the government's budget constraint, (14), can be rewritten as:

(21) $\dot{b}(t) = \rho(t)\, b(t) + \rho(t)\, k(t) + \alpha(t)\omega(t)\, h_0 - f\left(k(t), \alpha(t) h_0\right) + g(t)$

We may write the government's current value Hamiltonian as follows (ignoring the time indices from now on):

(22)
$$\begin{aligned} H^g &= \alpha u\,(c^1, 1-h_0) + (1-\alpha) u(c^2, 1) + \mu\left\{\rho\, b + \rho\, k + \alpha\omega h_0 - f\left(k, \alpha h_0\right) + g\right\} \\ &+ \lambda\left\{f\left(k, \alpha h_0\right) - \alpha c^1 - (1-\alpha) c^2 - g\right\} + \psi(\theta - \rho)\, q + \nu\rho \end{aligned}$$

In principle, the government faces the states, k, b and q, and chooses the controls ρ and τ^L. For algebraic convenience, we pose the government's problem as follows. The government chooses the controls ρ and α. Then using the equilibrium sequence of α as in (19), one can determine the optimal labour tax, τ^L. [11] Denoting $u^1 = u(c^1, 1-h_0)$ and $u^2 = u(c^2, 1)$, the first-order conditions facing the government are as follows:

(23) $$\frac{\partial H^g}{\partial \alpha} = u^1 - u^2 + \mu\left[\omega h_0 - f_2 . h_0\right] + \lambda\left[f_2 h_0 + c^2 - c^1\right] = 0$$

(24)
$$\begin{aligned} &\frac{\partial H^g}{\partial q} = -\dot{\psi} + \theta\psi \\ &\Rightarrow \dot{\psi} = \rho\psi - \alpha u_c^1 c_q^1 - (1-\alpha) u_c^2 c_q^2 - \mu\alpha\Omega'(q) h_0 + \lambda\left[\alpha c_q^1 + (1-\alpha) c_q^2\right] \end{aligned}$$

(25) $$\frac{\partial H^g}{\partial \rho} = \mu(b+k) - \psi q + \nu = 0$$

[11] It is straightforward to verify that for given k and q, $\alpha'(\tau^L) < 0$ and hence $\alpha(\cdot)$ can be inverted with respect to τ^L.

(26) $$\frac{\partial H^g}{\partial k} = \mu(\rho - f_1) + \lambda f_1 = \theta\lambda - \dot{\lambda}$$

(27) $$\frac{\partial H^g}{\partial b} = \mu(\rho - \theta) = -\dot{\mu}$$

3.2. *Preliminaries*

The multiplier ν in equation (25) is of importance in proving our results. As Chamley (1986) notices ν(0)>0, meaning that the no-confiscation constraint binds at date zero. This can be seen from (25), since the multiplier associated with public debt is negative and the multiplier associated with the jump variable $q(0)$ is zero. A lot of insight regarding the time path can be gained by studying the law of motion for ν. We know that ν has to reach zero in finite time (to avoid marginal utility of consumption, q, going to infinity).

We may write ν as a system of differential equations (see Appendix A):

(28) $$\dot{\nu} = \theta\nu + Z$$

(29) $$\dot{Z} = (\theta - \rho)Z + (\rho - f_1)(\lambda - \mu)S + M$$

where

(30) $$Z \equiv (\lambda - q)S + \mu c^2$$

(31) $$S \equiv \alpha \overset{1}{C_q} q + (1-\alpha) \overset{2}{C_q} q$$

(32) $$M \equiv \mu \overset{2}{C_q} \dot{q} + (\lambda - q)\dot{S}$$

At some point in time the no-confiscation constraint ceases to bind, say at t_1. Then, for $t \geq t_1$,

$$\nu(t) = \dot{\nu}(t) = 0$$

Next, (28) implies $Z(t_1) = 0$, and consequently $\dot{Z}(t) = 0$ for $t \geq t_1$. From (29) we can then see when $\rho(t) = f_1(t)$, for $t < \infty$ (i.e. when capital is untaxed in finite time). This happens when $M(t)$=0, at least for $t \geq t_1$. We shall explore the conditions on preferences for this to be the case.

Notice, $Z(t)$ =0 in (30) helps signing a multiplier combination from date t_1 and onwards

$$(33) \qquad \frac{q-\lambda}{-\mu} = \frac{c^2}{-S}$$

Thus, private marginal utility of assets exceeds public marginal utility of capital. This is helpful in exploring the labour tax later on.

Taking the time derivative of (33) gives

$$(34) \qquad \dot{S} = \frac{\dot{q}}{q}\left(S + \alpha \overset{1}{c}_{qq}(q)^2 + (1-\alpha)\overset{2}{c}_{qq}(q)^2\right) + \dot{\alpha}\left(\overset{1}{c}_q q - \overset{2}{c}_q q\right)$$

Consequently

$$(35) \qquad M = \frac{\dot{q}}{q}\left[\mu \overset{2}{c}_q q + (\lambda - q)\left(S + \alpha \overset{1}{c}_{qq}(q)^2 + (1-\alpha)\overset{2}{c}_{qq}(q)^2\right)\right] + \dot{\alpha}(\lambda - q)\left(\overset{1}{c}_q q - \overset{2}{c}_q q\right)$$

Since the time derivative of α is a function of technology (as well as preferences), a certain path of α that makes M zero at all dates cannot be guaranteed by restricting to a particular class of preferences. Therefore necessary and sufficient for M=0 at all dates is that the terms in square brackets sum to zero and either

$$(36) \qquad \overset{1}{c}_q q = \overset{2}{c}_q q$$

or α is constant. However, it turns out that preferences guaranteeing a constant α are inconsistent with making the terms within square brackets summing to zero. Therefore, only condition (36) is of interest.

3.1. *Optimal tax results*

Proposition 1. *The optimal capital-income tax reaches zero in finite time if, and only if, either* (i) *or* (ii) *holds:*

(i) *the utility function is of the following class*

(37) $$u(c,1 - h) = D + \frac{c^{1-\pi}}{1-\pi} + \phi(1-h_0)$$

(ii) *the economy reaches a steady state in finite time.*

Proof. If (36) holds, S, as defined in (31), reduces to

(38) $$S = c_q^2 q$$

Taking the time derivative of (38) and combining with (32) gives

(39) $$M = \dot{q}\left[\mu c_q^2 + (\lambda - q)\left(c_q^2 + c_{qq}^2 q\right)\right]$$

Using (33) gives

(40) $$M = \mu c^2 \frac{\dot{q}}{q}\left[\frac{c_q^2 q}{c^2} - \frac{c_{qq}^2 q}{c_q^2} - 1\right]$$

Setting the bracketed term to zero and integrating gives (37) (see Appendix B). ‖

The preferences in (37) imply full consumption insurance, i.e. consumption is equalized across states (this because it is additively separable in consumption and leisure, implying that marginal utility of consumption can only be equalized across states if consumption itself is equalized across states). We will next look at the labour tax implied by those preferences.

Proposition 2. *If preferences are of the class* (37), *then the labour tax is positive (at least from the date at which the non-confiscation constraint does not bind), and is constant over time from the date at which the non-confiscation constraint ceases to bind.*

Proof. Plugging (12') into (23) and using (11), one obtains,

(41) $$f_2 - \omega = \frac{(q-\lambda)y}{(\lambda-\mu)h_0}$$

Full insurance (implied by (37)) in (11) gives $y=\omega h_0$., $f_2=\omega$ which means $\tau^L=0$.
Then (41) becomes

(42) $$\frac{\tau^L}{1-\tau^L} = \frac{q-\lambda}{\lambda-\mu}$$

Equation (31) proves that the right-hand side of (42) is positive. Taking the time derivative of (42) using (10b), (26), and (27), one obtains:

(43) $$\frac{d}{dt}\frac{\tau^L}{1-\tau^L} = \frac{d}{dt}\frac{q-\lambda}{\lambda-\mu} = (f_1-\rho)\frac{q-\mu}{\lambda-\mu}$$

Appendix C outlines the steps in deriving the second equality. Proposition 1 proved that the capital income tax is zero from date t_0 and onwards, consequently the right-hand side of (43) is zero. ‖

We will next explore the possibility of the economy reaching a steady state in finite time. We then have to derive preferences that make real variables (prices and quantities) constant in finite time. For example, it is necessary that the after-tax wage becomes constant in finite time. However, from equation (12) we see that if preferences guarantee a constant after-tax wage, then it has to be constant at all dates. We find those preferences in the next Lemma:

Lemma 1. *The after-tax wage is constant if, and only if, the utility function is of the following exponential class*

(44) $$u(c, 1-h) = D - \phi^h e^{-Ac}$$

where D and A are constants (A>0*), and* $\phi^h = \phi(h)$, *with* $\phi'(h)>0$ *and* $\phi''(h)>0$.

Proof. Taking the time derivative of (12) and setting it to zero gives

(45) $\quad -\dot{q}[c^1 - c^2 - \omega h_0] + q\,\dot{\omega} h_0 = 0$

Thus, only c^1- c^2 - ωh_0 = 0 (i.e. zero insurance demand) guarantees a constant after-tax wage. Then (12) implies $u^1 = u^2$, i.e. utilities are equalized across states. Together with marginal utility equalization, we have $u^1/u_c^1 = u^2/u_c^2$. For this to hold at all dates the utility-marginal utility ratio has to be equal to a constant, say $-1/A$.
Integrating $u^1/u_c^1 = -1/A$ gives (44), (see Appendix B). ‖

Proposition 3. *If preferences are of the class* (44), *then the labour tax is zero at all dates.*

Proof. Follows by setting y=0 in (41). ‖

Corollary 1. *If preferences are of the class* (44), *then capital's and labour's marginal products are constant at all dates.*

Proof. By Lemma 1 the after-tax wage is constant. Since labour income is untaxed (by Proposition 3) the pre-tax wage (labour's marginal product) is also constant, as is capital's marginal product (by constant returns-to-scale). ‖

Proposition 4. *If preferences are of the class* (44), *then the economy reaches a steady state in finite time, say at* t_1. *Capital is taxed at* 100% *up until date* t_1, *and is untaxed thereafter.*

Proof. For preferences of the class in (44), S as defined in (31) is constant, and in particular $S = -1/A$. Then M as defined in (32) becomes (N.B. $c_q^2 q = q/u_{cc}^2 = u_c^2/u_{cc}^2 = -1/A$):

(46) $\quad M = \mu \dfrac{\rho - \theta}{A}$

From time $t \geq t_1$ we have $Z(t) = \dot{Z}(t) = 0$, then (29) becomes (note that S=-1/A)

(47) $$(\rho - f_1)(\lambda - \mu) = \mu(\rho - \theta)$$

where (46) has been used. Obviously, a steady state (ρ=θ) implies that capital is untaxed, see (47). But if capital is untaxed and there is a steady state we must have f_1= θ. Since f_1 is constant (Corollary 1) already at t1 it must equal θ at all times. Then (47) implies that capital is untaxed, and the economy is at a steady state, from time t_1 and onwards. This implies that asymptotic convergence to a steady state does not happen under the second-best tax program. The only possibility left is that a steady state is not reached and that $f_1 \neq \theta$. The remainder of the proof investigates that.

Equation (33) gives λ-μ = q-μ+μAc^2, which substituted into (47), and using the definition of ρ, gives

(48) $$-\tau^k f_1\left(\frac{q}{\mu} - 1 + Ac^2\right) = f_1 - \theta - f_1\tau^k$$

Taking the time derivative of (48) (f_1 is constant by Corollary 1, q/μ is also constant) and using (48) gives

(49) $$\dot{\tau}^k = (\tau^k)^2 f_1\left[1 - \frac{f_1}{f_1 - \theta}\tau^k\right]$$

This differential equation has two steady states: τ^{k*}={0, 1-θ/f_1}. If $f_1 > \theta$, then regardless the level of τ^k the capital tax converges to 1-θ/f_1. This means that $\rho = f_1(1 - \tau^k) = f_1\theta/f_1 = \theta$. However, this would imply convergence to a steady state, and thus contradicts the first part of the proof (asymptotic convergence does not happen). If $f_1 < \theta$, the level of τ^k must fall below 1-θ/f_1, otherwise the tax rate would explode according to (49). For levels of below 1-θ/f_1 all τ^k go to zero asymptotically. However, if τ^k goes to zero asymptotically, then (47) would imply an asymptotic convergence to a steady state. Again, this is a contradiction. ‖

4. CONCLUSION

There are two main contributions of this paper. First is to extend the analysis of Chamley and Judd to indivisible labour and analyze the dynamics of capital income tax. Second, we unravel a new class of preferences for which labour may remain untaxed in the short run and long run. These results have useful tax policy implications. Whether it is optimal to tax labour and/or capital typically boils down to the form of the individual preferences. The second-best labour-tax depends crucially on the degree of complementarity between consumption and leisure. If leisure is a not neutral, there is scope for government intervention in the form of labour taxation or subsidy in a complete market environment.

APPENDIX A

Derivation of (28).

Premultiply (24) by q and exploiting the fact that $\Omega'(q)= [c^1-c^2-\omega h_0]/qh_0$, one obtains

$$\text{(A.1)}\quad q\dot{\psi}=q\rho\psi+(\lambda-q)[\alpha c_q^1 q+(1-\alpha)c_q^2 q]+\mu\alpha\left[\omega h_0-c^1+c^2\right]$$

Next note that

$$\text{(A.2)}\quad \frac{d}{dt}\left(\psi q\right)=\dot{\psi}q+\psi\dot{q}=\dot{\psi}\,q+\psi(\theta-\rho)q$$

Plugging (A.2) into (A.1) gives

$$\text{(A.3)}\quad \frac{d}{dt}(\psi q)=\theta\psi q+(\lambda-q)\left[\alpha c_q^1 q+(1-\alpha)c_q^2 q\right]+\mu\alpha\left[\omega h_0-c^1+c^2\right]$$

Next note that

$$\text{(A.4)} \quad \frac{d}{dt}(\mu a) = \dot{\mu}\, a + \mu \dot{a}$$

Plugging (27) into (A.4)

$$\text{(A.5)} \quad \frac{d}{dt}(\mu a) = \mu(\theta - \rho) a + \mu \dot{a}$$

Using (2), (3) and (10a), the household's budget constraint can be rewritten as:

$$\text{(A.6)} \quad \dot{a} = \rho a + \alpha \omega h_0 - \alpha c^1 - (1-\alpha) c^2$$

which after plugging into (A.5) gives

$$\text{(A.7)} \quad \frac{d}{dt}(\mu a) = \theta \mu a + \mu \left[\alpha \omega h_0 - \alpha c^1 - (1-\alpha) c^2 \right]$$

Next noting that a=b+k, rewrite (25) as

$$\text{(A.8)} \quad \nu = \psi q - \mu a \,.$$

Taking the time derivative of (A.8), one obtains

$$\text{(A.9)} \quad \dot{\nu} = \frac{d}{dt}(\psi q) - \frac{d}{dt}(\mu a)$$

Using (A.3) and (A.7) in (A.9) one obtains equation (28).

APPENDIX B

Derivation of equation (44). Inverting gives

$$\text{(B.1)} \qquad u_c^s / u^s = -A$$

for s={1,2}. Or equivalently

$$\text{(B.2)} \qquad \frac{d \ln(-u^s)}{d\, c^s} = -A$$

Integrating both sides with respect to c^s gives

$$\text{(B.3)} \qquad \ln(-u^s) = N^s - A\, c^s$$

where N^s is any constant, possibly dependent on s.
Taking exponents of both sides gives

$$\text{(B.4)} \qquad -u^s = B^s e^{-Ac^s}$$

where B^s=exp{N^s}, and consequently is positive, and is the only term which can be a function of leisure, consequently yielding (44).‖

APPENDIX C

Derivation of equation (43). Note that

$$\text{(C.1)} \qquad \frac{d}{dt}\left[\frac{q-\lambda}{\lambda-\mu}\right] = \left[\frac{(dq/dt)-(d\lambda/dt)}{\lambda-\mu}\right] - \left[\frac{q-\lambda}{\lambda-\mu}\right] \cdot \left[\frac{(d\lambda/dt)-(d\mu/dt)}{\lambda-\mu}\right]$$

Next plug in (10b), (26) and (27) into the right hand side of (C.1) to obtain (43). ‖

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