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ABSTRACT

Optimal Capital Allocation Using RAROC™ and EVA®*

This Paper analyses firms' capital allocation decisions when optimal capital structure is linked to the risk of underlying assets and when equity capital is costly and cannot be raised instantaneously. In the model, division managers receive private information and authority is delegated to them over risky project choices. The optimal mechanisms are related to EVA compensation and RAROC performance measurement systems. In the optimal mechanism, position limits will be employed but are not always completely utilized. Hurdle rates reflect capital allocation through a division-specific capital structure. In the multidivisional context the optimal capital allocation mechanism incorporates valuable externalities leading to overall firm EVA maximization.

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1 Introduction

In frictionless markets without asymmetric information the theory of corporate finance provides clearly specified rules for firms' optimal capital budgeting decisions. In such an environment capital is flexible and management simply distributes funds to the divisions as if they were autonomous entities. This paradigm contrasts sharply with the observed budgeting procedures and incentive schemes actually utilized by firms, especially those with multiple divisions. A critical ingredient of these procedures involves the allocation of capital from the central management to the business units.

Capital allocation therefore plays an important role in the overall level and mixture of activities when firms face imperfections in capital markets that limit their ability to continuously and costlessly raise funds and reconfigure their capital structure. This paper derives optimal capital allocation and performance measurement and their implications for capital budgeting if firms face financial market imperfections of the following sort: (1) debt is favored over equity up to some limit based on the riskiness of the underlying assets and (2) equity capital must be raised in advance of knowledge of precise investment opportunities.

A good example of firms in this environment are financial institutions such as banks, who issue equity infrequently but have continuous access to debt capital. Banks, by their very nature, are in the business of accepting deposits, which implies a bias in favor of debt financing. The provision of services such as liquidity through demand deposits, letters of credit etc. imply that depositors are willing to loan their funds at rates below those prevailing in money markets. Thus, in the absence of regulation, banks will prefer high leverage. The capital structure limitation is determined by the risk of the bank's assets, which can change quickly due to asset allocation decisions and the volatility of asset markets. In fact, explicit regulations have been promulgated by the Basle Committee to deal with this effect.

Many countries have implemented the recommendations of the Basle Banking Committee and define banks' capital requirements on the basis of various risk measures. In addition to monitoring risks to comply with regulation, many banks and financial institutions have also adopted their own internal risk management systems designed to measure and limit their risks in accordance with their equity capital.[1]

Although in such an environment equity capital requirements will be determined at the firm level, it is

[1]James (1996) and Zaik, Walter, Kelling and James (1996) analyze the capital budgeting process at Bank of America where equity capital is defined by a value at risk measure.

important to allocate capital at the divisional or project level. The key question involved in capital allocation is how much equity capital to charge each unit with so that the effect of individual investment decisions on the overall level of risk is internalized. In this way the capital charge to the division is not determined by the actual amount of capital required by this division's investments. The most dramatic example is a division entirely engaged in derivatives trading for which no literal cash investment may be required.

In standard capital budgeting contexts the need to assess project-specific systematic risks for the purpose of deciding on an appropriate cost of capital is well-documented. Less well-understood, however, is the need to incorporate project-specific capital structure weights. Stulz (2003) discusses these issues in the context of risk management and demonstrates that the capital budgeting process must incorporate an adjustment to net present value to represent the impact of project-specific unsystematic risk on the capital of the firm.

While our results are more general, for clarity we present them in the context of a financial institution.[2] Our paper addresses several central issues related to capital budgeting and performance measurement. We demonstrate how two important concepts from the theory of shareholder value, Economic Value Added (EVA) and risk adjusted return on capital (RAROC) can be justified on the basis of optimal investment decisions.[3] Essentially the optimal capital allocation mechanism we derive requires the institution to compute an *economic capital*, rather than a book capital for use in the EVA and RAROC computations. Moreover, we show that the economic capital is related to the widespread use of value-at-risk (VaR).

We extend our results to a multidivisional setting, where risk management and investment activities need to be coordinated. Under conditions where an agency problem exists with respect to divisional management we find an appropriate allocation of economic capital such that externalities are incorporated and overall EVA is maximized.[4]

Froot and Stein (1998) discuss this problem of divisional interdependence in a model in which risk management arises endogenously from the need to avoid an adverse selection problem with respect to external finance. We extend this research by showing that the conditions under which EVA and RAROC can be justifiably employed include those where divisional management is privately informed about their own

[2]Cuoco and Liu (2003) considers the impact on the portfolio of a division of a financial institution when capital is adjusted infrequently relative to trading activities.

[3]See, for example, Uyemura, Kantor and Pettit (1996), in the banking context.

[4]Ad hoc procedures for capital allocation, such as those in Kimball (1997) may create significant distortions in the decisions taken.

investment opportunities and exercises local control. We derive RAROC hurdle rates and relate them to the amount of equity raised, external costs of capital, the extent of private information and the nature of the managers' outside opportunities.

The method we propose involves the central authority of the institution specifying a mechanism under which divisions are charged an internal price for economic capital. In some ways our results are therefore reminiscent of the literature on internal capital markets (Stein, 1997). However an important distinction is that the "price" must be personalized for the division's own investment opportunities. The use of this risk pricing mechanism "separates" the investment decisions in a way that allows each division to act independently of the others.

As in other models of capital budgeting under asymmetric information, we derive distortions relative to *first best* in the firms investment policies. One of the original papers to look at capital budgeting under asymmetric information and solve for the amount of inefficiency induced was Harris, Kriebel and Raviv (1982). Harris and Raviv (1996) and Harris and Raviv (1998) analyze capital budgeting decisions in the presence of asymmetric information about project quality and empire building preferences by divisional managers. At a cost, headquarters can obtain information about a division's investment opportunity set. The paper demonstrates under which circumstances headquarters will delegate the decision how to allocate capital across projects and what form this delegation may take. In these papers, distortions can either be in the form of under or overinvestment. Bernardo, Cai and Luo (2001) and Bernardo, Cai and Luo (2003) discuss this issue using optimal compensation for the manager and argue that underinvestment will always prevail. By contrast, in our model we argue that the magnitude of these distortions are related to the extent of managerial participation in the cash flows, whereas the direction is determined by the outside opportunities. When outside opportunities are increasing significantly with the division manager's inside investment opportunities, such as when managers may be able to retain a client list if they move to another firm, we find that overinvestment, i.e., excessive risk-taking obtains.[5]

A further contribution of our paper is that it derives endogenously the degree of capital rationing amongst the divisions and also determines the degree to which capital is underutilized. The mechanisms we propose feature both position or risk limits as well as performance measurement based on ex post risk, rather than

[5] Milbourn and Thakor (1996) consider an asymmetric information model of capital allocation and compensation. They focus on the moral hazard problem of managerial effort as well as enhanced bargaining opportunities for the central authority.

ex ante values. In our model, managers will not necessarily hit their limits, even though it would appear inefficient to do otherwise. Here we find that outside options enjoyed by incumbent divisional managers make it more likely that top management will employ stricter risk limits across the spectrum of possibilities.

The remainder of the paper is structured as follows. Section 2 discusses the economic and institutional environment and develops the model. Section 3 provides the analysis for the case of a single division. Section 4 extends the model to the case of multiple divisions. Section 5 concludes.

2 Model Development

Financial institutions and banks, in particular, face market imperfections such as costs of financial distress, transactions costs in accessing capital markets, or simply regulatory constraints. These frictions imply that risk management, capital structure and capital budgeting are interdependent. We begin by discussing the nature of investment opportunities faced by the financial institution.

2.1 Investment Opportunities

We first specify how a business unit's investment opportunities are modeled. A financial institution consists of n divisions, each of which may choose investment projects, defined by their standard deviations of gross cash flows. Expected cash flows at the end of the period are given by

$$\mu_i = \mu_i(\sigma_i)\theta_i \tag{1}$$

where σ_i is the standard deviation of cash flows of division i and θ_i represents an information variable that represents the profitability of the investment technology specified below.[6]

We assume that, *ceteris paribus*, more "aggressive" risk taking by a division translates into higher expected returns, i.e.

$$\frac{\partial \mu_i}{\partial \sigma_i} \equiv \mu_{i\sigma} > 0. \tag{2}$$

To ensure interior solutions we also assume that the investment technology is concave, $\partial^2 \mu_i / \partial \sigma_i^2 < 0$, and

[6] In order to generate the functional representation of investment opportunities embodied in (1) it may be necessary to apply an efficiency criterion to a more general investment opportunity set.

that $\lim_{(\sigma_i \to 0)} \ \partial\mu_i/\partial\sigma_i = \infty$ and $\lim_{(\sigma_i \to \infty)} \ \partial\mu_i/\partial\sigma_i = 0$.

The parameter θ_i defines the functional relationship between risk and expected return. Higher values of θ_i correspond to greater cash flows per unit of cash flow risk. Note that the well-known Spence (1973) - Mirrlees (1971) *sorting condition* reduces to:

$$\frac{\partial^2\mu_i}{\partial\sigma_i\partial\theta_i} = \mu_{i\sigma} > 0. \tag{3}$$

This condition is critical in the analysis of mechanisms under asymmetric information. Condition (3) implies that a higher θ_i makes "risk" more productive and makes a higher σ_i more desirable.

We make the additional expositional simplification that the investment activity of division *i* requires a total financing requirement of $A_i\sigma_i$ dollars at the initial time period, where $A_i \geq 0$ is a constant coefficient for division *i* representing the amount of *physical* investment capital required. Depending on the nature of activities for each division, this coefficient could be very different. For instance lending requires substantial financing requirements. On the other hand, for certain derivatives trading such as swaps the funding requirements are virtually zero, in principle. The assumption of proportionality between physical capital and cash flow risk is commonly employed when the investment involves a holding in frictionless capital markets, such as in the case of equity or bond trading. In such cases, the constant A_i is equal to the reciprocal of the standard deviation in rate of return units.[7]

2.1.1 Examples

Equation (1) is a reduced form, representing the composition of a number of concurrent operating activities of financial institutions. We provide two specific descriptions of activities that underlie the specification in equation (1). The first example is that of a bank which engages in deposit-financed lending activity. We hereby assume that deposit rates are below the expected return on loans. This may be due to imperfect competition between banks or due to fixed costs in setting up a bank. We normalize the interest rate paid to depositors to zero.

Let *L* denote the amount of lending undertaken and assume that the marginal loan quality is decreasing in the aggregate amount of lending. We can then represent the expected net cash flows from lending activities

[7]For example, if the standard deviation of equity returns is 0.20, the coefficient would be $A_i = 1/.2 = 5$.

by $\mu = f(L)$ with f being an increasing concave function. Suppose, for instance, that the aggregate risk is increasing linearly in aggregate lending, so that $\sigma = sL$ with $s > 0$. We can therefore rewrite the expected cash flows in the following way

$$\mu = f(\frac{\sigma}{s}) \equiv \mu(\sigma).$$

Therefore equation (1) characterizes the deposit-financed lending activity described above.

The next situation we consider is that of a trading division engaged in self-financing derivatives trading. That is zero net investment positions are taken in financial instruments such as forwards, futures, swaps etc. These activities may either be thought of as arising from offering structured products to corporate customers or from trading in the capital markets based on proprietary valuation models. Let σ represent the risk of the cash flows generated by the derivatives exposure. Similar to the previous example, suppose that σ is linear in the face value and that the expected cash flow is an increasing and concave function of the face value of the derivative position. This concavity may be due to a number of sources, such as price impact, or estimation risk in the valuation model used to derive expected cash flows.

Although we have given two examples of activities that financial institutions are involved in, our model can be interpreted to reflect a variety of other investment activities. Positive initial cash investments can be modeled by assuming that they are partly debt financed. Cash flows are always defined after appropriate interest costs.

2.2 Capital

We now specify the objective function of the financial institution. The institution is being run in an environment where shareholders interests are paramount, as is the focus in most of the well-developed economies today. The institution's first step is to raise equity capital, C. The cost of equity capital is denoted by r_E. To keep the analysis as parsimoniuous as posssible, we assume that r_E is constant and independent of the firms' investment opportunities. We shall motivate this assumption below in consideration of the role that outside regulation plays in restricting the risk that the institution may take. The second source of funds is debt or deposits, the cost of which we denote by r_D. Summing the gross cash flows over all divisions less the costs associated with debt financing, gives the cash flows attributable to equity capital. This net cash flow minus the cost for equity capital is denoted as EVA, in accordance with the modern perspective of shareholder value.

$$EVA = \sum_i \mu_i(\sigma_i)\theta_i - r_D(\sum_i A_i\sigma_i - C) - r_E C. \tag{4}$$

That is, the total financing requirements are $\sum_i A_i\sigma_i$, out of which C is made up of equity and the rest debt. EVA represents the annual contribution to shareholder value. It is well-known that the net present value equals the discounted sum of EVAs.[8]

Financial institutions frequently find it advantageous to minimize their use of equity capital. This occurs for a number of reasons. First, due to liquidity and security offered to depositors, bank deposits are considered to be a cheap source of capital. Second, as in other industries, debt offers a tax subsidy due to deductibility of interest. Third, as emphasized by Merton and Perold (1993), a bank's customers are frequently also its debtholders so that the nature of banking requires leverage. The more "customers" a bank wants to attract, the more debt it must be prepared to accept in its capital structure. In our model, we capture these effects by assuming that the (after-tax) cost of deposits, r_D, is less than the cost of equity, r_E.

On the other hand, depositors and other debtholders are only willing to lend their capital to a bank if it has a sufficient amount of equity capital to ensure its solvency. In addition regulators specify minimum equity standards which are based on various risk measures. Thus, a bank's capital structure is determined on the one hand by several advantages of debt, and on the other hand either by the bank's own desire to limit costs of financial distress or by regulatory constraints. Normally the equity capital is less than the total amount of physical financing requirements, $C < \sum_i A_i\sigma_i$. However if the equity capital required is greater than the financing requirements, it is invested in a riskless asset, paying the same rate of return, r_D, as the cost of debt. The fact that this "extra" equity is invested with a lower rate of return than its cost essentially means that the financial institution incurs deadweight costs which, as in Froot and Stein (1998) may be interpreted as a tax.

To enable the bank to realize the maximum benefits of leverage without violating regulatory constraints or incurring excessive transactions costs, most financial institutions have adopted sophisticated risk man-

[8]The correspondence between EVA and net present value (NPV) was perhaps first discussed by Preinreich (1937). The precise specifications of EVA and the corresponding depreciation schedule are discussed in Rogerson (1997) and Reichelstein (1997), who showed that EVA-type measures represent the unique optimum among the class of linear measures. Unfortunately this uniqueness result depends critically on an assumption that cash flows are certain. The results are extended to the case of moral hazard in Reichelstein (2000).

agement systems to monitor and limit their risks in accordance to their equity capital. The basic concept is to specify a maximum loss, which may only be exceeded with a specified probability. This loss is referred to as *Value at Risk* (VaR). The value at risk is a function of the entire cash flow distribution and is measured by looking at the point on the lower tail where the probability is equal to a specified threshold value. When cash flows are normally distributed, the Value at Risk can be expressed as

$$\text{VaR} = \alpha\sigma, \tag{5}$$

where α is a factor defined by the probability with which the actual loss may not exceed the VaR; σ is the standard deviation of cash flows on the institutions's portfolio over a specified time period.[9] Note that the definition of VaR in equation (5) does not take the expected return on investments into account. There are several reasons why VaR is generally defined ignoring the mean return. First, since VaR calculations are usually made for short periods of time, i.e., for one to ten day periods, the first moments are of second order. Second, most VaR models used in practice do not take the mean return into account when calculating risk and, finally, regulators are conservative with respect to allowing banks' capital standards to be reduced by modeling the mean returns.

Bank management as well as regulators increasingly define capital requirements in terms of its VaR. More precisely, the equity capital of the bank, C, is given by the constraint[10]

$$C \geq \text{VaR} = \alpha\sigma. \tag{6}$$

According to equation (6) a bank is limited in its risk taking such that the resulting VaR does not exceed the amount of equity capital. This capital structure constraint ultimately determines the required capital allocation to investment projects or divisions.[11]

[9] For a detailed analysis of the concept of Value at Risk, see Jorion (2001). Stulz (2003) demonstrates the applicability of VaR in risk management contexts.

[10] Our formulation can be easily extended to the case where a non-regulated firm wishes to satisfy a constraint on its debt rating; here one may wish to redefine the VaR as mean-adjusted.

[11] The Basle Accord has been amended as of January 1, 1998 to allow banks to use their own internal models to assess risk. In this case a standard of 99% for ten days times a multiplier of between 3 and 4 is used for the bank's trading activities. That is, α is between 6.9 and 9.2, and σ is defined as the ten day standard deviation.

2.3 The Agency Problem

In our model the divisions of the financial institution are run by individual managers who have better information about investment opportunities than the top management, once they have observed the parameter θ_i. If divisional managers' compensation does not depend in an increasing way on the expected cash flow generated by their investment decision, they would not have any incentive to collect information and observe the realization of θ_i.

We therefore assume that the divisional managers receive a fraction γ of their division's cash flow minus a transfer T_i which is chosen optimally by the central management to account for the cost of financing the investment and thus to maximize the total EVA to the firm minus the compensation paid to the divisional managers. Formally, managerial utility is therefore modeled as:

$$U_i = \gamma_i \mu_i(\sigma_i)\theta_i - T_i, \tag{7}$$

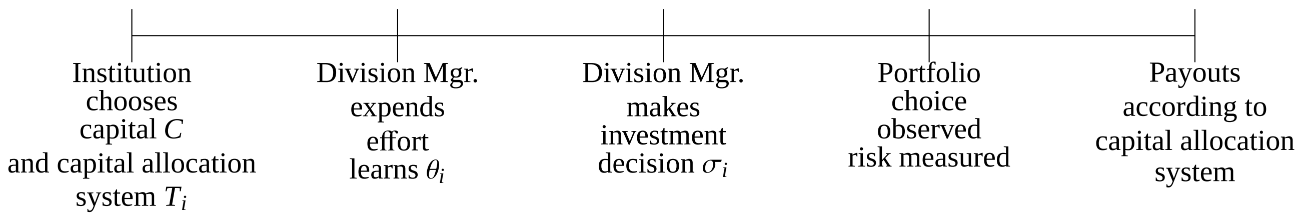
where γ_i is some fixed fraction less than one, and T_i is interpreted as a capital charge assigned by the *capital allocation system*. The coefficient γ_i is the fraction of expected cash flows given to the manager to motivate effort to gather information, as mentioned previously. Generally the capital allocation implied by T_i will be a function of the decisions taken by the manager insofar as risk is concerned as well as the information conveyed by the manager to the central management of the institution. The principle goal of this paper is to derive the functional form of T_i and to show how it can be interpreted in terms of a divisional EVA and RAROC.

Since the capital allocation and optimal compensation are being chosen by the institution, it is necessary to specify some *reservation*, or default utility that the manager would otherwise be able to obtain. It is of fundamental importance how this utility varies with the private information of the manager. We specify this reservation utility as:

$$U_i(\theta_i) \geq \underline{U} + \eta_i \mu_i(\sigma_i)\theta_i. \tag{8}$$

We consider two polar situations. The first, $\eta_i = 0$, is where the manager has very limited bargaining power, in that his *outside options* are not a function of private information. This represents an economic situation in which the information might be thought of as specific to the parent institution and not transferable. The

Figure 1: Sequence of Events



second situation, $\eta_i > \gamma_i$, holds when the manager has outside options which are more significant than the incentives generated by the firm internally via the fraction of the cash flows paid to the manager, γ_i. An example for such a situation may be a manager who is managing an equity portfolio, for instance, that could easily be transferred to a competing institution. Considering only the two polar cases $\eta_i = 0$ and $\eta_i > \gamma_i$ greatly simplifies the mathematical analysis of asymmetric information in the next section.

The first step in the institution's risk management problem is to select the amount of equity capital, C, to raise. Then the institution elicits information from each of the managers about their investment opportunities based on the commitment to a mechanism involving the capital allocation. Finally, the institution allocates capital to each of the divisions and they decide upon their optimal levels of risk. This problem is documented in Figure 1.

In the next section we solve for the optimal capital allocation function in the situation where there is a single division manager with private information and where the risk taken is delegated at the divisional level.

3 Single Division Capital Allocation

We first consider a setting in which there is a single division run by a manager engaged in risk-taking activities. The financial institution delegates the investment decision to the manager, since he has private information about the investment opportunities.

We assume that the divisional manager must expend some unobservable effort to learn the information parameter, θ.[12] Information is modeled as a single-dimensional draw, $\theta \in [\underline{\theta}, \overline{\theta}]$ given distribution function $F(\theta)$.

As is well-known in the theory of private information and agency (Fudenberg and Tirole, 1992), the optimal mechanism may be derived by using the revelation principle (Myerson, 1979). The direct revelation mechanism is $\sigma(\hat{\theta})$ and $T(\hat{\theta})$, where $\hat{\theta}$ represents the division's *report* of information or "type". The Bayesian Nash equilibria of the asymmetric information game are only those that can be supported by the *incentive compatibility* condition that $\hat{\theta} = \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

The overall problem may therefore be expressed as maximizing expected EVA minus the compensation given to the manager or:

$$\max_{T,\sigma(\hat{\theta}),C} I = E\left[\mu(\sigma(\hat{\theta}))\theta - r_D(A\sigma(\hat{\theta}) - C) - r_E C - U\right], \tag{9}$$

subject to the incentive compatibility condition that the manager truthfully reports his private information in the optimal mechanism:

$$\arg\max_{\hat{\theta}} U(\sigma, \theta, \hat{\theta}) = \gamma\mu(\sigma(\hat{\theta}))\theta - T(\hat{\theta}), \tag{10}$$

subject to reservation utility:

$$U(\theta) \geq \underline{U} + \eta\mu(\sigma(\hat{\theta}))\theta \tag{11}$$

and the regulatory constraint on overall firm equity capital:

$$\alpha\sigma(\theta) \leq C. \tag{12}$$

3.1 No Outside Options

We now solve problem (9) in the situation where the manager has outside options independent of the risk level chosen, i.e., $\eta = 0$. The major step in solving problem (9) is to first-convert the global incentive-compatibility condition (10) into a local representation. Following Fudenberg and Tirole (1992, p. 264) it

[12] In this section we drop the subscript i since we are only considering a single division.

can be shown that a necessary and sufficient condition for (10) to hold is that

$$U(\theta) = \underline{U} + \int_{\underline{\theta}}^{\theta} \gamma\mu(\sigma(\hat{\theta}))d\hat{\theta}, \tag{13}$$

and $\sigma(\theta)$ non-decreasing.[13] This also implies that the reservation utility constraint is binding only at the lower endpoint, $U(\underline{\theta}) \geq \underline{U}$. Using this representation, the optimal risk of the institution solves the following problem:

$$\max_{C,\sigma(\theta)} \quad \int_{\underline{\theta}}^{\bar{\theta}} [\mu(\sigma(\theta))\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)(\alpha\sigma(\theta) - C) - U(\theta)]dF(\theta), \tag{14}$$

subject to (13) where $\lambda(\theta)$ represents the Lagrange multiplier on the total capital constraint, (12).

The solution to this problem is mostly standard within the field of mechanism design and is relegated to the appendix. Proposition 1 gives the solution.

Proposition 1. *There exists a threshold value, $\theta^* \in [\underline{\theta}, \bar{\theta}]$ such that optimal risk for $\theta \in [\underline{\theta}, \theta^*]$ satisfies*

$$\mu_\sigma(\sigma(\theta))\theta - r_D A = \frac{\gamma(1 - F(\theta))}{F'(\theta)}\mu_\sigma(\sigma(\theta)). \tag{15}$$

The optimal risk level for $\theta \in [\theta^*, \bar{\theta}]$ is constant, $\sigma(\theta) = \sigma^*$, where θ^* satisfies:

$$\mu_\sigma(\sigma^*) \int_{\theta^*}^{\bar{\theta}} \theta dF(\theta) = (r_E - r_D)\alpha + r_D A(1 - F(\theta^*)) + \gamma\mu_\sigma(\sigma^*) \left[\int_{\theta^*}^{\bar{\theta}} \theta dF(\theta) - (1 - F(\theta^*))\theta^* \right]. \tag{16}$$

Proposition 1 shows that when there is asymmetric information, optimal risk is reduced relative to the level that would pertain in a first-best case where capital could be raised instantaneously at the cost of capital, r_D. This can be seen from (15) by noting that the right-hand side is positive. Therefore, the marginal benefit of taking on increased risk at the second-best optimum is greater than the marginal cost. The extent of this deviation is greater for lower types. This is intentional on the part of the institution as it strives to make it more costly for the better divisions to misreport.

Equation (16) determines the optimal amount of capital that is raised. The left hand side represents the

[13]The sorting condition (3) is the critical assumption that is necessary to obtain this representation of the incentive compatibility conditions.

benefit from raising α units of capital. This benefit is generated from taking on one additional unit of risk (standard deviation) in the states between θ^* and $\overline{\theta}$. This increases the cash flow in these states by μ_σ.

The right hand side of equation (16) represents the costs of raising one additional unit of capital. First, the net cost of equity is $r_E - r_D$. This is so since equity capital can be thought of as being invested in the riskless asset. Second, the additional investment of A in the risky asset must be financed via deposits in the states between θ^* and $\overline{\theta}$. Third, the additional investment in the risky asset leads to additional payments to the divisional manager. This is captured in the third expression.

Thus, if the amount of equity capital raised is chosen so that equation (16) holds, then the marginal benefit of equity capital just equals its marginal costs.

Proposition 1 implies the following corollary:

Corollary 1.

$$C^* < \alpha\sigma^*(\overline{\theta}), \tag{17}$$

i.e. the risk limit is binding in some states.

Proof. Suppose that $C^* = \alpha\sigma^*(\overline{\theta})$. In this case $\theta^* = \overline{\theta}$. This implies that equation (16) cannot hold since the left hand side is zero and the right hand side is $(r_E - r_D)\alpha$. □

3.2 Implementation via EVA and RAROC

We now interpret these results by showing that the optimal risk level can be implemented by providing the manager with an appropriate incentive schedule and delegating the decision to him. We show that the incentive schedule can be interpreted as a *divisional EVA* compensation system, where economic capital is appropriately computed.

First, note that since risk is bounded by σ^* in the optimal mechanism, the institution must impose a risk limit $\sigma \leq \sigma^*$ for all information types θ. Whenever the risk limit is not binding, consider the implementation of the optimal $\sigma(\theta)$ via the following (linear) incentive schedule:

$$\hat{T}(\hat{\theta}, \sigma) = \nu(\hat{\theta}) + \kappa(\hat{\theta})\sigma. \tag{18}$$

Suppose now that the division has "reported" $\hat{\theta}$, thereby determining the functional form of (18). Consider the sub-problem where the institution now allows the division to select the risk level by maximizing utility:

$$\max_{\sigma} \quad \gamma\mu(\sigma)\theta - \hat{T}(\hat{\theta}, \sigma)). \tag{19}$$

Definition The indirect mechanism $\hat{T}(\hat{\theta}, \sigma)$ *implements* the direct mechanism $\langle \sigma(\hat{\theta}), T(\hat{\theta}) \rangle$ whenever the solution to (19), $\hat{\sigma}(\hat{\theta}) = \sigma(\hat{\theta})$ and $\hat{T}(\hat{\theta}, \hat{\sigma}(\hat{\theta})) = T(\hat{\theta})$ for all $\hat{\theta}$.

Using this definition, we now see that a necessary condition for implementation of the optimal second-best mechanism is that

$$\gamma\mu_{\sigma}\theta - \hat{T}_{\sigma} = 0$$

coincides with the optimal decision according to (15). Substituting for the optimality condition of (15) and the definition of $\hat{T}$ from (18) we arrive at

$$\kappa(\theta) = \gamma r_D A + \gamma^2 \mu_{\sigma}(\sigma(\theta)) \frac{(1 - F(\theta))}{F'(\theta)}. \tag{20}$$

This result leads to the next proposition.

Proposition 2. *Under asymmetric information, the optimal mechanism may be implemented via a risk limit accompanied by a capital allocation schedule such that*

$$\hat{T}(\theta, \sigma) = \nu(\theta) + \left[r_D A + \mu_{\sigma}\gamma \frac{(1 - F(\theta))}{F'(\theta)} \right] \sigma, \tag{21}$$

where

$$\nu(\theta) = \gamma\mu(\sigma(\underline{\theta}))\underline{\theta} - \kappa(\underline{\theta})\sigma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \sigma(\hat{\theta}) d\kappa(\hat{\theta}) - \underline{U}. \tag{22}$$

Proof. The proof of proposition 2 is in the appendix. □

This capital allocation schedule can now be interpreted in terms of Economic Value Added from the manager's perspective. Using the functional form for (21) in the utility function (7) gives a utility interpretation as a share of overall EVA minus an adjustment function:

$$\begin{aligned} U(\theta) &= \gamma\mu(\sigma)\theta - \nu - \kappa\sigma \\ &= \gamma\mu(\sigma)\theta - \nu - \gamma r_D A\sigma - \gamma^2\mu_\sigma\frac{(1-F(\theta))}{F'(\theta)}\sigma \\ &= \gamma EVA - \nu + \gamma(r_E - r_D)C - \gamma^2\mu_\sigma\frac{(1-F(\theta))}{F'(\theta)}\sigma. \end{aligned}$$

In this sense the manager essentially receives a share of overall EVA minus two adjustments independent of risk, ν, and $\gamma(r_E - r_D)C$, and a deduction for risk undertaken, represented by the last asymmetric information term.

We now extend these results to provide a Risk Adjusted Return on Capital interpretation. In this vein, the standard way in which RAROC is applied is such that

$$EVA = (RAROC)(EC),$$

where EC represents economic capital. Then the realized RAROC is compared to zero and shareholder value creation is is achieved if and only if $RAROC > 0$. Alternatively, one can use the notion of Return on Risk Adjusted Capital or RORAC and define a hurdle rate, r^*, such that shareholder value creation is equivalent to $RORAC > r^*$.

It is common to define economic capital as the VaR criterion and apply the hurdle rate, r^*, to this amount as a charge for the usage of capital; therefore we set $EC = \alpha\sigma$, giving a definition of RAROC as follows:

$$RAROC = \frac{\mu\theta - \delta - r^* EC}{EC}, \tag{23}$$

where δ is an adjustment to the cash flows independent of risk, defined such that the numerator of the RAROC ratio equals EVA. Using the RAROC criterion, a division will continue to make risky investments as long as the RAROC of the marginal project is greater than zero. Proposition 3 now derives the hurdle rate applicable to the RAROC performance measure.

Proposition 3. *Suppose that RAROC is defined in terms of economic capital using a VaR criterion based on* *the amount of capital utilization. Then when the hurdle rate, r^*, is given by*

$$r^* = r_D\left[\frac{A}{\alpha} + \frac{\gamma\mu_\sigma(1 - F(\theta))}{r_D\alpha F'(\theta)}\right], \tag{24}$$

shareholder value is created whenever the change in the marginal RAROC for the incremental project is greater than zero and is optimized at the point where the marginal RAROC = 0.

Proof. Consider the RAROC of the marginal investment, given by the change in the numerator in equation (23) divided by the marginal change in economic capital, i.e., the denominator in (23):

$$\frac{\frac{d\mu(\sigma)\theta}{d\sigma} - \frac{d(r^*\alpha\sigma)}{d\sigma}}{\frac{d(\alpha\sigma)}{d\sigma}} = \frac{\mu_\sigma\theta - \alpha r^*}{\alpha}.$$

This expression is greater than zero whenever

$$\frac{\mu_\sigma\theta}{\alpha} - r^* > 0.$$

But from the optimal capital allocation mechanism,

$$\gamma\mu_\sigma\theta - \kappa > 0,$$

along the optimal investment path. Therefore the optimum will be achieved using a RAROC hurdle rate as long as $r^*\alpha = \kappa/\gamma$, which is the same as (24). □

According to equation (24) the hurdle rate to be used for an additional unit of capital allocated to a manager is determined by two components. The first component is $r_D\, A/\alpha$. Note that for one additional unit of capital allocated, the division can invest A/α additional units of physical capital which will be financed through deposits at cost r_D. In other words, A/α should be interpreted as one plus the leverage ratio, where equity is defined as the economic capital. The second component is $\gamma\mu_\sigma(1-F(\theta))/(r_D F'(\theta))$. This represents the increase in the required manager compensation due to an additional capital unit allocated.

It is interesting to note that the hurdle rate does not depend on the cost of equity, r_E. This is so since, at the time of the investment decision, the amount of equity is fixed. The correct hurdle rate for an additional unit of capital is therefore determined by the cost of funding the additional physical capital through deposits plus the additional management compensation.

3.3 Outside Options

Now we reconsider the situation where the manager has outside options that increase with the information at a sufficiently high rate. Since most of the analysis mirrors that of the previous case, we shall be brief in deriving the results and then turn to an explanation of the differences and the implications.

In dealing with this case we follow the approach of Maggi and Rodriguez-Clare (1995), who show that one can define the manager's rent,

$$V(\theta) = U(\theta) - \eta\mu(\sigma)\theta. \tag{25}$$

Using this definition, note that

$$\begin{aligned} \frac{dV}{d\theta} &= \frac{\partial U}{\partial \theta} - \eta\mu_\sigma\theta\frac{d\sigma}{d\theta} - \eta\mu \\ &= \gamma\mu - \eta\mu - \eta\mu_\sigma\theta\frac{d\sigma}{d\theta}. \end{aligned}$$

Since $d\sigma/d\theta$ is non-decreasing from the incentive compatibility constraint, it follows that a sufficient condition for $dV/d\theta < 0$ is that $\eta > \gamma$. The implication of this result is that the reservation utility constraint, $V(\theta) + \eta\mu(\theta)\theta \geq \underline{U}$, is binding at the upper endpoint, $\theta = \bar{\theta}$.[14]

The equivalent problem to be solved is now:

$$\max_{C,\sigma(\theta)} \quad \int_{\underline{\theta}}^{\bar{\theta}} [\mu(\sigma(\theta))\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)(\alpha\sigma(\theta) - C) - (V(\theta) + \eta\mu(\sigma(\theta))\theta)]dF(\theta), \tag{26}$$

subject to (13) and $V(\bar{\theta}) = \underline{U}$. The solution to this problem is given in the following proposition.

Proposition 4. *There exists a threshold value, $\theta^* \in [\underline{\theta}, \bar{\theta}]$ such that optimal risk for $\theta \in [\underline{\theta}, \theta^*]$ satisfies*

$$\mu_\sigma(\sigma(\theta))\theta - r_D A = -\frac{\gamma F(\theta)}{F'(\theta)}\mu_\sigma(\sigma(\theta)). \tag{27}$$

[14] Maggi and Rodriguez-Clare (1995) consider results intermediate to the two polar cases considered here and illustrate how countervailing incentives leads to situations of pooling amongst the types.

The optimal risk level for $\theta \in [\theta^*, \bar{\theta}]$ is constant, $\sigma(\theta) = \sigma^*$, where θ^* satisfies:

$$\mu_\sigma(\sigma^*) \int_{\theta^*}^{\bar{\theta}} \theta dF(\theta) = (r_E - r_D)\alpha + r_D A(1 - F(\theta^*)) + \mu_\sigma(\sigma^*) \left[\gamma(\bar{\theta} - F(\theta^*)\theta^*) - \gamma \int_{\theta^*}^{\bar{\theta}} \theta dF(\theta) - \eta\bar{\theta} \right]. \quad (28)$$

Proof. The proof is very similar to Proposition 1. The main differences are outlined in the appendix. □

In comparison to the case without outside options, we see that once again there is a threshold level, θ^*, which is in general different with the capital constraint binding above this region. Below this region, there is a greater amount of risk taken in comparison with the case without outside options. The reason for this again has to do with information rent. Now the institution desires to prevent managers with less desirable investment opportunities from proclaiming more positive information (because of the outside options being greater for good information). This means that greater degrees of distortion will occur for higher θ and the distortion takes the form of overinvestment or greater risk taking than would occur in a first-best case.

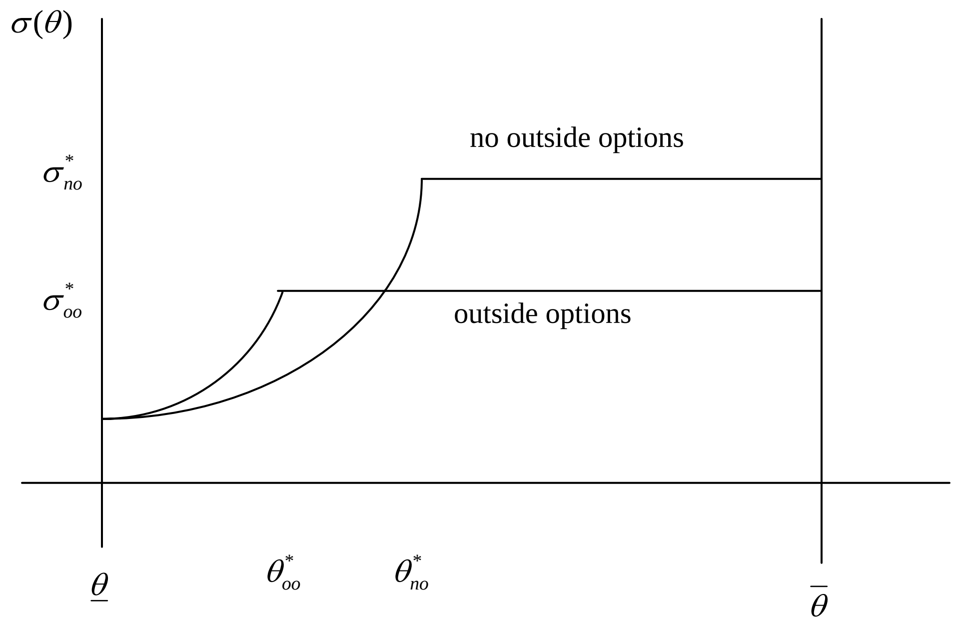
It is also instructive to contrast the conditions governing the threshold level and therefore the optimal amount of capital raised ex ante with those when the divisional manager has no increasing outside options. Comparing expression (16) to (28) we see that the only difference is in the form of the information rent term. In the following proposition, we show how this difference in information rent influences the optimal capital raised in the two cases.

Proposition 5. *When the manager has outside options, the range of states for which the risk limit is binding is greater than when outside options are nonexistent.*

Proof. See the appendix. □

The justification for the result in Proposition 5 derives from the commitment properties of capital. Compared to a symmetric information situation, underinvestment occurs in the case without outside options, while overinvestment occurs in the case with outside options. This means that in the latter case, by precommitting through the risk limit the institution mitigates the distortion problem that occurs in high states. In the former case, the risk limit plays no role in mitigating the distortion problem. This is the essential reason why the extent of limiting risk through the ex ante capital constraint is greater in the situation with outside options. The two situations are illustrated in Figure 2

Figure 2: Risk Limits in the Two Mechanisms



Our model also allows us to make predictions about capital utilization. First, for both the case of outside options and no outside options we get regions in which capital is not fully utilized. This acords well with actual experience in which risk limits are often underutilized. Second, comparing the two cases we find that the region in which capital is not fully utilized is smaller in the case with outside options. The empirical prediction is thus that underutilization of risk limits occurs more frequently in situations in which managerial skills are more easily transferrable.

The method for implementing EVA and RAROC in the case with outside options mirrors the previous case almost exactly. Once again we can define a capital allocation function,

$$\hat{T}(\hat{\theta}, \sigma) = \nu(\hat{\theta}) + \kappa(\hat{\theta})\sigma.$$

Analogous to Proposition 2 the optimal capital allocation is then determined.

Proposition 6. *Under asymmetric information, the optimal mechanism with increasing outside options may* *be implemented via a risk limit accompanied by a capital allocation schedule such that*

$$\hat{T}(\theta,\sigma) = \nu(\theta) + \left[r_D A - \mu_\sigma \gamma \frac{F(\theta)}{F'(\theta)} \right] \sigma, \tag{29}$$

where

$$\nu(\theta) = \gamma\mu(\sigma(\underline{\theta}))\underline{\theta} - \kappa(\underline{\theta})\sigma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \sigma(\hat{\theta}) d\kappa(\hat{\theta}) - \underline{U}. \tag{30}$$

It is therefore clear that RAROC is also implemented using economic capital, $EC = \alpha\sigma$, with a hurdle rate defined as:

$$r^* = r_D \left[\frac{A}{\alpha} - \frac{\gamma\mu_\sigma F(\theta)}{r_D \alpha F'(\theta)} \right]. \tag{31}$$

Once again, the hurdle rate depends on the leverage ratio times the cost of debt, adjusted for information rent. However, in this case the hurdle rate is lower than the case under symmetric information. Further, Proposition 3 applies to this case showing that the use of (31) as a hurdle rate leads to optimal performance measurement of shareholder value.

4 Multidivisional Capital Allocation

We now consider the problem of a multidivisional firm with a central authority under incomplete information. As before we derive the optimal mechanism and show how it can be implemented in a delegation environment. The basic aspects of the single divisional model are preserved in this environment.

We consider two divisions for simplicity; extension to more than two is straightforward. The expected cash flows of the two divisions are given by equation (1). The overall risk of the portfolio of investments from the two divisions is defined to be σ_p where

$$\sigma_p(\sigma_1,\sigma_2)^2 = \sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2. \tag{32}$$

For simplicity, we assume that information θ_i pertains only to trading division i and that it is independent across divisions.

The problem is formulated as a direct revelation game in which divisions each report the value of their private information subject to a Bayesian Nash incentive compatibility condition.[15] The direct revelation mechanism with two divisions is defined by the functions $\sigma_i(\theta_1, \theta_2); i = 1, 2$, denoting the risk level as a function of the information of the two divisions, and $T_i(\theta_1, \theta_2); i = 1, 2$, the capital allocation function. Information is represented by the joint distribution function, $F(\theta_1, \theta_2) \in [\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2]$, exhibiting independence with respect to (θ_1, θ_2). Given the Bayesian Nash structure of the problem, we use the following notation: a bar over a variable (e.g., $\overline{\mu_i}$) indicates that expectations are taken by division i with respect to the private information of division j. With these features, the multidivisional problem for two divisions under joint asymmetric information is:

$$\begin{aligned} \max_{\sigma_i(\theta_1,\theta_2), T_i(\theta_1,\theta_2), C} \quad & I = E[\mu_1(\sigma_1(\theta_1, \theta_2))\theta_1 + \mu_2(\sigma_2(\theta_1, \theta_2))\theta_2 - r_D(A_1\sigma_1(\theta_1, \theta_2) - A_2\sigma_2(\theta_1, \theta_2) - C) \\ & - r_E C - \overline{U}_1(\theta_1) - \overline{U}_2(\theta_2)] \end{aligned} \tag{33}$$

subject to

$$\theta_i \in \arg\max_{\hat{\theta}_i} \quad \gamma_i \overline{\mu}_i(\sigma_i(\hat{\theta}_i, \theta_j))\theta_i - \overline{T}_i(\hat{\theta}_i, \theta_j), \tag{34}$$

$$\overline{U}_i = \gamma_i \overline{\mu}_i(\sigma_i(\theta_1, \theta_2))\theta_i - \overline{T}_i(\theta_1, \theta_2) \geq \underline{U}_i + \eta_i \overline{\mu}_i(\sigma_i(\theta_1, \theta_2)\theta_i \tag{35}$$

$$C \geq \alpha\sigma_p(\sigma_1(\theta_1, \theta_2), \sigma_2(\theta_1, \theta_2)). \tag{36}$$

The objective function of the institution, (33) reflects the need to take expectations over the joint distribution of divisional types, $F(\theta_1, \theta_2)$. The set of equations embodied in (34) indicates that each division reports the "true" value of private information while taking the truthful report of the other division as given. Similarly in (35) each division must take expectations over the other division's information parameter. The total capital regulatory constraint, (36) is as before.

Given the structure of the problem, it is fairly straightforward to apply analysis similar to that of section 3, except with respect to the two divisions. The incentive compatible representation condition for equations

[15] Darrough and Stoughton (1989) discuss a joint venture mechanism design problem in which there is mutivariate private information. They derive the functional form of optimal screening mechanisms. Mookherjee and Reichelstein (1992) discuss the formulation of a multi-agent screening problem and the conditions under which implementation via *dominant* strategies is possible. Unfortunately due to the joint dependence through the portfolio effects their "condensation" condition does not hold. As a result we consider Nash implementation.

(34) is

$$\bar{U}_i(\theta_i) = \underline{U} + \int_{\underline{\theta}_i}^{\theta_i} \gamma_i \bar{\mu}(\sigma_i(\hat{\theta}_i, \theta_j)) d\hat{\theta}_i, \tag{37}$$

and the expectation (over θ_j) of the designated risk level for division i, $\bar{\sigma}_i(\theta_i)$, is non-decreasing in θ_i.

Again, when $\eta_i = 0$ so that division i enjoys no increasing outside options, by analogy with equation (15), we therefore find that the optimal joint levels of risk obtained in the multidivisional mechanism satisfies:

$$\mu_{i\sigma}(\sigma_i)\theta_i - r_D A_i - \gamma_i \mu_{i\sigma} \left[\frac{1 - F_i(\theta_i)}{F_i'(\theta_i)} \right] - \lambda(\theta_1, \theta_2) \alpha \frac{\partial \sigma_p}{\partial \sigma_i} = 0, \tag{38}$$

where F_i denotes the marginal distribution of θ_i, $\lambda(\theta_1, \theta_2)$ is the Lagrange multiplier on the total capital constraint and $\sigma_i(\theta_i, \theta_j)$ depends jointly on the two divisions' private information.

4.1 Incremental Value at Risk

To interpret these results, we now utilize the concept of *incremental value at risk* (IVaR) as defined here:

Definition The incremental value at risk, $\varsigma_i(\sigma_i, \sigma_j)$ for division i is defined as

$$\varsigma_i(\sigma_i, \sigma_j) = \alpha \sigma_i \frac{\partial \sigma_p}{\partial \sigma_i} = \alpha \frac{\sigma_i^2 + \rho \sigma_i \sigma_j}{\sigma_p}. \tag{39}$$

The incremental value at risk is proportional to the regression coefficient from a regression of the cash flows of division i on the institution's overall portfolio. Specifically, if β_{ip} is the regression coefficient, then $\varsigma_i = \alpha \beta_{ip} \sigma_p$. Not surprisingly therefore, the incremental value at risk has the property that $\alpha \sigma_p = v_1 + v_2$, i.e., the sum of the IVaRs equals the institution's overall VaR.

Substituting this definition into the first-order condition above gives the following representation for the optimal investment decision of each division:

$$\mu_{i\sigma}(\sigma_i)\theta_i - r_D A_i - \gamma_i \mu_{i\sigma} \left[\frac{1 - F_i(\theta_i)}{F_i'(\theta_i)} \right] - \lambda(\theta_1, \theta_2) \frac{\varsigma_i}{\sigma_i} = 0, \tag{40}$$

That is, investment occurs up to the point where the marginal increase in expected cash flows is balanced by the costs of capital, for both the physical investment required as well as the incremental contribution to the risk of the overall institution. In addition, due to asymmetric information, the marginal benefit is reduced by the rent paid out to the manager of division *i*, as in the case of the single division.

4.2 Implementation

In order to operationalize the above mechanism through an indirect mechanism where the risk level is delegated to each divisional manager, we propose the following multivariate mechanism: (1) the central authority asks each division manager to make a report of their information, $\hat{\theta}_i$; (2) based on the joint set of reports, the central authority selects a capital allocation function, $\hat{T}_i(\hat{\theta}_i, \hat{\theta}_j, \sigma_i)$ a function of the reports and the risk level for division *i*; (3) delegate the decision, σ_i to each divisional manager *i* so as to solve their individual economic value added as in

$$\max_{\sigma_i} \quad EVA_i = \gamma_i \mu_i(\sigma_i)\theta_i - \hat{T}_i(\hat{\theta}_i, \hat{\theta}_j, \sigma_i), \tag{41}$$

where

$$\hat{T}_i(\hat{\theta}_i, \hat{\theta}_j, \sigma_i) = \overline{\nu}_i(\hat{\theta}_i) + \kappa_i(\hat{\theta}_i, \hat{\theta}_j)\sigma_i. \tag{42}$$

That is, each division is presented with a linear capital allocation schedule with a fixed component, $\overline{\nu}_i$ that does not depend on risk taken, the reports or the action of the other division. The risk charge, κ_i, however is impacted by the joint set of reports. Generally, with better private information of division j, the risk charge for division *i* will be greater. This induces a kind of internal capital market within the financial institution.

Proposition 7 establishes the functional form of this optimal indirect mechanism. Its proof is essentially the same as that of proposition 2.

Proposition 7. *The optimal multidivisional capital allocation mechanism can be implemented by a modified IVaR schedule such that*

$$\kappa_i(\theta_i, \theta_j) = \gamma_i r_D A_i + \gamma_i^2 \mu_{i\sigma}(\sigma_i(\theta_i, \theta_j)) \frac{(1 - F_i(\theta_i))}{F_i'(\theta_i)} + \gamma_i \lambda(\theta_i, \theta_j) \frac{\varsigma_i(\theta_i, \theta_j)}{\sigma_i(\theta_i, \theta_j)}, \tag{43}$$

and

$$\overline{\nu}_i(\theta_i) = \gamma_i \overline{\mu}_i(\sigma_i(\underline{\theta}_i))\underline{\theta}_i - \overline{\kappa}_i(\underline{\theta}_i)\overline{\sigma}_i(\underline{\theta}_i) - \int_{\underline{\theta}_i}^{\theta_i} \overline{\sigma}_i(\hat{\theta}_i) d\overline{\kappa}_i(\hat{\theta}_i) - \underline{U}, \tag{44}$$

where $\overline{\kappa_i}$ denotes expectations of κ_i with respect to θ_j and likewise $\overline{\sigma_i}$ also denotes expectations with respect to the information variable of division j.

The only major difference between the single and multiple division capital allocation schedules lies in the last term in equation (43). This is the IVaR term and indicates where the interactive effect is present. Recall that in the single division case, the capital allocation mechanism is utilized for risk selection only when the capital constraint is not binding. Here in the multiple division problem the capital allocation mechanism will need to be utilized even when the capital constraint is binding. This is because even if capital is constrained, it must be allocated optimally across divisions and the externality of one division's risk choice on the other must be internalized. The 'price' of risk, $\lambda\varsigma_i/\sigma_i$ has both a common and a divisional specific component. The common component which will impact both divisions evaluation is λ, the shadow price of the capital constraint.[16] If one division contributes more in terms of IVaR than another division, its own internal price for risk will be adjusted higher. However each division takes its own risk charge as fixed and applies it to its own risk level to make the optimal joint decision from the point of the institution.

4.3 RAROC

The goal of a RAROC hurdle rate in the multiple division case is to get each division acting in its own interest to select the overall optimal level of aggregated risk for the institution. As in the single division case, this requires an initial phase in which the hurdle rate is established based on reports or selections of capital allocation mechanisms by the two divisions. However, the divisions should be judged using only their own contribution to risk. Therefore we utilize the IVaR concept in our definition of RAROC in the multidivisional situation. Let $\varsigma_i^* = \varsigma_i(\theta_i, \theta_j)$ and $\sigma_i^* = \sigma_i(\theta_i\theta_j)$ stand for the values of the IVaR and risk levels at the *optimal* risk decisions based on truthful reports. Then define RAROC as

$$RAROC = \frac{\mu_i(\sigma_i)\theta_i - \overline{\delta}_i - r^*[\varsigma_i^*/\sigma_i^*]\sigma_i}{[\varsigma_i^*/\sigma_i^*]\sigma_i}. \tag{45}$$

Considering the RAROC of the marginal project, there are two cases. Under typical circumstances, the IVaR will be positive, indicating that cash flows of division i are positively correlated with overall cash flows. Then risky investments will continue to be made as long as $\mu_{i\sigma}\theta_i - r^*(\varsigma_i^*/\sigma_i^*) > 0$. This is consistent

[16] If the capital constraint is not binding, for low joint values of θ_i and θ_j, the price will be zero.

with optimality, (38), if the hurdle rate, r^* is given by $r^*(\varsigma_i^*/\sigma_i^*) = \kappa_i/\gamma_i$. Therefore we find from (43) that the RAROC hurdle rate satisfies

$$r^* = (\sigma_i^*/\varsigma_i^*) r_D A_i + \gamma_i \mu_{i\sigma} \frac{(1 - F(\theta_i))}{F'(\theta_i)} (\sigma_i^*/\varsigma_i^*) + \lambda(\theta_i, \theta_j). \tag{46}$$

On the other hand, if the IVaR is negative, indicating that division i is serving as a type of "hedge" for division js risks, then it turns out that the optimal investment policy is to invest whenever the marginal change in the numerator of (45) is negative: $\mu_{i\sigma}\theta_i - r^*/(\varsigma_i^*/\sigma_i^*) < 0$.

There are some interesting differences implied by this hurdle rate compared to the single division case. Essentially the ratio, ς_i/σ_i, the IVaR to risk ratio, replaces α from before. This ratio can be interpreted as α times the fraction of divisional risk in the cash flows, and is normally strictly less than α. To interpret this difference, recall that the hurdle rate is used to measure performance with respect to physical capital. Suppose that the capital constraint is binding. Then, for every unit of equity capital allocated, the amount of physical investment can be increased by $1/\alpha$ in the case of a single isolated division. But in the case of multiple divisions, the amount of physical investment can increase by $1/(\alpha(\partial\sigma_p/\partial\sigma_i)) = 1/(\varsigma_i/\sigma_i)$. This implies that in ordinary circumstances a multiple division firm may utilize a greater amount of debt financing as a percentage of physical investment requirements. This greater amount of debt financing implies that the hurdle rate is higher for such divisions, *cet. par.*.

As with the single division case, there is also another positive increment to the hurdle rate from asymmetric information in this case and finally a charge for the impact of the capital constraint, λ, when it is binding. This charge is identical for both divisions i and j. Otherwise the hurdle rate will be larger for a division whose IVaR at the optimal level of risk is lower, since that division utilizes a higher percentage of debt financing for its investment activities.

4.4 Increasing Outside Options

Most of the analysis when the manager of division i has increasing outside options, $\eta_i > \gamma_i$ is handled analogously with the single division case. All of the results go through concerning the form of the capital allocation schedule; now the risk charge for allocation to division i becomes:

$$\kappa_i(\theta_i, \theta_j) = \gamma_i r_D A_i - \gamma_i^2 \mu_{i\sigma}(\sigma_i(\theta_i, \theta_j)) \frac{F_i(\theta_i)}{F_i'(\theta_i)} + \gamma_i \lambda(\theta_i, \theta_j) \frac{\varsigma_i(\theta_i, \theta_j)}{\sigma_i(\theta_i, \theta_j)}. \tag{47}$$

The RAROC hurdle rate in this case should be defined as:

$$r^* = (\sigma_i/\varsigma_i) r_D A_i - \gamma_i \mu_{i\sigma} \frac{F(\theta_i)}{F'(\theta_i)} (\sigma_i/\varsigma_i) + \lambda(\theta_i, \theta_j). \tag{48}$$

As in the single division case, this is lower than the hurdle rate without outside options in order to promote higher risk taking.

5 Implications and Conclusions

Discussions with practitioners support the view that corporate CFOs as well as their counterparts at financial institutions view capital budgeting, risk management and capital structure as being inextricably linked. They seem less concerned with the determination and maintenance of an optimal debt to equity ratio in response to capital budgeting decisions. Rather, the main issue is to optimally allocate capital to various business units or investments and manage the resulting risks for a given financial structure. Although firms focus considerable attention on this internal capital allocation process, very little research has been done to provide management with robust normative rules. That is the main purpose of our paper.

As in other theoretical work such as Kashyap, Rajan and Stein (2002) and Diamond and Rajan (2000), we view banks as both benefiting from and providing liquidity in financial markets. The perspective we take is also consistent with some of the evidence concerning capital market imperfections, internal capital markets and the impact on financial institutions. For instance, Lamont (1997) finds evidence of intrafirm investment dependence during oil shocks. In the banking context, Houston, James and Marcus (1997) find that subsidiary lending activities are more sensitive to the cash flow and capital of the holding company than to their own cash flow and capital.

Although the important work by Froot and Stein (1998) documents some of the stringent conditions required for a RAROC system to work within a financial institution, we are more sanguine about the applicability of such an approach. We find that even with asymmetric information and an agency setting in which division managers appropriate a share of cash flows revenues, there exists an appropriate linear incentive schedule which we interpret as a capital allocation mechanism. This linear incentive schedule charges each division manager with a cost of capital multiplied by the divisions actual economic capital utilization as measured by the contribution to (incremental) value at risk. Further this capital allocation mechanism can also be translated into an appropriate hurdle rate that the return on economic capital must overcome in order to be optimal. In a multidivisional setting, the central authority of the institution plays an important role in designing the appropriate channels of communication and setting the transfer price for internal capital. Nevertheless, the actual investment and risktaking decisions can be delegated in an independent manner to the respective divisions.

The optimal capital allocation system we derive also features a role for absolute position or risk limits. Risk limits are more significant in terms of the management of risk when firms are more focused in their lines of business and less diversified. This implies that the EVA/RAROC incentive scheme we discuss here will be more important for larger financial institutions with a more comprehensive scope of activities. Nevertheless, even when risk limits are utilized, we find that managers will optimally choose not to utilize them fully. We relate the usage of risk limits to the outside opportunities of managers.

In contrast to the situation with perfect capital markets, we find that the use of internal capital markets and financing imperfections implies distortions in investment policy. We relate these distortions to the environment that the division is in, as well as the outside opportunities of the manager running it. When the firm has a single division, there will generally be underinvestment in risk-taking activities as compared to the first best when the manager has little or no outside options (with another institution). On the other hand, when managers have outside options that increase sufficiently with favorable information, overinvestment obtains. This implies that hurdle rates are upwardly biased in the case of bank-specific lending activity for instance, but would be downwardly biased in the case of equity management activity, since managers may be able to retain their clients when they move to a competitor firm. However in both situations of a single divisional institution, the hurdle rate is related directly to the cost of debt (deposits) for the institution multiplied by a fixed leverage ratio.

When the institution consists of multiple divisions, the cost of capital and the associated hurdle rate has to reflect the externality implied by the joint set of risk-taking activities. We indicate here how this externality can be incorporated using the concept of incremental value at risk, which is the marginal contribution to overall value at risk from the activities of a single division. Once again, the hurdle rate for investments can be related to the cost of debt, but the leverage ratio is endogenized such that divisions that are less correlated with the overall firm cash flows use higher amounts of debt relative to firm capital. That is, there is a division-specific 'capital structure' that is implied by the optimal capital allocation process. Nevertheless, in addition to the cost of debt, there is an additional (common) charge for capital utilization across divisions. This could be one reason why actual observations of hurdle rates and target rates of return are more similar across multidivisional financial institutions than they would be in a world of perfect capital markets where divisions hurdle rates would only reflect their own business risks.

Our results concerning multidivisional firms imply a motive for bank mergers even when operational synergies may not exist. Empirical evidence such as that in DeLong (2001) has established that mergers created for diversification purposes do not enhance shareholder value while those resulting in enhanced focus do. Diversifying mergers can be rationalized in our model as a way to accomplish a more flexible approach to capital allocation when outside capital is difficult to raise on short intervals.

We believe that optimal capital allocation in the presence of capital adequacy regulation is a fruitful area for future research. For instance there are clear multiperiod effects from results in one period on the ability to raise capital in the next. Although tremendous advances have been made (and a Nobel prize awarded) recently in the area of measuring and analyzing risk of prices and portfolios held by financial institutions, questions have arisen about application of these modern methodologies. This paper represents a first step in developing a normative theory of the specialized nature of risk management in financial institutions.

A Appendix

A.1 Proof of Proposition 1

The solution to this problem is standard within the screening literature (Guesnerie and Laffont, 1984).

Using the representation of the incentive compatibility constraints (13), and integration-by-parts, we find that

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)dF(\theta) &= U(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)dU(\theta)\\ &= U(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu(\sigma(\theta))F(\theta)d\theta\\ &= U(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu(\sigma(\theta))(1-F(\theta))d\theta,\end{aligned}$$

We now rewrite (14) as follows:

$$\max_{C,\sigma(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\mu(\sigma(\theta))\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)(\alpha\sigma(\theta) - C) - \gamma\mu(\sigma(\theta))\frac{(1-F(\theta))}{F'(\theta)}\right] dF(\theta) - \underline{U}. \quad (49)$$

The determination of the optimal $\sigma(\theta)$ can now be accomplished in a pointwise manner. This gives the following set of first order conditions:

$$\mu_\sigma(\theta) - r_D A - \lambda(\theta)\alpha - \gamma\mu_\sigma(\theta)\frac{(1-F(\theta))}{F'(\theta)} = 0. \quad (50)$$

and

$$\int_{\underline{\theta}}^{\bar{\theta}} [r_D - r_E + \lambda(\theta)]dF(\theta) = 0. \quad (51)$$

When the constraint on capital is non-binding, $\lambda = 0$ and equation (50) gives the first condition, (15), in the proposition. Now write (51) as

$$(r_E - r_D)\alpha = \int_{\underline{\theta}}^{\bar{\theta}} \lambda(\theta)\alpha dF(\theta),$$

and notice that over the range where the capital constraint is binding, $\lambda(\theta) > 0$, $\sigma(\theta)$ and hence $\mu_\sigma(\sigma(\theta))$ is also constant. Because $\sigma(\theta)$ must be non-decreasing, the capital constraint is only binding over an interval

$[\theta^*, \bar{\theta}]$. From (50),

$$\lambda(\theta)\alpha = \mu_\sigma \theta - r_D A - \gamma \mu_\sigma \frac{(1 - F(\theta))}{F'(\theta)},$$

in which case

$$\begin{aligned}
&\int_{\underline{\theta}}^{\bar{\theta}} \lambda(\theta)\alpha dF(\theta) \\
&= \int_{\theta^*}^{\bar{\theta}} \left\{ \left[\theta - \gamma \frac{(1 - F(\theta))}{F'(\theta)} \right] \mu_\sigma - r_D A \right\} dF(\theta) \\
&= (1 - \gamma)\mu_\sigma \int_{\theta^*}^{\bar{\theta}} \theta dF(\theta) + \gamma \mu_\sigma (1 - F(\theta^*))\theta^* - r_D A(1 - F(\theta^*)) \\
&= (r_E - r_D)\alpha.
\end{aligned}$$

Equation (16) is a rearrangement of this latter expression. □

A.2 Proof of Proposition 2

The first statement of the proposition (equation (21)) follows from the definition of $\hat{T}$ and equation (22).

To derive the optimal ν, substitute into the definition of the objective of the division to get

$$U(\theta) = \gamma \mu(\sigma(\theta))\theta - \nu(\theta) - \kappa(\theta)\sigma(\theta)).$$

Substituting the representation of utility under incentive compatibility, (13),

$$\underline{U} + \int_{\underline{\theta}}^{\theta} \gamma \mu(\sigma(\hat{\theta}))d\hat{\theta} = \gamma \mu(\sigma(\theta)) - \nu(\theta) - \kappa(\theta)\sigma(\theta).$$

Rearranging provides the following sequence of expressions:

$$\begin{aligned}
\nu(\theta) &= \gamma \int_{\underline{\theta}}^{\theta} \hat{\theta} d\mu(\sigma(\hat{\theta})) - \kappa(\theta)\sigma(\theta) + \gamma \mu(\sigma(\underline{\theta}))\underline{\theta} - \underline{U} \\
&= \int_{\sigma(\underline{\theta})}^{\sigma(\theta)} \kappa(\sigma^{-1}(\sigma'))d\sigma' - \kappa(\theta)\sigma(\theta) + \gamma \mu(\sigma(\underline{\theta}))\underline{\theta} - \underline{U} \\
&= -\kappa(\underline{\theta})\sigma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \sigma(\hat{\theta})d\kappa(\hat{\theta}) + \gamma \mu(\sigma(\underline{\theta}))\underline{\theta} - \underline{U}.
\end{aligned}$$

Equation (22) is derived from the above equation. □

A.3 Outline of Proof of Proposition 4

First, we use the definition of V to show that

$$\int_{\underline{\theta}}^{\overline{\theta}} V(\theta)dF(\theta) = V(\overline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} \gamma\mu(\sigma(\theta))F(\theta)d\theta + \overline{\theta}\eta\mu(\sigma(\overline{\theta})) - \int_{\underline{\theta}}^{\overline{\theta}} \eta\theta\mu(\sigma(\theta))dF(\theta).$$

Then this is substituted into the objective function of (26) which can be rewritten as:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\mu(\sigma(\theta))\theta - r_D A\sigma(\theta) - (r_E - r_D)C - \lambda(\theta)(\alpha\sigma(\theta) - C) + \gamma\mu(\sigma(\theta))\frac{F(\theta)}{F'(\theta)}\right] dF(\theta) - V(\overline{\theta}) - \overline{\theta}\eta\mu(\sigma(\overline{\theta})).$$

Equation (27) is given by the pointwise derivative of this expression when $\lambda(\theta) = 0$. Equation (28) is derived in a manner similar to that of proposition 1 from the derivative with respect to C:

$$\int_{\underline{\theta}}^{\overline{\theta}} [-(r_E - r_D) + \lambda(\theta)]dF(\theta) - \overline{\theta}\eta\mu_\sigma(\sigma(\theta^*))\frac{d\sigma(\theta^*)}{dC} = 0.$$

One also uses the relationship for the region where $\theta \geq \theta^*$ that the capital constraint is binding, $C = \alpha\sigma(\theta^*)$.
□

A.4 Proof of Proposition 5

Let θ^*_{oo} be the threshold level with outside options; θ^*_{no} the threshold level with no outside options, and σ^*_{oo} and σ^*_{no} the corresponding risk limits with and without outside options.

Suppose that $\theta^*_{oo} > \theta^*_{no}$; then comparing (15) with (27), $\sigma^*_{oo} > \sigma^*_{no}$. By concavity of the cash flow return functions using (16), we know that

$$(1-\gamma)\mu_\sigma(\sigma^*_{oo}) \int_{\theta^*_{oo}}^{\overline{\theta}} \theta dF(\theta) + \gamma\mu_\sigma(\sigma^*_{oo})(1 - F(\theta^*_{oo}))\theta^*_{oo} < r_D A(1 - F(\theta^*_{oo})) + (r_E - r_D)\alpha.$$

However using (28),

$$r_D A(1 - F(\theta^*_{oo})) + (r_E - r_D)\alpha = (1-\gamma)\mu_\sigma(\sigma^*_{oo}) \int_{\theta^*_{oo}}^{\overline{\theta}} \theta dF(\theta) - \gamma\mu_\sigma(\sigma^*_{oo})F(\theta^*_{oo})\theta^*_{oo} - (\eta - \gamma)\overline{\theta}\mu_\sigma(\sigma^*_{oo}).$$

Using these last two expressions would imply that

$$\gamma\mu_\sigma(\theta^*_{oo})\theta^*_{oo} + (\eta - \gamma)\overline{\theta}\mu_\sigma(\sigma^*_{oo}) < 0$$

which is a contradition. □

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