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ABSTRACT

Cross-Border Tax Externalities: Are Budget Deficits too Small?*

In a dynamic optimizing model with costly tax collection, a tax cut by one nation creates positive externalities for the rest of the world if initial public debt stocks are positive. By reducing tax collection costs, current tax cuts boost the resources available for current private consumption, lowering the global interest rate. This pecuniary externality benefits other countries because it reduces the tax collection costs for foreign governments of current and future debt service. In the non-cooperative equilibrium, nationalistic governments do not allow for the effect of lower domestic taxes on debt service costs abroad. Taxes are too high and government budget deficits too low compared to the global cooperative equilibrium. Even in the cooperative equilibrium, complete tax smoothing is not optimal: current taxes will be lower than future taxes.

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1. Introduction

Does one country's borrowing in an integrated global financial market impose externalities on other countries? If so, are these spillovers welfare-enhancing or welfare-reducing? The issue figures prominently in the debate about the merits of the European Union's Stability and Growth Pact and is part of the debate on the merits of G3 policy coordination. Two broad classes of cross-border public debt externalities are recognised in the literature. The first are externalities associated with either the occurrence of sovereign debt default or with actions undertaken by the debtor country or by others to prevent sovereign defaults. The second are cross-border externalities associated with the transmission of national public debt policies through their effect on the global risk-free real interest rate; it is this second type of externality that is the focus of this paper.

We provide an explicitly intertemporal equilibrium model with optimising households and governments, in which public debt and the intertemporal budget constraint of the government provide an explicit link between tax decisions today and tax decisions tomorrow. Such intertemporal models are analytically difficult, especially if they do not exhibit first-order Ricardian equivalence or debt neutrality. The combination of budget constraints (where changes in asset stocks enter additively) and equilibrium determination of intertemporal relative prices (which enter multiplicatively with asset stocks) means that non-linearities are intrinsic. This has led us to specify the simplest possible 'supply side' for the national economies (a perishable endowment technology), simple household preferences, a representative infinite-lived consumer with log-linear preferences and a simple source of Ricardian non-equivalence, or absence of debt neutrality: *fiscal transfer costs.* We assume that there are increasing and strictly convex real resource costs of administering and collecting taxes.

The focus of this paper is on real interest rate cross-border spillovers that occur in the absence of sovereign default risk and without strategic interactions between a national fiscal authority and a national or supranational monetary authority. Our formal model is that of a non-monetary economy in which every national fiscal authority satisfies its intertemporal budget constraint. Government spending on goods and services is exogenous. We assume that each government can commit to a path of taxes, taking the taxes of the other governments as given. Thus, there is commitment but no international cooperation. We show this non-cooperative behaviour results in inefficient global equilibria.

Households are infinite-lived and there are no overlapping generations features that could cause problems of dynamic inefficiency. Each country's supply side is a simple endowment economy with a single perishable good. Resources are always fully utilised. There is perfect international mobility of financial capital. International transmission of national fiscal policy is only through interest rates. The assumption that prevents our model from exhibiting Ricardian equivalence, is the presence of increasing and strictly convex tax administration and collection costs.[1] Taxes are *lump-sum*; their incidence can not be altered through changes in private behaviour, but because of the strict convexity of the tax administration and collection costs, the timing of taxes matters in this model, just as it would with conventional distortionary taxes on labour income or asset income in models with endogenous labour supply and capital accumulation. In the formal model, these administration and collection costs are all located in the public sector. When taxes are negative (government subsidies or transfers) real resource costs result from private rent-seeking behaviour. Extending the model to include compliance costs

[1]Tax systems have both administrative and enforcement costs borne by the public sector and compliance, avoidance and evasion costs born by the private sector. To keep the notation simple, we have chosen to model the fiscal transfer cost here as an administrative cost, borne solely by the public sector. Slemrod and Yitzhaki (2000) report that the administrative cost of the US tax system is 0.6 cents per dollar of revenue raised. Slemrod (1996) estimates the compliance costs to be about 10 cents per dollar collected.

borne by the private tax payers, either to comply with or to avoid or evade taxes, would add notational complexity without changing our qualitative conclusions.

Without these tax administration and collection costs, our model, with its representative private agent, would exhibit Ricardian equivalence: any sequence of lump-sum taxes and debt that satisfies the intertemporal budget constraints would support the same equilibrium for any given sequence of public spending on goods and services. There would be no international spillovers.

If the representative agent assumption were replaced by that of overlapping generations without a bequest motive, alternative rules for financing a given public spending programme would give rise to pure *pecuniary* externalities if there were no tax administration and collection costs. That is, even with symmetric countries, there could be distributional effects between generations, but, as long as dynamic inefficiency does not occur, any feasible sequences of lump-sum taxes and debt support equilibria that are Pareto efficient.[2]

In the single country special case of our model, the presence of tax administration and collection costs does not give rise to an inefficiency, as long as one assumes that, in the counterfactual command economy, resource transfers between the private and public sector would be subject to the same real fiscal transfer costs as in our market economy. Inefficiency comes when there is more than one country and each country influences the choice set of the other countries in a way that is not adequately reflected in market prices.

Without costly tax administration and collection (or conventional distortionary taxes),

[2]See Buiter and Kletzer (1991). If there is dynamic inefficiency (the real interest rate is almost always below the real growth rate), then fiscal policy that causes redistribution from the young can lead to a Pareto improvements. With asymmetric countries, alternative deficit financing policies would, in general, have international as well as intergenerational distributional implications.

alternative government financing rules either have no externalities associated with them (in models such as ours, which would exhibit debt neutrality) or only purely pecuniary externalities (in OLG models). These are external effects that, first, are transmitted only through a market price - the global real interest rate and, second, do not have efficiency implications.[3] Obviously, these pecuniary externalities will have distributional consequences if some countries are net lenders while others are net borrowers.

Distributional effects from policies that change the global interest rate need not have efficiency implications. It is an implication of the first welfare theorem, that all competitive equilibria supported by different lump-sum tax-transfer and borrowing schemes are Pareto efficient.[4] This is true even if a country is large in the world capital market and exploits its monopoly power. All that is required is that taxes and transfers be lump-sum. Because our model has a representative agent and taxes are lump-sum, there would be debt neutrality without fiscal transfer costs. However, with strictly convex fiscal transfer costs, there will be interest rate spillovers. Our national economies are symmetric and, even when there is costly tax administration and collection causing alternative government financing policies to affect the global rate of interest, there are no distributional consequences. Changes in the global interest rate brought about by domestic tax policy have efficiency effects because they affect the interest bill faced by governments with outstanding debt. All governments must meet their intertemporal budget constraint and changes in the interest bill require changes in taxes, now and/or in the future. We assume that national governments maximise the welfare of their representative

[3]Again, in the case of OLG models, we rule out dynamically inefficient equilibria. With optimising governments, dynamic inefficiency does not occur in equilibrium (Buiter and Kletzer (1991)).

[4]See the previous footnote.

national consumer and do not internalise any fiscal transfer costs they may impose on foreign nations. In the presence of fiscal transfer costs, a national government's financing decision that raises the world interest rate inflicts a negative externality on the rest of the world if other governments have positive stocks of debt outstanding. Thus far our model supports the conventional wisdom.

Where our model departs radically from conventional wisdom is through the mechanism by which different government financing choices affect interest rates. The conventional wisdom associates policies that result in larger government deficits with 'financial crowding out'. That is, given public spending, larger deficits raise interest rates. In our thoroughly neo-classical intertemporal model the opposite is true. Lower taxes and larger deficits early on result in a *lower* global rate of interest.

We show that if governments are too small to affect the global interest rate, they minimise the costs of collecting taxes by smoothing the taxes over time. If they are able to influence the interest rate and countries have a positive initial stock of debt, then they set a lower tax in the initial period than in subsequent periods. This is because lower tax distortions in the initial period than in latter periods imply that aggregate consumption is higher in the initial period than in later periods. Thus, the interest rate at which the country can borrow in the initial period is lower than with perfectly smooth taxes and this lowers the debt service on its outstanding debt and, hence, future tax collection costs.

Relative to the global (cooperative) optimum, non-cooperative countries tax too much and issue too little debt in the initial period. Reducing current taxes has a positive welfare spillover, even though it requires the issuance of more debt. Lowering the current interest rate by lowering current taxes lowers the cost of servicing all countries' debt and thus reduces all countries' need to collect costly taxes. In a non-cooperative equilibrium, countries do not take into account this benefit to other countries and they tax too much in the initial period.

Our conclusion that lack of international cooperation leads to taxes that are initially too high and public deficits that are initially too small seems to contradict the presumption reflected in the debt and deficit ceilings of the Stability and Growth Pact that deficits are apt to be too large. However, we do not want to make too much of the size of the externalities associated with alternative tax and borrowing policies of national governments in EMU; even the larger EMU countries are rather small fish in the global financial pond. Our analysis is more relevant to interaction between the United States, the European Union as a whole and, possibly, Japan and China.

There are few papers analysing the welfare economics of international interest-rate spillovers from national tax and borrowing strategies of national governments using optimising sequential general equilibrium models. Hamada (1986) and Buiter and Kletzer (1991) state the problem but do not develop the excessive deficits bias issue. Kehoe (1987,1989) considers the welfare economics of international fiscal policy cooperation, but in a model where government budgets are always balanced.

In section 2 we present the model. In section 3 we extend the model to consider small l variations in the households' intertemporal elasticity of substitution. We show that as this elasticity falls, the deviation between cooperative and noncooperative taxes rises. In section 4 we consider a production economy and constant elasticity of intertemporal substitution preferences. We show that if the world economy is at a steady state with positive debt, a coordinated reduction in the current tax financed by higher future taxes improves welfare. Section 5 concludes.

2. The Model

The model comprises $N \geq 1$ countries, each inhabited by a representative infinite-lived household and a government. Each period, each household receives an endowment of the single private tradeable, non-storable consumption good and each government purchases an exogenous amount of the private good to produce a public good. The governments finance their purchases by issuing debt or by taxing their resident households. We assume that the tax system is costly to administer; the government uses up real resources collecting taxes. All savings is in the form of privately or publicly issued real bonds. We assume that households are symmetric and that endowments and government purchases are constant over time. There is perfect international integration of the national financial markets, and hence, a common world interest rate..

2.1 The households

The country-i household, $i = 1,...,N$, has preferences over its consumption path given by

$$u^i = \sum_{t=0}^{\infty} \beta^t \ln c_t^i, \tag{1}$$

where c_t^i is its period-t consumption and $\beta \in (0,1)$ is its discount factor.

The household's period-t, $t = 0,1,...$, budget constraint is

$$c_t^i + a_{t+1}^i = W - \tau_t^i + R_t a_t^i. \tag{2}$$

where a_t^i is the household's stock of assets (in the form of real bonds) at the start of period t, R_t is (one plus) the interest rate between period t - 1 and period t, W is the household's per-period endowment of the good and τ_t^i is it time-t tax bill. The household's initial assets, a_0 , are given.

In addition to satisfying its within-period budget constraint, the household must satisfy the long-run solvency condition that the present discounted value of its assets is non-negative as time goes to infinity. The transversality condition associated with its optimisation problem ensures that the present discounted value of its assets is not strictly positive. Thus,

$$\lim_{t\to\infty}\left(a_{t+1}^{i} / \prod_{s=0}^{t} R_s \right) = 0. \quad \textbf{(3)}$$

Equations (2) and (3) imply that the present discounted value of the household's consumption equals the present discounted value of its (after-tax) income plus its initial assets:

$$a_0 + \sum_{t=0}^{\infty} (W - \tau_t^i)/\prod_{s=0}^{t} R_s = \sum_{t=0}^{\infty} c_t^{\,i}/\prod_{s=0}^{t} R_s, \quad \textbf{(4)}$$

The household chooses its path of asset holdings and its consumption stream to maximise its utility function (1) subject to its intertemporal budget constraint (4). The solution to its problem satisfies equation (4) and the Euler equation

$$c_{t+1}^{\,i} = \beta R_{t+1} c_t^{\,i},\ t = 0,1,\ldots\ . \quad \textbf{(5)}$$

Solving the difference equation (5) yields the household's time-t consumption as a function of its initial consumption and the t-period interest factor

$$c_t^{\,i} = \beta^t \left(\prod_{s=1}^{t} R_s \right) c_0^{\,i},\ t = 1,2,\ldots\ . \quad \textbf{(6)}$$

Substituting equation (6) into equation (4) yields the household's initial consumption as a function of its taxes and the interest factors

$$c_0^{\,i} = (1 - \beta)\left[R_0 a_0 + W - \tau_0^i + \sum_{t=1}^{\infty} (W - \tau_t^i)/\prod_{s=0}^{t} R_s \right]. \quad \textbf{(7)}$$

Substituting equation (6) into equation (1) yields the household's indirect utility as a function of initial consumption and the interest factors

$$u^i = \ln c_0^i + (1 - \beta)\sum_{t=1}^{\infty} \beta^t \ln\left(\prod_{s=1}^{t} R_s\right), \tag{8}$$

where constants that do not affect the household's optimisation problem are ignored.

2.2 The government

The country-i, $i = 1,...,N$, government's period-t, $t = 0,1,...$, budget constraint is

$$\tau_t^i - (\varphi/2)\tau_t^{i\,2} + b_{t+1}^i = G + R_t b_t^i, \tag{9}$$

where b_t^i is the government's outstanding debt at the start of period t, $G > 0$ is its per-period purchase of the consumption good. The amount of resources used up in collecting a tax of τ (or administering a surplus of $-\tau$) is given by and $(\varphi/2)\tau^2$, where $\varphi > 0$. The government's initial debt (or credit, if negative), b_0, is given. We restrict the parameters of the model so that satisfying equation (9) is feasible; these restrictions are detailed later in this section.

In addition to satisfying its within-period budget constraint, the government also satisfies

$$\lim_{t\to\infty}\left(b_{t+1}^i / \prod_{s=0}^{t} R_s\right) = 0. \tag{10}$$

As with the household, this is an implication of the long-run solvency constraint and the transversality condition associated with the government's optimisation problem.

Equations (9) and (10) imply that the present discounted value of the government's purchases, plus its initial debt, equals the present discounted value of its tax stream, net of collection costs:

$$\tau_0^i - (\varphi/2)\tau_0^{i\,2} - g_0 + \sum_{s=0}^{\infty} [\tau_t^i - (\varphi/2)\tau_t^{i\,2} - g_t]/\prod_{s=1}^{t} R_s = 0,$$
$$\text{where } g_t := \begin{cases} G + R_0 b_0^i, & t = 0 \\ G, & t > 0. \end{cases} \quad \textbf{(11)}$$

2.3 Market clearing

Market clearing requires that the sum of the N households' asset holdings equals the sum of the N governments' debt. Thus,

$$a_t = b_t, \quad \textbf{(12)}$$

where variables without a superscript denote global averages.

The global resource constraint requires that total household consumption plus total government purchases of the good plus total resources used up paying the tax equals total endowments. Thus,

$$c_t = W - G - \frac{\varphi}{2N}\sum_{j=1}^{N} \tau_t^{j\,2}, \; t = 0,1,\dots . \quad \textbf{(13)}$$

Equation (13) is, of course, also implied by equations (2), (9) and (12).

Averaging both sides of the Euler equation (6) over the N countries gives the period-t real interest factor as a function of aggregate consumption in periods 0 and period t.

$$\prod_{s=1}^{t} R_s = \frac{c_t}{\beta^t c_0}, \; t = 1,2,\dots. \quad \textbf{(14)}$$

Equations (13) and (14) imply that in equilibrium, the time-t interest factor is solely a function of time-0 and time-t taxes.

Substituting equation (14) into equation (8) gives the country-i household's indirect utility as a function of its initial consumption and the path of aggregate consumption:

$$u^i = \ln c_0^i - \beta \ln c_0 + (1 - \beta)\sum_{t=1}^{\infty} \beta^t \ln c_t. \tag{15}$$

Substituting equations (12) and (14) into equation (7) gives the household's initial consumption as a function of the paths of taxes. The predetermined value of initial government debt enters as a parameter.

$$\begin{aligned} c_0^i &= (1 - \beta) c_0 \sum_{t=0}^{\infty} \beta^t (w_t - \tau_t^i)/c_t, \\ \text{where } w_t &:= \begin{cases} W + R_0 b_0, & t = 0 \\ W, & t > 0. \end{cases} \end{aligned} \tag{16}$$

Substituting equation (16) into the indirect utility function (equation (15)) yields

$$u^i = \ln\left[\sum_{t=0}^{\infty} \beta^t (w_t - \tau_t^i)/c_t\right] + (1 - \beta)\sum_{t=0}^{\infty} \beta^t \ln c_t. \tag{17}$$

Substituting the global resource constraint (equation (13)) into equation (17) would allow the household's indirect utility to be expressed solely as a function of the paths of taxes in the N countries.

Substituting equation (14) into equation (11) yields the country-i government's budget constraint as a function of its own taxes and aggregate consumption

$$\sum_{t=0}^{\infty} \beta^t s_t^{\,i} = 0, \text{ where}$$

$$s_t^{\,i} = \frac{\tau_t^i - (\varphi/2)\tau_t^{i^2} - g_t}{c_t}, \; t = 0,1,\ldots \,. \tag{18}$$

Substituting equation (13) into equation (17) would allow the budget constraint to be expressed solely as a function of the paths of the taxes in the *N* countries.

2.4 Taxes and revenues

With tax revenues (net of collection costs) bounded in our model and with exogenous real public consumption spending and endowments, we must impose further restrictions on the parameter space to ensure the existence of an equilibrium. Thus, we assume

$$\max\{G, g_0\} \le 1/(2\varphi), \; w_0 \ge 1/\varphi, \; W - G > 2/\varphi, \; g_0 > 0. \tag{19}$$

The net tax revenue function, $\tau - (\varphi/2)\tau^2$ looks like a Laffer curve, although its shape is the result of tax collection costs and not the distortions associated with non-lump sum taxes and subsidies. It is maximised at $\tau = 1/\varphi$ and (net) revenue equals $1/(2\varphi)$ at this point. The first inequality in assumption (19) ensures that it is possible to finance expenditures of G and g_0. This implies that an equilibrium with no government borrowing from time-1 on is feasible.

The time-0 budget is balanced with a tax of $\tau_0^- := (1 - \sqrt{1 - 2\varphi g_0})/\varphi$ or a tax of $\tau_0^+ := (1 + \sqrt{1 - 2\varphi g_0})/\varphi$. Likewise, the time-$t$, $t > 0$, budget is balanced with taxes of $\tau^- := (1 - \sqrt{1 - 2\varphi G})/\varphi$ or $\tau^+ := (1 + \sqrt{1 - 2\varphi G})/\varphi$. The taxes τ_0^- and τ^- are on the "right", or upward-sloping part of the time-0 and time-t, $t > 0$, net tax revenue curves, respectively. the taxes τ_0^+ and τ^+ are on the "wrong" or downward-sloping parts of the time-0 and time-t, $t > 0$, net tax revenue curves, respectively. There is a conventional government budget surplus in country i in period 0 if and only if $\tau_0^i \in [\tau_0^-, \tau_0^+]$ and there is a primary (that is, net of interest payments) surplus in period

t if and only if $\tau^i \in [\tau^-, \tau^+]$.

By the resource constraint, equation (13), the upper bound on feasible taxes in a symmetric equilibrium is $\bar{\tau} := \sqrt{(2/\varphi)(W - G)}$. Assumption (19) implies that the taxes τ_0^+ and τ^+ are strictly less than $\bar{\tau}$, and hence are feasible. By equations (13) and (18), a symmetric equilibrium with constant taxes has taxes of $(1 \pm \sqrt{1 - 2\varphi[G + (1 - \beta)R_0 b_0]})/\varphi$. By assumption (19), this is feasible.

We allow for negative taxes, or subsidies. In this case the collection cost is viewed as the cost of administering the surplus. We rule out, however, the empirically implausible case of an initial stock of credit that is so large that the government can achieve a balanced budget (including interest payments) in period zero with a subsidy. The necessary condition for this, $g_0 > 0$, is included in (19).

The variable s_t^i, $t = 0,1,2,\ldots$, in equation (18) can be interpreted as the period-0 value of the government's time-t budget surplus (or deficit, if negative), divided by c_0. For period 0, this surplus is the total surplus and for periods $t > 0$ it is the primary surplus. We will refer to s_t^i as country i's *discounted time-t surplus*.

From the government's budget constraint, equation (18), it appears that a rational government with market power may set taxes on the "wrong" side of the net tax revenue curve, that is, at a tax higher than $1/\varphi$, the tax that maximises net revenue. To see this suppose that $\tau_t^i = 1/\varphi$. Market power gives a country the ability to influence the global interest factor. Holding other taxes positive, a marginal increase in τ_t^i causes aggregate period-t consumption to fall. As net revenues are insensitive to taxes at $1/\varphi$, they are unaffected by a marginal increase in the tax above this point. Thus, a marginal increase in τ_t^i above $1/\varphi$ causes the discounted time-t surplus to rise.

We have the following result. Proofs of this and all other propositions are in the

Appendix.

***Proposition* 1.** *Given* τ_t^j, $j \neq i$, s_t^i *has a unique maximum in* τ_t^i. *The maximising tax is decreasing in the number of countries; as* $N \to \infty$ *it goes to* $1/\varphi$. *If* $t > 0$, *the maximising tax is an element of* $[1/\varphi, \tau^+]$ *and* s_t^i *is increasing (decreasing) below (above) the maximising tax on* $[\tau^-, \tau^+]$. *If* $t = 0$, *the maximising tax is an element of* $[1/\varphi, \tau_0^+]$ *and* s_0^i *is increasing (decreasing) below (above) the maximising tax on* $[\tau_0^-, \tau_0^+]$.

Denote the tax that maximises s_t^i when taxes are symmetric and there are N countries by τ_0^{*N} if $t = 0$ and by τ^{*N} if $t > 0$. If $N = 1$ and taxes are symmetric then the maximising tax is $w_0 - \sqrt{w_0^2 - \tau^2}$ if $t = 0$ and it is $W - \sqrt{W^2 - \tau^2}$ if $t > 0$. The relationship between important time-t values is shown in Figure 1. The position of the corresponding time-0 values is similar.

We now show that it cannot be part of an equilibrium for any government to ever set a tax above the one that maximises its discounted surplus. The strategy of the proof is to show that if it did so, it could always pick a tax on the other side of the net tax revenue curve that would provide the same discounted surplus and higher utility.

Proposition 2. *It cannot be part of an equilibrium for government i to set its time-t tax higher than the one that maximises* s_t^i.

3. Dynamic Optimal Taxation

We assume that at time zero, the government in country i can commit to a tax plan $\{\tau_t^i\}_{t=0}^{\infty}$. It takes the tax plans of the other governments as given and maximises the indirect utility of its household (equation (17)) subject to its budget constraint (equation (18)).

We begin analysing the problem by considering the effects of time-t taxes on welfare and the government's fiscal position. By equations (13) and (17), the marginal change in utility from a marginal increase in the time-t tax is

$$\frac{\partial u^i}{\partial \tau_t^i} = \beta^t m_t^i, \ where \ m_t^i := \frac{-1 + \dfrac{\varphi\tau_t^i}{N}\dfrac{w_t - \tau_t^i}{c_t}}{c_t \sum_{s=0}^{\infty} \beta^s \dfrac{w_s - \tau_t^s}{c_s}} - \frac{(1-\beta)\varphi\tau_t^i}{Nc_t}, \ t = 0,1,... \ . \qquad \textbf{(20)}$$

Let taxes be constant across countries. Then equation (16) implies $\Sigma_{t=0}^{\infty}\beta^t(w_t - \tau_t)/c_t = 1/(1 - \beta)$. Equation (13) and the definition of s_t (in equation (18)) implies $(w_t - \tau_t)/c_t = 1 - s_t$. These results and equation (20) imply that with symmetric taxes

$$m_t^i = m_t = -\frac{1-\beta}{c_t}\left(1 + \frac{\varphi\tau_t s_t}{N}\right), \ t = 0,1,... \ . \qquad \textbf{(21)}$$

We next show that the marginal (indirect) utility of taxes must be strictly negative.

***Proposition* 3**. *A symmetric equilibrium must have* $m_t < 0$, $t = 0,1,...$.

Holding other taxes constant, an increase in time-t taxes has the direct effect of lowering consumer income in period t and this tends to lower welfare. Suppose $\tau_t > 0$. Then a tax increase lowers available global resources in period t because of the higher collection costs. Thus, it raises the relative price of consumption in period t. This interest rate effect has a positive effect on welfare if consumers lend to the government in period t ($s_t < 0$). The direct dominates the interest rate effect; an increase in the time-t tax lowers within-period welfare at time t. Suppose $\tau_t < 0$. Then a tax increase *increases* available resources in period t because of the lower administration costs. Thus, the relative price of consumption in period t falls. As consumers must be lending to the government in this case, the interest rate effect also tends to lower welfare and marginal utility must be negative.

By equations (13) and (18), the marginal increase in the discounted value of the budget surplus (the left-hand side of equation (18)), that results from a marginal increase in the time-t tax when taxes are identical across countries is

$$\beta^t n_t,\ where\ n_t := (1/c_t)(1 - \varphi\tau_t + \varphi\tau_t s_t/N),\ t = 0,1,\ldots\ . \tag{22}$$

We show that in equilibrium, a marginal increase in the time-t tax must increase the discounted value of the government's stream of budget surpluses.

***Proposition* 4**. *A symmetric equilibrium must have* $n_t > 0$, $t = 0,1\ \ldots$.

After the first period, the government smooths taxes.

***Proposition* 5.** *A symmetric Nash equilibrium has constant taxes after period zero.*

Let $\tau_t^i = \tau^i$, $c_t^i = c^i$, and $s_t^i = s$, $t > 0$. Then equations (17) and (18) imply that the government maximises

$$\ln[(1 - \beta)(w_0 - \tau_0^i)/c_0 + \beta(W - \tau^i)/c] + (1 - \beta)\ln c_0 + \beta\ln c \tag{23}$$

subject to

$$B^i := (1 - \beta)s_0^i + \beta s^i = 0. \tag{24}$$

By equations (13) and (23), the marginal utilities associated with τ_0^i and τ^i are

$$m_0^i = -\frac{1-\beta}{c_0}\left(\frac{1 - \frac{\varphi\tau_0^i}{N}\frac{w_0 - \tau_0^i}{c_0}}{C} + \frac{\varphi\tau_0}{N}\right),\ m_1^i = -\frac{\beta}{c}\left(\frac{1 - \frac{\varphi\tau^i}{N}\frac{w - \tau^i}{c}}{C} + \frac{\varphi\tau}{N}\right), \tag{25}$$

where $C := (1 - \beta)(w_0 - \tau_0^i)/c_0 + \beta(W - \tau^i)/c$,

respectively.

By equations (13) and (24), the marginal increases in B^i associated with increases in τ_0^i

and τ^i are

$$n_0^i = \frac{1 - \beta}{c_0}\left(1 - \varphi\tau_0^i + \frac{\varphi\tau_0^i s^i}{N}\right), \; n_1^i = \frac{\beta}{c}\left(1 - \varphi\tau^i + \frac{\varphi\tau^i s^i}{N}\right), \tag{26}$$

respectively.

If the countries act symmetrically, then equation (16) implies that $C = 1$. Equation (13) implies that $(W - \tau_0)/c_0 = 1 - s_0$ and $(W - \tau)/\tau = 1 - s$. These results and equations (25) and (26) imply

$$\begin{aligned} m_0 &= -\frac{1 - \beta}{c_0}\left(1 + \frac{\varphi\tau_0 s_0}{N}\right), \; m_1 = -\frac{\beta}{c}\left(1 + \frac{\varphi\tau s}{N}\right) \\ n_0 &= \frac{1 - \beta}{c_0}\left(1 - \varphi\tau_0 + \frac{\varphi\tau_0 s_0}{N}\right), \; n_1 = \frac{\beta}{c}\left(1 - \varphi\tau + \frac{\varphi\tau s}{N}\right). \end{aligned} \tag{27}$$

The first-order conditions for an equilibrium imply $m_1/m_0 = n_1/n_0$. This and equations (27) imply[5]

$$\tau_0/(1 + \varphi\tau_0 s_0/N) = \tau/(1 + \varphi\tau s/N). \tag{28}$$

Symmetry and equation (24) imply

$$(1 - \beta)s_0 + \beta s = 0. \tag{29}$$

The second-order condition requires that the bordered Hessian matrix associated with the optimisation problem has a strictly positive determinant. This requires

[5]In deriving equation (28), both sides were divided by φ. If $\varphi = 0$, the timing of taxes is irrelevant as long as the government satisfies its intertemporal budget constraint.

$$n_0(m_{00}m_1^2 - 2m_{01}m_0m_1 + m_{11}m_0^2) - m_0(n_{00}m_1^2 + n_{11}m_0^2) < 0, \qquad \textbf{(30)}$$

where m_{t0} and m_{t1} are the derivatives of m_t^i (as given by equation (25)) with respect to τ_0^i and τ^i, respectively, when taxes are symmetric and where n_{t0} and n_{t1} are the derivatives of n_t^i (as given by equation (26)) with respect to τ_0^i and τ^i, respectively, when taxes are symmetric. It is straightforward, but *exceedingly* tedious to demonstrate that symmetric taxes which satisfy equations (28) and (29) also satisfy equation (30).[6]

Definition 1. *A symmetric* ***equilibrium*** *is a pair of taxes* $\{\tau_0,\tau\}$ *such that*

(*i*) *the feasibility condition* (29) *is satisfied*

(*ii*) *the optimality condition* (28) *is satisfied.*

We first establish that taxes are always positive.

***Proposition* 6**. *An equilibrium cannot have subsidies*. ($\tau_0 < 0$ *or* $\tau < 0$).

We analyse the equilibrium by graphing equations (29) and (28) in Figure 2. This figure is drawn for strictly positive taxes that are less than the ones that maximise the within-period discounted surpluses as we have shown that no other taxes can be part of an equilibrium.

The feasibility condition (29) is represented by the solid curves F_-, F, and F_+; the different curves representing different initial stocks of debt. By Proposition 4, these curves are downward sloping; an increase in the future tax allows the government to reduce the current tax and still balance its budget.[7] The curve representing an initial debt of zero, F_0, goes through the point (τ^-,τ^-), labelled A. The curve corresponding to a strictly positive stock of initial debt, F_+, lies above the curve with zero debt and the curve corresponding to a negative stock of initial debt, F_-,

[6]Details available on request.

[7]The curves are drawn as convex to the origin. This is true if N is sufficiently large, but need not be true otherwise.

lies below it. Both curves pass through the point $(\bar{\tau}_0, \bar{\tau})$. With a positive initial stock of debt, this point lies above *A*; with a negative initial stock of debt it lies below *A*.

The curves representing equation (28) in Figure 2 are represented by the dashed lines. The curve O_0 represents the case of no initial debt or an infinite number of countries. The curves labelled $O_+^{N'}$ and $O_+^{N''}$ represent the case strictly positive initial debt and $N = N'$ and $N = N''$, respectively, where $1 \leq N'' < N' < \infty$; the curves labelled $O_-^{N'}$ and $O_-^{N''}$ represent the case strictly negative initial debt when $N = N'$ and $N = N''$, respectively.

***Proposition* 7**. *The curves representing the optimality conditions in Figure* 2 *have the following properties:*

(*i*) *The curve O_0 is the* 45° *line.*

(*ii*) *All of the optimality curves are upward sloping and pass through the origin.*

(*iii*) *$O_+^{N'}$ and $O_+^{N''}$ lie below the* 45° *line; $O_+^{N'}$ and $O_+^{N''}$ lie above the* 45° *line.*

(*iv*) *$O_+^{N'}$ lies above $O_+^{N''}$ when $\tau > \bar{\tau}$ and $\tau_0 < \bar{\tau}_0$; $O_-^{N'}$ lies below $O_-^{N''}$ when $\tau < \bar{\tau}$ and $\tau_0 > \bar{\tau}_0$.*

The intuition behind the optimality curves in Figure 2 is that the government trades off two objectives. First, it wants to smooth consumption by smoothing taxes, and hence tax distortions, over time. If this were its sole objective, optimality would be represented by O_0. Second, it wants to lower the discounted value of the tax collection costs through its influence on the global rate of interest. If it has an initial stock of debt, it does this by lowering initial taxes and raising future taxes. Through the global resource constraint (equation (13)) this raises initial consumption and lowers future consumption, thus lowering the interest rate on government debt between periods zero and one. Thus, its required tax revenue falls. Likewise, if the government is an initial creditor it can lower its required discounted tax revenue, and thus its tax collection costs, by raising initial taxes and lowering future taxes, thus raising the interest rate on government savings between periods zero and one.

This second objective means that the curve representing the optimality condition in Figure 2 is flatter than O_0 when there is initial debt and it is steeper than O_0 when there is an initial surplus. The more market power a country has (that is, the smaller is *N*) the greater is its ability to affect the global interest rate and the more important this second motive becomes. Thus, as the number of countries falls, the optimality curve becomes flatter if the country is an initial debtor and steeper if the country is an initial creditor. When $N \to \infty$ countries have no market power. Only the first objective matters and the optimality equation is represented by O_0.

Equilibrium occurs at the intersection of the relevant feasibility and optimality curves. We show that a unique intersection must occur.

***Proposition* 8.** *A unique symmetric equilibrium exists.*

Different equilibria are represented by the points *A* - *G* in Figure 2. Point *A* is the equilibrium when there is no initial stock of debt. In this case there is tax smoothing and the budget is balanced each period. Points *B*, *C* and *D* represent equilibria when there is a positive stock of initial debt. If $N = \infty$, the equilibrium is represented by point *B* and there is tax smoothing. Points *C* and *D* lie below the 45° line; hence, if $N < \infty$ and there is a positive initial stock of debt, $\tau > \tau_0$. As *N* falls, the negative slope of the curve representing equation (29) ensures that the initial tax declines and the future tax rises.

Likewise, points *E*, *F* and *G* represent equilibria when there is an initial negative stock of debt. If $N = \infty$ (point *G*), there is complete tax smoothing. Points *E* and *F* lie above the 45° line; hence, if $N < \infty$ and there is a negative initial stock of debt, $\tau > \tau_0$. As *N* falls, the initial tax rises and the future tax falls.

These results are summarised below.

***Proposition* 9.** *If countries have no market power* ($N = \infty$) *or if the value of the initial debt is zero, then there is complete tax smoothing. If countries have some market power* ($N < \infty$), *then the* *initial tax is strictly less (greater) than the subsequent tax if there is a strictly positive (negative) stock of initial debt.*

When countries have no market power, we derive the same result as Barro (1979). Taxes result in resource losses, here because they are costly to collect and in Barro (1979) because they are distortionary. If these costs are convex, then an optimising government smooths them over time. If, however, the government can affect the interest rate and it has an initial stock of debt, then it lowers the discounted value of its required tax revenue by reducing initial taxes and raising future taxes. If it is an initial creditor it raises its return to its savings by increasing the initial tax and lowering future taxes.[8]

The case of $N = 1$ corresponds to the social planner's outcome. Hence, we have the following result.

***Proposition* 10.** *Suppose that* $N > 1$. *If there is a positive (negative) initial stock of debt, then the initial tax is too high (low) relative to the social optimum. The subsequent tax is too low (high) relative to the social optimum.*

If there is a positive stock of initial debt, lowering initial taxes causes a positive externality by decreasing all country's borrowing costs. Countries do not take into account the social benefit and they do not decrease initial costs enough.

4. CES Preferences

The log-linear preference specification of the previous section is a special case of CES

[8]If the government begins with a strictly positive (negative) initial stock of debt, then Proposition 9 says that the initial tax is lower (higher) than subsequent taxes. This implies that the government enters period one with a strictly positive (negative) stock of initial debt. Thus, if the government could re-optimise, beginning in period one, Proposition 9 implies that it would set a lower (higher) tax in period one then in later periods. This implies that the equilibrium, which features constant taxes from period one on, is not time consistent unless $R_0 b_0 = 0$. The time inconsistency arises because the initial debt is taken as exogenous, and hence, unaffected by taxes.

preferences for an elasticity of intertemporal substitution equal to one. In this section, we look at how small changes in the value of the elasticity of substitution in the neighbourhood of 1 effect the results of the last section. Let

$$u^i = \frac{1}{1-\theta}\sum_{t=0}^{\infty} \beta^t ({c_t^i}^{1-\theta} - 1),\ 0 < \beta < 1,\ \theta > 0, \tag{31}$$

where θ is the reciprocal of the elasticity of intertemporal substitution. As $\theta \to 1$, the above preferences become the logarithmic specification of the previous sections. We assume that θ is arbitrarily close to one.[9]

Given the above preferences, the Euler equation of the consumer's optimisation problem becomes

$$c_{t+1}^i = (\beta R_{t+1})^{1/\theta} c_t^i,\ t = 0,1,\dots\ . \tag{32}$$

Solving the difference equation (32) yields the household's time-*t* consumption as a function of its initial consumption and the interest rate

$$c_t^i = \left(\beta^t \prod_{s=1}^{t} R_s\right)^{1/\theta} c_0^i,\ t = 1,2,\dots\ . \tag{33}$$

Averaging both sides of equation (33) across countries yields

[9] It is easy to generalise the results of the last section to $\theta < 1$, and by continuity arguments, to θ within a right-hand side neighbourhood of one. It appears analytically intractable to extend them to θ sufficiently greater than one. In this section we are concerned with marginal changes at $\theta = 1$.

$$\prod_{s=1}^{t} R_s = \frac{1}{\beta^t}\left(\frac{c_t}{c_0}\right)^{\theta}, \; t = 1,2,\ldots\,. \tag{34}$$

Substituting equations (33) and (34) into the household's budget constraint yields

$$\frac{c_0^i}{c_0} = \sum_{t=0}^{\infty} \beta^t \frac{w_t - \tau_t^i}{c_t^{\theta}} \Big/ \sum_{t=0}^{\infty} \beta^t c_t^{1-\theta}, \; t > 0 \tag{35}$$

Substituting equations (33) - (35) into equation (31) and ignoring constants that are unimportant to the optimisation problem yields the indirect utility function

$$\frac{1}{1-\theta}\left(\sum_{t=0}^{\infty} \beta^t \frac{w_t - \tau_t^i}{c_t^{\theta}}\right)^{1-\theta}\left(\sum_{t=0}^{\infty} \beta^t c_t^{1-\theta}\right)^{\theta}. \tag{36}$$

Substituting equation (34) into the government's budget constraint (equation (11)) yields

$$\sum_{s=0}^{\infty} \beta^t \hat{s}_t^{\,i} = 0, \text{ where } \hat{s}_t^{\,i} = \frac{\tau_t^i - (\varphi/2)\tau_t^{i\,2} - g_t}{c_t^{\theta}}, \; t = 1,2,\ldots\,.. \tag{37}$$

In the previous section we demonstrated that a symmetric equilibrium must have constant taxes after period zero. Substitute $\tau_t = \tau$ into equations (36) and (37) . Then the optimisation problem of the government is to choose τ_0 and τ to maximise

$$[(1-\beta)(w_0 - \tau_0^i)/c_0^{\theta} + \beta(W - \tau^i)/c^{\theta}]^{1-\theta}[(1-\beta)c_0^{1-\theta} + \beta c^{1-\theta}]^{\theta}/(1-\theta) \tag{38}$$

subject to

$$(1-\beta)\hat{s}_0^{\,i} + \beta\hat{s}^{\,i} = 0. \tag{39}$$

The first-order conditions evaluated at a symmetric equilibrium imply

$$\tau_0/(1 + \varphi\theta\tau_0 s_0/N) = \tau/(1 + \varphi\theta\tau s/N). \quad \textbf{(40)}$$

By equation (39) and symmetry

$$(1 - \beta)\hat{s}_0 + \beta\hat{s} = 0. \quad \textbf{(41)}$$

The feasibility constraint and the optimality condition are represented graphically in Figure 3. In this figure, F^k represents the feasibility constraint and O^k represents the optimality constraint for the case of $\theta = \theta^k$, $k = 0,1$, where $\theta^0 < \theta^1$. The properties of the curves are summarised in the following proposition.

***Proposition* 11**. *The curves F^0 and F^1 are downward sloping and intersect at $(\bar{\tau}_0, \bar{\tau})$ and on the* 45° *line. F^0 lies above F^1 to the left of $(\bar{\tau}_0, \bar{\tau})$ and below the* 45° *line; it lies below F^1 to the right of $(\bar{\tau}_0, \bar{\tau})$ and above the* 45° *line. The curves O^0 and O^1 are upward sloping and lie below the* 45° *line. The curve O^1 lies below O^0.*

To see the properties of the feasibility curves, first suppose there is no initial debt. Then increasing θ would make the government's tradeoff over feasible current and future taxes more favourable. To see the intuition suppose that the government is running a time-zero deficit. It finances this deficit by borrowing. With lower period-zero taxes than future taxes, consumption is higher in period zero than in period one. Consumers smooth their consumption by lending to governments in period zero. This higher is θ, the lower is the intertemporal elasticity of substitution and the greater is their desire to smooth their consumption. Thus, the higher is θ, the less costly is it for the government to trade off future tax increases for tax cuts in period zero. The intuition is similar for the case of an initial surplus.

When $R_0 b_0 > 0$, the government's tradeoff is more favourable with a higher value of θ

than with a lower value of θ if consumption is higher is higher in the period in which the government runs a deficit. With an initial positive stock of debt, however, it is possible for the country to be running a deficit in period zero, even though consumption is lower in period zero than in period one. This corresponds to the parts of the curves between the two intersecting points. In this case, reducing current taxes requires higher future taxes and the higher is θ the higher are these future taxes. Consumption is made less smooth by the government's borrowing and the higher is θ, the more the government must pay to borrow.

To see the shape of the optimality curves, suppose there is an initial stock of government debt and that taxes are constant across periods. Then the governments run a deficit in the current period and must borrow. If first-period taxes were lowered, this would increase current consumption and lower the interest rate that the government must pay on its debt. If this interest rate effect is taken into account, then taxes will be lower in the first period than if the interest rate effect is not taken into account. This is the argument of the previous section.

The lower is θ, the less consumers want to smooth their consumption and the less is the interest rate effect. Thus, the bigger is θ the greater is the socially optimal reduction in the first-period tax below the feasible constant tax.

Given Proposition 11 we have the following.

***Proposition* 12.** *Suppose that $N > 1$. If there is a positive initial stock of debt, then the initial tax is too high relative to the social optimum and the subsequent taxes are too low relative to the social optimum. An increase in θ causes the socially optimal value of the initial tax to fall.*[10]

As well as considering marginal changes in θ around one, we can consider the polar cases

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[10]As noted, this theorem is for marginal changes at $\theta = 1$. It is straightforward to generalise it to large changes for $\theta < 1$, but it is not analytically tractable to consider large changes above one.

where θ goes to zero and to infinity. In the limit as θ falls to zero, there is no interest rate effect as consumers do not care at all about smoothing their income. Thus, the socially optimal and uncoordinated outcomes coincide and taxes are smoothed over time. In the limit as θ goes to infinity, indifference curves for current and future consumption become right angles and only the minimum consumption matters. If $R_0 b_0 > 0$, cooperating governments should borrow marginally less than their outstanding debt in period one and set the current tax marginally higher than the one that balances the future primary deficit. Current consumption is then marginally lower than future consumption so the required gross interest rate on the borrowing is zero. The future tax is thus the one that balances the future primary deficit. Minimum consumption over the current and future can be made arbitrarily close to $W - G - (\varphi/2)\tau^{-2}$.

4. Production and capital accumulation

An important simplifying feature of the model is that varying the pattern of taxes over time (and thereby changing the pattern of real fiscal transfer costs over time) is the only way to transfer real resources between periods. Households and governments make saving decisions, but in equilibrium, net global saving (private plus public) is always zero and investment is always zero for each individual country because real goods are perishable. Reducing taxes in any given period will increases the real resources available that period. In equilibrium, private consumption in that period will therefore increase. In our benchmark model, the interest rate on current savings falls as a result of the current tax cut.

In this section, we extend the model to allow for production. When capital formation is added to the model, real resources can be transferred between periods not only by shifting the pattern of taxation (and of fiscal transfer costs) but also by capital formation.

Formally, we assume that the household has the CES preferences of the last section and that the single good in the model is both a capital and a private and public consumption good.

The representative households each supply one unit of labour inelastically each period and save both bonds and the output of the current good in the form of capital. The savings of capital are loaned to the firms to be used in the next-period's production process. The firms transform capital and labour into output via a Cobb-Douglas production function with a capital share of output of $\alpha \in (0,1)$. Then if k is the capital-labour ratio, the output per unit of labour is $f(k) = Ak^{\alpha}$, where $A > 0$. We suppose that labour is immobile across countries, physical capital is perfectly mobile and capital depreciates completely. Then perfect mobility of capital and perfect competition imply that capital-labour ratios and wages are equalised across countries and $k_t = k(R_t) = [A(1 - \alpha)/R_t]^{1/\alpha}$.

A symmetric equilibrium is characterised by the Euler equation (32), the government budget constraint (37) and the global resource constraint, which is now

$$f(k(R_t)) - G - (\varphi/2)\tau_t^2 - c_t - k(R_{t+1}) = 0, \; t = 0,1,... \; . \qquad \textbf{(42)}$$

The model with capital is far more difficult to analyse than the one without. To obtain an analytical result, we restrict ourselves to a simple experiment. Imagine that the world is at a symmetric steady state with constant taxes and a positive initial stock of debt. Can policy makers raise welfare with a coordinated symmetric marginal tax cut?

Proposition 13. *Suppose countries are at a symmetric steady state with constant taxes and strictly positive debt. Then it is possible to increase welfare with a coordinated marginal tax cut in the current period.*

We show in the proof that welfare is improved if the current tax cut is financed with future tax rises that leave consumption constant from period one on. The intuition is that lowering the current tax and raising future taxes raises current consumption and lowers future consumption, thus lowering the current interest rate as in the previous sections. This lowers the cost of servicing the debt and reduces future tax collection costs. To see that the interest rate must fall, suppose that it did not. Then next period's marginal product of capital will rise so current capital accumulation falls. With lower tax collection costs and fixed current output, this implies current consumption rises. This is inconsistent with the interest rate falling in the current period unless next period's, and hence every future period's, consumption rises by more than current consumption. However, with lower current capital accumulation and higher future tax collection costs this is impossible. Thus, we have a contradiction.

Conclusion.

We have demonstrated that, in our baseline model, optimising governments will perfectly smooth taxes if they have no market power or if they have no initial debt. If countries are large enough to affect the world interest, then they will set lower taxes in the current period than in the future if they have a positive initial stock of debt. If they have an initial stock of credit they will set higher taxes in the current period than in the future.

We show that, relative to the first-best, co-operative outcome, with positive initial debt, countries set their current taxes too high. Thus, relative to the optimum, initial budget deficits are too low. Similarly, if countries are initial creditors, initial budget deficits are too high.

We extend our baseline model, which features log-linear preferences, to the case of CES preferences. We show that a marginal fall in the intertemporal elasticity of substitution increases the deviation between the uncoordinated outcome and the first-best outcome; a marginal rise decreases the deviation. We also consider the case of production and capital accumulation. We show that if there is a steady state with constant taxes and strictly positive debt, then it is possible to increase welfare with a coordinated cut in the current tax.

Appendix

Proof of Proposition 1. We show this for $t > 0$; the proof for $t = 0$ is similar.

A tax that maximises s_t^i must be in $[\tau^-,\tau^+]$ and it must satisfy

$$\frac{ds_t^{\,i}}{d\tau_t^i} = \frac{1 - \varphi\tau_t^i}{c_t} + \frac{\varphi\tau_t^i s_t^{\,i}}{Nc_t} = 0 \tag{43}$$

$$d^2 s_t^{\,i}/d\tau_t^{i^2} = \frac{2\varphi\tau_t}{Nc_t}\frac{ds_t^{\,i}}{d\tau_t^i} - \frac{\varphi}{c_t} + \frac{\varphi s_t^{\,i}}{Nc_t} < 0. \tag{44}$$

When $\tau_t^i \in [\tau^-,\tau^+]$, then $\tau_t s_t^{\,i} \geq 0$; hence a solution of (43) must have $\tau_t^i \geq 1/\varphi$. When $\tau_t^i = 1/\varphi$, $ds_t^{\,i}/d\tau_t^i > 0$; when $\tau_t^i = \tau^+$, $ds_t^{\,i}/d\tau_t^i < 0$; hence (43) has a solution on $[1/\varphi,\tau^+]$. By (42) and (43), $d^2 s_t^{\,i}/d\tau_t^{i^2} = 1/(\tau_t^i c_t^{\,i}) < 0$ at this solution; hence, the solution is unique and it is a maximum. For $\tau_t^i \in [\tau^-,1/\varphi]$, $ds_t^{\,i}/d\tau_t^i > 0$; hence, $s_t^{\,i}$ is increasing below the maximising tax on $[\tau^-,\tau^+]$ and decreasing above the maximising tax.

It is obvious from (43) and (44) that the maximising tax is decreasing in N and goes to $1/\varphi$ as N goes to ∞.

Proof of Proposition 2. Suppose to the contrary that $\exists\, t > 0$ such that at least one of the countries sets its tax on the "wrong" side of the net tax revenue curve. Without loss of generality, suppose the country with the highest tax is country i and let $\tau_t^i = \tau^W$. We suppose $t > 0$; the argument for $t = 0$ is similar. Let $\hat{\tau}$, the average tax in the other countries in period t, be given. Let s^W be the value of $s_t^{\,i}$ at τ^W. Let τ^* be the tax that maximises $s_t^{\,i}$, let the value of $s_t^{\,i}$ at this tax be s^*, let the positive value of τ_t^i for which $c_t = 0$ be $\bar{\tau}^*$. Then $\tau^W > \tau^*$ and $s^W \in (-\infty, s^*)$. We have that $s_t^{\,i}$, maps $(-\tau^*,\tau^*)$ onto $(-\infty, s^*)$; hence, $\exists\, \tau^R \in (-\bar{\tau}^*,\tau^*)$ such that $s_t^{\,i} = s^W$ at τ^R. By equation (18), a switch from τ^W to τ^R has no effect on the government's intertemporal budget constraint and, hence, does not require a change in any other tax. Thus, the government prefers τ^R to τ^W if indirect utility (given by (17) is higher when $\tau_t^i = \tau^R$ than when $\tau_t^i = \tau^W$.

We have $\tau^{R^2} < \tau^{W^2}$; hence, $c^R > c^W$, where c^k is c_t when $\tau_t^i = \tau^k$, $k = R,W$ Thus, by (17), indirect utility is higher when $\tau_t^i = \tau^R$ than when $\tau_t^i = \tau^W$ if $(W - \tau_t^R)/c^R > (W - \tau^W)/c^W$. By (13),

$$\frac{W - \tau^k}{c_t^k} = 1 - s^W + \frac{N-1}{N}\frac{\varphi}{2}\frac{\hat{\tau}^2 - \tau^{k^2}}{c_k}. \tag{45}$$

Thus, we need to show that $(\hat{\tau}^2 - \tau^{R^2})/c^R > (\hat{\tau}^2 - \tau^{W^2})/c^W$. By (13), this is true iff $\hat{\tau} < \overline{\tau}$ which must be the case as the other countries have lower taxes than country *i*. As the country with the highest tax cannot set its tax on the wrong side of the net tax revenue curve, then neither can any other country.

Proof of Proposition 3**.** Suppose $t > 0$. By (21), m_t cannot be positive unless $\tau_t s_t < 0$. By Proposition 1, $\tau_t \in]-\overline{\tau},\tau^{*N}]$; hence, m_t cannot be positive unless $\tau_t \in]0,\tau^-]$. In this case, $m_t < 0$ if $L(\tau_t) := W - G + (\varphi/2)\tau_t^2 - (\varphi^2/2)\tau_t^3 - \varphi G\tau_t > 0$. The function L has an interior minimum at $\underline{\tau}$ iff $\exists\underline{\tau}$ such that $L'(\underline{\tau}) = \underline{\tau} - (3\varphi/2)\underline{\tau}^2 - G = 0$ and $L''(\underline{\tau}) = 1 - 3\varphi\underline{\tau} > 0$. If $1 - 6\varphi G < 0$, then no such $\underline{\tau}$ exits and $L(0) = L(\tau^-) = W - G > 0$ ensures the proposition holds. If $1 - 6\varphi G \geq 0$, then an interior minimum exists. It is sufficient to show that L is strictly positive at this point. Using $(3\varphi/2)\underline{\tau}^2 = \underline{\tau} - G$, we have $L(\underline{\tau}) = W - G + (\varphi/2)\underline{\tau}(\underline{\tau} - \varphi\underline{\tau}^2 - 2G) = (1/3)[3(W - G) + (\varphi/2)\underline{\tau}(\underline{\tau} - 4G)]$ $= (1/9)[9W - 10G + (1 - 6\varphi G)\underline{\tau}]$. Assumption (19) ensures $9W > 10G$; hence this is true. The proof for $t = 0$ is similar.

Proof of Proposition 4. Suppose to the contrary that $\exists u \geq 0$ such that, $n_u < 0$. By Proposition 1 and 2, $\tau_u < \tau^-$ (or $\tau_0 < \tau_0^-$ if $u = 0$). Thus, $s_u^i < 0$ and there must be some period v where $s_v^i > 0$. By propositions 1 and 2 $\tau_v \in (\tau^-,\tau^{*N}]$ (or $\tau_0 \in (\tau_0^-,\tau_0^{*N}]$ if $v = 0$) and $n_v > 0$. Suppose that the government of country *i* were to lower τ_u^i marginally and to change τ_v^i to satisfy its intertemporal budget constraint. Then $d\tau_v^i = -\beta^{u-v}(n_u/n_v)d\tau_u^i < 0$. By marginally lowering taxes in both periods, Proposition 3 ensures that utility rises. Thus τ_u cannot be part of an equilibrium. This is a contradiction.

*Proof of Proposition 5***.** Suppose to the contrary that $\tau_u < \tau_v$, $u,v \geq 1$. If the government of country i marginally increases τ_u, changing τ^v to satisfy its budget constraint, then $du^i = \beta^u m_u d\tau_u^i + \beta^v m_v d\tau_v^i$, where $d\tau_v^i = -\beta^{u-v}(n_u/n_v)d\tau_u$. Then $du^i = \beta^u(m_u - m_v n_u/n_v)d\tau_u^i$. By Proposition 1, $n_u > 0$; hence, country i would defect from the equilibrium if $m_u/n_u > m_v/n_v$. This is true if m_t/n_t is strictly decreasing in τ_t. By (21) and (22), this is true if $\tau_t/(1 + \varphi\tau_t s_t/N)$ is increasing in τ_t. This is true if $1 > (\varphi\tau_t^2/N)\, ds_t/d\tau_t$. Proposition 4 and (18) ensure that $ds_t/d\tau_t > 0$ in an equilibrium. Thus, if $\tau_t < 0$ the result must hold. Suppose $\tau_t > 0$. We show the result holds when $t > 0$; the proof for $t = 0$ is similar.

The result is true if $1 > (\varphi\tau_t^2/N)(W - G - \varphi\tau_t w + \varphi\tau_t^2/2)/c_t^2$ on $[0,\tau^{*1}] \supseteq [0,\tau^{*N}]$. If the left-hand side is negative, this is true. If it is positive, it is true if it is true for $N = 1$. This is true if $G(\tau) := (\bar{\tau}^2 - \tau^2)^2 - 2\tau^2(\bar{\tau}^2 - 2W\tau + \tau^2) > 0\ \forall\tau \in [0,\tau^{*1}]$. For G to have an interior minimum on $[0,\tau^{*1}]$ requires $(3W - \sqrt{9W^2 - 8\bar{\tau}^2})/2 \leq \tau^{*1}$. By the definition of τ^{*1}, this is impossible. $G(0) = \bar{\tau}^4 > 0$ and $G(\tau^{*1}) = (\bar{\tau}^2 - \tau^{*1^2})^2 > 0$; hence $G > 0\ \forall\tau \in [0,\tau^{*1}]$.

Proof of Proposition 6. Assumption (19) rules out negative taxes in both periods. Rearranging (28) yields $N(\tau_0 - \tau) = \varphi\tau\tau_0(s_0 - s)$. Substituting in $s_0/s = -\beta/(1 - \beta)$ (from (29)) yields

$$N(1 - \beta)(\tau - \tau_0) = \varphi\tau\tau_0 s. \tag{46}$$

Suppose $\tau > 0 > \tau_0$. Then (19) and (29)imply $s > 0$. The left-hand side of (46) is positive, the right-hand side is negative. This is a contradiction. Suppose $\tau_0 > 0 > \tau$. Then $s < 0$, the left-hand side of (46) is negative and the right-hand side is positive. This is a contradiction.

Proof of Proposition 7. Let the right-hand side of (28) be represented by $h(\tau;N) := \tau(1 + \varphi\tau s/N) > 0$ for $\tau \in [0,\tau^{*1}]$; the left-hand side by $h_0(\tau_0;N) := \tau(1 + \varphi\tau_0 s_0/N) > 0$ for $\tau_0 \in [0,\tau_0^{*1}$.] These functions have the following properties:

(a) $h(0;N) = h_0(0;N) = 0$

(b) $dh(\tau;N)/d\tau > 0$; $dh_0(\tau_0;N)/d\tau_0) > 0$. (The first inequality follows from an argument in the proof of Proposition 5, the second by a similar argument)

(c) $h(\tau;N') > (=,<)\ h(\tau;N'')$ when $\tau < (=,>)\ \bar{\tau}$, $h_0(\tau_0;N') > (=,<)\ h_0(\tau_0;N'')$ when $\tau_0 < (=,>)\ \bar{\tau}_0$.

(d) $h_0(\tau;N) > (=,<)\ h(\tau;N)$ when $R_0 b_0 > (=,<)\ 0$.

Result (i) follows from (d) and $h_0(\tau) \rightarrow h(\tau)$ when $N \rightarrow \infty$. Result (ii) follows from (a) and (b). Result (iii) follows from (b) and (d). Result (iv) follows from (b) and (c).

Proof of Proposition 8. Uniqueness follows from the strictly negative slope of the feasibility curves and the positive slope of the optimality curves.

Suppose $R_0 b_0 \geq 0$. An equilibrium fails to exist iff the F_+ curve lies above O_+^N at the highest period-1 tax that is consistent with equilibrium, τ^{*N}. By (43), this tax satisfies $1 - \varphi\tau^{*N} + \varphi\tau^{*N} s^{*N}/N = 0$, where s^{*N} denotes s, evaluated at this tax. Thus, the right-hand side of (28) evaluated at τ^{*N} equals $1/\varphi$. For (28) to hold, (43) implies that $\tau_0 = \tau_0^{*N}$. Assumption (19) implies that surpluses are positive in both periods at the point (τ_0^{*N}, τ^{*N}). Hence, this point must lie above F_+ and an equilibrium exists. The proof for $R_0 b_0 < 0$ is similar.

Proof of Proposition 11. Let $f(\tau) := \tau - (\varphi/2)\tau^2 - g$, $f_0(\tau) := \tau - (\varphi/2)\tau^2 - g_0$, $c(\tau) := W - G - (\varphi/2)\tau^2$ and $B(\tau_0,\tau;\theta) := (1 - \beta)f_0(\tau_0)/c(\tau_0)^\theta + \beta f(\tau)/c(\tau)^\theta$. Then F^k represents the feasibility constraint when $\theta = \theta^k$, $k = 0,1$. Clearly $B(\bar{\tau}_0,\bar{\tau};\theta^0) = B(\bar{\tau}_0,\bar{\tau};\theta^1) = 0$ and $B(\tau,\tau;\theta^0) = B(\tau,\tau,\theta^1)$. Thus F^0 and F^1 pass through $(\bar{\tau}_0,\bar{\tau})$ and intersect on the 45° line. Let (τ_0',τ') be such that $B(\tau_0',\tau';\theta^0) = 0$. Then $B(\tau_0',\tau';\theta^1) = [\beta f(\tau')/c(\tau_0')^{\theta_1}]\{[c(\tau_0')/c(\tau')]^{\theta_1} - [c(\tau_0')/c(\tau')]^{\theta_0}\} > 0$ iff $\tau' > (<)\ \bar{\tau}$ and $c(\tau_0')/c(\tau') > (<)\ 1$. This is true iff $\tau' > \bar{\tau}$ and $\tau_0' < \tau'$ or $\tau' < \bar{\tau}$ and $\tau_0' > \tau'$.

Redefine the curves h and h_0 in Proposition 7 as $h(\tau;\theta) := \tau(1 + \theta\varphi\tau s/N)$ and $h_0(\tau_0;\theta) := \tau(1 + \theta\varphi\tau_0 s_0/N)$. It is easy to establish that property (b) in Proposition continues to hold if $\theta < 1$ and (by a continuity argument) if θ is sufficiently close to one. In addition, property (d) holds and h is decreasing in θ when $\tau > \tau^-$ and h_0 is increasing n θ when $\tau_0 < \tau^-$. These properties ensure that the O^0 and O^1 have the stated properties.

Proof of Proposition 13. Suppose the initial tax is less than 1/φ, and hence is on the "right" side of the tax revenue curve. If this were not true, with constant taxes welfare could be improved by moving to the lower revenue-equivalent tax. Let the initial period be denoted by $t = 0$. Suppose the coordinated marginal fall in the initial tax $d\tau_0 < 0$ is financed by a sequence of future tax changes $\{d\tau_t\}_{t=1}^{\infty}$ such that $dc_t^i = dc_t = dc,\ t = 1,2,...$. Differentiating (31) and evaluating at the initial steady state yields

$$dU^i = \sum_{t=0}^{\infty} \frac{\beta^t dc_t^i}{c_t^{i\,1-\theta}} = \frac{dc_0}{c^{1-\theta}} + \frac{\beta}{1-\beta}\frac{dc}{c^{1-\theta}} > 0 \Leftrightarrow dc_0 + \frac{\beta dc}{1-\beta} > 0 \qquad \textbf{(47)}$$

Differentiating (37) and evaluating at the initial steady state yields

$$\sum_{s=0}^{\infty} \beta^t \frac{(1-\varphi\tau_t^i)d\tau_t^i}{c_t^\theta} - \theta \sum_{s=0}^{\infty} \beta^t \frac{s_t^i dc_t}{c_t^\theta} = 0 \Leftrightarrow$$

$$(1-\varphi\tau)\sum_{t=0}^{\infty}\beta^t d\tau_t - \left(s - \frac{R_0 b_0}{c}\right)dc_0 - \frac{\beta s dc}{1-\beta} = 0 \qquad \textbf{(48)}$$

$$\text{where } s = \frac{\tau - (\varphi/2)\tau^2 - G}{c}.$$

At a steady state the gross interest rate must equal 1/β. Thus, evaluating (37) at the steady state yields $s = (1-\beta)b_0/(\beta c)$. Substituting this into (48) yields

$$(1-\varphi\tau)\sum_{t=0}^{\infty}\beta^t d\tau_t + b_0 dc_0/c - b_0 dc/c = 0. \qquad \textbf{(49)}$$

Differentiating (32) and evaluating at the steady state yields

$$dR_1 = \theta(dc - dc_0)/(\beta c), \; dR_t = 0 \;\; t = 2,3... \, . \tag{50}$$

Differentiating (42), employing $f'(k_t) = R_t$ and $dk_t/dR_t = -k_t/(\alpha R_t)$, substituting in (42) and (49) and evaluating at a steady state yields

$$\begin{aligned} \varphi\tau d\tau_0 &= \theta(dc - dc_0)k_1/(\alpha c) - dc_0 \\ \varphi\tau d\tau_1 &= -\theta(dc - dc_0)k_1/(\alpha\beta c) - dc \\ \varphi\tau d\tau_t &= -dc, \; t = 2,3... \, . \end{aligned} \tag{51}$$

Substituting (51) into (48) yields

$$dc/dc_0 = -\left(\frac{1-\varphi\tau}{\varphi\tau} - \frac{b_0}{c}\right) \Big/ \left(\frac{1-\varphi\tau}{\varphi\tau}\frac{\beta}{1-\beta} + \frac{b_0}{c}\right). \tag{52}$$

Substituting (52) into (47) yields that utility rises if and only if true iff

$$b_0 > 0, \; \frac{1-\varphi\tau}{\varphi\tau}\frac{\beta}{1-\beta} + \frac{b_0}{c} > 0. \tag{53}$$

The second expression is true by $\tau < 1/\varphi$. This gives us our result.

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Figure 1. Important Tax Values

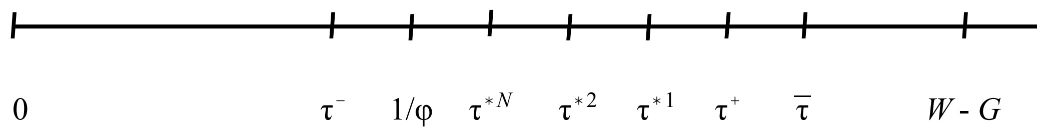


Figure 2. Equilibrium Taxes

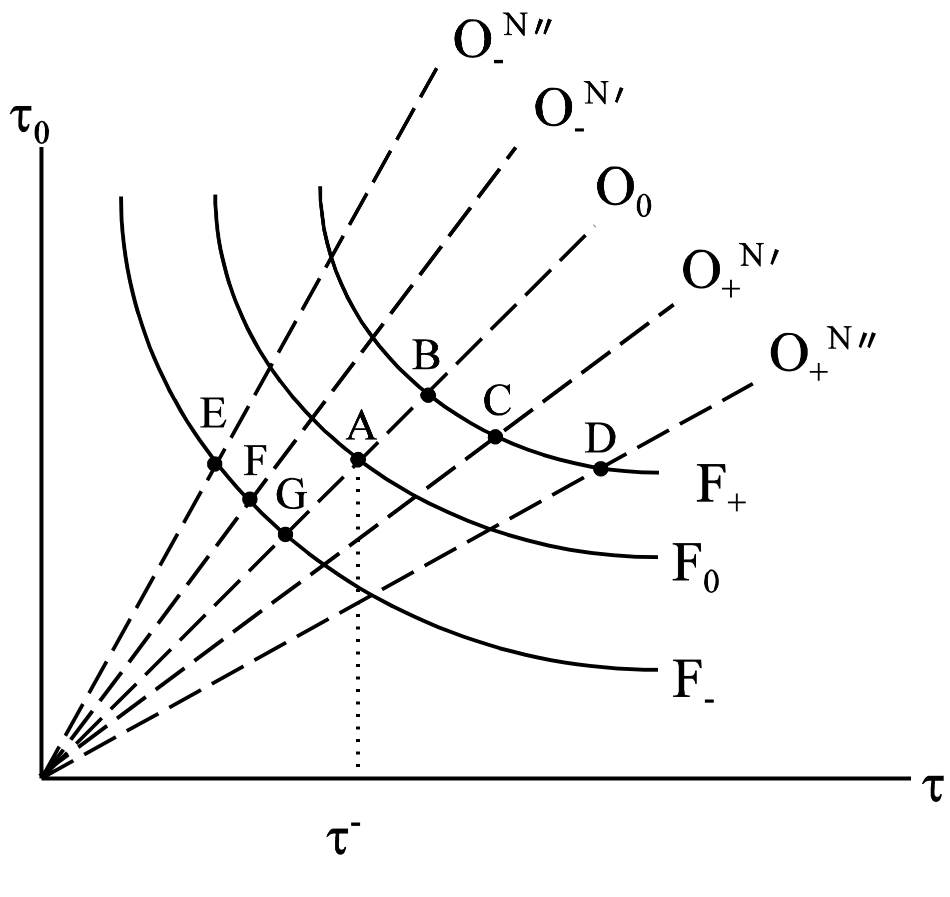
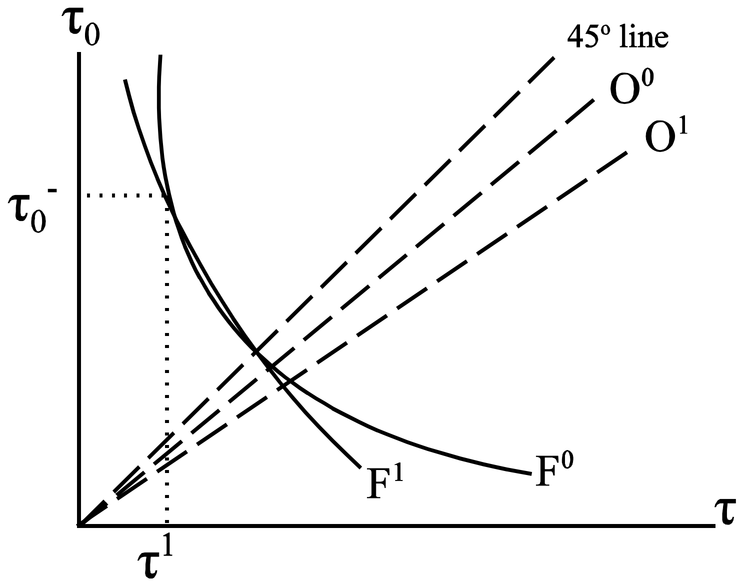


Figure 3 Equilibrium with CES Preferences and Initial Debt



$\theta^0 < \theta^1$