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ABSTRACT

Wage Bargaining with On-The-Job Search: Theory and Evidence*

The Nash wage bargaining model is ubiquitous in modern labour economics. Yet most applications of this model ignore inter-employer competition for labour services and attribute all of the workers’ rent to their bargaining power. In this Paper, we write and estimate an equilibrium model with strategic wage bargaining and on-the-job search and use it to take another look at the determinants of wages in France. There are three essential determinants of wages in our model: productivity, competition between employers resulting from on-the-job search, and the workers’ bargaining power. We find that between-firm competition matters a lot in the determination of wages, as it is quantitatively more important than wage bargaining à la Nash in raising wages above the workers’ ‘reservation wages’, defined as out-of-work income. In particular, we detect no significant bargaining power for intermediate- and low-skilled workers, and a modestly positive bargaining power for high-skilled workers. In addition, the Paper provides some empirical information on the nature of sorting of workers by firms.

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1 Introduction

Given productivity, the mechanisms that determine a worker’s wage can be categorized into two broad classes. The first class contains the various forms of between-employer competition for labor services. Models of the labor market based on perfect competition, wage posting or Bertrand competition recognize between-firm competition as the workers’ main rent provider (Burdett and Mortensen, 1998; Moen, 1997; Postel-Vinay and Robin, 2002a,b; Burdett and Coles, 2003). The second class encompasses all “noncompetitive” wage setting mechanisms. In particular, workers can bargain for their wage with their employers either individually or through a trade union, or they can take advantage of information asymmetries to force employers to pay efficiency wages. Within the latter class, the (Nash) bargaining model is—by far—the one that has received most attention from applied labor economists in recent years. In particular, the famous Mortensen-Pissarides model of matching and bargaining has had an enormous influence on both micro and macro labor economics over the last 15 years (see Pissarides, 2000 or Mortensen and Pissarides, 1999 for literature reviews).1

While both competitive and noncompetitive mechanisms have been extensively analyzed and evaluated, it is striking to observe that those two classes of wage theories have always been considered separately, and that no attempt has been made so far at assessing their relative importance in a unified framework. This is what we want to do in this paper.

More specifically, we construct and estimate a structural labor market equilibrium model in which workers explicitly negotiate over wages with their employers, and are also allowed to bring several would-be employers into competition through on-the-job search. When an employed worker receives an outside job offer, a (3-player) bargaining process is started between the worker, his/her initial employer and the employer who made the outside offer. We explicitly model this bargaining process using a somewhat modified version of the Osborne and Rubinstein

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1Matching models of the labor market are in fact consistent with virtually any wage setting mechanism. However, it is most frequently combined with some form of the Nash bargaining model.
finite horizon bargaining game. Between-employer competition for labor services and wage bargaining thus consistently interact as determinants of wages in our approach, which offers a synthetic theoretical framework that introduces labor market competition in the context of an otherwise conventional wage bargaining model.

Our approach is closely related to the paper by Dey and Flinn (2000). Like us, they represent the negotiation process by the Nash bargaining solution in the presence of between-firm competition for workers. Their idea is that a worker who is currently employed at a wage $w_0$ and receives an outside offer bargains with the poaching firm on the basis that if the negotiation fails the current wage contract $w_0$ will prevail. Let $w_1$ be the wage thus negotiated with the poacher. The worker uses this offer $w_1$ as an outside option to negotiate a new wage, $w_2$, with his/her current employer. A sequence of bilateral Nash bargaining games is played instantaneously until one of the firms can no longer bid up, that is when the sequence of wage offers has reached the smallest firm reservation value or match productivity. Our contribution provides an explicit non-cooperative bargaining game that relies on a precise definition of the strategic interactions at work in the wage renegotiation. Moreover, Dey and Flinn consider a more complex framework, with multidimensional employment contracts stipulating wages and health insurance provisions, that does not yield wage equations allowing the estimation of the worker’s bargaining power. Our economic model is simple enough to deliver closed-form wage equations, relating a worker’s wage to this worker’s innate ability, the productivity levels of his/her current and past employers, the frequency at which s/he receives outside job offers (which is a measure of the intensity of labor market competition), and of course on his/her bargaining power.

Beyond the qualitative analysis, our main goal in this paper is a quantitative assessment of the relative importance of labor market competition and bargaining in wage determination. We estimate our structural model on a 1993-2000 panel of matched employer-employee French administrative data. This data contains firm-level information on value added, wages and hours
worked by labor category (based on occupation). One of the important empirical novelties of this paper is that we are able to use wage data on one side and productivity data on the other and see whether our wage equation correctly captures the link between the two. Since it is precisely the link between wages and productivity that identifies the workers’ bargaining power, it clearly matters that both be observed if one wants to obtain credible bargaining power estimates.\(^2\)

Our estimated model proves to correctly capture the relationship between wages and labor productivity. In particular, we find that firm-level mean wages are below labor productivity, with a mark-up increasing from zero at low-productivity firms to about 100% at high-productivity firms.

Our main findings, though, indicate that between firm competition puts significant upward pressure on wages, thus permitting the workers to obtain a substantially larger share of the job surplus than what their mere bargaining power would predict. This phenomenon is particularly important for the “unskilled” workers (workers with no managerial tasks) who tend to have very low bargaining power—between 0 and 0.2, on a \([0, 1]\) interval, depending on the particular industry and labor category considered—whereas “skilled” workers (supervisors, managers of all ranks and engineers) generally have somewhat higher bargaining power—between 0.2 and 0.4.\(^3\)

That labor market competition is found to matter a lot in wage determination in France can sound somewhat surprising. The reputed “sclerosis” of the French labor market, corroborated by its rather low estimated rates of job-to-job transitions leads to the presumption that the intensity of labor market competition is not what French employers should complain about. An

\(^2\)The conventional approach in the structural job search literature—which is forced by the absence of data on value added—is to use the structural wage equation to infer productivity from wages, as in Eckstein and Wolpin’s (1995) seminal paper (see Mortensen, 2002, for a survey). Even though the structure of our wage equation theoretically allows identification of the workers’ bargaining power even without resorting to productivity data, it would take strong faith in the model’s structure to believe in the estimates obtained with that approach. More on this below.

\(^3\)Up to the remarkable exception of top-skilled workers in the Construction sector for which we estimate a bargaining power of 0.97. This special finding is discussed in more detail in the paper.
unexpected implication of our model, however, is that a relatively mild level of labor market
competition—i.e. a relatively low frequency of outside job offers accruing to employed workers—
is enough to put strong upward pressure on wages.

These results suggest that most existing empirical studies overestimate the bargaining power. Those studies (a far from exhaustive list of which includes Abowd and Lemieux, 1993; Blanchflower, Oswald and Sanfrey, 1996; Van Reenen, 1996; Margolis and Salvanes, 2001; Kramarz, 2002), are based on static models where some bargaining process leads to splitting the job surplus, typically defined as the difference between productivity and an outside wage that depends on worker characteristics and selected labor market variables such as the (local) unemployment rate and the industry- or economy-wide mean wage. This whole approach is based on the prior that the surplus share obtained by workers is entirely explained by their bargaining power. Our contribution shows that this is not the case and that between-firm competition for labor services also plays a prominent role in the wage setting process as it raises the workers’s threat point in bargaining.

The plan of the paper is as follows. In the following section, we develop formal non cooperative negotiation and renegotiation games which allow us to express wages as functions of worker ability, firm productivity, matching frictions and the bargaining power of workers. The empirical implementation of the model is presented in section 3. We use the framework of section 2 to estimate the influence of productivity, between-firm competition and the bargaining power of workers on wages. Because it is particularly important to separately identify the different sources of wage dispersion, we choose to implement a multi-stage estimation procedure. We first estimate the friction parameters (arrival rate of job offers, job destruction rate) from worker data on employment spell durations. We then use a firm panel of data on value-added and employment differentiated by skill to estimate labor productivity at the firm level. Lastly, we relate mean wages per firm to productivity to estimate bargaining powers. Section 4 presents some empirical applications related to the evaluation of the bargaining power, the importance
of between-firm competition and the allocation of workers across firms. Section 5 concludes.

2 Theory

We first describe the characteristics and objectives of workers and firms. The matching process and the negotiation game that workers and firms play to determine wages is then explained. In the third and last subsection, the steady-state general equilibrium of this economy is characterized.

2.1 Workers and firms

We consider a labor market in which a measure $M$ of atomistic workers face a continuum of competitive firms, with a mass normalized to 1, that produce one unique multi-purpose good. Time is continuous, workers and firms live forever. The market unemployment rate is denoted by $u$. The pool of unemployed workers is steadily fueled by layoffs that occur at the exogenous rate $\delta$.

Workers have different professional skills. A given worker’s ability is measured by the amount $\varepsilon$ of efficiency units of labor s/he supplies per unit time. The distribution of ability values in the population of workers is exogenous, with cdf $H$ over the interval $[\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]$. We only consider continuous ability distributions and designate the corresponding density by $h$.

Summing over all employee ability values for a given firm defines the efficient firm size. The marginal productivity of efficient labor is denoted as $p$. Firms differ in the technologies that they operate, meaning that parameter $p$ is distributed across firms with a cdf $\Gamma$ over the support $[p_{\text{min}}, p_{\text{max}}]$. This distribution is assumed continuous with density $\gamma$. The marginal productivity of the match $(\varepsilon, p)$ of a worker with ability $\varepsilon$ and a firm with technology $p$ is $\varepsilon p$.

A type-$\varepsilon$ unemployed worker receives an income flow of $\varepsilon b$, with $b$ a positive constant, which s/he has to forgo upon finding a job. Being unemployed is thus equivalent to working at a “virtual” firm with labor productivity equal to $b$ that would operate on a Walrasian labor market, therefore paying each employee their marginal productivity, $\varepsilon b$. 


Workers discount the future at an exogenous and constant rate $\rho > 0$ and seek to maximize the expected discounted sum of future utility flows. The instantaneous utility flow enjoyed from a flow of income $x$ is $U(x) = x$.\textsuperscript{4} Firms maximize profits.

2.2 Matching and wage bargaining

Firms and workers are brought together pairwise through a sequential, random and time consuming search process. Specifically, unemployed workers sample job offers sequentially at a Poisson rate $\lambda_0$. As in the original Burdett and Mortensen (1998) paper, employees may also search for a better job while employed. The arrival rate of offers to on-the-job searchers is $\lambda_1$. The type $p$ of the firm from which a given offer originates is assumed to be randomly selected in $[p_{\min}, p_{\max}]$ according to a sampling distribution with cdf $F$ (and $F \equiv 1 - F$) and density $f$. The sampling distribution is the same for all workers irrespective of their ability or employment status. Note that we a priori assume no connection between the probability density of sampling a firm of given type $p$, $f(p)$, and the density $\gamma(p)$ of such types in the population of firms. When a match is formed, the wage contract is negotiated between the different parties according to the rules that we now explain.

The bargaining games. Wages are bargained over by workers and employers in a complete information context. In particular, all agents that are brought to interact by the random matching process are perfectly aware of one another’s types. All wage and job offers are also perfectly observed and verifiable. Specifically, we make the following three assumptions about wage strategies and wage contracts:

Assumption 1 Wage contracts stipulate a fixed wage that can be renegotiated by mutual agreement only.

\textsuperscript{4}This is merely for simplicity. The theoretical model is tractable with an arbitrary utility function (provided that intertemporal transfers are ruled out), and the empirical analysis can in principle be conducted for any CRRA specification (see Postel-Vinay and Robin, 2002b).
Assumption 1 implies that renegotiations occur only if one party can credibly threaten the other to leave the match for good if the latter refuses to renegotiate. In our framework, renegotiations can be triggered only when employees receive outside offers. The assumption of renegotiation by mutual agreement captures an important and often neglected feature of employment contracts (see the enlightening survey by Malcomson, 1999).

The following two assumptions describe the structure of the negotiation game that is played by an unemployed worker and an employer (Assumption 2), and that of the renegotiation game that is played by a currently employed worker, his/her current employer and a poaching employer (Assumption 3).

**Assumption 2** When an unemployed worker meets a firm, the wage is determined according to the following bargaining game:

1. The firm makes a wage offer;

2. The worker either accepts the offer and signs the contract, or s/he rejects it;

3. In case of rejection at step 2, some time elapses. Then:
   - With probability $\beta$, the worker makes a wage offer;
   - With probability $1 - \beta$, the firm makes a wage offer;

4. The player who has received the offer at step 3 either accepts it and signs the contract, or rejects it. In case of rejection the match ends and the worker remains unemployed.

**Assumption 3** An employed worker who receives an outside job offer renegotiates his/her wage according to the following game:

1. The firms make simultaneous noncooperative wage offers;

2. The worker either chooses one wage offer and signs a new contract or keeps the pre-existing contract;
3. If the worker has chosen one wage offer at step 2, some time elapses. Then the players can participate in the following game:

- With probability $\beta$, the worker makes separate wage offers to both employers;
- With probability $1 - \beta$, the firms make simultaneous non-cooperative wage offers.

4. Any player who has received an offer at step 3 either accepts or rejects it. In case of disagreement at step 4, the worker’s decision at step 2 prevails. In case of agreement between the worker and either firm, a new contract is signed. The worker chooses among the firms if both accept the offer s/he made at step 3.

Assumptions 2 and 3 describe two very simple strategic negotiation games adapted from Osborne and Rubinstein (1990). The seminal contributions of Binmore, Rubinstein and Wolinsky (1986) and Osborne and Rubinstein (1990) have shown that the Nash sharing rule can be derived from strategic bargaining games that are very useful to properly define the threat payoffs. Obviously, any strategic bargaining game is necessarily peculiar. Our game has been designed to provide a simple and tractable tool to understand the renegotiation process in the presence of between-firm competition for workers.

The negotiation game that is played between two firms and an initially employed worker looks like the game between a firm and an unemployed worker except that the former has three players instead of two. Steps 1 and 2 have been modified to enable the worker to maneuver in order to get him/herself an optimal credible threat point in the renegotiation subgame (steps 3 and 4). Namely, if the worker accepts the offer of the poaching firm at step 2, s/he quits the incumbent firm and this offer becomes his/her threat point in the renegotiation. Conversely, his/her threat point is the offer of the incumbent employer if that offer is accepted at step 2. This game can appear somewhat unrealistic at first glance, as it gives the employee the option to momentarily quit his/her initial employer to eventually return with a new contract at the end of the renegotiation. Such back-and-forth worker movements don’t happen in the real world.
Neither do they in our game, as we wish to emphasize, since temporarily quitting to a less attractive employer is only a threat available for the worker to use, which is never implemented in equilibrium.

It is also worth insisting on the fact that whenever the worker receives an outside offer, the pre-existing contract with the incumbent employer prevails if no agreement is reached (at step 2). This is an important difference with the negotiation on new matches—between unemployed workers and firms—that are dissolved in case of disagreement. We view this assumption as more in accordance with actual labor market institutions than the usual one according to which matches always break up in case of renegotiation failure (Pissarides, 2000, Mortensen and Pissarides, 1999). It is indeed legally considered in most OECD countries, and especially in France, that an offer to modify the terms of a contract does not constitute a repudiation. Accordingly, a rejection of the offer by either party leaves the pre-existing terms in place, which means that the job continues under those terms if the renegotiation fails (Malcomson, 1999, p. 2321).

**Wage and job mobility.** We now exploit the preceding series of assumptions to derive the precise values of wages and the job mobility patterns.

The subgame perfect equilibria of the two bargaining games described above are characterized in Appendix A.1. In both games the worker receives a share $\beta$ of the match rent. Let $V_0(\varepsilon)$ denote the lifetime utility of an unemployed worker of type $\varepsilon$ and $V(\varepsilon, w, p)$ that of the same worker when employed at a firm of type $p$ and paid a wage $w$. The rent of a match between a type-$\varepsilon$ unemployed worker and a type-$p$ job amounts to $V(\varepsilon, \varepsilon p, p) - V_0(\varepsilon)$.\(^5\) It is shown in the Appendix that the wage bargained on a match between a type-$\varepsilon$ unemployed worker and a

\(^5\) $V(\varepsilon, \varepsilon p, p)$ is the match value for the worker when s/he gets paid his/her marginal productivity $\varepsilon p$. In this case, the employer makes zero marginal profit on this worker who therefore receives the entire match surplus. $V(\varepsilon, \varepsilon p, p)$ is thus equal to the match surplus. If the match is not consummated, the worker’s only option is to stay in unemployment, which has a value of $V_0(\varepsilon)$, and the employer shuts down the job, therefore making a value of 0. It follows that the total match surplus is equal to $V(\varepsilon, \varepsilon p, p) - V_0(\varepsilon)$.\(^5\)
type-\( p \) firm, denoted as \( \phi_0(\varepsilon, p) \), solves:

\[
V(\varepsilon, \phi_0(\varepsilon, p), p) = V_0(\varepsilon) + \beta \left[ V(\varepsilon, \varepsilon p, p) - V_0(\varepsilon) \right].
\]

This equation merely states that a type-\( \varepsilon \) unemployed worker matched with a type-\( p \) firm gets his reservation utility, \( V_0(\varepsilon) \), plus a share \( \beta \) of the job surplus.

The assumption of long term contracts, renegotiated by mutual agreement only, implies that wages can be renegotiated only if employees receive new job offers. We thus now describe what happens when an employee paid a wage \( w \) in a type-\( p \) firm receives an outside offer from a type-\( p' \) firm.

- If \( p' \leq p \), the worker stays at the type-\( p \) firm, because the match with the type-\( p' \) firm is associated with a lower rent. However, the employee can get wage increases if \( p' \) is sufficiently high in regard of his/her current wage, \( w \). If the employee triggers a renegotiation (by accepting the poacher’s first offer at step 2), s/he eventually stays at his/her initial firm (the type \( p \) firm) with a new wage \( \phi(\varepsilon, p', p) \) as defined by:

\[
V(\varepsilon, \phi(\varepsilon, p', p), p) = V(\varepsilon, \varepsilon p', p') + \beta \left[ V(\varepsilon, \varepsilon p, p) - V(\varepsilon, \varepsilon p', p') \right].
\]

Obviously, the employee decides to trigger a renegotiation only if it is a way to get a wage increase, i.e. if the productivity parameter of the new match, \( p' \), exceeds a threshold value \( q(\varepsilon, w, p) \), that satisfies:\footnote{Note that}

\[
\phi(\varepsilon, q(\varepsilon, w, p), p) = w.
\]

\( V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p)) = V(\varepsilon, w, p) - \frac{\beta}{1 - \beta} \left[ V(\varepsilon, \varepsilon p, p) - V(\varepsilon, w, p) \right] \leq V(\varepsilon, w, p) \)

(with strict inequality if \( w < p \)). The observant reader will thus have noticed that an outside offer from a type \( p' \) firm can result in a wage increase even when \( V(\varepsilon, \varepsilon p', p') < V(\varepsilon, w, p) \), i.e. even when the poacher’s productivity is so low that it can’t even afford to compensate the worker for his/her pre-existing value \( V(\varepsilon, w, p) \). This results from the existence of steps 3 and 4, which ensure that the worker can credibly threaten to accept the weaker firm’s offer at step 2, even in cases where that offer is lower than what s/he would have gotten at status quo. In other words, in order to force his/her incumbent employer to renegotiate, the worker is willing to “take the chance” of accepting a very unattractive offer from the poacher because s/he knows that it is then in the interest of her incumbent employer to attract him/her back with a wage increase at later stages of the renegotiation game.
- If $p' > p$, the outside offer creates a (private) rent supplement equal to $V(\varepsilon, \varepsilon p', p') - V(\varepsilon, \varepsilon p, p)$. The renegotiation game thus implies that the worker moves to the type-$p'$ job, where s/he gets a wage $\phi(\varepsilon, p, p')$ that solves:

$$V(\varepsilon, \phi(\varepsilon, p, p'), p') = V(\varepsilon, \varepsilon p, p) + \beta [V(\varepsilon, \varepsilon p', p') - V(\varepsilon, \varepsilon p, p)].$$

(4)

It can be seen that an employee who moves from a type-$p$ to a type-$p'$ firm gets a value equal to the maximum that s/he could get from staying at the type-$p$ firm, plus a share $\beta$ of the new match rent. Note that the wage $\phi(\varepsilon, p, p')$ obtained in the new firm can be smaller than the wage $w$ paid in the previous job, because the worker expects larger wage raises in firms with higher productivity. In other words, workers may be willing to take wage cuts just to move from a low- to a high-productivity firm.

To sum up, one of the following three situations may arise when a type-$\varepsilon$ worker, paid a wage $w$ by a type-$p$ firm, receives a type-$p'$ job offer:

(i) $p' \leq q(\varepsilon, w, p)$, and nothing changes.

(ii) $p \geq p' > q(\varepsilon, w, p)$, and the worker obtains a wage raise $\phi(\varepsilon, p', p) - w > 0$ from his/her current employer.

(iii) $p' > p$, and the worker moves to firm $p'$ for a wage $\phi(\varepsilon, p, p')$ that may be greater or smaller than $w$.

Before we go any further, we should note that Dey and Flinn (2000) have reached similar sharing rules to those just derived by applying the Nash bargaining solution. Our contribution shows that this result can be derived from a precisely defined strategic bargaining game compatible with job continuation when renegotiations fail.\footnote{Moreover, Dey and Flinn focus on the renegotiation issue in a more complex framework with multidimensional employment contracts stipulating wages and health insurance provisions. Due to this added complexity, they are unable to come up with closed-form expression for wages and wage distributions.}
The wage equation. The precise form of wages can be obtained from the expressions of lifetime utilities (see Appendix A.2 for the corresponding algebra). The wage $\phi(\varepsilon, p', p)$ of a type-$\varepsilon$ worker, currently working at a type-$p$ firm and whose last job offer emanated from a type-$p'$ firm, is defined by:

$$\phi(\varepsilon, p', p) = \varepsilon \cdot \left( p - (1 - \beta)\int_{p'}^{p} \frac{\rho + \delta + \lambda_1 F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx \right).$$  \hspace{1cm} (5)

This expression shows that the returns to on-the-job search depend on the bargaining power parameter $\beta$. It can be seen that outside offers cause wage increases within the firm only if employers have some bargaining power. In the limiting case where $\beta = 1$, the worker appropriates all the surplus up-front and gets a wage equal to $\varepsilon p$, whether or not s/he searches on the job.

In the opposite extreme case, where $\beta = 0$, the wage increases as outside offers come since all offers from firms of type $p' \in \{q(\varepsilon, w, p), p\}$ cause within-firm wage raises.

The wage $\phi_0(\varepsilon, p)$, obtained by a type-$\varepsilon$ unemployed workers when matched with a type-$p$ firm, writes as:

$$\phi_0(\varepsilon, p) = \varepsilon \cdot \left( p_{\text{inf}} - (1 - \beta)\int_{p_{\text{inf}}}^{p} \frac{\rho + \delta + \lambda_1 F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx \right),$$  \hspace{1cm} (6)

where $p_{\text{inf}}$ is the lowest viable marginal productivity of labor. The latter is defined as the productivity value that is just sufficient to compensate an unemployed worker for his/her forgone value of unemployment, given that s/he would be paid his marginal productivity, thus letting the firm with zero profits. Analytically:

$$V(\varepsilon, \varepsilon p_{\text{inf}}, p_{\text{inf}}) = V_0(\varepsilon)$$

$$\downarrow$$

$$p_{\text{inf}} = b + \beta(\lambda_0 - \lambda_1)\int_{p_{\text{inf}}}^{p_{\text{max}}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx$$  \hspace{1cm} (7)

It appears that $p_{\text{inf}}$ differs from the unemployment income if workers have positive bargaining power. For instance, $\varepsilon p_{\text{inf}}$ is greater than the unemployment income flow $\varepsilon b$ if the arrival rate of job offers to unemployed workers $\lambda_0$ is larger than the arrival rate to employees, $\lambda_1$. In
that case, accepting a job reduces the efficiency of future job search. The worker needs to be compensated for this loss through a wage strictly above unemployment income. Operating firms thus have to be able to afford wages at least equal to $p_{\text{inf}}$, which imposes the obvious condition that they be at least as productive as $p_{\text{inf}}$. It is worth noting that the lower support of observed marginal productivities, that we denote by $p_{\text{min}}$, can be strictly above the lower support of viable productivities $p_{\text{inf}}$, for instance if free entry is not guaranteed on the search market.

The definition (6) of $\phi_0(\varepsilon,p)$ together with the definition (7) of $p_{\text{inf}}$ shows that entry wages, received by individuals who exit from unemployment, are not necessarily higher than the unemployment income. It actually appears that those wages are always smaller than the unemployment income if workers have no bargaining power, because accepting a job is a means to obtain future wage raises. Entry wages obviously increase with the bargaining power parameter $\beta$.

We conclude this Section by commenting on comparative statics. The wage function $\phi(\varepsilon,p,p')$ decreases with $\lambda_1$ and $\mathcal{F}$ (in the sense of first-order stochastic ordering), and increases with $\delta$. These properties reflect an option value effect: workers are willing to pay today for higher future earnings prospects. Of course $\phi(\varepsilon,p,p')$ increases with the bargaining power, $\beta$. It also increases with worker ability $\varepsilon$ and the type $p$ of the less competitive employer, as both Bertrand competition and Nash-bargaining work in tandem to push wages up. However, we note an ambiguous effect of the type $p'$ of the employer winning the auction: $\phi(\varepsilon,p,p')$ decreases with $p'$ if $\beta$ is small enough for the option value effect to dominate. A high $p'$ means that the upper bound put on future renegotiated wages is more remote (as it is equal to $p'$) and the worker is thus willing to trade lower present wages for a promise of higher future wages. However, $\phi(\varepsilon,p,p')$ increases with $p'$ if $\beta$ is large enough for the bargaining power effect on rent sharing to take over the option value effect.
2.3 Steady-state equilibrium

We know from what precedes that a type $\varepsilon$ employee of a type $p$ firm is currently paid a wage $w$ that is either equal to $\phi_0(\varepsilon, p) = \phi(\varepsilon, p_{\text{inf}}, p)$, if $w$ is the first wage after unemployment, or is equal to $\phi(\varepsilon, q, p)$, with $p_{\text{inf}} \leq p_{\text{min}} < q \leq p$, if the last wage mobility is the outcome of a bargain between the worker, the incumbent employer and another firm of type $q$. The cross-sectional distribution of wages therefore has three components: a worker fixed effect ($\varepsilon$), an employer fixed effect ($p$) and a random effect ($q$) that characterizes the most recent wage mobility. In this section we determine the joint distribution of these three components.

In a steady state a fraction $u$ of workers is unemployed and a density $\ell(\varepsilon, p)$ of type-$\varepsilon$ workers is employed at type-$p$ firms. Let $\ell(p) = \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} \ell(\varepsilon, p) d\varepsilon$ be the density of employees working at type-$p$ firms. The average size of a firm of type $p$ is then equal to $\overline{M}(p)/\gamma(p)$. We designate the corresponding cdfs with capital letters $L(\varepsilon, p)$ and $L(p)$, and we let $G(w|\varepsilon, p)$ represent the cdf of the (not absolutely continuous, as we shall see) conditional distribution of wages within the set of workers of ability $\varepsilon$ within type-$p$ firms.

We now proceed to the derivation of these different distributional parameters by increasing order of complexity. The steady state assumption implies that inflows must balance outflows for all stocks of workers defined by a status (unemployed or employed), a personal type $\varepsilon$, a wage $w$, an employer type $p$. The relevant flow-balance equations are spelled out in Appendix A.3. They lead to the following series of definitions/results:

- **Unemployment rate:**
  \[ u = \frac{\delta}{\delta + \lambda_0}. \] (8)

- **Distribution of firm types across employed workers:** The fraction of workers employed at a firm with mpl less than $p$ is
  \[ L(p) = \frac{F(p)}{1 + \kappa_1 F(p)}, \] (9)

14
\( \kappa_1 = \frac{\lambda_1}{\delta} \), and the density of workers in firms of type \( p \) follows from differentiation as

\[
\ell(p) = \frac{1 + \kappa_1}{[1 + \kappa_1 F(p)]^2} f(p).
\]

**Distribution of matches:** The density of matches \( (\varepsilon, p) \) is

\[
\ell(\varepsilon, p) = h(\varepsilon) \ell(p).
\]

**Within-firm distribution of wages:** The fraction of employees of ability \( \varepsilon \) in firms with \( \text{mpl}_p \) is

\[
G(w|\varepsilon, p) = \left( \frac{1 + \kappa_1 F(p)}{1 + \kappa_1 G(q(\varepsilon, w, p))} \right)^2 = \left( \frac{1 + \kappa_1 L[q(\varepsilon, w, p)]}{1 + \kappa_1 L(p)} \right)^2.
\]

where \( q(\varepsilon, w, p) \), defined equation (3), stands for the threshold value of the productivity of new matches above which a type-\( \varepsilon \) employee with a current wage \( w \) can get a wage increase.

Equation (8) is standard in equilibrium search models (see Burdett and Mortensen, 1998) and merely relates the unemployment rate to unemployment in- and outflows. Equation (9) is a particularly important empirical relationship as it will allow us to back out the sampling distribution \( F \) from its empirical counterpart \( L \).

Equation (11) implies that, under the model’s assumptions, the within-firm distribution of individual heterogeneity is independent of firm types. Nothing thus prevents the formation of highly dissimilar pairs (low \( \varepsilon \), high \( p \), or low \( p \), high \( \varepsilon \)) if profitable to both the firm and the worker. This results from the assumptions of constant returns to scale, scalar heterogeneity and undirected search.

Finally, equation (12) expresses the conditional cdf of wages in the population of type \( \varepsilon \) workers hired by a type \( p \) firm. What the pair of equations (11,12) shows is that a random draw from the steady-state equilibrium distribution of wages is a value \( \phi(\varepsilon, q, p) \) where \( \varepsilon, p, q \) are three random variables such that

\[\text{It is exactly the same equilibrium relationship as between the distribution of wage offers and the distribution of earnings in the Burdett and Mortensen model.}\]
(i) $\varepsilon$ is independent of $(p, q)$,

(ii) the cdf of the marginal distribution of $\varepsilon$ is $H$ over $[\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]$,

(iii) the cdf of the marginal distribution of $p$ is $L$ over $[p_{\text{min}}, p_{\text{max}}]$, and

(iv) the cdf of the conditional distribution of $q$ given $p$ is $\tilde{G}(q|p)$ over $\{p_{\text{inf}}\} \cup [p_{\text{min}}, p]$ such that

$$
\tilde{G}(q|p) = G(\phi(\varepsilon, q, p)|\varepsilon, p) = \frac{1 + \kappa_1 F(p)}{1 + \kappa_1 F(q)}
$$

for all $q \in \{p_{\text{inf}}\} \cup [p_{\text{min}}, p]$. The latter distribution has a mass point at $p_{\text{inf}}$ and is otherwise continuous over the interval $[p_{\text{min}}, p]$.

3 Estimation

In this section, we describe the data and the estimation procedure and we discuss the results.

3.1 Data

We use a matched employer-employee panel of French data collected by the French National Statistical Institute (INSEE) and covering the period 1993-2000. This panel contains standard accounting information drawn from the BRN ("Bénéfices Réels Normaux") firm data source: total compensation costs, value added, current operating surplus, gross productive assets, etc. The BRN data are supposedly exhaustive of all private companies (not establishments) with a sales turnover of more than 3.5 million FRF (about 530,000 Euros) and liable to corporate taxes. In addition, we use the DADS ("Déclarations Annuelles de Données Sociales") worker data source to compute labor costs and employment at the company level for various worker (skill) categories. The DADS data are based on mandatory employer (establishments) reports of the earnings of each salaried employee of the private sector subject to French payroll taxes.

9The BRN is a subset of a larger firm sample, the BIC, "Bénéfices Industriels et Commerciaux".
over one given year. This is a very large dataset, which we “collapse” by firm and worker category and then merge with the BRN dataset to obtain our base sample.\textsuperscript{10}

Our base sample thus essentially contains firm-level data on value added, capital, and employment and labor costs by labor category over the period 1993-2000. Regarding labor categories, we have arranged workers into the following four distinct categories, based on occupation:\textsuperscript{11}

\begin{itemize}
  \item Executives
  \item Managers
  \item Engineers
\end{itemize}

\begin{itemize}
  \item Technicians
  \item Foremen, supervisors of all kinds
\end{itemize}

\begin{itemize}
  \item Clerical employees
  \item Skilled production workers
\end{itemize}

\begin{itemize}
  \item Sales workers
  \item Unskilled production and service employees
\end{itemize}

\textit{Category #1.}

\textit{Category #2.}

\textit{Category #3.}

\textit{Category #4.}

In the sequel, we shall refer to “workers of observed skill level \(s\)”, for \(s = 1,\ldots,4\), where our prior is a worker’s observed “skill level” (loosely defined though it may be) is a decreasing function of the worker’s category index, \(s\).

Given this classification of workers, we then split our base panel into four panels of firm data on value-added, employment and average labor costs by skill category covering the period 1993-2000 and corresponding to four distinct industries: Manufacturing, Construction, Trade and Services. Finally, these four panels were balanced and firms with strictly less than 10 employees in total were removed. This final trimming leaves us with four seven-year panels, involving an approximate total of just under 3 million workers distributed into 50,000 firms each year.

Table 1 contains some descriptive statistics for selected variables. From that table, we see that our four industries are somewhat different in size (as measured by the total number

\textsuperscript{10}For more information on these datasets, we refer to Crépon and Desplat (2002) who were the first to construct a similar matched panel covering the period 1993-97. See also Abowd, Kramarz and Margolis (1999) for another very precise description of the same data sources and others.

\textsuperscript{11}Apart from age, gender and place of birth, occupation is the only personal characteristic that is available in our worker panel.
of either firms or workers), and in the structure of their workforce. In this last respect, the Construction sector stands out in that it seems to employ an especially large share of medium-skilled production workers ($s = 3$), and very few of the extreme categories ($s = 1$ or $4$) within relatively small firms. In spite of these differences, the skill category $s = 3$ is by a substantial amount the most numerous—and therefore presumably the most heterogeneous—in all four industries. A last feature of Table 1 that may be worth mentioning at this point is the numbers in parentheses in the rightmost column. Those are the relative mean wages of labor categories 1, 2 and 3 to category 4. We see that the wage hierarchy follows our prior about the ranking of the observed skill levels. There are cross-sectoral differences in those wage ratios, with the Construction sector once more being remarkable in that it is the sector where cross-occupational wage inequality is most important. The main issue addressed in this paper is to understand what lies behind those cross-sectoral/occupational wage differentials.

< Table 1 (descriptive statistics) about here. >

Finally, estimating the model requires data on worker mobility. We use the French Labor Force Survey ("Enquête Emploi") which is a three-year rotating panel of individual professional trajectories similar to the American CPS ("Current Population Survey"). We prefer to use the LFS panel instead of the larger DADS panel as the latter is known to be affected by large attrition biases. Moreover, the LFS is precisely designed to study unemployment and worker mobility.

3.2 Productivity

The values and distribution of firm marginal productivity values $p$ are crucial determinants of wages in the structural model. Since these values are not directly observed in the data, their construction is a key step in the estimation procedure. A central principle that we want to stick with in the design of this procedure is that the productivity parameters $p$ should not be constructed to a priori fit the wage data, but should rather be identified from value-added
data. This, we believe, is the only way to get credible estimates of the bargaining power $\beta$, which in turn will be identified by the connection that exists in the data between wages and productivity.

The production data is a set of $NT$ observations of value-added ($Y_{jt}$), the book value of capital ($K_{jt}$) and the number of working hours (divided by $2028 = 52 \times 39$) of skill category $s = 1, ..., 4$ used by firm $j$ in year $t$ ($M_{sjt}$), where $j = 1, ..., N$ is the firm index and $t = 1, ..., T$ the time index.

The observed skill type $s$ does not necessarily capture all the productive heterogeneity of workers. Specifically, the $M_{sjt}$ workers in skill category $s$ may have different unobserved individual ability levels $\varepsilon$. We assume, as in the theory laid out in the preceding section, that the labor markets are perfectly segmented between skill categories and that there is no sorting within each observationally homogeneous category of workers. We also assume that firm decisions follow a stationary equilibrium path. The distribution of abilities in the $s$th skill category within each firm should therefore fluctuate around some fixed density, say $h_s(\varepsilon)$. Assuming further that workers are perfectly substitutable between skill categories as well as within, we define the total amount of efficient labor employed at firm $j$ at time $t$ as

$$L_{jt} = \sum_{s=1}^{4} \alpha_s M_{sjt},$$

where $\alpha_s = \int \varepsilon h_s(\varepsilon) d\varepsilon$ is the steady-state mean ability in category $s$.

We then specify firm $j$’s total per-period output (value-added) as the following Cobb-Douglas function of capital and efficient labor:

$$Y_{jt} = \theta_j K_{jt}^\chi L_{jt}^\xi \exp(\eta_{jt}),$$

where $\theta_j$ is a firm-specific productivity parameter and $\eta_{jt}$ is a zero-mean, stationary productivity shock; $\xi$ and $\chi$ are between 0 and 1 and are also common to all firms within a given industry.

The rent that is shared between the entrepreneur $j$ and one single type-$s$ worker with ability value $\varepsilon$ (drawn from the support of $h_s$) should be this worker’s marginal contribution to firm
that is

$$\frac{\partial Y_{jt}}{\partial [h_s(\varepsilon) M_{jst}]} = \varepsilon \xi \theta_j K_{jt} L_{jt}^{\varepsilon - 1} \exp \left( \eta_{jt} \right) = \varepsilon \xi Y_{jt} L_{jt}. \quad (15)$$

The marginal productivity of the match thus multiplicatively depends on the worker’s ability $\varepsilon$ and the firm’s mean value-added $p_{jt} = \frac{Y_{jt}}{L_{jt}}$. Under the stationarity assumption, we shall thus define the firm type $p_j$ by $\ln p_j = \mathbb{E} \ln p_{jt}$, where $\mathbb{E}$ denotes the mathematical expectation operator, and the steady-state marginal productivity of a match $(\varepsilon, p_j)$ as $\xi \varepsilon p_j$.

### 3.3 Worker mobility

A key determinant of the market equilibrium is the parameter $\kappa_1 = \lambda_1 / \delta$, which measures the average number of outside job offers a worker receives between two unemployment spells. Since outside job offers are the source of wage increases in the model, we expect that more “mobile” workers (those with higher values of $\kappa_1$) should on average exhibit steeper wage-tenure profiles. However, what $\kappa_1$ essentially determines is the duration of job spells. We shall thus identify $\kappa_1$ exclusively from job duration data rather than wage data which would certainly buy us sizeable efficiency gains but would also increase the risk of misspecifications biases.

The likelihood function. As all job transition processes are Poisson, all corresponding durations are exponentially distributed. In this Section we are interested in the distribution of


---

12This result is independent of the timing of the investment decision. To see that, let $F(K, L)$ define the output as a function of capital $K$ and labor $L$. If investment is made before wage setting, the marginal surplus of one unit of labor is $\frac{\partial F(K, L)}{\partial L}$. If the capital is adjusted after wage setting, its optimal value, $K^*$, satisfies $\frac{\partial F(K, L)}{\partial K} = r$, where $r$ stands for the user cost of capital. In that case, the marginal surplus of one unit of labor writes as

$$\frac{d}{dL} \left[ F(K, L) - rK \right] = \frac{\partial F(K, L)}{\partial L} + \frac{dK^*}{dL} \left( \frac{\partial F(K, L)}{\partial K} - r \right) = \frac{\partial F(K, L)}{\partial L}.$$  

13Note that we completely neglect the sort of externality problems pointed out by Stole and Zwiebel (1996), Wolinsky (2000) and Cahuc and Wasmer (2001) resulting from diminishing marginal returns to labor. With nonconstant returns to scale, the hiring decisions of firms affect their levels of productivity, and consequently their labor costs. We simply assume that the firms’ hiring decisions are exogenous.

14We consider each particular market segment and each skill category $s = 1, ..., 4$ separately. For notational ease, however, we omit the skill category index $s$ in this subsection (i.e., for example we write $\kappa_1$ for any $\kappa_{1s}$).
job spell durations \( t \), which have the following density, conditional on \( p \):

\[
\mathcal{L}(t|p) = \left[ \delta + \lambda_1 F(p) \right] \cdot e^{-[\delta + \lambda_1 F(p)]t},
\]

where we know from equation (10) that \( p \) is distributed in the population of employed workers according to the density:

\[
\ell(p) = \frac{1 + \kappa_1}{[1 + \kappa_1 F(p)]^2} f(p).
\]

Because it is impossible to match the LFS worker data with the BRN firm data, we shall treat \( p \) as an unobserved heterogeneity variable, that is: we integrate out \( p \) from the joint likelihood of \( p \) and \( t \), \( \ell(p) \mathcal{L}(t|p) \), and maximize the unconditional likelihood,

\[
\mathcal{L}(t) = \int_{p_{\text{min}}}^{p_{\text{max}}} \ell(p) \mathcal{L}(t|p) \, dp = \frac{\delta(1 + \kappa_1)}{\kappa_1} \int_{1}^{1+\kappa_1} e^{-\delta xt} \frac{dx}{x} dx = \frac{\delta(1 + \kappa_1)}{\kappa_1} \left[ \text{Ei}(\delta t) - \text{Ei}(\delta t (1 + \kappa_1)) \right],
\]

to get an estimate of \( \delta \) and \( \kappa_1 \).\(^{15}\) This method of unconditional inference applied to labor market transition parameters was first explored by van den Berg and Ridder (2003). As we already mentioned, it has the additional advantage of yielding estimates of the transition rate parameters that are robust to any specification error in the estimation of the productivity parameters \( \theta_j \) for all firms \( j \).

**Results.** The unconditional ML estimates of \( \delta \), \( \lambda_1 \) and, most importantly, \( \kappa_1 \) are reported in Table 2. In terms of \( \kappa_1 \), i.e. the average number of outside contacts that an employed worker can expect before the next unemployment period, higher skill categories tend to be more mobile than lower skilled ones (with the remarkable exception of the Construction sector, where category 1 turns out to have the lowest value of \( \kappa_1 \)). Now looking at the sheer frequency

\(^{15}\)The exponential integral function \( \text{Ei}(t) = \int_{t}^{\infty} e^{-x} \, dx \) is tabulated in many statistical softwares (such as GAUSS and MATLAB). Note that the exact likelihood that we maximize does take into account the fact that the panel covers a fixed number of periods so that some job durations are censored. It is easy to account for such right censoring. Moreover, the unconstrained likelihood can be analytically developed into simple combinations of exponentials and exponential-integral functions, just as well as in the no-censoring case that we develop in this subsection.
of such contacts, which is measured by $\lambda_1$, we find a similar pattern, in which workers with higher observed skill levels tend to get more frequent outside offers than less skilled workers. Finally, the rate of job termination $\delta$ is everywhere a decreasing function of the skill index $s$ (except again for Construction where categories 1, 2 and 3 exhibit values of $\delta$ that are roughly equal).

< Table 2 (transition parameter estimates) about here. >

The average duration of an employment spell (i.e. the average duration between two unemployment spells), $1/\delta$, ranges from 10 to 35 years, while the average waiting time between two outside offers, $1/\lambda_1$, lies between 3.5 and 19 (!) years. The average number of outside contacts, $\kappa_1$, that results from these estimates is never very large (between 1 and 6.36) which confirms the relatively low degree of worker mobility in the French labor market. Workers are relatively less mobile in Manufacturing than elsewhere, where they tend to have both lower job separation and job-switching rates.

### 3.4 The wage equation

We now turn to the next step of our estimation procedure, in which we combine wage and productivity data according to the wage equation delivered by our structural model in order to estimate the remaining parameters: the bargaining power $\beta$, the elasticity of efficient labor, $\xi$, and the relative efficiency of each skill category, $\alpha_s$, $s = 1, ..., 4$.

**Estimation procedure.** Consider again a market segment consisting of workers all in the same skill category.\(^\text{16}\) Using the theory of wage determination and equilibrium wage distributions laid out in the preceding section, we can derive the steady-state mean wage $\mathbb{E}(w|p)$ paid by any firm of type $p$ for each skill category $s$, the empirical counterpart of which is the

\(^{16}\)Again, to simplify the notation, we omit when we can the skill index $s$, keeping in mind that the transition parameters $\lambda_s$, $\delta_s$ and $\kappa_s$, the relative productivity parameter $\alpha_s$, the offer sampling distribution $F_s(\cdot)$ and, of course, the bargaining power $\beta_s$ are all skill level-specific.
firm-level average wage. Equation (A16) in the Appendix shows that:

\[
\mathbb{E}(w|p) = \xi \mathbb{E}(\varepsilon) \cdot \left( p - \frac{(1 + \kappa_1 \overline{F}(p))^2}{(1 + \kappa_1)^2} \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} \left[ \phi(\varepsilon, p_{\text{min}}, p) - \phi(\varepsilon, p_{\text{inf}}, p) \right] h(\varepsilon) d\varepsilon \right.
\]

\[
\left. - \left[ 1 + \kappa_1 \overline{F}(p) \right]^2 \int_{p_{\text{min}}}^{p} \frac{(1 - \beta) \left[ 1 + (1 - \sigma)\kappa_1 \overline{F}(q) \right] \left[ 1 + \kappa_1 \overline{F}(q) \right]^2 dq \right),
\]

where \( \mathbb{E}(\varepsilon) = \alpha \) is the mean efficiency of workers in that market and \( \sigma = \frac{-\beta}{\rho + \beta} \).

This expression can be further simplified by noticing that if the lower support of viable productivities \( p_{\text{inf}} \) equals the lower support of observed productivities \( p_{\text{min}} \) (which amounts to assuming free entry and exit of firms on the search market), then the second term in the right hand side vanishes. We shall henceforth adopt this assumption.17 We thus now have:

\[
\mathbb{E}(w|p) = \xi \alpha \left( p - \frac{(1 + \kappa_1 \overline{F}(p))^2}{(1 + \kappa_1)^2} \int_{p_{\text{min}}}^{p} \frac{(1 - \beta) \left[ 1 + (1 - \sigma)\kappa_1 \overline{F}(q) \right] \left[ 1 + \kappa_1 \overline{F}(q) \right]^2 dq \right),
\]

or again

\[
\mathbb{E}(w|p) = \xi \alpha \left( p - \frac{1 - \beta}{[1 + \kappa_1 L(p)]^2} \int_{p_{\text{min}}}^{p} \frac{[1 + \kappa_1 (1 - \sigma) + \sigma \kappa_1 L(q)] [1 + \kappa_1 L(q)]^2 dq}{1 + \beta \kappa_1 (1 - \sigma) + (1 - \beta + \beta \sigma) \kappa_1 L(q)} \right). \quad (17)
\]

using equation (10) to substitute \( F(p) \) by \( L(p) \) according to the formula:

\[
\overline{F}(p) = \frac{1 - L(p)}{1 + \kappa_1 L(p)}.
\]

The advantage of this substitution is that the density of firm mean productivities \( p_j \) has an obvious data analog.

The production function parameters (\( \alpha_1, \ldots, \alpha_4 \) and \( \xi \)) are involved in the wage equation (17), both through the intercept—the \( \ln \alpha_s \) term appearing in \( \ln \mathbb{E}(w|p) \)—and through the firms’ productivity parameters—the \( p_j \)'s, as

\[
\ln p_j = \mathbb{E} \ln \left( \frac{Y_{jt}}{L_{jt}} \right) = \mathbb{E} \ln \left( \frac{Y_{jt}}{\sum_{s=1}^{4} \alpha_s M_{sjt}} \right).
\]

One way to proceed is to estimate the production function (14) directly, plug the estimates of \( \hat{\alpha}_s \) and \( \hat{\xi} \) into (17) and use this equation for the estimation of \( \beta_s \) only. Yet, one may worry that

17 Yet, we tried to estimate the unconstrained equation, but these unconstrained estimations always lead to the conclusion that \( p_{\text{inf}} \) indeed equals \( p_{\text{min}} \).
estimates of \( \beta_s \) obtained in this way only reflect measurement errors in \( \alpha_s \) and \( \xi \). Fortunately, looking more closely at equation (17), one sees that \( \mathbb{E}(w|p = p_{\min}) = \xi \alpha p_{\min} \), so that \( \xi \alpha \) is identified from the wage equation. Once \( \xi \alpha \) is fixed, the way in which \( \mathbb{E}(w|p) \) varies with \( p \) is entirely determined by \( \beta \). So the wage equation identifies both \( \beta \) and \( \xi \alpha \), a property that we shall now exploit.

Let \( \overline{w}_{sjt} \) denote the observed firm-level mean wage of labor category \( s \), at date \( t \), in firm \( j \). We estimate both \((\xi, \alpha_1, \alpha_2, \alpha_3)\) and \((\beta_1, \ldots, \beta_4)\) simultaneously by iterating the following procedure until numerical convergence: For starting values \( \xi_0, \alpha_0^1, \alpha_0^2, \alpha_0^3, \alpha_0^4 = 1 \) of \( \xi, \alpha_1, \alpha_2, \alpha_3, \alpha_4 = 1 \):

1. Estimate \( p_j \) as

   \[
   \hat{p}_j = \exp \left[ \frac{1}{T} \sum_t \ln \left( \frac{Y_{jt}}{\sum_{s=1}^4 \alpha_s^0 M_{sjt}} \right) \right]
   \]

   and estimate the steady-state distribution \( \hat{L}_s \) of workers of skill category \( s = 1, \ldots, 4 \) at firms of any productivity \( p \), by the empirical distribution of \( \hat{p}_j \), weighting each firm in the sample by the average amount of type \( s \) labor in that firm over the \( T \) observations periods (\( \overline{M}_{sj} = \frac{1}{T} \sum_t M_{sjt} \)):

   \[
   \hat{L}_s(p) = \frac{\sum_{j=1}^N \overline{M}_{sj} 1 \{ \hat{p}_j \leq p \}}{\sum_{j=1}^N \overline{M}_{sj}},
   \]

   or any smooth version of this formula.

2. Estimate \( \xi \alpha_1, \ldots, \xi \alpha_4 = \xi \) and \( \beta_1, \ldots, \beta_4 \) by applying Nonlinear Least Squares to the system of regressions: for \( s = 1, \ldots, 4 \), \( j = 1, \ldots, N \), \( t = 1, \ldots, T \),

   \[
   \ln \overline{w}_{sjt} = \ln(\xi \alpha_s) + \ln \left[ \hat{p}_j - \frac{1 - \beta}{1 + \kappa_1 \hat{L}_s(\hat{p}_j)} \int_{\min(\hat{p}_j)}^{\hat{p}_j} \left[ 1 + \kappa_1(1 - \sigma) + \sigma \kappa_1 \hat{L}_s(q) \right] \left[ 1 + \kappa_1 \hat{L}_s(q) \right]^2 dq \right] + \text{residual},
   \]

The production technology is defined by the pair of equations (13,14). Looking at those equations, one clearly sees that a normalization is needed, either on one of the \( \alpha_s \) or on the mean firm fixed effect, \( \mathbb{E}(\theta_j) \). We choose to impose \( \alpha_4 = 1 \), so that \( \alpha_s \) measure the relative mean ability of labor categories \( s \) and 1.
imposing the normalization $\alpha_4 = 1$. (We set the discount factor $\rho$ to an annual value of 5% for everyone, i.e. $e^\rho = 0.95$.)

3. Compare the thus obtained quadruple $(\hat{\xi}, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)$ to $(\xi^0_0, \alpha^0_1, \alpha^0_2, \alpha^0_3)$. If different, start over at step 1 using $(\hat{\xi}, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)$ as a starting guess.

**Results.** The estimation results are gathered in Table 3. The numbers in brackets are the bootstrap standard errors based on 1,000 bootstrap replications of our entire estimation procedure, i.e. including the estimation of the transition parameters. Thus, the reported standard errors do account for the presence of nuisance parameters $\kappa_1$ and $L_s$ and for the fact that mean productivity $\tilde{p}_j$ depends on ability parameters $\alpha_s$. We note that despite the number of nuisance parameters, those estimators are remarkably precise.

The first four columns of Table 3 display the bargaining power estimates and the last five columns the estimates of the production function parameters. Bargaining power is found to be an increasing function of observable ability, the least skilled two categories being endowed with a bargaining power close to zero. There are some small discrepancies across sectors but the most striking one is the bargaining power of the first category of workers (managers) in the Construction sector: it is close to one whereas it is never higher than a third in all other sectors. Also, bargaining power seems to be uniformly low for all labor categories in the Service sector.

<Table 3 (wage equation estimates) about here.>

Looking at productivity parameter estimates, we find that the less skilled categories 3 and 4 display values of $\alpha_s$ very close to (yet slightly lower than) the wage ratios displayed in Table 1. This is not the case of categories 1 and 2 where the ability ratios $\alpha_s$ are estimated substantially lower than the corresponding wage ratios. Productivity differences thus only account for a fraction of inter-occupational wage differentials. Other nonproductive factors have to be appealed to in order to explain cross-occupation wage inequality. The Construction sector keeps being remarkable in this respect as this is the sector where inter-occupational wage
dispersion is highest. Productivity differences across labor categories also seem larger here than in the other sectors, but still the productivity ratio of managers in the Construction sector is not nearly large enough to explain the relative wage of that category of workers.

To show how well equation (17) fits the data we have plotted the predicted and observed (log) mean wages against (log) firm productivity levels $\ln \hat{p}_j$ for our four industries on Figure 1. Each column pertains to one given industry, and each row to a given skill level. The solid curves represent the log wages predicted by the structural model. The dashed curve are nonparametric regressions of log wages on log labor productivity, which is what the model’s prediction should be compared with. The gray dots correspond to the scatterplot. Finally, the solid lines represent the firms’ (log) productivity parameters $\ln \hat{p}_j$.

A glance at the various panels of Figure 1 shows that the model is reasonably good at predicting wages. More specifically, two remarkable stylized facts are brought about by those Figures. One is that the wage paid by the lowest-$p$ firms in our four samples and for all categories of workers is always very close to match productivity (solid line) at $p_{\text{min}}$. The second observation that one can make is that profit rates are strongly increasing with productivity: the gap between wages and productivity—which as we just saw is close to zero at $p_{\text{min}}$—becomes substantial at higher values of $p$. Our structural model correctly captures this phenomenon.

< Figures 1 (wage-productivity relationship) about here. >

3.5 A consistency check: direct estimation of the production technology

As we mentioned in the previous subsection, the production function parameters $\xi, \alpha_1, \alpha_2, \alpha_3$ are identified using the wage equation (17). The corresponding regression essentially fits wage data to data on value added per worker, and one may wonder whether this approach is consistent with a direct estimation of the production function (14), that would make no use of wage data. The aim of this subsection is to carry out this consistency check.

\footnote{In order to keep the graph readable, only a small subsample of observations was used for the scatterplot.}
Estimation procedure. To this end, we take the following logged version of (14) to the data:

\[ y_{jt} = \ln \theta_j + \chi \ln K_{jt} + \xi \ln \left( \sum_{s=1}^{4} \alpha_s M_{sjt} \right) + \eta_{jt}, \]  

(18)

where \( y_{jt} = \ln Y_{jt} \) is the log value-added of firm \( j \) at date \( t \), and, as defined earlier, \( M_{sjt} \) is the number of workers from category \( s \) employed by firm \( j \) at date \( t \), and \( \eta_{jt} \) is an error term independent of the fixed effect \( \ln \theta_j \).

Under the steady-state assumption (which is necessary to apply our theoretical economic model), equation (18) can be estimated in levels, using lagged first differences of the RHS variables as instruments (see Arellano and Bover, 1995). The model being nonlinear, it is well known since Chamberlain (1992) that the optimal vector of instruments is equal to the conditional expectation of the gradient of the production function (with respect to the parameters) given all instruments. To obtain an implementable GMM estimator, Chamberlain suggests that one should in practice use standard GMM, and “complete” the set of instruments by adding polynomial functions to form a vector space approximately \( L_2 \)-dense in the set of all \( C^2 \) functions of the instruments. The problem with this approach is that the number of moment conditions quickly becomes very large, which we know causes finite sample biases and is computationally costly.

As an alternative, we restrict the set of instruments to lagged first-differences of the production function gradient evaluated at the values of the \( \alpha_s \)'s previously estimated. This method proved to work well in all the simulations that we programmed to test it.

To sum up, we estimate equation (18) by GMM under the following sets of moment restrictions:

\[
\begin{align*}
(ln \theta_j + \eta_{jt}) \perp & \left\{ \Delta \ln \left( \sum_{s=1}^{4} \alpha_s^0 M_{sjt-\tau} \right) ; \left\{ \Delta \left( \frac{M_{sjt-\tau}}{M_{jt-\tau}} \right) , s = 1, 2, 3 \right\} ; \Delta \ln K_{jt-\tau} \right\} , \quad \tau \geq 4,
\end{align*}
\]

(19)

where \( \alpha_s^0 \) (\( s = 1, \ldots, 4 \)) is the initial guess about \( \alpha \) (again, the value of which is taken to equal \( ^{20} \)

We also tried cross-category mean wage ratios. The estimation results are not too sensitive to variations in this guess within a “reasonable” range.
the estimates reported in Table 3). We use instruments lagged four times based on the Sargan overidentification test. We observed that the Sargan test statistic is a decreasing function of $\tau$ up to $\tau = 4$. For longer lags, the Sargan statistic decreases more slowly when on increments $\tau$ while the precision of the obtained estimates dwindles very quickly.

**Results.** The GMM estimation results of equation (18) are reported for our 4 sectors in Table 4. More precisely, this table has six columns for each sector. The first column reports the GMM estimates obtained on the full sample. In order to detect potential biases and obtain confidence intervals, we ran 1,000 estimations on artificial samples obtained by random resampling (with replacement) of firms from our base panel. For each sector, columns 2 to 5 report the mean, 2.5th percentile, median and 97.5th percentile of the thus obtained distributions of parameter estimates. Finally, column 6 reports the bootstrap and asymptotic $p$-values of the Sargan test.

< Table 4 (production function estimates) about here. >

The asymptotic $p$-value of the Sargan test and the bootstrap $p$-value are very close, meaning that standard first-order asymptotic expansions of the moment conditions provide a good approximation of asymptotic standard errors also at the second order. The null hypothesis is not rejected at the 5% level for the Construction and Services sectors. For the Manufacturing and Trade sectors, the null is rejected at all levels greater than about 3 per thousand. Now, given the size of the sample (more than 10,000 firms) we take these estimates of the production functions as reasonably well validated by the Sargan specification test.

The 95% bootstrap confidence intervals, the bounds of which are displayed in columns 3 and 5, show that in spite of the large number of observations, the precision with which we are able to estimate our production function is rather poor, in particular for the highest skilled category.

---

21 We follow the bootstrap method of Freedman (1984) for the linear model estimated by Two-stage Least Squares, which proceeds in the following steps: 1) estimate the model by 2SLS, 2) compute residuals, 3) regress residuals on instruments and compute new orthogonalized residuals to force the null hypothesis to be verified in the sample, 4) resample from the orthogonalized residuals. This technique has been extended to GMM-based tests by Hall and Horowitz (1996).
This sharply contrasts with the very precise estimates we were able to obtain in Table 3, which is what governed our choice not to constrain the values of $\alpha_s$ and $\xi$ in the wage equations to equal the production function estimates. However, one can see that the estimates of $\alpha_s$ and $\xi$ obtained from fitting the wage data lay perfectly well within those confidence intervals. We take this as quite remarkable a result. One set of estimates helps predicting wages from mean value-added per worker while the other set helps predicting value-added from labor and capital inputs: As these are really two very different relationships, we view this result as a clear support to the theory.

Looking at the estimated returns to labor, capital and scale, one first sees that the estimated returns to capital are very low (between 3 and 9%, depending on the industry and estimator considered). Zero returns to capital cannot even be rejected in Construction or Trade. Conversely, the estimated returns to labor are high (between 0.9 and 0.97 with 1 often being comprised in the confidence interval). As a result, the constancy of returns to scale is rejected in none of our four sectors.

3.6 Distributions

Figure 2 plots densities $\gamma(p)$, $f_s(p)$ and $\ell_s(p)$ for all categories of workers in all four industries. The overall shape is log-normal-like. The sampling distribution $f_s(p)$ is more concentrated than the distribution of productivity $\hat{\rho}_j$ across firms, which is itself more concentrated than the distribution of employer productivities across workers, $\ell_s(p)$. A clear stochastic dominance pattern appears: $f_s(\cdot)$ is systematically to the left of $\gamma(\cdot)$ which is in turn first-order stochastically dominated—albeit to a lesser extent—by $\ell_s(\cdot)$.

< Figures 2 (productivity densities) about here. >

4 Applications

In this section, we use our framework to shed light on two issues. First, we disentangle the respective influence of the bargaining power and the between-firm competition on wage deter-
mination within each sector. Then we turn to the allocation of worker types across firms.

4.1 Assessing the importance of between-firm competition

As we argued in the Introduction, the conventional approach to evaluating the workers’ bargaining power ignores job-to-job mobility. Our model offers a simple way of assessing the bias in the estimation of $\beta$ resulting from this simplification. It is this bias that we examine in this subsection.

What fraction of rent sharing is due to bargaining power and what fraction is due to between-firm competition? Ignoring job-to-job mobility amounts to forcing $\kappa_1 = 0$ in the wage equation (17) that now reads

$$E(w|p, \kappa_1 = 0) = \beta_0 \alpha p + (1 - \beta_0) \alpha p_{\text{min}}.$$  

(20)

Absent of on-the-job search, the bargaining power thus simply measures the mean worker share of match rent, $E[E(w|p) - \alpha p_{\text{min}}]/E(\alpha p - \alpha p_{\text{min}})$. We obtain an estimator $\hat{\beta}_0$ of the bargaining power in the absence of on-the-job search simply by computing this mean share separately for each industry and skill level.\(^{22}\)

The values of $\hat{\beta}_0$ are gathered in the first column of Table 5. Comparing the bargaining power estimates with and without on-the-job search—i.e. comparing the values of $\hat{\beta}_0$ to the values of $\beta$ from Table 3—immediately shows that the bargaining power is always overestimated when one ignores job-to-job mobility. The magnitude of this upward bias varies across skill groups and sectors, but the bias always seems to be there, and is always important. This was expected as on-the-job search is a means by which an employee can force her employer to renegotiate her wage upward. Neglecting on-the-job search biases the workers’ bargaining power upward to make it fit the actual share of compensation costs in value-added.

Table 5’s column 2 reports estimates $\hat{\beta}_0$ of the mean worker share of match rents obtained as

\(^{22}\)The values of the production function parameters $\alpha_1, \ldots, \alpha_4$ and $\xi$ that we use for this exercise are those reported in Table 3.
from predicted (rather than observed) firm level mean wage data. That is, we simulated firm mean wages using our set of parameter estimates, and re-ran the estimation of equation (20) using those simulated data. As can be seen by comparing the values $\hat{\beta}_0$ and $\hat{\beta}_0$, we tend on average to overestimate wages a bit. Simulated and observed data otherwise produce reasonably similar results.

< Table 5 (Mean worker share of match rents) about here. >

What we want to know next is how much of this estimated rent share $\hat{\beta}_0$ is explained by “noncompetitive wage setting” (i.e. the bargaining power that workers may have), versus how much of it is due to between-firm competition. What we are looking for here is another answer to the question of knowing to what extent an extra rent sharing device (in addition to between-firm competition) is needed to explain wages. In search of this answer we simulate new wage data, again using our previously obtained estimates for all parameters but $\beta$, which we force to equal zero. That is, we produce the wage data that one would collect from the French labor market if French workers had no bargaining power at all, i.e. if the only source of rent acquisition by workers was between-employer competition. That being done, we estimate equation (20) using this set of artificial wage data, and compare the rent share obtained to $\hat{\beta}_0$. This tells us how much of $\hat{\beta}_0$ is explained by between-firm competition alone.

The results are in the last column of Table 5. Clearly, competition between firms explains one hundred percent of the workers’ share of match rents everywhere where the bargaining power $\beta$ was estimated to be zero. What that means is that between-firm competition alone is enough to explain wages in all industries except Construction, for the less skilled workers. We also find from looking at the other categories of workers that the rent share they are able to capture is explained in large part by between-employer competition. Even though we undoubtedly do need some noncompetitive wage formation device such as wage bargaining to reproduce skilled wages, sheer labor market competition explains way over half (in fact, about 60% on average if one believes the figures in Table 5) of the rents accruing to these workers.
Counterfactual evaluation of the effect of on-the-job search on rent sharing. The claim that between-firm competition plays a prominent role in the wage formation in France may sound surprising since the arrival rate of job offers is very low: as shown by Table 2, the average waiting time between two outside offers lies between 3.5 and 19 years. However, we shall now see that it only takes very little between-firm competition as measured by worker mobility—i.e. it only takes small values of the parameter $\kappa_1$—to provide the workers with a large share of the match rent.

This is illustrated on Figure 3, which is constructed as follows: First, we simulate artificial wages using our wage equation (17) and our estimates as parameter values, with the exception that we force $\beta$ to equal zero and $\kappa_1$ to cover the interval $[0, 15]$. That is, we simulate the wages that workers would receive if they had zero bargaining power, in various competitive environments ranging from $\kappa_1 = 0$—job-to-job mobility is ruled out, implying no between-employer competition—to $\kappa_1 = 15$—job-to-job mobility is very easy, implying fierce between-employer competition. We then compute the mean worker share of match rent—i.e., the $\beta_0$—corresponding to each value of $\kappa_1$ within our range. Figure 3 plots this share against $\kappa_1$, for all skill categories and industries.

We see on Figure 3 that the dependence on $\kappa_1$ of the workers’ rent share is upward sloping (this is no big surprise), and highly concave: while the workers’ rent share increases very steeply—from 0 to a typical 20-25%—as $\kappa_1$ rises from zero to a value of about 2 or 3, it only increases by a few extra percentage points as one takes $\kappa_1$ to values as unrealistically high as 15. This finding has two implications. First, as we said earlier, relatively modest values of $\kappa_1$ are enough to guarantee a large share of the match rent for the workers. In other words, it only takes little between-firm competition to raise the workers’ wages by a substantial amount. A

\footnote{We arbitrarily take $\kappa_1 = 15$ as an upper bound for our illustrative purpose because we deem it to be an already unrealistically high value of that parameter.}
candidate explanation of that phenomenon goes as follows. When a worker finds his/her first job, s/he is initially unemployed. At that point the negotiation outcome is favorable to the employer as the worker’s only outside option is to remain unemployed. The first outside offer the new employee obtains is of great (expected) value as it allows him/her to renegotiate his/her wage under much more favorable conditions. The second outside offer is already less valuable (still in expected terms), as the worker’s wage was already raised due to the first offer, and it is therefore less likely that the second offer will get the worker a substantial additional wage raise. As new outside offers come along, the worker’s situation improves, and the expected gain from the next outside offer declines (especially if the distribution of firm productivities is not very dispersed). Generally speaking, the returns to on-the-job search are expected to be rapidly decreasing with the number of outside offers already raised. This suggest that endogenizing on-the-job search intensity so as to make it a function of current wage as in Christensen et al. (2001) is certainly a sensible thing to do.

Second, ignoring on-the-job search altogether (i.e. assuming that $\kappa_1 = 0$ in the wage equation) causes large downward biases in the assessment of the workers’ rent share. It is therefore likely to cause sizeable upward biases in the estimation of the workers’ bargaining power. However, an imprecise assessment of between-firm competition (in the sense of an imprecise value of $\kappa_1$) is unlikely to have a large impact on the estimation of the workers’ rent share or bargaining power, so long as this imprecise assessment is within a reasonable order of magnitude (typically, from Figure 3, $\kappa_1 \geq 2$).

Figure 3 also brings about a final comment. The solid vertical lines on the four panels indicate the values of $\kappa_1$ as we estimated them from the LFS data and the dotted vertical lines locate the 95% bootstrap confidence interval around the estimated value (see Table 2). We see that those values are typically at the very beginning of the “flat region” of the $\kappa_1$ – workers’ rent share relationship. A way to express this result is to say that it may be true that between-firm competition on the French labor market is not very lively, but encouraging it probably would
not have a large impact on wages.

4.2 The allocation of workers across firms

In this subsection, we ask two questions: How does firm productivity relate to firm size? How do workers with different (observable) skills allocate across firms of different productivity? These are important questions for at least two reasons. First, addressing them should yield interesting insights about the vacancy-posting behavior of firms (to the extent that vacancy-posting determines firm size). Second, a positive size-productivity relationship has been put forward in a recent literature as potentially underpinning the widely documented positive size-wage premium (Oi and Idson, 1999). Third, although positive assortative matching seems to be a rather popular and consensual idea in the profession there is still little evidence either in favor or against it.

The size-productivity relationship. The evidence contained in our French data is gathered on Figure 4, which shows scatterplots as well as nonparametric regressions of log-firm sizes, \( \ln \left( \sum_{s=1}^{4} M_{sj} \right) \), against log firm productivity levels, \( \ln \hat{p}_j \). The solid line features the nonparametric (Kernel) regression. Using a magnifier, one could see that this relation is increasing in the Manufacturing sector, very slightly increasing in Construction and Trade, and slightly decreasing in the Service sector. A fair conclusion that can be drawn from this figure is that firms size is largely independent of \( p \) (except maybe for Manufacturing). At the very least, productivity explains very little of firm size.

< Figure 4 (size-productivity relationship) about here. >

How should one interpret this finding? Our model predicts the following relationship between the measure of workers of skill \( s \) in one firm and its type:

\[
M_s(p) = \frac{M_s \ell_s(p)}{\gamma(p)} = \frac{M_s (1 + \kappa_{1s}) f_s(p)}{1 + \kappa_{1s} F_s(p)}.
\] (21)
where \( f_s(p) \triangleq v_s(p) \) (say) is the sampling weight of type \( p \) firms. It is interpretable as the relative hiring effort of type \( p \) firms or as the fraction of the total number of vacancies of a given skill type posted by a type \( p \) firm \((\int v_s(p)d\Gamma(p) = 1)\). Our model does not provide a theory for \( \kappa_{1s} \) and \( v_s(p) \) (or \( f_s(p) \)). This requires a more complete matching model à la Diamond-Mortensen-Pissarides maybe along the line of Mortensen (2000). Nevertheless, \( \kappa_{1s} \) and \( v_s(p) \) are free parameters which can be consistently estimated.

The first fraction on the right-hand side of equation (21) is increasing in \( p \), reflecting the fact that higher \( p \) firms are more attractive for workers. What the model therefore tells us is that the seemingly absent relationship between size and productivity is the result of two conflicting effects: high-productivity firms make less job offers (post less vacancies) and enjoy higher acceptance rates of their job offers. Figure 5 plots the vacancy functions \( v_s(p) \) for each industry and each skill category as a function of \( \Gamma(p) \) instead of \( p \) to emphasize that vacancy functions are decreasing over the whole sample except, maybe, for a small fraction of low productivity firms. It therefore seems that the higher the productivity the bigger the firm’s labor market power and the lower the hiring effort the firm has to make in order to build the desired labor force size.

< Figure 5 (job vacancies) about here. >

**Sorting on observables.** As we saw in subsection 2.3, our theory predicts no sorting of workers on unobserved ability: for any observed skill level \( s \), the within-firm distribution of individual abilities is \( h_s(\varepsilon) \), independently of the firm’s type \( p \). This, however, doesn’t rule out sorting on observable characteristics.

Evidence on such sorting on observables can be presented as follows. For each firm \( j \), the data allow us to construct the mean share of each labor category \( s \) employed at that particular firm, \( \hat{m}^s_j \) as \( \hat{m}^s_j = \hat{M}_{sj}/\sum_{\sigma=1}^4 \hat{M}_{\sigma j} \) (where one should recall that \( \hat{M}_{sj} = \hat{p}_j \sum_t M_{sjt} \)), and then look at how these shares are related to firm productivity, \( \hat{p}_j \). The result is displayed on Figure

35
6, which plots the nonparametric kernel regressions of \( \hat{m}_j^s \) \((s = 1, \ldots, 4)\) on the estimated productivity parameters \( \hat{p}_j \) against \( \ln \hat{p}_j \).

< Figure 6 (sorting on observables) about here. >

Figure 6 shows clear evidence of sorting on observables in Manufacturing, Trade and Services. In these three industries, the share of type 4 labor (sales workers and unskilled production workers) declines sharply with \( \hat{p}_j \) while the shares of type 1 and 2 labor (executives, managers, engineers, technicians and supervisors) increases with \( \hat{p}_j \). In other words, high-productivity firms hire proportionately less observationally unskilled workers and more observationally skilled workers. There is no clear or uniform pattern for the dependence on \( \hat{p}_j \) of the share of medium-skilled (type \( s = 3 \)) workers, which goes either way, depending on the industry. Category 3, which again is probably the most heterogeneous of all, seems to be “pivotal” in this sense. Finally, we can look at the Construction sector which once more seems somewhat exceptional. As in the other three sectors, the shares of type 1 and 2 labor increase slightly with \( \hat{p}_j \) (from around 5.5% at \( p_{\text{min}} \) to 6.5% at \( p_{\text{max}} \) and from around 12.5% at \( p_{\text{min}} \) to 15% at \( p_{\text{max}} \), respectively). Where Construction differs from other sectors is in that the share of type 4 labor also tends to rise with \( \hat{p}_j \) (from about 10.5% to 14%). Finally, the share of type 3 labor decreases with \( \hat{p}_j \) from 71% to 64%. The sorting pattern is thus somewhat less clear in Construction than in the other 3 sectors, even though the structure of a typical firm’s labor force varies with this firm’s productivity in Construction as in every other sector.

Overall, we see Figure 6 as evidence of “positive assortative matching” of workers and firms, with workers being sorted according to their observable skill levels.\(^{24}\) It is worth noting that this phenomenon is closely related to the differences in the arrival rates of job offers. Employees enjoying a high arrival rate of job offer tend to reach high productivity firms more quickly.

Looking at Table 2, it turns out that \( \hat{k}_{18} \), the estimated number of job offers per employment spell, is higher for more observationally skilled workers in all sectors, except in Construction

\(^{24}\)Haltiwanger, Lane and Spletzer (1999) reach similar conclusions using U.S. matched employer-employee data.
where the opposite holds true. Accordingly, it is not surprising to see that there is positive assortative matching in Manufacturing, Trade and Services but not in Construction.

Friction parameters $\kappa_{1s}$ thus seem to vary across worker skills in a way that accounts, at least partly, for the observed allocation of workers by skill and employer productivity. It is interesting to test whether or not the same holds true for the (normalized) vacancy functions $v_s(p)$. A rapid glance at Figure 5 gives the impression that estimated vacancy functions do not vary much across skill categories within each industry. To test this statement more formally we provide in Figure 7 bootstrap confidence intervals for $\hat{v}_s(p_j) - \hat{v}_{s'}(p_j)$ for each $j = 1, \ldots, N$ and each pair $(s, s') \in \{1, \ldots, 4\}^2$. In the Manufacturing and Construction sectors, the formal test confirms the eye-ball test: $v_s(p)$ varies with $p$ but not with $s$. In the Trade sector, the difference between the vacancy functions of skill categories 1, 2 and 4 are not statistically significant but category 3 (clerical employees and skilled blue collar workers) is significantly different from the other categories. In the Service sector, the shapes of vacancy functions look different for all four skill categories.

What we learn from that exercise is that different sorting patterns exist which require more or less complex matching mechanisms, even in the context of undirected search and matching. If neither workers nor firms can direct their search, then the way employer-employee matches are formed only depends on the way firm vacancies depend on firm and worker types. In general the number of vacancies posted by type $p$ firms on the type $s$ labor market, say $V_s(p) \equiv \overline{V}_s \cdot v_s(p)$, where $\overline{V}_s$ is the total number of vacancies posted by all firms in market $s$, can depend on $s$ and $p$ in an arbitrary way. Yet, we have found empirical evidence in the Manufacturing and Construction sectors that those functions are proportional in such a way that $V_s(p)/V_{s'}(p)$ is (possibly) function of $(s, s')$ but is independent of $p$. In other words, it seems reasonable as representation of the allocation process of workers into firms to assume that the relative number of type $s$ and $s'$ vacancies posted by a firm is independent of the firm’s productivity level. In this case, sorting only results from different aggregate levels of vacancies implying different job
offer arrival rates $\lambda_1$ (or $\kappa_1$). A natural issue, which we leave open, is that of the theoretical foundations of this representation.

5 Conclusion

This paper is the first attempt at estimating the influence of productivity, the bargaining power and between-firm competition on wages in a unified framework. We use an original equilibrium job search model with on-the-job search and wage bargaining as a theoretical structure, which we bring to the data. The combined use of a panel of matched employer-employee data and of LFS data allows us to implement a multi-stage estimation procedure that yields separate estimates of the search friction parameters (job destruction rates, arrival rates of job offers) and labor productivity at the firm level. These estimated values of the friction parameters and firm productivity levels are then used to estimate the bargaining power that shows up in the wage equation delivered by the theoretical model.

Our main finding is that between-firm competition plays a prominent role in wage determination in France over the period 1993-2000. The bargaining power of workers turns out to be very low—typically between 0 and a third—in all industries, up to a few exceptions among high-skilled workers. However, workers are able to capture a substantial share of the job surplus—typically between 20 and 50%, sometimes outside this range depending on the industry and skill category—as they benefit from the between-firm competition caused by on-the-job search. In other words, it turns out that the less skilled workers have very little “voice” within the firm and definitely need to brandish the threat of “exit” if they want to retain a share of their match rent. Only for the highest skill category do we find that the reverse is true, i.e. that “voice” matters more than “exit”.

Our results rely on simplifying assumptions that would need further scrutiny. We mentioned that allowing for endogenous search intensity is a useful thing to do. Another aspect of the model which merits a lot of thinking is residual wage dynamics. We already pointed out in
Postel-Vinay and Robin (2002b) that the model only partially captured the very large earnings fluctuations that could be observed both within and between employment spells. It is very unlikely that the current extension do a much better job.

However, maybe the most urgent task that awaits us is to model labor demand. Although the punchline of this paper was definitely on wage setting, we ended up looking at the allocation of workers across firms. We showed that there likely exist different patterns of assortative matching which require explanations that are not yet available. Mortensen (2000, 2003) already bridged the gap between equilibrium search models and search-matching models. We need to follow him into that direction, hunting down labor market imperfections to their very source, the matching function, and come up with an imperfect equilibrium model of the labor market where every noncompetitive aspect of the labor market can be tracked back to this black box.
Appendix

A Details of some theoretical results

A.1 Wage bargaining

A.1.1 Bargaining with unemployed workers

The subgame perfect equilibrium of the strategic negotiation game on matches with an unemployed worker is obtained by backward induction. For the sake of simplicity, it is assumed that the value of a vacant job, $\Pi_0$ is always zero. At step 4, the type-$p$ firm accepts any offer $w$ such that $w \leq \varepsilon p$, and the type-$\varepsilon$ worker accepts any offer $w$ yielding $V(\varepsilon, w, p) \geq V_0(\varepsilon)$. Therefore, at step 3, the worker offers $w = \varepsilon p$, and the employer offers $w$ such that $V(\varepsilon, w, p) = V_0(\varepsilon)$. At step 2, the worker refuses any offer that leaves him with less than his expected discounted utility, which amounts to $e^{-\rho \Delta} \cdot [\beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V_0(\varepsilon)]$, where $\Delta \rightarrow 0$ denotes the delay between steps 2 and 3. At step 1, the employer offers the lowest possible wage $\phi_0(\varepsilon, p)$ that the worker will accept, which satisfies:

$$V(\varepsilon, \phi_0(\varepsilon, p), p) = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V_0(\varepsilon). \quad (A1)$$

The worker accepts the wage $\phi_0(\varepsilon, p)$ in step 2 because he prefers to secure this offer rather than going on a process that does not raise his expected utility. Notice that it is the existence of a short delay between steps 2 and 3 that ensures existence and uniqueness of this subgame perfect equilibrium with instantaneous agreement at step 2 (see Osborne and Rubinstein, 1990).

A.1.2 Renegotiations

Renegotiations on continuing jobs occur when employees receive job offers and use them to claim wage increases. The renegotiation game is also solved by backward induction. Let us consider a situation in which a type-$\varepsilon$ employee on a type-$p$ job and earning a wage $w$ receives a job offer from a type-$p'$ employer. Let us denote by $w'_1$ and $w_1$ the wage offer made at step 1 by firm $p'$ and $p$ respectively. We assume that if the worker receives two offers yielding the same value, s/he chooses to stay with the incumbent employer.

Step 4. Decisions at step 4 are straightforward: firms accept any offer increasing their profits, and the worker accepts any offer increasing his/her contract values, in comparison to their fallback payoffs.

Step 3. At step 3, the worker makes offers with probability $\beta$, and the firms make simultaneous offers with probability $1 - \beta$. 

40
Claim 1 If the worker makes the offers, s/he moves to or stays at the firm with highest mpl, \( \max(p, p') \), and obtains a contract value depending on his/her decision at step 2 as in the following table

<table>
<thead>
<tr>
<th>Worker’s decision at step 2:</th>
<th>( p' &gt; p )</th>
<th>( p \geq p' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Accepts } V(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, \varepsilon p', p') ) if ( V(\varepsilon, \varepsilon p, p) &gt; V(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, \varepsilon p', p') ) if ( V(\varepsilon, \varepsilon p', p') &gt; V(\varepsilon, w_1, p) )</td>
</tr>
<tr>
<td>( \max(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, \varepsilon p, p) ) if ( V(\varepsilon, \varepsilon p, p) \leq V(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, w_1, p) ) if ( V(\varepsilon, \varepsilon p', p') \leq V(\varepsilon, w_1, p) )</td>
</tr>
</tbody>
</table>

**Proof of this claim.** The worker offers \( V(\varepsilon, \varepsilon p, p) \) to the type-\( p \) firm and \( V(\varepsilon, \varepsilon p', p') \) to the type-\( p' \) firm. The firm with highest market power (\( \max(p, p') \)) eventually wins the worker as \( p < p' \) implies \( V(\varepsilon, \varepsilon p, p) < V(\varepsilon, \varepsilon p', p') \).

As to the value of the resulting contract, one can derive it as follows: If \( p' > p \), \( p' \) accepts the wage \( \varepsilon p' \) offered by the worker only if, at step 2, the worker has not already signed with firm \( p' \) a contract \( w_1' \) such that \( V(\varepsilon, \varepsilon p, p) \leq V(\varepsilon, w_1', p') \). In such a case, if \( p' \) rejects the worker’s offer at step 4, the employee still prefers to stay at \( p' \) with wage \( w_1' \). Conversely, if \( p' \leq p \), a contract with associated wage \( \varepsilon p \) is effectively signed with firm \( p \) if, at step 2, the worker has accepted either the offer \( w_1' \) made by firm \( p' \) or the wage \( w_1 \) offered by firm \( p \) such that \( V(\varepsilon, \varepsilon p', p') > V(\varepsilon, w_1, p) \). Otherwise, if firm \( p \) rejects the worker’s offer at step 4, the employee still prefers to stays at firm \( p \) with wage \( w \) if \( V(\varepsilon, w_1, p) \geq V(\varepsilon, \varepsilon p', p') \).

Claim 2 If firms make offers, they enter a Bertrand game won by the firm with highest mpl, \( \max(p, p') \), at the end of which the worker obtains the contract value depending on his/her decision at step 2 as in the following table

<table>
<thead>
<tr>
<th>Worker’s decision at step 2:</th>
<th>( p' &gt; p )</th>
<th>( p \geq p' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Accepts } V(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, \varepsilon p, p) ) if ( V(\varepsilon, \varepsilon p, p) &gt; V(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, \varepsilon p', p') ) if ( V(\varepsilon, \varepsilon p', p') &gt; V(\varepsilon, w_1, p) )</td>
</tr>
<tr>
<td>( \max(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, \varepsilon p, p) ) if ( V(\varepsilon, \varepsilon p, p) \leq V(\varepsilon, w_1', p') )</td>
<td>( V(\varepsilon, w_1, p) ) if ( V(\varepsilon, \varepsilon p', p') \leq V(\varepsilon, w_1, p) )</td>
</tr>
</tbody>
</table>

**Proof of this claim.** Let us first consider this game when \( p' > p \). Since it is willing to extract a positive marginal profit from every match, the best the type-\( p \) firm can do to keep its employee is to offer him a wage exactly equal to \( \varepsilon p \) yielding the value \( V(\varepsilon, \varepsilon p, p) \) to the worker. Accordingly, the employee accepts to move to (or to stay at) firm \( p' \) if firm \( p' \) offers at least \( V(\varepsilon, \varepsilon p, p) \) (or \( \max[V(\varepsilon, \varepsilon p, p), V(\varepsilon, w_1', p')] \)).

Now consider the case \( p' \leq p \). The type-\( p \) firm can keep its employee by offering \( \max[V(\varepsilon, \varepsilon p', p'), V(w_1, p)] \) and can attract him/her back, if s/he moved to firm \( p' \) at step 2, by offering \( V(\varepsilon, \varepsilon p', p') \).
Step 2. At step 2, the worker chooses the offer that yields the highest expected utility in the continuing negotiation game. If s/he refuses both offers, s/he gets the value \( V(\varepsilon, w, p) \) of the pre-existing contract with the incumbent firm. Otherwise s/he gets the values \( EV \) as in the following table:

<table>
<thead>
<tr>
<th>( p' &gt; p )</th>
<th>Worker's decision at step 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( EV = \beta V(\varepsilon, \varepsilon p', p') + (1 - \beta)V(\varepsilon, \varepsilon p, p) ) if ( V(\varepsilon, \varepsilon p, p) &gt; V(\varepsilon, w_1', p') )</td>
</tr>
<tr>
<td></td>
<td>( EV = V(\varepsilon, w_1', p') ) if ( V(\varepsilon, \varepsilon p, p) \leq V(\varepsilon, w_1', p') )</td>
</tr>
</tbody>
</table>

| \( p \geq p' \) | \( EV = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta)V(\varepsilon, \varepsilon p', p') \) if \( V(\varepsilon, \varepsilon p', p') > V(\varepsilon, w_1, p) \) |
|             | \( EV = V(\varepsilon, w_1, p) \) if \( V(\varepsilon, \varepsilon p', p') \leq V(\varepsilon, w_1, p) \) |

Step 1. At step 1, employers make simultaneous offers. Both employers offer the lowest possible wage that attracts the worker (and still yields nonnegative profits).

Claim 3 At step 1, the firm with highest mpl, \( \max(p, p') \), makes an offer immediately accepted provided that it increases the worker’s value. This offer defines a wage \( \phi(\varepsilon, p, p') \) which solves:

\[
V(\varepsilon, \phi(\varepsilon, p, p'), p') = \beta V(\varepsilon, \varepsilon p', p') + (1 - \beta)V(\varepsilon, \varepsilon p, p) \quad \text{if} \quad p' > p
\]
\[
V(\varepsilon, \phi(\varepsilon, p, p'), p) = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta)V(\varepsilon, \varepsilon p', p') \quad \text{if} \quad p \geq p'
\]  

(A2)

Proof of this claim.

- If \( p' > p \), the preceding table of expected outcomes implies that the worker can get at least \( EV = \beta V(\varepsilon, \varepsilon p', p') + (1 - \beta)V(\varepsilon, \varepsilon p, p) \) in the continuing negotiation game by accepting any offer made by the type-\( p \) firm. In order to avoid a waste of time in unnecessary negotiation, firm \( p' \) offers a wage \( w_1' = \phi(\varepsilon, p, p') \), that the worker accepts at step 2 and that solves:

\[
V(\varepsilon, \phi(\varepsilon, p, p'), p') = \beta V(\varepsilon, \varepsilon p', p') + (1 - \beta)V(\varepsilon, \varepsilon p, p).
\]

Firm \( p \) cannot bid this wage that is bigger than \( \varepsilon p \).

- If \( p' \leq p \), the worker can get at least \( EV = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta)V(\varepsilon, \varepsilon p', p') \) by accepting any offer made by the type-\( p' \) firm. In order to avoid a waste of time, firm \( p \) offers a wage \( w_1 = \phi(\varepsilon, p', p) \), that the worker accepts at step 2 and that solves:

\[
V(\varepsilon, \phi(\varepsilon, p', p'), p) = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta)V(\varepsilon, \varepsilon p', p').
\]

Firm \( p' \) cannot bid this wage that is bigger than \( \varepsilon p' \).
This completes the characterization of the subgame perfect equilibrium of our bargaining game. It is worth introducing some extra notation at this point (for later use): we see that the minimal value of $p'$ for which “something happens” (i.e. either causing a wage increase or an employer change) is $q(\varepsilon, w, p)$ such that
\[
V(\varepsilon, w, p) = \beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p)).
\] (A3)

Note that
\[
V(\varepsilon, \varepsilon q(\varepsilon, w, p), q(\varepsilon, w, p)) = V(\varepsilon, w, p) - \beta \left[ V(\varepsilon, \varepsilon p, p) - V(\varepsilon, w, p) \right]
\] whenever $w < p$.

\section{Equilibrium wage determination}

Here we derive the precise closed-form of equilibrium wages $\phi_0(\varepsilon, p)$ and $\phi(\varepsilon, p, p')$ defined in equations (1) and (2) respectively. The first step is to derive the value functions $V_0(\cdot)$ and $V(\cdot)$. Time is discounted at rate $\rho$. Since offers accrue to unemployed workers at rate $\lambda_0$, $V_0(\varepsilon)$ solves the following Bellman equation:
\[
(\rho + \lambda_0) V_0(\varepsilon) = \varepsilon b + \lambda_0 \mathbb{E}_F \left\{ \max \left[ V(\varepsilon, \phi_0(\varepsilon, X), X), V_0 \right] \right\},
\] (A4)

where $\mathbb{E}_F$ is the expectation operator with respect to a variable $X$, which has distribution $F$. Using the definition (A1) to replace $V(\varepsilon, \phi_0(\varepsilon, X), p)$ by $\beta V(\varepsilon, p, p) + (1 - \beta) V_0(\varepsilon)$ in the latter equation, we then show that:
\[
\rho V_0(\varepsilon) = \varepsilon b + \lambda_0 \mathbb{E}_F \left\{ \max \left( \beta \left[ V(\varepsilon, \phi_0(\varepsilon, X), X) - V_0(\varepsilon) \right], 0 \right) \right\}.
\] (A5)

We thus find that an unemployed worker’s expected lifetime utility depends on his personal ability $\varepsilon$ through the amount of output he produces when engaged in home production, $\varepsilon b$, but also on labor market parameters such as the distribution of jobs and his bargaining power $\beta$.

Now turning to employed workers, consider a type-\varepsilon worker employed at a type-\(p\) firm. Since layoffs occur at rates $\delta$, we may now write the Bellman equation solved by the value function $V(\varepsilon, w, p)$:
\[
[\rho + \delta + \lambda_1 \mathbb{F}(q(\varepsilon, w, p))] V(\varepsilon, w, p) = w
+ \lambda_1 [F(p) - F(q(\varepsilon, w, p))] \mathbb{E}_F \left\{ V(\varepsilon, \phi(\varepsilon, X, p), X) | q(\varepsilon, w, p) \leq X \leq p \right\}
+ \lambda_1 \mathbb{F}(p) \mathbb{E}_F \left\{ V(\varepsilon, \phi(\varepsilon, p, X), X) | p \leq X \right\} + \delta V_0(\varepsilon).
\] (A6)
Let us denote by $p_{\max}$ the upper support of $p$. Equations (A3) and (A6), together with the bargaining rule (A2) allow us to rewrite (A6) as follows:

$$
[w + \delta + \lambda_1 F(q(\varepsilon, w, p))] V(\varepsilon, w, p) = w + \delta V_0(\varepsilon) + $$

$$
\lambda_1 \int_0^{p_{\max}} [(1 - \beta) V(\varepsilon, \varepsilon x, x) + \beta V(\varepsilon, \varepsilon p, p)] dF(x) + $$

$$
\lambda_1 \int_p^{p_{\max}} [(1 - \beta) V(\varepsilon, \varepsilon p, p) + \beta V(\varepsilon, \varepsilon x, x)] dF(x). \quad (A7)
$$

Imposing $w = \varepsilon p$ in (A7), taking the derivative, and noticing that the definition (A3) of $q(\varepsilon, w, p)$ implies that $q(\varepsilon, \varepsilon p, p) = p$, one gets:

$$
\frac{dV(\varepsilon, \varepsilon p, p)}{dp} = \frac{\varepsilon}{\rho + \delta + \lambda_1 \beta F(p)}. \quad (A8)
$$

Then, integrating by parts in equation (A7):

$$
(\rho + \delta) V(\varepsilon, w, p) = w + \delta V_0(\varepsilon) + \beta \lambda_1 \varepsilon \int_0^{p_{\max}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx $$

$$
+ (1 - \beta) \lambda_1 \varepsilon \int_q^{p_{\max}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A9)
$$

Again imposing $w = \varepsilon p$, the last equation in turn implies that

$$
(\rho + \delta) V(\varepsilon, \varepsilon p, p) = \varepsilon p + \delta V_0(\varepsilon) + \beta \lambda_1 \varepsilon \int_0^{p_{\max}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A10)
$$

Noticing that $q(\varepsilon, \phi(\varepsilon, p', p), p) = p'$, an expression of $V(\varepsilon, \phi(\varepsilon, p', p), p)$ can be obtained from (A9):

$$
(\rho + \delta) V(\varepsilon, \phi(\varepsilon, p', p), p) = \phi(\varepsilon, p', p) + \delta V_0(\varepsilon) + \beta \lambda_1 \varepsilon \int_0^{p_{\max}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx $$

$$
+ (1 - \beta) \lambda_1 \varepsilon \int_{p'}^{p} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A11)
$$

But, following the bargaining rule (A2), $(\rho + \delta) V(\varepsilon, \phi(\varepsilon, p', p), p)$ should also equal

$$
(\rho + \delta) [\beta V(\varepsilon, \varepsilon p, p) + (1 - \beta) V(\varepsilon, \varepsilon p', p')]$$

which, using (A10), writes as:

$$
\beta \varepsilon p + (1 - \beta) \varepsilon p' + \delta V_0(\varepsilon) + \beta^2 \lambda_1 \varepsilon \int_0^{p_{\max}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx + \beta (1 - \beta) \lambda_1 \varepsilon \int_{p'}^{p_{\max}} \frac{F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx.
$$

Equating this expression with the right hand side of equation (A11), one gets the following expression for the wage $\phi(\varepsilon, p', p)$:

$$
\phi(\varepsilon, p', p) = \beta \varepsilon p + (1 - \beta) \varepsilon p' - (1 - \beta)^2 \lambda_1 \varepsilon \int_{p'}^{p} \frac{\varepsilon F(x)}{\rho + \delta + \lambda_1 \beta F(x)} dx. \quad (A12)
$$
The lower support of the distribution of marginal productivities, \( p_{\text{min}} \), cannot fall short of the value \( p_{\text{inf}} \) such that \( V(\varepsilon, \varepsilon p_{\text{inf}}, p_{\text{inf}}) = V_0(\varepsilon) \). Using the definitions (A5), of \( V_0(\varepsilon) \), and (A9), of \( V(\varepsilon, w, p) \), this identity yields:

\[
p_{\text{inf}} = b + \beta(\lambda_0 - \lambda_1) \int_{p_{\text{inf}}}^{p_{\text{max}}} \frac{\bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx. \quad (A13)
\]

(Note that the value of \( p_{\text{inf}} \) is independent of \( \varepsilon \). This result holds true for any homogeneous specification of the utility function.) Finally, as the bargaining outcome implies (A12), the identity \( V(\varepsilon, \varepsilon p_{\text{inf}}, p_{\text{inf}}) = V_0(\varepsilon) \) implies the following alternative definition of \( \phi_0(\varepsilon, p) \):

\[
\phi_0(\varepsilon, p) = \phi(\varepsilon, p_{\text{inf}}, p) = \beta \varepsilon p + (1 - \beta) \varepsilon p_{\text{inf}} - (1 - \beta)^2 \lambda_1 \int_{p_{\text{inf}}}^{p} \frac{\varepsilon \bar{F}(x)}{\rho + \delta + \lambda_1 \beta \bar{F}(x)} dx. \quad (A14)
\]

**A.3 Equilibrium wage distributions**

The \( G(w|\varepsilon, p) \ell(\varepsilon, p) (1 - u) \bar{M} \) workers of type \( \varepsilon \), employed at firms of type \( p \), and paid less than \( w \in [\phi_0(\varepsilon, p), \varepsilon p] \) leave this category either because they are laid off (rate \( \delta \)), or because they receive an offer from a firm with \( mpl \geq q(\varepsilon, w, p) \) which grants them a wage increase or induces them to leave their current firm (rate \( \lambda_1 \bar{F}[q(\varepsilon, w, p)] \)). On the inflow side, workers entering the category (ability \( \varepsilon \), wage \( \leq w \), \( mpl \) \( p \)) come from two distinct sources. Either they are hired away from a firm less productive than \( q(\varepsilon, w, p) \), or they come from unemployment. The steady-state equality between flows into and out of the stocks \( G(w|\varepsilon, p) \ell(\varepsilon, p) \) thus takes the form:

\[
\{\delta + \lambda_1 \bar{F}[q(\varepsilon, w, p)]\} G(w|\varepsilon, p) \ell(\varepsilon, p) (1 - u) \bar{M} = \left\{\lambda_0 u \bar{M} h(\varepsilon) + \lambda_1 (1 - u) \bar{M} \int_{p_{\text{min}}}^{q(\varepsilon, w, p)} \ell(\varepsilon, x) dx \right\} f(p) = \left\{\delta h(\varepsilon) + \lambda_1 \int_{p_{\text{min}}}^{p} \ell(\varepsilon, x) dx \right\} (1 - u) \bar{M} f(p), \quad (A15)
\]

since \( \lambda_0 u = \delta (1 - u) \). Applying this identity for \( w = \varepsilon p \) (which has the property that \( G(\varepsilon p|\varepsilon, p) = 1 \) and \( q(\varepsilon, \varepsilon p, p) = p \)), we get:

\[
\{\delta + \lambda_1 \bar{F}(p)\} \ell(\varepsilon, p) = \left\{\delta h(\varepsilon) + \lambda_1 \int_{p_{\text{min}}}^{p} \ell(\varepsilon, x) dx \right\} f(p),
\]

which solves as

\[
\ell(\varepsilon, p) = \frac{1 + \kappa_1}{[1 + \kappa_1 \bar{F}(p)]^2} h(\varepsilon) f(p).
\]

This shows that \( \ell(\varepsilon, p) \) has the form \( h(\varepsilon) \ell(p) \) (absence of sorting), and gives the expression of \( \ell(p) \). Hence the equations (10) and (11). Equation (10) can be integrated between \( p_{\text{min}} \) and \( p \) to obtain (9). Substituting (9), (10) and (11) into (A15) finally yields equation (12).
A.4 Derivation of \( \mathbb{E}[T(w)|p] \) for any integrable function \( T(w) \)

The lowest paid type-\( \varepsilon \) worker in a type-\( p \) firm is one that has just been hired, therefore earning \( \phi_0(\varepsilon, p) = \phi(\varepsilon, p_{\text{haf}}, p) \), while the highest-paid type-\( \varepsilon \) worker in that firm earns his marginal productivity \( \varepsilon p \). Thus, the support of the within-firm earnings distribution of type \( \varepsilon \) workers for any type-\( p \) firm belongs to the interval \( [p_{\text{min}}, p] \). Noticing that \( G(q|p) = G(\phi(\varepsilon, q, p)|\varepsilon, p) \) has a mass point at \( p_{\text{haf}} \) and is otherwise continuous over the interval \( [p_{\text{min}}, p] \), we can readily show that for any integrable function \( T(w) \),

\[
\mathbb{E}[T(w)|p] = \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} \left( \int_{\phi(\varepsilon, p_{\text{min}}, p)}^{\varepsilon_{\text{max}}} T(w) \, G(dw|\varepsilon, p) + T(\phi_0(\varepsilon, p)) \, G(\phi_0(\varepsilon, p)|\varepsilon, p) \right) h(\varepsilon) \, d\varepsilon
\]

\[
= \left[ 1 + \kappa_1 F(p) \right]^2 \left\{ 1 + \frac{\kappa_1^2}{(1 + \kappa_1)^2} \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T(\phi_0(\varepsilon, p)) \, h(\varepsilon) \, d\varepsilon \right\}
\]

\[
+ \int_{p_{\text{min}}}^{p} \left[ \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T(\phi(\varepsilon, q, p)) \, h(\varepsilon) \, d\varepsilon \right] \frac{2\kappa_1 f(q)}{[1 + \kappa_1 F(q)]^3} \, dq
\]

\[
= \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T(\varepsilon) h(\varepsilon) \, d\varepsilon + \frac{1 + \kappa_1 F(p)}{1 + \kappa_1} \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} [T(\phi_0(\varepsilon, p)) - T(\phi(\varepsilon, p_{\text{min}}, p))] \, h(\varepsilon) \, d\varepsilon
\]

\[\quad - \left[ 1 + \kappa_1 F(p) \right]^2 \int_{p_{\text{min}}}^{p} \left[ \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} T'(\phi(\varepsilon, q, p)) \, \varepsilon h(\varepsilon) \, d\varepsilon \right] \frac{1 - \beta}{1 + (1 - \sigma) \kappa_1 F(q)} \, dq. \tag{A16}\]

The first equality follows from the definition of \( G(w|\varepsilon, p) \) as

\[
G(w|\varepsilon, p) = \frac{[1 + \kappa_1 F(p)]^2}{[1 + \kappa_1 F(q(\varepsilon, w, p))]^2}
\]

yielding

\[
G'(w|\varepsilon, p) = \left[ 1 + \kappa_1 F(p) \right]^2 \frac{2\kappa_1 f(q)}{[1 + \kappa_1 F(q)]^3} \frac{\partial q(\varepsilon, w, p)}{\partial w}.
\]

The second equality is obtained with an integration by parts, deriving the partial derivative of \( \phi(\varepsilon, q, p) \) with respect to \( q \) from (A12) as

\[
\frac{\partial \phi(\varepsilon, q, p)}{\partial q} = (1 - \beta) \varepsilon \frac{1 + (1 - \sigma) \kappa_1 F(q)}{1 + (1 - \sigma) \kappa_1 F(q)}.
\]

References


<table>
<thead>
<tr>
<th>Industry</th>
<th>No. of Firms</th>
<th>Total no. of workers</th>
<th>Mean share of labor in v.a. (%)</th>
<th>Mean annual labor cost (1000 Euros)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>4,127</td>
<td>1,402,367</td>
<td>7.3</td>
<td>36.6</td>
</tr>
<tr>
<td>Construction</td>
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<td>68.0</td>
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<tr>
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<td>49.4</td>
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<td>Total</td>
<td>4,254</td>
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<td>7.3</td>
<td>36.6</td>
</tr>
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Notes:
1. In 1000 Euros.
TABLE 2: Transition Parameter Estimates$^{1,2}$

<table>
<thead>
<tr>
<th>Industry</th>
<th>Labor category</th>
<th>$\lambda_1$</th>
<th>$1/\lambda_1$</th>
<th>$\delta$</th>
<th>$1/\delta$</th>
<th>$\kappa_1 = \lambda_1/\delta$</th>
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<td></td>
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<td>(1.50)</td>
<td>(0.002)</td>
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Notes:  
$^1$Per annum.  
$^2$Standard errors in parentheses.
### TABLE 3: Wage Equation Estimates\(^1,2\)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Bargaining power</th>
<th>Productivity</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \alpha_4 )</th>
<th>( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>( \beta_1 = 0.35 )</td>
<td>( \beta_2 = 0.13 )</td>
<td>( \beta_3 = 0.00 )</td>
<td>( \beta_4 = 0.00 )</td>
<td>2.54</td>
<td>1.51</td>
<td>1.16</td>
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<td>0.15</td>
<td>0.17</td>
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<td>0.00</td>
<td>2.47</td>
<td>1.28</td>
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<td>0.00</td>
<td>0.08</td>
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<td>2.57</td>
<td>1.55</td>
<td>1.07</td>
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Notes:  
\(^1\)Discount rate \( e^{\rho} = 0.95 \).  
\(^2\)Bootstrap standard errors in parentheses.
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<tr>
<th>Industry</th>
<th>Parameter Estimate</th>
<th>Full Sample Mean</th>
<th>2.5th %ile</th>
<th>50th %ile</th>
<th>97.5th %ile</th>
<th>p-values</th>
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<td>α₂</td>
<td>α₃</td>
<td>α₄</td>
<td>ξ</td>
<td>χ</td>
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<td>0.90</td>
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</tr>
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Sargan test p-values:

Asymptotic: 0.003, 0.003, 0.003, 0.003, 0.003, 0.003
Bootstrap: 0.091, 0.091, 0.091, 0.091, 0.091, 0.091
TABLE 5: Mean worker share of match rents

<table>
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<tr>
<th>Industry</th>
<th>Labor category</th>
<th>observed: $\hat{\beta}_0$</th>
<th>predicted: $\hat{\beta}_0$</th>
<th>share of $\hat{\beta}_0$ due to between-firm competition</th>
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<td>39%</td>
</tr>
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<tr>
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<td>4</td>
<td>0.15</td>
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<td>100%</td>
</tr>
<tr>
<td>Construction</td>
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Note: $^1$Discount rate $e^\rho = 0.95.$
Figure 1: The wage-productivity relationship
Figure 2: Productivity densities
Figure 4: The size–productivity relationship
Figure 5: Job vacancies
Figure 6: Sorting on observables
Figure 7: Cross-skill differences in vacancies