DISCUSSION PAPER SERIES

No. 4125

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INTERNATIONAL MACROECONOMICS

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ISSN 0265-8003

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Discussion Paper No. 4125
December 2003

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CEPR Discussion Paper No. 4125

December 2003

ABSTRACT

Information Variables for Monetary Policy in a Small Structural Model of the Euro Area*

This Paper estimates a small New-Keynesian model with imperfect information and optimal discretionary policy using data for the euro area. The model is used to assess the usefulness of monetary aggregates and unit labour costs as information variables for monetary policy. The estimates reveal that the information content of the M3 monetary aggregate is limited. A more useful role emerges for the unit labour costs indicator, which contains information that helps to reduce the volatility of the output gap. Finally, the estimated weights for the objectives of monetary policy indicate that considerable importance is attributed to interest-rate smoothing, greater than the importance attributed to the output gap stabilization. This finding indicates that the welfare gains of commitment may be smaller than is suggested by typical parameterizations of New-Keynesian models.

JEL Classification: E50
Keywords: inflation, Kalman filter, monetary policy and output gap

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* This Paper is produced as part of a CEPR research network on 'Specialization Versus Diversification: The Microeconomics of Regional Development and the Spatial Propagation of Macroeconomic Shocks in Europe', funded by the European Commission under the Research Training Network Programme (Contract No: HPRN-CT-2000-00072). We thank Lars Svensson for a stimulating discussion at the Sveriges Riksbank conference 'Small Structural Models for Monetary Policy Analysis. Progress, Puzzles and Opportunities'. We are also grateful to Mike Woodford and seminar participants at the Banca d'Italia and the Banco de España for helpful comments. The views are personal and do not involve the responsibility of the institutions with which we are affiliated.

Submitted 14 October 2003

1. Introduction

Dynamic stochastic models of the "new keynesian" variety developed by Rotemberg and Woodford (1997), Woodford (1999) and Clarida, Gali and Gertler (1999) have acquired a solid position in the analysis of monetary policy. Such models have proved useful, e.g. to analyze the properties of various interest rate rules (Jensen, 2002), to quantify the welfare effects of simple versus optimal policy (Dennis and Söderström, 2002) and of imperfect information (Ehrmann and Smets, 2003). Several central banks employ variants of these models to inform the policy analysis.

This paper estimates the structural parameters of such a model using data for the euro area, under the assumptions of imperfect information and optimal discretionary policy. This exercise adds useful elements to existing analyses.

First, integrating imperfect information in the new keynesian model is important because one of its key variables, "potential output" (i.e. the flexible price level of output), is not observable. This aspect adds to the fact that information about several other variables of interest, such as contemporaneous GDP or inflation, is available to policy makers only with lags and subject to statistical revisions. Previous quantitative analyses that deal with this problem, e.g. Ehrmann and Smets (2003) and Coenen, Levin and Wieland (2002), proceed by separately estimating the information structure and the structural parameters (estimated and/or calibrated). This separation is in principle problematic because, as Svensson and Woodford (2000) show, the equilibrium motion of all variables depends on both the structural parameters and the information structure when information is imperfect. An advantage of the maximum likelihood estimation pursued here is that it allows this issue to be dealt with in a consistent way, by jointly determining the economy's structural parameters and the noisiness of each indicator.[1] Another advantage, recently highlighted by Lindé (2002), is that maximum likelihood yields more precise estimates than limited information methods in the presence of measurement error.

Second, since the model's quantitative predictions hinge upon some key parameters, estimation is important. For instance, Dennis and Söderström (2002)

[1]The joint presence of an optimization and a filtering problem is an important difference with respect to Ireland (2001), who estimates a small structural model for the US assuming perfect information and an exogenous policy rule.

show that the welfare gains delivered by commitment in comparison to discretion vary significantly, from almost nil to very large, depending on the degree of forward-looking behavior in the inflation equation and on the weight attached to the interest rate stabilization objective by the monetary authority.

Finally, the estimation of the monetary authorities' objectives, obtained under the assumption of optimal discretionary policy, distinguishes this paper from previous pioneering estimation exercises, e.g. Ireland (2001) for the United States or Smets and Wouters (2003) for the euro area, in which a "simple" instrument rule (i.e. restricted to depend on a few key variables) is used to describe monetary policy.[2]

The estimation results show that monetary aggregates contain little information about the state variables of interest for the conduct of stabilization policy. M3 turns out to have basically no usefulness for stabilization policy. The unit labor cost indicator, instead, helps improving the inference about potential output. This reduces the volatility of the output gap, increasing the policy maker's welfare. Moreover, the estimates for the monetary authority's objectives show that a large weight is attached to inflation, followed closely by the interest-smoothing target and by a small output-gap weight. Several previous papers use a non-zero weight on interest-smoothing in order to fit the persistence of short term rates, though the values chosen are usually much smaller than the estimated ones. Our estimates, similar to the ones by Dennis (2002) for the United States, imply that commitment gains are smaller than suggested by typical calibrations.

The paper is organized as follows. The next section specifies a dynamic stochastic monetary policy model that incorporates an imperfect information problem, based on Ehrmann and Smets (2003). The solution of this model, following Svensson and Woodford (2000), maps the structural parameters into a vector autoregression. Section 3 discusses how to estimate the model parameters using the Kalman filter following a methodology proposed by Sargent (1989) and Ireland (2001) and presents the data, the estimation results and their robustness. Section 4 utilizes the estimated model to quantify the welfare effects of the various indicators and the welfare gains delivered by commitment. Section 5 summarizes main findings.

[2]Our endevour is similar to Dennis (2002), who estimates the policy preferences of the US Federal Reserve. One remaining difference is our consideration of imperfect information aspects.

2. The model

We model policy by assuming that the central banks aims at minimizing the intertemporal loss function:

$$\Lambda_t = E[\sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau} \mid I_t] \tag{2.1}$$

where $\beta \in (0,1)$ is the intertemporal discount factor and period losses are given by:

$$L_t \equiv \left[(\pi_t)^2 + \lambda(y_t - \bar{y}_t)^2 + \nu(i_t - i_{t-1})^2\right]. \tag{2.2}$$

where π_t, y_t, $\bar{y}_t$ and i_t denote, respectively, inflation, output, potential output and the nominal short term interest rate.

Our benchmark model, taken from Ehrmann and Smets (2003), consists of the following structural equations:

$$\begin{aligned}
y_t &= \delta y_{t-1} + (1-\delta)\, y_{t+1|t} - \theta \left(i_t - \pi_{t+1|t}\right) + u_{p,t} && (2.3)\\
\pi_t &= \alpha \pi_{t-1} + (1-\alpha)\, \pi_{t+1|t} + \kappa \left(y_t - \overline{y}_t\right) + u_{c,t} && (2.4)\\
\overline{y}_t &= \rho \overline{y}_{t-1} + u_{\overline{y},t} && (2.5)\\
m_t &= \gamma_1 m_{t-1} + \gamma_2 m_{t+1|t} + \gamma_y y_t - \gamma_i i_t + u_{m,t} && (2.6)
\end{aligned}$$

where m_t is real money and the subscript $_{t+1|t}$ denotes the expected value of a variable in period $t+1$ conditioned on information as of time t. There are four structural i.i.d. innovations in the model with covariance matrix Σ_u^2: a preference shock $u_{p,t}$, a cost-push shock $u_{c,t}$, a potential output shock $u_{\overline{y},t}$ and a money demand shock $u_{m,t}$.

One reason for choosing this model is its relative simplicity, which allows for a clear interpretation of the transmission mechanism of structural shocks. Moreover, the specification encompasses purely forward looking models, as the ones used, for example, by Rotemberg and Woodford (1997) and Clarida, Galí and Gertler (1999), and more backward looking models as the one described in Rudebusch (2002).

The presence of lagged values in the output, inflation and real money equations has been shown to be important to fit the dynamics of the data. Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2001) show that lagged terms in the output and inflation equations arise in the presence of, respectively, habits-in-consumption and Calvo-pricing firms with indexation to last period inflation. Similarly, lagged and future real money in the money demand equation can be introduced by assuming costly adjustment for money holdings. The shocks can also be given a microfoundation. The cost-push shock $u_{c,t}$ that appears in the inflation equation emerges with a time-varying mark-up in the goods market (e.g. Smets and Wouters (2003)), while the shock $u_{p,t}$ is obtained by introducing a random disturbance to the utility function of the representative households. The money demand shock can be justified as a shock to the real balances component of the utility function.

Information about the variables in the economy is obtained from the following vector of measurables:

$$y_t^o = y_{t-1} + v_{y,t} \tag{2.7a}$$

$$\pi_t^o = \pi_t + v_{\pi,t} \tag{2.7b}$$

$$m_t^o = m_t + v_{m,t} \tag{2.7c}$$

$$\zeta_t^o = \zeta_{t-1} + v_{x,t} \tag{2.7d}$$

where y_t^o is the indicator output variable, given by a noisy observation of the previous period output level. This assumption models the fact that information on output y_t in a given quarter is not contemporaneously available and that, moreover, output observations are subject to revisions, which justifies the existence of noisy measurement ($v_{y,t}$). The indicators π_t^o and m_t^o posit that inflation and real money balances are observed contemporaneously, possibly with noise. Although no direct role for money exists in this model, as it does not affect any of the payoff relevant variables or their transmission mechanism, the monetary indicator may contain useful information on current output through the money demand equation (2.6), which may help reducing the imperfect information problem.

The last indicator, ζ_t^o, is a noisy measure of the previous period real unit labor cost. Rotemberg and Woodford (1997) show, among others, that such costs are proportional to the output gap, which we capture by assuming:

$$\zeta_t = \mu(y_t - \bar{y}_t) \tag{2.8}$$

The measurement errors in the vector v are assumed to be i.i.d. with covariance matrix Σ_v^2.

2.1. The Economy under a Discretionary Equilibrium

We focus on the discretionary (i.e. Markov perfect) equilibrium, whereby the strategies of both the policy maker and the agents are constrained to be functions of the predetermined state variables alone (i.e. history-dependent strategies are ruled out).[3]

To solve the above model it is convenient to rewrite the system in the state-space form following a Svensson and Woodford (2000), defining the vector $X_t' \equiv \left[\begin{array}{ccccccccc} y_{t-1} & \pi_{t-1} & m_{t-1} & \overline{y}_t & u_{p,t} & u_{c,t} & u_{m,t} & i_{t-1} & \overline{y}_{t-1} \end{array}\right]$ of predetermined state variables and the vector $x_t' \equiv \left[\begin{array}{ccc} y_t & \pi_t & m_t \end{array}\right]$ of non-predetermined (forward looking) variables (see Appendix B).

Information is described by the set $J_t \equiv \{Z_\tau, \Omega;\ \tau = t, t-1, ..., 0\}$ i.e. all agents in the model are supposed to know the model parameters

$$\Omega \equiv [\alpha, \beta, \delta, \gamma_1, \gamma_2, \gamma_y, \gamma_i, \lambda, \nu, \mu, \kappa, \theta, \rho, \Sigma_u^2, \Sigma_v^2]$$

and the history up to and including period t of the four observable variables (2.7), stacked in the vector:

$$Z_t' \equiv [y_t^o, \pi_t^o, m_t^o, \zeta_t^o].$$

We use the algorithms of Gerali and Lippi (2003) to solve for the optimal Markov perfect policy ($i_t = FX_{t|t}$) and compute the equilibrium representation of the model, i.e. the law of motion of the state variables (X_t), forward-looking (x_t) variables and the optimal prediction for X_t computed by the Kalman filter:

[3] Alternatively, the model could be solved and estimated for the optimal Ramsey policy, under the assumption that the central bank can commit (see section 4.2).

$$
\begin{aligned}
X_{t+1} &= HX_t + JX_{t|t} + C_u u_{t+1} && \text{(2.9a)}\\
x_t &= GX_{t|t} + G^1(X_t - X_{t|t}) && \text{(2.9b)}\\
X_{t|t} &= X_{t|t-1} + K[L(X_t - X_{t|t-1}) + v_t] && \text{(2.9c)}
\end{aligned}
$$

where the matrices F, H, J, C_u, G, G^1, L and K depend on the primitive parameters in Ω (see Svensson and Woodford, 2000).

The linear quadratic structure of this problem and the certainty equivalence principle imply that the optimal interest rate rule in this model, $i_t = FX_{t|t}$, is a linear function of the estimate of the states which does not depend on the uncertainty in the system. Of course uncertainty affects the way in which an innovation in the observables is mapped into an updated estimate of the state variables, which occurs through the Kalman gain matrix: K.

3. Bringing the model to the data

The evolution of the whole economic system (2.9) can be expressed in a compact notation using the vector autoregression:

$$
Q_{t+1} = \hat{A}Q_t + \hat{G}w_{1,t+1} \qquad (3.10)
$$

where

$$
Q_{t+1} \equiv \begin{bmatrix} X_{t+1} \\ X_{t+1|t} \end{bmatrix} \qquad \hat{A} \equiv \begin{bmatrix} H + JKL & J(I - KL) \\ (H+J)KL & (H+J)(I-KL) \end{bmatrix}
$$

$$
w_{1,t+1} \equiv \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix} \qquad \hat{G} \equiv \begin{bmatrix} C_u & JK \\ 0 & (H+J)K \end{bmatrix}
$$

In particular, note that (3.10) allows the dynamics of the observable variables Z_t

and the nominal interest rate i_t to be expressed as a function of Q_t as:

$$\begin{bmatrix} Z_t \\ i_t \end{bmatrix} = \hat{L}Q_t + \hat{M}v_t \tag{3.11}$$

where

$$\hat{L} \equiv \begin{bmatrix} L + MKL & M(I - KL) \\ FKL & F(I - KL) \end{bmatrix} \text{ and } \hat{M} \equiv \begin{bmatrix} MK + I \\ FK \end{bmatrix}.$$

The data, represented by the vector d_t used in the estimation, are given by the 3-month interest rate, taken to be a noisy measure of the monetary policy control variable and the four observables of the theoretical model, which are taken as noisy measures of the true (lagged) output, inflation, money and the (lagged) output gap (hence $d_t' = [Z_t'\ i_t]$). Using (3.11):

$$\begin{aligned} d_t &= \hat{L}Q_t + w_{2,t} \\ w_{2,t} &\equiv \hat{M}v_t + e_t \end{aligned} \tag{3.12}$$

where $e_t \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & e_{i,t} \end{bmatrix}'$ is a vector of measurement errors in the data. Since we already have measurement errors in the theoretical model (the vector v), the measurement errors in e_t associated to the Z_t variables are assumed to be identically zero to avoid redundancy. Instead, the introduction of a measurement error for the interest rate is needed to avoid a stochastic singularity problem, as the theoretical model predicts that the interest rate is a linear function of the state variables. By introducing the measurement error $e_{i,t}$ we create a wedge between the optimal rate predicted by the model and the actual rate recorded in the data. This makes estimation possible. The standard deviation of the measurement error $e_{i,t}$ can be interpreted as a measure of the distance between actual policy and the optimal one prescribed by the model.

Equations (3.10) and (3.12) represent a state space system to which a Kalman filter can be applied to estimate the structural model parameters, Ω. The basic insight rests on the fact that the solution of the theoretical model maps the structural parameters Ω into the matrices $\hat{A}$, $\hat{G}, \hat{L}$, $\hat{M}$, Σ_u^2 and Σ_v^2 which fully characterize the system (3.10) and (3.12). Given this system, the Kalman filter provides a convenient method to compute the likelihood function associated to a vector of observations on d_t. The estimation problem thus consists in finding the vector of parameters Ω that maximizes the likelihood function. The idea, originally due to Sargent (1989), McGrattan (1994) and Ireland (2001), is illustrated in more detail in Appendix C.[4]

3.1. The data

The data used in the estimation are the euro area counterparts of the variables in the vector Z_t and i_t: output, which is measured by real GDP, the inflation rate, measured by the quarterly changes in the GDP deflator, real money, measured by the stock of nominal M3 divided by the GDP deflator, the (lagged) output gap indicator, measured by (lagged) real unit labor costs and the nominal short-term interest rate. These data, which run from 1981:1 to 2002:2, contain a subsample during which the euro area was not formally established (until the end of 1998). Euro area data for this subsample are taken from Fagan, Henry and Mestre (2001) who aggregate national data (See Appendix A).

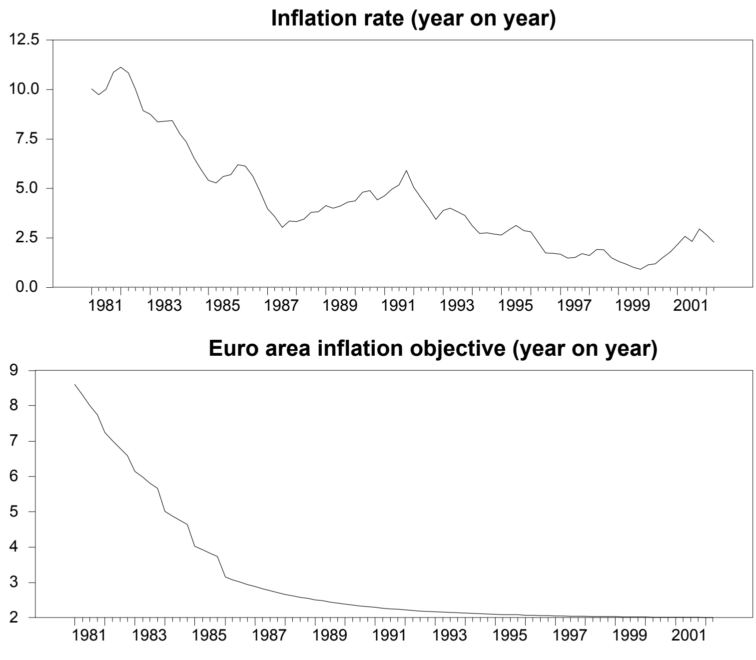

Figure 1: Inflation (actual and objective)

[4]One difference in comparison to previous contributions is that we integrate measurement errors in the theoretical specification of the structural model, not just as a wedge between the data and the theoretical model.

Stationarity of the time series is achieved by means of the Hodrick-Prescott filter with the only exception of the inflation rate and the short-term interest rate. This procedure was preferred over linear detrending since it ensured the elimination of unit roots. With respect to the inflation rate we follow Gerlach and Svensson (2003) in modelling an implicit time-varying inflation objective for the euro area as a whole. These authors assume that the euro area inflation objective converged gradually to the Bundesbank's one according to the following partial adjustment mechanism:

$$\pi_t^{uem} = \pi_t^b + \gamma^\pi \left(\pi_{t-1}^{uem} - \pi_{t-1}^b\right) \tag{3.13}$$

where π_t^b and π_t^{uem} denote, respectively, the Bundesbank and the euro area inflation objectives. This specification introduces two additional parameters that need to be estimated: the speed of adjustment, γ^π, and the difference between the Bundesbank and the euro area objective in the first period of the sample: $\pi_0^{uem} - \pi_0^b$. Both parameters are estimated jointly with the other model parameters. Figure 1 reports the estimated inflation objective for the euro area and the inflation rate (annualized, in per cent).[5]

Given the estimated inflation objective we construct the implicit objective for the short-term interest rate dividing π_t^{uem} by the discount rate β =0.9949, calibrated to match the average real interest rate between 1998 and 2002 (a period in which inflation fluctuates around the target; the implied real three-month interest rate is 2.0 per cent). The data used in the estimation are shown in Figure 2.

[5] Our estimates of γ^π and $\left(\pi_0^{uem} - \pi_0^b\right)$ (see Table 1) are remarkably similar to those of Gerlach and Svensson (2003).

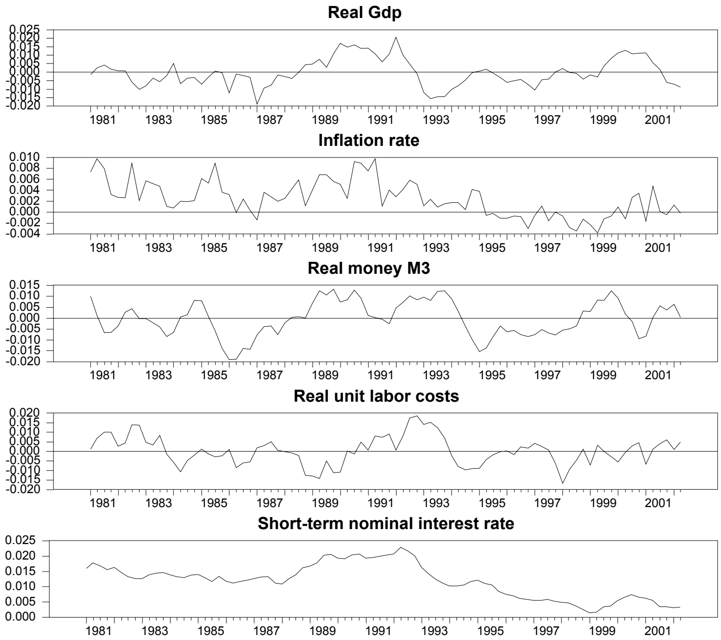

Figure 2: Detrended data

The likelihood function is constructed using the Kalman filter and is maximized with respect to the structural parameters in Ω and (3.13). The first four observations of the data (from 1981:1 to 1981:4) are used to initialize the Kalman filter as in Smets and Wouters (2003). The parameter μ linking the output gap to unit-labor cost (equation 2.8) cannot be pinned down precisely by the data independently of the value of κ (see equation 2.4). A unit value was therefore chosen for μ in the estimation, which amounts to a normalization on the value of κ. Estimation of the model parameters with alternative values of μ does not significantly alter the quantitative conclusions of this paper. The estimates of the parameters are reported in Table 1.[6] With the exception of γ_2 (the forward component of money demand) and $\sigma_{v,m}$ (the measurement error of money), all parameters are statistically significant at conventional 5 per cent confidence level.

3.2. Estimation results

The wide range of calibrated values used in the literature on optimal monetary policy to describe the preferences of the monetary authority makes estimation

[6]The likelihood function is maximized using the algorithm `csminwel.m` by C. Sims. This routine is robust to discontinuities in the objective function.

interesting. In most calibrations the coefficient of the output gap ranges between 0 and 1 while a smaller coefficient, between 0 and 0.5, is chosen for the weight on interest rate changes (e.g. Ehrmann and Smets (2003) and Dennis and Söderström (2002)). These parameters are crucial in quantifying, for example, the gains from commitment and in shaping the dynamics of output, inflation and the nominal interest rate. To the best of our knowledge, this is the first attempt to measure these parameters for the euro area.[7] The estimates indicate a small weight for the output gap (0.06) and a large weight for the interest rate adjustment term (0.74). This empirical finding, similar to the one by Dennis (2002) for the United States, suggests that the monetary authority is much more concerned with fluctuations in the inflation rate and the interest rate than with the output gap. Note how the estimated values differ substantially from the ones used in calibrations: for example in the benchmark calibration in Ehrmann and Smets (2003) the weights are set to 1 and 0.1 for, respectively, the output gap and the changes in the interest rate.

The estimates concerning the structural equations show that *both* the forward and backward components are important of output and inflation dynamics. This finding is consistent with previous studies, e.g. Gali and Gertler (1999) and Lindé (2002), who reject an either fully-backward of fully-forward specification. In particular, the estimated degree of backwardness is quite large (high δ and α) for both variables. With respect to the interest rate elasticity (θ) our estimate is smaller than the ones in Andres et al. (2001) and Smets and Wouters (2003), but within the range of values found in the literature. Moreover, we find a low elasticity of inflation to the output gap (κ) as in Smets and Wouters (2003).[8] Finally, the estimated money demand equation detects a large degree of backwardness (large γ_1), a small interest rate elasticity and a rather small elasticity with respect to current output (γ_y). In principle a greater value of γ_y implies that monetary aggregates contain a clearer signal about current output. The finding of a small coefficient suggests that information on monetary aggregates is unlikely to be of

[7] Dennis (2002) develops a similar exercise for the US.

[8] There is little consensus on the value of the slope of the new Phillips curve in the literature. The estimates range from a minimum value of 0.015 for the U.S. (Galí and Gertler (1999)) to a maximum of 0.39 (Orphanides and Wieland (2000)). With respect to the euro area Smets and Wouters (2002) estimate a slope of 0.007, which is pretty close to our estimate. Calibrated values for this parameter for the euro area range between 0.03 and 0.08 (Casares (2001, 2002)).

Table 1. Parameter estimates. Full sample: 1981:1-2002:2

	Estimates	Standard errors
λ	0.06	0.03
ν	0.74	0.07
δ	0.75	0.25
θ	0.08	0.01
α	0.71	0.28
κ	0.002	0.0002
ρ	0.79	0.22
γ_1	0.80	0.003
γ_2	0.05	0.05
γ_y	0.06	0.01
γ_i	0.009	0.001
$\sigma_{u,p}$	0.39	0.01
$\sigma_{u,c}$	0.03	0.00
$\sigma_{u,\overline{y}}$	0.81	0.05
$\sigma_{u,m}$	0.39	0.02
$\sigma_{v,y}$	0.24	0.05
$\sigma_{v,\pi}$	0.26	0.01
$\sigma_{v,m}$	0.02	0.02
$\sigma_{v,x}$	0.10	0.05
$\sigma_{e,i}$	0.52	0.13
$\pi_0^{uem} - \pi_0^b$	4.9	0.43
γ^π	0.93	0.11

Note: Standard deviations in percentage points much use as an information variable (see Section 4).

The standard deviations of the structural shocks (σ_u) indicate that innovations in potential output are the most volatile. This result is in line with the empirical findings of Ireland (2001) for the United States and of Smets and Wouters (2003) for the euro area. Quite importantly, the estimation shows that measurement errors in the observables (σ_v) play a rather minor role, as their standard deviation is smaller than the structural innovations. Real money is the variable which is measured with the highest precision (the standard deviation is not significantly different from zero), while the largest measurement error is detected for inflation (0.26 percentage points). Finally, the discrepancy between the model-based optimal interest rate and the one in the data, σ_{ei}, is on average of about half of a percentage point.

Figure 3 reports the actual and predicted values for the variables used in the maximum likelihood estimation. The model forecasting performance within sample is modest for what concerns the inflation rate. The correlation between the one-step-ahead prediction and actual inflation is around 0.5, this shows that the estimated equations are unable to capture the high frequency fluctuations that occur in the data for the inflation rate (see the second box in the figure). This indicates that while model captures the dynamics of a "core inflation" component, a large portion of the high frequency movements of quarterly inflation is affected by measurement error. A similar result is found by Eichenbaum and Fisher (2003) who show that integrating measurement errors for inflation within the standard Calvo pricing model helps improving the estimation of the new-keynesian Phillips curve.

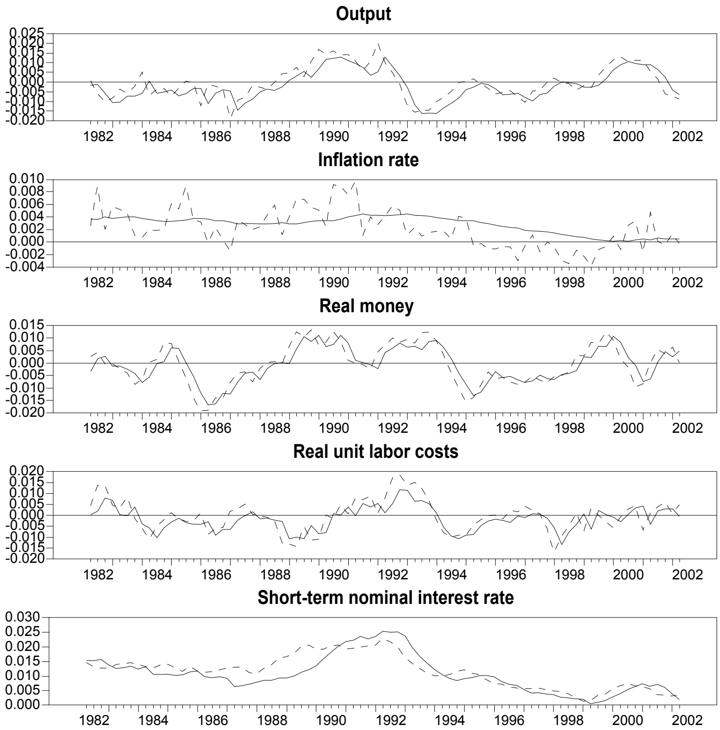

Figure 3: Actual (dashed line) and fitted data

The model forecasting performance is substantially better with respect to real GDP (0.81), real money M3 (0.86), the short-term nominal interest rate (0.86) and real unit labor cost (0.75).[9] Significant improvements in performance can be obtained by enriching the model specification with additional equations, along the lines followed by Smets and Wouters (2003). For the purposes of this paper, however, we chose to develop the analysis using the parsimonious model presented in Section 2.

3.3. Robustness of the estimates

The robustness of the estimates was tested along two dimensions. First, we re-estimated the model over a shorter period (1990:1-2002:2), during which the euro-area is characterized by a greater degree of homogeneity, such as the adoption

[9]The ratio between the standard deviation of the forecast errors in a given variable and the standard deviation of the same variable is equal to 0.59 for output, 0.86 for inflation, 0.50 for real money, 0.66 for real unit labor costs and 0.57 for the interest rate.

Table 2. Parameter estimates. Subsample: 1990:1-2002:2

	Estimates	Standard errors
λ	0.001	0.029
ν	1.11	0.35
δ	0.68	0.29
θ	0.13	0.07
α	0.56	0.30
κ	0.003	0.003
ρ	0.75	0.30
γ_1	0.83	0.02
γ_2	0.05	0.07
γ_y	0.000	0.001
γ_i	0.01	0.002
$\sigma_{u,p}$	0.40	0.02
$\sigma_{u,c}$	0.03	0.00
$\sigma_{u,\overline{y}}$	0.79	0.05
$\sigma_{u,m}$	0.37	0.02
$\sigma_{v,y}$	0.00	0.05
$\sigma_{v,\pi}$	0.20	0.01
$\sigma_{v,m}$	0.01	0.04
$\sigma_{v,x}$	0.05	0.15
$\sigma_{e,i}$	0.13	0.01

Note: Standard deviations in percentage points

of similar inflation targets. The convergence in inflation objectives is a direct consequence of the Maastricht Treaty which imposed to the candidate countries several criteria for accessing the third phase of the European Monetary Union. Based on this observation, the model estimation over this subperiod imposes a common 2.0 per cent inflation objective. This choice is consistent with the implicit inflation objective that was estimated above for the full sample (1981:1-2002:2; see Figure 1).

Table 2 reports the estimated parameters with the corresponding standard errors. A comparison of Tables 1 and 2 shows that most parameters are quite stable across the two samples, even though a few parameters (in particular α and κ) become insignificant in the shorter sample. Some differences emerge for what concerns the measurement errors of output, which is smaller in the short sample.

The standard deviations of the structural shocks are essentially unchanged. The parameter measuring the degree of backwardness of inflation is smaller in the short sample (0.56 compared with 0.71) and the same is true for output (0.68 compared with 0.75). The parameters in the money demand equation are stable, even though the output elasticity becomes not significantly different from zero. The largest difference between the two sets of estimates regards the central bank's preferences: in the short sample the weight attached to the output gap is zero while the weight on the changes in the interest rate is larger than in the full sample (1.11 compared with 0.74).

As a second robustness check, two alternatives were considered to measure inflation deviations from target over the full sample. The first one assumes a constant inflation target of 2 per cent, the second uses HP-filtered inflation as the measure of inflation deviations. The estimates of most structural parameters are not substantially affected by these alternatives, with the exceptions of the degree of inflation backwardness and the parameters of the central bank loss function. When the constant inflation objective of 2 per cent is used the output gap weight increases (to around 0.25) and the interest-adjustment weight decreases (to around 0.5) in comparison to the benchmark estimates of Table 1. When HP filtered inflation is used, both the inflation backwardness and the output-gap weight decrease in comparison to the benchmark.[10]

These robustness checks reveal that the degree of inflation backwardness and the weight attached to the output gap in the central bank loss function are sensitive to the sample period and to the method chosen to detrend inflation. However, it is important to stress for the purposes of this paper that the instability in these parameters does not affect the qualitative and quantitative assessment of the information content of indicators presented in the next Section.

3.4. The optimal monetary policy rule in the estimated model

The estimated model is characterized by the optimal monetary policy rule $i_t = FX_{t|t}$, the coefficients of which are reported in the Table 3. The standard errors are computed by means of Monte Carlo simulations. The optimal rule reacts strongly to the cost-push shock which has important effects on inflation. The

[10] More detailed results on these estimates are available from the authors upon request.

weight on lagged inflation is also large. The coefficient on potential output is negative and significant: an increase in potential output requires the central bank to accommodate the shock to stabilize inflation and the output gap. The coefficients on lagged real money and the money demand shock are zero: these two variables have no direct effect on the target variables. Therefore it is optimal for the central bank not to react to them.

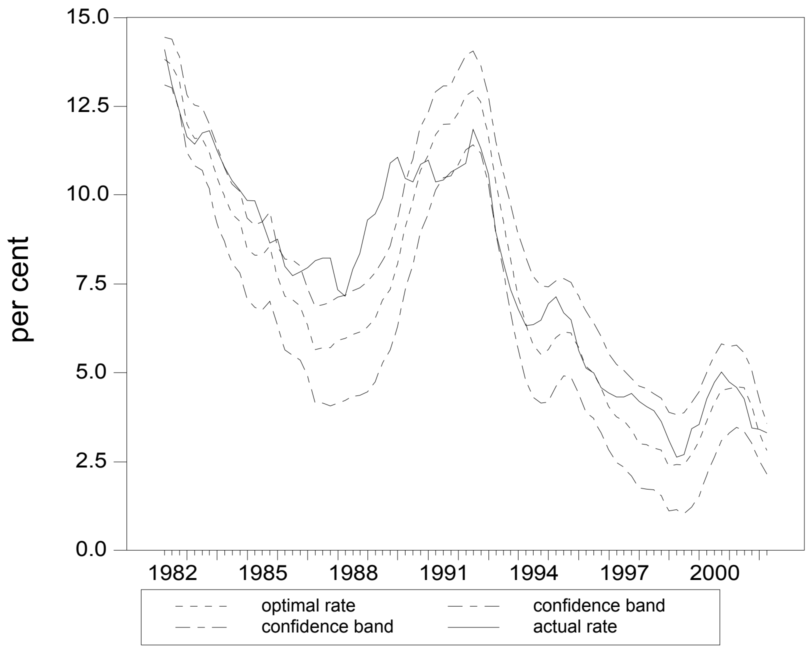

Figure 4: Optimal and realized short-term interest rate

Figure 4 reports the time series for the interest rate that is implied by the optimal rule, together with 95 per cent confidence bands (dashed lines) and the realized 3-month interest rate (solid line) over the estimation period. It shows that the optimal rate implied by the theoretical model tracks the actual interest rate sufficiently well on average.

In order to provide an intuitive interpretation of the optimal monetary policy rule reported in Table 3, we estimated a "Taylor" rule over the data generated by a simulation of the model under the optimal discretionary policy. The chosen specification of the "Taylor" rule constrains the interest rate to be a linear function of the contemporaneous estimates of the output gap, inflation and the lagged interest rate: $i_t = \phi_x x_{t|t} + \phi_\pi \pi_{t|t} + \phi_\imath i_{t-1}$. The ordinary least square regression explains more than 97 per cent of the variability of the optimal rule. The estimated

Table 3. The optimal policy function

	Coefficient	Standard error
$y_{t-1\|t}$	0.25	0.05
$\pi_{t-1\|t}$	1.00	0.13
$m_{t-1\|t}$	-	-
$\overline{y}_{t\|t}$	-0.09	0.03
$u_{p,t\|t}$	0.33	0.07
$u_{c,t\|t}$	1.40	0.19
$u_{m,t\|t}$	-	-
i_{t-1}	0.75	0.03
$\overline{y}_{t-1\|t}$	-	-

coefficients, which are all significant at conventional 5 per cent confidence level, are 0.13 on the output gap, 0.52 on inflation and 0.87 on the lagged interest rate. These coefficients imply a strong long-run response of the interest rate to inflation, a milder one to the output gap (the coefficients are, respectively, around 5 and 1).[11]

4. The Effects of Information

The estimates provide quantitative information on the extent of the imperfect information problem in the model. Column {1} of Table 4 reports the (unconditional) standard deviation of the contemporaneous forecast errors about the fundamental shocks that the agents in the model face when information is processed optimally (using the Kalman filter) and both the monetary and the unit labor cost indicators are used. It shows that the largest forecast errors pertain to the innovations in potential output. This is partly due to the relatively large size of the innovations hitting this variable (see Table 1) and partly to the noisiness of the unit labor cost indicator used to forecast this variable.

The other columns of the Table analyze how the forecast errors change as we vary the information available to agents. When the monetary indicator is taken

[11] Other properties of the estimated model can be described by means of impulse response functions, omitted here for reasons of space. These are available from the authors upon request.

Table 4. Information and forecast error about fundamental shocks

	With Measurement Error			Without Meas. Error
Indicator:	*ULC and Money*	*ULC*	*Money*	
	{1}	{2}	{3}	{4}
$std\left[u_{p,t}-u_{p,t\vert t}\right]$	0.39	0.40	0.39	0.39
$std\left[u_{c,t}-u_{c,t\vert t}\right]$	0.04	0.04	0.04	0.01
$std\left[\overline{y}_{t}-\overline{y}_{t\vert t}\right]$	0.83	0.83	1.31	0.80
$std\left[u_{m,t}-u_{m,t\vert t}\right]$	0.04	0.39	0.04	0.04

out of the vector of observables Z_t, the forecast errors about the money demand innovation naturally increase (see column {2}) but the forecast errors about the innovations in output (the preference shock) potential output, and the cost push shock are virtually unaffected. While the M3 indicator may allow a better identification of the fundamental shocks in principle (including potential output and cost push shocks), the analysis reveals that its use improves the identification of money demand shocks, slightly improves the identification of output shocks but does not contribute to the identification of potential output and cost push shocks. The reason lies in the low signal to noise ratio of the monetary indicator, which originates from the small elasticity of money demand to output (γ_y), the large standard deviation of the money demand equation ($\sigma_{u,m}$), and the small inflation elasticity to the output gap (κ).

The next experiment, reported in column {3}, considers dropping the unit labor cost indicator from the information set of the policy maker. It shows that the this indicator contains significant information on potential output, as the forecast errors about this variable increase significantly. As expected, no information of interest is contained about the other fundamental variables, whose forecast errors are essentially unchanged.

As a benchmark of comparison, column {4} reports the standard deviation of the forecast errors that is obtained in the absence of measurement error (i.e. when $\Sigma_v^2 = 0$). It shows that even with perfect measurement an incomplete information problem persists about actual and potential output. This is due to the assumption that information on these variables is available only with a lag. This benchmark also shows that when both indicators are used the forecast errors on output and money demand shocks are as small as they would be if there was no measurement error. Forecast errors about potential output remain slightly above this benchmark even when the unit labor cost indicator is used (columns {1} and {2}).

4.1. Effects of information on outcomes and welfare

The forecast errors discussed above influence the unconditional volatility of the main variables in the model. Table 5 reports the standard deviation of the three goal variables (output gap, inflation and interest rate changes) together with the unconditional value of expected losses. The table considers three alternative information assumptions. As before, the spirit of the exercise is to use the estimated model to analyze how economic performance (volatilities, welfare) changes in each of these scenarios.

The results for the benchmark case in which both the monetary and the unit labor cost indicator are used appear in column {1}. Let us compare the volatility of the targets in this case with the one obtained when no monetary indicator is available and only the unit labor cost indicator is used. As shown in column {2} of Table 4, this change in the information set causes forecast errors about innovations in current and potential output to increase only by a tiny amount. The target volatilities reported in Table 5 indicate that economic targets are basically unaffected by the removal of monetary aggregates from the information set. A smaller variance in one of the three goal variables leads to a moderate decrease in the losses enjoyed by the policy maker. Less information about output innovations turns out to be good for welfare as it results in a smaller volatility of target variables.[12]

Quantitatively noticeable consequences emerge, instead, when the output gap indicator is taken away from the agent's information set (column {3}). In this case, the greater uncertainty surrounding the estimate of potential output leads to a significant reduction in monetary policy activism (e.g. a smaller volatility of interest rate changes) and to a greater output gap volatility. These effects arise entirely from the effect of uncertainty on the estimates of the states (i.e. through

[12]In a model with forward-looking variables less information (for both the agents and the policy maker) may cause endogenous variables to be less responsive to the new information reducing their volatility and increasing welfare. Further results on this point are available from the authors upon request.

Table 5. Targets volatility and the value of losses

	With Measurement Error		
Indicators:	*ULC and Money*	*ULC*	*Money*
	{1}	{2}	{3}
$std\,[y_t - \overline{y}_t]$	1.5	1.5	1.6
$std\,[\pi_t]$	0.13	0.13	0.11
$std\,[i_t - i_{t-1}]$	0.17	0.16	0.14
Λ_t	36.13	36.10	36.56
% change in Λ_t w.r. to {1}	-	-0.1	1.2

the matrix K in the updating equation (2.9c)), since the vector F of the optimal control rule ($i_t = FX_{t|t}$) does *not* depend on the uncertainty.[13] As shown in the bottom line of the table, these changes increase the losses of the policy maker, in comparison to the benchmark case by approximately 1 per cent. This finding suggests that the unit labor cost indicator is useful as it allows the policy maker to implement a welfare superior stabilization policy.

4.2. The welfare gains of commitment

Svensson (1997), Clarida et al. (1999) and Woodford (1999) highlighted that the forward-looking elements in the new keynesian model give rise to a form of time inconsistency.[14] This is also known as the problem of the "stabilization bias". In a model with forward-looking agents the optimal prescription is that shocks should be stabilized gradually, i.e. policy is displays a form of history dependence. This allows stabilization costs to be smoothed over time. One interpretation of the commitment solution is that, after a shock hits the economy, the policy maker announces a path of current and future policy responses to this shock and sticks to it afterwards. Unfortunately, this plan cannot be implemented under a markov-perfect equilibrium in which policy cannot be made contingent on past "promises".[15] Under discretion, inflation in the new-keynesian model is excessively

[13] Due to the certainty equivalence feature of this problem (see Svensson and Woodford, 2000).

[14] This is an instance of the more general time-inconsistency problem brought out by Kydland and Prescott (1977).

[15] Technically, under discretion (markov-perfect equilibrium) policy is function of the state variables alone while, under commitment, the lagrange multipliers of the forward-looking vari-

Table 6. The welfare effects of commitment

	Our Model ($\lambda = 0.06$ and $\nu = 0.74$)		Model with: $\lambda = 0.5$ and $\nu = 0.1$	
	Discretion	Commitment	Discretion	Commitment
	{1}	{5}	$\{W_d\}$	$\{W_c\}$
$std\,[y_t - \overline{y}_t]$	1.5	1.5	1.02	1.02
$std\,[\pi_t]$	0.13	0.12	0.17	0.15
$std\,[i_t - i_{t-1}]$	0.17	0.18	2.5	2.6
Λ_t	36.1	36.0	118.3	117.8
% change w.r. to *discr.*	-	-0.2	-	-0.4

volatile in comparison to the commitment benchmark. The estimated model allows us to quantify the welfare gains of commitment.

Columns {1} and {5} of Table 6 report the outcomes of the target variables which originate, respectively, under discretion and commitment.[16] It appears that under commitment interest-rate changes are more volatile while inflation and the output gap are more stable. This reduces losses of about 0.2 per cent.

Columns $\{W_d\}$ and $\{W_c\}$ of Table 6 compare the outcomes of discretion and commitment for a model in which the preference weights of the monetary authority are closer to the values usually employed in the literature, i.e. a relatively small weight is chosen for interest smoothing ($\nu = 0.1$) and a relatively large one is given to the output gap ($\lambda = 0.5$). The bottom line of the table shows that under this parametrization the gains from commitment are twice as large than in the estimated model: losses are reduced by about 0.4 per cent.

5. Concluding remarks

This paper used maximum likelihood to estimate a small New-Keynesian model for the euro area. The model accounts for imperfect information about the state of the economy and is used to assess the role of imperfect information and the

ables (costate variables) also enter the optimal policy rule.

[16]The "separation principle" which holds for this model implies that the information content of indicator variables (e.g. the mean square forecast error of the state variables) is not affected by the equilibrium notion (commitment/discretion).

usefulness of indicators, namely monetary aggregates and unit labor costs, in the conduct of monetary policy. Moreover, by assuming optimal discretionary policy, the estimates allow us to characterize policy in terms of the relative weights assigned to the targets: inflation, the output gap and the volatility of the short term interest rate.

The equilibrium characterization involves the solution of an optimal control and filtering problem, as in Svensson and Woodford (2000). Once this is done, the model is estimated using maximum likelihood. Overall, the broad indications that emerge from the estimates are consistent with previous findings. In particular, both forward and backward looking components are important to characterize the dynamics of inflation and output, as found by Gali and Gertler (1999) and Lindé (2002). Moreover, the estimates show that measurement error in inflation is useful in bringing the new Keynesian model to the data, as found by Eichenbaum and Fisher (2003).

As to the role of indicator variables, the analysis shows that M3 does not help in a better identification of the state variables of interest for stabilization policy. In particular, it delivers no information about potential output and cost push shocks. Therefore, the model suggests a limited usefulness of the M3 indicator. A more useful role emerges for the unit labor costs indicator, which contains information on potential output that helps reducing the volatility of the output gap. These findings, which lay at the centre of our analysis, are robust to the re-estimation of the model over the more recent sample (1990-2002) and to alternative detrending of the data.

Finally, the estimated weights for the targets of monetary policy suggest that the largest weight is attached to inflation, followed closely by the interest-smoothing target and by a small weight for the output-gap. These values, which are quite different from the ones employed in calibration exercises, are similar to the ones estimated by Dennis (2002) for the United States. The relatively large weight on interest-smoothing suggested by our estimates implies that the welfare gains delivered by commitment are smaller than in typical parametrizations. This happens because it makes the discretionary monetary response to shocks gradual, replicating an essential feature of commitment policy (Woodford, 1999).

A. Appendix: Data source

All data are quarterly and seasonally adjusted. The source for the real GDP, the GDP deflator and unit labour costs is EUROSTAT for the period running from 1991:1 to 2002:3. The data for the period from 1981:1 to 1990:4 are constructed recursively using the starting value (1991:1) of the EUROSTAT series and the growth rates of the corresponding series from the Area Wide Model (AWM) database constructed by Fagan, Henry and Mestre (2001). The source for the stock of nominal money M3 is the European Central Bank. The source for the short-term nominal interest rate is the AWM database for the period running from 1981:1 to 1998:4. For the period up to 2002:2 the interest rate is taken to be the three-month Euribor rate. The source is the European Central Bank.

B. Appendix: State-space formulation of the ESM model

The model can be represented in state-space formulation:

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + Bi_t + \begin{bmatrix} C_u \\ 0 \end{bmatrix} u_{t+1}$$

where the vector $X_t' \equiv \begin{bmatrix} y_{t-1} & \pi_{t-1} & m_{t-1} & \overline{y}_t & u_{p,t} & u_{c,t} & u_{m,t} & i_{t-1} & \overline{y}_{t-1} \end{bmatrix}$ and $x_t' \equiv \begin{bmatrix} y_t & \pi_t & m_t \end{bmatrix}$ denote, respectively, predetermined and non-predetermined (forward looking) variables at time t and i_t is the instrument controlled by the central bank.

The observables are stacked in the vector $Z_t \equiv \begin{bmatrix} y_t^o & \pi_t^o & m_t^o & x_t^o \end{bmatrix}$ according to:

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t$$

and target variables are collected in the vector $Y_t \equiv \begin{bmatrix} y_t - \overline{y}_t & \pi_t & i_t - i_{t-1} \end{bmatrix}$:

$$Y_t = C^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + C^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + C_i i_t$$

Mapping the model of Section 2 into this formulation yields the following matrices:

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\delta}{1-\delta} & \frac{\alpha\theta}{\xi} & 0 & -\frac{\theta\kappa}{\xi} & -\frac{1}{1-\delta} & \frac{\theta}{\xi} & 0 & 0 & 0 & \frac{1-\alpha+\theta\kappa}{\xi} & -\frac{\theta}{\xi} & 0 \\ 0 & -\frac{\alpha}{1-\alpha} & 0 & \frac{\kappa}{1-\alpha} & 0 & -\frac{1}{1-\alpha} & 0 & 0 & 0 & -\frac{\kappa}{1-\alpha} & \frac{1}{1-\alpha} & 0 \\ 0 & 0 & \frac{\gamma_1}{\gamma_2} & 0 & 0 & 0 & -\frac{1}{\gamma_2} & 0 & 0 & -\frac{\gamma_y}{\gamma_2} & 0 & \frac{1}{\gamma_2} \end{bmatrix},$$

where $\xi \equiv (1-\alpha)(1-\delta)$

$$A_2 = [0], \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \frac{\theta}{1-\delta} \\ 0 \\ \frac{\gamma_i}{\gamma_2} \end{bmatrix} \qquad C_u = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \qquad D_2 = [0]$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad C_2 = [0] \qquad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \nu \end{bmatrix}$$

C. Appendix: Computing the likelihood function

In this section we describe how to compute the likelihood function for the model described in Section 2. Recall the state state-space representation of the model (equations 3.10 and 3.12 in Section 3) is:

$$\begin{aligned} Q_{t+1} &= \hat{A}Q_t + \hat{G}w_{1,t+1}, \qquad w_{1,t+1} \equiv \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix} \\ d_t &= \hat{L}Q_t + w_{2,t} \qquad w_{2,t} \equiv \hat{M}v_t + e_t \end{aligned}$$

where the first equation is the law of motion of the unobserved states Q_{t+1} and the second is the observation equation linking observed variables d_t to the unobserved states.

The vector of structural shocks u_t and the measurement errors v_t are assumed to be independent i.i.d. processes with covariance matrices Σ_u^2 and Σ_v^2. The Kalman filter consists in a system of recursive equations that allows to forecast the unobserved state vector using the information contained in the observed variables.

The recursive system for computing the Kalman filter is given by the following equations:

$$\begin{aligned} Q_{t+1|t} &= \hat{A}Q_{t|t-1} + K_t\left(d_t - d_{t|t-1}\right) && \text{(C.1)} \\ & \quad \textit{where} \\ K_t &\equiv \left(\hat{A}\Sigma^2_{t|t-1}\hat{L}' + \hat{G}V_3\right)\left(\hat{L}\Sigma^2_{t|t-1}\hat{L}' + V_2\right)^{-1} && \text{(C.2)} \\ \Sigma^2_{t+1|t} &\equiv \left(\hat{A}\Sigma^2_{t|t-1}\hat{A}' + \hat{G}V_1\hat{G}'\right) - K_t\left(A\Sigma^2_{t|t-1}\hat{L}' + \hat{G}V_3\right)' && \text{(C.3)} \end{aligned}$$

where the matrix K_t is the Kalman gain and $\Sigma^2_{t+1|t} \equiv cov(Q_{t+1} - Q_{t+1|t})$. The covariance matrices V_1, V_2 and V_3 are given by:

$$V_1 \equiv E\left(w_{1,t+1}w'_{1,t+1}\right) = \begin{bmatrix} \Sigma_u^2 & 0 \\ 0 & \Sigma_v^2 \end{bmatrix} \tag{C.4}$$

$$V_2 \equiv E\left(w_{2,t}w'_{2,t}\right) = \hat{M}\Sigma_v^2\hat{M}' + E\left(e_t e'_t\right) \tag{C.5}$$

$$V_3 \equiv E\left(w_{1,t+1}w'_{2,t}\right) = E\begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix}\left(v'_t\hat{M}' + e'_t\right) = \begin{bmatrix} 0 \\ \Sigma_v^2\hat{M}' \end{bmatrix} \tag{C.6}$$

The prediction errors of the observed variables d_t, which are used to compute the likelihood function, are given by:

$$a_t \equiv d_t - d_{t|t-1} = d_t - \hat{L}Q_{t|t-1} \tag{C.7}$$

where $d_{t|t-1}$ is the forecast of the observed variables based on the information available up to period $t-1$ and:

$$\Omega_t \equiv E\left(a_t a'_t\right) = \hat{L}\Sigma_{t|t-1}^2\hat{L}' + V_2 \tag{C.8}$$

is the covariance matrix of the vector of prediction errors.

Finally, the likelihood function is given by:

$$\log L = -\frac{nT}{2}ln\left(2\pi\right) - \frac{1}{2}\sum_{t=1}^{T} ln|\Omega_t| - \frac{1}{2}\sum_{t=1}^{T} a'_t\Omega_t^{-1}a_t \tag{C.9}$$

where n is the number of variables in vector d_t and T is the sample size.

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