DISCUSSION PAPER SERIES

No. 4091

DOES ANONYMITY MATTER IN ELECTRONIC LIMIT ORDER MARKETS?

Thierry Foucault, Sophie Moinas and Erik Theissen

FINANCIAL ECONOMICS

Centre for Economic Policy Research

www.cepr.org

DOES ANONYMITY MATTER IN ELECTRONIC LIMIT ORDER MARKETS?

Thierry Foucault, Hautes Etudes Commerciales (HEC), Jouy-en-Josas and CEPR
Sophie Moinas, Doctorat HEC, Jouy-en-Josas
Erik Theissen, Rheinische Fr-Wilhelms-Universität, Bonn

This Discussion Paper is issued under the auspices of the Centre's research programme in **FINANCIAL ECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

ABSTRACT

Does Anonymity Matter in Electronic Limit Order Markets?*

We analyse the effect of concealing limit order traders' identities on market liquidity. We develop a model in which limit order traders have asymmetric information on the cost of limit order trading (which is determined by the exposure to informed trading). A thin limit order book signals to uninformed bidders that the profitability of limit orders is small. This deters uninformed bidders from improving upon the posted quotes. Informed bidders exploit this effect by bidding as if the cost of liquidity provision were large when indeed it is small. This bluffing strategy is less effective when traders cannot distinguish between informative and uninformative limit orders. Hence informed bidders act more competitively in the anonymous market. For this reason, concealing limit order traders' identities affects market liquidity in our model. We test this prediction using a natural experiment. On April 23, 2001, the limit order book for stocks listed on Euronext Paris became anonymous. We find that following this change, the average quoted spreads declined significantly whereas the quoted depth decreased.

Thierry Foucault
HEC, School of Managment
1 rue de la Libération
78351 Jouy-en-Josas
FRANCE
Tel: (00 33 1) 39 67 94 11
Fax: (00 33 1) 39 67 70 85
Email: foucault@hec.fr

Sophie Moinas
Doctorat HEC
1 rue de la Libération
78351 Jouy-en-Josas
FRANCE
Fax: (00 33 1) 39 67 70 85
Email: moinass@hec.fr

Erik Theissen
Rheinsiche Fr-Wilhelms-Universität
Bonn
BWL 1
Adenauerallee 24-42
53113
Germany

Tel: (00 49) 228 73 9208
Fax: (00 49) 228 73 59 24
Email: thiessen@uni-bonn.de

Submitted Tuesday, July 15, 2003

1   Introduction

In the last decade, the security industry has witnessed a proliferation of electronic trading systems. Several of these new trading venues (e.g. Island for equity markets or Reuters D2000-2 for the foreign exchange market) are organized as limit order markets where traders can either post quotes (submit limit orders) or hit posted quotes (submit market orders). In some of these markets (e.g. the Hong Kong Stock Exchange), the identities of the traders with orders standing in the limit order book are disclosed whereas in other markets (e.g. Island), these identities are concealed. Does it matter? Is market liquidity affected by the disclosure of limit order traders' identities? Our objective in this paper is to investigate this issue.

Two types of information regarding market participants' identities can be disclosed *before* a transaction occurs: (i) *supply-side information:* information on the *identities* of the traders setting prices (liquidity suppliers, i.e. traders using limit orders), or (ii) *demand-side information:* information on the identities of the traders who demand immediate execution at standing quotes (liquidity demanders, i.e. traders using market orders).[1] Several authors have analyzed the effects of providing demand side information.[2] Much less is known on the effects of providing supply-side information.

Our paper fills this gap in two ways. First we study a simple theoretical model in which we show why and how market liquidity can be affected by information on liquidity suppliers' identities. Second, using a natural experiment, we test the model prediction that market liquidity is affected by the disclosure of limit order traders' IDs. This experiment takes opportunity of a change in the anonymity of the trading system owned by Euronext Paris (the French stock exchange). Euronext Paris operates an electronic limit order market where brokerage firms (henceforth *broker-dealers*) can place orders for their own account or on behalf of their clients.[3] Until April 23, 2001 the identification codes for broker-dealers submitting limit orders were displayed to all brokerage firms. Since then, the limit order book is anonymous. Thus, using

---

[1]The identities of the contraparties to each transaction constitute a third type of information that can be provided to market participants. Obviously, this information can only be distributed *after* the transaction took place.

[2]These include Seppi (1990), Forster and Georges (1992), Benveniste et al. (1992), Madhavan and Cheng (1997), Theissen (2001) and Garfinkel and Nimalendran (2002).

[3]Many electronic limit order markets (e.g. the Toronto Stock Exchange, the Stockholm Stock Exchange or Island) have a design which is very similar to the trading system used in Euronext Paris.

Euronext Paris data, we can empirically test whether concealing liquidity suppliers' identities impacts market liquidity or not.

Obviously, a non-anonymous limit order book enables traders to adopt bidding strategies which are contingent on *the identities* of broker-dealers with orders standing in the book. This is worthwhile if some broker-dealers have a superior expertise in choosing their quotes. For illustration, in our model, this expertise stems from knowledge of whether or not an information event will occur (but other scenarii are possible; see Section 3.2). The key point is that expert dealers can better assess the exposure to informed trading, that is the cost of providing liquidity.[4] Hence their bids are informative about this cost. In particular, cautious bidding by expert traders *signals* that the cost of liquidity provision is large and it deters non-expert traders from improving upon the offers posted in the book. In turn expert dealers recognize and exploit the fact that their bids are viewed as being informative. They sometimes try to "fool" non-expert traders by bidding as if the cost of liquidity provision were large (by posting steep limit order schedules) when indeed it is small. When their bluff is successful, i.e. deters non-experts from improving upon posted quotes, experts earn larger profits.[5]

These strategic interactions exist whether limit order traders' IDs' are concealed or not since the limit order book remains informative even when trading is anonymous. In this case as well, a thin limit order book foreshadows an information event and signals that the exposure to informed trading is large. However, the informational content of the book is lessened when trading is anonymous as non-expert traders cannot distinguish informative orders (those placed by expert traders) from non-informative orders (those placed by non-expert traders or for liquidity reasons). Thus, expert traders' choices have less influence on non-expert traders' decision to improve upon the best quotes, i.e. bluffing strategies are less effective. We show that this effect induces expert dealers to bid more aggressively (i.e. to bluff less frequently) when their identities are concealed than when they are not. It follows that concealing liquidity suppliers' identities affects measures of liquidity such as (i) the size of the quoted spread and (ii) the quantities offered at the best

---

[4]A trader who submits a sell (resp.buy) limit order is implicitly selling a call (resp.put) option with a strike price equal to the price of the limit order. Thus information on the likelihood or the magnitude of future price changes helps limit order traders to better assess the option value of their limit orders.

[5]In our model, a steep limit order schedule signals to potential competitors that the profitability of limit orders within the best quotes is small. This signal reduces potential competitors' incentive to enter more competitive orders in the book. This line of reasoning is reminiscent of Milgrom and Roberts (1982)'s analysis of limit pricing. See also Harrington (1986).

quotes (the quoted depth). Interestingly we find cases in which the anonymous market features a smaller bid-ask spread but less quoted depth.

The empirical analysis supports our prediction that concealing liquidity suppliers' IDs' affects the liquidity of a limit order market. Our experiment reveals a significant decrease in various measures of the quoted spreads after the switch to an anonymous limit order book. This result is robust even after controlling for changes in other variables which are known to affect bid-ask spreads (such as volatility and trading volume). Furthermore we find that the variability of the bid-ask spread is significantly smaller in the anonymous trading environment (another prediction of our model). Finally the multivariate analysis shows that the switch to an anonymous limit order book has impaired the quoted depth for the stocks in our sample. Overall these results suggest that a switch to anonymity has an ambiguous impact on liquidity: it reduces bid-ask spreads but it also reduces the quantities offered at the best quotes.

Our findings underscore the complex nature of the issues related to anonymity in financial markets. In the extant literature, the consensus is that concealing information about *liquidity demanders*' identities results in larger bid-ask spreads. In this case, anonymity reduces liquidity suppliers' ability to screen informed and non-informed liquidity demanders. In contrast, our theoretical and empirical findings show that concealing information on *liquidity suppliers'* identities can reduce bid-ask spreads. In this case, anonymity reduces the informational content of the book. This effect lessens expert liquidity suppliers' ability to use bluffing strategies and works to yield more competitive outcomes.

The remainder of the paper is organized as follows. Section 2 discusses the relevant literature. Section 3 describes a theoretical model of trading in a limit order market. In Section 4, we solve for equilibrium bidding strategies and we compare trading outcomes when liquidity suppliers' identities are disclosed and when they are concealed. Section 5 derives the empirical implications of our model and briefly discusses possible extensions. In Section 6, we empirically analyze the effect of concealing liquidity suppliers' identities using data from Euronext Paris. Section 7 concludes. The proofs which do not appear in the text are collected in the appendix. The notations used in the theoretical model are listed in Table 1 just before the Appendix.

2 A Review of the Literature

The provision of information on traders' identities improves market transparency. For this reason our paper is related to the longstanding controversy regarding the desirability of transparency in security markets (see O'Hara (1995) for a review). Recent papers have analyzed theoretically and empirically the effect of providing information regarding the prices and sizes of limit orders standing in the book (respectively Baruch (1999), Madhavan, Porter and Weaver (2002) and Boehmer, Saar and Yu (2003)). However, none of these papers analyze the effect of disclosing information on limit order traders' identities, *holding information on limit order sizes and prices constant*.[6]

Waisburd (2003) analyzes empirically the effect of revealing traders' identities *post-trade*, using data from Euronext Paris. He considers a sample of stocks which trade under two different anonymity regimes: one in which the identities of the brokers involved in a transaction are revealed post trade and one in which they are concealed. He finds that the average bid-ask spread is larger and quoted depth is smaller in the post-trade anonymous regime. In contrast, we focus on the effect of revealing liquidity suppliers'IDs' *before* a transaction. Interestingly, we find empirically that the average bid-ask spread is smaller when liquidity suppliers' IDs are concealed. Hence post-trade and pre-trade anonymity have different effects.

Simaan, Weaver and Whitcomb (2003) argue that non-anonymous trading facilitate collusion among liquidity suppliers. Actually it is easier to detect and retribute dealers who breach a non-competitive pricing agreement when dealers' IDs' are displayed. Simaan et al. (2003) find that dealers post more aggressive quotes in ECNs' than in Nasdaq, as predicted by the collusion hypothesis (dealers' IDs' are displayed on Nasdaq but not in ECNs').[7]

Collusion among liquidity suppliers is unlikely in limit order markets like Euronext. Actually, in this market, intermediaries act as broker-dealers, i.e. they place orders for their own accounts

---

[6]In Euronext Paris, intermediaries can observe all limit orders standing in the book (except hidden orders). This feature of the market has not been altered by the switch to anonymous trading.

[7]Albanesi and Rindi (2000) also consider the effect of anonymity in a dealership market. The screen-based trading system used in the Italian bond market became anonymous in 1997. Albanesi and Rindi (2000) compare the time-series properties of transaction prices in this market before and after 1997. Due to data constraints, they cannot report results on direct measures of market liquidity such as quoted spread and depth, as we do in this paper.

(principal orders) or on behalf of their clients (agency orders). The organization of Euronext Paris has always been such that no information is provided on the nature (agency or principal) of the orders placed in the book. Hence it is difficult for broker-dealers to distinguish between voluntary and unvoluntary price cuts. This feature hinders collusion by making it harder to detect broker-dealers who breach the collusive agreement. Our model does not rely on collusion among liquidity suppliers and thereby provides an alternative to Simaan et al. (2003)'s collusion hypothesis.

Rindi (2002) considers a rational expectations model (à la Grossman and Stiglitz (1980)). In the non-anonymous market, uninformed traders can make their offers contingent on the demand function of informed traders (their "limit orders") whereas they cannot in the anonymous market. With exogenous information acquisition, she shows that market liquidity is always smaller in the anonymous market. With endogenous information acquisition, she finds parameter values for which liquidity is higher in the anonymous market.

Our approach is different from Rindi (2002). First, the nature of private information for liquidity suppliers is different. In our model, informed liquidity suppliers have information on the likelihood of a price movement but not on the direction of this price movement (more on this in Section 3.2). Second the trading mechanism considered in this paper is different. Rindi (2002) analyzes a batch auction in which all orders are submitted simultaneously and are executed at a single clearing price. In contrast, in our model, liquidity suppliers submit their orders sequentially and, importantly, market orders can execute at different prices (they can "walk up" or "walk down" the book). This is closer to the actual operations of limit order markets.

For this reason, our paper is related to the recent literature on price formation in limit order markets (in particular Glosten (1994), Seppi (1997) and Sandås (2001)). Our baseline model can be seen as a (very) simplified version of Glosten (1994), with *sequential bidding* (as in Seppi (1997) or Sandås (2001)). In contrast with the extant literature however, we assume that some traders posting limit orders are better informed about the exposure to informed trading ("the risk of being picked-off"). Hence the state of the book in our model provides information on the cost of liquidity provision (which increases with the risk of being picked-off). This signaling role for the state of the book is new to this paper and is key for our results regarding anonymity.[8]

---

[8]Madhavan and Panchapagesan (2000) and Kalay and Wohl (2003) empirically study the informational content of the book.

5

3 The Model

3.1 Timing and Market Structure

We consider the following model of trading in a security market. There are 3 dates. At date 2, the final value of the security, which is denoted $\widetilde{V}_2$, is realized. It is given by

$$\widetilde{V}_2 = v_0 + \widetilde{\epsilon}_1, \tag{1}$$

where $\widetilde{\epsilon}_1$ is a random variable with zero mean. For simplicity we assume that $\widetilde{\epsilon}_1$ takes one of two values: $+\sigma$ or $-\sigma$ with equal probabilities. If an information event occurs at date 1, a trader (henceforth a speculator) observes the innovation, ϵ_1, with probability α. Upon becoming informed, the speculator can decide to trade or not. If, as happens with probability $(1 - \alpha)$, no trader observes ϵ_1 or if no information event occurs at date 1, a liquidity trader submits a buy or a sell market order with equal probabilities. Each order must be expressed in terms of a minimum unit (a round lot) which is equal to q shares. In the rest of the paper, we normalize the size of 1 round lot to 1 share ($q = 1$). The order size submitted by a liquidity trader is random and can be equal to 1 or 2 round lots with equal probabilities.

Following Easley and O'Hara (1992), we assume that there is *uncertainty* on the occurrence of an information event at date 1. Specifically, we assume that the probability of an information event is $\pi_0 = 0.5$. Figure 1 depicts the tree diagram of the trading process at date 1. Liquidity suppliers (described below) post limit orders for the security at date 0. A sell (buy) limit order specifies a price and the maximum number of round lots a trader is willing to sell (buy) at this price. In the rest of this section we describe in more detail the decisions which are taken at dates 1 and 0. Our modeling choices are discussed in detail in the next subsection.

Speculators. The speculator submits a buy or a sell order depending on the direction of his information. If ϵ_1 is positive (negative), the speculator submits a buy (sell) market order so as to pick off all sell (buy) limit orders with a price below (resp. above) $v_0 + \sigma$ (resp. $(v_0 - \sigma)$).[9]

---

[9] An information event can be seen, for instance, as the arrival of public information (corporate announcements, price movements in related stocks, headlines news etc...). In this case, the probability α is the probability that a trader reacts to the new information before mispriced limit orders disappear from the book (either because a market order arrived or because limit order traders cancelled their orders). This probability depends on the intensity with which traders monitor the flow of information (as in Foucault, Roëll and Sandås (2003) for instance).

Liquidity Suppliers. There are two kinds of liquidity suppliers: (a) risk-neutral *value traders* who post limit orders so as to maximize their expected profits and (b) *pre-committed traders* who have to buy or to sell a given number of round lots. Value traders can be viewed as brokers who trade for their own account. Pre-committed traders represent brokers who seek to execute an order on behalf of a client (e.g. an institutional investor who rebalances his portfolio). They place limit orders so as to minimize their client's execution costs.[10] Henceforth we will refer to the value traders as being "the dealers".

We assume that dealers are not equally informed on the likelihood of an information event. There are two types of dealers: (i) informed dealers who know whether or not an information event will take place at date 1 (but they do not know the direction of the event) and (ii) uninformed dealers who do not have this knowledge. Of course the exposure to informed trading and thereby the cost of providing liquidity are larger when an information event is about to occur. For this reason, the schedule of limit orders posted by informed dealers is informative about the cost of liquidity provision.

Dealers post their limit orders *sequentially*, in 2 stages denoted L (first stage) and F (second stage). Figure 2 describes the timing of the bidding game which takes place at date 0. With probability $(1-\beta)$, the price schedule (*the limit order book*) posted in the first stage is established by an informed dealer. Otherwise the limit order book is established by precommitted liquidity suppliers. In the second stage, an uninformed dealer observes the limit order book, updates her beliefs on the cost of liquidity provision (i.e. the likelihood of an information event) and decides to submit limit orders or not. This timing gives us the possibility to analyze how uninformed dealers react to the information contained in the limit order book. In the rest of the paper, we sometimes refer to the liquidity supplier acting in stage L as being the *leader* and to the liquidity supplier acting in stage F as being the *follower*.

At date 1, the incoming buy (sell) market order is filled against the sell (buy) limit orders posted in the book. Price priority is enforced and each limit order executes at its price. Furthermore, *time priority* is enforced. That is, at a given price, the limit order placed by the leader is executed before the limit order placed by the follower. Table 1 below summarizes the different

---

[10]Foucault, Kadan and Kandel (2003) show that it can be optimal for pre-committed traders to use limit orders instead of market orders. See Harris and Hasbrouk (1996) for a discussion of the differences between value traders and pre-committed traders.

types of traders in our model.

Table 2: The Traders

Liquidity Suppliers (date 0)	Liquidity Demanders (date 1)
Precommitted Limit Order Traders	Liquidity Traders
Uninformed Dealers	Speculators
Informed Dealers	

Limit Order Book. Modeling price formation in limit order markets quickly becomes very complicated. In order to keep the model tractable, we make the following assumptions. Liquidity suppliers can post sell limit orders at prices A_1 and A_2. We assume that

$$A_2 - A_1 = A_1 - v_0 = \Delta. \tag{2}$$

Parameter Δ can be seen as the tick size, i.e. the minimum variation between two consecutive quotes in the book : A_1 is the smallest eligible price above the unconditional expected value of the asset and A_2 is the second smallest eligible price above this value. We describe the price schedule posted by liquidity supplier j by the pair (x_{1j}, x_{2j}) where x_{kj} denotes the number of round lots offered by liquidity supplier j at price A_k, $k \in \{1, 2\}$. We also assume that

$$A_1 < v_0 + \sigma < A_2. \tag{3}$$

This assumption implies that limit orders posted at price A_1 are exposed to the risk of an information event but limit orders posted at price A_2 are not. Two implications follow. First, collectively, dealers will never supply more than 2 round lots at price A_1 since this is the maximal demand of a liquidity trader.[11] Second, dealers (informed or uninformed) can safely offer to trade any quantity at price A_2.

Thus, we can restrict our attention to the case in which the leader chooses one of 3 price schedules on the sell side: (a) $(0, 2)$,(b) $(1, 2)$ and (c) $(2, 2)$ that we denote T, S and D, respectively. At the end of the first stage, the limit order book can be in one of 3 states: (a) *"thin"* if the leader posts schedule T, (b) *"shallow"* if the leader posts schedule S or (c) *"deep"* if the

---

[11]The informed dealer has no incentive to submit limit orders for more than 2 round lots since any additional round lot would only execute against orders sent by a speculator and therefore would lose money. The same argument applies for the follower since time priority is enforced.

leader posts schedule D. The informed dealer chooses the schedule which maximizes his expected profit. The choice of pre-committed liquidity suppliers is exogenous: they choose schedule $K \in \{T, S, D\}$ with probability $\Phi_K > 0$. Given the state of the book, the uninformed dealer has 3 possibilities: (1) add 1 round lot at price A_1, (2) add 2 round lots at price A_1 or (3) do nothing. She never submits a limit order at price A_2 since this order has a zero execution probability (the leader always offer 2 round lots at price A_2). In summary, the follower chooses one of the following price schedules: (a) $(1, 0)$, (b) $(2, 0)$ or (c) $(0, 0)$.

We make symmetric assumptions on the buy side. This symmetry implies that the equilibrium price schedules on the buy side are the mirror image of the equilibrium price schedules on the sell side. Thus from now on we focus on the sell limit orders chosen by the dealers exclusively.[12]

We denote by $Q_1 \in \{0, 1, 2\}$ the number of round lots offered at price A_1 *at the end* of the bidding stage. Let $\widetilde{Q}_s$ and $\widetilde{Q}_l$ be the size of buy market orders submitted at date 1 by a speculator and a liquidity trader, respectively. If $\epsilon_1 = \sigma$, a speculator optimally submits a buy market which exhausts the quantity offered at price A_1 (since $A_1 < v_0 + \sigma < A_2$). Hence $\widetilde{Q}_s = Q_1$. This means that the probability distribution for the size of the buy market order which is submitted at date 1 depends on Q_1. Specifically we obtain:

$$\widetilde{Q}(Q_1) = I Q_1 + (1 - I)\widetilde{Q}_l, \tag{4}$$

where (i) $\widetilde{Q}(Q_1)$ is the size of the market order at date 1 and (ii) I is an indicator variable equal to 1 if the trader who submits a buy market order at date 1 is informed and zero otherwise.

Anonymous and Non-Anonymous Limit Order Markets. We shall distinguish two different trading systems: (i) the *anonymous limit order market* and (ii) the *non-anonymous limit order market*. In the non-anonymous trading system, the follower observes the identity of the leader, that is she can distinguish between informative and non-informative orders. In the anonymous market, she cannot. In both cases, however, the follower observes the price schedules posted in the first stage (i.e. the book is "open").

---

[12]As we restrict bidders to 2 prices on each side of the book, our model is best viewed as a model of competition at the inner quotes in the book. Several empirical papers (e.g. Biais, Hillion and Spatt (1995)) find that most of the activity is at or close to the best quotes.

Measures of Market Liquidity We will compare the liquidity of these two trading systems for *fixed values* of the exogenous parameters $(\sigma, \alpha, \beta, \Delta)$. To this end, we consider 2 different measures of market liquidity: (a) the *small trade spread* (or quoted spread) which is the difference between the best ask price and the unconditional expected value of the security and (b) *the large trade spread* which is the difference between the average execution price of a market order for 2 round lots and the the unconditional expected value of the security. For instance, if the first round lot executes at price A_1 and the second round lot executes at price A_2, the large trade spread is $(\frac{A_1+A_2}{2} - v_0)$. As shown in the next section, for some parameter values, a switch to anonymity reduces the small trade spread but simultaneously increases the large trade spread. Actually, this switch affects both the probability distribution of the quoted spread *and* the probability distribution of the number of round lots offered at price A_1. Market liquidity unambiguously improves when both the small trade spread and the large trade spread decrease.

3.2 Discussion.

Informed Dealers. Several empirical findings suggest that some liquidity suppliers have a superior expertise in choosing their quotes. For instance, Blume and Goldstein (1997) find that the NYSE specialist establishes the best quotes much more frequently than regional exchanges' specialists. The latter tend to match the NYSE specialist's quotes and rarely improve upon his spread. This leader-follower type of behavior suggests that regional exchanges' specialists view the NYSE specialist as knowledgeable for the determination of the "right" spread.

Lee, Mucklow and Ready (1993) obtain findings which support this interpretation. They show that the reduction in quoted depth and the increase in spread which precede earnings announcements are greater for earnings announcements which trigger large price movements. They conclude (p.368) that: "*Both findings suggest a market in which the liquidity suppliers are able to anticipate, to some extent, the price informativeness of an upcoming earnings release.*"

Anand and Martell (2001) find that limit orders placed by institutional investors on the NYSE perform better than those placed by individuals, even after controlling for order characteristics (such as order aggressiveness or order size). They argue (page 2) that institutional investors are better able "*to predict at least the flow of information and use this knowledge to submit trades, which avoid adverse selection associated with limit orders*". Finally, for Euronext Paris, Declerk (2001) shows that there are substantial variations in the trading profits of the intermediaries who

actively trade for their own account. This finding suggests that some intermediaries (those with superior profits on average) have more expertise.

Here we capture variations in expertise across liquidity suppliers with the assumption that some dealers have information on the true cost of liquidity provision. This information gives them an edge in the positioning of their limit orders. Information on the cost of liquidity provision may come from several sources in practice. One possibility (suggested by Lee et al. (1993)) is that some dealers receive information on the likelihood of an information event, as we assume here. Intuitively this information helps a dealer to assess his exposure to the risk of being picked off and therefore to better position his quotes. Superior information on the magnitude of upcoming price changes (i.e. σ) would have the same effect.

It is worth stressing that informed dealers have information on the likelihood or magnitude of a price movement during the trading day, *not on the direction* of this price movement (in contrast to speculators who intervene at date 1).[13] In particular, observe that the expected value of the security at date 0 is the same (and equal to v_0) for the informed and the uninformed dealers *alike*. Hence the informed dealer will never find it profitable to submit market orders at date 0. In other words, information on the likelihood of an information event is useful to position quotes but useless for the decision to trade at these quotes.

As an example, consider the case of a dealer who knows that a merger announcement is pending. Numerous empirical studies have shown that this type of announcement has no impact on the price of the acquiring firm, *on average*. Thus a dealer with this information can correctly anticipate that the announcement will trigger a price reaction for the acquiring firm without being able to predict the direction of the price reaction.

Timing. The timing of our model (the uninformed dealer is always the follower) may look artificial. A more general formulation would allow the sequence in which the informed and the uninformed dealer choose their price schedules to be random.[14] This formulation however would

---

[13]Kyle (1989), Calcagno and Lovo (2001) or Rindi (2002) consider models in which liquidity suppliers possess private information on the final payoff of a security. In these models, informed traders have information on both the direction and the magnitude of the price change.

[14]In auctions with fixed end times, expert bidders may choose to place their bids in the closing seconds of the auction to avoid revealing their information (see the empirical study of Roth and Ockenfels (2002)). In limit order markets, the notion of *fixed end time* does not apply since the times at which market orders arrive are random. Thus an informed bidder who chooses to wait in order to avoid revealing his information runs the risk of missing

obscure the presentation of our results without adding new insights. Actually, the follower's bidding strategy depends on the identity of the leader only when (i) the leader has a chance to be informed and (ii) the follower is uninformed. This configuration is therefore the only case in which concealing the leader's identity has an effect, if any. The timing of our model is thus a way to focus the attention on the interesting case.

Pre-committed Traders. Uninformed dealers' bidding strategy will depend on the informativeness of the book. In Euronext Paris, we expect a large fraction of agency orders to be non-informative on the risk of being picked off. This source of noise stems from the limit orders placed by precommitted liquidity suppliers in our model. These orders blur the inferences which can be drawn by uninformed dealers from the limit order book when it is anonymous. Thus they reduce the informational content of equilibrium prices. In this sense, pre-committed traders play the role ascribed to noise traders in Noisy Rational Expectations models (e.g. Hellwig (1980)). As in many of these models, the behavior of these traders is exogenous in our model.

Finally we assume that the informed dealer always trades for his own account. In reality, all intermediaries operating in Euronext Paris conduct agency trades. The model can easily be amended to accomodate this uncertainty, however. For instance the informed dealer might post quotes on behalf of another investor (acts as a precommitted liquidity supplier) with probability $\gamma < 1$. In this case, the results are qualitatively unchanged since limit orders placed by the informed dealer remain more informative than those placed by precommitted liquidity suppliers. This is the feature of our model which drives the results.

3.3   The Follower's Optimal Reaction

As a building block, we first study the follower's optimal reaction in each possible state of the book for *given*, but arbitrary, beliefs π about the occurrence of an information event. Suppose that the book is thin ($K = T$) at the end of the first stage. The follower can profitably add one round lot at price A_1 if this price is larger than her valuation of the asset, conditional on

the next trade. In addition, he cannot be certain that an uninformed bidder will not react before the arrival of the next market order. In these conditions, the natural modelling strategy is to assume that bidders' arrival times are random.

execution of one round lot or

$$A_1 - E_\pi(V \mid \widetilde{Q}(1) \geq 1) \geq 0. \tag{5}$$

In case of execution, the follower knows that the size of the market order is at least equal to 1 round lot but she does not know the exact size of the order. This explains why the follower's valuation is given by an "upper-tail expectation" (see Glosten (1994)). Computations yield

$$E_\pi(V \mid \widetilde{Q}(1) \geq 1) = v_0 + \pi\alpha\sigma. \tag{6}$$

Using the same logic, we conclude that the follower can profitably offer another round-lot at price A_1 when one is already offered if

$$A_1 - E_\pi(V \mid \widetilde{Q}(2) \geq 2) \geq 0.$$

Computations yield

$$E_\pi(V \mid \widetilde{Q}(2) \geq 2) = v_0 + (\frac{2\pi}{\pi\alpha + 1})\alpha\sigma. \tag{7}$$

It is useful to interpret $E_\pi(V \mid \widetilde{Q}(1) \geq 1)$ as the "cost" of providing 1 round lot at price A_1 for a dealer who assigns a probability π to the occurrence of an information event. Similarly $E_\pi(V \mid \widetilde{Q}(2) \geq 2)$ is the cost of providing one additional round lot at price A_1 when one is already offered.[15] For this reason, we refer to the cost schedule defined by Equations (6) and (7) as being the *expected cost of liquidity provision*.[16] This schedule is increasing (in the quantity) since

$$(\frac{2\pi}{\pi\alpha + 1})\alpha\sigma > \pi\alpha\sigma \qquad \forall \pi > 0.$$

One implication is that it may be optimal (depending on parameter values) to offer 1 round lot but not more at price A_1. The informed speculator always exhausts the depth available at price A_1. In contrast, a liquidity trader always trades at least 1 round lot but trades 2 round lots with probability 0.5 only. Thus the second round lot offered at price A_1 is relatively more exposed to informed trading than the first round lot. This explains why the cost of providing this second round lot is larger than the cost of providing the first one.

---

[15]For a given π, the cost of providing a second round lot at price A_1 does not depend on whether the trader offering the second round lot is the trader offering the first round lot or not. Actually if the two traders are different, the first one has time priority. Thus the first round lot will be executed before the second. Thus execution of the second round lot means that the market order size is larger than or equal to 2 round lots.

[16]The *actual* cost is either high if an information event occurs or low (and equal to zero here) if there is no information event.

13

The follower's belief about the occurence of an information event, π, will in general depend on the state of the book just before she submits (or not) her limit order. Henceforth, to make this linkage explicit, we denote by π_K the follower's belief when the state of the book is K (π_K is endogenized in the next section). The follower's optimal behavior given the state of the book immediately derives from the previous remarks. For a given state of the book at the end of stage L, the follower will fill the book up to the point where an additional round lot offered at price A_1 would lose money. This means that the number of shares offered at price A_1 at the end of the bidding stage is the *largest* $Q_1 \in \{1, 2\}$ such that

$$A_1 - E_{\pi_K}(V \mid \widetilde{Q}(Q_1) \geq Q_1) \geq 0. \tag{8}$$

If this inequality cannot be satisfied for $Q_1 > 0$ then no limit order can be profitably placed at price A_1. In this case $Q_1 = 0$ (the book is empty at price A_1). The follower's optimal behavior for each possible state of the book is then given by the next lemma.

Lemma 1 :

1. *When the follower observes a thin book, she submits a limit order at price A_1 for 2 round lots if $\frac{2\pi_T \alpha \sigma}{\pi_T \alpha + 1} < \Delta$, 1 round lot if $\pi_T \alpha \sigma < \Delta < \frac{2\pi_T \alpha \sigma}{\pi_T \alpha + 1}$ and does nothing otherwise.*

2. *When the follower observes a shallow book, she submits a limit order at price A_1 for 1 round lot if $\frac{2\pi_S \alpha \sigma}{\pi_S \alpha + 1} < \Delta$ and does nothing otherwise.*

3. *When the follower observes a deep book, she does nothing.*

The risk of being picked off is large when the likelihood of an information event is large. For this reason the expected cost of liquidity provision increases with the likelihood of an information event (see Equations (6) and (7)). Hence the follower's inclination to add depth to the book is smaller when she assigns a large probability to the occurrence of an information event. This mechanism explains why, for a given state of the book, the follower acts less and less aggressively as the likelihood of an information event, π_K, increases (consider the case in which the book is thin for instance).

4 Equilibria in Anonymous and Non-Anonymous Limit Order Markets

In this section, we analyze the nature of equilibria in the anonymous and in the non-anonymous market. As a benchmark, we first study the case in which dealers have symmetric information (the leader and the follower are uninformed).

4.1 A Benchmark

When dealers have symmetric information, the state of the book at the end of the first stage does not convey any information on the actual cost of liquidity provision. Furthermore the follower's beliefs about this cost are identical in the anonymous and in the non-anonymous trading system since the leader's identity (combined with his bidding strategy) does not convey information. Therefore $\pi_S = \pi_T = \pi_0 \overset{def}{=} 0.5$ in both the anonymous and the non-anonymous trading systems. It follows that the equilibrium is not affected by the anonymity of the trading system.

Proposition 1 *(Benchmark): When dealers have symmetric information, market liquidity (i.e. the small trade spread and the large trade spread) is identical in the anonymous and in the non-anonymous trading system.*

This result will not hold when there is asymmetric information among dealers, as shown in Corollary 2 below. In the rest of the paper we concentrate on the case in which

$$\frac{2\pi_0\alpha\sigma}{\pi_0\alpha + 1} = \frac{2\alpha\sigma}{\alpha + 2} < \Delta. \tag{9}$$

Under this condition, in absence of asymmetric information, the follower always fills the book so that the maximal possible depth (2 round lots) is offered at price A_1 (Lemma 1). This yields the next proposition.

Proposition 2 *(Benchmark): Suppose that dealers have symmetric information. When $\frac{2\alpha\sigma}{\alpha+2} < \Delta$, the unique subgame perfect equilibrium is as follows: (i) the dealer acting in stage F chooses schedule D and (ii) the follower acts as described in Lemma 1 for $\pi_S = \pi_T = 0.5$. In equilibrium, the book obtained at the end of the second stage is always deep (2 round lots are offered at price A_1), i.e. the small trade spread and the large trade spread are equal to $A_1 - v_0$.*

Why does the dealer acting in stage F post the most competitive schedule (namely D)? Suppose to the contrary that he chooses a less competitive schedule, say schedule S. This schedule gives him a strictly larger profit than schedule D when a large market order arrives and walks up the book (the second round lot executes against his limit order placed at price A_2). However, this will never occur since the follower will seize the profit opportunity left at price A_1. Anticipating this reaction, the dealer acting in the first stage fills the book so as to leave no profit opportunity at price A_1.

4.2 The Anonymous Limit Order Market

Now we turn to the case in which there is asymmetric information among dealers. Throughout we focus on Perfect Bayesian equilibria of the bidding game at date 0, as usual in analyses of signaling games. We denote by Ψ an indicator variable which is equal to 1 if there is an information event and zero otherwise. To make things interesting, we focus on the case in which:

$$\frac{2\alpha\sigma}{\alpha + 2} < \Delta < \alpha\sigma \tag{10}$$

This condition guarantees that the follower's reaction is influenced by her belief on the occurence of an information event.[17] Actually, the L.H.S inequality implies that the uninformed dealer will submit a limit order at price A_1 (when the book is not deep) if her posterior belief about the occurrence of an information event is not too different from her prior belief (consider Equation (7) with $\pi = 0.5$). But the R.H.S implies that the uninformed dealer will find suboptimal to submit a limit order at price A_1 if her posterior belief is large enough compared to her prior belief (consider Equation (6) with $\pi = 1$). The rest of this subsection analyzes equilibrium bidding strategies when the limit order market is anonymous.

When there is an information event, the informed dealer cannot profitably place a limit order at price A_1. Actually, the actual cost of providing 1 round lot at price A_1 is then (see Eq.(6)):

$$E_1(V \mid \widetilde{Q}(1) \geq 1) = v_0 + \alpha\sigma$$

which is larger than $A_1 = v_0 + \Delta$ since $\Delta < \alpha\sigma$. Thus we shall focus on equilibria in which the informed dealer chooses schedule T when there is an information event. When there is *no*

---

[17]Clearly the set of parameters such that Condition (10) is satisfied is never empty. We have also assumed: $\sigma \leq 2\Delta$. This condition combined with the R.H.S of Condition (10) imposes $\alpha > \frac{1}{2}$. This condition can be relaxed if the condition $\sigma \leq 2\Delta$ is relaxed. Intuitively, the risk of informed trading matters only if α or σ are large enough.

information event, the informed dealer can profitably establish the deep book. He then obtains an expected profit equal to:

$$\Pi^L(D,0) \stackrel{def}{=} (A_1 - v_0)E(\widetilde{Q}_u) = \frac{3(A_1 - v_0)}{2} > 0. \tag{11}$$

But he may also try to reap a larger profit by establishing a thin book. For the follower, a thin book constitutes a warning and she revises upward the probability she assigns to an information event (see Eq. (12) below). If this revision is large enough, she is deterred from submitting a limit order within the best quotes and the informed dealer clears all the market orders at price $A_2 > A_1$. If the informed dealer sometimes behaves in this way, we say that he follows a *bluffing strategy*.

Formally let m be the probability with which the informed dealer chooses schedule D when $\Psi = 0$. With the complementary probability, he chooses schedule T when $\Psi = 0$. The next proposition describes the conditions under which there exists an equilibrium with bluffing (i.e. $0 \leq m < 1$). Let $\beta^* \stackrel{def}{=} \frac{(\alpha - r)}{(\alpha - r) + \Phi_T(2r - \alpha)}$ and $r \stackrel{def}{=} \frac{\Delta}{\sigma}$.

Proposition 3 : *When $0 \leq \beta \leq \beta^*$ and $\frac{2\alpha\sigma}{\alpha + 2} < \Delta < \alpha\sigma$, the following bidding strategies constitute an equilibrium:*

1. *When there is an information event, the informed dealer posts schedule T. When there is no information event, the informed dealer posts schedule D with probability $m^*(\beta) = (\frac{(1 - \beta + \beta\Phi_T)}{1 - \beta})(\frac{2r - \alpha}{r})$ and schedule T with probability $(1 - m^*(\beta))$, with $0 < m^*(\beta) < 1$.*

2. *When the book is thin, the follower submits a limit order for 1 round lot at price A_1 with probability $u_T^* = \frac{3}{4}$ and else does nothing. When the book is shallow, the follower adds 1 round lot at price A_1. When the book is deep, the follower does nothing.*

3. *The average small trade spread and the average large trade spread are greater than in the benchmark case.*

There is a non-empty set of parameters for which the equilibrium described in the proposition is obtained since (a) the condition $\beta < \beta^*$ implies that $m^*(\beta) < 1$ and (b) the condition $\Delta < \alpha\sigma$ implies that $\beta^* > 0$. Observe that both dealers (informed and uninformed) use mixed strategies in equilibrium.

We now explain in detail the intuition behind the last proposition. The key point is that the state of the book contains information on the likelihood of an information event. In particular, when she observes a thin book, the uninformed dealer revises upward the probability she assigns to an information event. Formally, for a given m, the uninformed dealer's posterior belief is

$$\pi_T(m, \beta) \stackrel{def}{=} prob(\Psi = 1 \mid K = T) = \frac{\beta \Phi_T + (1 - \beta)}{2\beta \Phi_T + (1 - \beta)(2 - m)} \geq \pi_0 = 0.5. \qquad (12)$$

As the uninformed dealer revises upward the probability she assigns to an information event, she marks up the cost of liquidity provision. We refer to this effect as being the *deterrence effect* of cautious bidding by the leader since it reduces the uninformed dealer's incentive to submit a limit order at price A_1.

The larger is the follower's posterior belief $(\pi_T(m, \beta))$, the larger is the deterrence effect. Thus the deterrence effect increases in m and decreases in β. Actually these two variables determine *the informativeness of the limit order book*. If $m = 0$, the book is not informative since the informed dealer establishes a thin book whether an information event occurs or not. As m enlarges, the informativeness of the book improves since the informed dealer chooses a thin book less and less frequently when there is no information event. As β increases, the informational content of the book decreases since it is more and more likely that it has been established by a precommited trader. Hence the follower's belief about the occurence of an information event is less and less sensitive to the state of the book. To sum up the deterrence effect is strong when the informational content of the book is large.

Conditional on the state of the book being thin $(K = T)$, the uninformed dealer estimates the cost of offering one round lot at price A_1 to be :

$$E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1) = v_0 + \pi_T(m, \beta)\alpha\sigma. \qquad (13)$$

A graphical representation of this conditional expectation as a function of m is given in Figure 3. The perceived cost of offering 1 round lot at price A_1 for the uninformed dealer becomes larger as m enlarges. This reflects the fact that the deterrence effect increases with m.

INSERT FIGURE 3 ABOUT HERE

Figure 3 helps to understand how the equilibrium is obtained. Observe that $m^*(\beta)$ is the value of m such that the follower is just indifferent between submitting a limit order for 1 round

18

lot at price A_1 or doing nothing. That is $m^*(\beta)$ is such that:

$$A_1 - E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1) = \Delta - \pi_T(m^*, \beta)\alpha\sigma = 0. \tag{14}$$

Suppose that the informed dealer chooses schedule D with probability $m > m^*$. In this case a thin book induces a relatively large revision in the follower's estimation of the cost of liquidity provision. So large that she never finds it optimal to submit a limit order at price A_1 (see Figure 3). But then the informed dealer should choose to submit limit orders only at price A_2 (i.e he should always choose schedule T), whether an information event took place or not (i.e. $m = 0$). This deviation precludes the existence of an equilibrium in which $m > m^*$. Suppose then that the informed dealer chooses schedule D with probability $m < m^*$. In this case a thin book induces a relatively small revision in the follower's estimation of the cost of liquidity provision. So small that she always finds it optimal to submit a limit order at price A_1. But then the informed dealer is strictly better off if he chooses schedule D when there is no information event (i.e. $m = 1$). This deviation precludes the existence of an equilibrium in which $m < m^*$.

When $m = m^*$, the follower is just indifferent between undercutting a thin book or doing nothing. Thus she follows a mixed strategy. She undercuts the thin book sometimes but not always. The leader is then confronted with a trade off between certain execution at price A_1 and uncertain execution at a more profitable price, A_2. In fact, when there is no information event, the informed dealer's expected profit if he establishes a thin book is:

$$\Pi^L(T,0) \stackrel{def}{=} (1-u_T)(A_2-v_0)E(\widetilde{Q}_u) + \frac{u_T}{2}(A_2-v_0) = ((1-u_T)\frac{3}{2} + \frac{u_T}{2})(A_2-v_0), \tag{15}$$

where u_T is the probability that the follower undercuts the thin book with a limit order for 1 round lot at price A_1. In contrast, if the informed dealer chooses the deep book, he obtains an expected profit equal to

$$\Pi^L(D,0) = \frac{3(A_1-v_0)}{2}. \tag{16}$$

It is immediate that the informed dealer is better off choosing a thin (resp.a deep) book iff $u_T < \frac{3}{4}$ (resp.$u_T > \frac{3}{4}$). For $u_T = \frac{3}{4}$, he is just indifferent and therefore he uses a mixed strategy, as described in the proposition.[18]

---

[18]One might be surprised by the fact that u_T does not depend on the model parameters. It does not depend on β because this parameter does not per se affect the informed dealer's expected profits ($\Pi^L(T,0)$ and $\Pi^L(D,0)$). It does not depends on the tick size because the informed dealer's expected profit with a thin or a deep book is a linear function of the tick size.

Interestingly, these order placement strategies imply that the state of the book at the end of the bidding stage is random. For instance, suppose that the leader establishes a thin book. The follower reacts by improving upon the quotes with probability $\frac{3}{4}$ and does nothing otherwise. The book faced by market order submitters might then be shallow or thin. Thus the book is not necessarily deep at date 1, in contrast with the benchmark case. For this reason the liquidity of the market is smaller than in the benchmark case (last part of the proposition).

Observe that the informed dealer bids more aggressively when β enlarges ($m^*(\beta)$ increases with β). The intuition is as follows. Other things equal (m^* fixed), the larger is β, the smaller is the informational content of the book. As we already explained, this relaxes the deterrence effect. Accordingly, the informed dealer must choose schedule D more frequently (m^* increases) in order to strengthen the informational content of the book and thereby the deterrence effect.

For β large enough ($\beta > \beta^*$), the follower cannot be deterred from submitting a limit order for 1 round lot at price A_1, even if $m = 1$. In this case, there is no equilibrium in which the informed dealer uses a bluffing strategy. The equilibrium bidding strategies are described in the following proposition. Let $\beta^{**} \overset{def}{=} \frac{((2-r)\alpha-r)}{((2-r)\alpha-r)+\Phi_T(2r-(2-r)\alpha)} < 1$.

Proposition 4 : *When $\beta^* < \beta \leq \beta^{**}$ and $\frac{2\alpha\sigma}{\alpha+2} < \Delta < \alpha\sigma$, the following bidding strategies constitute an equilibrium:*

1. *When there is an information event, the informed deader chooses schedule D. When there is no information event, the informed deader chooses schedule T.*

2. *When the book is thin or shallow, the follower submits a limit order for 1 round lot at price A_1. When the book is deep, the follower does nothing.*

3. *The average small-trade spread is as in the benchmark case but the average large trade spread is greater than in the benchmark case.*

When she observes a thin book, the follower revises upward her belief regarding the likelihood of an information event. The revision is too small to deter her from submitting a limit order for 1 round lot at price A_1 but large enough to deter her from posting a larger size. In fact it is easily checked that :

$$A_1 - E_{\pi_T}(V \mid \widetilde{Q}(2) \geq 2) = \Delta - (\frac{2\pi_T(1,\beta)}{\pi_T(1,\beta)\alpha + 1})\alpha\sigma \leq 0, \quad for \quad \beta \leq \beta^{**}, \tag{17}$$

which means that the uninformed dealer perceives the cost of offering 2 round lots at price A_1 as being larger than A_1 (see Figure 4).

INSERT FIGURE 4 ABOUT HERE

The uninformed dealer bids more aggressively than in the equilibrium described in Proposition 3 but still more cautiously than in the benchmark case. This explains the last part of the proposition.

Proposition 5 : *When* $\beta > \beta^{**}$ *and* $\frac{2\alpha\sigma}{\alpha+2} < \Delta < \alpha\sigma$ *then the following bidding strategies constitute an equilibrium:*

1. *When there is no information event, the informed deader chooses schedule T. When there is an information event, the informed deader chooses schedule D.*

2. *When the book is thin, the follower submits a limit order for 2 round lots. When the book is shallow, the follower submits a limit order for 1 round lot at price A_1 and when the book is deep, the follower does nothing.*

3. *The average small-trade spread and the average large-trade spread are as in the benchmark case.*

Intuitively, when β is very large (greater than β^{**}) the informational content of the book is too small to influence the follower's beliefs on the profitability of adding depth to the book. Thus the latter behaves as in the benchmark case, that is she fills the book so that eventually 2 round lots are offered at price A_1. Anticipating this behavior, the leader establishes a deep book whenever this is profitable. In contrast with the equilibria described in Propositions 3 and 4, the state of the book at the end of the bidding stage is not random when $\beta > \beta^{**}$ (the book is deep with probability 1 at date 1 in this case).

A Technical Remark. In equilibrium, the follower's posterior belief about the occurrence of an information event is determined by Bayes rule whenever this is possible. As usual in signaling games, there is a difficulty if some states of the book are out-of-the equilibrium path. By definition these states have a zero probability of occurence in equilibrium. Hence in these

states the follower's posterior belief cannot be determined by Bayes rule. This problem does *not* arise when $\beta > 0$ (all states of the book are on the equilibrium path). When $\beta = 0$, the shallow book is out-of the equilibrium path since the informed dealer never chooses a shallow book in the equilibria that we described previously. In this case, we assume that the follower does not revise her prior belief about the occurence of an information event when she observes a shallow book.[19] In this way, the follower's posterior belief conditional on observing a shallow book is a continuous function of β (it does not "jump" at $\beta = 0$).

4.3 The Non-Anonymous Limit Order Market

The equilibrium in the non-anonymous market can be derived by considering a special case of the analysis for the anonymous market. Consider the polar situation in which $\beta = 0$. In this case, in the anonymous market, the uninformed dealer knows that the leader is an informed dealer, even though she does not directly observe his identity. Thus, in this case, the game in the anonymous market is identical to the game played in the non-anonymous market when the leader is informed. This remark yields the next corollary.

Corollary 1 : *Consider the case in which the leader is the informed dealer. In this case, the dealers' bidding strategies described in Proposition 3 when $\beta = 0$ form an equilibrium of the non-anonymous market. In particular, the informed dealer uses a bluffing strategy: when there is no information event, he chooses schedule D with probability $m^*(0) < 1$.*

Now consider the case in which the leader is a precommitted liquidity supplier. In this case, the limit orders posted in the first stage contains no information and the uninformed dealer optimally behaves as in the benchmark case. Thus she fills the book so that 2 round lots are offered at price A_1 (see Proposition 2).

Anonymity and Bidding Aggressiveness. It is useful to analyze in detail how dealers' bidding behavior differs in the anonymous market and in the non-anonymous market. Ultimately this helps understanding how a switch to anonymity affects liquidity in our model. Observe that

---

[19]The Perfect Bayesian Equilibrium concept does not put restrictions on how players' beliefs should be formed when they observe actions which are out-of-the equilibrium path (actions which have a zero probability of occurence in equilibrium). For these actions, players' beliefs can be specified arbitrarily. See Fudenberg and Tirole (1991), Chapter 8.

for a given value of β, the informed dealer chooses to establish a deep book with probability $m^*(\beta)$ in the anonymous market and probability $m^*(0)$ in the non-anonymous market, when there is no information event. Thus, as $m^*(\beta) > m^*(0)$, the informed dealer behaves more competitively in the anonymous market than in the non-anonymous market. From the point of view of the informed dealer, the effect of a switch to anonymity is similar to the effect of an increase in β. This is intuitive: a switch to anonymity reduces the informational content of the book, other things equal. As explained in the previous section, a noisier book induces the informed dealer to post more aggressive limit orders.

The effect of anonymity on the uninformed dealer's bidding behavior is more complex. Consider the case in which the uninformed dealer faces a thin book (for the other states of the book, the uninformed dealer's behavior is not affected by the anonymity regime). In the non-anonymous market, the uninformed dealer undercuts the best offer with probability $u_T^* = \frac{3}{4}$ if the leader is informed and with probability 1 if the leader is a precommitted trader. Thus the probability of observing a limit order improving upon the quotes (conditional on the book being thin) is:

$$\Pr{}_{na}^{F}(\text{undercut} \mid K = T) = \frac{3(1-\beta)}{4} + \beta = \frac{(3+\beta)}{4}, \tag{18}$$

in the non-anonymous market. In the anonymous market, the uninformed dealer undercuts the best offer with probability $u_T^* = \frac{3}{4}$ if $\beta \leq \beta^*$ and probability 1 if if $\beta > \beta^*$, whether the leader is informed or not. As $\frac{3}{4} < \frac{(3+\beta)}{4} < 1$, we conclude that the likelihood that the uninformed dealer improves upon the best quotes can be smaller or larger in the anonymous market depending on the value of β. Another measure of the follower's aggressiveness is the probability that she will offer two round lots at price A_1 if she undercuts a thin book. This probability is β in the non-anonymous market. In the anonymous market, this probability is equal to zero if $\beta \leq \beta^{**}$ and 1 otherwise. Thus the follower can offer more or less depth at price A_1 in the anonymous market, depending on the value of β. To sum up, the impact of anonymity on the uninformed dealer's bidding aggressiveness is ambiguous.

4.4 Anonymity and Market Liquidity

In this section, we compare our measures of market liquidity in the anonymous and in the non-anonymous trading mechanism. Formally, the expected small trade spread in a given trading

mechanism is given by:

$$
\begin{aligned}
ES_{small} &= \Pr(Q_1 \geq 1)A_1 + \Pr(Q_1 = 0)A_2 - v_0 \\
&= \Delta(1 + \Pr(Q_1 = 0)).
\end{aligned}
\tag{19}
$$

The expected large trade spread is given by

$$
ES_{l\arg e} = \Pr(Q_1 = 2)A_1 + (\frac{A_1 + A_2}{2})\Pr(Q_1 = 1) + \Pr(Q_1 = 0)A_2 - v_0,
$$

which rewrites

$$
ES_{l\arg e} = \frac{\Delta}{2}(3 + \Pr(Q_1 = 0) - \Pr(Q_2 = 2)).
\tag{20}
$$

We obtain the following result.

Corollary 2 : *A switch to an anonymous limit order book reduces the expected small and large trade spreads only when β is large enough ($\beta \geq \beta^{**}$). When β is small ($\beta < \beta^*$), a switch to an anonymous limit order book enlarges the expected small and large trade spreads. When $\beta^* < \beta < \beta^{**}$, a switch to anonymity: (i) reduces the expected small trade spread and (ii) increases the expected large trade spreads if $Min\{\overline{\beta}, \beta^*\} < \beta$ (the cut-off $\overline{\beta}$ is defined in the appendix).*

Thus a switch to an anonymous limit order book should affect liquidity. The impact, however is ambiguous and depends on β. Recall that the informed trader behaves more competitively in the anonymous market. However when β is small, the uninformed trader bids more conservatively (undercuts a thin book less frequently) in the anonymous market (see the previous subsection). These two effects have opposite impacts on market liquidity and the second effect dominates when β is small. When β is large enough, a switch to anonymity makes both the informed dealer and the uninformed dealer more aggressive. This explains why it reduces the small and the large trade spread.

Interestingly, for intermediate values of β ($\beta^* < \beta < \beta^{**}$), a switch to anonymity is beneficial to traders who submit small market orders (since it reduces the average small trade spread) but not necessarily to traders who submit large orders. Notice that the average large trade spread is determined by the average number of round lots offered at price A_1 (the "quoted depth", $E(\widetilde{Q}_1)$) since:

$$
ES_{l\arg e} = \frac{\Delta}{2}(3 + \Pr(\widetilde{Q}_1 = 0) - \Pr(Q_1 = 2)) = \frac{\Delta}{2}(4 - E(\widetilde{Q}_1)).
\tag{21}
$$

24

For $\beta^* < \beta < \beta^{**}$, the switch to anonymity reduces the probability that no round lots will be offered at price A_1 (i.e. $\Pr(Q_1 = 0)$ decreases). But, simultaneously, it reduces the probability that the uninformed dealer will offer 2 round lots at price A_1 (see previous subsection). Overall the probability that 2 round lots will be offered at price A_1 (i.e. $\Pr(\widetilde{Q}_1 = 2)$) is smaller. For $Min\{\overline{\beta}, \beta^*\} < \beta < \beta^{**}$, the second effect dominates and the average quoted depth at price A_1 is reduced. Accordingly large market orders bear larger trading costs.

5 Empirical Implications and Extensions

5.1 Empirical Implications

Corollary 2 and the discussion following the corollary suggest to analyze the effect of a switch to anonymity on both the average quoted spread and the quoted depth. Specifically it yields two testable hypotheses:

H.1: *The average quoted spread in a trading system where liquidity suppliers' identities are concealed is significantly different from the average quoted spread in a trading system where liquidity suppliers' identities are disclosed.*

H.2: *The average quoted depth in a trading system where liquidity suppliers' identities are concealed is significantly different from the average quoted depth in a trading system where liquidity suppliers' identities are disclosed.*

An interesting feature of our model is that, for given parameter values, the quoted spread at the end of the bidding stage is random (for $\beta \leq \beta^{**}$) whether the market is anonymous or not. Thus we can analyze the effect of a switch to an anonymous limit order book on the variability of the inside spread.

Corollary 3 *:The variance of the small trade spread is smaller in the anonymous market if* $\beta \geq \beta^*$.

Hence we will test the following hypothesis:

H.3: *The variability of the quoted spread in a trading system where liquidity suppliers' identities are concealed is significantly different from the variability of the quoted spread in a trading*

system where liquidity suppliers' identities are disclosed.

We test these predictions by considering the switch to an anonymous order book which took place on Euronext Paris in April 2001. In our model the effects of a switch to anonymity depend on the proportion of precommitted liquidity suppliers (see the two previous corollaries). Given our interpretations (see Section 3.1), a natural proxy for β would be the proportion of agency limit orders. Unfortunately, in our dataset, we cannot disentangle principal from agency orders. For CAC40 stocks, Declerck (2001) finds that the 6 intermediaries which handled 71% of all principal principal trades accounted for only 39% of all orders during her study period. Furthermore, principal trading accounted for 27% of the trading volume, on average. These findings suggest that β is relatively high for CAC40 stocks (which are part of our sample). Thus, at least for CAC40 stocks, we expect a decrease in the mean and the variance of the quoted spread following the switch to anonymity.

5.2   Extensions

In our model, a switch to anonymity induces the informed dealer to behave more competitively. This effect explains why the quoted spread is smaller on average in the anonymous market when $\beta \geq \beta^*$. This result might be an artifact of the assumption that the informed dealer faces competition from only one uninformed dealer. This is not the case. The informed dealer will use a bluffing strategy (i.e. behaves non-competitively) when $\beta \leq \beta^*$ (in particular if $\beta = 0$) even if he competes with (i) several uninformed dealers or (ii) with uninformed and informed dealers. We briefly explain why below.

Several Uninformed Dealers. Suppose that $N \geq 1$ uninformed dealers observe the limit orders posted in the initial stage. They submit their limit orders *sequentially.* Now consider the following course of actions when $0 \leq \beta \leq \beta^*$:

1. The informed dealer acts as described in Proposition 3.

2. When she faces a thin book, the uninformed dealer who reacts first submits a limit order for 1 round lot at price A_1 with probability $p_T^*(N) = 1 - (\frac{1}{4})^{1/N}$ and does nothing otherwise. For other states of the book, she acts as described in Proposition 3.

3. An uninformed dealer who does not react first submits a limit order for 1 round lot at price A_1 with probability $p_T^*(N) = 1 - (\frac{1}{4})^{1/N}$ if she faces a thin book and does nothing otherwise.

It is readily shown that these bidding strategies constitute an equilibrium (we omit the detailed proof for brevity). When they observe a thin book, uninformed dealers revise upward their beliefs about the occurence of an information event in such a way that they are **all** indifferent between submitting a limit order at price A_1 or not (exactly as described in Section 4.2). Hence they play a mixed strategy when they observe a thin book. Their mixed strategy is such that the probability of being undercut for the informed dealer if he posts a thin book is:

$$u_T^* = p_T^*(N) + (1 - p_T^*(N))p_T^*(N) + ... + (1 - p_T^*(N))^{N-1}p_T^*(N) = 3/4.$$

Thus, when there is no information event, the informed dealer is just indifferent between posting a deep book or a thin book (as explained in Section 4.2). Therefore he uses the bluffing strategy described in Proposition 3.

Competition between Informed Dealers. Suppose that the follower can be informed on the likelihood of an information event with probability θ. Otherwise she is uninformed. Consider the following course of actions when $0 \leq \beta \leq \beta^*$ and $\theta \leq \frac{3}{4}$.

1. The informed dealer who acts in stage L bids as described in Proposition 3.

2. When she faces a thin book, the uninformed dealer submits a limit order for 1 round lot at price A_1 with probability $p_T^* = \frac{(\frac{3}{4} - \theta)}{(1 - \theta)}$ and does nothing otherwise. For other states of the book, she acts as described in Proposition 3.

3. When there is an information event, the informed dealer who acts in stage F does nothing. If there is no information event, the informed dealer who acts in stage F submits a limit order at price A_1 for (a) 1 round lot if she faces a shallow book and (b) 2 round lots if she faces a thin book.

It is straightforward to show that these bidding strategies form an equilibrium. For brevity, we just show that it is optimal for the informed dealer to use a bluffing strategy. When there is no information event, the probability that the follower undercuts a thin book is:

$$u_T^* = \theta + (1 - \theta)p_T^* = 3/4.$$

Thus the informed dealer is just indifferent between posting a deep book or a thin book when there is no information event. Therefore he uses a bluffing strategy, as described in Proposition 3. It follows that a switch to anonymity will induce the informed dealer acting in the first stage to bid more aggressively, exactly as in the case in which $\theta = 0$. Notice that this result holds for all values of $\theta \leq \frac{3}{4}$. For larger values of θ, the informed dealer does not use a bluffing strategy (he behaves as described in Propositions 4 and 5) and a switch to anonymity has no effect on market liquidity. This is intuitive. For instance if $\theta = 1$, dealers have symmetric information and market liquidity is not affected by the anonymity regime (see Proposition 1).

Other Parameter Values. In the previous sections, we have analyzed in detail the equilibria which emerge when $\frac{2\alpha\sigma}{\alpha+2} < \Delta < \alpha\sigma$. Analysis of other parameter values yields similar conclusions. For instance, we have studied the case in which $\alpha\sigma < \Delta < \frac{2\alpha\sigma}{\alpha+1}$. In this case, it is profitable to offer 1 round lot (but no more) at price A_1 if there is an information event. Thus, the informed dealer posts a shallow book (rather than a thin book) when there is an information event. For β small enough, the informed dealer uses a bluffing strategy (he sometimes posts the shallow book when there is no information event). In this case the small trade spread is not affected by the switch to anonymity. But this is an artifact of the condition $\alpha\sigma < \Delta$. We have focused on the case $\Delta < \alpha\sigma$ to show that a switch to anonymity affect both the quoted spread and the quoted depth, in general.

6 Empirical Analysis

6.1 Institutional Background and Dataset

6.1.1 Euronext Paris

In March 2000, the Amsterdam Stock Exchange, the Brussels Stock Exchange and the Paris Bourse decided to merge. This merger (which took place in September 2000) gave birth to Euronext, a holding with 3 subsidiaries: Euronext Amsterdam, Euronext Brussels and Euronext Paris. Since the merger, the 3 exchanges have strived to create a unique trading platform (called NSC).[20] This goal is achieved since October 29, 2001. However, as of today, the 3 exchanges still have separate limit order books for each stock (mainly because clearing houses for French,

---

[20]NSC is an acronym which stands for "Nouveau Système de Cotation".

Dutch and Belgian stocks still differ). Euronext Paris was first to adopt the new trading platform on April 23, 2001, soon followed by Brussels on May 21, 2001 and Amsterdam on October 29, 2001. For Euronext Paris, the trading rules were very similar before and after the switch to NSC. Indeed, for very liquid stocks, the switch to an anonymous limit order book was the only significant change (see below).

NSC is an electronic limit order market (see Biais, Hillion and Spatt (1995) for a complete description of this market). Trading occurs continuously from 9:00 a.m. to 5:25p.m. for most of the stocks.[21] The opening and the closing prices are determined by a call auction. All orders are submitted through brokers who trade for their own account or on behalf of other investors. Traders mainly use two types of orders: (a) limit orders and (b) market orders. Limit orders specify a limit price and a quantity to buy or to sell at the limit price. Limit orders are stored in the limit order book and executed in sequence according to price and time priority. If the limit price crosses a limit on the opposite side of the book (so called "marketable limit orders") then the limit order is immediately executed (entirely or partially depending on its size). Market orders execute upon arrival against the best price on the opposite side of the book. Any quantity in excess of the depth available at this price is transformed into a limit order at that price. Marketable limit orders can walk up or down the book (if they are large enough) whereas market orders do not not (they can be viewed as marketable limit orders at the best price on the opposite side of the book).

All limit orders must be priced on a pre-specified grid. The tick size is a function of the stock price level. At the time of our study, the tick size is 0.01 Euros for prices below 50 Euros, 0.05 Euros for prices between 50.05 and 100 Euros, 0.1 Euros for prices between 100.1 Euros and 500 Euros and 0.5 Euros for prices above 500 Euros.[22]

The transparency of the market is quite high. Brokers observe (on their computer terminals) all the visible limit orders (price and associated depth) standing in the book at any point in time. The 5 best limits on each side of the book, the total depth available at these limits and the number of orders placed at each limit are disclosed to the public. The depth available in the book can be larger than the visible depth. Actually NSC enables traders to display only a portion of

---

[21]Less liquid stocks trade in call auctions which take place at fixed points in time during the trading day. All stocks in our sample are traded continuously.

[22]In April 2001, the value of the euro in dollar was approximately 0.86 Dollar / Euro.

their limit order by submitting hidden orders. The hidden portion retains price priority but loses time priority. A fraction of the hidden quantity becomes visible only when the quantity initially disclosed is fully executed.

Until April 23, 2001, *but not after that date*, the identification code of the issuing broker was also displayed for each order standing in the book. We refer to this change in the trading organization as a *switch to supply side anonymity*. This switch applied to **all** stocks listed on Euronext Paris. The objective of market organizers was to harmonize the trading rules in Euronext Paris and Euronext Amsterdam (in which trading was anonymous). The electronic limit order book used in the Paris Bourse has been non-anonymous for a long period of time (since 1986). Over this period, brokerage firms had the opportunity to identify those who have a greater expertise in choosing their limit orders. Hence in the non-anonymous environment, they were able to sort out informative orders from non-informative orders, as we assumed in our model.

Euronext Paris classifies stocks which trade continuously in 2 different groups, called "Continu A" and "Continu B". Stocks are assigned to one group based on measures of market activity (transaction and order frequency, trading volume). Stocks in Continu A feature a higher level of market activity. For stocks in Continu B, the switch to supply side anonymity was accompanied by another major change. For these stocks, counterparty IDs' used to be disclosed immediately after completion of their transaction until April 23, 2001. This is not the case anymore since this date. Thus stocks in Continu B have experienced a change in both pre- trade and post-trade anonymity. For this reason, it is difficult to isolate the effects of supply side anonymity on measures of market liquidity for these stocks. Fortunately, counterparty IDs' have always been concealed for stocks in Continu A. Consequently, our empirical analysis focuses only on stocks that belong to Continu A group (which include CAC40 stocks).

6.1.2 The Dataset

The data are provided by Euronext Paris. Our dataset contains a time stamped record of all transactions and orders (price and quantities) submitted to the market from March 1 to May 30, 2001. As explained previously we focus on stocks in Continu A group. We will distinguish between two types of stocks: (i) the constituent stocks of the CAC 40 index and (ii) the remaining stocks in Continu A (which are not part of the index). We refer to the first subsample as the CAC40 subsample and to the second subsample as the "restricted Continu A" subsample.

In order to avoid contamination of our findings due to the proximity of the event date, we drop two weeks of observations around April 23, 2001. Finally we drop all observations after May 20, 2001 in order to avoid confounding effects due to the adoption of NSC by Euronext Brussels.[23] Eventually our data set contains 129 stocks and 28 trading days: (i) 14 trading days before the event from March 26 to April 12, 2001 and (ii) 14 trading days after the event from April 30 to May 20, 2001.[24] We conduct our experiment on this dataset.

Additional but minor changes in trading rules took place for the stocks in our sample on April 23, 2001. Firstly, the Bourse changed some of the criteria which are used to select the opening price when there is a multiplicity of clearing prices at the opening. Secondly, it advanced by 5 minutes the end of the continuous trading session in order to facilitate the organization of the call auction determining the closing price. In our empirical analysis, we exclude observations collected during the first and the last 5 minutes of the continuous trading period. Thus our findings should not be contaminated by changes which affect the determination of opening and closing prices.

The Bourse also changed the treatment of orders which can trigger a trading halt. In Euronext Paris (as in many other exchanges) trading halts occur when price changes exceed pre-specified thresholds. Before April 23, 2001 traders had the possibility to submit marketable limit orders resulting in a halt without partial execution of their order. Thus traders could suspend the trading process without bearing any direct cost. In contrast, since April 23, 2001 marketable limit orders triggering a halt are partially executed up to the threshold price. This change in the handling of trading halts applied to all stocks. Hence there is no obvious way to control for its possible effects.

Table 3 presents some summary statistics (number of trades, average price, trading volumes, average trade sizes, daily return volatility and market capitalization) for the whole sample (Panel A) as well as for the subsample of CAC 40 stocks (Panel B) and the subsample of Continu A stocks, excluding CAC40 stocks (Panel C). Separate figures are given for the pre-event period (March 26 to April 12) and the post-event period (April 30 to May 20). We further report t-values for a test for the equality of means and z-values for a Wilcoxon test for equality of medians.

---

[23] Arguably, this switch facilitated the access of Belgian traders to the French market. Thus, it may have increased the number of participants to Euronext Paris.

[24] In the CAC40 subsample, we drop one stock which was withdrawn from the index during our sample period. Thus the CAC40 subsample contains 39 stocks.

The figures reveal a high level of trading activity for the stocks in our sample. The average daily number of transactions per stock is in the range of 2300 for the CAC 40 subsample and in the range of 330 for the restricted Continu A subsample. The number of transactions is slightly lower in the post-event period. On the other hand, the trading volume (in number of shares and in Euro) is higher in the post-event period. None of the differences are significant, however. Return volatility, defined as the standard deviation of 30 minute midquote returns, is significantly lower in the post-event period. Thus, in our empirical analysis we will have to control for the possible effect of lower volatility on spread and depth.

6.2   Empirical Findings

6.2.1   Univariate Analysis

Our first testable hypothesis is that the switch to pre-trade anonymity should affect the size of the bid-ask spread. To test whether this is the case we proceed as follows. We first calculate an average spread for each stock and each trading day. Then we average over the 14 days of the pre-event period and the 14 days of the post-event period. This results in two observations for each stock, one pre-event observation and one post-event observation. Finally, we average over the sample stocks.

We use three measures of the bid-ask spread, namely, the quoted spread in Euro, the quoted percentage spread, and the effective spread.[25] When estimating the average quoted spread we use two weighting schemes. The first gives each observation equal weight. The second assigns each observation a weight that corresponds to the time span during which the respective spread was valid. We thus have a total of five metrics for the effect of the switch to anonymity on the bid-ask spread.

The results are shown in Table 4. We first observe that the different weighting schemes do not materially affect the spread estimates. Spreads for the CAC 40 subsample are markedly lower than those for the restricted Continu A subsample. Most importantly, spreads in the post-event period are lower than those in the pre-event period. This holds for all three samples, and

---

[25]In order to compute the quoted spread, we collect the value of the bid-ask spread each time there is a change in the size of the inside spread or in the quantities offered at the best quotes. For the effective spread, we collect the value of the bid-ask spread 5 seconds before each transaction.

irrespective of the spread measure used. For instance, for the full sample, the quoted spread in Euro has decreased by 0.15 euros on average (32% of the average quoted spread in the pre-event period). We apply a t-test to investigate whether the reduction in spread is significant. The test statistics, also shown in Table 2, indicate that the reduction is significant for the full sample and for the restricted Continu A subsample. When considering the CAC 40 subsample, we find that the reduction is significant for the percentage quoted spread but not for the quoted spread in Euro and the effective spread. One possible explanation is that the tick size is frequently binding for CAC40 stocks. Besides the t-test we use a Wilcoxon test for equality of the medians. The resulting z-values are very similar to the t-values reported in Table 3 and are, therefore, omitted from the table.

Our second testable hypothesis is that a switch to anonymity should affect the number of shares offered at the best quotes (the "quoted depth"). Thus, in a second step, we analyze whether the quoted depth is significantly different after the switch to anonymity. We proceed as described above. We first calculate the average depth per stock and per trading day, then average over the 14 days of the pre- and the post-event period and finally aggregate over the sample stocks. Depth is measured in Euro, not in number of shares. We further apply the two weighting schemes described previously. Table 5 reports separate results for the depth at the best bid, the depth at the best ask and the average depth at the best bid and ask prices.

The results indicate that the depth at the best quotes is larger in the post-event period. This is true for all samples, and irrespective of the depth measure used. However, the change in depth is generally not statistically significant, although some t-values indicate significance at the 10% level.

Our third hypothesis is that the variability of the quoted spread should be affected by the switch to supply side anonymity. To test this prediction, we use two measures for the variability of the spreads. Our first measure is simply the standard deviation of the quoted spread. As the mean quoted spread has changed, the standard deviation may not be appropriate, however. Our second measure is therefore the coefficient of variation, the ratio of the standard deviation to the mean. Results are reported in Table 6. All measures indicate that the variability of the spreads is significantly smaller in the post-event period.

This finding does not support the collusion hypothesis: a switch to pre-trade anonymity reduces the bid-ask spread because it hinders collusion. In a collusive environment, liquidity

suppliers coordinate (implicitly or explicitly) on given spreads levels and do not undercut each other. Accordingly the "collusion hypothesis" predicts that the variability of the spread should be smaller in the non-anonymous environment.

6.3 Multivariate Analysis

The changes in spreads and depth documented in the preceding section may be caused by variables we have not controlled for. In particular, Table 1 reveals that volatility is systematically lower in the post-event period. We use a regression framework to analyze whether the switch to anonymity affects spreads and depth once we control for variables which are known to affect market liquidity.

Numerous empirical studies find that spreads depend on trading volume, the price level, and return volatility (see Stoll (2000)). We therefore include the log of the trading volume (in euro), the average price level and the standard deviation calculated from 30-minute midquote returns as control variables. As noted previously, the minimum tick size is a function of the price level of the stock. As the tick size potentially affects the size of the spread, we include the effective average tick size for stock i as explanatory variable. It is defined as

$$TS_{i,j} = \frac{1}{n} \sum_{j=1}^{n} \left[\frac{TS(A_j) + TS(B_j)}{2} \right] \qquad (22)$$

where $A_j[B_j]$ denotes the jth ask (bid) price ($j = 1, ..., n$) observed on day t and $TS(.)$ denotes the minimum tick size associated with the ask and bid price, respectively.[26] To sum up, the regression model is

$$s_{i,t} = \gamma_0 + \gamma_1 \log(Vol_{i,t}) + \gamma_2 TS_{i,t} + \gamma_3 P_{i,t} + \gamma_4 \sigma_{i,t} + \gamma_5 D + \varepsilon_{i,t} \qquad (23)$$

where D is a dummy variable which captures the effect of supply-side anonymity on the bid-ask spread (it takes on the value 1 for the observations in the anonymous regime). All variables are first calculated for each stock and each day and are then aggregated over the 14 days of the pre- and the post-event period. We thus have two observations for each stock, one pre-event and one post-event.

---

[26]The tick size for a given stock is not constant over time in our sample since it is a function of the stock price. Thus it changes whenever a stock's bid and ask prices rise above, or fall below, one of the price thresholds which determine the tick size on which the stock trades. Furthermore, the tick size can be different on each side of the book if the ask and the bid price are respectively above and below a threshold price. .

We estimate separate regressions for the five spread measures described above. The results are reported in Table 7. The independent variables explain a large part of the variation in bid-ask spreads, as evidenced by R^2s ranging from 0.54 to 0.94. All spread measures are negatively related to volume and are positively related to volatility. Quoted spreads measured in Euros and effective spreads are positively related to the price level. For the CAC 40 stocks, we find a significant positive relation between spreads and effective tick size. This supports the conjecture that, for these stocks, the tick size may often been binding for the inside spread.

We now turn our attention to the effect of the post-event dummy. The coefficient on this variable is negative in each case, indicating that spreads are lower after the switch to anonymity. In 12 out of 15 cases the reduction in spreads is statistically significant. For the CAC40 subsample and the restricted Continu A subsample, the switch to anonymity reduces the quoted spread by 0.01 euro and 0.09 euros, respectively. Overall, the multivariate analysis confirms the univariate results. The switch to anonymity is associated with lower quoted spreads.

We run a similar regression for the quoted depth. The set of explanatory variables is the same as in the spread regression. Glosten (1994)'s model predicts that the quoted depth should increase with the tick size and decrease with volatility. Results are shown in Table 8. The explanatory power of the regression is high for the CAC 40 subsample ($R^2 = 0.8$) but is far lower for the restricted Continu A subsample ($R^2 = 0.1$). As expected, the quoted depth is positively related to the tick size and negatively related to volatility. However, the relationship is statistically significant only for the CAC 40 subsample.

Contrary to the univariate results, we do not find that depth is higher in the post-event period. The coefficient on the post-event dummy is always negative. But it is statistically significant only for the CAC 40 subsample. The difference with the univariate results can be ascribed to the smaller volatility in the post event period. The smaller volatility leads to larger depth on average. Once we control for the effect of volatility, the switch to anonymity appears to have reduced the quoted depth. Notice that a switch to anonymity can simultaneously reduce the average quoted spread and the average quoted depth when $\overline{\beta} < \beta < \beta^{**}$ in our model.

In their empirical study of the Paris Bourse, Biais et al (1995) find that 43% of the orders get immediate execution. Among these orders, 11% are large marketable orders (their execution consume liquidity behind the best quotes). Our empirical findings suggest that the switch to anonymity has reduced trading costs for small orders but not necessarily for large marketable

orders. We cannot investigate empirically this question since we do not have data on the quantities offered behind the best quotes.

7 Conclusions

We have analyzed the effect of concealing limit order traders' identities on market liquidity. In our theoretical model limit order traders have asymmetric information on the likelihood of an information event (which determines the cost of liquidity provision). Informed dealers know whether an information event is about to occur or not. Of course they bid more conservatively in the former case. Thus the state of the book contains information on the likelihood of an information event. In particular a thin book signals that the cost of liquidity provision is large (because an information event is about to occur) and thereby it reduces uninformed dealers' inclination to improve upon posted offers. This effect induces informed dealers to employ bluffing strategies: sometimes, they bid as if the cost of liquity provision were large when indeed it is small.

We show that these strategic interactions imply that information on limit order traders' IDs affects market liquidity. When these IDs are concealed, it is more difficult for uninformed dealers to draw inferences from the book since they do not perfectly observe informative limit orders (those place by informed dealers). Hence bluffing strategies are less effective at deterring uninformed dealers from entering new orders within the quotes. For this reason, informed limit order traders bid more aggressively in the anonymous market. The impact of anonymity on uninformed dealers' aggressiveness is ambiguous: they may bid more or less aggressively in the anonymous market. Hence, depending on parameter values, a switch to anonymity can result in smaller or larger bid-ask spread on average. The impact of anonymity on quoted depth is ambiguous as well. We also find cases in which a switch to anonymity improves the bid-ask spread but impairs the quoted depth.

Thus a switch to anonymity has an impact on market liquidity but the direction of the impact is an empirical question. On April 23, 2001, the limit order book for stocks listed on Euronext Paris became anonymous. We compare quoted spreads and quoted depth before and after this event for a sample of 129 stocks. This natural experiment indicates that quoted spreads are significantly smaller and less variable in the anonymous market. We also find a decrease in

quoted depth after the switch to anonymity, once we control for the effect of volatility on this variable. This decrease is significant only for CAC40 stocks, however.

In our model, the limit order book contains information on the likelihood and/or the magnitude of future price changes. This suggests that the state of the limit order book may be used to forecast price volatility. More specifically steep limit order books should foreshadow large price movements. Extracting information on future price volatility from the limit orders placed in the book is an interesting venue for research. This might be of interest, for instance, for option traders.

References

[1] Albanesi, S. and Rindi, B. (2000):"The Quality of the Italian Treasury Bond Market, Asymmetric Information and Transaction Costs", *Annales d'Economie et Statistique*, 60, 1-19.

[2] Anand, A. and Martell, T. (2001):"Informed Limit Order Trading", mimeo, Syracuse University.

[3] Baruch, S. (2002): "Who Benefits from an Open Limit Order Book", Working Paper, University of Utah.

[4] Benveniste, L., Marcus, A. and Wilhelm, W. (1992):"What 's Special about the Specialist?", *Journal of Financial Economics*, 32, 61-86.

[5] Biais, B. Hillion, P. and Spatt, C. (1995): "An Empirical Analysis of the Limit Order Book and the Order Flow in the Paris Bourse", *Journal of Finance,* **50**, 1655-1689.

[6] Blume, M. and Goldstein, M. (1997):"Quotes, Order Flow, and Price Discovery", *Journal of Finance*, **52**, 221-244.

[7] Boehmer, E. Saar, G. and Yu, L. (2003):"Lifting the Veil: An Analysis of Pre-Trade Transparency at the NYSE", mimeo, NYSE and NYU.

[8] Calcagno, and Lovo, S. (2001): "Market Efficiency and Price Formation when Dealers are Asymmetrically Informed", Working Paper, HEC School of Management.

[9] Declerk, F. (2001): "Dual Trading and Order-Driven Market Liquidity: An Empirical Analysis of Euronext Members' Principal Trading", Working Paper, Toulouse University.

[10] Easley, D. and O'Hara, M. (1992): "Time and the Process of Security Price Adjustment", *Journal of Finance*, **57**,577-605.

[11] Foucault, T., Roëll, A. and Sandas, P. (2002): "Market-Making with Costly Monitoring: An Analysis of the SOES Controversy", *Review of Financial Studies,* **16**, *345-384.*

[12] Foucault, T., Kandel, E. and Kadan, O. (2003): "Limit Order Book as a Market for Liquidity", Working Paper, CEPR.

[13] Forster, M. and Georges, T. (1992): "Anonymity in Securities Market", *Journal of Financial Intermediation*, **2**, 168-206.

[14] Fudenberg, D. and Tirole, J. (1991), *Game Theory*, MIT Press.

[15] Garfinkel, J. and Nimalendran, N. (2002):"Market Structure and Trader Anonymity: An Analysis of Insider Trading", forthcoming *Journal of Financial and Quantitative Analysis*.

[16] Glosten, L. (1994):"Is the Electronic Limit Order Book Inevitable", *Journal of Finance*, 49, 1127-1161.

[17] Harrington, J. (1987):"Oligopolistic Entry Deterrence under Incomplete Information", *Rand Journal of Economics*, 18, 211-231.

[18] Harris, L. and Hasbrouk, J. (1996):"Market vs. limit Orders: the SuperDot Evidence on Order Submission Strategy", *Journal of Financial and Quantitative Analysis,* **31**, 213-231.

[19] Hellwig, M. (1980):"On the Aggregation of Information in Competitive Markets", *Journal of Economic Theory*, **22**, 477-498.

[20] Kalay, A. and Wohl, A. (2002):"The Information Content of the Demand and Supply Schedules of Stocks", mimeo, Tel Aviv University.

[21] Kyle, A. (1989):"Informed Speculation with Imperfect Competition", *Review of Economic Studies*, **56**, 317-356.

[22] Lee, C., Mucklow, B. and Ready, M. (1993):"Spreads, Depths and the Impact of Earnings Information: An Intraday Analysis", *Review of Financial Studies*, **6**, 345-374.

[23] Madhavan, A. and Cheng, M. (1997): "In search of liquidity: block trades in the upstairs and downstairs markets", *Review of Financial Studies*, **10**, 175–203.

[24] Madhavan, A. and Penchapagesan, V. (2000):"Price Discovery in Auction Markets: a Look Inside the Black Box", *Review of Financial Studies*, **13**, 627–658.

[25] Madhavan, A., Porter, D. and Weaver, D. (2002): "Should Securities Markets be Transparent", Working Paper, Baruch College.

[26] Milgrom, P. and Roberts, J. (1982):"Limit Pricing and Entry under Incomplete Information: an Equilibrium Analysis", *Econometrica*, 50, 443-459.

[27] O'Hara, M. (1995), *Market Microstructure* Theory, Blackwell.

[28] Rindi, B. (2002), "Transparency, Liquidity and Price Formation", Working Paper, Bocconi University.

[29] Roth, A. and Ockenfels, A. (2002), "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet", forthcoming *American Economic Review.*

[30] Sandås, P., (2000), "Adverse Selection and Competitive Market Making: Evidence from A Limit Order Market", *Review of Financial Studies,* **14**, 705-734.

[31] Seppi, D. (1990):"Equilibrium block trading and asymmetric information", *Journal of Finance*, **45**, pp 73–94.

[32] Seppi, D. (1997):"Liquidity-Based Competition for Order Flow", *Review of Financial Studies*, 11, 789-816.

[33] Simaan, Y., Weaver, D. and Whitcomb, D. (2003): "Market Maker Quotation Behavior and Pre-Trade Transparency", Working Paper, Baruch College.

[34] Stoll, H. (2000):"Friction", *Journal of Finance*, 1479-1514.

[35] Theissen, E. (2001): "Trader anonymity, price formation and liquidity", *European Finance Review*, **7**, 1-26.

[36] Waisburd, A. (2003):"Anonymity and Liquidity: Evidence from the Paris Bourse", mimeo, Texas Christian University.

$\widetilde{V}_2$	Final value of the security at Date 2
ϵ_1	Innovation at date 1
v_0	Unconditional expected value of the security
α	Probability of order submission by a speculator if information event
q	Size of 1 round lot
π_0	Prior probability of an information event
σ	Size of an innovation
β	Probability that the leader is a precommited trader
Δ	Tick size
A_j	j^{th} ask price on the grid above the unconditional expected value
K	State of the book at the end of the first stage
Φ_K	Probability that the state of the book is K if the leader is a pre-commited trader
Q_1	Depth of the book at price A_1
Q_s	Size of the market order submitted by a speculator
Q_l	Size of the market order submitted by a liquidity trader
π_K	Follower's belief about the occurence of an information event
Ψ	Indicator variable (0 if there is no information event; 1 otherwise)

8   Appendix

Preliminary Remarks. Let $\Pi^F(n, K)$ be the follower's expected profit if she offers n round lots at price A_1 conditional on the state of the book being K at the end of stage L and conditional on the arrival of a buy order at date 1 (this is the expected profit on a sell limit order). Obviously $\Pi^F(0, K) = 0$. Furthermore, we have

$$\Pi^F(2, T) = [\pi_T[2\alpha(A_1 - (v_0 + \sigma)) + \frac{3}{2}(1 - \alpha)(A_1 - v_0)] + \frac{3}{2}(1 - \pi_T)(A_1 - v_0)],$$

which rewrites (using the expressions for $E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1)$ and $E_{\pi_T}(V \mid \widetilde{Q}(2) \geq 2)$ given in Equations (6) and (7)):

$$\Pi^F(2, T) = A_1 - E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1) + \Pr(\widetilde{Q}(2) \geq 2 \mid K = T)(A_1 - E_{\pi_T}(V \mid \widetilde{Q}(2) \geq 2)),$$
$$(24)$$

where $\Pr(\widetilde{Q}(2) \geq 2 \mid K = T) = \frac{\alpha\pi_T + 1}{2}$. Using the same type of reasoning we also obtain:

$$\Pi^F(1, T) = A_1 - E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1) \tag{25}$$

and

$$\Pi^F(1, S) = \Pr(\widetilde{Q}(2) \geq 2 \mid K = S)(A_1 - E_{\pi_S}(V \mid \widetilde{Q}(2) \geq 2)). \tag{26}$$

These expressions will be used in the proofs below.

Proof of Lemma 1. The proof follows directly from the arguments in the text. The reader can also check the claim by using the follower's expected profits given in Equations (24), (25) and (26).■

Proof of Proposition 1. It follows from the argument before the proposition.■

Proof of Proposition 2. We denote by $\Pi^L(K)$, the leader's expected profit if he posts schedule K conditional on the arrival of a buy order at date 1. The follower's reaction is given in Lemma 1 for $\pi_S = \pi_T = 0.5$ (since dealers have symmetric information). It follows that the book at the end of the bidding stage will be deep (since $\frac{2\alpha\sigma}{\alpha+2} < \Delta$). *Given the follower's reaction*, we deduce that

$$\Pi^L(T) = 0,$$

$$\Pi^L(S) = \pi_0[\alpha(A_1 - (v_0 + \sigma)) + (1 - \alpha)(A_1 - v_0)] + (1 - \pi_0)(A_1 - v_0) = A_1 - E_{\pi_0}(V \mid \widetilde{Q}(1) \geq 1),$$

$$\Pi^L(D) = \pi_0[2\alpha(A_1 - (v_0 + \sigma)) + \frac{3}{2}(1 - \alpha)(A_1 - v_0)] + \frac{3}{2}(1 - \pi_0)(A_1 - v_0),$$

which rewrites (using the expressions for $E_{\pi_0}(V \mid \widetilde{Q}(1) \geq 1)$ and $E_{\pi_0}(V \mid \widetilde{Q}(1) \geq 2)$):

$$\Pi^L(D) = A_1 - E_{\pi_0}(V \mid \widetilde{Q}(1) \geq 1) + \Pr(\widetilde{Q}(2) \geq 2)(A_1 - E_{\pi_0}(V \mid \widetilde{Q}(2) \geq 2)),$$

where $\Pr(\widetilde{Q}(2) \geq 2) = \frac{\alpha\pi_0 + 1}{2}$ is the probability that a buy order at date 2 is larger than 2 round lots (when 2 round lots are offered at price A_1). Recall that

$$E_{\pi_0}(V \mid \widetilde{Q}(2) \geq 2) = v_0 + \frac{2\pi_0\alpha\sigma}{\pi_0\alpha + 1} = v_0 + \frac{2\alpha\sigma}{\alpha + 2} < v_0 + \Delta,$$

and that $E_{\pi_0}(V \mid \widetilde{Q}(1) \geq 1) < E_{\pi_0}(V \mid \widetilde{Q}(2) \geq 2)$. Therefore we conclude that

$$E_{\pi_0}(V \mid \widetilde{Q}(1) \geq 1) < E_{\pi_0}(V \mid \widetilde{Q}(2) \geq 2) < A_1.$$

It immediately follows that

$$\Pi^L(T) < \Pi^L(S) < \Pi^L(D),$$

which proves that the dealer acting in stage L chooses schedule D.∎

Proof of Proposition 3.

Step 1. We show that the follower's bidding strategy is a best response to the informed dealer's bidding strategy. First consider the case in which the book is thin at the end of the first stage. The follower's expected profit if she submits a limit order for 1 round lot at price A_1 is (see Eq. (25) in the preliminary remarks):

$$\Pi^F(1, T) = A_1 - E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1),$$

that is (using Equation (6)):

$$\Pi^F(1, T) = \Delta - \alpha\pi_T(m^*, \beta)\sigma. \tag{27}$$

Substituting $m^*(\beta)$ by its expression in $\pi_T(m^*, \beta)$ (given by Eq.(12)) and then substituting $\pi_T(m^*, \beta)$ in Equation (27), we find that

$$\Pi^F(1, T) = \Delta - \alpha\pi_T(m^*, \beta)\sigma = 0.$$

Furthermore Equation (24) yields

$$\Pi^F(2, T) = A_1 - E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1) + \Pr(\widetilde{Q}(2) \geq 2 \mid K = T)(A_1 - E_{\pi_T}(V \mid \widetilde{Q}(2) \geq 2)).$$

As $A_1 = E_{\pi_T}(V \mid \widetilde{Q}(1) \geq 1)$ (as we just have shown) and since $E_{\pi_T}(V \mid \widetilde{Q} \geq 2)) > E_{\pi_T}(V \mid \widetilde{Q} \geq 1)$, we deduce that $\Pi^F(2, T) < 0$. Hence we have shown that:

$$\Pi^F(1, T) = \Pi^F(0, T) > \Pi^F(2, T).$$

Thus, when she observes a thin book, the follower's optimal reaction is either to submit a limit order for 1 round lot or to do nothing. As she is indifferent, the mixed strategy given in the proposition is a best response for the follower. In equilibrium, the informed dealer never chooses a shallow book (whether $\Psi = 1$ or not). Thus when she observes a shallow book, the follower does not update her beliefs and behaves as in the benchmark case.[27] These arguments establish the second part of the proposition.

Step 2. We show that the informed dealer's bidding strategy is a best response. We denote by $\Pi^L(K, \Psi)$, the leader's expected profit in state Ψ if he posts schedule K conditional on the arrival of a buy order at date 1. When $\Psi = 0$, straightforward computations yield (taking into account the follower's reaction):

$$\Pi^L(T, 0) = (1 - u_T^*)(A_2 - v_0)E(\widetilde{Q}_u) + \frac{u_T^*}{2}(A_2 - v_0) = (1 - u_T^*)(A_2 - v_0)\frac{3}{2} + \frac{u_T^*}{2}(A_2 - v_0).$$

and

$$\Pi^L(S, 0) = A_1 - v_0,$$

and

$$\Pi^L(D, 0) = E(\widetilde{Q}_u)(A_1 - v_0) = \frac{3}{2}(A_1 - v_0).$$

Using the fact that $u_T^* = \frac{3}{4}$, we obtain

$$\Pi^L(D, 0) = \Pi^L(T, 0) > \Pi^L(S, 0).$$

Thus when $\Psi = 0$, the leader optimally chooses schedule D or schedule T. As she is indifferent between these two schedules, choosing schedule D with probability $m^*(\beta)$ and schedule T with probability $(1 - m^*(\beta))$ is a best response. Notice that $m^*(\beta) < 1$ if $\beta < \beta^*$.

---

[27]The informed dealer never chooses a shallow book. Thus when $\beta = 0$, the probability of observing a shallow book at the end of the first stage of the bidding stage is zero. The follower's posterior belief after observing a shallow book cannot be computed by bayes rule in this case. In this case (see remark at the end of Section 4.2), we assume that the follower's belief on the occurence of an information event is given by her prior belief. This guarantees continuity with respect to β of the follower's posterior belief conditional on observing a shallow book.

Now we consider the informed dealer's optimal reaction when $\Psi = 1$. Given the follower's reaction and the informed trader's behavior, we deduce that:

$$\Pi^L(T, 1) = (1 - \alpha)[(1 - u_T^*)(A_2 - v_0)\frac{3}{2} + \frac{u_T^*}{2}(A_2 - v_0)] > 0.$$

and

$$\Pi^L(S, 1) = A_1 - E_1(V \mid \widetilde{Q}(1) \geq 1)$$

and

$$\Pi^L(D, 1) = A_1 - E_1(V \mid \widetilde{Q}(1) \geq 1) + \Pr(\widetilde{Q}(2) \geq 2)(A_1 - E_1(V \mid \widetilde{Q}(2) \geq 2))$$

Using Eq.(6) and (7), we obtain $E_1(V \mid \widetilde{Q}(1) \geq 1) = v_0 + \alpha\sigma$ and $E_1(V \mid \widetilde{Q}(2) \geq 2) = v_0 + \frac{2\alpha\sigma}{\alpha+1}$. Hence when

$$\Delta \leq \alpha\sigma,$$

we have

$$\Pi^L(T, 1) \geq 0 > Max\{\Pi^L(S, 1), \Pi^L(D, 1)\}.$$

Thus when $\Psi = 1$, the leader optimally chooses schedule T.

Finally observe that there cases in which the book will be thin at the end of the bidding stage. This happens when (i) the informed dealer chooses a thin book and the follower does not undercut or (ii) a pre-commited trader establishes a thin book and the follower does not undercut. Thus there are cases in which large or small orders will execute at price A_2. In the benchmark case, all orders execute at price $A_1 < A_2$. This remark yields the last part of the proposition.∎

Proof of Proposition 4.

Part 1. We first show that the follower's bidding strategy of the follower is a best response. First consider the case in which the book is thin. The follower's expected profit if she submits a limit order for 1 round lot at price A_1 is (using Equation (27) in the proof of the previous proposition):

$$\Pi^F(1, T) = \Delta - \alpha\pi_T(1, \beta)\sigma \tag{28}$$

Given the informed dealer's bidding behavior, bayesian calculus yields:

$$\pi_T(1, \beta) = prob(\Psi = 1 \mid K = T) = \frac{\beta\Phi_T + (1 - \beta)}{2\beta\Phi_T + (1 - \beta)}.$$

It is then easily checked that

$$\Pi^F(1,T) = \Delta - \alpha\pi_T(1,\beta)\sigma = \Delta - \alpha[\frac{\beta\Phi_T + (1-\beta)}{2\beta\Phi_T + (1-\beta)}]\sigma > 0,$$

iff $\beta^* < \beta$. Furthermore the follower's expected profit if she submits a limit order for 2 round lots (given that the book is thin) can be written (see Equation (24)):

$$\Pi^F(2,T) = \Pi^F(1,T) + \Pr(\widetilde{Q}(2) \geq 2 \mid K = T)(A_1 - E_{\pi_T}(V \mid \widetilde{Q}(2) \geq 2)).$$

Recall that

$$A_1 - E_{\pi_T(1)}(V \mid \widetilde{Q}(2) \geq 2) = \Delta - (\frac{2\pi_T(1,\beta)}{\pi_T(1,\beta)\alpha + 1})\alpha\sigma.$$

It is easily checked that $\pi_T(1,\beta)$ is such that

$$A_1 - E_{\pi_T(1)}(V \mid \widetilde{Q}(2) \geq 2) \leq 0,$$

iff $\beta \leq \beta^{**}$. Thus the follower never submit a limit order for two round lots at price A_1 since she expects to lose money on the second round lot. Hence we have shown that the follower's best response when the book is thin is to submit a limit order for 1 round lot. In equilibrium, the informed dealer never chooses a shallow book (whether $\Psi = 1$ or not). Thus when she observes a shallow book, the follower does not update her beliefs and behaves as in the benchmark case. These arguments establish the second part of the proposition.

Part 2. Next we show that the informed dealer's bidding strategy is a best response. When $\Psi = 1$, the argument is identical to the argument developed in the previous proposition (with $u_T^* = 1$). When $\Psi = 0$, straight forward computations yield (taking into account the follower's reaction):

$$\Pi^L(T,0) = \frac{1}{2}(A_2 - v_0) = \Delta.$$

and

$$\Pi^L(S,0) = A_1 - v_0 = \Delta,$$

and

$$\Pi^L(D,0) = E(\widetilde{Q}_u)(A_1 - v_0) = \frac{3}{2}(A_1 - v_0) = \frac{3}{2}\Delta.$$

Thus the informed dealer's best response when there is no information event is to post schedule D.

Part 3. On the equilibrium path, there is at least 1 round lot offered at price A_1. Thus the small trade spread is $(A_1 - v_0)$ with certainty, as in the benchmark case. There are cases,

however, in which the book will be shallow at the end of the bidding stage (instead of deep in the benchmark case). This occurs when the leader (informed or not) chooses a thin book. In this case the follower submits a limit order for 1 round lot at price A_1 and at the end of the bidding stage the book is shallow. Thus there are cases in which the marginal execution price for large market orders is A_2. This implies that the large trade spread is greater than in the benchmark case, on average. ∎

Proof of Proposition 5

The proof is identical to the proof of Proposition 4. The only difference is that

$$A_1 - E_{\pi_T(1,\beta)}(V \mid \widetilde{Q}(2) \geq 2) = \Delta - (\frac{2\pi_T(1,\beta)}{\pi_T(1,\beta)\alpha + 1})\alpha\sigma > 0,$$

since $\beta > \beta^{**}$. It immediately follows that $\Pi^F(2,T) > \Pi^F(1,T)$. This means that the follower submits a limit order for 2 round lots when the book is thin. Notice that in this case, the book is deep with certainty at the end of the bidding stage, as in the benchmark case. This yields the last part of the proposition. ∎

Proof of Corollary 1. It follows immediately from the arguments in the text. ∎

Proof of Corollary 2.

In what follows, a superscript "a" (resp. "na") indexes the value of a variable in the anonymous (resp. non-anonymous) market.

Part 1. The Small Trade Spread. The expected small trade spread is given by:

$$ES^j_{small} = \Delta(1 + \Pr(Q^j_1 = 0)), \text{ for } j \in \{a, na\}.$$

We deduce that the difference between the expected small trade spread in the anonymous market and the expected small trade spread in the non-anonymous markets is:

$$ES^a_{small} - ES^{na}_{small} = \Delta(\Pr(\widetilde{Q}^a_1 = 0) - \Pr(\widetilde{Q}^{na}_1 = 0)).$$

When $\beta > \beta^*$, we have $\Pr(\widetilde{Q}^a_1 = 0) = 0$. This follows from Propositions 4 and 5. Furthermore we deduce from Corollary 1 that:

$$\Pr(\widetilde{Q}^{na}_1 = 0) = (1 - \beta)(\frac{1}{8} + \frac{1 - m^*(0)}{8}) > 0. \tag{29}$$

Thus for $\beta > \beta^*$, $ES^a_{small} - ES^{na}_{small} < 0$. When $\beta \leq \beta^*$, using the equilibrium bidding strategies described in Proposition 3, we obtain:

$$\Pr(\widetilde{Q}^a_1 = 0) = \beta(\frac{\Phi_T}{4}) + (1 - \beta)(\frac{1}{8} + \frac{1 - m^*(\beta)}{8}). \tag{30}$$

Thus

$$\Pr(\widetilde{Q}^a_1 = 0) - \Pr(\widetilde{Q}^{na}_1 = 0) = \beta(\frac{\Phi_T}{4}) + \frac{(1 - \beta)}{8}(m^*(0) - m^*(\beta)).$$

Using the expression for $m^*(\beta)$, we rewrite this equation:

$$\Pr(\widetilde{Q}^a_1 = 0) - \Pr(\widetilde{Q}^{na}_1 = 0) = \frac{\beta\Phi_T\alpha}{8r} > 0,$$

which means that $ES^a_{small} - ES^{na}_{small} > 0$ when $\beta \leq \beta^*$.

Part 2. The Large Trade Spread.

The expected large trade spread is given by

$$ES^j_{l\arg e} = \frac{\Delta}{2}(3 + \Pr(Q^j_1 = 0) - \Pr(Q^j_2 = 2)), \, for \, j \in \{a, na\}.$$

We deduce that the difference between the expected small trade spread in the anonymous market and the expected small trade spread in the non-anonymous markets is:

$$ES^a_{l\arg e} - ES^{na}_{l\arg e} = \frac{\Delta}{2}(\Pr(\widetilde{Q}^a_1 = 0) - \Pr(\widetilde{Q}^{na}_1 = 0) + \Pr(\widetilde{Q}^{na}_1 = 2) - \Pr(\widetilde{Q}^a_1 = 2)).$$

Using Corollary 1, we obtain

$$\Pr(\widetilde{Q}^{na}_1 = 2) = \beta + \frac{(1 - \beta)m^*(0)}{2} < 1.$$

When $\beta > \beta^{**}$, we have $\Pr(\widetilde{Q}^a_1 = 2) = 1$ (see Proposition 5). Furthermore, we have already shown that $\Pr(\widetilde{Q}^a_1 = 0) - \Pr(\widetilde{Q}^{na}_1 = 0) < 0$ in this case (see Part 1). Thus $S^a_{l\arg e} - S^{na}_{l\arg e} < 0$ for $\beta > \beta^{**}$.

For $0 \leq \beta < \beta^*$, we deduce from Proposition 5 that:

$$\Pr(\widetilde{Q}^a_1 = 2) = \beta(\Phi_S + \Phi_D) + \frac{(1 - \beta)m^*(\beta)}{2}.$$

Hence

$$\Pr(\widetilde{Q}^{na}_1 = 2) - \Pr(\widetilde{Q}^a_1 = 2) = \frac{(1 - \beta)}{2}(m^*(0) - m^*(\beta)) + \beta\Phi_T.$$

Using the expression for $m^*(.)$ and rearranging, we rewrite this equation:

$$\Pr(\widetilde{Q}_1^{na} = 2) - \Pr(\widetilde{Q}_1^a = 2) = \frac{\beta \Phi_T \alpha}{2r} > 0.$$

Since $\Pr(\widetilde{Q}_1^a = 0) - \Pr(\widetilde{Q}_1^{na} = 0) > 0$ (see Part 1), we deduce that $ES_{l \arg e}^a - ES_{l \arg e}^{na} > 0$ for $\beta < \beta^*$.

For $\beta^* < \beta \leq \beta^{**}$, we deduce from Proposition 4 that:

$$\Pr(\widetilde{Q}_1^a = 2) = \beta(\Phi_S + \Phi_D) + \frac{(1-\beta)}{2}.$$

Thus

$$\Pr(\widetilde{Q}_1^{na} = 2) - \Pr(\widetilde{Q}_1^a = 2) = \frac{(1-\beta)}{2}(m^*(0) - 1) + \beta \Phi_T.$$

Hence, using the expression for $\Pr(\widetilde{Q}_1^{na} = 0)$ given in Part 1, we obtain after some manipulations

$$ES_{l \arg e}^a - ES_{l \arg e}^{na} = \frac{\Delta}{2}(-\frac{(1-\beta)\alpha}{8r} + \frac{(1-\beta)}{2}(m^*(0) - 1) + \beta \Phi_T).$$

Substituting $m^*(0)$ by its expression, we obtain after some algebra that $ES_{l \arg e}^a - ES_{l \arg e}^{na} > 0$ iff

$$\beta > \overline{\beta},$$

where $\overline{\beta} = \frac{8(\alpha - r) + 2\alpha}{16r\Phi_T + 2\alpha + 8(\alpha - r)}$. Straight forward computations show that $\overline{\beta} < \beta^{**}$ and that $\overline{\beta} > \beta^*$ iff $\alpha \sigma > \frac{6\Delta}{5}$. These remarks yield the last part of the proposition.■

Proof of Corollary 3

The quoted spread at the end of the bidding stage is equal to (i) $A_1 - v_0 = \Delta$ or (ii) $A_2 - v_0 = 2\Delta$. The second event occurs with probability $\Pr(Q_1 = 0)$. Hence the variance of the quoted spread is:

$$Varspread = \Delta^2 \Pr(Q_1^j = 0)(1 - \Pr(Q_1^j = 0)), \quad \text{for } j \in \{a, na\},$$

Observe that the variance of the quoted spread increases with $\Pr(Q_1^j = 0)$ for $\Pr(Q_1^j = 0) < \frac{1}{2}$. Now, using the expressions for $\Pr(Q_1^a = 0)$ and $\Pr(Q_1^{na} = 0)$(see Equations (30) and (29)), we obtain:

$$\Pr(\widetilde{Q}_1^j = 0) < \frac{1}{2}.$$

Furthermore we know from the proof of Corollary2 that $\Pr(\widetilde{Q}_1^a = 0) < \Pr(\widetilde{Q}_1^{na} = 0)$ if and only if $\beta > \beta^*$. The corollary follows from these remarks.■

Figure 1

Date 1 : Tree Diagram of the Trading Process.
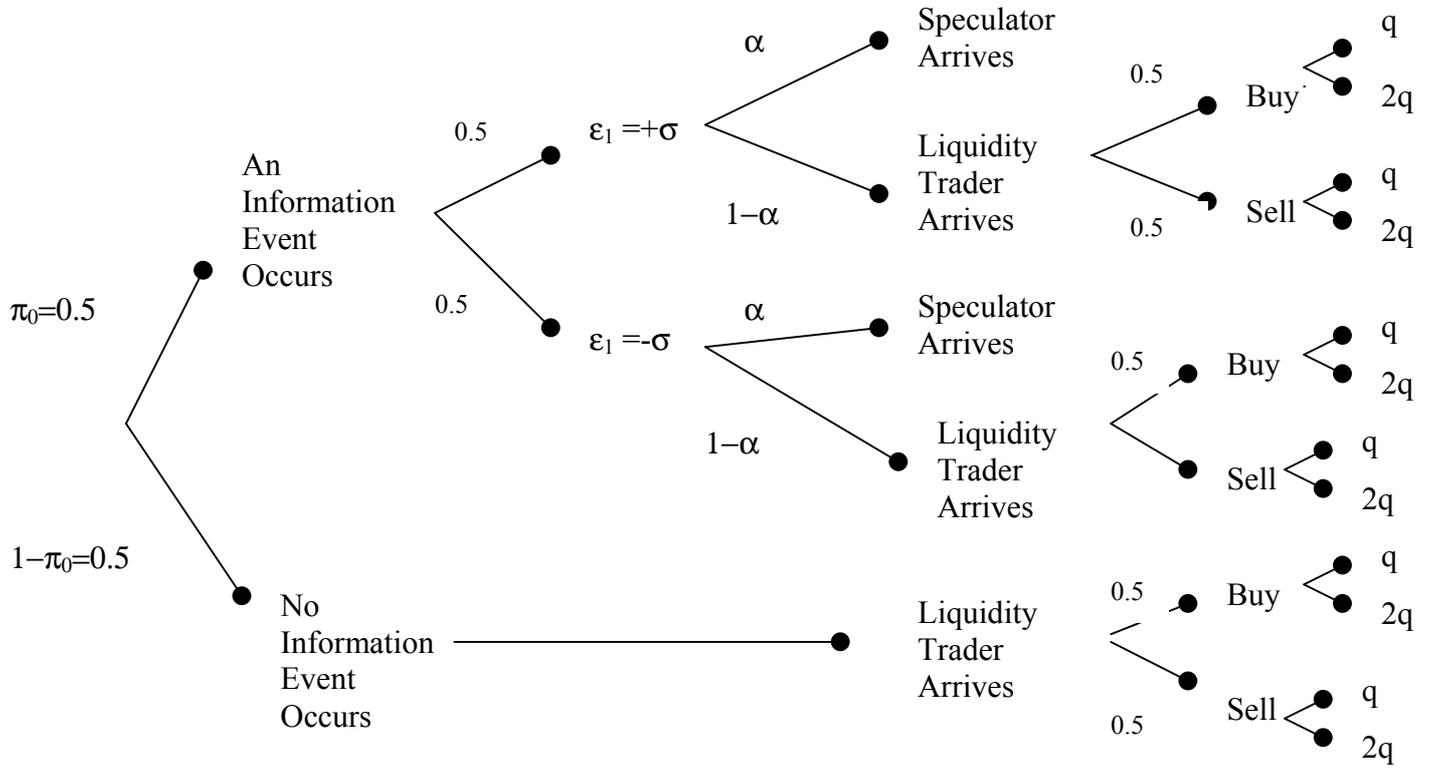
Figure 2 : Building the Limit Order Book

Stage L   **Stage F**

1-ß   Informed Dealer   ⟶   Uninformed Dealer

ß   Pre-Committed   ⟶   Uninformed Dealer
             Trader
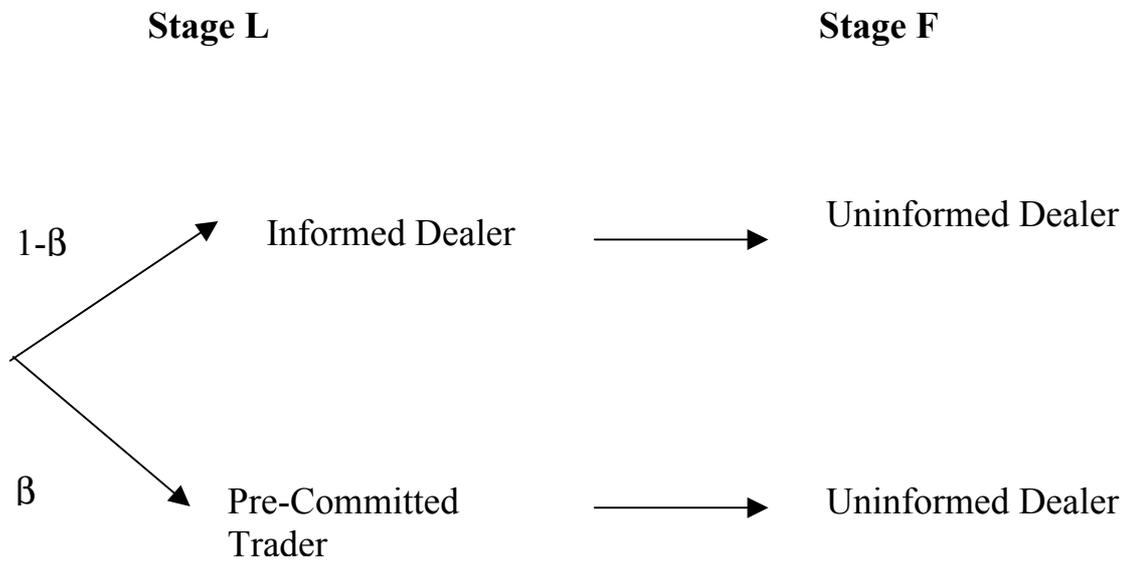
Figure 3 : Equilibrium when $\beta < \beta^*$

Perceived Cost of
Submitting a
Limit order for 1
round lot at price
A_1

$E_{\pi T}(V|Q\geq 1)=v_0+\alpha\pi_T(m,\beta)\sigma$

The curve shifts
to the right
when β
increases

$A_1=v_0+\Delta$

$v_0+\alpha\pi_0\sigma$

0

$m^*(\beta)$

1

m
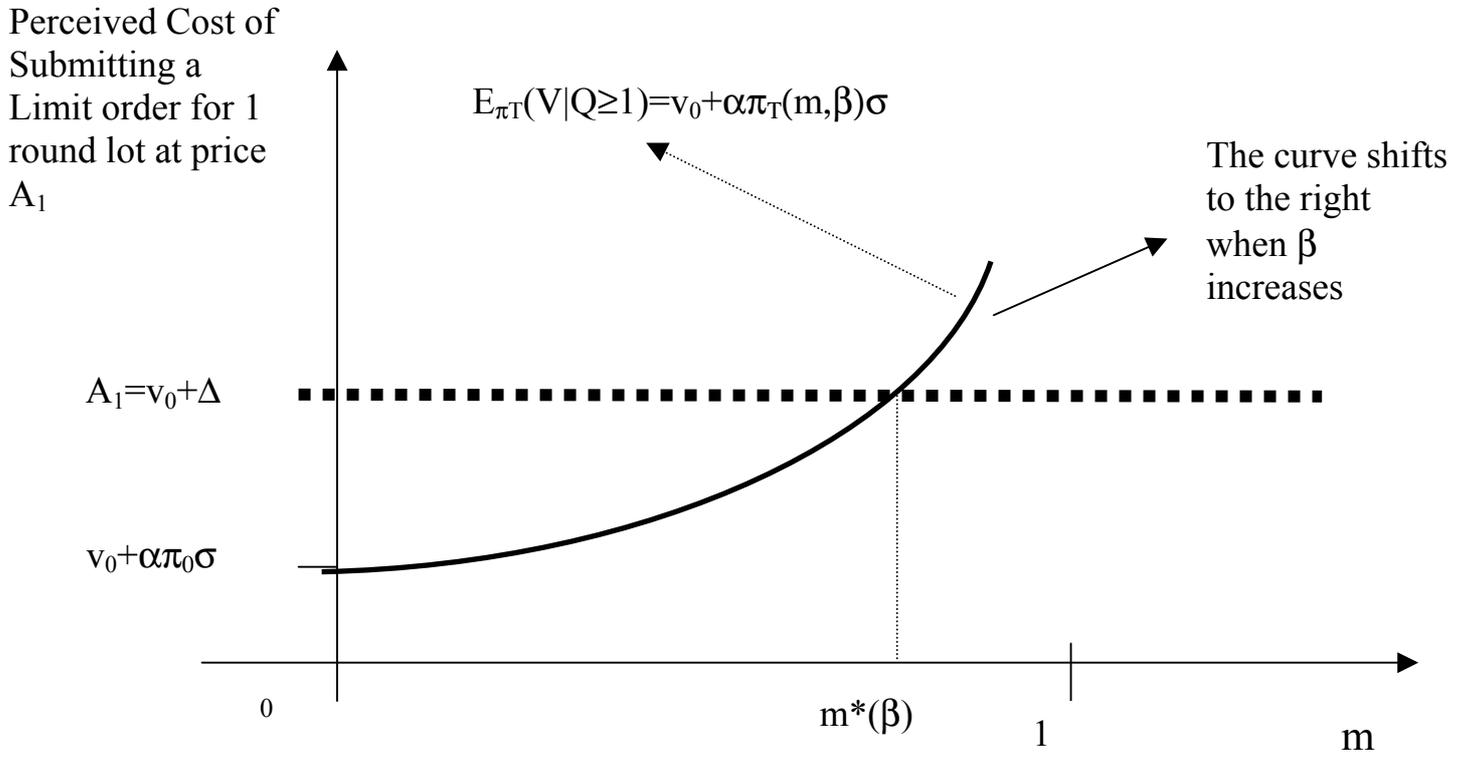
Figure 4 : Equilibrium when $\beta^* < \beta < \beta^{**}$

Perceived Costs
of Submitting a
Limit orders for 1
or 2 round lot at
price A_1

$E_{\pi T}(V|Q \geq 2) = v_0 + \alpha \pi_T(m,\beta)\sigma/(\alpha \pi_T(m,\beta)+1)$

The curve
shifts to the
right when β
increases

$A_1 = v_0 + \Delta$

$v_0 + \alpha \pi_0 \sigma/(\alpha \pi_0 + 1)$

$E_{\pi T}(V|O \geq 1)$

$v_0 + \alpha \pi_0 \sigma$

0

$m^*(\beta) = 1$

m
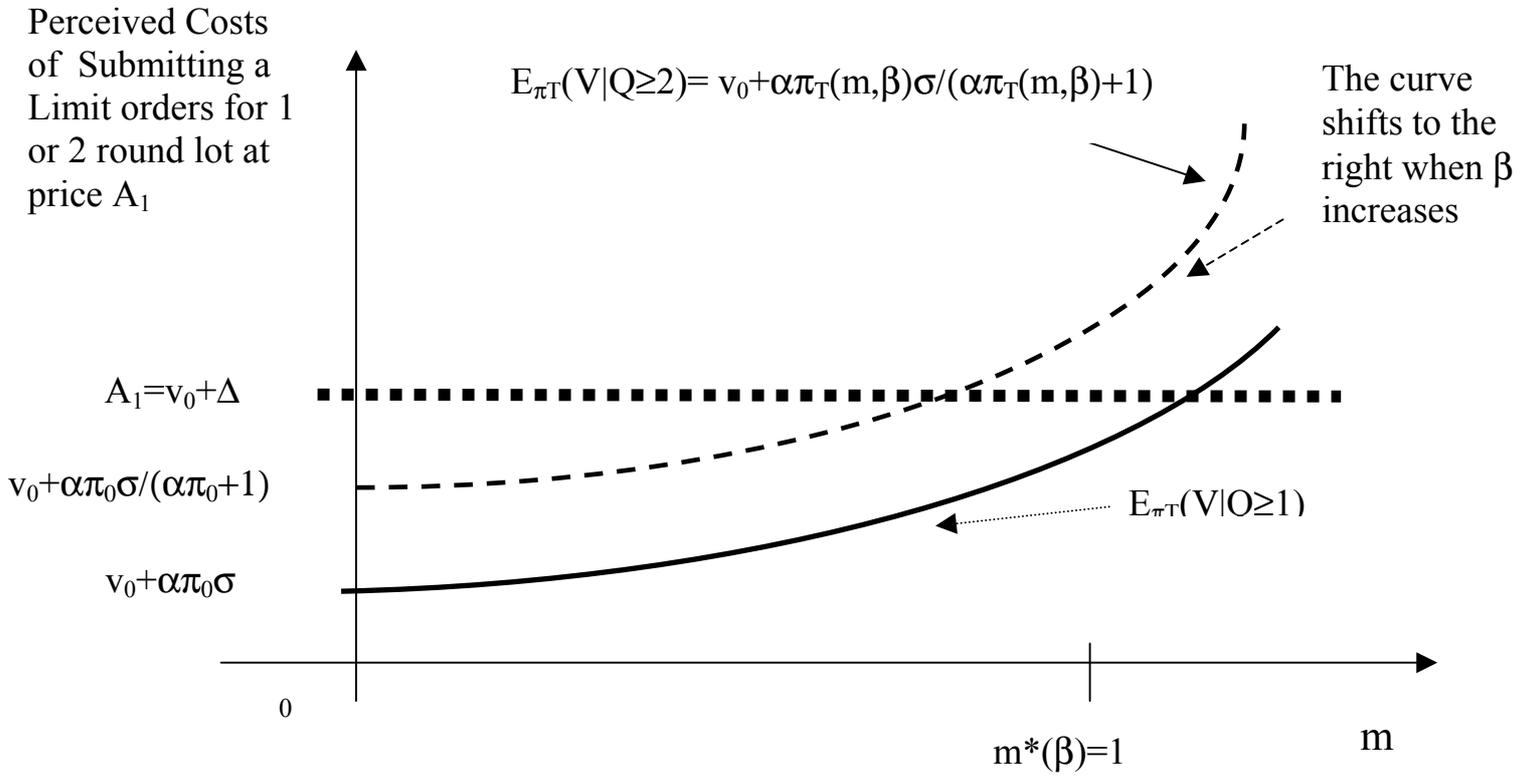
Table 3
Summary Statistics

The table reports averages for the variables listed in the first column. We first calculated averages for each stock and each day. Then, we average over the 14 days of the pre-event period and the post-event period, respectively. Volatility is measured by the standard deviation of 30 minute midquote returns. The last two columns report the test statistics (t-test and z-value of a Wilcoxon test) of the null hypothesis that the differences in means and medians, respectively, are zero.

Panel A: All stocks

	pre-event	post-event	t-value	z-value
Number of trades	937.82	924.99	0.07	0.77
Price	74.2	78.67	0.59	0.67
Trading volume (shares)	523011.6	562277.0	0.26	0.47
Trading volume (€ 1000s)	28252.5	33494.4	0.65	0.66
Average Trade Size (shares)	418.75	388.57	0.45	0.1
Volatility	0.00526	0.00377	6.61	6.05
Market Capitalization	9692.4	12069.3	0.86	0.54

Panel B: CAC40

	pre-event	post-event	t-value	z-value
Number of trades	2316.5	2286.1	0.07	0.57
Price	85.43	89.87	0.33	0.62
Trading volume (shares)	1323177	1433757	0.26	0.91
Trading volume (€ 1000s)	83390.5	98644.7	0.73	1.00
Average Trade Size (shares)	459.93	515.28	0.90	0.92
Volatility	0.00608	0.00443	5.76	4.90
Market Capitalization	26482.3	33857.4	0.99	0.63

Panel C: Restricted Continu A

	pre-event	post-event	t-value	z-value
Number of trades	333.7	328.5	0.07	1.20
Price	69.28	73.76	0.49	0.51
Trading volume (shares)	172377.2	180392.5	0.13	0.48
Trading volume (€ 1000s)	4090.9	4945.3	0.79	0.71
Average Trade Size (shares)	400.70	333.05	0.73	0.51
Volatility	0.00490	0.00348	4.96	4.48
Market Capitalization	2335.0	2521.7	0.50	0.67

Table 4
Quoted Spread

The table reports averages for the variables listed in the first column. We first calculated averages for each stock and each day. Then, we average over the 14 days of the pre-event period and the post-event period, respectively. Z-values for a test for equality of the medians are very similar to the t-values and are, therefore, omitted from the table.

	all stocks			CAC40			Restricted Continu A		
	pre-event	post-event	t-value	pre-event	post-event	t-value	pre-event	post-event	t-value
quoted spread €, equally-weighted	0.457	0.300	2.78	0.177	0.146	1.36	0.580	0.368	2.79
quoted spread €, time-weighted	0.433	0.294	2.61	0.177	0.147	1.27	0.545	0.358	2.60
quoted percentage spread, equally-weighted (in %)	0.676	0.416	3.98	0.220	0.171	3.67	0.876	0.524	4.25
quoted percentage spread, time-weighted (in %)	0.641	0.401	3.90	0.219	0.171	3.56	0.826	0.501	4.13
effective spread	0.369	0.259	2.33	0.138	0.119	1.06	0.470	0.320	2.35

Table 5
Quoted Depth

The table reports averages for the variables listed in the first column. We first calculated averages for each stock and each day. Then, we average over the 14 days of the pre-event period and the post-event period, respectively. Z-values for a test for equality of the medians are very similar to the t-values and are, therefore, omitted from the table.

	all stocks			CAC40			Restricted Continu A		
	pre-event	post-event	t-value	pre-event	post-event	t-value	pre-event	post-event	t-value
quoted depth at ask, €, equally-weighted	38968.2	49858.7	1.39	76879.3	94191.8	1.26	22355.4	30431.8	0.98
quoted depth at bid, €, equally-weighted	37977.2	59066.9	1.63	71473.2	92919.6	1.62	23299.2	44232.5	1.21
quoted depth average, €, equally-weighted	38472.7	54462.8	1.79	74176.3	93555.7	1.44	22827.3	37332.2	1.31
quoted depth at ask, €, time-weighted	37483.1	49117.1	1.49	72595.7	91942.5	1.46	22096.6	30350.9	0.98
quoted depth at bid, €, time-weighted	37205.5	57452.6	1.61	69576.9	90219.1	1.62	23020.2	43094.3	1.20
quoted depth average, €, time-weighted	37344.3	53284.9	1.76	71086.3	91080.8	1.54	22558.4	36722.8	1.29

Table 6
Variance of Quoted Spreads

The table reports the coefficients of variation and the standard deviation of the quoted spreads in the pre- and the post-event period. The third column reports the z statistic for a test for equality of the coefficients of variation. The last two columns report the test statistics for a test of equality of variances. The Brown-Forsythe test is more robust against deviations from normality than the F-test.

	coefficient of variation			standard deviation			
	pre-event	post-event	z-value	pre-event	post-event	F-value	Brown-Forsythe test statistic
quoted spread €, equally-weighted	1 367	1 118	8.41	0.622	0.339	3.38	61.63
quoted spread €, time-weighted	1 342	1 169	5.80	0.578	0.345	2.80	51.72
quoted percentage spread, equally-weighted	1 141	0.803	14.46	0.768	0.337	5.20	104.70
quoted percentage spread, time-weighted	1 128	0.799	14.21	0.720	0.322	5.01	100.31

Table 7
Regression Model for the Quoted Spread

The table reports regression results for the various spread measures denoted in line 1. Volume is measured in thousand of Euros. The tick size variable measures the average effective tick size. The tick size is 1 €-Cent (5 Cents, 10 Cents, 50 Cents) for stocks trading at prices below 50 € (between 50 and 100 €, between 100 and 500 €, above 500€). The effective tick size can take on intermediate values if a stock trades at prices in more than one tick size range. Volatility is measured by the standard deviation of 30-minute midquote returns. Numbers in italics are heteroscedasticity-consistent t-values.

Panel A: All stocks

	quoted spread in €, equally-weigted	quoted spread in €, time-weigted	percentage quoted spread, equally-weigted	percentage quoted spread, time-weigted	effective spread
Constant	0.843	0.813	1.733	1.666	0.751
	10.45	*10.73*	*13.10*	*13.72*	*10.43*
Log(volume)	-0.133	-0.126	-0.194	-0.182	-0.108
	6.48	*6.42*	*7.20*	*7.12*	*6.24*
Ticksize	0.780	0.703	0.809	0.778	0.457
	1.48	*1.41*	*1.46*	*1.49*	*0.88*
Price	0.0042	0.0041	-0.0001	-0.0002	0.0034
	5.33	*5.66*	*0.34*	*0.40*	*4.75*
Volatility	67.576	59.700	98.416	85.516	44.338
	2.65	*2.49*	*3.07*	*2.79*	*2.03*
Post-Event	-0.0552	-0.0499	-0.0832	-0.085	-0.043
	1.66	*1.56*	*2.29*	*2.42*	*1.54*
R2	0.60	0.62	0.56	0.55	0.54

Panel B: CAC40

	quoted spread in €, equally-weigted	quoted spread in €, time-weigted	percentage quoted spread, equally-weigted	percentage quoted spread, time-weigted	effective spread
Constant	0.316	0.330	0.702	0.734	0.187
	9.15	*8.44*	*17.11*	*16.55*	*6.66*
Log(volume)	-0.034	-0.0347	-0.052	-0.054	-0.022
	11.03	*10.547*	*14.64*	*14.53*	*8.89*
Ticksize	0.291	0.247	0.264	0.254	0.320
	2.90	*2.659*	*2.90*	*2.71*	*4.11*
Price	0.0015	0.0016	-0.0003	-0.0003	0.0011
	10.42	*12.45*	*2.64*	*2.38*	*9.87*
Volatility	12.326	10.830	13.943	11.196	10.146
	5.79	*4.54*	*5.46*	*4.07*	*6.05*
Post-Event	-0.0118	-0.0129	-0.0149	-0.0179	-0.0036
	2.04	*2.10*	*2.52*	*2.80*	*0.79*
R2	0.94	0.94	0.85	0.84	0.94

Panel C: Restricted Continu A

	quoted spread in €, equally-weigted	quoted spread in €, time-weigted	percentage quoted spread, equally-weigted	percentage quoted spread, time-weigted	effective spread
Constant	0.911	0.903	2.148	2.082	0.824
	6.01	*6.29*	*9.11*	*9.68*	*6.57*
Log(volume)	-0.158	-0.152	-0.265	-0.252	-0.129
	4.60	*4.63*	*5.92*	*5.98*	*4.39*
Ticksize	0.770	0.642	-0.0595	-0.072	0.302
	1.27	*1.10*	*-0.09*	*0.12*	*0.46*
Price	0.0053	0.0051	0.0007	0.00069	0.0044
	5.69	*5.97*	*1.17*	*1.22*	*4.93*
Volatility	80.933	71.797	128.142	112.410	53.456
	2.63	*2.48*	*3.42*	*3.14*	*2.00*
Post-Event	-0.0998	-0.088	-0.136	-0.132	-0.0765
	2.44	*2.25*	*2.93*	*2.96*	*2.22*
R2	0.64	0.65	0.56	0.55	0.56

Table 8

Regression Model for the Quoted Depth

The table reports regression results for the depth measures denoted in line 1. Volume is measured in thousand of Euros. The tick size variable measures the average effective tick size. The tick size is 1 €-Cent (5 Cents, 10 Cents, 50 Cents) for stocks trading at prices below 50 € (between 50 and 100 €, between 100 and 500 €, above 500€). The effective tick size can take on intermediate values if a stock trades at prices in more than one tick size range. Volatility is measured by the standard deviation of 30-minute midquote returns. Numbers in italics are heteroscedasticity-consistent t-values.

Panel A: All stocks

	average quoted depth in € 1000, equally-weighted	average quoted depth in € 1000, time-weighted
Constant	-55.962	-52.174
	3.02	*2.85*
Log(volume)	19.317	18.623
	8.36	*8.32*
Ticksize	56.647	60.955
	0.46	*0.51*
Price	-0.141	-0.142
	1.57	*1.60*
Volatility	-11908.2	-11792.1
	2.93	*2.99*
Post-Event	-4.506	-4.275
	0.57	*0.54*
R2	0.27	0.26

Panel B: CAC40

	average quoted depth in € 1000, equally-weighted	average quoted depth in € 1000, time-weighted
Constant	-366.03	-335.74
	8.45	*7.68*
Log(volume)	47.440	44.497
	11.11	*10.41*
Ticksize	220.292	236.365
	3.47	*3.73*
Price	-0.151	-0.160
	2.68	*2.91*
Volatility	-13896.34	-14.260.05
	5.01	*5.19*
Post-Event	-14.216	-13.583
	2.28	*2.26*
R2	0.81	0.80

Panel C: Restricted Continu A

	average quoted depth in € 1000, equally-weighted	average quoted depth in € 1000, time-weighted
Constant	-12.982	-11.571
	0.93	*0.82*
Log(volume)	13.063	12.751
	2.96	*2.97*
Ticksize	-150.310	-143.909
	1.02	*1.02*
Price	-0.0164	-0.0212
	0.14	*0.18*
Volatility	-9507.960	-9422.017
	1.95	*2.00*
Post-Event	-0.367	-0.537
	0.04	*0.05*
R2	0.10	0.10