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**COMPETITIVE NON-LINEAR  
PRICING IN DUOPOLY EQUILIBRIUM:  
THE EARLY US CELLULAR  
TELEPHONE INDUSTRY**

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# **COMPETITIVE NON-LINEAR PRICING IN DUOPOLY EQUILIBRIUM: THE EARLY US CELLULAR TELEPHONE INDUSTRY**

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## ABSTRACT

### Competitive Non-Linear Pricing in Duopoly Equilibrium: The Early US Cellular Telephone Industry\*

This Paper estimates an equilibrium oligopoly model of horizontal product differentiation where firms compete in non-linear tariffs. The estimation explicitly incorporates the information contained in the shape of the tariffs offered by competing duopolists to recover the structural parameters associated to the distribution of consumers' unobserved heterogeneity. The model identifies the determinants of the non-uniform equilibrium markups charged to consumers who make different usage of cellular telephone services. Estimates are then used to evaluate the welfare effects of competition, a reduction of the delay in awarding the second cellular license, and alternative linear and non-linear pricing strategies. Our policy evaluations reveal that a single two-part tariff achieves 63% of the potential welfare gains and 94% of the profits of a more complex fully non-linear tariff.

JEL Classification: D43, D82 and L96

Keywords: common agency, competitive non-linear pricing and estimation of equilibrium oligopoly models

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# 1 Introduction

Competing firms frequently offer sophisticated tariffs to sell their products and discriminate among consumers with different purchasing patterns and willingness to pay. Since firms are only able to approximate consumers' individual valuations for their products, they may resort to use these complex pricing mechanisms to induce them to self-select through the purchase of different amounts at different unit prices. There are interesting questions that arise naturally in this framework. How are the gains of introducing competition distributed among different consumers when nonlinear pricing practices are prevalent? Who benefits the most, large or small consumers? How large are profits and welfare relative to the standard case of linear pricing?

Although competitive nonlinear pricing is a very common practice, empirical economists have found serious difficulties in evaluating the extent of competition in markets where pricing strategies of firms are drawn, in general, from a space of nonlinear functions. Such general strategies raise many technical obstacles that may explain the delay in addressing a structural estimation of a nonlinear pricing competition model. Among these difficulties we identify the following: the lack, until very recently, of theoretical solutions to the problem of competitive second degree price discrimination; the prohibiting amount of individual information required to identify demand levels of different consumer types; and some ambiguity and arbitrariness in the choice of the quality dimensions that the monopolist utilizes to screen consumers. In addition to the typical identification issues surrounding empirical oligopoly equilibrium models, –including firms with market power and generally unobservable marginal costs–, we also need to account for the fact that in the case of nonlinear pricing competition, the equilibrium markup is different for each type of consumer. Therefore, a proper modeling approach should be able to identify the heterogeneity of individual demands that sellers face.

Our approach recognizes that optimal tariffs offered by sellers necessarily account for the unobserved heterogeneity of individual demands. The main contribution of this paper is to make use of the shape of tariffs as an equilibrium feature that accounts for the informational asymmetries in the competitive environment where these firms interact. The theory of nonlinear pricing is used to identify, in an equilibrium framework, three critical structural parameters that are not observable to econometricians: marginal costs (that equals the marginal tariff offered to the highest consumer type); cutoff consumer type (determined by the intercept of the tariff); and the distribution of types (through the one-to-one relationship between the shape of the tariff and the actual distribution of types that determines the optimal price distortion at each consumption level in order to minimize the informational rents of consumers while inducing self-selection). We offer a framework that incorporates –in a meaningful way– the information contained in the shape and position of tariffs

in the estimation of a structural model of nonlinear pricing competition that explicitly accounts for the fact that in equilibrium each tariff is the best response to the competitor’s tariff and *vice versa*.

The optimal screening of consumers leads to concave tariffs under very common conditions for demand and distribution of individual types. Different market features and the common knowledge distribution of relevant consumer characteristics in such markets critically condition the design of the tariff offered, and therefore its curvature. Thus, the shape of the actual tariff conveys important information regarding the decision making behind the design of nonlinear tariffs. While this issue has been frequently overlooked in the empirical analysis of second degree price discrimination, we intend to make full use of the tariff shape information contained in our data to identify the distribution of consumer types and evaluate welfare accounting for the heterogeneity of consumers.

This paper is, to our knowledge, the first attempt to explicitly incorporate the information contained in the shape of the nonlinear tariffs offered by sellers as part of the estimation of a structural model of nonlinear pricing. This approach makes use of readily available tariff information and does not rely on observations of individual purchases of different quantities and prices from several sellers, although if such information were available our estimates would be more efficient. Our approach overcomes common difficulties related to the amount and detail of data necessary to estimate a competitive nonlinear pricing model. Thus, we expect that the present setup can be easily “exported” to study other markets or industries where competition takes place through the use of nonlinear pricing, such as electricity, gas, and other utilities recently deregulated and open to competition. An added advantage of the application of the present model is that the econometric methods employed are quite straightforward.

⇒ INSERT Figure 1 ⇐

The idea behind our econometric approach can be easily summarized in Figure 1. The set of dotted lines represents the actual menu of tariff plans offered by a firm in a particular market and time. The concave function is the prediction of the tariff lower envelope according to our model of nonlinear pricing competition. At the econometric stage, predictions are made conditional on market and firm specific characteristics and will not impose symmetric pricing solutions neither within or across markets. Our estimates minimize the distance between the predicted and actual lower envelope of the tariff. Since there is a one-to-one correspondence between the shape of the optimal tariff and the distribution of consumers’ types for any given demand specification, the parameters summarizing the shape of the tariff function provide useful information to characterize the distribution of consumers’ types. The theoretical framework of Section 3 allows us to summarize this information in a sufficient statistic and recover the value of the fundamental structural parameters

from the estimates of the shape and position of the offered tariff by assuming a Nash equilibrium in quadratic tariff strategies.

We study the early U.S. cellular telephone industry. During the 1980s, the Federal Communications Commission (FCC) divided the U.S. into 305 non-overlapping markets defined to fit the geographic boundaries of standard metropolitan statistical areas (SMSAs).<sup>1</sup> In each market, two cellular carriers were allowed to operate. This industry is particularly well suited for our study for several reasons. First, markets are well defined. Second, the number of competitors in each market is limited to two, although we also have information for periods where only one firm operates in each market. Thus, we can compare how the gains of switching from monopoly to competition are distributed across different use-intensive consumers. Third, tariffs mainly consist of few calling plans that target consumers with different use intensity. Ours is a model of quantity discounts instead of quality discounts/premia. Thus, we do not need to argue whether consumers have a particular preference for size, weight, or appearance of the product. Valuations of the cellular service are mostly responsive to airtime usage. And last, but not least, we have information covering the complete tariff of each firm (not just the bill for a given number of minutes of airtime) in 113 markets for almost four years. Thus, we know the shape of tariffs for each firm-market-time combination. The substantial variation of tariffs across time and markets allows us to identify the structural parameters of a model of nonlinear pricing competition.

Using the different samples of our data, we estimate two specifications: a model of nonlinear pricing when there is only a monopolist in each market and a model of nonlinear pricing competition when there is a duopoly in each market. For the monopoly phase, the solution of a standard mechanism design problem characterizes the optimal tariff and its features (intercept, slope at the top, and curvature) for a linear demand model and a given family of well behaved distributions. For the duopoly phase, the theoretical model similarly characterizes the predicted tariffs although they are now a Nash equilibrium within the space of (quadratic) nonlinear strategies. Nash equilibrium in nonlinear tariffs imposes nonlinear identifying restrictions that allow us to recover the structural parameters of interest. We can incorporate such restrictions into the estimation because we observe the tariffs of all firms in each market. Had we observed only the tariff of one of the two firms, we would not be able to account for the strategic interaction of these firms when they design their nonlinear tariffs.

The estimation is carried out in two stages. In the first stage, we fit a quadratic polynomial on airtime usage to each available nonlinear tariff. We then recover the structural parameters

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<sup>1</sup> Each SMSA includes a central city of at least 50,000 people or an urbanized area of at least 50,000. It also includes the county containing the central city and other contiguous counties with strong economic and social ties to the central city. SMSAs accounted for about 76% of the population and 16% of the land of the U.S. in the 1990 Census.

by making use of the nonlinear restrictions among optimal nonlinear tariffs as derived from our theoretical model. In the second stage, we relate the estimated distribution of structural parameters to numerous market specific demand and cost characteristics.<sup>2</sup> The existence of these differentiated market regimes allows us to analyze the distribution of optimal monopoly markups across different levels of consumption as well as to compare how this distribution of markups is altered by the introduction of competition.

The estimated structural parameters allow us to identify whether markups are higher or lower, and how they vary across different volumes of consumption for markets with specific characteristics. Similarly, we can evaluate the welfare effects of introducing competition when firms engage in nonlinear pricing either for any specific market, or evaluated at the sample average of market characteristics. We thus use the structure of the model to evaluate the potential welfare effects of hypothetical restrictions in the type of pricing strategies that firms may use.

## 1.1 Literature Review

Notwithstanding our previous comments, there is an increasing number of empirical papers dealing with second degree price discrimination in competitive environments. Many of these papers focus on price discrimination based on different qualities of the products or services sold to consumers, and many of them adopt a reduced form approach to document the existence of pricing components that are aimed to screen different consumer types. Thus, for instance Shepard (1991) finds that the pricing of full and self-service in the retail gasoline market is consistent with second degree price discrimination. Two related papers are those of Borenstein (1991), dealing with the choice between leaded and unleaded gasoline, and Cohen (2000) regarding packaging size of paper towels. Borenstein (1989) first documents that the introduction of competition in the airline market does not reduce the markups for different services evenly. Busse and Rysman (2001) also study extensively this issue in relation with the pricing of advertisements of different sizes in yellow pages directories. Finally, Busse (2000), using our same data set, conducts an extensive analysis of the similarity of tariff shapes to conclude that this was an effective way to achieve collusion.

All these studies take the tariffs of firms as given. On the contrary, equilibrium models assume that the actual seller's pricing and consumers' purchases are optimal decisions. The structural parameters are estimated, through revealed preference arguments, by explicitly assuming that the optimality conditions are satisfied. Clerides (2002) studies the optimal inter-temporal pricing in the book publishing industry; Leslie (2000) studies the price discrimination practices of a Broadway

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<sup>2</sup> The structural parameters recovered in the first stage convey all the information contained in the offered nonlinear tariffs. Thus, there is no loss of efficiency in the estimation. Furthermore, in practical terms, the estimation then turns out to be a sequence of ordinary least squares regressions.

Theater; and Cohen (2001) focuses on the pricing by paper towel manufacturers. These papers have in common that although they all consider oligopolistic equilibrium models, the estimation and identification of structural parameters is achieved with data from either one or a subset of firms in each industry (Yale University Press, the *Seven Guitars* show, and the top seven paper towel manufacturers, respectively). McManus (2001) gets around this problem by defining a geographic market for specialty coffee in the surroundings of the campus of the University of Virginia. The pioneering work of Ivaldi and Martimort (1994) is still the only one that includes data for all the suppliers of different types of energy to the French dairy industry. They estimate a structural model where the equilibrium nonlinear tariffs are indeed best responses to the strategies implemented by competitors and *vice versa*.

Our paper can be viewed as combining the analytical structure of Ivaldi and Martimort (1994) to address the effects of competition on the shape of the tariffs as in Busse and Rysman (2001). We observe the pricing decisions of all firms in the market, and furthermore, we can control for firm and market specific information relevant for pricing decisions. Relative to Ivaldi and Martimort, our work makes use of the whole tariff information across markets, but we do not observe individual transactions by customers, which imposes limits to our ability to empirically identify demand. The model, including the distribution of consumer types, is identified up to any given specification of demand. However, since the cellular carriers of our data set change these tariffs over time, we are able to identify the reasons driving the actual pricing decisions. Similarly, we do not just describe why markups are higher or smaller for different volumes of consumption as in Busse and Rysman (2001), but we rather use this observed equilibrium behavior to identify and estimate the structural parameters that drive the pricing decisions. We thus incorporate the information contained in the shape of the tariffs, while our estimation explicitly accounts for the existence of a competing tariff that affects (and is affected) by the firm's offered schedule. The estimation of such structural model opens the possibility to evaluate alternative pricing strategies that are different from those actually implemented in the markets of our sample.

## 1.2 Main Results

There are several results reported in the paper. Our model allows us to recover the equilibrium distribution of marginal costs and distributions of consumers' tastes parameters both under monopolistic and competitive environments. Simple statistical analysis of the distributions of structural parameters indicate that there is significant asymmetry of information in the U.S. cellular market during the mid 1980s. Taste parameters linked to the service produced by the incumbent and the entrant are neither perfect substitutes nor identically distributed. The incumbent *wireline* firm appears to realize important efficiency gains when the market turns competitive after the entry of

the *nonwireline* company. Finally, for most markets, the entrant faces significant cost disadvantages as it enters the market. These cost disadvantages get reduce somewhat over the period covered in our sample but are far from vanishing by middle of 1988.

The distributions of the structural parameters carry all the relevant information for our analysis, and since we recover their distribution across time and markets we can relate them to observable market characteristics. Thus we learn for instance that markups tend to be larger in populated markets, the longer the average commute, and as the market matures, but on the contrary it is smaller the higher the income per capita in a particular market. These estimates also allow us to measure the welfare loss of delay due to the FCC's license awarding method. We estimate them at about 6% of the FCC's induced \$86bn in cost due to an over-a-decade deliberation about granting permission and setting standards for cellular telephone industry.

Competition lowers marginal tariffs, thus inducing further consumption, and also reduces subscription fees very significantly, therefore fostering participation. The combined effect of this is that welfare increases at the same time that consumers do not realize major benefits. Profits increase significantly due to efficiency gains together with the additional sales due to lower marginal rates. Consumers capture much of the benefits although gains from competition are quite moderate in this less than mature market: about 1% or (\$9.25m) per market on average.

Finally, we conduct some policy evaluations of alternative pricing strategies that firms could have followed in order to screen consumers. It is well known that two-part tariffs account for most of the efficiency gains of screening. In our case, a two-part tariff would generate 63% of the potential welfare gains of a fully nonlinear tariff and 94% of the potential profits. The low foregone profits of further screening may explain why in over a third of the markets only a two-part tariff is offered.

### 1.3 Road Map

In Section 2, we describe in detail the relevant features of the U.S. cellular telephone industry during the 1980s as well as the different sources of our data. Solving a common agency problem in closed form requires several functional form assumptions. Our approach requires a closed form solution in order to impose the nonlinear restrictions among the estimated parameters necessary to ensure that we can interpret the observed tariffs as Nash equilibrium strategies, and thus provide a way to identify the structural parameters of interest. In Section 3, we focus in reviewing each one of these functional form assumptions, discuss their economic content, and connect our data to the different elements of the model. The technical questions arising in the development and estimation of the model are relegated to the appendix at the end of the paper. Section 4 describes the econometric approach to estimate the structural parameters of the model, reports the results,

and evaluates alternative hypothesis regarding pricing behavior. Section 5 evaluates the welfare effect of implementing alternative pricing strategies such as a single two-part tariff, standard linear pricing, or a common flat tariff. Section 6 concludes.

## 2 The Early U.S. Cellular Telephone Industry

Cellular phones are a quintessential part of the Information Technology revolution of the 1990s. With 1.3 billion subscribers worldwide, wireless telephones surpassed the number of fixed-line subscribers in 2002. It currently accounts for more than 30% of the \$1 trillion total worldwide telecommunications revenues. In the U.S., with 136 million subscribers in 2002—a market penetration of 45%—sales of wireless telephony amounted to \$60bn and generated almost 200,000 direct jobs. For the earlier period studied in this paper these numbers are more modest. By mid 1988, which is at the end of our sample of data, there were 1.6 million subscribers in the U.S., who purchased services of almost \$2bn a year. The U.S. cellular telephone industry employed about 9,000 people and the average bill reached its historical peak of \$98.02 a month by mid 1988.<sup>3</sup>

At the beginning of the 1980s, technology was a barrier for competition, essentially because of the amount of bandwidth needed for transmission and the scarce radio spectrum available. The FCC divided the U.S. into 305 non-overlapping markets corresponding to SMSAs. In 1981, the FCC set aside 50 MHz of spectrum in the 800 MHz band for cellular services. One of the two cellular channel blocks in each market—the B block or *wireline* license—was awarded to a local *wireline* carrier, while the A block—the *nonwireline* license—was awarded by comparative hearing to a carrier other than a local *wireline* incumbent. After awarding the first thirty SMSA licenses by means of this expensive and time consuming *beauty contest*, rules were adopted in 1984 and 1986 to award the remaining *nonwireline* licenses through lotteries. Depending on the market, there were between 6 and 579 contenders for the *nonwireline* license.<sup>4</sup> Licenses were awarded in ten tiers from more to less populated markets, beginning in 1983. In general the *wireline* licensee offered the service first and enjoyed a temporary monopoly position until the holder of the *nonwireline* license entered the

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<sup>3</sup> See the 1997 Economic Census of U.S. Census Bureau; the 2001 Telecommunications Indicators Update of the International Telecommunications Union; and the 2002 Semiannual Wireless Survey of the Cellular Telecommunications & Internet Association.

<sup>4</sup> See the detailed description of this market in Busse (2000, §2.1) as well as an account of the entry policy and regulation of these markets in Shew (1994). The large number of applicants is partly due to the low cost of completing the paperwork to enter the lottery—between \$250 and \$5,000—, but also by the high expected immediate return since there was no restriction to resale the *nonwireline* license other than not selling it to the operator of the *wireline* license. The FCC itself did not obtain any revenue for awarding the licenses. Hazlett and Michaels (1993) study the economic consequences of this inefficient license awarding mechanism. Cellular technology uses low powered transmitters that exhaust the allocated bandwidth within small cells. A single conversation using the older high powered transmitters of car phones used a channel within a radius of about 75 miles. The FCC required that the low powered transmitters of the new cellular technology used these channels within a maximum radius of 30 miles. Hausman (2002, §2.1) describes in detail how the combination of cell splitting and sectorization increased capacity by a factor of 8 relative to pre-existing, non-cellular, car telephone technology.

market. By the early 1990s markets had two operators. For our sample, this was the case in 1988. We thus have data on monopoly and dupoly markets.

## 2.1 Data

The data used in this study cover four areas: (1) service prices; (2) input prices; (3) output measures; and (4) demand variables. Appendix A1 describes each variable, provides its average value for the pool sample and details its source. The study period begins in December 1984 and ends in July 1988. During this period most cellular markets experienced both monopoly and duopoly phases. Our data include information for 113 markets covering over 40% of the U.S. population. In general, these are among the largest markets, with an average 1.6 million residents *vs.* the national average of 542,000 inhabitants across all SMSAs in 1987.

1. Information on Individual Price Plans: During the period of study, cellular operators self-reported price and product offering information. Prices were collected repeatedly for each operator. Cellular companies offer nonlinear prices in the form of multiple price plans from which consumers self-select the one that is expected to minimize their monthly bill. A typical price plan is a two-part tariff with a peak-load component. A plan has a monthly access fee, a price per minute of peak-hour airtime usage, a price per minute for off-peak usage, and, in some cases, a some peak and/or off-peak minutes included without charge.<sup>5</sup> The number of plans that define the nonlinear tariff vary across markets, time, and carriers with some firms offering only one plan, and few others offering as many as nine plans –although the maximum number of effective (non-dominated) plans is only four. Our analysis uses peak retail prices only.<sup>6</sup>

2. Factor Prices: Inputs include labor, energy, capital equipment (radio and switching equipment), and general overhead (leases, office expenses, administrative costs, etc.). Factor prices were collected for each of these inputs across markets and over time. Among the determinants of costs we include other non-price related variables to control for the possibility of economies of density, to account for the potential savings in the deployment of antennas in heavily populated areas, as well as the potential cost reductions due to external factors as the market matures or due to learning by doing. All money valued magnitudes are expressed in dollars of July 1986.

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<sup>5</sup> The tariff allowance was rarely part of the tariff lower envelope. In general, there was another least expensive tariff option for the range of usage of the tariff allowance of the most expensive plans. Only the small allowance of the option with the least expensive monthly fee –and higher rate per minute– was part of the tariff lower envelope.

<sup>6</sup> Peak time spanned between 11 and 13 hours a day during the sample period and included the vast majority of airtime traffic. Wholesale prices offered by operators during the study period consisted of quantity discounts for users purchasing more than one subscription. These prices were mostly offered to larger corporate accounts, as opposed to small business and individuals.

3. *Output*: Output levels are not directly observable since operators are not required to report the airtime traffic or their subscriber levels to public authorities. Instead, data were collected on the number of cellular antenna sites used by operators over time and across markets. Each cell site accounts for 1,100 to 1,300 subscribers each, depending on the engineering configuration of the local network and the usage pattern common at that time. As such, the total number of cells in a given network can act as a proxy for the industry’s output in each market.<sup>7</sup>

4. *Demand*: Variables used in the model to account for demand shifts include market population (overall demand effect); market coverage (network externalities or bandwagon effects); average growth rate of population during the 1980s; total number of high potential business establishments; a time trend to proxy for market growth due to anything else other than population growth; as well as the average commuting time in each area, median income, percent households with income below poverty level, median age of population, and median number of years of education. Other exogenous demand information most likely linked to subscription decisions includes weather and crime indicators.

### 3 The Model

This section describes the different elements of the model and relate them to the information contained in our data set. There are three basic elements in a model of nonlinear pricing: demand, costs, and distribution of consumer types. The following subsections make specific functional form assumptions that allow us solving the standard monopoly pricing in closed form, as well as the common agency problem that characterizes the equilibrium tariff in a duopoly.

#### 3.1 Demand

Consumers may purchase telecommunication services from any of the two competing cellular operators. These services may be differentiated, or perceived as different by consumers. Brand loyalty, whether it is positive or negative, may affect the relative valuation of subscribing to one cellular firm over the other since the owner of the *wireline* license may also be a known provider of the fixed-line local telephony service. We will include a *wireline* dummy in the estimation of the model to capture this effect. Notice that differences between the cellular carriers in the quality of the transmission, the area of coverage, the handset bundled with the subscription to a long term contract, the availability and cost of additional features such as voice-mail, wake-up calls, or roaming chargers among others

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<sup>7</sup> In order to support the assumption that cell-sites are a reasonable proxy for subscriber output we conducted an additional analysis based on a very limited sample of markets where both subscriber and cell-site data were available at the firm and market level. Based on 22 observations for 8 markets in 1985–1987, the linear correlation amounts to 0.916 (p-value < 0.0001) between cell sites and number of subscribers at the market level.

may also affect the intercept of potential demand for each product as well as to the substitution parameter  $\kappa$  below. We represent the utility function of users of cellular services as follows:<sup>8</sup>

$$U(x_1, x_2, \theta_1, \theta_2, \kappa) = \theta_1 x_1 + \theta_2 x_2 - \frac{b_1}{2} x_1^2 - \frac{b_2}{2} x_2^2 + \kappa x_1 x_2. \quad (1)$$

Services  $x_1$  and  $x_2$  are substitutes when  $\kappa < 0$  and complements if  $\kappa > 0$ , while condition  $b_1 b_2 - \kappa^2 > 0$  suffices to ensure that the utility function is concave. All other parameters are positive. If  $\kappa = 0$ , these products are independent. If  $\theta_1 = \theta_2$  and  $b_1 = b_2 = -\kappa$ , then  $x_1$  and  $x_2$  are perfect substitutes [Vives (1999, §6.1)]. Type dimensions  $\theta_1$  and  $\theta_2$  denote the intensity of preferences for product varieties offered by firm 1 and 2, respectively.<sup>9</sup>

We associate index  $i=1$  with the first service available in each metropolitan area, which is normally, but not always, owned by a fixed-line local telephone carrier in that market. This service is the only available until the second cellular carrier, the *nonwireline* licensee, who offers service  $i=2$ , enters.<sup>10</sup> Furthermore, data from the monopoly phase allow us to identify the parameters behind the optimal nonlinear pricing rule by monopolists. In this monopoly phase ( $x_2 = 0$ ), consumers utility is represented by:

$$U(x_1, \theta_1) = \theta_1 x_1 - \frac{b_1}{2} x_1^2, \quad (2)$$

which leads to a linear demand for  $x_1$  with individual specific satiation usage  $\theta_1/b_1$ .

### 3.2 Distribution of Types

Consumer types are distributed according to a Burr type XII distribution of the form:<sup>11</sup>

$$\theta_i \sim F_i(\theta_i) = 1 - \left[ 1 - \frac{\theta_i - \underline{\theta}_i}{\bar{\theta}_i - \underline{\theta}_i} \right]^{1/\lambda_i}, \quad \theta_i \in [\underline{\theta}_i, \bar{\theta}_i]. \quad (3)$$

⇒ INSERT Figure 2 ⇐

<sup>8</sup> Consumers subscribe only to one carrier. However, the existence of multi-line business contracts and resellers that purchase capacity from the two licensed carriers justify, at least partially, that a non-negligible proportion of subscribers purchase airtime from both carriers through resellers. However, as we have indicated previously, individual purchases are not observable in this data set. In this paper we adopt a common agency approach and postpone the estimation of an exclusive agency model for future research. Section 3.5 further discusses this methodological approach.

<sup>9</sup> Cellular services may be differentiated for a variety of reasons such as the own network coverage, the bundled handset, calling features, carrier reputation and others. Heterogeneity of service quality among cellular carriers has been documented by Marciano (2000).

<sup>10</sup> The FCC moved away from comparative hearing to lotteries as reviewing applications for each market amounted to a considerable burden. In many cases, the winner of the lottery never intended to operate a cellular telephone company, but rather to resale such license to another firm. This resale process explains most of the delay in entry by the *nonwireline* firm. From an equilibrium perspective, even if the incumbent was guided by any attempt to prevent the *nonwireline* firm from entering, such strategy should be ruled out as the second carrier always entered. Furthermore, the timing of entry is exogenously given by the FCC decision to award the license in each tier of markets. Firms were required to offer the service within six months of having been awarded one of the licenses.

<sup>11</sup> The notation in this section is closely attached to the monopoly solution. For duopoly, these distributional assumptions refer to the “effective types”  $z_i$  defined in (15) instead of  $\theta_i$ . For further results and properties of the Burr type XII see Johnson, Kotz, and Balakrishnan (1994, §12.4.5). Appendix 1 of Miravete (2002b) discusses the properties of this distribution within the context of nonlinear pricing models.

There are several advantages of using this distribution. First, this distribution allows for an explicit closed form solution for the pricing problem.<sup>12</sup> Second, different values of  $\lambda_i$  identify whether high valuation consumers are more or less numerous than low valuation consumers. Figure 2 shows that  $\lambda_i = 1$  corresponds to the case of uniformly distributed types. If  $\lambda_i > 1$ , consumers are more concentrated around higher values of  $\theta_i$  and *vice versa* when  $\lambda_i < 1$ . If  $\lambda_i = 0$ , distribution (3) becomes degenerate at  $\theta_i = \underline{\theta}_i$ . Third, the hazard rate of this distribution is:

$$r_i(\theta_i) = \frac{1}{\lambda_i(\bar{\theta}_i - \theta_i)}, \quad (4)$$

where value increases monotonically over  $\Theta_i = [\underline{\theta}_i, \bar{\theta}_i]$ . Furthermore, higher values of  $\lambda_i$  lead to distributions with uniformly lower hazard rate.<sup>13</sup> The hazard rate function is increasing and the optimal markup is decreasing for  $\lambda_i > 0$ . The position and curvature of the lower envelope of tariff options in the data is a key element to identify the parameter  $\lambda_i$ . Figures 3 and 4 illustrate this argument. Figure 3 shows that the hazard rate ordering of the Burr type XII distributions with respect to parameters  $\lambda_i$ , a critical feature in nonlinear pricing models since the optimal price markup is inversely related to the hazard rate of the distribution of types. This feature allow us to characterize optimal tariffs with different degree of concavity that do not cross each other as shown in Figure 4.

⇒ INSERT Figure 3 and Figure 4 ⇐

Therefore, given the individual demand function associated with the quadratic utility (1) and the Burr type XII distribution (3), the solution of the optimal pricing mechanism presented below identifies  $\lambda_i$  from the shape of every tariff offered by each firm in different markets and time. This is because the inverse of the hazard rate of the distribution enters the expression of the optimal marginal tariff explicitly and hazard rates and tariffs can be unequivocally ordered with respect to  $\lambda_i$ . Thus, given the ratio of high to low valuation customers, firms have to introduce more or less powerful contracts to induce self-selection and thus minimize the informational rents kept by consumers. More or less powerful contracts translate into tariff functions with different degrees of concavity. Thus, distributional assumption (3) allows us to summarize in a single sufficient statistic the degree of concavity of the tariff over the relevant consumption range.<sup>14</sup>

<sup>12</sup> We include the monopoly solution in Appendix A2 and the duopoly solution in Appendix A3.

<sup>13</sup> For  $\lambda_1 \geq \lambda_2$ ,  $r_1(\theta) \leq r_2(\theta) \forall \theta \in \Theta \subseteq \mathbb{R}$ , and thus  $F_1(\theta)$  dominates  $F_2(\theta)$  in hazard rate.

<sup>14</sup> The hazard rate is a nonlinear function of the moments of any distribution. As it is shown in Figure 2, the Burr type XII distribution can be indexed by a single parameter  $\lambda_i$  directly linked to the value of the hazard function of this distribution. Alternatively, we could consider the more general family of beta distributions since the above Burr type XII distribution is indeed a beta distribution with parameters 1 and  $1/\lambda_i$ . However, while the shape of the tariff may serve to recover the relative proportion of high to low types, it is unreasonable that such tariff information could also serve to recover higher moments of the distribution of the types.

### 3.3 Cost

The cost structure is simple but adequate for this industry. It is characterized by an important fixed cost (that is not identifiable from our data) and a marginal cost that depends on output capacity, and other firm and market specific characteristics that may affect its level:

$$C_i(Y, \mathbf{W}_i) = \mathbb{C}_i + c_i(Y, \mathbf{W}_i) \cdot Y, \quad (5)$$

where  $Y$  is the aggregate output capacity level of the firms in a given market and time period, and  $\mathbf{W}_i = \{W_{i1}, \dots, W_{iK}\}$  is the corresponding vector of factor prices that firms face in this particular market as well as other firm specific cost elements.<sup>15</sup>

#### 3.3.1 *Scale-Dependent Marginal Costs*

Equation (5) allows marginal costs to be a function of the market output in order to allow for either increasing or decreasing returns to scale in the early cellular industry. But since marginal costs depend on the total output, the characterization of the tariff becomes contingent on the assumed level of aggregate output. As Wilson (1993, §4.2) indicates, this characterization of the equilibrium has to be iteratively applied until the anticipated marginal cost of the aggregate used to construct the tariff coincides with the marginal cost predicted from the aggregation of purchases induced by the tariff design. In monopoly equilibrium, tariff (7a) below is such that the total output  $Y$  coincides with the sum of individual purchases (7b) given the distribution (3) and firms' marginal costs (5). The same applies under competition. As for the estimation, we assume that markets are in equilibrium, with total sales coinciding with the sum of unobservable individual purchases. Thus, we can account for the effect of the aggregated level of output on the marginal cost function of sellers through variations across markets and time.<sup>16</sup>

### 3.4 Monopoly Solution

During the monopoly phase, the only seller –firm 1– offers a contract  $\{T_1(\theta_1), x_1(\theta_1)\}$  to his customers, *i.e.*, a contingent plan that details the payments associated to each consumption level. Consumers implicitly reveal their type  $\theta_1$  when they make their consumption and payment decisions.<sup>17</sup>

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<sup>15</sup> We use the aggregate capacity  $Y$  instead of the individual sales  $Y_i$  because we only observe firm's capacities at the market level (with the exception of very few markets and time periods).

<sup>16</sup> The total number of antenna sites in each market is the right measure of capacity because the FCC required the *wireline* company to offer unrestricted resale of its service until the *nonwireline* company was fully operational in order to foster competition and usage of the cellular service. See Vogelsang and Mitchell (1997, p.207).

<sup>17</sup> Consumers first subscribe to one of the available tariff plans and later decide how much to consume. It is therefore possible that consumers do not always choose the cost minimizing tariff option for their telephone usage. However, in the long run we expect that consumers learn which is their cost minimizing tariff option. The empirical analysis of Miravete (2003) supports this view. Thus, to simplify the model, the payment and consumption indicated in the contract  $\{T_1(\theta_1), x_1(\theta_1)\}$  are implicitly assumed to be simultaneous decisions.

Given the demand function and the distribution of types, the monopolist designs the schedule  $T_1[x_1(\theta_1)]$  to maximize his expected profits provided that consumers truthfully reveal their type through the choice of consumption and tariff payment subject to the incentive compatibility constraint (IC) of consumers. As it is shown in Appendix A2, the optimal tariff is such that:

$$\theta_1 - b_1 x_1(\theta_1) = c_1(Y, \mathbf{W}_1) + \lambda_1(\bar{\theta}_1 - \theta_1). \quad (6)$$

From here, the optimal contract  $\{T_1(\theta_1), x_1(\theta_1)\}$  for the monopolist is:

$$T_1(\theta_1) = \underline{\mathcal{U}}_1 + \left[ \frac{c_1(Y, \mathbf{W}_1) + \lambda_1 \bar{\theta}_1}{1 + \lambda_1} \right] x_1(\theta_1) - \left[ \frac{b_1 \lambda_1}{2(1 + \lambda_1)} \right] x_1^2(\theta_1), \quad (7a)$$

$$x_1(\theta_1) = \frac{(1 + \lambda_1)\theta_1 - c_1(Y, \mathbf{W}_1) - \lambda_1 \bar{\theta}_1}{b_1}. \quad (7b)$$

These two expressions are the predictions of the model. If data on individual consumption were available we could use it to estimate more efficiently the structural parameters of this model. However, only aggregate data are available, and we have to combine equations (7a) and (7b) to identify the different elements of our model.

Equation (6) shows –as it is well known in the literature–, that all consumer types are inefficiently priced with the exception of the highest type  $\bar{\theta}_1$ . This *efficiency at the top* allows us to identify the marginal cost from the marginal rate offered for very high use levels. The magnitude of the optimal distortion is negatively related to the value of the hazard rate of the distribution for each consumer type and determines the curvature of the optimal tariff. Therefore, the optimal distortion defines the non-homogeneous markup for different consumers and critically determines the shape of the optimal tariff. Because of this one-to-one relationship between the hazard rate of the distribution and the curvature of the optimal tariff, the value of  $\lambda_1$  can be recovered by minimizing the distance between the actual tariff offered by the monopolist and the tariff solution of our model that, as equation (7a) shows, is just a general quadratic approximation to any concave function and whose degree of concavity depends on  $\lambda_1$ . However, many aspects of the problem still need to be addressed before we estimate the model. We now explain in detail these issues with reference to the monopoly solution. All these issues generalize to the duopoly case.

### 3.4.1 Stochastic Structure

Given the quadratic nature of the tariff prediction (7a) we fit the lower envelope of the actual tariff for the peak-period (of each firm in every market and quarter) on a second order polynomial over a grid of airtime usage:<sup>18</sup>

$$T_1(x_1) = \alpha_1 + \beta_1 x_1 + \frac{\gamma_1}{2} x_1^2 + \varepsilon_{T_1}, \quad (8)$$

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<sup>18</sup> We acknowledge that cellular firms will design simultaneously their pricing for peak and off-peak periods. However, we make the simplifying assumption that these two markets are independent and that firms design tariff

so that:

$$\alpha_1 = \underline{U}_1, \quad \beta_1 = \frac{c_1(Y, \mathbf{W}_1) + \lambda_1 \bar{\theta}_1}{1 + \lambda_1}, \quad \gamma_1 = -\frac{b_1 \lambda_1}{1 + \lambda_1}, \quad (9)$$

while  $\varepsilon_{T_1}$  is an error term that accounts for the discrepancy between the predicted and the actual tariff. This particular stochastic structure of our problem is appropriate because approximation errors arise in practice when we use a continuous, quadratic, tariff prediction to fit the lower envelope of a menu of few optional two-part tariffs. The difference between the fully nonlinear tariff and the approximation with a menu of two-part tariff options is small and decreases with the number of options offered. There is therefore no reason to expect that the variance of  $\varepsilon_{T_i}$  is very different over the consumption range we can avoid the use of potentially complex, airtime-dependent, weighting rules for the computation of  $\alpha_1$ ,  $\beta_1$ , and  $\gamma_1$ , that it is attained by means of ordinary least squares to estimate (8) on a grid of usage for each firm-market-quarter combination.

### 3.4.2 Continuous Tariff vs. Discrete Number of Plans

Is it reasonable to implement a continuous tariff approximation to a menu of two-part tariffs when firms actually offered menus of optional two-part tariffs? If the distribution of consumer types has a compact support, the optimal tariff is always a nonlinear tariff. If firms offer a menu with a discrete number of options, they will not extract the maximum informational rents from consumers. Furthermore, consumer surplus is not maximized because within each option different consumers will be treated identically, thus leading to bunching. In the absence of computation, marketing, or search costs, welfare is increasing in the number of options offered, although at a decreasing rate.<sup>19</sup> Under monopoly, we have to compare smaller incremental profits of adding an additional tariff option with the unobservable cost of offering it. In the competitive oligopoly case, this problem becomes much more complex because firms do not only solve for the optimal nonlinear tariffs that are best response to each other, but also the number of two-part tariffs that are “best response” to the approximate implementation of the tariff by the competitor and *vice versa*. This is impractical, since we would need to define what constitutes a Nash equilibrium in strategies that only approximate optimal nonlinear equilibrium tariffs.

Unless we account for some costs associated with the introduction of a large number of tariff options, the equilibrium number of tariff options will never be finite. Indicators related to these billing or marketing costs are not available, and thus, we are not able to determine endogenously the optimal number of optional two-part tariffs. We therefore fit the continuous tariff predicted by our

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optimally in each market separately. The assumption is reasonable if, as in the present case, the bulk of consumers are business customers with very low elasticity of substitution between peak and off-peak consumption. Solving a problem of multiproduct nonlinear pricing competition is beyond the scope of this paper. Furthermore, we do not have detailed data, even at the aggregate level, to distinguish between cellular telephone utilization during peak and off-peak periods.

<sup>19</sup> This result was first shown by Faulhaber and Panzar (1977). See also Wilson (1993, §8.3).

model to the lower envelope of tariff options. The major reason justifying our approach is that the number of options needed to “almost” mimic the continuous solution is quite small.<sup>20</sup> In general as the number of tariff options exceeds three, the approximation becomes quite accurate. Over 60% of the firms offer either 3 or more tariff options.

### 3.4.3 *Identification of Structural Parameters*

Despite the algebra, the identification of the basic structural model from the offered nonlinear tariff follows through very easily, provided that we know the values of the parameters of the utility function  $\{b_1, b_2, \kappa\}$ . The intercept of the least expensive tariff option determines the rent of the marginal consumer type, and therefore it identifies which customer type is indifferent between participating in the market or not. Second, the slope of the tariff for the highest type identifies the marginal cost –equation (6)– because, as it is well known, this is the only consumer type that is efficiently priced. And finally, the degree of concavity of the tariff identifies the indexing parameter of the distribution of types. Thus, first, from equation (9) we have that:

$$\lambda_1 = \frac{-\gamma_1}{\gamma_1 + b_1}, \quad (10)$$

so that the indexing parameter of the distribution of types is recovered as a nonlinear transformation of the estimate of  $\gamma_1$ , which accounts for the degree of concavity of the offered tariff.

Second, in order to identify the marginal cost we need to address how to determine the maximum minutes of airtime of the grid used to fit the quadratic polynomial (8) to recover parameters (9). With a potentially unbounded upper limit for airtime usage, the more minutes we include, the larger the proportion of the sample used in the regression that corresponds to the tariff option designed for high consumption. Thus, the higher the maximum consumption considered in the regression,  $\mathbb{X}$ , the flatter the fitted tariff, and the more likely it becomes that we overestimate the corresponding value of  $\lambda_i$ .<sup>21</sup> We choose the value of  $\mathbb{X}$  to reflect the average pattern of consumption. Reliable data on individual average airtime consumption is not available. We only know that the national average monthly bill was \$96.83 in 1987 and \$98.02 in 1988. The distribution of airtime consumption generates these average bills for a level of 200 minutes of airtime use. Since we need 471 minutes (1987) and 321 (1988) to reach the level of the average bill plus once its standard deviation, we set  $\mathbb{X} = 500$  as a safe upper bound for maximum airtime consumption for regression purposes. Results are robust to alternative values of  $\mathbb{X}$  in a wide range around 500 minutes. Thus, making  $x_1(\bar{\theta}_1) = \mathbb{X}$  in (7b) we obtain the following expression for the highest consumer type:

$$\bar{\theta}_1 = c_1(Y, \mathbf{W}_1) + b_1\mathbb{X}. \quad (11)$$

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<sup>20</sup> This topic is extensively discussed in Wilson (1993, §6.4 and §8.3). Appendix A5 evaluates the magnitude of the specific error of approximation of our model depending on the number of options offered by firms.

<sup>21</sup> Equation (12) below shows that a larger  $\mathbb{X}$  will also bias upwards our estimate of the marginal cost.

This relation is key to express the marginal cost as a function of the estimated parameters equation (8). After substituting equation (11) into the definition of  $\beta_1$  in equation (9) and making use of the definition of  $\gamma_1$  we obtain:

$$c_1 = \beta_1 + \gamma_1 \mathbb{X}. \tag{12}$$

Therefore, the present model offers a simple way to identify the structural parameters of interest from almost trivial regressions of the tariff payment on airtime usage. Provided any estimates of  $\alpha_1$ ,  $\beta_1$ , and  $\gamma_1$ , we recover the estimate of the marginal cost as a simple linear combination of  $\beta_1$  and  $\gamma_1$ . The right hand side of (12) coincides with the marginal tariff paid by the highest type,  $\bar{\theta}_1$  when purchasing  $x_1(\bar{\theta}_1) = \mathbb{X}$ , and thus, the identification of the marginal cost relies on the well known result of *no distortion at the top*.

Finally, we have to address the identification of the participation decision. For this early stage of development of the cellular industry, it is unreasonable to assume that all consumers were served. Therefore, we account for the fact that sellers' pricing decisions may be aimed to exclude some customers. Exclusion, however, does not have to do with the concavity of the tariff –aimed to screen among the active consumers– but rather with the fixed fee of the service that determines the marginal participating consumer.<sup>22</sup>

The non–usage dependent component in the tariff (7a),  $\underline{\mathcal{U}}_1$ , plays the role of separating the participation incentives from those of screening. It is standard to assume that consumers' reservation value is independent of their type. Because of monotonicity of the solution  $x_1(\theta_1)$ , it suffices to assume that  $\mathcal{U}(\theta_1) \geq \underline{\mathcal{U}}_1$ . While it is also customary to normalize the rent of the lowest participating consumer,  $\theta_1^\circ$  as  $\mathcal{U}(\theta_1^\circ) = 0$  in deriving the optimal tariff, we do not follow such approach because it would imply that our tariff prediction (7a) always start at the origin. We know that this is not the case because the observed tariff options always include positive fees. Assuming that  $\mathcal{U}(\theta_1^\circ) = 0$  would result in a misspecified model that will overestimate the curvature of the lower envelope of the tariff and, consequently, underestimate the value of  $\lambda_1$ . We therefore do not assume that the rent of the lowest type is necessarily zero, but rather  $\alpha_1 = \mathcal{U}(\theta_1^\circ) = \underline{\mathcal{U}}_1 \geq 0$ . In such a case, as Fudenberg and Tirole (1991, §7.1.1) indicate, an increase of the fixed payment will suffice to make the individual participation constraint binding for the lowest type, thus maximizing the expected profits of the monopolist. Alternatively (and equivalently), we study how the marginal type  $\theta_1^\circ$  changes across

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<sup>22</sup> Alternatively we could have focused on the market of active consumers only. In such a case, we should have considered the upper truncated distribution function  $F_1(\theta_1 | \theta_1 \geq \theta_1^\circ)$ . But the hazard rate of this truncated distribution is identical to (4), and therefore the estimation of  $\lambda_i$  does not depend on the truncation parameter  $\theta_1^\circ$ . Therefore, the estimation of  $\lambda_1$  from the curvature of the existing tariff is robust regardless of whether markets are fully served or not.

different markets. The lowest type  $\theta_1^\circ$  is indifferent between participating in the market or not. Thus,  $\mathcal{U}(\theta_1^\circ) = \underline{\mathcal{U}}_1 \geq 0$  whenever  $x_1(\theta_1^\circ) = 0$ , which requires:<sup>23</sup>

$$\theta_1^\circ = \frac{c_1(Y, \mathbf{W}_1) + \lambda_1 \bar{\theta}_1}{1 + \lambda_1} = \beta_1. \quad (13)$$

### 3.5 Duopoly

Solving a nonlinear pricing competition problem is considerably more complicated than characterizing the monopolist's optimal tariff. However, some few papers have successfully shown that there may exist oligopoly models where the equilibrium strategies are characterized with non-homogeneous markups for different purchase levels. There are two modeling decisions that we need to justify. First, why we prefer a model with two instead of one type dimension, and second, why we favor a common agency over the exclusive agency framework.

First, if we choose a model with single-dimensional types, equation (1) should be re-written with  $\kappa = -b_1 = -b_2$  and  $\theta_2 = \bar{\theta}_1 - \theta_1$ . In this case we would consider a model of vertical product differentiation where, perhaps for technical reasons, one of the two networks, is preferred by most customers over the other.<sup>24</sup> Perhaps, this model may be reasonable to address the European cellular industry where the former state-run local monopolist always entered first and always exceeded new entrants in financial and technical capability. The U.S. cellular industry does not fit this description. While the *wireline* firm was most likely owned by one of the Baby Bells, the *non-wireline* firm is frequently owned by another Baby Bell company, with the same know-how and considerable financial capability. There is no obvious criteria to rank these two networks and a two-dimensional type model appears more realistic in addressing these horizontally differentiated products. As documented by Marciano (2000) consumers may prefer one firm over the other depending on the area of coverage (normally different for each firm), location of home residence or business, roaming charges, equipment, or other features.

Second, the choice between a common agency model such as those of Biais, Martimort, and Rochet (2000); Ivaldi and Martimort (1994); and Martimort and Stole (2002) *vs.* the exclusive agency models of Armstrong and Vickers (2001) or Rochet and Stole (2002) is a difficult one. Each of these approaches implies a different mechanism of transmission of the strategic interactions between the principals, *i.e.*, through the purchasing decisions of consumers among the existing services *vs.* their subscription decision to a particular carrier. However, characterizing a competitive

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<sup>23</sup> In particular, notice that when there is no asymmetry of information  $\lambda_1 = 0$  and  $\theta_1$  becomes degenerate at  $\theta_1^\circ = \underline{\theta}_1 = 0$ . In such a case, condition (13) just requires that demand for the lowest consumer type exceeds the marginal cost  $c_1(Y, W_1)$  at  $x_1(\underline{\theta}_1) = 0$ .

<sup>24</sup> Rey (2000) discusses a model of this sort.

nonlinear tariff in an exclusive agency framework involves solving a two–point boundary problem with a second–order nonlinear differential equation. This is the consequence of introducing random participation constraints that breaks the recursive nature of the standard monopolist’s nonlinear pricing solution –*e.g.*, Maskin and Riley (1984)–, where the fulfillment of the participation constraint for the lowest type ensures the participation of all other consumer types. Solving the non–recursive pricing problem does not lead to tractable closed form solutions. However, the most important reason for not embracing this approach at this stage is because the nonparametric identification between shape of tariffs and the distribution of the asymmetric information parameter may not longer hold.<sup>25</sup>

In our institutional framework consumers may subscribe to one of the two licensed carriers through exclusive agents or buy from resellers who purchase large amounts of airtime capacity at wholesale price from either one or both licensed carriers. We recognize that both strategic interactions –common and exclusive agency– are present in the design of the tariff. However, since we do not observe individual purchases, we turn our attention to common agency models in this paper and postpone the more complex empirical estimation of an exclusive agency for future research.<sup>26</sup>

Among the models of common agency, Ivaldi and Martimort (1994) present a competing mechanism where strategies are quadratic and the aggregation of types across differentiated products enables the use of the Revelation Principle even within a common agency problem. This setup is also quite suitable for empirical estimation, and we borrow heavily from it in order to specify our econometric model for the duopoly phase of our data.<sup>27</sup>

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<sup>25</sup> Rochet and Stole (2002) show that if the market is fully covered and duopolist have the same marginal cost, then the equilibrium is characterized by a symmetric two–part tariff regardless of the distribution of consumer types, thus effectively breaking the link between the curvature of the tariff and the distribution of consumer tastes.

<sup>26</sup> There is a further, more theoretically based, argument to justify our modeling choice. In models of common agency it is normally required that all consumers purchase strictly positive amounts of all products so that by buying them in different proportions, they can provide the strategic linkage that affect the incentives of sellers to design their contracts as the optimal response to the contract of the rival firm. However, the assumption of an interior solution is only necessary if the first order approach is used to solve the common agency problem. It should be noted that the strategic linkage of common agency models does not necessarily disappear because some consumers only purchase one product. It is certainly more difficult to handle it using a first–order approach to solve this agency problem as the non–negativity constraint of purchases may not allow using the marginal payment on the rival’s tariff to identify the optimal purchase of the rival’s product due to the existence of a corner solution. But corner solutions do not rule out tangency of a consumer’s indifference curve and her budget constraint at zero consumption of one of the products, *e.g.*, Silberberg (1990, §14.1). In the absence of resellers, or for those customers that purchase through a dedicated agent, it is theoretically possible to find a large enough value of  $z_1$  such that  $x_2 = 0$  and the marginal rate of substitution between  $x_1$  and  $x_2$  equals the ratio of marginal tariffs even at such boundary point. This result is proved in Appendix A7. Since the range of acceptable values of  $z_1$  and  $z_2$  depends on the values of the structural parameters of the model, we have evaluated whether these conditions are fulfilled at our particular parameter estimates. We find that in 83% of the duopoly markets, our structural estimates allow for such tangency at the corner to occur.

<sup>27</sup> The main difference between ours and the approach of Ivaldi and Martimort is due to the fact that they observe individual transactions, –price paid and quantity purchased by each customer (244 French dairy firms; 74% of the market)–, but ignore the actual tariff used by the competing firms. Our case is just the opposite. We, on the contrary, have rich information regarding the tariffs offered by competing firms in several markets and time periods, but ignore individual transactions by the millions of potential customers that these firms may serve.

### 3.5.1 Definition of Effective Types

In addition to equation (1) that characterizes the preferences of consumers, we need to assume that the competing firm offers a nonlinear tariff that is quadratic in the firm's own sale:

$$T_i(x_i) = \alpha_i + \beta_i x_i + \frac{\gamma_i}{2} x_i^2, \quad (14)$$

Observe that the tariff schedule depends only on the output sold by firm  $i$ , thus excluding the possibility that the payment of each firm depends on the schedule offered by the competitor.

The problem defined by equations (1) and (14) cannot be directly solved using the standard mechanism design approach because preferences are not ordered along a single dimension. Since in addition, preferences are defined over services offered by competing firms, the well known *Revelation Principle* cannot be directly applied in a common agency context.<sup>28</sup> Solving this problem takes advantage of the fact that firm 1 is not able to screen consumers with respect to type dimension  $\theta_2$  and *vice versa*. This leads to a redefinition of the “effective types” of consumers, where we account for the substitution effect between products 1 and 2, as well as for consumers' optimal responses to changes in the pricing of the competitive firm. The equilibrium distribution of the “effective” willingness to pay for a particular service is now also affected by the pricing decision of the competing firm. The equilibrium distribution of types will therefore be endogenously determined by the nonlinear tariffs offered by the competing firms and *vice versa*. The sufficient statistic that captures consumers heterogeneity of preferences for product 1 is:<sup>29</sup>

$$z_1 = \theta_1 + \frac{\kappa \theta_2}{b_2 + \gamma_2}, \quad (15)$$

### 3.5.2 Joint Distribution of Effective Types

In order to solve this model in closed form we assume that  $z_1$  and  $z_2$  have a flexible Sarmanov distribution with the following density function:<sup>30</sup>

$$f(z_1, z_2) = \prod_{i=1}^2 \frac{\left[1 - \left(\frac{z_i - \underline{z}_i}{\bar{z}_i - \underline{z}_i}\right)\right]^{\lambda_i - 1}}{\lambda_i (\bar{z}_i - \underline{z}_i)} \cdot \left(1 + \varphi \prod_{i=1}^2 \left[z_i - \frac{\lambda_i \bar{z}_i - \underline{z}_i}{1 + \lambda_i}\right]\right), \quad (16a)$$

<sup>28</sup> See footnote 12 of Ivaldi and Martimort (1994), as well as Martimort and Stole (2002, 2003) for a detailed discussion on the effect of the dimensionality of the message space and out-of-equilibrium messages in characterizing the equilibrium of a common agency game.

<sup>29</sup> See discussion in Rochet and Stole (2003, §4) on the aggregation of these types into a sufficient single statistic. Appendix A3 shows how to transform the parameter space defined by  $\{\theta_1, \theta_2, \kappa\}$  into  $\{z_1, z_2\}$  where  $z_i$  represents the type associated to one single firm. The advantage of this approach is that it captures the effect of the substitution pattern between goods through  $\kappa$ , together with the effect of pricing of the competing firm 2 on the distribution of consumers that purchase good 1.

<sup>30</sup> See Sarmanov (1966) as well as Johnson, Kotz, and Balakrishnan (2000, §44.7). Lee (1996) studies the case leading to marginal beta distributions of interest in our model, as well as multivariate extensions that would allow to extend our duopoly model to oligopoly markets with any given number of competing firms. Appendix A4 further discusses this choice of multivariate distribution for the types.

which leads to the following marginal Burr type XII distributions:

$$z_i \sim F_i(z_i) = 1 - \left[ 1 - \left( \frac{z_i - \underline{z}_i}{\bar{z}_i - \underline{z}_i} \right) \right]^{1/\lambda_i}, \text{ for } \underline{z}_i < z_i < \bar{z}_i. \quad (16b)$$

This particular joint distribution of types does not only contribute towards the robustness of our results in the presence of correlated types but also ensures the tractability of the duopoly solution. Since each firm is going to solve the optimal mechanism  $\{T_i(z_i), x_i(z_i)\}$ , payments and purchases will be a function of  $z_i$  and its marginal distribution only. Notice also that the aggregation rule (15) defines the distribution of  $z_1$  as the convolution of the distributions of  $\theta_1$  and  $\theta_2$ .<sup>31</sup> The major advantage of the Sarmanov distribution is that it allows for general correlation patterns among types—regardless of whether they are identifiable or not with our data—and at the same time, it ensures that the monopoly and duopoly solution are comparable because its marginal distributions are Burr type XII as in the monopoly case.

Another important feature of the adopted approach is that it allows for different values of  $\lambda_i$  for each firm. This is an important departure from Ivaldi and Martimort (1994) that may avoid potential misspecification bias (see discussion in Appendix A4). Parameter  $\lambda_i$  is closely related to the non-homogeneous markup that competing firms charge to different customers. By allowing a different value of this parameter for each firm, we can identify carrier-specific effects on markups associated to the *wireline* and *nonwireline* firms. The difference in markups could perhaps be explained by something additional to the lead of the *wireline* carrier such as some brand loyalty or valuation of their expertise—since they also offer local telephone service in the area—or perhaps reflect a better knowledge of the communications behavior of customers in such market. Allowing for different  $\lambda_i$  enables us to identify which firm charges relatively higher markups to different types of customers. We can also test whether identical pricing is rejected or not by the data rather than imposing such assumption to conduct the estimation.

### 3.5.3 Solution of the Duopoly Pricing Game

The solution of the nonlinear competition pricing problem is a pair of nonlinear tariffs that are best response to each other firm’s nonlinear tariff strategy. Furthermore, in equilibrium, these tariffs are

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<sup>31</sup> Miravete (2002a) shows in general that convolution distributions as (16b) inherit the increasing hazard rate property necessary to ensure a separating equilibrium of the screening game as long as the distributions of its components,  $\theta_1$  and  $\theta_2$  in our case, have both increasing hazard rates. The particular distribution (16b) has the increasing hazard rate property as long as  $\lambda_i > 0$ , and can be understood as such convolution distribution. Observe that all expressions for the duopoly solution include those of the monopoly phase. To obtain the latter, we just need to consider that the distribution of  $\theta_2$  is degenerate. Similarly, if these products are independent,  $\kappa = 0$ , then  $z_1 = \theta_1$ , and the distribution (16b) becomes identical to (3).

quadratic in consumption as in (14). Appendix A3 offers a detailed derivation of the equilibrium nonlinear tariffs that leads to the following pair of functions (for firm 2 just reverse the indices):

$$T_1(z_1) = \underline{\mathcal{W}}_1 + \frac{c_1(Y, \mathbf{W}_1) + \lambda_1 \left[ \bar{z}_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} \right]}{1 + \lambda_1} x_1(z_1) - \frac{\lambda_1}{2(1 + \lambda_1)} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1^2(z_1), \quad (17a)$$

$$x_1(z_1) = \frac{(1 + \lambda_1)z_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} - c_1(Y, \mathbf{W}_1) - \lambda_1\bar{z}_1}{\left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right]}. \quad (17b)$$

### 3.5.4 Using Nash Equilibrium to Identify Structural Parameters

The similarity between the duopoly solution (20) and the monopoly solution (9) allows us to quickly review the identification issues relevant for the estimation of this nonlinear pricing competition model. Thus, we again invoke an error of approximation argument to add the stochastic structure to the theoretical model. Indeed, competing firms also offer a low number of tariff plans to implement their optimal nonlinear tariff. Therefore, we fit the actual tariff payments with a quadratic polynomial defined on airtime usage as in (8) for each competing firm in each market and time period. Equating coefficients we get (again, reverse the indices for firm 2):<sup>32</sup>

$$\alpha_1 = \underline{\mathcal{W}}_1; \quad \beta_1 = \frac{c_1(Y, \mathbf{W}_1) + \lambda_1 \left[ \bar{z}_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} \right]}{1 + \lambda_1}; \quad \gamma_1 = -\frac{\lambda_1}{(1 + \lambda_1)} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right]. \quad (18)$$

Thus, the quadratic definition of tariff (14) requires that some constraints among the structural parameters hold for each nonlinear tariff to be an optimal best response to the nonlinear tariff of the other firm. We are primarily interested in the indexing parameters of the distribution of types. After estimating the degree of concavity of both tariffs,  $\gamma_i$ , equation (18) solves for these structural parameters as follows:<sup>33</sup>

$$\lambda_i = \frac{-\gamma_i}{\gamma_i + \left[ b_i - \frac{\kappa^2}{b_j + \gamma_j} \right]} \geq 0, \quad \text{for } i, j = 1, 2. \quad (19)$$

Observe that in order to recover the structural estimate of  $\lambda_1$  we need to account for the concavity of both tariffs as both,  $\gamma_1$  and  $\gamma_2$ , appear in equation (19). Given estimates of  $b_1$  and  $b_2$  (to be addressed in Section 4), equation (19) implements the nonlinear restrictions between  $\gamma_1$  and  $\gamma_2$  that need to be fulfilled for each tariff to be the best response to each other firm's tariff, *i.e.*, to implement

<sup>32</sup> Observe that the corresponding relations for the monopoly case are obtained by making  $\kappa = 0$ , thus leading to the coefficients of the quadratic solution (9).

<sup>33</sup> As in Ivaldi and Martimort, (1994) functional form assumptions ensure that there is an equilibrium in quadratic strategies. However the possibility of an equilibrium in more general strategies cannot be ruled out.

the Nash equilibrium perfection in quadratic tariffs.<sup>34</sup> Next, after making  $x_1(\bar{z}_1) = \mathbb{X}$ , the upper bound of the effective type can be written as:

$$\bar{z}_1 = c_1(Y, \mathbf{W}_1) + \frac{\kappa\beta_2}{b_2 + \gamma_2} + \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] \mathbb{X}. \quad (20)$$

Once we substitute this expression into the definition of  $\beta_i$  in (18), and making use of the definitions of  $\gamma_1$  and  $\gamma_2$  we can easily recover the estimate of the marginal cost as:

$$c_i = \beta_i + \gamma_i \mathbb{X}, \quad \text{for } i, j = 1, 2. \quad (21)$$

Finally, we need to address the issue of participation. As in the monopoly case, we allow for the possibility of a nonnegative rent at the bottom. Thus, analogously to the monopoly case,  $\alpha_1 = \mathcal{W}(z_1^\circ) = \underline{\mathcal{W}}_1 \geq 0$ . Again, the lowest type  $z_1^\circ$  is indifferent between purchasing product 1 or not. Therefore,  $\mathcal{W}(z_1^\circ) = \underline{\mathcal{W}}_1 \geq 0$  whenever  $x_1(z_1^\circ) = 0$ , leading to a cutoff type given by:

$$z_1^\circ = \beta_1 + \frac{\kappa\beta_2}{b_2 + \gamma_2}. \quad (22)$$

## 4 Estimation

In this section we first describe how to estimate the model. We pay particular attention to the identification of the remaining structural parameters not discussed in Section 3. We then present the distribution of the structural parameters and some welfare comparisons. We finally show how the structural parameters, markup, and welfare magnitudes vary across markets and time.

### 4.1 Descriptive Statistics and Tariff Parameters

Quarterly data include demographic and economic information, as well as tariff information as offered by firms by the beginning of each quarter.<sup>35</sup> In Appendix A1 we define all variables and report their mean value for the pool sample. Table 1 details these averages for each year of the sample. Table 2 further analyzes the data by presenting a histogram of each variable.

⇒ INSERT Table 1 and Table 2 ⇐

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<sup>34</sup> From this expression it is evident that in equilibrium the degree of concavity of each tariff –given by  $\gamma_1$  and  $\gamma_2$ – must keep some relationship with the underlying structural parameters of the model. Our approach assumes that such tariff shape restrictions –represented here by equation (19)– hold, and thus the Nash perfection condition helps to recover the rest of structural parameters. However, the scale of the structural parameters has to be such that  $\lambda_i \geq 0$  when tariffs are concave as in our data. This requires that the units of measurement of the airtime usage are appropriately re-scaled to provide meaningful results. These scale issues are addressed in Appendix A6.

<sup>35</sup> In some few cases we have more than one observation per firm and quarter. We decided to take only the earlier observation to avoid promotions and sales common in Christmas and summer.

We first obtain the tariff parameters  $\{\alpha_i, \beta_i, \gamma_i\}$  for every firm–market–time tuple. For each firm, we fit the total payments of each active tariff with a quadratic polynomial on a uniform grid of airtime according to equation (8). For the monopoly phase this is just a single equation estimation for each market and time. For the duopoly phase we estimate a system of equations for the tariffs of the two competing firms. This procedure generates a distribution of estimates for  $\{\alpha_i, \beta_i, \gamma_i\}$ . In particular we obtain 479 estimates of  $\{\alpha_1, \beta_1, \gamma_1\}$  for the monopoly phase and 551 estimates of  $\{\alpha_1, \beta_1, \gamma_1\}$  and  $\{\alpha_2, \beta_2, \gamma_2\}$ , respectively for the duopoly phase.<sup>36</sup> The average of these estimates for monopoly, duopoly *wireline*, and *non-wireline* are reported at the top of Table 5.

The quadratic polynomial approximation used to summarize the basic features of tariffs does not induce misspecification bias because tariffs are indeed concave. Table 3 reports the average monthly fee and marginal charge of the different tariff plans. Subscribers to tariff plans with higher monthly fees enjoy lower charges per minute. Entry of the *nonwireline* firm appears to trigger an important reduction of the monthly fee of the plans offered by the *wireline* firm, although marginal charges remain quite stable. The lowest marginal charge appears to be within the range of 30–35 cents per minute of peak airtime.

⇒ INSERT Table 3 ⇐

Once we have obtained the empirical distributions of the tariff parameters,  $\{\alpha_i, \beta_i, \gamma_i\}$ , we can recover the structural parameters  $\{c_1, \lambda_1, \theta_1^\circ\}$  for monopoly and  $\{c_1, \lambda_1, z_1^\circ, c_2, \lambda_2, z_2^\circ\}$  for duopoly, according to the nonlinear identification restrictions of Section 3. We compute these estimates for each market and quarter of our sample. Some of these structural parameters also depend on the value of other structural parameters that we have so far neglected: the slopes of the linear demand functions,  $b_i$ , and the substitution parameter  $\kappa$ . These are difficult parameters to identify empirically because of lack of individual consumption data. The following subsection explores the empirical identification of these parameters.

## 4.2 Further Identification Issues

Our model includes the following nine structural parameters:  $\{c_1, c_2, \lambda_1, \lambda_2, z_1^\circ, z_2^\circ, b_1, b_2, \kappa\}$ . We have so far identified six of them, although  $\{c_1, c_2, \lambda_1, \lambda_2, z_1^\circ, z_2^\circ\}$  still depend on the values of the other structural parameters  $\{b_1, b_2, \kappa\}$ . We will use the available data to estimate these parameters. Regrettably, these estimates are obtained using a very few observations that include market share

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<sup>36</sup> We fit the quadratic polynomials after normalizing both airtime usage and tariff payment. Airtime usage is re-scaled to the unit interval by dividing by  $\bar{X} = 500$ . Similarly, tariff payments are re-scaled to the unit interval as well by dividing actual payments by  $T(\bar{X}) = 270$ . This double normalization re-scales the magnitudes of the estimates to make them independent of the units of measurement of airtime usage and money payments. This ensures that restrictions discussed in Appendix A6 are not violated because of the particular units used to measure airtime consumption and payments.

information. Our conclusions will therefore be contingent on the validity of these estimates, but we prefer to make use of the data to identify them than assuming some arbitrary values for  $\{b_1, b_2, \kappa\}$ . We however explore two alternative identification assumptions that define two scenarios, as each identification assumption will generate a different distribution for the structural parameters  $\{\lambda_1, \lambda_2, z_1^\circ, z_2^\circ\}$ .

We estimate parameter  $\kappa$  by making use of only 22 observations in 8 markets between 1985 and 1987 that include the number of subscribers of the two competing firms.<sup>37</sup> Our basic identifying assumption is that conditional on observed demand characteristics, consumers do not differ in the substitution pattern among services. Since market shares of each product will coincide with the proportions in which consumers subscribe to each service, the estimation of  $\kappa$  is then obtained by evaluating the demand system at the average marginal tariffs for this small sample of markets with subscription information.<sup>38</sup>

$$p_1 = a_1 - b_1 x_1 + \kappa x_2, \tag{23a}$$

$$p_2 = a_2 - b_2 x_2 + \kappa x_1, \tag{23b}$$

where  $p_1, p_2$  are simple sample averages of the marginal tariffs offered by each firm in the range of 0 – 500 minutes;  $x_1, x_2$  are the subscribers to the *wireline* and *nonwireline* carrier, respectively; and  $a_1, a_2$  are the net effects of non-price related elements on demand. We further assume that  $b_1 = b_2$  and use the population and few cost variables of these SMSAs as instrumental variables in the system estimation of (23a) – (23b) in order to control for endogeneity bias as individual consumption and marginal tariffs effectively paid are determined simultaneously.

⇒ INSERT Table 4 ⇐

The first column of Table 4 shows these system estimates for Scenario 1. On average consumers' willingness to pay is higher for the *nonwireline* carrier. The estimate of  $\kappa \simeq -1.56$  together with  $b_1 = b_2 = 4.71$  reveals that consumers regard *wireline* and *nonwireline* services as far substitutes to each other (the degree of substitution  $\kappa^2/(b_1 b_2) = 0.11$ ). Scenario 2 assumes independent goods, thus breaking the strategic linkage between the design of the tariffs by these competing firms. Considering these two scenarios will allow us to evaluate how markups and welfare measures change with  $\kappa$ , as well as the response of our estimates for different degrees of substitutability among competing products.

⇒ INSERT Table 5 ⇐

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<sup>37</sup> These markets are among the earliest to operate and among the largest in population size.

<sup>38</sup> These are indeed the first order conditions of consumers' maximization of utility (1) constrained by some individual budget constraint.

We use these demand estimates to recover the structural parameters  $\lambda_i$  and  $z_i^\circ$  under each alternative hypothesis (marginal costs do not depend on the estimated value of  $\{a_1, a_2, b, \kappa\}$ ). Given the estimates of Table 4, and making use of the identification restrictions described in Section 3 for each market and quarter, we obtain an empirical distribution of structural parameters  $\{\alpha_i, \beta_i, \gamma_i, c_i, \lambda_i\}$ . Table 5 reports the mean and standard deviation of the distribution of these estimated structural parameters for the monopoly phase, as well as for each of the competing firms during the duopoly phase.

### 4.3 Welfare Analysis

Similarly to any other empirical oligopoly equilibrium model, we have jointly estimated marginal costs and markups. The major difference in the present paper is that such markups are not uniform for different use levels because of the existence of quantity discounts that characterize the nonlinear tariffs offered by these cellular carriers. Our setup allows us to summarize the determinants of these different markups in a single statistic,  $\lambda_i$ , for each firm. Thus, differentiating (17a) with respect to  $x_1$  and substituting (17b) we get that the price markup is:

$$\frac{p_1(z_1) - c_1}{p_1(z_1)} = \frac{\lambda_1(\bar{z}_1 - z_1)}{c_1 + \lambda_1(\bar{z}_1 - z_1)}, \quad (24)$$

which is increasing in  $\lambda_1$ . Thus, the larger  $\lambda_i$  is, *i.e.*, the larger the proportion of high valuation consumers is, firms charge a uniformly higher markup to every consumer type except the highest one,  $\bar{z}_i$ . This is necessary for incentive compatibility to hold. This more powerful mechanism with higher markups reduces the incentives of high valuation customers (now more numerous) to imitate low valuation customers. Such deviation is not in their self-interest because they benefit from a minimal tariff reduction only when they reduce their consumption very substantially.

However, it is not clear what the total effect on expected consumer surplus, profits, and welfare would be. This is because the larger value of  $\lambda_i$  together with a lower  $c_i$  and the possibilities of substitution to the other product determine in the end the participation of consumers through  $z_i^\circ$ . We thus first analyze how the basic structural parameters are correlated with observable market specific characteristics, and later study how optimal markups and welfare magnitudes behave. We finally compute empirical comparative statics of the different parameters of the model.

#### 4.3.1 *Marginal Cost, Market Penetration, and Distribution of Individual Demands*

We are not going to suggest any particular theory to explain how different features of these markets affect marginal costs, market penetration, and the ratio of high-to-low valuation customers. We feel however, that the market specific information available can be used to evaluate the effect of

introducing competition in the cellular industry. Since it is not common to have information available on how problems of asymmetric information are solved in multitude of markets, this empirical analysis may provide useful information about how firms and consumers condition their pricing and consumption decisions on characteristics whose distribution may be known to all relevant agents.

As we have repeatedly argued, tariffs actually offered by either monopolists or competing firms carry important information on the actual marginal cost that firms face, the distribution of their demand, as well as the determinants of market penetration. These three elements have been summarized in or model in the distribution of the price independent indicators  $\{c_i, \lambda_i, z_i^\circ\}$ .

In order to analyze how these structural parameters vary with the features of different markets we regress the estimates of the structural parameters on demographics, market specific variables and cost indicators according to equations (25) – (27) presented below. Identification of the effect of different cost magnitudes is achieved through variation of the marginal tariff for the very high level of consumption across markets and time where these carriers operate. The analysis focuses on those markets where a phase of monopoly preceded the entry of the *nonwireline* carrier. There are 834 firms–market–quarter observations. Simple OLS regressions are presented in Table 6. We regress the logarithm of  $\{c_i, \lambda_i, z_i^\circ\}$  on demand and cost related characteristics of markets over time. Thus, most of the reported estimates are semi–elasticities.

⇒ INSERT Table 6 ⇐

In accordance with equation (5) we consider that firms’ marginal costs are a function of firms’ output capacity as well as of the input prices and related firm specific cost indicators:

$$\ln(c_i) = \psi_{i0}Y + \sum_{k=1}^K \psi_{ik}W_{ik} + \varepsilon_{c_i}, \quad (25)$$

so that we can identify the possibility of economies or diseconomies of scale. The set of input prices includes the price of energy, labor, office space rent, and operating costs. We also include the population density of each market to control for the effect on costs of a dispersed base of customers that requires numerous antenna sites with low powered transmitters to have proper coverage and quality of transmissions. All input price related estimates are elasticities as all cost indices used as regressors are measured in logarithms (since they are originally reported as indices).

The picture arising from the results of column 1 of Table 6 is that the entrant faces a significant cost disadvantage. It is interesting to notice that although marginal costs tend to grow over time, there appears to be some significant learning as indicated by the negative effect of the market age and the time lead of the *wireline* over the *nonwireline* carrier in entering a particular market. It also appears that capacity constraints are not binding since marginal costs can be considered independent of scale. The need to finance these important investments is also an

important element of costs –something that the literature on cellular business has long recognized in the case of small independent entrants–, as are the operating costs of running an office. Surprisingly, there appears to be significant diseconomies of density. The advantage of building and maintaining sites in a close area does not compensate the high cost of finding adequate places where to set up antenna sites. Health concerns and regulations on where and how to install these sites were very frequent.

In the second column of Table 6 we address how the ratio of high–to–low valuation customers vary across markets. This is a price independent measure of the magnitude of asymmetric information of each market. Although firms do not know the type of a particular consumer, they can condition the distribution on a set of observable socioeconomic magnitudes likely to be related to the average willingness to pay for cellular services. These variables differ from market to market. For instance, in markets with a larger proportion of high income households or with a large number of business, we may perhaps expect to observe a higher proportion of large customers. These arguments indicate that the value of  $\lambda_i$  can be related to observable market specific characteristics. In order to disentangle this market specific heterogeneity –in a very reduced form manner–, we assume the following specification for  $\lambda_i$  in order to ensure that its value is always positive, so that the pricing solution is always well behaved:

$$\ln(\lambda_i) = \sum_{j=1}^J \phi_{ij} Z_{ij} + \varepsilon_{\lambda_i}, \quad (26)$$

where  $\mathbf{Z}_i = \{Z_{i1}, \dots, Z_{iJ}\}$  includes market specific characteristics such as population, growth, median income, average commuting time, and number of business in the area among others.

The number of business affects  $\lambda_i$  positively. The same happens in markets with fast growing population, and as market matures which, perhaps capture the positive effects of learning about this experience good. The effect of income appears to be nonlinear as both income and the percentage of population below poverty level have a negative effect on  $\lambda_i$ . Another reasonable result (frequently found in the telecommunications literature) is that younger customers tend to be among the most intensive users. Network externalities are not present. This is a result that should also be expected as these markets are in their infancy. This result is consistent with the fact that in the U.S. there is no distinction in the numbering of fixed and cellular phones, and also consistent with the fact that the receiver also pays, which triggered consumers to switch off their telephones when they are not calling another party.

Parameter  $\lambda_i$ , and therefore the markup, is higher on average in the duopoly phase as it becomes smaller over time. The latter is the expected effect following the establishment of an effective competition regime. Accounting for the total effect of opening markets to competition is more involved. On the one hand the entrant faces some cost disadvantage thus inducing an increase

of the average tariff. But in addition competition incorporates new customers by effectively lowering the most affordable subscription fee. Firms may then find optimal to increase their markups to discourage the numerous high valuation customers from deviating from their optimal intensive use of the cellular service.

Finally, the value of consumers' outside options may also vary across markets and time periods and thus, the marginal type will also vary across markets and time. Similarly to the previous specifications for the other structural parameters, (26) and (25) respectively, we assume the following relationship between market demographics and determinants of market participation:<sup>39</sup>

$$\ln(Z_i^\circ) = \sum_{g=1}^G \zeta_{ig} M_{ig} + \varepsilon_{Z_i}, \quad (27)$$

where  $\mathbf{M}_i = \{M_{i1}, \dots, M_{iG}\}$  includes all demographics  $\mathbf{Z}_i$ , all cost variables  $\mathbf{W}_i$ , as well as some other variables, potentially more closely related to subscription decisions than to usage such as crime indices to control for precautionary motives to subscribe to the cellular service.<sup>40</sup>

The widening of the market is perhaps the most important effects of competition in terms of welfare. Entrants increase their base of subscribers among low valuation customers. Finally, we find some support for the marketing strategies followed by cellular carriers at the time of introduction of cellular telephones. Although very small, it appears that precautionary motives might be behind subscription to the cellular service. Subscription is more frequent in areas of high but non-violent crime.

#### 4.3.2 *Welfare Evaluations*

The change of markups after the *nonwireline* carrier enters the market does not capture all the potential gains from competition. Two additional effects are present: there is an increase in variety and a larger market penetration due to the well documented lower cost of equipment and the overall reduction of subscription charges. In order to evaluate welfare magnitudes using our estimates, we simulate a distribution of consumers according to the estimated values of  $\lambda_1$  and  $\lambda_2$  for each

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<sup>39</sup> Estimating a similar regression for  $\alpha_i$  is redundant because for well behaved, concave, tariffs, as the ones predicted by this model for  $\lambda_i \geq 0$ , there is a one-to-one inverse relationship between the intercept of any tariff option,  $\alpha_i$ , and the slope of such tariff option,  $\beta_i$ .

<sup>40</sup> Crime may induce subscription to cellular service as a way to facilitate access to emergency numbers. Murray (2002, p.212–213) reports that the marketing strategies of this early market emphasized the personal security added by carrying a handy cellular phone. Crandall and Waverman (2000) and Riordan (2002, §2) documents that climatology and location affects the decision to subscribe to local telephone service. Regarding fixed telephony, the “cold states” effect has been interpreted as the greater need that customers have to communicate while perhaps having to remain at home. The negative effect of the precipitation and temperature variables in Table 6 indicates that in those states there appears to be an added demand component in extreme weather conditions, perhaps because of the convenience and dependability of cellular telephony.

market and quarter. Table 7 presents the expected dollar value of these simulations per customer for different scenarios.<sup>41</sup>

⇒ INSERT Table 7 ⇐

Comparing the first and second column we can evaluate the magnitude of the effect of competition. These are unconditional magnitudes, and in this case they involve comparing welfare components at different moments in time. The average \$1.62 welfare increase per customer due to the introduction of competition represents an average increase in welfare of almost \$2.6m per market. Competition benefits consumers and reduces profits slightly. The average consumer surplus increase per market amounts to \$4.4m while profits get reduced by \$1.8m. While the average marginal tariff is similar in both market configurations, their distribution has changed, and competition induces many new customers to sign up for service by lowering fixed fees.

We can easily evaluate other scenarios. In the third column of Table 7 we report the magnitude of the welfare measures for the case of  $\kappa = 0$ , *i.e.*, when cellular companies become local monopolists offering independent products. We recompute all the equilibrium strategies for  $\kappa = 0$  and simulate the optimal purchase and payments for the distributions of  $z_1$  and  $z_2$  given by our estimates of  $\lambda_1$  and  $\lambda_2$ . In the remaining columns of Table 7 we proceed similarly but assume that either the distributions of tastes or, alternatively the marginal costs, are identical to either those of the *wireline* or the *non-wireline* carrier, respectively. The second column of Table 7 now serves as reference since all these evaluations are made for the duopoly phase.

The case of  $\kappa = 0$  is interesting because firms behave as local monopolists in such a scenario. Markets are then independent and the strategic effect of nonlinear pricing competition disappears. The absence of a substitute service increases the market power of sellers. They increase their profits by about \$2.62 per customer but at the cost of reducing the average consumer surplus by \$78.49 not only because of higher tariffs that follow the disappearance of a substitute product, but also because of the reduction of variety itself. The value of the existence of a second variety amounts to \$121m in the average market, an important magnitude when compared to the modest welfare gain due to competition presented above.

The remaining cases are counterfactuals of symmetric solutions. The fourth and fifth column assume that the distribution of both types are identical at the estimated value of either  $\lambda_1$  or  $\lambda_2$ , respectively. In both cases, imposing similar markups does not increase profits relative to the duopoly solution. This calls into question Busse’s (2000) result on the effectiveness of identical pricing as a way of firms to collude. On the contrary, consumer surplus always increase although

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<sup>41</sup> We performed all the simulations assuming that the correlation between types,  $\rho(z_1, z_2) = 0.8$ . We have chosen this positive number to acknowledge that tastes are likely correlated. Still, we computed the equilibrium for  $-1 \leq \rho \leq 1$ . The largest elasticity of any welfare magnitude with respect to changes in  $\rho$  is only 0.0008.

more importantly in the second case (fifth column) as the average  $\lambda_2$  is lower than the average  $\lambda_1$ , thus reducing the average markup relative to the non-symmetric duopoly solution.

The last two columns of Table 7 conduct a similar analysis for symmetric marginal costs. In the sixth column, expected profits increase because the marginal cost of the entrant has been reduced to the level of the *wireline* carrier. The effect of reducing the cost disadvantage between the *wireline* and *nonwireline* carriers amounts to \$9.36m in profits in an average market. The opposite result happens if the marginal cost of the incumbent equals that of the *nonwireline* firm.

### 4.3.3 Comparative Statics

The information of Table 7 measures the welfare effect of discrete changes of structural parameters to evaluate some counterfactuals regarding symmetric solutions. In order to account for the relative impact of changes of the different structural parameters of the model, Table 8 computes welfare, sales, and tariff related elasticities with respect to all of them. These elasticities are evaluated in the neighbourhood of the duopoly solution. Each parameter is allowed to change within a  $\pm 5\%$  of its estimated value. The model is then solved for the new values and the corresponding welfare, sales, and tariffs computed and averaged out across duopoly markets and time.

⇒ INSERT Table 8 ⇐

Most comparative statics results have the intuitive sign. Table 8 shows that there is a negligible response to changes in the correlation among type dimensions  $\theta_1$  and  $\theta_2$ . As products become closer substitutes (larger  $\kappa$ ) firms lose market power and consumers keep a larger proportion of their informational rent. The overall effect is an important positive effect on welfare. Similarly, any increase of  $\lambda_1$  or  $\lambda_2$  can be interpreted as shifting to environments with “more” asymmetric information about consumers’ tastes. The effect of these changes (ignoring changes of marginal costs and participation mentioned in the previous section) is to increase both consumer surplus and profits although a 1% change in  $\lambda_2$  leads to changes in these magnitudes that more than double a 1% change in  $\lambda_1$  of the same sign. Increases of marginal costs  $c_1$  and  $c_2$  reduce both consumer surplus and profits. The effect of  $c_1$  is more important on consumer surplus but  $c_2$  affects more to total profits.

### 4.3.4 Distributional Issues

Does competition affect all consumers evenly? In the presence of nonlinear pricing, competition may have quite different effects on heterogeneous consumers with substantially different elasticity of demand. Some studies, such as those of Borenstein (1989) and Busse and Rysman (2001), have accounted for differences in pricing across customers. The structural model of the present

paper suggest an operationally feasible framework where this question can be addressed easily. Furthermore, since we observe these cellular markets under monopoly and duopoly, we can also study the effect of entry on the optimal markup for different usage levels.

⇒ INSERT Table 9 ⇐

Table 9 presents the average tariff payment, marginal rate, and price markup at five different airtime levels as induced by our estimates. While total tariff payments and marginal rates could be computed directly from the firms schedules, the model allows us to easily recover the markup at each usage level by providing a structural interpretation of the quadratic polynomial approximation to the concave tariff offered by firms. Thus, by making use of (14) and (21), the price–markup (24) can simply be written as:

$$\frac{p_i - c_i}{p_i} = \frac{\gamma_i (\bar{X} - x_i)}{\beta_i + \gamma_i x_i}. \quad (28)$$

To further investigate the effect of introducing competition in markets where pricing is characterized with quantity discounts we compare the pricing under monopoly with the duopoly pricing both immediately after the entry of the second carrier and at the end of the sample. Although our model is strictly static, and we do not consider intertemporal linkages of strategies –only because the solution of such model is infeasible–, we believe that such comparison may shed some light on the dynamics of pricing and provide with some guidance on what we should expect after entry is allowed in industries similar to cellular telephony.

Tariffs are lower both in the short run –immediately after the entry of the *nonwireline* carrier–, and in the long run –at the end of our sample–. The reduction is generalized across all airtime usage levels, although it is more important for low valuation customers than for intensive consumers. The monthly bill of 50 minutes of airtime consumption is 21% lower in the short run and 16% lower in the long run for those customers who remain subscribed to the *wireline* carrier. At the other end, consumers of 450 minutes a month enjoy a tariff reduction of only 9%–10% in the short and long run, respectively. As for marginal tariffs, the effect of competition is just the opposite. Large customers enjoy more important tariff reductions both in the short and long run. Competition induces further participation by lowering the fixed monthly fee, while it also prompts an increase in consumption by passing to large consumers a significant fraction of the efficiency gains attained. In general, this effect intensifies over time.

While tariffs and marginal rates are quite similar (but not identical) across firms, markups are quite different. Markups, on the other hand, show quite a different behavior over time. After competition is introduced, the markup of the *wireline* carrier increases between 55% for low users to 63% for high users. On the contrary, *nonwireline* companies, endowed with a significant cost disadvantage, tend to price their service in line with the previous pricing strategy of the incumbent.

As time passes, markups bounce back and the initial increase is halved for *wireline* carriers while *nonwireline* carriers offer, in general, a markup at the end of our sample that is lower than the one offered by the incumbent at the end of the monopoly phase. As in the case of marginal rates, markups are smaller for larger consumers, thus being in accordance with the prediction of the theory.

#### 4.3.5 The Cost of Not Awarding Both Licenses Simultaneously

To conclude we relate the total gains of introducing competition to all observable market characteristics (as these effects depend both on demand and cost arguments). These results are summarized in Table 10. It appears that gains in welfare were most common in large and densely populated markets with few business.

⇒ INSERT Table 10 ⇐

Perhaps the most interesting result that Table 10 provides is a first approximation to the cost of delaying the award of the second license through the estimated effect of LEAD on welfare. It was only after the massive number of applications for the *nonwireline* license received in the first round of license awards for the largest markets, that the FCC decided to award all future *nonwireline* licenses through a lottery instead of a comparative hearing procedure. For the markets included in the first tier of awards, the delay was further increased by the applicants repeated court appeals to each FCC administrative decision to award a license.<sup>42</sup>

The welfare effect of an additional month of lead time between the *wireline* and *nonwireline* firms is negative. The dollar value of such monthly loss is \$1.7 per customer ( $-0.0029 \times 578.38$ ). With an average population of 1.6 million potential customers per market, the loss reaches \$2.68m per month. We observe 98 markets where there is some delay between the entry of the *wireline* and the *nonwireline* firm. Arguably, delays were less important for later markets as the FCC moved on to awarding licenses through lotteries, and thus we will assume that delays did not occur in any other market. For these 98 markets there is an average of 19.73 months of delay, thus leading to a total potential cost of about \$5.19bn for not awarding both licenses simultaneously.

Is this an exorbitant estimate of the cost of delay induced by the FCC's *beauty contest*? To put this number into perspective we compare it to the existing estimate of the cost of authorizing the introduction of cellular service in the U.S. Rohlfs, Jackson, and Kelly (1991) have estimated that the delay in licensing cellular telecommunications reached \$86bn. Thus, the additional cost of delay induced by the process adopted to allocate the licenses only added a 6% to the monthly cost of introducing the cellular technology into the market.

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<sup>42</sup> The chaos surrounding the award of these licenses and the FCC's failure to capture the economic rents of these scarce rights has been extensively studied. See Hazlett and Michaels (1993), Murray (2002), and Shew (1994).

## 5 Policy Evaluations: Alternative Pricing Strategies

Flat tariffs are popular pricing strategies in telecommunications. Although there was an evident shortage of broadband capacity in the late 1990s, some European governments required internet providers to offer a simple flat tariff aiming to widen the use of internet. This somewhat populist policy lowered the marginal tariff that consumers faced, thus fostering usage among subscribers. However, the optimal single flat tariff may be high enough that excludes a significant portion of customers, and thus, the welfare effects of these government driven policy restriction are unclear. For the early cellular industry of our study we show that such policy would have been very costly.

In this section we use our model to address welfare and profit evaluations of pricing policy restrictions regardless of whether they are the consequence of a government regulation or of a common practice in the industry. The major advantage of a structural estimation method is the possibility to evaluate strategies different from the ones actually used by the firms. Here we address the welfare and profitability effects of alternative pricing strategies. These alternative pricing strategies may respond to hypothetical restriction imposed by regulators on firms nonlinear pricing strategies. They could also be the outcome of a particular game, and thus freely chosen by the firms.<sup>43</sup> In particular, we consider three alternatives. Firms may be restricted to offer either a single two-part tariff, a single rate without fixed fee payment (linear pricing), or a flat tariff. Without loss of generality, if a firm wants to offer the optimal two-part tariff to account for the heterogeneity of his customers, it will choose the fixed fee  $A(p_i, z_i^\circ)$  and marginal rate  $p_i$  in order to maximize:

$$\Pi_i = A(p_i, z_i^\circ)[1 - F_i(z_i^\circ)] + (p_i - c_i) \int_{z_i^\circ}^{\bar{z}_i} x_i(p_i, z_i) f_i(z_i) dz_i, \quad \text{for } i = 1, 2. \quad (29)$$

The fixed fee that extracts all the informational rent of the marginal consumer type who is indifferent to subscribe the cellular telephone service is:

$$A(p_i, z_i^\circ) = \int_{p_i}^{\infty} x_i(\varepsilon, z_i^\circ) d\varepsilon, \quad \text{for } i = 1, 2. \quad (30)$$

The optimal two-part tariff is found by solving the following two conditions:<sup>44</sup>

$$\frac{\partial \Pi_i}{\partial p_i} = \int_{z_i^\circ}^{\bar{z}_i} \left[ (p_i - c_i) \frac{\partial x_i(p_i, z_i)}{\partial p_i} + \frac{1 - F_i(z_i)}{f_i(z_i)} \frac{\partial x_i(p_i, z_i)}{\partial z_i} \right] f_i(z_i) dz_i = 0, \quad (31a)$$

$$\frac{\partial \Pi_i}{\partial z_i} = \int_{p_i}^{\infty} \left[ (\varepsilon - c_i) \frac{\partial x_i(\varepsilon, z_i^\circ)}{\partial p_i} + \frac{1 - F_i(z_i^\circ)}{f_i(z_i^\circ)} \frac{\partial x_i(\varepsilon, z_i^\circ)}{\partial z_i} \right] d\varepsilon = 0. \quad (31b)$$

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<sup>43</sup> We will not discuss under which circumstances these simpler tariffs arise as equilibrium features of the model. As we mentioned earlier, only the existence of unobserved billing and marketing costs may explain that firms depart from offering the fully nonlinear tariff. Here we concentrate in a few alternative ways to screen consumer heterogeneity with less powerful mechanisms.

<sup>44</sup> Appendix A8 solves this problem explicitly and discusses how to compute the marginal tariff  $p_i$  and the cutoff type  $z_i^\circ$ . Appendices A9 and A10 also solve the linear and the flat tariff case, respectively.

Once the optimal  $p_i$  and  $z_i^\circ$  are found, equation (30) provides with the solution for  $A(p_i, z_i^\circ)$ . The case of standard linear pricing assumes  $A(p_i, z_i^\circ) = 0$  in (29) and it is characterized by solving (31a) subject to the condition that  $x(p_i, z_i^\circ) = 0$ . Finally, a flat tariff assumes that  $p_i = 0$  in (29). Thus, the cutoff type  $z_i^\circ$  is characterized using a particular version of (31b) with  $p_i = 0$ .<sup>45</sup> We do this starting from the estimated values of  $\{c_1, c_2, \lambda_1, \lambda_2, b_1, b_2, \kappa\}$  in duopoly markets, we compute the optimal two-part, linear, and flat tariffs for these demand, cost, and distribution parameters of consumers' tastes. Each solution involves different average and marginal tariffs as well as different market penetration levels. Then, each solution is evaluated, as in Section 4, using random draws from the corresponding distribution of consumers' preferences given by the estimated  $\lambda_i$  for each firm-market-quarter tuple. The support of this distribution is defined by  $\bar{z}_i$  as in equation (20) and  $z_i^\circ$  according to (31a) – (31b) as discussed in the appendices A8–A10. We therefore control for the effect that each pricing strategy has on participation decisions as well as on consumption due to changes in the marginal tariff that each consumer type faces. Results are presented in Table 11.

⇒ INSERT Table 11 ⇐

The first column of Table 11 reports the expected welfare, sales, and tariff components of the duopoly markets under the optimal nonlinear tariff solution of the present model. This is identical to the first column of Table 7 and it is reported here to ease the comparison with the other pricing strategies. The other columns of Table 11 report these same magnitudes for the simulated cases corresponding to a single two-part tariff, standard linear pricing, and a flat tariff, respectively. Welfare decreases with all simulated pricing strategies as they represent less powerful mechanisms unable to provide the necessary incentives to separate different types of consumers. All of them are actually constrained versions of the fully nonlinear tariff solution of our model.

The worst performing mechanism is the standard linear pricing strategy. Although consumers do not have to pay monthly fees (similar to the popular pre-paid phone cards in today's cellular telephone industry), they have to pay a price per minute that quadruples the average price per minute under fully nonlinear tariff. This reduces sales by almost 90% and billing by 44%. Thus, consumer surplus reaches only 17% of its potential, while profits and welfare amount to 52% and 28% of their values under fully nonlinear tariff, respectively.

The main effect of flat tariff is to exclude a large base of low valuation customers by quintuplicating the fixed fee. Those willing to pay such subscription fee will consume up to their satiation level. Consumers are almost twice better off than under linear pricing, but they only reach 28% of their potential surplus under nonlinear pricing. Profits and welfare are also lower, about 57% and 37% of their potential under nonlinear pricing, respectively.

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<sup>45</sup> Extensive explanations on these issues can be found in sections 4.5, 6.3, 6.4, and 8.3 of Wilson (1993).

Similar results hold if a two-part tariff is used. Here there is a milder exclusion effect due to the smaller increase in the fixed fee and an ambiguous effect of the single marginal tariff since it will induce further consumption by low end consumers but will restrain usage for high valuation customers. Overall it reaches 49% of the consumer surplus and 63% of welfare relative to the potential levels under nonlinear pricing. But perhaps the most remarkable result is that a single two-part tariff achieves 94% of the profit level of nonlinear tariffs. The foregone profits of not screening consumers with more options is very small, about \$9.51 per customer. It is then understandable that firms do not engage in very complex screening mechanisms when potential gains are so limited.

## 6 Concluding Remarks

The major contribution of this paper is to recognize that the shape of the offered tariff carries valuable information about the distribution of consumers' asymmetric information parameters, something that has been overlooked in the empirical literature on nonlinear pricing. The identification of the structural parameters relies on nonlinear transformations of the parameters of the curve that fit the lower envelope of the offered tariff options in such a way that each nonlinear tariff is taken to be the best response to the nonlinear tariff offered by the competing firms and *vice versa*.

The paper provides with strong evidence in favor of a flexible approach that avoids imposing any symmetry assumption in the equilibrium tariffs. We find that the marginal costs of the competing firms are slightly different, providing the incumbent with a relative cost advantage. Similarly, the distribution of taste parameters for the two cellular services are not identically distributed, and the willingness to pay for the *nonwireline* product is "more concentrated" in a area close to the highest valuation possible.

Finally, we want to identify in this last paragraph few directions for future research that may use the framework of this paper as a starting point. One potential extension of the model could address the profits and welfare foregone by not screening consumers optimally, *i.e.*, using a menu of two-part tariffs instead of a fully nonlinear tariff. A second potentially interesting line of research could compare the performance of the present model, based on a common agency approach, with a solution based on an exclusive agency (random participation) approach. Still, the most interesting application would be to apply our model to a different data set where individual purchasing information were available, thus making full use of the predictions of the model, and not only those related to the optimal tariff.

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## Appendices

### • A1: Definition of Variables, Mean Values, and Data Sources

#### *OUTPUT CAPACITY*

CELLS.1	Number of cells owned by the incumbent.	17.12
CELLS.2	Number of cells owned by the entrant.	13.14

*Source:* Cellular Business, various issues, 1984–1988.

#### *DEMAND*

POPULATION	Market Population in thousands.	1,598.53
COVERAGE	$1200 \times \text{CELLS} / \text{POPULATION} (\times 100)$ .	2.24
GROWTH	Average percent growth of population in the 1980's.	1.07
BUSINESS	Thousands of high potential business establishments, including firms engaged in business services, health care services, professional and legal services, contract construction, transportation, finance, insurance, and real estate).	42.24
COMMUTING	Average commuting time in minutes.	25.73
INCOME	Median income in thousand of dollars.	27.87
POVERTY	Percentage of households with income below poverty level.	10.41
POP-AGE	Median age of the population in years.	32.65
EDUCATION	Median number of years of education.	12.55
CRIME	Number of offenses per 100,000 inhabitants. Missing values are interpolated with information from adjacent years.	6,524.50
VIOLENT	Number of violent offenses (murder, nonnegligent manslaughter, forcible rape, robbery, and aggravated assault) per 100,000 inhabitants.	715.99
PROPERTY	Number property offenses (burglary, larceny-theft, motor vehicle theft, and arson) per 100,000 inhabitants.	5,808.51
SVCRIME	Percent share of violent crimes.	10.84
TEMPERATURE	Average quarterly temperature in Fahrenheit degrees recorded by the closest station to each SMSA.	57.47
RAIN	Average quarterly precipitation in inches recorded by the closest station to each SMSA	3.29

*Sources:* 1989 Statistical Abstracts of the United States; U.S. Department of Commerce, Bureau of the Census, using the FCC Cellular Boundary Notices, 1982–1987, available in *The Cellular Market Data Book*, EMCI, Inc. Latitude and longitude of each SMSA is available in the 2000 U.S. Census. Crime is obtained from the Uniform Crime Report, FBI, 1984–1988. Weather data is available on the web, <http://cdiac.esd.ornl.org>, and includes average temperature and precipitation for 1221 stations in the contiguous continental states plus those of Alaska. See Easterling, D.R., T.R. Karl, E.H. Mason, P.Y. Hughes, D.P. Bowman and R.C. Daniels, *United States Historical Climatology Network (U.S. HCN) Monthly Temperature and Precipitation Data. ORNL/CDIAC-87, NDP-019/R3, 1996. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, Oak Ridge, Tennessee.*

*FACTOR PRICES AND COST VARIABLES*

WAGES	Index of average annual wages per employee for the cellular industry. Prior to 1988, wage rates were calculated for communications employees in general. Wages for cellular employees were back-extrapolated over time and across states for the missing years of the cellular index using the general communications index. State-level wage rates are then applied to markets located in each state. The reference is the average wage in New York City during the first quarter of 1986 ( $\times 10$ ).	7.25
ENERGY	State average electricity prices measured in dollars per kilowatt/hour.	1.71
PRIME	Cost of capital; one-period lagged prime lending rate which reflects the costs of financing cellular equipment, a common practice in this industry.	9.35
OPERATE	Building Owners and Managers Association International's index of operating expenses (cleaning, repair and maintenance, administrative costs, utilities, local taxes, security and ground services, office payroll, as well as other leasing expenses associated with running an office) on a per-square-foot basis across U.S. metropolitan areas.	6.47
RENT	Index of average monthly rent per square-foot of office space in each of the cellular markets.	16.04
DENSITY	Thousands of inhabitants per square-mile in each SMSA.	17.59
LEAD	Number of months where the market was served only by a monopolist.	9.41
MKT-AGE	Age of the market in months.	37.07

*Sources:* Bureau of Labor Statistics; U.S. Department of Energy; BOMA Experience Exchange Report: Income/Expense Analysis for Office Buildings, various issues, 1985–1989; and Cellular Price and Marketing Letter, Information Enterprises, various issues, 1984–1988; 1990 U.S. Census.

*TARIFF INFORMATION*

DUOPOLY	Dummy equals one for the duopoly phase.	0.54
PLANS.M	Number of tariff plans offered by the incumbent under monopoly.	2.55
PLANS.1	Number of tariff plans offered by the incumbent under duopoly.	3.58
PLANS.2	Number of tariff plans offered by the entrant under duopoly.	3.53
PAY200.M	Average bill for 200 minutes under monopoly.	105.52
PAY200.1	Average bill for 200 minutes with the incumbent under duopoly.	92.39
PAY200.2	Average bill for 200 minutes with the entrant under duopoly.	94.18

*Sources:* Cellular Price and Marketing Letter, Information Enterprises, various issues, 1984–1988.

• **A2: Nonlinear Pricing by a Monopolist**

For the most part of this Appendix we adapt Tirole (1988, §3.5) to the particular single-product demand case of equation (2). The monopolist offers a contract  $\{T_i(\theta_i), x_i(\theta_i)\}$ . A consumer with taste parameter  $\theta_i$  purchases  $x_i(\theta_i)$  units and pays  $T_i[x_i(\theta_i)]$ . The monopolist's expected profit is:

$$\pi^M = \int_{\theta_i^c}^{\bar{\theta}_i} \{T_i[x_i(\theta_i)] - K - c(Y_i)x_i(\theta_i)\} f_i(\theta_i) d\theta_i. \quad (A.1)$$

Where  $\theta_i^\circ$  is the lowest participating consumer type that defines the proportion of consumers that participate in the market. Let define the rent of a consumer of type  $\theta_i$  as:

$$\mathcal{U}(\theta_i) = \theta_i x_i(\theta_i) - \frac{b_i}{2} x_i^2(\theta_i) - T_i[x_i(\theta_i)]. \quad (\text{A.2})$$

For the marginal consumer  $\theta_i^\circ$ , the individual rationality (*IR*) constraint is:

$$\underline{\mathcal{U}}_i = \mathcal{U}(\theta_i^\circ) = \theta_i^\circ x_i(\theta_i^\circ) - \frac{b_i}{2} x_i^2(\theta_i^\circ) - T_i[x_i(\theta_i^\circ)] \geq 0, \quad (\text{A.3})$$

which holds whenever  $x_i(\theta_i^\circ) = 0$ . Incentive compatibility (*IC*) is a key property because it leads to separating equilibria. Different consumers will purchase different  $x_i$  depending on their type  $\theta_i$ . This is the principle of truth-telling: it must be optimal for a consumer of type  $\theta_i$  to consume  $x_i(\theta_i)$ . This condition can be written globally, *i.e.*,  $\forall \theta_i, \theta_i'$ :

$$\mathcal{U}(\theta_i) = \theta_i x_i(\theta_i) - \frac{b_i}{2} x_i^2(\theta_i) - T_i[x_i(\theta_i)] \geq \theta_i x_i(\theta_i') - \frac{b_i}{2} x_i^2(\theta_i') - T_i[x_i(\theta_i')]. \quad (\text{A.4})$$

But *IC* can also be enforced locally because in equilibrium  $x_i(\theta_i)$  and  $T_i(\theta_i)$  are strictly increasing and differentiable. Thus:

$$\theta_i \in \operatorname{argmax}_{\theta_i'} \left\{ \theta_i x(\theta_i') - \frac{b_i}{2} x_i^2(\theta_i') - T_i[x_i(\theta_i')] \right\}, \quad (\text{A.5})$$

or equivalently:

$$\theta_i - b_i x_i(\theta_i) - T_i'[x_i(\theta_i)] = 0. \quad (\text{A.6})$$

Observe that applying the envelope theorem, this last condition leads to:

$$\mathcal{U}'(\theta_i) = \frac{\partial U(x_i(\theta_i), \theta_i)}{\partial \theta_i} = x_i(\theta_i), \quad (\text{A.7})$$

so that:

$$\mathcal{U}(\theta_i) = \underline{\mathcal{U}}_i + \int_{\theta_i^\circ}^{\theta_i} x_i(\xi) d\xi, \quad (\text{A.8})$$

Next, we substitute  $T_i[x_i(\theta_i)]$  from equation (A.4) and the *IC* condition contained in expression (A.8), into the expected profit equation (A.1), to get:

$$\pi^M = \int_{\theta_i^\circ}^{\bar{\theta}_i} \left\{ \theta_i x_i(\theta_i) - \frac{b_i}{2} x_i^2(\theta_i) - \underline{\mathcal{U}}_i - \int_{\theta_i^\circ}^{\theta_i} x_i(\xi) d\xi - c(Y_i) x_i(\theta_i) \right\} f_i(\theta_i) d\theta_i. \quad (\text{A.9})$$

Solving the double integral by parts we have:

$$\int_{\theta_i^\circ}^{\bar{\theta}_i} \int_{\theta_i^\circ}^{\theta_i} x_i(\xi) d\xi f_i(\theta_i) d\theta_i = \mathcal{U}(\bar{\theta}_i) - \int_{\theta_i^\circ}^{\bar{\theta}_i} x_i(\theta_i) F_i(\theta_i) d\theta_i = \int_{\theta_i^\circ}^{\bar{\theta}_i} x_i(\theta_i) [1 - F_i(\theta_i)] d\theta_i. \quad (\text{A.10})$$

Therefore:

$$\pi^M = \int_{\theta_i^\circ}^{\bar{\theta}_i} \left\{ \left[ \theta_i x(\theta_i) - \frac{b_i}{2} x_i^2(\theta_i) - c(Y_i) x_i(\theta_i) \right] f_i(\theta_i) - x_i(\theta_i) [1 - F_i(\theta_i)] \right\} d\theta_i - \underline{\mathcal{U}}_i. \quad (\text{A.11})$$

Finally, the optimal tariff is characterized by pointwise maximization of this expression with respect to  $x_i(\theta_i) \forall \theta_i$ :

$$\theta_i - b_i x_i(\theta_i) = c(Y_i) + \frac{1 - F_i(\theta_i)}{f_i(\theta_i)}, \quad (\text{A.12})$$

which for the particular family of Burr type XII distributions becomes:

$$\theta_i - b_i x_i(\theta_i) = c(Y_i) + \lambda_i(\bar{\theta}_i - \theta_i), \quad (\text{A.13})$$

Next, observe that because of the *IC* condition (A.6), the l.h.s. of (A.12) coincides with the marginal tariff, so that:

$$T'_i[x_i(\theta_i)] = p_i[x_i(\theta_i)] = c(Y_i) + \lambda_i(\bar{\theta}_i - \theta_i). \quad (\text{A.14})$$

Finally, we need to characterize the tariff function. Combining equation (7b) with equations (A.2) and (A.8) we obtain the following expression suitable to be estimated with the available data:

$$\begin{aligned} T_i[x_i(\theta_i)] &= \underline{\mathcal{U}}_i + \theta_i x_i(\theta_i) - \frac{b_i}{2} x_i^2(\theta_i) - \mathcal{U}(\theta_i) \\ &= \underline{\mathcal{U}}_i + \theta_i x_i(\theta_i) - \frac{b_i}{2} x_i^2(\theta_i) - \int_{\theta_i^o}^{\theta_i} \frac{(1 + \lambda_i)\xi_i - c(Y_i) - \lambda_i \bar{\theta}_i}{b_i} d\xi_i \\ &= \underline{\mathcal{U}}_i + \left[ \frac{c(Y_i) + \lambda_i \bar{\theta}_i}{1 + \lambda_i} \right] x_i(\theta_i) - \left[ \frac{b_i \lambda_i}{2(1 + \lambda_i)} \right] x_i^2(\theta_i). \end{aligned} \quad (\text{A.15})$$

The last line of (A.15) makes use of (7b) and (13) to write the tariff as an explicit function of the optimal consumption  $x_i(\theta_i)$ .

### • A3: Nonlinear Pricing by Duopolists

The results presented in this Appendix follow the work of Ivaldi and Martimort (1994) very closely. Without loss of generality we will focus on firm 1. The first order necessary conditions for consumption decisions are:

$$\theta_1 - b_1 x_1 + \kappa x_2 = T'_1(x_1), \quad (\text{A.16a})$$

$$\theta_2 - b_2 x_2 + \kappa x_1 = T'_2(x_2). \quad (\text{A.16b})$$

These conditions are assumed to hold even at  $x_1 = 0$  and  $x_2 = 0$ , respectively. Next, solve (A.16b) for  $x_2$  making use of (14) to get:

$$\theta_2 + \kappa x_1 - \beta_2 = (b_2 + \gamma_2)x_2, \quad (\text{A.16c})$$

which after substitution into (A.16a) leads to the following condition for the optimal consumption of good  $x_1$ :

$$\theta_1 - b_1 x_1 + \frac{\kappa [\theta_2 + \kappa x_1 - \beta_2]}{b_2 + \gamma_2} = T'_1(x_1). \quad (\text{A.16d})$$

This expression can be rewritten as follows:

$$z_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} = \left[ \theta_1 + \frac{\kappa\theta_2}{b_2 + \gamma_2} \right] - \frac{\kappa\beta_2}{b_2 + \gamma_2} = \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1 + T_1'(x_1), \quad (\text{A.16e})$$

which defines the single-dimensional sufficient statistic for the demand of product of firm 1, where the effective willingness to pay for product 1 is not only affected by the intrinsic valuation of this product  $\theta_1$ , but also by the degree of substitution  $\kappa$ , the intrinsic valuation of the alternative product  $\theta_2$ , as well as for the shape of the tariff offered by the competing firm through parameter  $\gamma_2$ .

The next step is to rewrite the problem in terms of  $\{z_1, \theta_2\}$ . By extension, first define  $\mathcal{U}(\theta_1, \theta_2) = \mathcal{W}(z_1, \theta_2)$  in a similar manner to (A.2). Since  $\theta_1$  enters linearly into the definition of  $z_1$ , it follows that the relevant *IC* condition in the first dimension of the type can be written as:

$$x_1(\theta_1, \theta_2) = \frac{\partial \mathcal{U}(\theta_1, \theta_2)}{\partial \theta_1} = \frac{\partial \mathcal{W}(z_1, \theta_2)}{\partial z_1} \frac{\partial z_1}{\partial \theta_1} = \frac{\partial \mathcal{W}(z_1, \theta_2)}{\partial z_1} = x_1(z_1, \theta_2). \quad (\text{A.17a})$$

The relation between  $z_1$  and  $\theta_2$  is however affected through (15) by the degree of substitutability  $\kappa$ , and the degree of concavity of the tariff offered by firm 2 through parameter  $\gamma_2$ . Therefore:

$$x_2(\theta_1, \theta_2) = \frac{\partial \mathcal{U}(\theta_1, \theta_2)}{\partial \theta_2} = \frac{\partial \mathcal{W}(z_1, \theta_2)}{\partial z_1} \frac{\partial z_1}{\partial \theta_2} = \frac{\partial \mathcal{W}(z_1, \theta_2)}{\partial \theta_2} + x_1(z_1, \theta_2) \frac{\kappa}{b_2 + \gamma_2}. \quad (\text{A.17b})$$

Combining this expression with (A.16c), the *IC* condition involving the second type dimension can be written as:

$$\frac{\partial \mathcal{W}(z_1, \theta_2)}{\partial \theta_2} = \frac{\theta_2 - \beta_2}{b_2 + \gamma_2}. \quad (\text{A.17c})$$

Next, the net rent of a consumer of type  $\{z_1, \theta_2\}$  can be written as follows by subtracting payments  $T_1(x_1)$  and  $T_2(x_2)$  from the utility (1), and once we substitute  $\theta_1$  for  $z_1$  according to (15) and write  $T_2(x_2)$  as in (14):

$$\mathcal{W}(z_1, \theta_2) = \max_{x_1, x_2} \left\{ \left[ z_1 - \frac{\kappa\theta_2}{b_2 + \gamma_2} \right] x_1 + (\theta_2 - \beta_2) x_2 - \frac{b_1}{2} x_1^2 - \frac{(b_2 + \gamma_2)}{2} x_2^2 + \kappa x_1 x_2 - \alpha_2 - T_1(x_1) \right\}, \quad (\text{A.18a})$$

After eliminating the cross-product term to fit (A.16e) this expression becomes:

$$\mathcal{W}(z_1, \theta_2) = \omega_2(\theta_2) + \max_{x_1} \left\{ \left[ z_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} \right] x_1 - \frac{1}{2} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1^2 - T_1(x_1) \right\}, \quad (\text{A.18b})$$

where  $\omega_2(\theta_2)$  denotes the value of consumers' outside option of buying an imperfect substitute offered by firm 2:

$$\omega_2(\theta_2) = \max_{x_2} \left\{ -\alpha_2 + (\theta_2 - \beta_2) x_2 - \frac{(b_2 + \gamma_2)}{2} x_2^2 \right\} = -\alpha_2 + \frac{(\theta_2 - \beta_2)^2}{2(b_2 + \gamma_2)}, \quad (\text{A.18c})$$

so that:

$$x_2(\theta_2) = \frac{\theta_2 - \beta_2}{b_2 + \gamma_2}, \quad (\text{A.18d})$$

which coincides with the expression of the *IC* constraint associated to the second type dimension. Since these products are substitutes, a subscriber of firm 1 must obtain a rent of at least  $\omega_2(\theta_2)$  because otherwise she will purchase from firm 2 only. A similar condition for subscribers of firm 2

could be written regarding the value of their outside option of purchasing the service from firm 1. Observe that for the optimization problem of the outside option to be concave we also need:

$$b_i + \gamma_i > 0, \quad (\text{A.18e})$$

a condition that we need to check whether it holds at the estimation stage. Therefore, competing firm 1 offers a nonlinear contract  $\{T_1(z_1, \theta_2), x_1(z_1, \theta_2), \mathcal{P}_1(z_1, \theta_2)\}$ . A consumer with taste parameter  $(z_1, \theta_2)$  purchases  $x_1(z_1, \theta_2)$  units from firm 1 and pays  $T_1[x_1(z_1, \theta_2)]$ . When the utility from purchasing from firm 1 equals that of purchasing from firm 2,  $\mathcal{P}_1(z_1, \theta_2)$  indicates the probability that consumers buy from firm 2. Duopolist 1's maximizes his expected profit:

$$\pi_1^D = \int_{\underline{z}_1}^{\bar{z}_1} \int_{\underline{\theta}_1}^{\bar{\theta}_1} \{T_1[x_1(z_1, \theta_2)] - K - c_1(Y, W)x_1(z_1, \theta_2)\} f(z_1, \theta_2) dz_1 d\theta_2. \quad (\text{A.19a})$$

subject to the following *IR* and exclusion constraints due to competition:

$$\mathcal{W}(z_1, \theta_2) \geq 0, \quad (\text{A.19b})$$

$$\mathcal{W}(z_1, \theta_2) \geq \omega_2(\theta_2), \quad (\text{A.19c})$$

and where:

$$\begin{aligned} \mathcal{W}(z_1, \theta_2) &= \max_{z'_1, \theta'_2} \left( \omega_2(\theta_2) \mathcal{P}_1(z'_1, \theta'_2) + [1 - \mathcal{P}_1(z'_1, \theta'_2)] \left\{ \omega_2(\theta_2) + \left[ z_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} \right] x_1(z'_1, \theta'_2) \right. \right. \\ &\quad \left. \left. - \frac{1}{2} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1^2(z'_1, \theta'_2) - T_1(z'_1, \theta'_2) \right\} \right) \\ &= \max_{z'_1, \theta'_2} [1 - \mathcal{P}_1(z'_1, \theta'_2)] \left\{ \left[ z_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} \right] x_1(z'_1, \theta'_2) - \frac{1}{2} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1^2(z'_1, \theta'_2) - T_1(z'_1, \theta'_2) \right\} \\ &\quad + \omega_2(\theta_2). \end{aligned} \quad (\text{A.19d})$$

This last relationship is key for the competitive nonlinear pricing problem to be solved in closed form. Observe that in the above optimization program firm 1 is not able to screen customers according to their second type dimension  $\theta_2$  through the design of the tariff for product 1. Therefore, in the context of this communication game,  $\theta'_2$  becomes a nuisance message with no value for the principal. Thus, instead of using the mechanism  $\{T_1(z_1, \theta_2), x_1(z_1, \theta_2), \mathcal{P}_1(z_1, \theta_2)\}$ , firm 1 can apply the *Revelation Principle* to design the simpler mechanism  $\{T_1(z_1), x_1(z_1)\}$  using only the marginal distribution  $f_1(z_1)$ . It follows from evaluating the term between curly brackets in the last part of (A.19d) at the true value  $z'_1 = z_1$ , that we can write the net surplus of a consumer of type  $z_1$  as:

$$\mathcal{W}(z_1) = \left[ z_1 - \frac{\kappa\beta_2}{b_2 + \gamma_2} \right] x_1(z_1) - \frac{1}{2} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1^2(z_1) - T_1[x_1(z_1)]. \quad (\text{A.20})$$

Observe that this expression for the rent of a consumer is a quadratic function of the newly defined type  $z_1$ . This expression is parallel to (A.2) which, for the monopoly case, characterized the rent of consumers as a function of  $\theta_1$ . In order to solve the case of competing duopolists, we need to

write down the *IR* and *IC* constraints as functions of  $z_1$ . Observe that the *IC* constraint is implicitly defined by (A.17a). The participation constraint, as before, should be independent of the type of consumers. According to (A.20), the *IC* and *IR* constraints become:

$$\mathcal{W}'(z_1) = x_1(z_1), \quad (\text{A.21a})$$

$$\mathcal{W}(z_1) = \underline{\mathcal{W}}_1 \geq 0. \quad (\text{A.21b})$$

As for the *IR* constraint, observe that none of these two duopolists should *a priori* exclude any customer. Remember that although exclusion might be optimal, we are not able to identify who subscribes which cellular telephone carrier. Only for some markets we know the market share of each carrier, but always at an aggregate level, without observing which consumers subscribe to each cellular carrier. The participating consumer with the lowest valuation for product 1 is:

$$z_1^\circ = \theta_1^\circ + \frac{\kappa \bar{\theta}_2}{b_2 + \gamma_2}. \quad (\text{A.22})$$

After rewriting (A.19a) in terms of  $z_1$ , duopolist 1 maximizes:

$$\pi_1^D = \int_{z_1^\circ}^{\bar{z}_1} \{T_1[x_1(z_1)] - K - c_1(Y, \mathbf{W}_1)x_1(z_1)\} f_1(z_1) dz_1, \quad (\text{A.23})$$

subject to (A.19b)–(A.19c), (A.20), and (A.21a)–(A.21b). Taking advantage of the formal similarity with the monopolist's problem, we can directly write the optimality condition of the competing nonlinear tariff as follows:

$$\left[ z_1 - \frac{\kappa \beta_2}{b_2 + \gamma_2} \right] - \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1(z_1) = c_1(Y, \mathbf{W}_1) + \lambda_1 (\bar{z}_1 - z_1). \quad (\text{A.24})$$

From here, the optimal purchase of product 1 for a consumer of type  $z_1$  becomes:

$$x_1(z_1) = \frac{(1 + \lambda_1)z_1 - \frac{\kappa \beta_2}{b_2 + \gamma_2} - c_1(Y, \mathbf{W}_1) - \lambda_1 \bar{z}_1}{\left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right]}, \quad (\text{A.25})$$

which defines  $z_1^\circ$  when  $x_1(z_1^\circ) = 0$ :

$$z_1^\circ = \frac{c_1(Y, \mathbf{W}_1) + \lambda_1 \bar{z}_1 + \frac{\kappa \beta_2}{b_2 + \gamma_2}}{1 + \lambda_1}. \quad (\text{A.26})$$

Equation (22) in the text substitute highest type and the marginal cost from equations (20) and (21), respectively into this expression. Finally, we can characterize the tariff function by combining equations (A.20) and (A.21a) – (A.21b) as well as (A.25) and (A.26):

$$\begin{aligned} T_1[x_1(z_1)] &= \underline{\mathcal{W}}_1 + \left[ z_1 - \frac{\kappa \beta_2}{b_2 + \gamma_2} \right] x_1(z_1) - \frac{1}{2} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1^2(z_1) - \int_{z_1^\circ}^{z_1} \frac{(1 + \lambda_1)\xi_1 - \frac{\kappa \beta_2}{b_2 + \gamma_2} - c_1(Y, \mathbf{W}_1) - \lambda_1 \bar{z}_1}{\left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right]} d\xi_1, \\ &= \underline{\mathcal{W}}_1 + \frac{c_1(Y, \mathbf{W}_1) + \lambda_1 \left[ \bar{z}_1 - \frac{\kappa \beta_2}{b_2 + \gamma_2} \right]}{1 + \lambda_1} x_1(z_1) - \frac{\lambda_1}{2(1 + \lambda_1)} \left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right] x_1^2(z_1). \end{aligned} \quad (\text{A.27})$$

• **A4: Bivariate Sarmanov Distribution with Correlated Types**

It is reasonable to think that types are most likely positively correlated. Customers with a high marginal willingness to pay for the cellular service of one of the carriers would also be willing to pay a high price for the service of the other carrier. We do not expect to observe individuals with a very high willingness to pay for the *wireline* service but almost nil for the *nonwireline* service. This is an important element that drives our choice of distribution (16a).

Two critical features of the model condition the extension to jointly distributed tastes with any correlation pattern. The first feature is that the tariff has to resemble equation (17a), which is a quadratic function of consumption. The second is that in the present model firms cannot make use of any incentive mechanism to effectively screen consumers with respect to the demand for the competitive good.<sup>46</sup> Therefore, firms make use of marginal distributions in computing their expected profits, thus making the identification of the correlation among types generally impossible. The clear advantage of the Sarmanov distribution (16a) is that it allows *any* pattern of correlation between taste parameters, including positive correlation. However, the identification of such correlation is not feasible from aggregated data because the marginal distributions do not depend on parameter  $\varphi$  that conditions the correlation between  $\theta_1$  and  $\theta_2$ . Still, assuming that  $z_1$  and  $z_2$  are jointly distributed according to a bivariate Sarmanov distribution turns our results to be robust to the existence of correlated types, regardless of whether such correlation is positive or negative.

To our knowledge there are two families of bivariate distributions that lead to marginal beta distributions, a family that includes the particular case of Burr type XII distribution used in the present model. The first is the family of Dirichlet distributions.<sup>47</sup> This distribution was used by Ivaldi and Martimort (1994) to avoid the assumption of independently distributed types. However, it has two serious drawbacks. First, it requires identical markups for both competing firms because condition  $\lambda_1 = \lambda_2 = \lambda < \frac{1}{2}$  is always needed for proper definition of such distribution. This restriction does not only lead to a misspecified model unless firms engage in identical pricing, but it also restricts the magnitude of markups –*e.g.*, excluding the possibility of uniformly distributed types– to ensure the integrability of the distribution. The second serious drawback is that the Dirichlet distribution always lead to negatively correlated types. In particular, the correlation between  $z_1$  and  $z_2$  will always be  $-\lambda$ . This contradicts the intuition on positively correlated types discussed above.

Regardless of whether the correlation is identifiable or not, the family of Sarmanov distributions is much more flexible because it does not impose any further constraint on the range of  $\lambda_1$  and  $\lambda_2$ . These parameters, and therefore the markups of the competing firms, do not need to be equal. And furthermore, it allows for both positive and negative correlation of types. In particular, for the particular case of distribution (16a) the correlation between  $z_1$  and  $z_2$  depends on the sign of  $\varphi$  alone. This correlation is:

$$\rho = \varphi \cdot \frac{\lambda_1[\bar{z}_1 - (2 + \lambda_1)z_1^\circ]\lambda_2[\bar{z}_2 - (2 + \lambda_2)z_2^\circ]}{(1 + \lambda_1)(1 + \lambda_2)\sqrt{(1 + 2\lambda_1)(1 + 2\lambda_2)}}. \quad (\text{A.28})$$

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<sup>46</sup> See the detailed discussion in Appendix 3 as well as in Ivaldi and Martimort (1994, §4.2).

<sup>47</sup> See Darroch and Ratcliff (1971), and Johnson, Kotz, and Balakrishnan (2000, §49.1).

• **A5: Approximation Through Menus of Two-Part Tariffs**

The goal of this appendix is to justify our approach of using the continuous tariff predicted by the model instead of its implementation by means of a menu of self-selecting two-part tariffs. The formal proof that the foregone welfare decreases with the number of tariff options offered is available in Wilson (1993, §8.3). Here, for illustration purposes we compare the optimal fully nonlinear tariff with menus of two-part tariffs of different number of options. To simplify we assume that consumer types are uniformly distributed in  $\Theta_i = [0, 1]$ . We also assume that  $b_i = 1$  and  $c_i = 0$ . We ignore any allowance that may exist (which is accurate for the “binding” case of low usage plans in our data). For this particular case of (7a), the optimal tariff is:

$$T(x) = \frac{1}{2}x - \frac{1}{4}x^2. \tag{A.29}$$

An  $n$ -part tariff is a fixed fee plus block-declining price schedule with  $n - 1$  segments such as  $A_k < A_{k+1}$  and  $p_k > p_{k+1}$ . Let  $p_0 \equiv \infty$  and  $\underline{U} = A_0 \equiv 0$  represent the option of not purchasing at all. Furthermore, consumers of type  $\theta$  choose the tariff  $k$  if  $\theta_k < \theta < \theta_{k+1}$ . The solution can then be written as:

$$p_k = \frac{n - k - 0.5}{2(n - 0.5) - 0.5}, \tag{A.30a}$$

$$A_k = \frac{k}{2(n - 0.5) - 0.5} - 2k \cdot \frac{n - (1 + k) \cdot 0.5}{[2(n - 0.5) - 0.5]^2}. \tag{A.30b}$$

**Table A1**

OPTIONS	One	Two	Three	Four	Five	Six
$A_1$	0.0800	0.0247	0.0118	0.0069	0.0045	0.0032
$A_2$		0.1481	0.0710	0.0415	0.0272	0.0192
$A_3$			0.1775	0.1038	0.0680	0.0480
$A_4$				0.1938	0.1270	0.0896
$A_5$					0.2041	0.1440
$A_6$						0.2112
$p_1$	0.0200	0.3333	0.3846	0.4118	0.4286	0.4400
$p_2$		0.1111	0.2308	0.2941	0.3333	0.3600
$p_3$			0.0769	0.1765	0.2381	0.2800
$p_4$				0.0588	0.1429	0.2000
$p_5$					0.0476	0.1200
$p_6$						0.0400
MISE	0.7613	0.0480	0.0089	0.0026	0.0010	0.0005

Row MISE reports the mean integrated square error ( $\times 10^3$ ) of fitting the lower envelope of each set of tariff options with the nonlinear tariff (A.29).

Table A1 presents the fixed fee and marginal tariffs of the optimal tariff options for different number of options that attempt to implement the optimal nonlinear tariff (A.29). Table A1 also reports a measure of how the approximation to a continuous tariff by means of optional two-part tariffs becomes quite accurate with relatively few tariff options. Figure A1 represents few approximations to the optimal nonlinear tariff as described in Table A1.

⇒ INSERT Figure A1 ⇐

• **A6: Domain of the Parameters of the Polynomial Approximation**

Parameter  $\lambda_i$  needs to be positive for the distribution of types to be properly defined. Provided a concave tariff function, the estimate of  $\gamma_i$  is negative so that according to equation (19) the following condition must hold:

$$b_1 + \gamma_1 \geq \frac{\kappa^2}{b_2 + \gamma_2}, \quad (\text{A.31a})$$

or alternatively:

$$\gamma_1\gamma_2 + b_2\gamma_1 + b_1\gamma_2 + (b_1b_2 - \kappa^2) \geq 0, \quad (\text{A.31b})$$

which is a particular case of the general quadratic form representation of a conic section. In particular it corresponds to a hyperbola with center at  $(\gamma_1, \gamma_2) = (-b_1, -b_2)$ . See McLenaghan and Levy (1996, §4.7.2). The principal axis of this hyperbola is a 45 degree line passing through the origin when  $b_1 = b_2$ . Figure A2 represents this hyperbola.

⇒ INSERT Figure A2 ⇐

The region to the left and below of the center of the hyperbola should not be considered because the concavity condition of the outside option (A.18e) is not fulfilled there. Only the region above the upper-right branch of the hyperbola but still in the third quadrant should be considered. The first, second, and fourth quadrant involve positive values for either  $\gamma_1$  or  $\gamma_2$ , only consistent with the existence of quantity premia. Positive values of  $\lambda_i$  implies quantity discounts, leading to concave tariffs and negative values for  $\gamma_i$ . As reported in Table 3, the concavity of the tariff is a feature present in all the markets and not a restriction artificially imposed upon the data.

Finally, observe that different values of  $\kappa$  affect the position of the branches of the hyperbola, and therefore the domain from which  $\gamma_1$  and  $\gamma_2$  can be estimated. If  $\kappa = 0$ , equation (A.31b) leads to two straight lines coinciding with the asymptotes represented in Figure A2. In this case, products are independent and the estimation would provide meaningful results as long as  $-1 < \gamma_i < 0$ . This is the case of monopoly pricing. However, if goods are perfect substitutes,  $\kappa = -1$  and the upper-right branch of the hyperbola would cross at the origin, thus limiting the admissible values to  $\gamma_i = 0$ . When  $\gamma_1 = \gamma_2 = 0$  both firms can only offer two-part tariffs in equilibrium.

• **A7: Common Agency: Tangency at the Corner**

Assume that a particular consumer does not purchase product 2, *i.e.*, her type can be written as  $(z_1, z_2^\circ)$  such that  $x_2(z_2^\circ) = 0$ . Then, tangency between the individual budget constraint and the indifference curve representing consumers' preferences occurs if:

$$\frac{T_1'(x_1)}{T_2'(0)} = \frac{\beta_1 + \gamma_1 x_1(z_1)}{\beta_2} = \frac{\theta_1(z_1, z_2^\circ) - b_1 x_1(z_1)}{\theta_2(z_1, z_2^\circ) + \kappa x_1(z_1)} = \frac{U_1(x_1, 0)}{U_2(x_1, 0)}. \quad (\text{A.32})$$

After inverting the type functions (15) we get:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} b_2 + \gamma_2 & \kappa \\ \kappa & b_1 + \gamma_1 \end{bmatrix} \cdot \begin{bmatrix} (b_2 + \gamma_2)z_1 \\ (b_1 + \gamma_1)z_2^\circ \end{bmatrix}, \quad (\text{A.33})$$

so that:

$$\theta_1(z_1, z_2^\circ) = \frac{(b_1 + \gamma_1)(b_2 + \gamma_2)z_1 - (b_1 + \gamma_1)z_2^\circ}{(b_1 + \gamma_1)(b_2 + \gamma_2) - \kappa^2}, \quad (\text{A.34})$$

and similarly for  $\theta_2(z_1, z_2^\circ)$ . Thus, the easiest way to prove that there are values of  $z_1$  that will attain the tangency result of (A.32) even when such consumer does not purchase product  $x_2$  is to rewrite equality (A.32) as:

$$g_1(z_1) = z_1 + \frac{\beta_2 [\theta_1(z_1, z_2^\circ) - b_1 x_1(z_1)]}{[\theta_2(z_1, z_2^\circ) + \kappa x_1(z_1)] \cdot [\beta_1 + \gamma_1 x_1(z_1)]} - 1. \quad (\text{A.35})$$

Since  $x_1(z_1)$ ,  $\theta_1(z_1, z_2^\circ)$ , and  $\theta_2(z_1, z_2^\circ)$  are continuous functions of  $z_1$ ,  $g_1(z_1) : \mathbb{R} \rightarrow \mathbb{R}$  is also a continuous functions of  $z_1$ , and by Brouwer's Fixed Point Theorem, there exists  $z_1^*$  such that  $g_1(z_1) = z_1$  for any  $z_2^\circ$ . The practical problem to justify the first order approach to solve our common agency problem is to determine whether  $z_1^* \in [z_1^\circ, \bar{z}_1]$ , and *vice versa*, for  $z_2^* \in [z_2^\circ, \bar{z}_2]$ , for every market-quarter combination since functions  $x_1(z_1)$  and  $x_2(z_2)$  depend on the estimated parameters  $\{c_i, \lambda_i\}$  for each firm-market-quarter combination. Given the estimates of the structural parameters, we have checked whether these conditions hold. We find that in 431 out of 521 markets it is possible to find a large enough  $z_1$  ( $z_2$ ) such that tangency condition (A.32) holds at the corner when  $x_2 = 0$  ( $x_1 = 0$ ).

• **A8: Optimal Two-Part Tariff**

The demand function for the competitive case is obtained from equation (A.16e) for the product of firm 1 (and similarly for firm 2):

$$x_1(p_1, z_1) = \frac{\left[ z_1 - \frac{\kappa \beta_2}{b_2 + \gamma_2} \right] - p_1}{\left[ b_1 - \frac{\kappa^2}{b_2 + \gamma_2} \right]}. \quad (\text{A.36})$$

Substituting the marginal distribution and density functions (19b) into (30) and after integrating by parts we obtain the optimality conditions by differentiating with respect to  $p_1$  and  $z_1^\circ$ , respectively:

$$(1 + \lambda_1)(p_1 - c_1) - \lambda_1(\bar{z}_1 - z_1) + \lambda_1(1 + \lambda_1)z_1^\circ \left[ \frac{\bar{z}_1 - \underline{z}_1}{\bar{z}_1 - z_1^\circ} \right] \frac{1}{\lambda_1} = 0, \quad (\text{A.37a})$$

$$[\lambda_1(\bar{z}_1 - \bar{z}_1^\circ) + c_1] - \frac{z_1^\circ + p_1}{2} = 0. \quad (\text{A.37b})$$

Combining these two expression, and after some algebra, we can write:

$$[\lambda_1^2 + (1 + \lambda_1)^2]z_1^\circ - (1 + \lambda_1)c_1 - \lambda_1(1 + 2\lambda_1)\bar{z}_1 + \lambda_1(1 + \lambda_1)z_1^\circ \left[ \frac{\bar{z}_1 - \underline{z}_1}{\bar{z}_1 - z_1^\circ} \right] \frac{1}{\lambda_1} = z_1^\circ. \quad (\text{A.38})$$

A solution can easily be found as long as  $\underline{z}_1$  exceeds  $[(1 + \lambda_1)c_1 + \lambda_1(1 + 2\lambda_1)\bar{z}_1]/[\lambda_1^2 + \lambda_1 + 1]$ .

• **A9: Optimal Linear Tariff**

Firm 1 now makes  $A(p_1, z_1^\circ) = 0$ . Following the same steps of the previous problem, the solution is found by solving:

$$(1 + \lambda_1) \left[ c_1 - z_1 + \frac{\kappa\beta_2}{b_2 + \gamma_2} \right] + \lambda_1(\bar{z}_1 - z_1) - \lambda_1(1 + \lambda_1)z_1^\circ \left[ \frac{\bar{z}_1 - z_1}{\bar{z}_1 - z_1^\circ} \right] \frac{1}{\lambda_1} = 0, \quad (\text{A.39})$$

which is ensured to have a solution if  $[z_1, \bar{z}_1] \in \mathbb{R}_+$ .

• **A10: Optimal Flat Tariff**

Firm 1 now makes  $p_1 = 0$ . Thus, the only problem is to find the optimal exclusion level given by  $z_1^\circ$ . This is done solving first order condition (31.b) for  $p_1 = 0$ . Thus:

$$(1 + \lambda_1)c_1 + \lambda_1(\bar{z}_1 - z_1) - \lambda_1(1 + \lambda_1)z_1^\circ \left[ \frac{\bar{z}_1 - z_1}{\bar{z}_1 - z_1^\circ} \right] \frac{1}{\lambda_1} = 0, \quad (\text{A.40})$$

which again is ensured to have a solution if  $[z_1, \bar{z}_1] \in \mathbb{R}_+$ .

**Table 1. Descriptive Statistics**

Variable	1984	1985	1986	1987	1988	All
POPULATION	2,709.43	2,312.74	1,618.41	1,436.73	1,291.99	1,598.53
COVERAGE	1.56	1.60	1.86	2.49	2.73	2.24
GROWTH	0.93	1.04	1.11	1.05	1.08	1.07
BUSINESS	75.12	63.42	43.16	37.73	32.99	42.24
COMMUTING	27.83	27.18	25.91	25.49	24.93	25.73
INCOME	30.48	29.69	28.29	27.69	26.55	27.87
POVERTY	9.77	10.09	10.51	10.39	10.54	10.41
POP-AGE	33.19	32.97	32.68	32.56	32.52	32.65
EDUCATION	12.62	12.61	12.56	12.55	12.52	12.55
CRIME	6,060.76	6,316.85	6,523.44	6,580.39	6,613.07	6,524.50
VIOLENT	714.76	731.22	749.72	703.08	691.09	715.99
PROPERTY	5,346.00	5,585.63	5,773.72	5,877.32	5,921.98	5,808.51
SVCRIMES	11.49	11.40	11.28	10.56	10.38	10.84
TEMPERATURE	49.38	54.03	57.13	56.82	61.03	57.47
RAIN	3.52	3.44	3.46	3.21	3.12	3.29
WAGE	7.13	7.13	7.18	7.32	7.31	7.25
ENERGY	1.82	1.78	1.76	1.68	1.64	1.71
PRIME	11.00	10.94	10.07	8.88	8.28	9.35
OPERATE	6.83	6.71	6.57	6.39	6.30	6.47
RENT	17.24	16.62	15.76	15.68	16.30	16.04
DENSITY	23.40	21.27	18.77	16.35	15.47	17.60
LEAD	3.71	5.65	9.32	10.54	10.61	9.41
MKT-AGE	1.00	6.83	13.03	21.54	27.08	18.39
CELLS.1	29.00	24.68	17.50	15.58	13.69	17.12
CELLS.2	17.14	17.12	14.18	13.68	11.41	13.14
PLANS.1	3.30	3.11	2.90	3.03	3.26	3.08
PLANS.2	3.57	3.74	3.57	3.52	3.41	3.50
PAY200.M	116.10	110.70	102.06	102.06	102.06	105.30
PAY200.1	102.06	97.02	91.80	91.80	91.80	91.80
PAY200.2	86.40	86.40	91.80	97.02	94.50	94.50
DUOPOLY	0.26	0.24	0.39	0.59	0.78	0.53
Monopoly Obs.	20	105	160	130	64	479
Duopoly Obs.	7	34	101	188	221	551

Sample means.

**Table 2. Distribution of Market Characteristics**

Variable	0%	25%	50%	75%	100%
POPULATION	Bridgeport, CT 1988:1 45.78	Knoxville, TN 1986:3 599.67	Buffalo, NY 1986:1 988.34	Cleveland, OH 1987:1 1,872.54	Los Angeles, CA 1988:3 8,659.88
COVERAGE	El Paso, TX 1986:2 0.44	Gary, IN 1987:4 1.17	Oxnard, CA 1988:3 1.69	San Antonio, TX 1988:3 2.30	Bridgeport, CT 1988:3 47.18
GROWTH	Youngstown, OH 1988:3 -0.70	Bridgeport, CT 1987:2 0.30	Memphis, TN 1987:1 0.80	Baton Rouge, LA 1986:1 1.90	Fort Myers, FL 1988:3 4.20
BUSINESS	Las Cruces, NM 1988:3 2.09	Tucson, AZ 1986:3 13.80	Bridgeport, CT 1987:3 24.45	Pittsburgh, PA 1986:1 46.26	New York City, NY 1984:4 394.44
COMMUTING	Billings, MT 1988:3 18.70	Hartford, CT 1987:2 23.40	Portland, OR 1986:3 25.40	Saint Louis MO 1988:3 27.30	New York City, NY 1984:4 36.50
INCOME	Brownsville, TX 1988:3 15.96	Sacramento, CA 1985:4 25.36	Harrisburg, PA 1986:1 27.37	Baltimore, MD 1987:2 30.04	Washington, DC 1985:1 42.32
POVERTY	New Brunswick, NJ 1988:1 6.00	Kansas City, MO 1987:4 8.50	Chattanooga, TN 1985:4 9.80	Orlando, FL 1987:1 11.40	McAllen, TX 1988:3 35.00
POP-AGE	McAllen, TX 1988:3 24.90	Columbus, OH 1987:2 31.50	Chicago, IL 1987:4 32.50	Philadelphia, PA 1985:2 33.70	Sarasota, FL 1988:3 48.40
EDUCATION	McAllen, TX 1988:3 9.30	New Orleans, LA 1986:2 12.40	New Haven, CT 1986:1 12.50	Oklahoma City, OK 1988:2 12.70	Washington, DC 1988:2 13.70
CRIME	Allentown, PA 1986:2 2,903.30	Tampa, FL 1985:3 5,095.60	Baltimore, MD 1988:2 6,463.50	Tucson, AZ 1985:4 7,871.90	Miami, FL 1988:1 13,062.40
VIOLENT	Billings, MT 1988:3 138.00	Des Moines, IA 1988:3 495.80	Washington, DC 1986:1 622.90	Fresno, CA 1987:3 922.40	New York City, NY 1988:1 1,949.50
PROPERTY	Pittsburgh, PA 1985:3 2,685.20	Washington, DC 1985:1 4,504.10	Chicago, IL 1985:2 5,623.50	Phoenix, AZ 1987:1 6,941.50	Miami, FL 1988:2 11,224.91
SVCRIMES	Billings, MT 1988:3 2.39	Cincinnati, OH 1985:1 8.79	Sacramento, CA 1986:2 10.45	Tampa, FL 1986:1 12.65	New York City, NY 1988:1 21.71
TEMPERATURE	Minneapolis-St.Paul, MN 1988:1 17.08	Chattanooga, TN 1986:1 43.42	San Antonio, TX 1987:4 60.06	Gary, IN 1987:3 70.28	Phoenix, AZ 1988:3 87.66
RAIN	Minneapolis-St.Paul, MN 1987:1 0.25	Nashville, TN 1988:2 2.21	Knoxville, TN 1986:1 3.18	Philadelphia, PA 1987:2 4.21	Mobile, AL 1988:3 10.71
WAGE	Louisville, KY 1984:4 3.84	Grand Rapids, MI 1988:2 6.11	Mobile, AL 1986:3 6.84	Oxnard, CA 1986:4 7.89	Washington, DC 1988:3 17.09
ENERGY	Columbia, SC 1987:1 0.70	Denver, CO 1987:3 1.45	Austin, TX 1987:4 1.67	Kansas City, MO 1985:3 1.94	Oklahoma City, OK 1986:4 4.40
PRIME	Oklahoma City, OK 1987:3 7.75	Orlando, FL 1988:2 8.35	Canton, OH 1987:1 9.25	Philadelphia, PA 1986:2 10.35	New York City, NY 1984:4 11.00
OPERATE	Columbia, SC 1987:1 2.69	Indianapolis, IN 1987:2 5.44	Louisville, KY 1987:4 6.09	New Brunswick, NJ 1988:1 7.39	New York City, NY 1988:3 13.57
RENT	Peoria, IL 1988:3 7.48	Milwaukee, WI 1985:1 12.90	Phoenix, AZ 1987:4 15.23	San Diego, CA 1987:4 18.77	New York City, NY 1985:4 36.18
DENSITY	Jacksonville, FL 1985:4 1.09	Omaha, NE 1986:4 5.21	Lansing, MI 1988:3 12.34	Rochester, NY 1987:4 27.38	Harrisburg, PA 1988:3 71.30
LEAD	Saint Louis, MO 1986:1 0.00	Washington, DC 1987:2 3.55	West Palm Beach, FL 1986:1 7.26	El Paso, TX 1986:4 13.68	Tampa, FL 1988:3 33.84
AGE	Flint, MI 1986:1 1.00	Orlando, FL 1986:2 7.00	Fresno, CA 1987:4 16.00	Wilmington, DE 1988:1 28.00	Washington, DC 1988:3 46.00
CELLS.1	Florence, SC 1988:3 1.00	New Orleans, LA 1986:4 6.00	Columbus, OH 1988:1 11.00	Norfolk, VA 1988:2 18.00	Los Angeles, CA 1988:1 81.00
CELLS.2	Roanoke, VA 1988:3 1.00	Long Branch, NJ 1987:3 5.00	Indianapolis, IN 1987:4 9.00	Cincinnati, OH 1988:1 14.00	New York City, NY 1986:2 46.00
PAY200	Chicago, IL 1988:3 43.20	Jacksonville, FL 1988:1 83.70	Washington, DC 1985:3 97.20	N.E. Pennsylvania, PA 1988:2 113.40	Los Angeles, CA 1984:4 140.40

Location, quarter, and value for different quartiles of the range of variation of each characteristic.

**Table 3. Concavity of Tariffs**

TARIFF OPTIONS		MONOPOLY	WIRELINE	NONWIRE.
ONE:	Monthly Fee 1	29.62 (10.86)	14.14 (10.37)	23.96 (9.56)
	Rate per minute 1	0.36 (0.07)	0.35 (0.07)	0.35 (0.05)
	Observations	294	120	110
TWO:	Monthly Fee 1	16.58 (6.76)	12.44 (8.05)	11.28 (6.12)
	Monthly Fee 2	41.29 (12.23)	35.49 (16.41)	31.43 (16.30)
	Rate per minute 1	0.55 (0.15)	0.50 (0.15)	0.57 (0.13)
	Rate per minute 2	0.35 (0.08)	0.32 (0.09)	0.35 (0.08)
	Observations	155	285	311
THREE:	Monthly Fee 1	4.33 (4.78)	7.39 (5.45)	5.76 (5.95)
	Monthly Fee 2	16.03 (6.89)	17.46 (7.04)	17.36 (10.31)
	Monthly Fee 3	30.93 (13.24)	34.52 (18.92)	42.45 (22.37)
	Rate per minute 1	0.66 (0.11)	0.67 (0.10)	0.69 (0.17)
	Rate per minute 2	0.43 (0.02)	0.44 (0.07)	0.40 (0.10)
	Rate per minute 3	0.36 (0.03)	0.34 (0.05)	0.31 (0.07)
	Observations	30	140	117
FOUR:	Monthly Fee 1		5.91 (1.41)	4.04 (3.89)
	Monthly Fee 2		13.33 (5.15)	28.43 (12.95)
	Monthly Fee 3		22.49 (11.72)	55.97 (26.96)
	Monthly Fee 4		81.67 (36.29)	80.96 (21.35)
	Rate per minute 1		0.77 (0.08)	0.67 (0.08)
	Rate per minute 2		0.42 (0.02)	0.45 (0.00)
	Rate per minute 3		0.36 (0.07)	0.34 (0.01)
	Rate per minute 4		0.22 (0.11)	0.28 (0.02)
Observations		6	13	

Mean and standard deviation of the monthly fee and marginal rate per minute of the effective number of plans offered by different types of firms. The number of observations reports the number of markets and quarters where these given number of tariffs were offered. All magnitudes are expressed in dollars of July 1986.

**Table 4. Demand System Estimates**

Estimates	SCENARIO 1	SCENARIO 2
$a_1$	0.1370 (26.67)	0.1651 (40.05)
$a_2$	0.2599 (50.61)	0.2707 (54.39)
$b$	4.7134 (28.88)	5.2979 (97.25)
$\kappa$	-1.5597 (9.56)	0.0000 —
Wireline: $R^2$	0.8462	0.8469
Nonwireline: $R^2$	0.5374	0.5396
Log-Likelihood	-0.0286	-0.0290

The estimation method is nonlinear two-stage least squares and the absolute t-statistics are reported between parentheses. We estimate the direct demand system dual to (23a) – (23b) and use population and cost variables of each market to instrument for potential endogeneity of prices. The sample includes a total of 22 observations of 8 markets between 1985 and 1987.

**Table 5. Distribution of Structural Parameters**

Estimates		MONOPOLY	WIRELINE	NONWIRELINE
$\alpha$		0.0966 (0.0448)	0.0534 (0.0314)	0.0612 (0.0308)
$\beta$		0.7689 (0.1949)	0.7533 (0.2086)	0.7439 (0.1824)
$\gamma$		-0.1315 (0.2018)	-0.2239 (0.2063)	-0.1733 (0.1921)
$c$		0.6374 (0.1484)	0.5294 (0.1439)	0.5705 (0.1327)
SCENARIO 1	$\lambda$	0.0351 (0.0556)	0.0598 (0.0583)	0.0460 (0.0542)
	$z^\circ$	0.7914 (0.1949)	0.5177 (0.1886)	0.5014 (0.1640)
SCENARIO 2	$\lambda$	0.0271 (0.0426)	0.0459 (0.0442)	0.0354 (0.0410)
	$z^\circ$	0.7914 (0.1949)	0.7758 (0.2086)	0.7664 (0.1824)
Observations		412	521	521

Values for  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  are the average and standard deviation of the distribution of OLS estimates of fitting a quadratic regression of tariff payment on a grid of airtime usage between 0 and 500 minutes. Parameters  $c_i$  and  $\lambda_i$  are computed using the nonlinear identification restrictions (10) – (12) for monopoly and (19) – (21) for duopoly, together with the corresponding demand estimates of Table 3. Cutoff types  $z^\circ$  are computed according to equation (22).

**Table 6. Basic Structural Parameters**

VARIABLES	$c_i$	$\lambda_i$	$z_i^0$
Constant	-1.6651 (4.99)	24.3291 (4.20)	-0.2491 (0.21)
NONWIRELINE	0.0728 (3.12)	0.1036 (0.64)	-0.1252 (4.04)
TIME	0.0254 (2.99)	-0.3188 (6.75)	-0.0207 (1.74)
MKT-AGE	-0.0047 (2.05)	0.1108 (6.74)	-0.0046 (1.29)
LEAD	-0.0084 (4.91)	0.0155 (1.29)	-0.0064 (2.55)
BUSINESS		0.0094 (5.62)	0.0014 (4.08)
POPULATION		-0.0002 (2.17)	0.0001 (2.70)
COMMUTE		-0.0414 (1.07)	0.0399 (4.57)
GROWTH		0.2446 (3.27)	0.0823 (4.46)
INCOME		-0.1001 (2.75)	-0.0182 (2.43)
EDUCATION		-1.1172 (2.54)	-0.0309 (0.34)
COVERAGE		0.8598 (0.46)	1.2070 (2.71)
POP-AGE		-0.2527 (8.39)	0.0159 (2.21)
POVERTY		-0.2547 (6.32)	-0.0264 (2.89)
TCELLS	-0.0006 (1.00)		-0.0098 (6.91)
WAGE	0.0200 (0.33)		0.1903 (2.06)
ENERGY	-0.0827 (1.48)		0.1369 (1.63)
OPERATE	0.2835 (4.06)		0.2034 (2.04)
RENT	-0.0369 (0.73)		-0.0347 (0.42)
PRIME	0.0612 (2.44)		-0.0532 (1.48)
DENSITY	0.0063 (6.99)		0.0027 (1.67)
TEMPERATURE			-0.0019 (2.00)
RAIN			-0.0155 (1.88)
CRIME			-0.0000 (0.48)
SVCRIMES			-2.9663 (4.16)
$R^2$	0.1672	0.2013	0.3625

OLS estimates and absolute t-statistics.

**Table 7. Overall Welfare Effects**

VARIABLES	MONOPOLY	DUOPOLY	$\kappa = 0$	$\lambda_i = \lambda_1$	$\lambda_i = \lambda_2$	$c_i = c_1$	$c_i = c_2$
CONS. SURPLUS	401.30	404.05	325.56	432.72	600.45	382.88	451.60
PROFITS	175.46	174.33	176.95	173.72	173.29	180.18	170.47
WELFARE	576.76	578.38	502.51	606.44	773.74	563.06	622.07
EXP. SALE	163.79	176.14	222.91	154.00	207.33	177.53	173.66
EXP. TARIFF	135.81	139.62	155.49	131.40	149.31	140.31	139.01
EXP. RATE	0.66	0.66	0.65	0.66	0.65	0.66	0.66
$c_1$	0.64	0.53	0.53	0.53	0.53	0.53	0.53
$\lambda_1$	0.04	0.06	0.06	0.06	0.06	0.06	0.06
$c_2$		0.57	0.57	0.57	0.57	0.57	0.57
$\lambda_2$		0.05	0.05	0.05	0.05	0.05	0.05

Expected welfare and tariff magnitudes per customer are expressed in dollars per active customer. Average marginal costs and distribution parameters are reported for each sample to ease comparison.

**Table 8. Welfare and Tariff Elasticities**

VARIABLES	$\rho$	$\kappa$	$\lambda_1$	$\lambda_2$	$c_1$	$c_2$
CONS. SURPLUS	0.0008	0.4054	0.1396	0.3384	-0.1425	-0.0398
PROFITS	0.0000	-0.0043	0.0278	0.0159	-0.2683	-0.3723
WELFARE	0.0006	0.2818	0.1060	0.2411	-0.1805	-0.1401
EXP. SALE	0.0000	-0.1081	0.0357	-0.1293	-0.1680	-0.1536
EXP. TARIFF	0.0000	-0.0441	0.0189	-0.0492	-0.0777	-0.0726
EXP. RATE	0.0000	0.0032	-0.0322	-0.0177	0.0192	0.0151

Average elasticities.

**Table 9. Price Distortion at Different Usage Levels**

VARIABLES	SAMPLE	50	150	250	350	450
TARIFF	<i>ENDM</i> :	46.60	87.05	126.03	163.51	199.51
	<i>DSR1</i> :	36.80	76.24	113.47	148.49	181.31
	<i>DSR2</i> :	40.12	77.10	112.86	147.43	180.80
	<i>DLR1</i> :	39.29	76.96	112.85	146.97	179.31
	<i>DLR2</i> :	40.00	77.03	112.96	147.79	181.52
MARGINAL RATE	<i>ENDM</i> :	0.75	0.72	0.70	0.67	0.64
	<i>DSR1</i> :	0.76	0.72	0.68	0.64	0.60
	<i>DSR2</i> :	0.72	0.70	0.67	0.65	0.62
	<i>DLR1</i> :	0.66	0.63	0.60	0.57	0.53
	<i>DLR2</i> :	0.66	0.64	0.62	0.60	0.58
PRICE MARKUP	<i>ENDM</i> :	13.95	11.96	9.54	6.50	2.52
	<i>DSR1</i> :	21.59	18.60	14.96	10.33	4.10
	<i>DSR2</i> :	13.33	11.19	8.73	5.85	2.34
	<i>DLR1</i> :	18.44	15.66	12.38	8.36	3.22
	<i>DLR2</i> :	11.58	9.61	7.39	4.88	1.94

The first line of each category, *ENDM*, averages across the last quarter of monopoly in all markets where entry of *wireline* and *nonwireline* carriers did not enter simultaneously. Line *DSR1* identifies the average of tariff, marginal rate and markup at different usage levels for the *wireline* firm in the very first quarter of effective competition. Index 2 identifies the *nonwireline* firm and *LR* indicates that the average is computed in the very last quarter of the data. The numbers in the first row indicate monthly usage levels in minutes. Tariff information is measured in dollars while markups are measured in percentages.

**Table 10. Markup and Welfare Components**

VARIABLES	MARKUP	CONS. SURPLUS	PROFITS	WELFARE
Constant	17.1009 (1.98)	-0.7084 (0.50)	-0.2931 (0.34)	1.3091 (1.61)
TIME	-0.4978 (5.89)	0.0558 (4.02)	-0.0487 (5.74)	0.0131 (1.65)
AGE	0.1773 (7.00)	-0.0182 (4.38)	0.0208 (8.20)	-0.0009 (0.37)
LEAD	0.0745 (4.20)	-0.0099 (3.39)	0.0117 (6.56)	-0.0029 (1.74)
BUSINESS	0.0032 (1.28)	-0.0039 (9.53)	0.0023 (9.04)	-0.0012 (4.87)
POPULAT	-0.0004 (2.04)	0.0002 (7.90)	-0.0002 (10.35)	0.0001 (3.38)
COMMUTE	-0.1345 (2.17)	0.0242 (2.38)	-0.0328 (5.28)	0.0081 (1.39)
GROWTH	-0.5056 (3.85)	-0.1470 (6.82)	0.0088 (0.67)	-0.0768 (6.22)
INCOME	-0.1224 (2.29)	0.0036 (0.41)	0.0076 (1.43)	0.0007 (0.13)
EDUCAT	0.5431 (0.83)	0.2093 (1.95)	0.0773 (1.18)	0.0800 (1.30)
COVERAGE	-3.6387 (1.15)	1.6218 (3.11)	-1.2115 (3.80)	0.4231 (1.42)
MEDINAGE	-0.3050 (5.96)	-0.0034 (0.41)	-0.0259 (5.04)	-0.0183 (3.80)
POVERTY	-0.0070 (0.11)	-0.0154 (1.44)	0.0144 (2.21)	-0.0129 (2.11)
TCELLS	0.0552 (5.55)	-0.0048 (2.94)	0.0118 (11.80)	0.0017 (1.87)
WAGE	-0.3698 (0.56)	-0.8690 (8.03)	0.0407 (0.62)	-0.5633 (9.08)
ENERGY	-3.2014 (5.35)	-0.2498 (2.54)	-0.3345 (5.57)	-0.2003 (3.56)
OPERATE	-1.3383 (1.88)	-0.6410 (5.48)	0.0575 (0.81)	-0.3414 (5.10)
RENT	1.6273 (2.76)	0.1393 (1.44)	0.1220 (2.06)	0.1621 (2.92)
PRIME	-0.6739 (2.63)	0.0225 (0.54)	-0.0524 (2.04)	0.0091 (0.38)
DENSITY	-0.0471 (4.05)	0.0117 (6.15)	-0.0076 (6.47)	0.0042 (3.88)
TEMPERAT	0.0046 (0.68)	-0.0009 (0.80)	0.0010 (1.55)	-0.0004 (0.66)
RAIN	0.1544 (2.63)	0.0094 (0.97)	-0.0022 (0.38)	0.0050 (0.90)
CRIME	-0.0002 (3.11)	0.0000 (3.39)	-0.0000 (6.61)	0.0000 (1.16)
SVCRIMES	-9.4650 (1.86)	-3.3144 (3.97)	1.7636 (3.46)	-1.7000 (3.56)
$R^2$	0.4684	0.4157	0.5967	0.3413

OLS estimates and absolute t-statistics.

**Table 11. Duopoly: Policy Experiments**

VARIABLES	NONLINEAR	TWO-PART	LINEAR	FLAT
CONS. SURPLUS	401.30	195.32	69.39	111.66
PROFITS	175.46	165.95	90.96	100.38
WELFARE	576.76	361.28	160.35	212.04
EXP. SALE	163.79	254.03	20.55	288.26
EXP. TARIFF	135.81	269.08	78.81	160.15
EXP. RATE	0.66	0.66	2.24	0.00
EXP. FEE	0.16	125.26	0.00	160.15

All magnitudes are measured in dollars per customer.

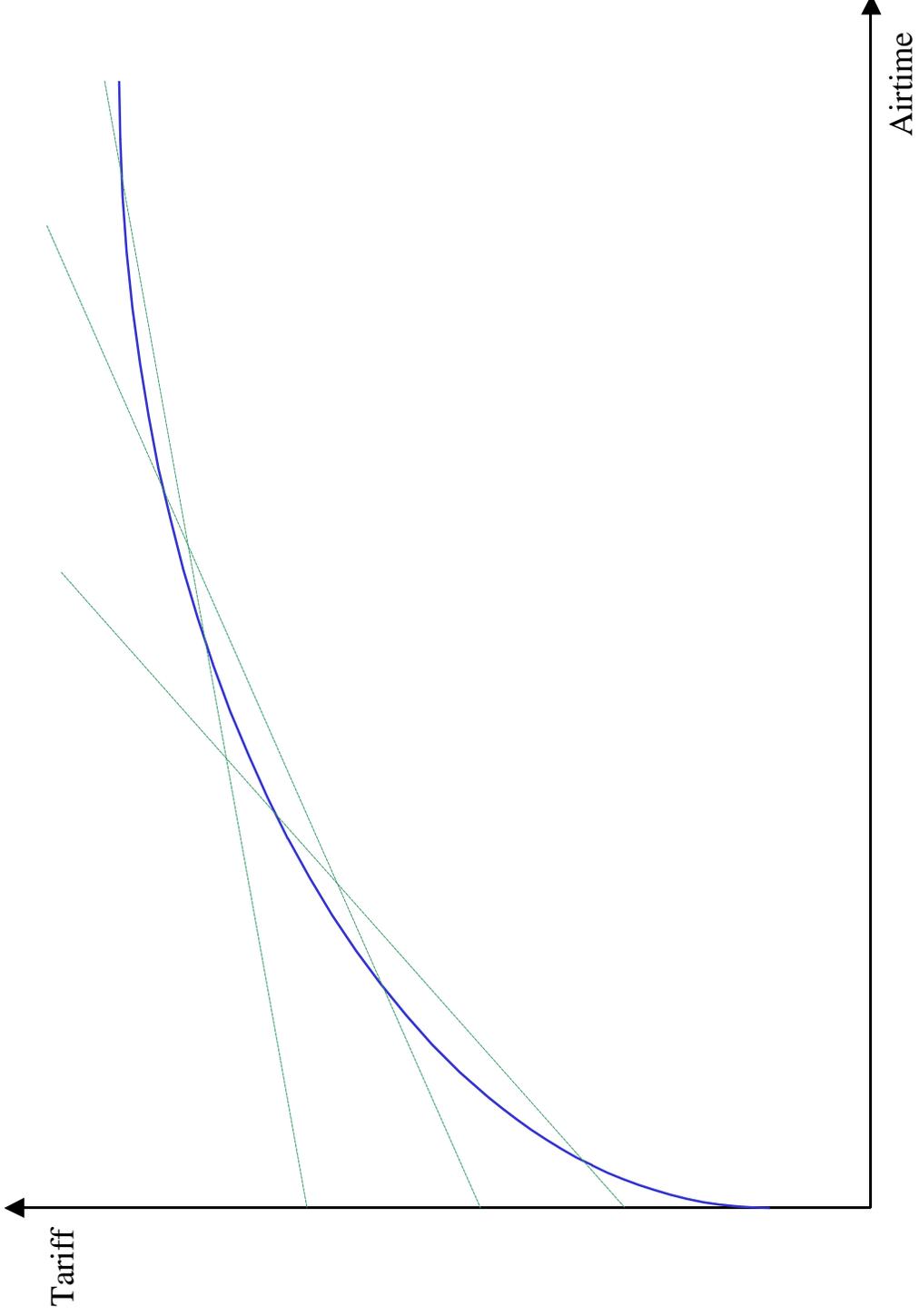


Figure 1: Basic Identification

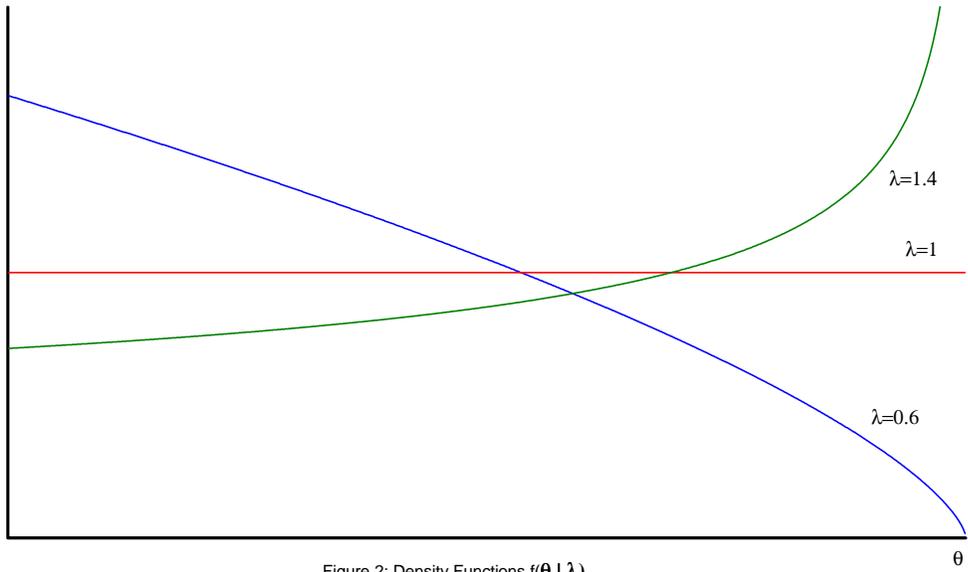


Figure 2: Density Functions  $f(\theta | \lambda)$

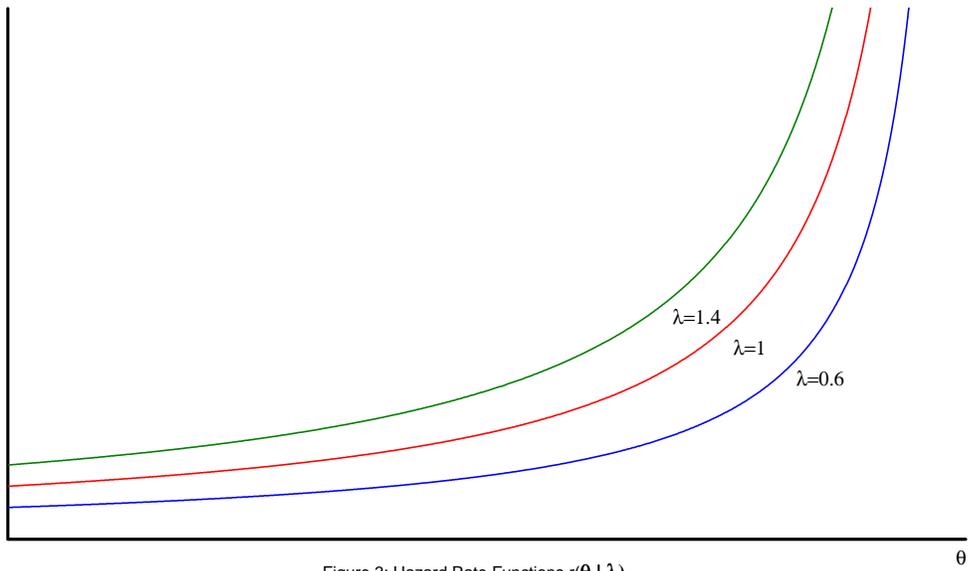


Figure 3: Hazard Rate Functions  $r(\theta | \lambda)$

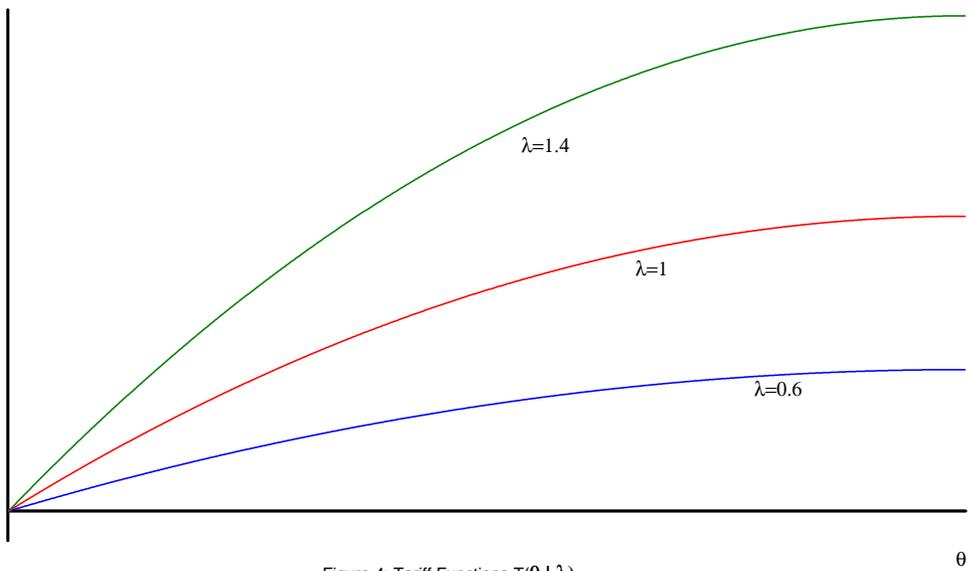
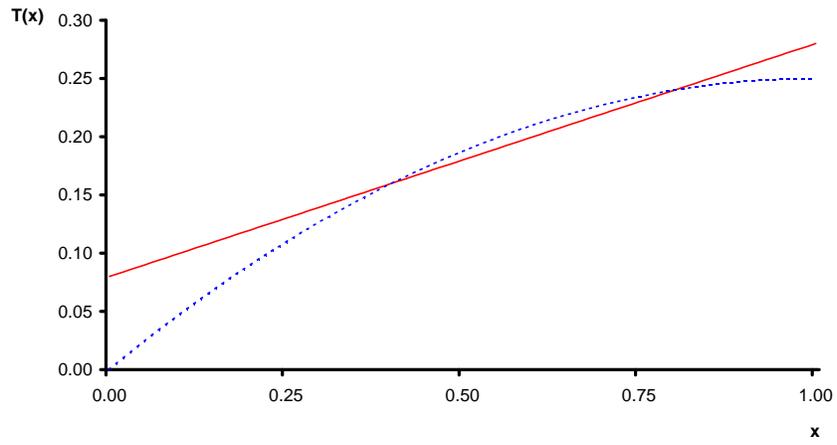
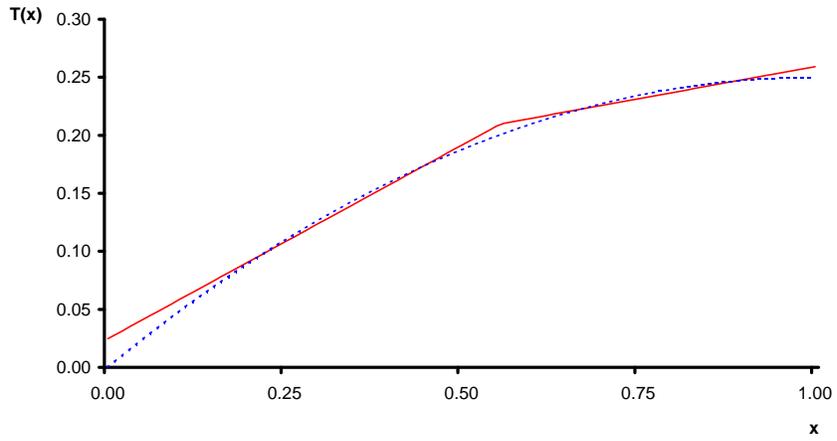


Figure 4: Tariff Functions  $T(\theta | \lambda)$

### One Optional Tariffs



### Two Optional Tariffs



### Three Optional Tariffs

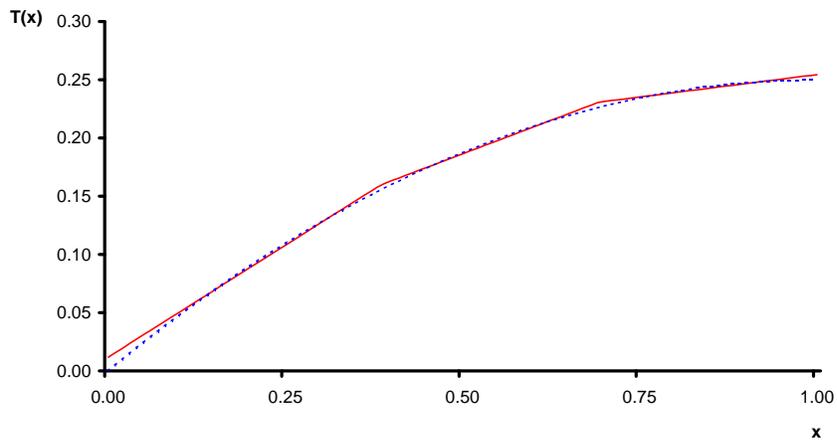


Figure A1: Approximation through Menus of Two-Part Tariffs

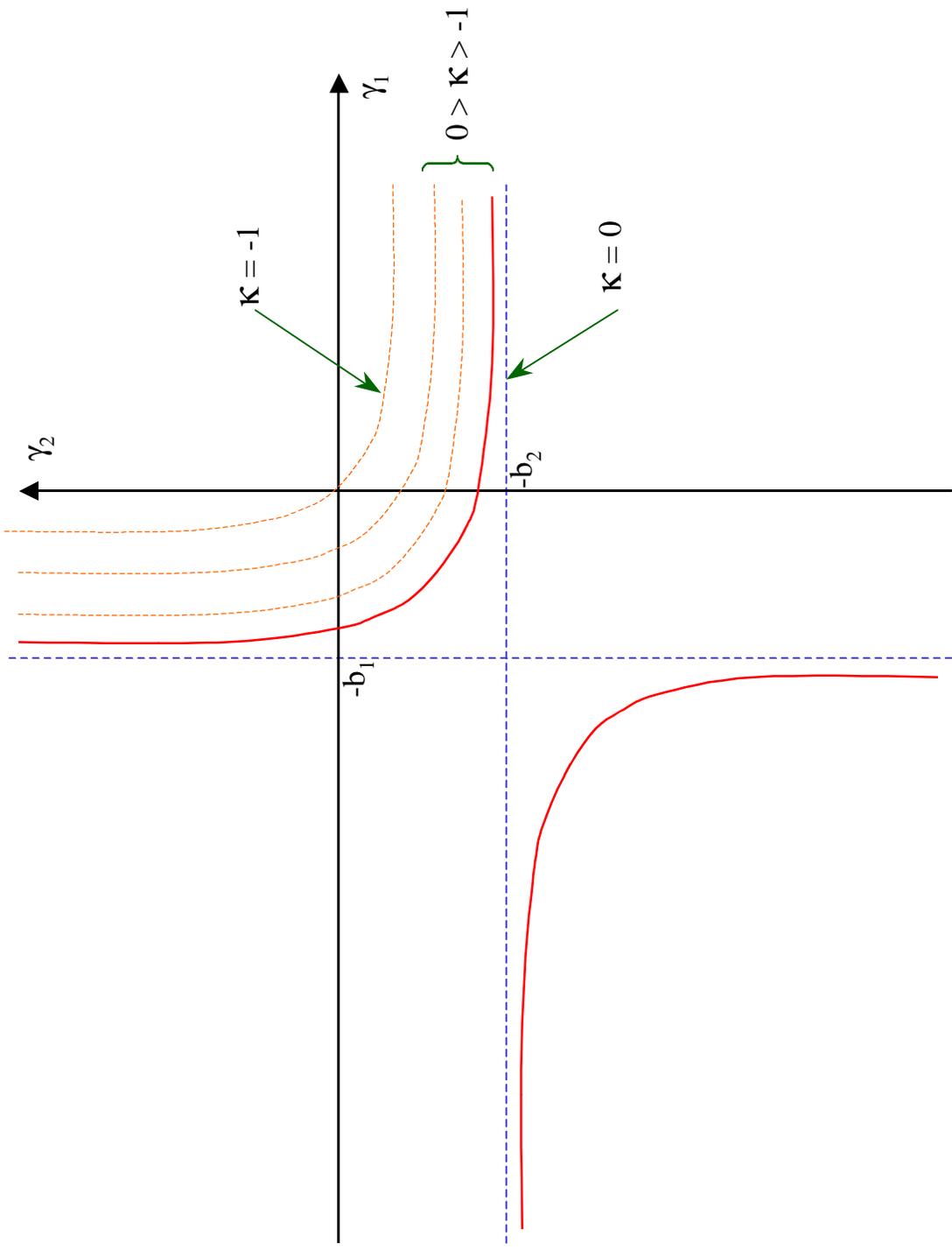


Figure A2: Domain of  $\gamma_1$  and  $\gamma_2$