

# DISCUSSION PAPER SERIES

No. 4067

## C-CAPM AND THE CROSS-SECTION OF SHARPE RATIOS

Paul Söderlind

*FINANCIAL ECONOMICS*



**Centre for Economic Policy Research**

[www.cepr.org](http://www.cepr.org)

Available online at:

[www.cepr.org/pubs/dps/DP4067.asp](http://www.cepr.org/pubs/dps/DP4067.asp)

# C-CAPM AND THE CROSS-SECTION OF SHARPE RATIOS

Paul Söderlind, University of St Gallen and CEPR

Discussion Paper No. 4067  
September 2003

Centre for Economic Policy Research  
90–98 Goswell Rd, London EC1V 7RR, UK  
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

This Discussion Paper is issued under the auspices of the Centre's research programme in **FINANCIAL ECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Paul Söderlind

CEPR Discussion Paper No. 4067

September 2003

## **ABSTRACT**

### **C-CAPM and the Cross-Section of Sharpe Ratios**

This Paper studies whether the consumption-based asset-pricing model can explain the cross-section of Sharpe ratios. The constant relative risk aversion (CRRA) model and several extensions (habit persistence, recursive utility and idiosyncratic shocks) all imply that the Sharpe ratio is linearly increasing in the asset's correlation with aggregate consumption growth. Results from quarterly data on 40 US portfolios (1947–2001) and ten international portfolios (1957/1971–2001) suggest that both the unconditional and conditional C-CAPM have serious problems: there is a great deal of variation in Sharpe ratios, but most portfolios have relatively similar and low correlations with aggregate consumption growth.

JEL Classification: E13, E32 and G12

Keywords: consumption-based asset pricing, habit persistence, idiosyncratic risk, multivariate GARCH and recursive utility

Paul Söderlind  
Swiss Institute for Banking & Finance  
University of St. Gallen  
Rosenbergstr. 52  
CH-9000 St. Gallen  
Switzerland  
Tel: (41 71) 224 7064  
Email: paul.soderlind@unisg.ch

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=114406](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=114406)

Submitted 01 September 2003

## 1 Introduction

It is well documented that the standard consumption-based asset pricing model (constant relative risk aversion, representative investor) has serious problems with matching the high risk premia on equity.<sup>1</sup> This paper takes a look at a related, but less studied, issue: can the consumption-based model explain the cross-sectional dispersion of Sharpe ratios—when we are agnostic about how to measure consumption volatility and the risk aversion?<sup>2</sup> That is, if we treat consumption volatility and risk aversion as free parameters, can C-CAPM then explain the dispersion of Sharpe ratios among different equity and bond portfolios?

There are good reasons to be uncertain about the relevant measure of consumption volatility and risk aversion. For instance, several recent extensions of the basic consumption-based model have argued that we should consider adding idiosyncratic risk (Mankiw (1986) and Constantinides and Duffie (1996)), recalibrate the risk aversion (Epstein and Zin (1989)) or assume habit persistence (Campbell and Cochrane (1999)).

Fortunately, we can test important implications of the consumption-based model quite independently of these issues. A basic feature of the model is that, in a cross-section, the Sharpe ratio is linearly related to the correlation of the return with aggregate consumption growth. This holds even if there is idiosyncratic risk and/or habit persistence: in all cases the slope of the relation should be positive (more so if consumption volatility and risk aversion are high) and the fit of the regression should be good (perfect if it wasn't for sampling uncertainty). The C-CAPM (with or without extensions) can thus be rejected if the slope is negative and the fit poor.

The outline of the rest of the paper is as follows: Section 2 derives the relation between the Sharpe ratio and correlation in the basic model and several extensions; Section 3 presents the results for 40 US portfolios and 10 international portfolios; and Section 4 concludes.

## 2 C-CAPM and the Cross-Section Sharpe Ratios

This section derives a simple relation between the an asset's Sharpe ratio and correlation with the consumption growth. This relation is later studied on some 50 US and interna-

---

<sup>1</sup>See, for instance, Campbell (2001) and Cochrane (2001) for overviews.

<sup>2</sup>For an early paper on a similar topic, see Breeden, Gibbons, and Litzenberger (1989).

tional portfolios.

The basic asset pricing equation says

$$E(Z_{it}M_t) = 0, \quad (1)$$

where  $Z_{it}$  is the excess return of holding asset  $i$  from period  $t - 1$  to  $t$ ,  $M_t$  is a stochastic discount factor, and  $E$  denotes the expectations (possibly conditional on some information set). To simplify the analysis, assume that the excess return,  $Z_{it}$ , and the log stochastic discount factor (SDF),  $\ln M_t$ , have a bivariate normal distribution. Use Stein's lemma (see Appendix B) and rearrange (1) to express the Sharpe ratio,  $SR_i = E(Z_{it})/\sigma(Z_{it})$ , as

$$SR_i = -\rho(Z_{it}, \ln M_t) \times \sigma(\ln M_t), \quad (2)$$

where  $\rho(Z_{it}, \ln M_t)$  is the correlation of the excess return and the log SDF, and  $\sigma(Z_{it})$  and  $\sigma(\ln M_t)$  are the standard deviations of the excess return and log SDF respectively. If the expectation in (1) is conditional, so are the moments in (2).

We can relax the assumption that the excess return is normally distributed: (2) holds also if  $Z_{it}$  and  $\ln M_t$  has a bivariate mixture normal distribution—provided  $\ln M_t$  has the same mean and variance in all the mixture components (see Appendix B). This restricts  $\ln M_t$  to have a normal distribution, but allows  $Z_{it}$  to have a distribution with fat tails and skewness.

## 2.1 The Standard CRRA Model

With constant relative risk aversion (CRRA), the SDF has the form  $M_t = \beta(C_t/C_{t-1})^{-\gamma}$ , where  $\gamma$  is the risk aversion parameter and  $C_t$  is the consumption level. The log SDF is therefore

$$\ln M_t = \ln \beta - \gamma \Delta c_t, \quad (3)$$

where  $\Delta c_t$  is the growth rate of consumption, that is,  $\Delta c_t = \ln(C_t/C_{t-1})$ . If we assume that that  $Z_{it}$  and  $\Delta c_t$  have a bivariate normal distribution (or a mixture normal as discussed above), then we can plug (3) into (2) to get

$$SR_i = \rho(Z_{it}, \Delta c_t) \times \sigma(\Delta c_t)\gamma. \quad (4)$$

Most studies of the consumption-based asset pricing model have focused on the prob-

lem of reconciling the low volatility of consumption growth,  $\sigma(\Delta c_t)$ , with the high excess return of an aggregate equity portfolios (this is the equity premium puzzle of Mehra and Prescott (1985)). This problem clearly becomes worse if the correlation,  $\rho(Z_{it}, \Delta c_t)$ , of the excess return and consumption growth is low (the correlation puzzle discussed by, among others, Cochrane (2001)).

This paper is instead about a broader cross-section of assets—and about the case when we are uncertain about how to measure the volatility of consumption growth and the risk aversion parameter. Here, the cross-sectional implication of (4) are studied by estimating the linear cross-sectional regression equation

$$SR_i = a + b\rho(Z_{it}, \Delta c_t) + v_i. \quad (5)$$

According to the model (4) the slope coefficient ( $b$ ) should be positive (since  $\sigma(\Delta c_t)\gamma$  is) and the coefficient of determination  $R^2$  high (unity, except for sampling variability). In a sense, this regression is similar to the traditional cross-sectional regressions of returns on factors with unknown factor risk premia (see, for instance, Campbell, Lo, and MacKinlay (1997)).

The advantage of this formulation is that it does not require any data on either risk aversion or consumption volatility—which will turn out to be especially convenient when we allow for habit persistence and idiosyncratic risk (see below).

The analysis has so far focused on unconditional moments. However, asset pricing theory is mostly about conditional moments. For instance, an investor's first order condition for optimal saving/consumption gives the basic asset pricing equation (1)—but where the expectation is conditional on the information set at the time of optimization,  $E_{t-1}(Z_{it}M_t) = 0$ .

Assuming that the conditional distribution of the excess return and the log SDF is bivariate normal (or a mixture normal as discussed above) gives the same kind of expression as before for the Sharpe ratio (4), except that all moments are now conditional (and therefore carry time subscripts)

$$SR_{t-1i} = \rho_{t-1}(Z_{it}, \Delta c_t) \times \sigma_{t-1}(\Delta c_t)\gamma, \quad (6)$$

where  $SR_{t-1i} = E_{t-1}(Z_{it})/\sigma_{t-1}(Z_{it})$ . This means that there is still a linear relation between the Sharpe ratio and the correlation—within a period. The slope can change

between periods (if  $\sigma_{t-1}(\Delta c_t)\gamma$  does). To test the conditional model we therefore have to run a cross-sectional regression like (5) for each period separately (using conditional Sharpe ratios and correlations).

The next few sections show that recent refinements of the consumption-based models (habit persistence, Epstein-Zin utility, idiosyncratic risk) typically give something similar to (4) or (6), except that the consumption growth volatility and risk aversion parameter have to be reinterpreted. However, the correlation is typically unchanged, so (5) (unconditional or conditional) is still a valid approach to study the cross-sectional implications.<sup>3</sup>

## 2.2 High Risk Aversion 1: Habit Persistence

The habit persistence model of Campbell and Cochrane (1999) has a CRRA utility function, but the argument is the difference between consumption and a habit level,  $C_t - X_t$ , instead of just consumption. The habit is parameterized in terms of the “surplus ratio”  $S_t = (C_t - X_t)/C_t$ , which measures how much aggregate consumption exceeds the habit. Since this ratio is external to the investor, marginal utility becomes  $(C_t - X_t)^{-\gamma} = (C_t S_t)^{-\gamma}$ . The log SDF is therefore

$$\ln M_t = \ln \beta - \gamma(\Delta s_t + \Delta c_t), \quad (7)$$

where  $s_t$  is the log surplus ratio. The process for  $s_t$  is assumed to be a non-linear AR(1) (constant suppressed)

$$s_t = \phi s_{t-1} + \lambda(s_{t-1})\Delta c_t, \quad (8)$$

where  $\lambda(s_{t-1}) \geq 0$  is a decreasing function of  $s_{t-1}$ .

Using these equations in the conditional version of (2) gives (see Appendix B)

$$SR_{t-1i} = \rho_{t-1}(Z_{it}, \Delta c_t) \times \sigma_{t-1}(\Delta c_t)\gamma[1 + \lambda(s_{t-1})]. \quad (9)$$

Within a period, the only difference to the CRRA model (6) is the very last term,  $1 + \lambda(s_{t-1})$ , which can be thought of as a time-varying risk aversion. This may help explain the equity premium puzzle (by making the effective risk aversion larger than  $\gamma$ ) and also predictability in risk premia (if any). However, it gives the same cross-sectional implications as the CRRA model:  $SR_{t-1i}$  is still linearly related to  $\rho_{t-1}(Z_{it}, \Delta c_t)$ .

<sup>3</sup>Lettau and Uhlig (2002) also provide simple (but different) analytical expressions for these models—mostly with the aim to discuss the equity premium and riskfree rate puzzles.

### 2.3 High Risk Aversion 2: Epstein-Zin Utility

The recursive utility function in Epstein and Zin (1989)<sup>4</sup> has different parameters for the risk aversion and the intertemporal elasticity of substitution. It is then possible to have a high risk aversion (to fit the Sharpe ratio) without playing havoc with the riskfree rate.

Epstein and Zin (1989) show that if all wealth is marketable, then the Euler equation for the excess return of asset  $i$  is

$$E_{t-1}[\beta^\theta (C_t/C_{t-1})^{-\theta/\psi} R_{mt}^{\theta-1} Z_{it}] = 0, \text{ where } \theta = (1 - \gamma)/(1 - 1/\psi), \quad (10)$$

where  $R_{mt}$  is the market gross return,  $\gamma$  the risk aversion, and  $\theta$  the elasticity of intertemporal substitution.

To illustrate that (10) has (approximately) the same implications for risky assets as the CRRA model, assume that consumption-wealth ratio is constant (for instance, because the market return is iid). This effectively closes down all interaction between the risk and intertemporal substitution (see Svensson (1989) and Campbell (1993)). It then follows that consumption growth and the market return are proportional so (10) can be written (see Appendix B)

$$E_{t-1}[(C_t/C_{t-1})^{-\gamma} Z_{it}] = 0, \quad (11)$$

which is exactly the same as the CRRA model. This holds as a good approximation as long as the consumption-wealth ratio is not too variable.

### 2.4 High Volatility of Consumption: Idiosyncratic Risk

The volatility of aggregate consumption may underestimate the risk faced by investors if there are uninsurable individual shocks. Such shocks mean that the consumption growth of investor  $j$  is the aggregate consumption growth plus an idiosyncratic component. His log SDF is therefore  $\ln M_{jt} = \ln \beta - \gamma \Delta c_t - \gamma u_{jt+\delta}$ , where  $u_{jt+\delta}$  is the idiosyncratic shock. For analytical convenience, I let the shock be realized a split second ( $\delta$ ) after the asset return and aggregate consumption.

Using the law of iterated expectations, the first order condition (the ‘‘asset pricing

---

<sup>4</sup>Utility is  $U_t = [(1 - \delta)C_t^{1-1/\psi} + \delta Z_t^{1-1/\psi}]^{1/(1-1/\psi)}$ , where  $Z_t$  is the certainty equivalent of future utility,  $Z_t = [E_t(U_{t+1}^{1-\gamma})]^{1/(1-\gamma)}$ .

equation”) of investor  $j$ ,  $E_{t-1}(Z_{it}M_{jt}) = 0$ , can now be written

$$E_{t-1}[Z_{it}M_t E_t \exp(-\gamma u_{jt+\delta})] = 0, \quad (12)$$

where  $M_t = \ln \beta - \gamma \Delta c_t$  (as in previous sections).

Let the distribution of the idiosyncratic shock, conditional on the information set in  $t$  be normal. It is important that the mean of this distribution does not depend on the return or consumption growth. If it did, the uninsurable idiosyncratic shock would be correlated with the return (which would make the shock insurable) or with aggregate consumption (which would make the shock non-idiosyncratic). We therefore assume that the mean is always zero. In contrast, the variance is assumed to be  $2\lambda(\varepsilon_t)$  where  $\lambda(\varepsilon_t)$  is some function of the aggregate shock,  $\varepsilon_t = \Delta c_t - E_{t-1} \Delta c_t$ .

With these assumptions, (12) can be written (see Appendix B)

$$E_{t-1}\{Z_{it} \exp[-\gamma(\varepsilon_t - \gamma\lambda(\varepsilon_t))]\} = 0. \quad (13)$$

To calculate this expectation, I use Lettau’s (2002) convenient approximation by assuming that  $\varepsilon_t$  and  $\lambda(\varepsilon_t)$  have a bivariate normal distribution. This is only an approximation since a normal distribution cannot guarantee that the variance is always non-negative.

When  $\lambda(\varepsilon_t)$  is a constant, then it can be cancelled from (13) and we are back in the standard CRRA model: idiosyncratic shocks have no effect on risk premia unless their variance depends on the aggregate shock (see Mankiw (1986) and Constantinides and Duffie (1996) for similar results).

The simplest model that gives an effect is  $\lambda(\varepsilon_t) = a + b\varepsilon_t$ . We can then replace  $\varepsilon_t$  in the standard CRRA model by  $\varepsilon_t(1 - \gamma b)$  to get the conditional Sharpe ratio

$$SR_{t-1i} = \rho_{t-1}(Z_{it}, \Delta c_t) \times \sigma_{t-1}(\Delta c_t)\gamma(1 - \gamma b). \quad (14)$$

If  $b < 0$ , so bad times are also risky times, then the idiosyncratic shocks add to the Sharpe ratio (the term  $1 - \gamma b$  is positive). However, the implication for the cross-sectional pattern of Sharpe ratios is the same as in the standard CRRA model without idiosyncratic shocks (6). This is likely to continue to hold as long as  $\lambda(\varepsilon_t)$  is not too non-linear.

### 3 Empirical Evidence

#### 3.1 Unconditional Sharpe Ratios

This section presents results for unconditional Sharpe ratios and correlations—estimated by the historical averages.

*Figure 1* shows results for fifty different portfolios—mostly US equity portfolios sorted on industry, dividend-price ratio, size, or book-to-market, but also some international bond and equity portfolios (dollar returns, for US, France, Germany, Japan, and UK). More details on data are given in Appendix A. In *Figure 1.a*, the vertical axis shows (annualized) unconditional Sharpe ratios and the horizontal axis shows unconditional correlations with consumption growth (here measured at  $t + 1$ ).<sup>5</sup> Results from a cross-section regression (5) are given at the bottom of the figure.

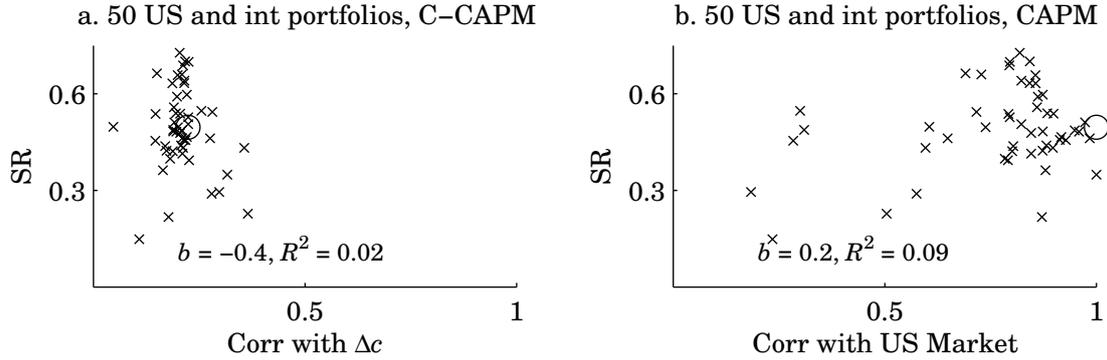
Most portfolios are clustered around a correlation of 0.2, but the Sharpe ratios vary a great deal. If anything, there is a weak negative relation between the correlation and the Sharpe ratio—and the  $R^2$  is virtually zero. This suggests that the consumption-based model has serious problems with explaining this broad cross-section. We will later study if it does a better job in smaller cross-sections.

As a comparison *Figure 1.b* shows results from a similar regression, but where the correlation with consumption growth is replaced by the correlation with the US equity market. This traditional CAPM seems to do a somewhat better job. The relation between the Sharpe ratio and the correlation with the US equity market is positive (although not significant at the 5% level; the test is based on a GMM approach that accounts for the generated regressors). However, the  $R^2$  remains very low. It could also be argued that the positive slope is due to a few particular portfolios with low correlations and low Sharpe ratios. We will later see that these are the international portfolios.

We now turn to studying smaller cross-sections, that is, subsets of the fifty portfolios. *Figure 2* shows the result for C-CAPM by plotting the Sharpe ratios against the correlations with consumption growth. For three of the four US cross-sections (industries, dividend-price ratio, and size) there is a weak negative relation, and it is only for the book-to-market portfolios that the C-CAPM is a success (significantly positive coefficient). For

---

<sup>5</sup>It is unclear if returns should be related to what is recorded as consumption this quarter or the next. The reason is that consumption is measured as a flow during the quarter, while returns are measured at the end of the quarter. I choose to show the results for  $\Delta c_{t+1}$  since they are somewhat more supportive of the consumption-based model—but the difference is not large.



**Figure 1: Sharpe ratio and correlation with consumption growth and US market (unconditional).** Subfigure a shows unconditional Sharpe ratios and correlations with  $\Delta c_{t+1}$  for 50 US and international portfolios. The large circle is the aggregate US stock market. The numbers at the bottom are slope coefficients and  $R^2$  from a regression of the Sharpe ratio on a constant and the correlation (5). Significance at the 5% level is marked by a star \*, and is based on a GMM approach that accounts for generated regressors. Subfigure b is similar but shows the correlation with the US equity market instead. Excess returns are real returns in excess of the real return on a one-month Treasury Bill. To annualize the Sharpe ratio, quarterly means are multiplied by 4 and quarterly standard deviations by 2. See Appendix A for details on data sources and transformations.

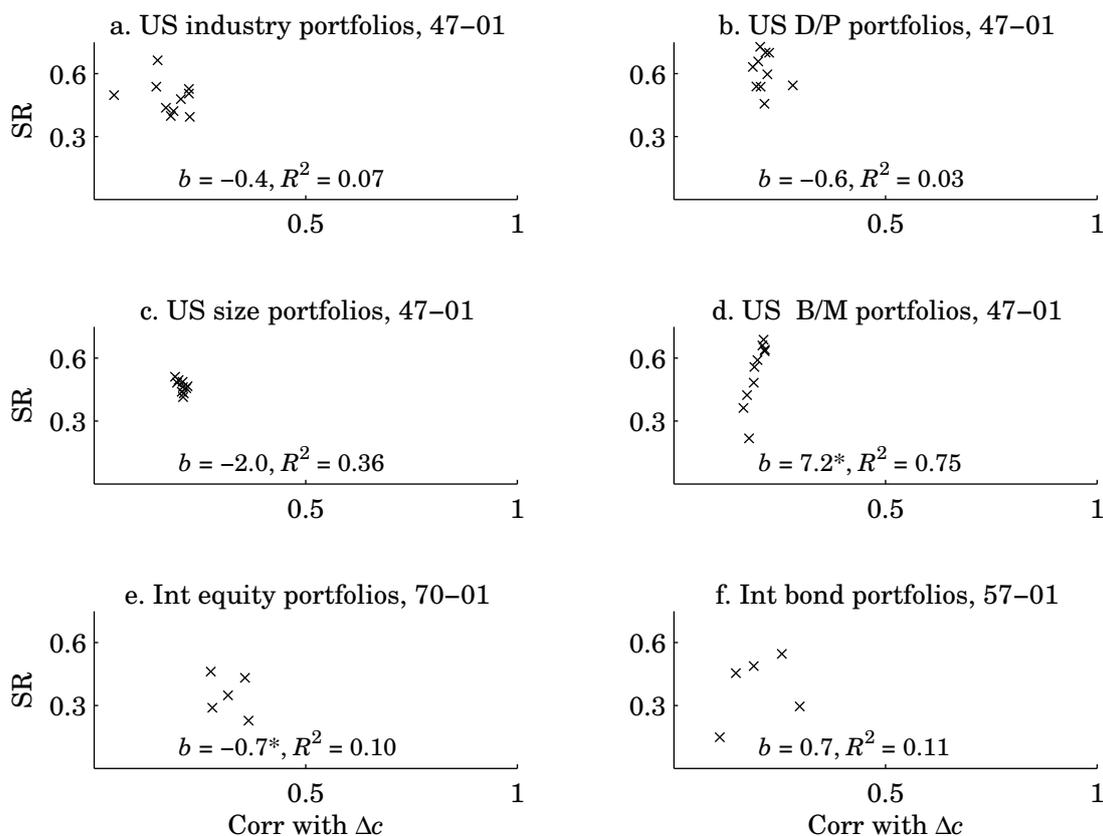
the international portfolios, the evidence is mixed: the relation is (significantly!) negative for equity portfolios, but positive for bond portfolios.

All in all, there is little support for the C-CAPM in either the large cross-section or in the smaller cross-sections. The main problem is that there are considerable differences in Sharpe ratios, but not in correlations with consumption growth. To put it bluntly: systematic risk seems to be something else than correlation with consumption growth.

Figure 3 shows CAPM result for the smaller cross-sections: the Sharpe ratio is plotted against the correlation with the US equity market. These correlations are typically much higher than the correlations with consumption growth, but the CAPM is about as bad as the C-CAPM at explaining the US returns (negative slope for three out of four cross-sections), although it does a better job with the international portfolios (positive slopes for both international equity and bond portfolios).

### 3.2 Conditional Sharpe Ratios

This section presents results for conditional (time-varying) Sharpe ratios and correlations. To estimate the conditional moments, I use a combination of a VAR model and a multi-

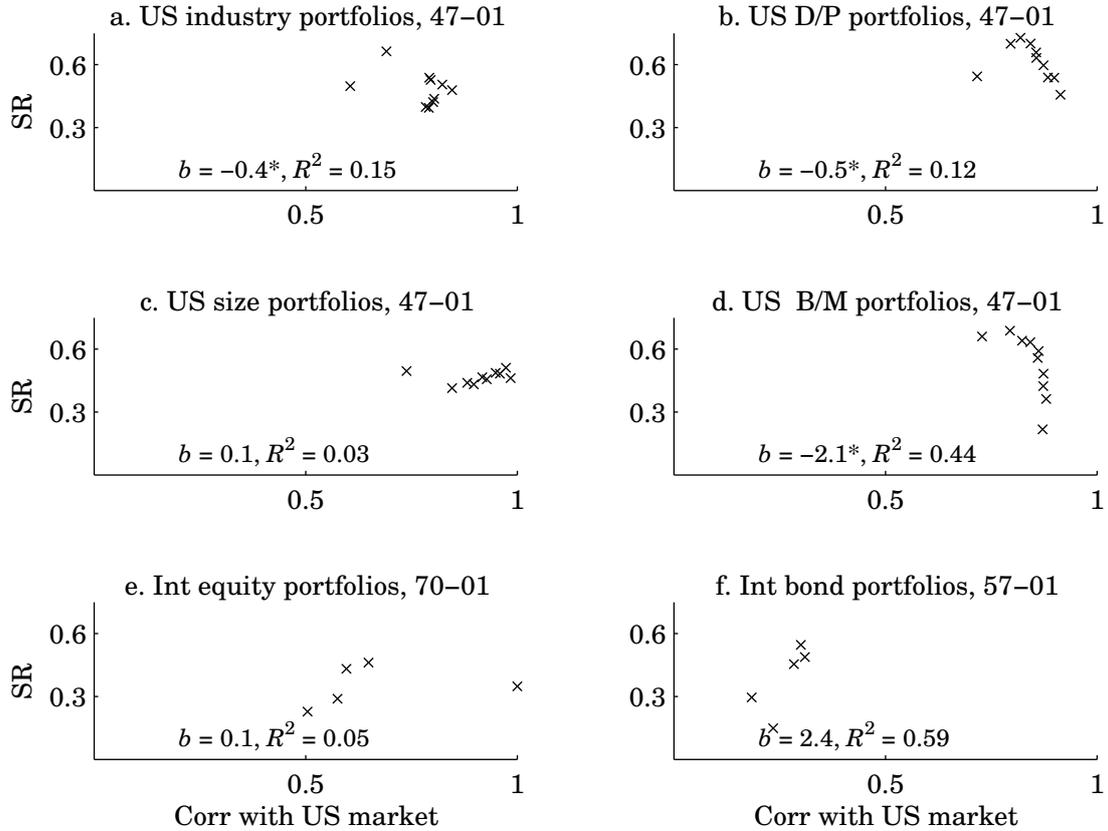


**Figure 2: Sharpe ratio and correlation with consumption growth (unconditional).** Subfigure a shows unconditional Sharpe ratios and correlations with  $\Delta c_{t+1}$  for 10 US industry portfolios for 1947Q1–2001Q4 (the sample period is given in the title). Subfigures b–d are for US dividend/price, size, and book-to-market portfolios respectively. Subfigures e–f are for international equity and bond portfolios respectively. See Figure 1 for more details on the figure and Appendix A for details on data sources and transformations.

variate GARCH model. These seem to be the most effective and widely used models for predicting first and second moments respectively.

For each asset, I estimate a “multivariate GARCH-in-mean” model with four variables: the excess return on asset  $i$ , consumption growth, the excess return on the US equity market, and the “cay” variable of Lettau and Ludvigson (2001) which has proven to be an interesting predictor of US returns. Let  $x_t$  be the vector  $[R_{it}^e, \Delta c_t, R_{mt}^e, \text{cay}_t]$ . The estimated model is

$$x_t = A_0 + A_1 x_{t-1} + A_2 x_{t-2} + B_m \text{Cov}_{t-1}(R_{it}^e, R_{mt}^e) + B_c \text{Cov}_{t-1}(R_{it}^e, \Delta c_t) + u_t, \quad (15)$$



**Figure 3: Sharpe ratio and correlation with US equity market (unconditional).** These figures show unconditional Sharpe ratios plotted against the correlation with the aggregate US equity market excess return. See Figure 2.

where the conditional covariance matrix of the residuals  $u_t$  is modelled as a “dynamic conditional correlation multivariate GARCH” (Engle (2002) and Engle and Sheppard (2001)). This multivariate GARCH model allows for dynamic correlations (which we certainly want), but is still fairly parsimonious.<sup>6</sup> The variables in the vector  $x_t$  are assumed to be predicted by two lags (cross effects are allowed) as well as the conditional covariances of asset  $i$  with consumption growth and the US market. The conditional covariances are from the GARCH model of the residuals. The estimation of this model is highly non-linear (since the covariances enter as “regressors”), but that is straightforward to handle in a quasi-maximum likelihood setting.

<sup>6</sup>Some details: the GARCH models are of order (1,1) and the DCC MVGARCH model is of order (2,1). Diagnostic tests suggest that this model is reasonable.

	$\Delta c_{t+1}$			US market		
	Slope	Rej %	$R^2$	Slope	Rej %	$R^2$
US industries, 47-01	0.08	0.33	0.19	-0.36	0.35	0.21
US D/P, 47-01	-0.75	0.32	0.21	-0.63	0.25	0.32
US size, 47-01	-0.47	0.38	0.22	-0.11	0.24	0.35
US B/M, 47-01	1.05	0.64	0.12	-2.13	0.56	0.38
Int equity, 70-01	-0.69	0.33	0.21	0.47	0.01	0.35
Int bond, 57-01	1.21	0.39	0.27	-0.29	0.47	0.24
All	-0.07	0.51	0.06	0.20	0.21	0.17

**Table 1: Results from period-by-period cross sectional regressions, average over time.** This table summarizes the results from period-by-period cross-sectional regressions of conditional Sharpe ratios on conditional correlations with  $\Delta c_{t+1}$  and the US market return. The slope coefficients and  $R^2$ s are time averages, and the Rej % is fraction of periods for which we reject the hypothesis that the slope coefficient is zero. The tests are based on standard errors from a Monte Carlo simulation of (15) with 2200 repetitions.

The essence of this reduced form model is that it allows the expected return (numerator of the Sharpe ratio) to vary with recent returns, consumption growth, and the cay variable as well as with the current conditional covariances (as theory predicts). The standard deviation of the asset return (the denominator in the Sharpe ratio) and the correlation are allowed to vary with recent volatility, which captures the notions of volatility clustering/spillover.

The estimation gives a time series of the conditional moments for every asset. These are used to study if the implication of (6) holds: within a period there should be a linear relation (in a cross-section) between the Sharpe ratio and the correlation of the asset with consumption growth or the US market. I study this by running a regression like (5) for every time period.

The slope coefficients vary a lot across time, and negative coefficients are not uncommon. *Table 1* summarizes the results by showing time averages. On average, the coefficients are slightly negative for the C-CAPM (both within the smaller cross-sections and the large cross-section of all 50 portfolios)—much like the results for the unconditional Sharpe ratios. This is also verified by simply taking time-averages of the conditional Sharpe ratios and correlations in (6) and then running the cross-sectional regression. The basic reason for why the results are similar is that there is relatively little predictability in these data so average conditional moments are fairly similar to unconditional moments.

These results are somewhat different from those obtained in Menzly, Santos, and Veronesi (2001) who get fairly positive results for a habit persistence model on 20 industry portfolios. The results are also different from those in Bansal, Dittmar, and Lundblad (2001) who get fairly good results from an Epstein-Zin utility on a fairly wide cross-section of portfolios. One possible reason for these differences is that I use a larger set of variables (“instruments”) to create the conditional moments.

The results for the CAPM on the smaller cross-sections are somewhat more discouraging than in the unconditional case, but we once again get a positive slope (time average) for the large cross-section—mostly because of the differences between the international and the US portfolios.

## 4 Summary

This paper studies if the consumption-based asset pricing model can explain the cross-sectional dispersion of Sharpe ratios—if we allow ourselves to be agnostic about the correct way of measuring consumption volatility and about the coefficient of relative risk aversion.

It is shown that the CRRA model and friends (Epstein-Zin utility, habit persistence, and idiosyncratic shocks) share the same implications for the cross-sectional dispersion of Sharpe ratios: the Sharpe ratio should be linearly increasing in the correlation of the asset return with aggregate consumption growth.

This is studied on quarterly data for 40 US portfolios (1947–2001) and 10 international portfolios (1957/1971–2001). The analysis is done in terms of both unconditional (time average) moments and conditional moments where the time-varying moments are estimated by a combination of a VAR model and a multivariate GARCH-in-mean model.

There is little support for the consumption-based model. The basic problem is that there is a great deal of variation in Sharpe ratios, but most portfolios have relatively similar (and low) correlations with aggregate consumption growth. The classical CAPM gets a bit more support, but mostly because the international portfolios tend to have lower Sharpe ratios and lower correlation with the US market.

## A Data Appendix

The nominal *US stock returns* are from French's (2001) website. These monthly returns are converted to quarterly returns by multiplying the monthly gross returns, for instance, the gross returns for January, February, and March are multiplied to generate a quarterly gross return. The portfolios are formed from NYSE, AMEX, and NASDAQ firms. The aggregate stock market return is a value weighted return of all available returns. The (equally-weighted) portfolios based on industry, size, dividend/price, or book-to-market are from the same data set, but with the firms sorted on the respective characteristic. The ten industry portfolios are for consumer nondurable, consumer durables, oil, chemicals, manufacturing, telecom, utilities, wholesale and retail, financial services, and other. The ten size portfolios are for deciles of firm market values; the D/P portfolios are for deciles of dividend/price; and the B/M portfolios are for deciles of book value/market values.

The nominal return on *long US government bonds* is from Ibbotson Associates.

The *international stock and long government bond returns* are from Ibbotson Associates, but come originally from Morgan Stanley and IMF respectively. The international data is for France, Germany, Japan, and United Kingdom, and has been converted into US dollar returns.

*Real returns* are calculated by dividing the nominal gross return by the gross inflation rate over the same period. Inflation is calculated from the seasonally adjusted CPI for all urban consumers (available at <http://www.stls.frb.org/fred/>).

Quarterly growth of *real consumption per capita* of nondurables and services is calculated from the seasonally adjusted number in NIPA Table 8.7 (available at <http://www.bea.doc.gov/bea/dn1.htm>). The growth rate is calculated as a weighted average of the growth rate of nondurables and the growth rate of services (chained 1996 dollars), where the (time-varying) weight is the relative (current dollar) size of nondurables in relation to services.

## B Derivations Appendix

### B.1 Derivation of (2)

If  $x$  and  $y$  have a bivariate normal distribution and  $h(y)$  is a differentiable function such that  $E[|h'(y)|] < \infty$ , then  $\text{Cov}[x, h(y)] = \text{Cov}(x, y) E[h'(y)]$ . See, for instance,

Cochrane (2001). Here, this gives  $\text{Cov}(Z_{it}, M_t) = \text{Cov}(Z_{it}, \ln M_t) E(M_t)$ . Since (1) can be written  $E(Z_{it}) E(M_t) = -\text{Cov}(Z_{it}, M_t)$ , (2) follows.

## B.2 Derivation of (9)

Use (8) in (7) to get  $\ln M_t = \ln \beta - \gamma(\phi - 1)s_{t-1} - [1 + \lambda(s_{t-1})]\gamma \Delta c_t$ . Since  $s_{t-1}$  is known in  $t - 1$ , the conditional standard deviation is  $\sigma_{t-1}(\ln M_t) = [1 + \lambda(s_{t-1})]\gamma \sigma_{t-1}(\Delta c_t)$ , and the conditional correlation is  $\rho_{t-1}(Z_{it}, \ln M_t) = -\rho_{t-1}(Z_{it}, \Delta c_t)$  which gives (9).

## B.3 Derivation of (11)

The budget restriction is  $W_t = R_{mt}(W_{t-1} - C_{t-1})$ , where  $W_t$  is wealth. If  $C_t/W_t = 1/\alpha$ , then we can substitute for wealth in the budget restriction to get  $C_t\alpha = R_{mt}(\alpha C_{t-1} - C_{t-1})$ , or  $C_{t+1}/C_t = R_{mt}(\alpha - 1)/\alpha$ . Using this in (10) gives (11).

## B.4 Derivation of (13)

We have that  $E_t \exp(-\gamma u_{jt+\delta}) = \exp[\gamma^2 \lambda(\varepsilon_t)]$ , and  $\ln M_t = \ln \beta - \gamma E_{t-1} \Delta c_t - \gamma \varepsilon_t$ . Equation (12) can therefore be written  $E_{t-1}\{Z_{it} \exp[(\ln \beta - \gamma E_{t-1} \Delta c_t - \gamma \varepsilon_t + \gamma^2 \lambda(\varepsilon_t))]\} = 0$ , which we can simplify by cancelling the non-random terms  $(\ln \beta - \gamma E_{t-1} \Delta c_t)$  to get (13).

## B.5 Derivation of Stein's Lemma for a Special Case of Mixture Normals

This section proves that Stein's lemma continues to hold if  $x$  and  $y$  has a bivariate mixture normal distribution, but that the marginal distribution of  $y$  is normal.

Let the pdf of  $(x, y)$  be a mixture of  $n$  bivariate normal distributions

$$pdf \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \sum_{i=1}^n \alpha_i \phi \left( \begin{bmatrix} x \\ y \end{bmatrix}; \begin{bmatrix} E_i(x) \\ E_i(y) \end{bmatrix}, \begin{bmatrix} \text{Var}_i(x) & \text{Cov}_i(x, y) \\ \text{Cov}_i(x, y) & \text{Var}_i(y) \end{bmatrix} \right), \text{ with } \sum_{i=1}^n \alpha_i = 1,$$

and where  $\phi(z; \mu, \Sigma)$  is the normal pdf with mean vector  $\mu$  and covariance matrix  $\Sigma$ .

Direct calculations give

$$\text{Cov}[x, h(y)] = \sum_{i=1}^n \alpha_i \{ \text{Cov}_i[x, h(y)] + E_i(x) E_i[h(y)] \} - E(x) E[h(y)].$$

If  $E_i[h(y)]$  is a constant  $E[h(y)]$ , then this simplifies to

$$\text{Cov}[x, h(y)] = \sum_{i=1}^n \alpha_i \text{Cov}_i[x, h(y)].$$

Since  $\text{Cov}_i[x, h(y)] = \text{Cov}_i(x, y) E_i[h'(y)]$  for all states, we get

$$\text{Cov}[x, h(y)] = E[h'(y)] \sum_{i=1}^n \alpha_i \text{Cov}_i(x, y).$$

If  $E_i(y) = E(y)$ , then the sum equals  $\text{Cov}(x, y)$ . Note, however, that the combination of  $E_i(y) = E(y)$  and  $E_i[h(y)] = E[h(y)]$  requires that  $\text{Var}_i(y) = \text{Var}(y)$ , which means that the marginal distribution of  $y$  is a normal distribution.

## References

- Bansal, R., R. F. Dittmar, and C. T. Lundblad, 2001, "Consumption, Dividend, and the Cross-Section of Equity Returns," mimeo, Duke University.
- Breeden, D. T., M. R. Gibbons, and R. H. Litzenberger, 1989, "Empirical Tests of the Consumption-Oriented CAPM," *Journal of Finance*, 44, 231–262.
- Campbell, J. Y., 1993, "Intertemporal Asset Pricing without Consumption Data," *American Economic Review*, 83, 487–512.
- Campbell, J. Y., 2001, "Consumption-Based Asset Pricing," in George Constantinides, Milton Harris, and Rene Stultz (ed.), *Handbook of the Economics of Finance*, North-Holland, Amsterdam.
- Campbell, J. Y., and J. H. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–251.
- Campbell, J. Y., A. W. Lo, and A. C. MacKinlay, 1997, *The Econometrics of Financial Markets*, Princeton University Press, Princeton, New Jersey.
- Cochrane, J. H., 2001, *Asset Pricing*, Princeton University Press, Princeton, New Jersey.
- Constantinides, G. M., and D. Duffie, 1996, "Asset Pricing with Heterogeneous Consumers," *The Journal of Political Economy*, 104, 219–240.
- Engle, R. F., 2002, "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models," *Journal of Business and Economic Statistics*, 20, 339–351.
- Engle, R. F., and K. Sheppard, 2001, "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH," Discussion Paper 2001-15, University of California, San Diego.
- Epstein, L. G., and S. E. Zin, 1989, "Substitution, Risk Aversion, and the Temporal Behavior of Asset Returns: An Empirical Analysis," *Journal of Political Economy*, 99, 263–286.

- French, K. R., 2001, "U.S. Research Returns Data," <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.
- Lettau, M., 2002, "Idiosyncratic Risk and Volatility Bounds, or Can Models with Idiosyncratic Risk Solve the Equity Premium Puzzle," *Review of Economics and Statistics*, forthcoming.
- Lettau, M., and S. Ludvigson, 2001, "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying," *Journal of Political Economy*, 109, 1238–1287.
- Lettau, M., and H. Uhlig, 2002, "Sharpe Ratios and Preferences: An Analytical Approach," *Macroeconomic Dynamics*, 6, 242–265.
- Mankiw, G. N., 1986, "The Equity Premium and the Concentration of Aggregate Shocks," *Journal of Financial Economics*, 17, 211–219.
- Mehra, R., and E. Prescott, 1985, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145–161.
- Menzly, L., T. Santos, and P. Veronesi, 2001, "Habit Formation and the Cross Section of Stock Returns," mimeo, University of Chicago.
- Svensson, L. E. O., 1989, "Portfolio Choice with Non-expected Utility in Continuous Time," *Economics Letters*, 40, 313–317.