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ABSTRACT

International Good Market Segmentation and Financial Market Structure*

While financial markets have recently become more complete and international capital flows well liberalized, markets for goods remain segmented. To investigate how more complete security markets may relieve the effects of this segmentation, we examine a series of two-country economies with internationally segmented good markets, distinguished by the available financial securities. We show that, under heterogeneity within countries, the financial structure matters: even with internationally complete financial markets, risksharing is limited and the equilibrium allocation may be inefficient, depending on the location of the securities. Sufficient conditions for efficiency include complete international financial markets together with liberalized international financial flows. Under these conditions, heterogeneous agents from the same country engage in 'financial shipping', using securities as a substitute for the international shipment of goods. This allows them to partially circumvent the segmentation, allowing for efficient risk sharing.

JEL Classification: F30, F36, G12 and G15
Keywords: financial innovation, international capital flows, market segmentation and risk sharing

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1. Introduction

The past 30 years or so have been characterized by an unprecedented development in the menu of financial instruments available and an increased liberalization of capital movements.[1] Progress in the free trade of goods, however, has been more limited, and there remain, even among developed countries, significant obstacles to the international shipment of goods.[2] Even with perfect financial markets, such good market segmentation would impose a burden on risk sharing by restricting the consumption of gains from financial trades with foreigners. A natural question then arises as to the extent to which the improvement in financial transfers can alleviate the burden on risk sharing imposed by good market segmentation.

The international finance, good market segmentation literature (Dumas (1992), Uppal (1992, 1993), etc.) has not thus far focused on the role of financial markets; equilibrium is typically solved via a central planner problem, where each country is modeled as a single representative agent, and the security market need not be modeled explicitly. The justification for this is that the equilibrium is assumed to be "Pareto optimal" (in the sense of Dumas (1992)) due to the financial market being "complete". There are, however, different notions of market completeness in an international setting. We shall refer to "international market completeness" if any cash-flow may be attained, but foreign securities may need to be used (and goods to be shipped). "Domestic market completeness" in a country is a more stringent requirement, defined as any cash-flow being attainable using the domestic securities alone.

Our objective is to examine the extent to which the financial innovation of new securities may alleviate the effects of segmentation in the good market. A part of this is to identify the financial market structure needed to obtain an efficient allocation and to justify the solution of equilibrium by means of a central planner. We consider a pure exchange economy with two countries, one having a homogeneous population proxied for by a single agent, but the other inhabited by two agents, heterogeneous in their preferences and endowments. There is a single consumption good, but the market therein is segmented, in that agents can only consume goods located in their own country, and must incur (proportional) shipping costs to transfer goods across borders. Securities are defined not only by their pay-offs, but also by their location, i.e., whether they can be redeemed for goods located in country 1 or country 2. It will appear that, because of the

[1] A limited amount of financial innovation took place as early as the 1960s, but the pace of innovation quickened considerably from the 1970s on; for a survey, see Allen and Gale (1994). There was a simultaneous evolution toward financial integration in developed countries, with such moves as the removal of capital controls following the end of the Bretton Woods system and the development of Euro-currency markets (see, e.g., Frankel (1986), Halliday (1989)). The liberalization is more recent in emerging economies (from the second half of the 1980s on) but is now well-documented, as are its effects in terms of financial integration (see, e.g., the chronologies in Bekaert and Harvey (2000) and Henry (2000), and the estimates of effects on the cost of capital in Bekaert and Harvey (2000), Errunza and Miller (2000)).

[2] Rogoff (1996) includes an overview of frictions in international trade; he estimates actual shipping costs to be approximately 10% for 1994. Tariffs have been declining but remain significant, ranging (on average) from 3 to 10% across countries (Whalley and Hamilton (1996, Table 3.1)). Rogoff (1996) concludes his analysis of the "purchasing power parity puzzle" by stating that there is no other satisfactory explanation for it than international good markets remaining quite segmented, due to the above frictions as well as nontariff barriers.

good market segmentation, the location of securities matters. Employing a general equilibrium approach, we solve for equilibrium under several financial market structures, distinguished by the securities that are available for trading; in all economies, however, markets are internationally complete, and trading in securities is unconstrained. This assumption is consistent with our earlier observation that, after decades of innovation, investors now face a rich menu of financial instruments, and international financial flows are quite well liberalized. Our approach is novel in that we explicitly model the security market, rather than deduce allocations from a central planner problem.

Our first task is to determine whether or not variations in the financial market structure (in terms of available securities) matter, despite our assumption of completeness at the international level, which is maintained throughout the paper. To our knowledge, this question is not addressed in the existing literature. Using a simple one-period example, we demonstrate that the answer is affirmative: because of the segmentation in the good market, additional securities are needed over an internationally complete market for an efficient allocation to result. Financial innovation has real effects, even under international market completeness, establishing that securities can indeed relieve imperfections in markets for goods, a fact little studied thus far. Stringent conditions are needed for efficiency, namely, the presence of a complete domestic market in any heterogeneous country. Securities settled abroad, even when they span the space of possible payoffs, do not suffice for the agents of a heterogeneous country to efficiently share risk; thus, the location of securities is revealed to matter.

To derive richer implications and characterizations of observable economic variables that can be compared to well-understood benchmarks, for the bulk of the paper we adopt a continuous-time setup. The segmentation makes the agents' optimization nonstandard. In the absence of any domestic financial market, an agent incurs shipping costs whenever he trades goods for securities. His dynamic budget constraint is therefore nonlinear in his consumption. He effectively faces a nonlinear price for consumption, and thus has a positive measure region (over his endowment) on which he does not trade. When securities are traded in both countries, however, an agent faces a profoundly different problem. Consider, for example, an agent who desires to consume from his holdings in foreign securities. Rather than sell these and ship the proceeds (and incur the shipping costs), the agent can exchange his foreign securities for securities traded in his home country, and then sell these and consume the proceeds. In other words, it is now possible for agents to use securities to transfer goods across countries and circumvent the shipping costs; we refer to this mechanism as *financial shipping* (as opposed to the "physical" shipping of goods, which is subject to shipping costs). Unlike with physical shipping, goods "shipped" financially are traded at the same real exchange rate no matter the direction of shipping (importing or exporting), resulting in an effective linearization in the transfer of goods from one country to the other. Any agent who has access to financial shipping is shown to face a linear problem, where the no-trade region is "knife-edge" as in a frictionless model.

Our notion of financial shipping captures in reduced form the role of international financial flows. Financial shipping cannot be modeled in a single-period framework, because no security trade occurs after endowments of the good are received, and so securities can never be exchanged for goods (and vice-versa). Because of this, our multiperiod model exhibits fundamentally different features from the single-period example; the conditions for efficient risk sharing are a lot less stringent. In this respect, our analysis provides an example where financial innovation plays a role specific to the dynamic setting (a feature missing in the existing literature, as pointed out by Allen and Gale (1994)). This striking difference between the single- and multi-period cases constitutes a highly unusual feature of our model.

Explicit characterization is provided for pertinent equilibrium economic quantities: interest rates, market prices of risk, imports/exports and exchange rates. We find that the key determinant of the nature of equilibrium is the availability of financial shipping. In its presence, the agents from the heterogeneous country can be aggregated into a single representative agent, and the allocation across countries is Pareto efficient and solves a central planner's problem with constant weights for the two countries. Thus, the conditions for efficiency coincide with those for the availability of financial shipping to the heterogeneous country agents. For efficiency, even a severely incomplete domestic market is sufficient, as long as securities can be freely exchanged for each other (i.e., international financial flows are unrestricted). In this respect, financial markets provide a remarkably effective relief of the burden imposed on risk sharing by the good market segmentation.

When financial shipping is not available, risk sharing is limited and the equilibrium allocation is inefficient, despite the presence of an internationally complete financial market. Here, a representative agent cannot be substituted for the heterogeneous country agents, and (a fortiori) the allocation across countries does not solve a central planner problem. Thus, our work clearly emphasizes the importance of investigating the properties of equilibrium with three agents (a case largely ignored in the literature) as, unless aggregation to the two-agent case is possible, the equilibrium is radically different. Our work also provides support for the body of literature originating in Dumas (1992), and solving for equilibrium via a central planner problem. We establish this approach to be valid as long as (unconstrained) financial markets are internationally complete and international financial flows are unconstrained (so that financial shipping is available to all agents), a set of assumptions consistent with the recent evolution of the international financial system. Dumas (1992), in a setup somewhat similar to ours, considers an economy with homogeneous linear production technologies in two countries, allowing an incorporation of an intertemporal capital investment decision. Unlike in our pure-exchange model, he must employ dynamic programming in his solution and hence his results are largely numerical. His focus is also different from ours, in that he primarily examines the behavior of the real exchange rate and considers a single financial market structure whose importance in enabling an efficient allocation is largely ignored.

The modeling strategy of Dumas (1992) is applied by various authors to several issues in international finance. Uppal (1992, 1993) examines agents' portfolio choice and international financial flows. His focus is on portfolio holdings, but he still solves for equilibrium by means of a central planner problem and does not examine the role of securities in making this possible. Sercu, Uppal and Van Hulle (1995), Hollifield and Uppal (1997) and Pavlova (2000) study other issues. Dumas and Uppal (2001), in a largely similar setting, but with recursive preferences, tackle a problem that can be seen as the mirror image of ours: rather than varying the financial market structure as we do, they vary the level of good market segmentation, and assess the benefits of international financial market integration in each case.

The literature on gains from international risk-sharing (e.g., Cole and Obstfeld (1991), Tesar (1995)) is also related to our work, in that it compares equilibria arising under different financial market structures, albeit in the polar cases of no international financial market and perfect integration. These papers also typically assume that a representative agent can be substituted for each country. As our work shows, however, this may not be justified in the absence of international financial trade. Casual empiricism suggests that domestic markets typically remain incomplete: if this were not the case, there would be no benefits to be reaped from international diversification, as the domestic securities would span all possible payoffs. But under domestic incompleteness, in the absence of international flows domestic allocations are inefficient, and a representative agent cannot be employed as a proxy for a country. Our work suggests that international financial markets may be used not only for trade across countries, but also by agents from a single country to trade risk with each other. This means that the existing literature may be understating the benefits of financial integration. This could explain why the gains from integration are typically estimated to be so small (about 0.20 percent of output according to Cole and Obstfeld (1991)).[3] This underscores the potential importance of allowing for intranational heterogeneity in international finance, a novelty of this work.

Our results could also help explain why, in the recent past, the growth in international financial flows has far outpaced the growth in international good trade, a fact often viewed as paradoxical. In our model, as financial markets become more complete and international financial flows are liberalized, some good trading is replaced by financial trading, as agents from the same country do not need to ship goods to trade risk with each other any more. They replace part of their "physical" shipping of goods by financial shipping.

The rest of the paper is organized as follows. Section 2 provides our single-period example. Section 3 presents the continuous-time economic setup, including the solution to the agents' optimization problems. Sections 4 and 5 analyze the equilibrium: Section 4 describes the main properties of the equilibria arising with and without financial shipping, while Section 5 provides additional characterizations of the economic quantities in both cases. Section 6 concludes, and the Appendix provides all proofs.

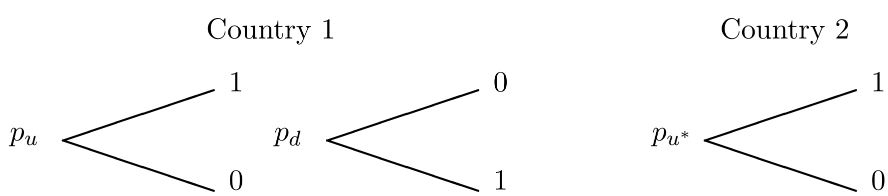
[3]Obstfeld (1994) obtains far higher estimates, but these are mostly due to the indirect effects, in a production economy, of international risk-sharing, via the switch it causes to riskier, more profitable projects.

2. A One-Period Example

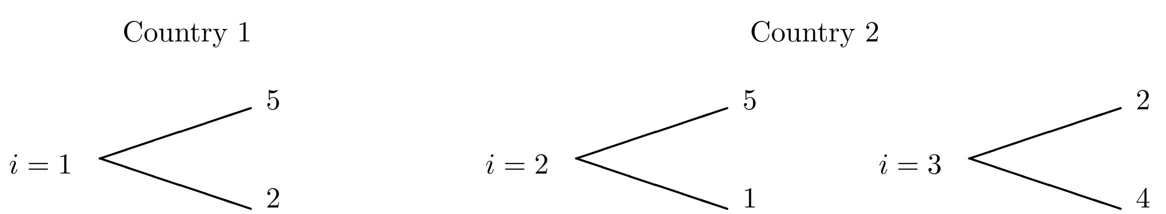
This section provides an example of the main ideas of the paper in the simplest possible setting. The example reveals that the financial market structure and the location of securities, not explicitly modeled in the previous good market segmentation literature, matter.

We consider a single good, single period model under uncertainty, with two equally probable states of nature, u ("up") and d ("down"). There are two countries 1, 2, populated by, respectively, one ($i = 1$) and two ($i = 2, 3$) agents. There are two dates, $t = 0$ (when agents choose their portfolio holdings) and $t = 1$ (when agents consume). Each agent maximizes the expected logarithm of time-1 consumption. Agents receive random time-1 endowments of the good located in their own country, and can only consume good located in their home country. There are shipping costs for transportation of the good between the two countries: if one unit of good is being shipped from a country, only 0.9 unit arrives (immediately) in the other. Hence, the good market is *segmented*.

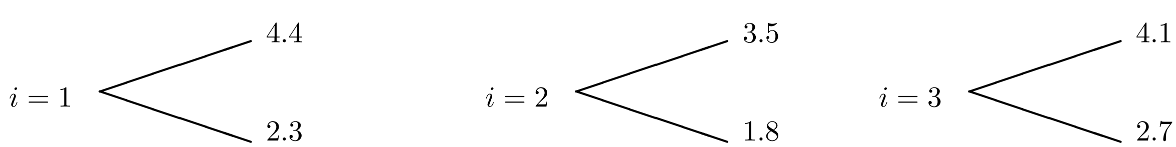
Three zero-net supply securities are traded, **u**, **d**, **u***, with prices p_u, p_d, p_{u^*}. Security **u** pays one unit of good located in country 1 if state u occurs and zero otherwise, security **d** pays one unit of good located in country 1 if state d occurs and zero otherwise, while **d*** pays one unit of good located in country 2 if state u occurs:



Under this financial market structure, although markets are complete internationally, country-2 agents face incomplete domestic markets. To make our point clear, we choose endowments that are highly heterogeneous among the country-2 agents:



Equilibrium consumption allocations can then be shown to be given by:



Country-2 agents' consumption in the two states reveals their marginal rates of substitution ($u'(c_u)/u'(c_d)$) to be different (as $3.5/1.8 \neq 4.1/2.7$), and so the allocation within country 2 is inefficient, in spite of the completeness of financial markets (at the international level). Thus, it does not prove possible to substitute a representative agent for the heterogeneous country and, a fortiori, the allocation across countries does not solve a central planner's problem either.

These inefficiencies arise from the fact that, under incompleteness of their domestic financial market, agents 2 and 3 may indulge in different directions of shipping and thus face effectively different prices for consumption. This is the case in the above example: agent 3, who has a low endowment in state u and hence is a net purchaser of consumption in this state, needs to purchase $1/0.9$ units of $\mathbf{u}$ to increase his consumption by one unit, hence facing a state price equal to $p_u/0.9$; agent 2 (short in $\mathbf{u}$) faces a state price equal to $0.9p_u$. The inefficiency that occurs manifests itself in two ways: the inequality in marginal rates of substitution across agents 2 and 3; and the "wasteful shipping". Goods are shipped back and forth wastefully when one agent in country 2 is importing and the other is simultaneously exporting: due to the absence of a complete domestic financial market, agents 2 and 3 need to go abroad to trade with each other. Under a complete country-2 domestic market, no wasteful shipping would occur, and hence an efficient allocation would obtain, which would also solve a central planner's problem. An incomplete market (such as the one in our example) provides only partial circumvention of the inefficiencies. This demonstrates that the financial structure does matter, even under internationally complete markets.

In short, to obtain an efficient allocation across countries and to justify the use of a central planner, strong assumptions (a complete domestic financial market within each country) seem to be needed. However, this may largely be an artifact of the single-period setup, in which agents decide on their portfolio at $t = 0$ and ship goods only at $t = 1$. In a multiperiod model, agents may trade in securities at the same dates as they ship goods. Then, in economies where securities are traded in both countries, each agent can, rather than "physically" shipping goods, do "financial shipping". Take the example of a country-2 agent who has made gains in country-1 settled securities and who desires to consume from these gains. Rather than export goods from country 1 to country 2 (and incur shipping costs), the agent can exchange his country-1 securities for country-2 securities, then exchange the country-2 securities for country-2 goods. These can then be consumed. This mechanism, which we refer to as "financial shipping", provides a viable alternative to the actual physical shipment of goods. The notion of financial shipping captures in reduced form the availability of unconstrained international financial flows.

Even though such spot good trades cannot be modeled independently of the financial market structure, they put only mild requirements on it. In particular, the availability of financial shipping is independent of market completeness: all that is needed is the presence in both countries of at least one security with a strictly positive value, as will be verified in the sequel. This suggests that, in a multiperiod model, securities play a dual role: as a tool for risk-sharing and as a substitute for the shipment of goods. Ignoring this, as a single-period model necessarily does, leads to inefficiencies that do not reflect any real-life imperfection. This suffices to make any multiperiod model qualitatively very different from the one-period model, and motivates our study of the continuous-time model to which we now turn.

3. The Economic Setting

We consider a continuous-time, pure-exchange economy with a finite horizon $[0, T]$ and a single consumption good. The uncertainty is represented by the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ (with $\mathcal{F} \equiv \mathcal{F}_T$) on which is defined a one-dimensional Brownian motion W.[4] The economy is populated by three agents, $i = 1$ (living in country 1) and $i = 2, 3$ (living in country 2), homogeneous in their (complete) information (represented by $\{\mathcal{F}_t\}$, the augmented filtration generated by W) and beliefs (represented by $\mathcal{P}$). All the stochastic processes introduced henceforth are assumed to be $\{\mathcal{F}_t\}$-progressively measurable, all equalities involving random variables hold $\mathcal{P}$-a.s., and all stochastic differential equations are assumed to have a solution.

3.1. Investment Opportunities

Investment opportunities consist of the following four securities. There are two zero-net supply "bonds" (money market accounts) with prices M and M^*, settled in country 1 and country 2 respectively (meaning that they can be exchanged for goods located in countries 1 and 2 respectively). In addition, there are two risky securities, each also in zero net supply, representing claims to the exogenously specified dividend processes δ and δ^*, and with prices P and P^*, settled in countries 1 and 2 respectively. The numeraire for securities M, P is the good located in country 1, and that for M^*, P^* is the good located in country 2. Security prices have dynamics

$$\begin{aligned} dM(t) &= M(t)r(t)dt\,, \\ dM^*(t) &= M^*(t)r^*(t)dt\,, \\ dP(t) + \delta(t)dt &= P(t)\left[\mu(t)dt + \sigma(t)dW(t)\right]\,, \\ dP^*(t) + \delta^*(t)dt &= P^*(t)\left[\mu^*(t)dt + \sigma^*(t)dW(t)\right]\,. \end{aligned}$$

All price parameters $(r, r^*, \mu, \mu^*, \sigma, \sigma^*)$ are to be determined endogenously in equilibrium.

We consider three economies, I, II, III, that differ in which securities are available for trading:

Economy	Available Securities
Economy I	M, P
Economy II	M, P, M^*
Economy III	M, P, M^*, P^*

[4] The extension to multiple sources of uncertainty, driven by a multi-dimensional Brownian motion, is straightforward and is discussed in Section 4.3. The main insights of our baseline, single Brownian motion economy remain valid.

Economy I has a complete financial market in one country and no financial market in the other. Economy II has a complete financial market in one country (1) and an incomplete market in the other, while Economy III has complete domestic markets in both countries. In all economies, however, markets are internationally complete for all agents, in the sense that any cash-flow can be perfectly replicated (at a cost that may differ across countries). The progression Economy I $\rightarrow$ Economy II $\rightarrow$ Economy III allows us to evaluate the effect of introducing more financial markets specifically in a country with heterogeneous agents. In particular, we may evaluate the extent to which financial securities allow circumvention of the good market segmentation (shipping costs).

In all economies, country 1's domestic financial market is complete; thus, all cash-flows paid in country 1 can be valued using the country-1 state price density process ξ with dynamics

$$d\xi(t) = -\xi(t)\left[r(t)dt + \theta(t)dW(t)\right],$$

where $\theta(t) \equiv (\mu(t) - r(t))/\sigma(t)$ denotes the market price of risk in country 1. This state price density is country-1 specific, in that it takes cash-flows paid in units of the good located in country 1 and yields a unique no-arbitrage price denominated in the same unit; accordingly, under standard regularity,

$$P(t) = \frac{1}{\xi(t)} E\left[\int_t^T \xi(s)\delta(s)ds \Big| \mathcal{F}_t\right];$$

and the deflated security gain processes $\xi P + \int_0^t \xi\delta dt$, ξM are $\mathcal{P}$-martingales. In Economy III, country 2's domestic market is also complete and we can similarly define a country-2 state price density process ξ^* by

$$d\xi^*(t) = -\xi^*(t)\left[r^*(t)dt + \theta^*(t)dW(t)\right], \tag{3.1}$$

where $\theta^*(t) \equiv (\mu^*(t) - r^*(t))/\sigma^*(t)$ denotes the market price of risk in country 2. Since the determination of ξ and ξ^* is sufficient to pin down all security prices, we focus our attention on these rather than the actual price processes M, M^*, P, P^*.

3.2. The Shipment of Goods and the Real Exchange Rate

While security markets are perfect, there are shipping costs for transportation of the good between the countries: if one unit of good is being shipped from a country, only k unit arrives (instantly) in the other, where $0 < k < 1$.[5] This (together with the fact that agents can only consume good located in their own country) makes the good market *segmented*. This segmentation is as in the international finance, good market segmentation literature (e.g., Dumas (1992)). We denote the amounts shipped, assuming the viewpoint of country 1, by x^E and x^I. $x^E(t) \geq 0$ is the amount of good exported, leaving country 1 at time t, and $x^I(t) \geq 0$ denotes the amount of good imported arriving in country 1. As we shall see, it may be necessary to distinguish country 1's exports and

[5] The shipping cost could be made specific to the direction of shipping (to account, for example, for the different nature of each country's exports) at only the cost of additional notational complexity.

imports depending on which country-2 agent they go to/originate from. Thus, we introduce the notations x_i^E, x_i^I, for $i \in \{2,3\}$, to denote, respectively, country 1's exports purchased by agent i and imports purchased from agent i. We have $x^I(t) = x_2^I(t) + x_3^I(t)$, $\forall t$.

Whenever securities are traded in both countries (Economies II and III), agents can freely exchange securities settled in one country for securities settled in the other (reflecting our observation that capital flows are now quite well liberalized). At the individual level, this is a substitute for the actual, "physical" shipment of goods, a notion we refer to as "financial shipping". The real exchange rate implicit therein (unique from no-arbitrage) is denoted $p(t)$ (expressed in units of good located in country 1 per unit of good located in country 2): for example, agents can exchange one unit of bond M^* for $(M^*(t)/M(t))\, p(t)$ unit(s) of bond M or $(M^*(t)/P(t))\, p(t)$ unit(s) of stock P; in both cases, $M^*(t)$ unit(s) of country-2 good are "financially shipped" and exchanged for $p(t)M^*(t)$ unit(s) of country-1 good. The exchange rate implicit in physical shipping equals either k or $1/k$, depending on the direction of shipment, whereas the rate for financial shipping equals p no matter the direction of shipment. No-arbitrage (that would be exploited by simultaneously shipping goods "financially" in one direction and "physically" in the other) implies:

$$k \leq p(t) \leq \frac{1}{k}. \tag{3.2}$$

The real exchange rate p will be shown in equilibrium to have dynamics:

$$dp(t) = p(t)\left[\mu_p(t)dt + \sigma_p(t)dW(t)\right].$$

3.3. Agents' Endowments and Preferences

Each of the three agents in the economy, $i = 1$ (living in country 1) and $i = 2, 3$ (country 2), is endowed with the (bounded) stochastic endowment process ϵ_i, with $\epsilon_i(t) > 0$, $\forall t$, expressed in units of the good located in his own country. ϵ_i has dynamics:

$$d\epsilon_i(t) = \mu_{\epsilon_i}(t)dt + \sigma_{\epsilon_i}(t)dW(t).$$

Agent i's portfolio holdings, expressed, for each security, in amounts of the good of the same nationality, are denoted by $\pi_i \equiv (\pi_{Mi}, \pi_{M^*i}, \pi_{Pi}, \pi_{P^*i},)^\top$, with $\pi_{M^*i} \equiv \pi_{P^*i} \equiv 0$ in Economy I and $\pi_{P^*i} \equiv 0$ in Economy II. We express agent i's wealth in units of the good located in country 1, $X_i \equiv \pi_{Mi} + \pi_{Pi} + p\left(\pi_{M^*i} + \pi_{P^*i}\right)$. A consumption-portfolio pair (c_i, π_i) is *admissible* if the associated wealth process X_i is bounded below, satisfies $X_i(T) \geq 0$ and obeys a dynamic budget constraint (such as (3.3)) that depends on the economy under consideration. Agent i derives time-additive, state-independent utility $u_i\left(c_i(t)\right)$ from intertemporal consumption of the good located in his own country in $[0, T]$. The function $u_i(\cdot)$ is assumed to be three times continuously differentiable, strictly increasing, strictly concave, and to satisfy $\lim_{c\to 0} u_i'(c) = \infty$ and $\lim_{c\to\infty} u_i'(c) = 0$. We denote the inverse of the first derivative of i's utility by $I_i(\cdot) \equiv \left(u_i'\right)^{-1}(\cdot)$. Agent i's optimization problem is to maximize $E\left[\int_0^T u_i\left(c_i(t)\right)dt\right]$ over all admissible (c_i, π_i) for which the expected integral is well defined.

Agents' optimization problems depend on their nationality and the financial market structure (Economies I-III) under consideration. The next two subsections are devoted to the solution to these problems. First, an agent with no domestic financial market is shown to face a nonlinear problem; his wealth dynamics are nonlinear in his consumption due to the shipping costs incurred whenever he trades in securities. However, in the presence of a domestic market, even incomplete, an agent is shown to face a linear problem; "financial shipping" allows him to purchase or sell consumption at a single rate. Accordingly, the solution techniques are different in the two cases.

3.4. Optimization without a Domestic Market

When agent i has no domestic financial market (agents 2 and 3 in Economy I), he must ship goods whenever he wants a consumption different from his endowment. For one unit of consumption in excess of his endowment, the agent needs to withdraw $1/k$ units of country-1 good from his financial wealth but, for one unit of endowment not consumed, he can only increase his financial wealth by k unit of country-2 good. The relevant dynamic budget constraint for his wealth is then nonlinear in consumption, and given by

$$\begin{aligned} dX_i(t) &= r(t)X_i(t)dt + \pi_{Pi}(t)\left(\mu(t) - r(t)\right)dt + \pi_{Pi}(t)\sigma(t)dW(t) \\ &\quad + k\left(\epsilon_i(t) - c_i(t)\right)^+ dt - \frac{1}{k}\left(\epsilon_i(t) - c_i(t)\right)^- dt, \qquad i = 2, 3. \end{aligned} \tag{3.3}$$

The second line, nonlinear in c_i, represents the agent's consumption "expenditures" (over his endowment), expressed in units of country-1 good. Re-formulating the problem as a linear one consisting for the agent in choosing his expenditure process e_i, where $e_i(t) \equiv k\left(\epsilon_i(t) - c_i(t)\right)^+ - \frac{1}{k}\left(\epsilon_i(t) - c_i(t)\right)^-$, leads to a simple solution of agent i's problem, using the martingale technology of Cox and Huang (1989), Karatzas, Lehoczky and Shreve (1987). His optimal consumption-portfolio policies are reported in Proposition 3.1.[6]

Proposition 3.1. *Assume that agent $i \in \{2, 3\}$ has no domestic security available. Then, his optimal consumption is given by*

$$c_i(t) = \begin{cases} I_i(ky_i\xi(t)) & \text{if} \quad \epsilon_i(t) > I_i(ky_i\xi(t)) & i \text{ exports (case S)} \\ \epsilon_i(t) & \text{if} \quad I_i(y_i\xi(t)/k) \le \epsilon_i(t) \le I_i(ky_i\xi(t)) & \text{no trade (case N)} \\ I_i(y_i\xi(t)/k) & \text{if} \quad \epsilon_i(t) < I_i(y_i\xi(t)/k) & i \text{ imports (case B)} \end{cases}, \tag{3.4}$$

where $y_i > 0$ satisfies the static budget constraint (with $c_i(t)$ given by (3.4))

$$E\left[\int_0^T \xi(t)\left\{k\left(\epsilon_i(t) - c_i(t)\right)^+ - \frac{1}{k}\left(\epsilon_i(t) - c_i(t)\right)^-\right\}dt\right] = 0. \tag{3.5}$$

The agent sells consumption and exports (S) in those times and states when his endowment is relatively high or the price of consumption ξ is relatively high, and buys consumption (B)

[6] Substituting $c_i(t)$ from (3.4) into (3.5) leads to a single equation where y_i is the only unknown.

when his endowment or the price of consumption is relatively low. Due to the shipping costs and the ensuing nonlinearity in the pricing of consumption, agent i will refrain from trading on an intermediate region of positive measure (N).

For $i = 2, 3$, we define agent-specific country-2 state price densities by $\xi_i^*(t) \equiv u_i'(c_i(t))/y_i$, and (individual-specific) shadow exchange rates by $p_i(t) \equiv \xi_i^*(t)/\xi(t)$. r_i and θ_i denote i's shadow interest rate and market price of risk and are defined by $d\xi_i^*(t) = -\xi_i^*(t)[r_i(t)dt + \theta_i(t)dW(t)]$. As will become clear in Section 3.5, p_2, p_3 are the exchange rates such that the agent would consume as he does here if foreign exchange were indeed allowed.

3.5. Optimization with a Domestic Market

This is the case for agent 1 in all Economies, and for agents 2 and 3 in Economies II and III. Financial shipping is now possible, at rate p, which by (3.2) ensures that financial shipping is either equivalent or preferable to physical shipping. Hence, agents 2 and 3's consumption expenditures (above their endowment), expressed in units of country-1 good, are linear in their consumption, and given by $p(t)(c_i(t) - \epsilon_i(t))$. Agent 1's are simply given by $(c_1(t) - \epsilon_1(t))$. Accordingly, agents' dynamic budget constraints, in units of country-1 good, are

$$\begin{aligned} dX_1(t) &= (\epsilon_1(t) - c_1(t))dt + \pi_{M1}(t)dM(t)/M(t) + \pi_{P1}(t)[dP(t) + \delta(t)dt]/P(t) \qquad (3.6) \\ &\quad + \pi_{M^*1}(t)d[p(t)M^*(t)]/p(t)M^*(t) + \pi_{P^*1}(t)[d(p(t)P^*(t)) + p(t)\delta^*(t)dt]/p(t)P^*(t) \\ &= [r(t)X_1(t) + (\epsilon_1(t) - c_1(t))]\,dt + [\pi_{P1}(t)(\mu(t) - r(t)) \\ &\quad + p(t)\pi_{M^*1}(t)(r^*(t) + \mu_p(t) - r(t)) + p(t)\pi_{P^*1}(t)(\mu^*(t) + \mu_p(t) + \sigma^*(t)\sigma_p(t) - r(t))]\,dt \\ &\quad + [\pi_{P1}(t)\sigma(t) + p(t)\pi_{M^*1}(t)\sigma_p(t) + p(t)\pi_{P^*1}(t)\sigma^*(t)\sigma_p(t)]\,dW(t), \qquad (3.7) \end{aligned}$$

and, for agents $i \in \{2, 3\}$,

$$\begin{aligned} dX_i(t) &= p(t)(\epsilon_i(t) - c_i(t))\,dt + \pi_{Mi}(t)dM(t)/M(t) + \pi_{Pi}(t)[dP(t) + \delta(t)dt]/P(t) \\ &\quad + \pi_{M^*i}(t)d[p(t)M^*(t)]/p(t)M^*(t) + \pi_{P^*i}(t)[d(p(t)P^*(t)) + p(t)\delta^*(t)dt]/p(t)P^*(t), \qquad (3.8) \\ &= [r(t)X_i(t) + p(t)(\epsilon_i(t) - c_i(t))]\,dt + [\pi_{Pi}(t)(\mu(t) - r(t)) \\ &\quad + p(t)\pi_{M^*i}(t)(r^*(t) + \mu_p(t) - r(t)) + p(t)\pi_{P^*i}(t)(\mu^*(t) + \mu_p(t) + \sigma^*(t)\sigma_p(t) - r(t))]\,dt \\ &\quad + [\pi_{Pi}(t)\sigma(t) + p(t)\pi_{M^*i}(t)\sigma_p(t) + p(t)\pi_{P^*i}(t)\sigma^*(t)\sigma_p(t)]\,dW(t), \qquad (3.9) \end{aligned}$$

with $\pi_{P^*i} \equiv 0$ in Economy II.

Although they enable financial shipping, the country-2 securities are equivalent to the country-1 risky security for purposes of exposure to the single factor of risk (driven by the one-dimensional Brownian motion). We thus introduce the risk-weighted sum Φ^i of i's holdings in the risky securities:

$$\Phi^i(t) \equiv \pi_{Pi}(t) + p(t)\pi_{M^*i}(t)\frac{\sigma_p(t)}{\sigma(t)} + p(t)\pi_{P^*i}(t)\frac{\sigma^*(t)\sigma_p(t)}{\sigma(t)}, \qquad (3.10)$$

also interpreted as i's composite risk exposure, in that all portfolio strategies leading to the same Φ^i yield the same volatility in i's dynamic budget constraint. No-arbitrage requires all risky securities to provide identical market prices of risk when expressed in the same unit (country-1 good), as reported by Proposition 3.2.

Proposition 3.2. *For agent $i = 1, 2, 3$'s optimization to have a solution, it is necessary that:*

$$\frac{\mu(t) - r(t)}{\sigma(t)} = \frac{r^*(t) + \mu_p(t) - r(t)}{\sigma_p(t)} = \frac{\mu^*(t) + \mu_p(t) + \sigma^*(t)\sigma_p(t) - r(t)}{\sigma^*(t) + \sigma_p(t)}, \qquad \forall t. \tag{3.11}$$

If (3.11) holds, agent i is indifferent between all $\pi^i(t)$ leading to the same value for $\Phi^i(t)$.

In Economy III, where state price densities are defined in both countries, (3.11) implies they are related by $\xi^*(t) = p(t)\xi(t)$, $\forall t$.

From Proposition 3.2, an agent's portfolio problem can be reduced to one involving a single, *composite* risky asset settled in country 1 and with price parameters as P. $\Phi^i(t)$ can be interpreted as the agent's amount invested therein. Agents' dynamic budget constraints in terms of Φ^i are:

$$\begin{aligned} dX_1(t) &= [r(t)X_1(t) + (\epsilon_1(t) - c_1(t))]\, dt + \Phi_1(t)\, [(\mu(t) - r(t))\, dt + \sigma(t) dW(t)] \,. \\ dX_i(t) &= [r(t)X_i(t) + p(t)\, (\epsilon_i(t) - c_i(t))]\, dt + \Phi_i(t)\, [(\mu(t) - r(t))\, dt + \sigma(t) dW(t)] \,, \quad i \in \{2, 3\}. \end{aligned}$$

Once reduced to the choice of (c_i, Φ_i), agent i's optimization problem can be solved using standard martingale techniques (Cox and Huang (1989), Karatzas, Lehoczky and Shreve (1987)), leading to the optimal policies in Proposition 3.3, assuming they exist.

Proposition 3.3. *Agent 1's optimal consumption in Economies I, II, III is given by*

$$c_1(t) = I_1\left(y_1 \xi(t)\right), \tag{3.12}$$

where $y_1 > 0$ satisfies the static budget constraint

$$E\left[\int_0^T \xi(t)\left\{I_1\left(y_1\xi(t)\right) - \epsilon_1(t)\right\} dt\right] = 0.$$

Agent i's, $i \in \{2, 3\}$, optimal consumption in Economies II, III is given by

$$c_i(t) = I_i\left(y_i p(t) \xi(t)\right), \tag{3.13}$$

where $y_i > 0$ satisfies the static budget constraint

$$E\left[\int_0^T p(t)\xi(t)\left\{I_i\left(y_i p(t)\xi(t)\right) - \epsilon_i(t)\right\} dt\right] = 0. \tag{3.14}$$

From (3.12), agent 1 sells consumption (S) for high enough endowment or price of consumption, and buys consumption (B) for low enough endowment or price of consumption. His no-trade region shrinks to a knife-edge. Agents 2 and 3's conditions are similar, only with $p(t)\xi(t)$ being substituted for $\xi(t)$. All agents now face a linear problem and, accordingly, trade everywhere except on a space of zero measure. In Economies II and III, agent $i \in \{2, 3\}$ faces the shadow state price density ξ_i^*, defined by $\xi_i^*(t) = u_i'\left(c_i(t)\right)/y_i$. From (3.13), $\xi_i^*(t) = p(t)\xi(t)$ for both agents (*not* individual-specific). In Economy III where country 2 is endowed with a complete domestic market, ξ_i^* coincides with the actual country 2 state price density ξ^* of (3.1).

4. Equilibrium under Differing Financial Market Structures

Equilibria of two types may occur in our model, depending on whether or not financial shipping is available to the country-2 agents: the financial structure and the location of securities matter despite international market completeness. In Economy I, the financial market structure does not allow for financial shipping, which leads to an equilibrium with fundamentally different features from Economies II-III where financial shipping is available. The next two subsections are devoted to an investigation of these two cases. Before engaging in it, we provide the definition of equilibrium in our setting.

Definition 4.1. *An equilibrium is a price system–(μ, r) in Economy I, (μ, r, r^*, p) in Economy II, (μ, r, μ^*, r^*, p) in Economy III–and consumption-portfolio processes (c_i, π_i) such that: (i) agents attain their optimal consumption-portfolio processes; (ii) the good markets in the two countries clear, i.e.,*

$$c_1(t) = \epsilon_1(t) + x_2^I(t) + x_3^I(t) - x_2^E(t) - x_3^E(t), \quad c_i(t) = \epsilon_i(t) + kx_i^E(t) - x_i^I(t)/k, \quad i = 2, 3; \tag{4.1}$$

(iii) security markets clear, i.e., $\sum_i \pi_{ji} = 0$, for $j \in \{M, P\}$ in Economy I, $j \in \{M, P, M^*\}$ in Economy II, $j \in \{M, P, M^*, P^*\}$ in Economy III.

4.1. Equilibrium Absent Financial Shipping

We first consider Economy I, where there is no domestic financial market and, a fortiori, financial shipping is not available to the country-2 agents. Then, the only way for agents 2 and 3 to trade (even with each other) is by importing or exporting, and necessarily incurring shipping costs. This admittedly extreme assumption of there being no financial market in country 2 captures in reduced form the situation of a country with imperfectly liberalized capital flows (preventing unconstrained financial shipping) together with incomplete domestic financial markets. In such an environment, the country-2 agents face a nonlinear problem (similar to that analyzed in Section 3.4) and individual-specific state prices. Highly heterogeneous country-2 agents will make different decisions as to import or export goods, leading to a situation analogous to the equilibrium cases described in this subsection. Such a situation (an incomplete domestic market together with imperfectly liberalized capital flows) corresponds to the reality of most countries before the recent (past 20 to 30 years) vague of financial innovation and liberalization. Even though the assumptions made here may not correspond to the full reality of current international economies, the results in this subsection serve as a benchmark to help us understand the effects of financial innovation and liberalization that will be analyzed further on.

Proposition 4.1 describes the equilibrium cases that are possible in this environment. We identify nine cases dependent upon the shipping situation of the nonlinear agents 2 and 3 (e.g., SB denotes agent 2 selling consumption and agent 3 buying consumption). Agent 1's situation need not be considered since he faces a linear problem and so his demand functions are identical whatever his shipping situation. All regions not presented (BS, NS, NB) are "mirror images" of regions presented (SB, SN, BN respectively).[7]

Proposition 4.1. *If equilibrium exists, in Economy I the amounts of good shipped and individual-specific real exchange rates are as follows:*

(SS) agents 2 and 3 export when

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t))}{u_1'(\epsilon_1(t)+x_3^I(t))} < k \quad \text{and} \quad \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t)+x_2^I(t))} < k, \tag{4.2}$$

and we have: $p_2(t) = p_3(t) = k$, $x_2^E(t) = x_3^E(t) = 0$, *and* $x_2^I(t), x_3^I(t) > 0$ *satisfy*

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t)-x_2^I(t)/k)}{u_1'(\epsilon_1(t)+x_2^I(t)+x_3^I(t))} = k \quad \text{and} \quad \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t)-x_3^I(t)/k)}{u_1'(\epsilon_1(t)+x_2^I(t)+x_3^I(t))} = k; \tag{4.3}$$

(NN) agents 2 and 3 do not trade when

$$k \le \frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t))}{u_1'(\epsilon_1(t))} \le \frac{1}{k} \quad \text{and} \quad k \le \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t))} \le \frac{1}{k},$$

and we have: $x_2^E(t) = x_3^E(t) = x_2^I(t) = x_3^I(t) = 0$,

$$p_2(t) = \frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t))}{u_1'(\epsilon_1(t))} \quad \text{and} \quad p_3(t) = \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t))};$$

(BB) agents 2 and 3 import when

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t))}{u_1'(\epsilon_1(t)-x_3^E(t))} > \frac{1}{k} \quad \text{and} \quad \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t)-x_2^E(t))} > \frac{1}{k},$$

and we have: $p_2(t) = p_3(t) = 1/k$, $x_2^I(t) = x_3^I(t) = 0$, *and* $x_2^E(t), x_3^E(t) > 0$ *satisfy*

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t)+kx_2^E(t))}{u_1'(\epsilon_1(t)-x_2^E(t)-x_3^E(t))} = \frac{1}{k} \quad \text{and} \quad \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t)+kx_3^E(t))}{u_1'(\epsilon_1(t)-x_2^E(t)-x_3^E(t))} = \frac{1}{k}; \tag{4.4}$$

(SB) agent 2 exports and agent 3 imports when

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t))}{u_1'(\epsilon_1(t)-x_3^E(t))} < k \quad \text{and} \quad \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t)+x_2^I(t))} > \frac{1}{k},$$

and we have: $p_2(t) = k$, $p_3(t) = 1/k$, $x_2^E(t) = x_3^I(t) = 0$, *and* $x_2^I(t), x_3^E(t) > 0$ *satisfy*

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t)-x_2^I(t)/k)}{u_1'(\epsilon_1(t)+x_2^I(t)-x_3^E(t))} = k \quad \text{and} \quad \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t)+kx_3^E(t))}{u_1'(\epsilon_1(t)+x_2^I(t)-x_3^E(t))} = \frac{1}{k}; \tag{4.5}$$

[7] Substituting x_i^E, x_i^I from (4.2)-(4.7) into (4.8) leads to two equations with two unknowns y_1/y_2 and y_1/y_3.

(SN) agent 2 exports and agent 3 does not trade when

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t))}{u_1'(\epsilon_1(t))} < k \quad \textit{and} \quad k \le \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t)+x_2^I(t))} \le \frac{1}{k},$$

and we have: $p_2(t) = k$, $x_2^E(t) = x_3^E(t) = x_3^I(t) = 0$, *and* $x_2^I(t) > 0$, $p_3(t)$ *satisfy*

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t) - x_2^I(t)/k)}{u_1'(\epsilon_1(t)+x_2^I(t))} = k \quad \textit{and} \quad p_3(t) = \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t)+x_2^I(t))}; \tag{4.6}$$

(BN) agent 2 imports and agent 3 does not trade when

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t))}{u_1'(\epsilon_1(t))} > \frac{1}{k} \quad \textit{and} \quad k \le \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t)-x_2^E(t))} \le \frac{1}{k},$$

and we have: $p_2(t) = 1/k$, $x_2^I(t) = x_3^E(t) = x_3^I(t) = 0$, *and* $x_2^E(t) > 0$, $p_3(t)$ *satisfy*

$$\frac{y_1}{y_2}\frac{u_2'(\epsilon_2(t) + x_2^E(t))}{u_1'(\epsilon_1(t)-x_2^E(t))} = \frac{1}{k} \quad \textit{and} \quad p_3(t) = \frac{y_1}{y_3}\frac{u_3'(\epsilon_3(t))}{u_1'(\epsilon_1(t)-x_2^E(t))}, \tag{4.7}$$

where y_1/y_2, y_1/y_3 *solve any two of the agents' budget constraints (with* x_i^E, x_i^I *as in (4.2)-(4.7)), i.e.,*

$$E\left[\int_0^T u_i'(\epsilon_i(t) + kx_i^E(t) - x_i^I(t)/k)\,(kx_i^E(t) - x_i^I(t)/k)\,dt\right] = 0, \quad i = 1,2. \tag{4.8}$$

The agent-specific state price densities and consumption allocations are given by

$$\begin{aligned} \xi(t) &= u_1'(\epsilon_1(t) - x_2^E(t) - x_3^E(t) + x_2^I(t) + x_3^I(t)), & c_1(t) &= \epsilon_1(t) - x_2^E(t) - x_3^E(t) + x_2^I(t) + x_3^I(t), \\ \xi_i^*(t) &= p_i(t)u_1'(\epsilon_1(t) - x_2^E(t) - x_3^E(t) + x_2^I(t) + x_3^I(t)), & c_i(t) &= \epsilon_i(t) + kx_i^E(t) - x_i^I(t)/k, \quad i = 1,2. \end{aligned} \tag{4.9}$$

Conversely, if there exist x_2^E, x_3^E, x_2^I, x_3^I, p_2, p_3, ξ, ξ_2^* *and* ξ_3^* *satisfying (4.2)-(4.9), then the associated optimal policies satisfy all market clearing conditions.*

Proposition 4.1 reveals that a country-2 agent, agent 2 or 3, will export (import) when the value of his contemporaneous endowment is high (low) relative to country 1's contemporaneous consumption, and if the difference is pronounced enough to justify incurring the shipping costs. If not, the agent neither imports nor exports. The value of the (individual-specific) shadow exchange rate p_i prevailing in equilibrium reveals the direction of the agent's shipping. No shipping by a country-2 agent takes place if the shadow exchange rate lies in the interior of $[k, 1/k]$ and the agent prefers consuming his own endowment.

As in a frictionless economy, the solution of equilibrium can be reduced to the determination of agents' relative weights y_1/y_2, y_1/y_3. Proposition 4.1 provides all other quantities as a function thereof. Nevertheless, in this setup one cannot solve for the equilibrium as is standard in international finance (Dumas (1992) and subsequent literature), by considering a central planner with constant weights for each *country*: this is because shipping costs now prevent efficient risk sharing within country 2. Whenever agents 2 and 3 are heterogeneous enough to make different decisions as to whether to buy or sell consumption, they face effectively different state prices (e.g., in SB, $\xi_1^* = k\xi$ and $\xi_2^* = \xi/k$) and so the allocation within country 2 is not Pareto optimal. As in a single period model, this reflects both the "wasteful shipping" in cases SB and BS and, more generally, agents 2 and 3 failing to equate their marginal rates of substitution in all cases but SS and BB. In this economy, the situation is essentially similar to the single period example of Section 2.

The implication that goods are simultaneously exported and imported (in regions SB and BS) may appear counter-intuitive. Our model, however, may be interpreted as a proxy for a multi-good model, where exports and imports consist of different goods, while imperfect domestic risk-sharing within country 2 (between agents endowed with different goods) leads to aggregate quantities as in our model. Another case of relevance for the results in this subsection is that of a 3-country model, where each agent represents a different country and only one country is endowed with financial markets. Characterization would then be identical to our case.

In short, the segmentation in the good market generates two types of inefficiencies: for an extended range of endowments, there is no shipment of goods between the two countries, leaving opportunities for risk-sharing *across* countries unexploited; and the segmentation effectively imposes a transaction cost on financial transactions between the country-2 agents. The next subsection demonstrates how the availability of "financial shipping" relieves this last type of inefficiency and how the equilibrium is affected.

4.2. Equilibrium with Financial Shipping

We now assume that there exists at least one security settled in country 2 (Economies II and III). Then, financial shipping is available to the country-2 agents and, from Section 3.5, agents 2 and 3 face a linear problem with a homogeneous state price density $\xi^* = p\xi$. Hence, we can substitute a representative agent (defined in a standard fashion) for the two agents in country 2. Proposition 4.2 describes the resulting equilibrium.

Proposition 4.2. *Assume there exists a financial market in country 2. Then, if equilibrium exists, in Economies II-III the amounts of good shipped and real exchange rate are as follows.* *When* $y_1 u_*' (\epsilon_2(t) + \epsilon_3(t)) / y_2 u_1' (\epsilon_1(t)) < k$, *country 1 imports and*

$$x^E(t) = 0, \quad x^I(t) > 0 \text{ solves } \frac{y_1}{y_2} \frac{u_*' (\epsilon_2(t) + \epsilon_3(t) - x^I(t)/k)}{u_1' (\epsilon_1(t) + x^I(t))} = k, \qquad p(t) = k. \tag{4.10}$$

When $k \leq y_1 u_*' (\epsilon_2(t) + \epsilon_3(t)) / y_2 u_1' (\epsilon_1(t)) \leq 1/k$, *there is no international trade and*

$$x^E(t) = x^I(t) = 0, \qquad p(t) = \frac{y_1}{y_2} \frac{u_*' (\epsilon_2(t) + \epsilon_3(t))}{u_1' (\epsilon_1(t))}. \tag{4.11}$$

When $y_1 u_*' (\epsilon_2(t) + \epsilon_3(t)) / y_2 u_1' (\epsilon_1(t)) > 1/k$, *country 1 exports and*

$$x^I(t) = 0, \quad x^E(t) > 0 \text{ solves } \frac{y_1}{y_2} \frac{u_*' (\epsilon_2(t) + \epsilon_3(t) + k x^E(t))}{u_1' (\epsilon_1(t) - x^E(t))} = \frac{1}{k}, \qquad p(t) = \frac{1}{k}, \tag{4.12}$$

where $u_*(\cdot)$ is the country 2 representative agent utility function defined by

$$u_*(c) \equiv \max_{c_2+c_3=c} u_2(c_2) + \frac{y_2}{y_3} u_3(c_3), \tag{4.13}$$

and y_1/y_2, y_2/y_3 solve any two of the agents' budget constraints (with x^I, x^E given by (4.10)-(4.12)), i.e.,

$$E\left[\int_0^T u_1'\left(\epsilon_1(t) - x^E(t) + x^I(t)\right)\left(x^E(t) - x^I(t)\right) dt\right] = 0; \tag{4.14}$$

$$E\left[\int_0^T p(t) u_1'\left(\epsilon_1(t) - x^E(t) + x^I(t)\right)\Big(I_2\left(y_2 p(t) u_1'\left(\epsilon_1(t) - x^E(t) + x^I(t)\right)\right) - \epsilon_2(t)\Big) dt\right] = 0. \tag{4.15}$$

The country-specific state price densities and consumption allocations are

$$\xi(t) = u_1'\left(\epsilon_1(t) - x^E(t) + x^I(t)\right), \quad \xi^*(t) = p(t) u_1'\left(\epsilon_1(t) - x^E(t) + x^I(t)\right), \tag{4.16}$$

$$c_1(t) = \epsilon_1(t) - x^E(t) + x^I(t), \quad c_2(t) = I_2\left(y_2 \xi^*(t)\right), \quad c_3(t) = I_3\left(y_3 \xi^*(t)\right). \tag{4.17}$$

Conversely, if there exist x^E, x^I, p, ξ and ξ^* satisfying (4.10)-(4.16), then the associated optimal policies satisfy all market clearing conditions.

As compared to Economy I (Proposition 4.1), the determination of equilibrium is considerably simplified. Once the relative weights (y_1/y_2, y_2/y_3) have been determined, the equilibrium can be solved for in two independent stages: the determination of international trade across countries and that of trade within country 2. The relative weight y_1/y_2 is set so that the flow of international trade satisfies a budget constraint that requires the present value of either country's exports, net of its imports, to be zero. The direction of international trade depends on the distribution of the contemporaneous endowment across countries. A country will import when it is relatively poor (as compared to the other country), and if the difference is pronounced enough to justify incurring the shipping costs. As could be expected, these conditions depend only on the *aggregate* endowment of country 2; they are as in a two-agent model, with one agent per country. The determination of trade within country 2, on the other hand, is as in a frictionless model; consequently, for $i \in \{2, 3\}$, whether agent i buys consumption, does not trade or sells consumption, depends on whether, respectively, $\epsilon_i(t) < I_i\left(y_i p(t)\xi(t)\right)$, $\epsilon_i(t) = I_i\left(y_i p(t)\xi(t)\right)$ or $\epsilon_i(t) > I_i\left(y_i p(t)\xi(t)\right)$. Since a representative agent can be substituted for the country-2 agents, only three cases obtain in equilibrium, depending on the direction of the trade of goods across countries (and not on the country-2 agents' individual decisions as to whether to buy or sell consumption).

In Economy II, where the country-2 domestic financial market is still incomplete, this considerable simplification in the determination of equilibrium relies on the availability of financial shipping. Now, the domestic security traded in country 2 can freely be exchanged for country-1 securities, allowing agents 2 and 3 to ship financially and, by so doing, to trade with each other without incurring shipping costs. A Pareto optimal allocation results within country 2, as does the possibility to substitute a country-2 representative agent. The availability of financial shipping ensures that no wasteful shipping takes place. This is because, when financial shipping opportunities are present, an agent will import or export physically only if it is not worse for him to do so rather than ship financially: e.g., agent 1 will export (physically) only if $p(t) = 1/k$ and import only if $p(t) = k$; otherwise, he would get a more favorable rate by doing financial shipping. Similarly, agents 2 and 3 will export only if $p(t) = k$ and import only if $p(t) = 1/k$. Hence, there can be a shipment of goods only in one direction simultaneously, because the exchange rate $p(t)$ is common to all agents. We note that financial shipping only facilitates risk-sharing at the intranational level (between agents 2 and 3). The actual, "physical" shipment of goods between countries still must take place and the corresponding transportation costs incurred, and hence there is an extended range of endowments for which there is no international trade (and reduced risk-sharing across countries relative to a frictionless model).

It is noteworthy that the same equilibrium obtains in Economy II (with an incomplete market in country 2) as in Economy III (with all domestic markets complete.) This reveals that efficient risk-sharing can result either from a complete domestic market in any heterogeneous country (Economy III) or from the availability of financial shipping (Economy II). In particular, domestically complete markets are not necessary for efficient risk-sharing, as long as markets are internationally complete and financial shipping is available to the agents. This result underscores the profound difference between a single-period model, where domestic market completeness is needed (Section 2), and the multiperiod model, where financial shipping allows for efficient risk-sharing under less stringent conditions on the financial structure.

Since financial shipping involves trade in securities (as described in Section 3.2), one might worry that this could interact with agents' portfolio strategies and perturb risk-sharing. This is not the case because any financial shipping takes place instantly: the agent purchases securities, immediately exchanges these for other securities and then exchanges these securities for goods. So, at the outset of the transaction, the agents' portfolio holdings are unaffected.

Proposition 4.2 implies that the equilibrium is "Pareto optimal" (using the term as in Dumas (1992)), in that it can be solved by means of a central planner problem (although marginal rates of substitution are not necessarily equated, due to the shipping costs).

Corollary 4.1. *If equilibrium exists, in Economies II and III, the equilibrium allocations solve the following maximization problem:*

$$\begin{array}{ll} \max_{c_1, c_*} & E\left[\int_0^T \left(\frac{1}{y_1} u_1\left(c_1(t)\right) + \frac{1}{y_2} u_*\left(c_*(t)\right)\right) dt\right] \\ \text{subject to} & c_1(t) = \epsilon_1(t) - x^E(t) + x^I(t) \\ & c_*(t) = \epsilon_*(t) + k x^E(t) - x^I(t)/k \\ & x^E(t) \geq 0, \quad x^I(t) \geq 0, \quad \forall\, t, \end{array} \tag{4.18}$$

where $c_*(t) = c_2(t) + c_3(t)$, $\epsilon_*(t) = \epsilon_2(t) + \epsilon_3(t)$ *and* $u_*(\cdot)$ *is as in Proposition 4.2.*

This finding provides support for the body of literature stemming out of Dumas (1992) (and more extensively reviewed in the Introduction), which employs this technique to determine equilibrium. It also offers reassurance as to the validity of this literature, since casual empiricism suggests that most domestic financial markets remain incomplete. The availability of internationally complete markets and unrestricted international financial flows, however, seems to fit the reality of today's financial system relatively well. This is consistent with the setting of Economy II, and the least stringent set of conditions for Corollary 4.1 to hold.

As discussed in Section 3, p is not only the shadow relative price of goods, but also the rate at which securities settled in different countries can be exchanged. When securities are only traded in one country (Economy I), p is a shadow exchange rate: the only rate that, if a foreign exchange contract were available to the agents, would induce identical allocations to our case. It is therefore pinned down uniquely, even in the absence of good trade across countries. Corollary 4.1 additionally allows us to verify that our notion of real exchange rate is consistent with the literature. Denoting the integrand in (4.18) by $V(t)$ and its partial derivatives with respect to $c_1(t)$ and $c_*(t)$ by $V_{c_1}(t)$ and $V_{c_*}(t)$ respectively, equations (4.10)-(4.12) reveal that $p(t) = V_{c_*}(t)/V_{c_1}(t)$, which is the notion of real exchange rate of Dumas (1992) and subsequent related work.

4.3. Extensions and Ramifications

Our model could easily be adapted to address a number of additional important issues.

(i) *Intranational homogeneity within country 2*. We now assume homogeneity (in preferences and endowments) within country 2, effectively populating it with a single agent. Then, we find that all financial market structures (Economies I-III) lead to the same equilibrium, as long as the assumption of international completeness is maintained. The equilibrium is identical to that of Section 4.2 (described in Proposition 4.2), with the country-2 representative agent utility u_* replaced by the utility of the single country-2 agent. Hence, under intranational homogeneity, the financial market structure has no impact on the real equilibrium quantities; the equilibrium allocation across countries is Pareto optimal in all cases.

To further investigate this irrelevance result, we consider an additional economy, I*, in which the only securities available for trading are M^* and P^*, so that country 2 is endowed with a complete market but no securities are traded in country 1. The comparison between Economy I (with a complete market in country 1) and Economy I* allows us to assess the effects of moving financial markets from country 1 to country 2. We find that there are no effects on the equilibrium consumption allocations. This may appear somewhat counterintuitive: one might expect an agent with a complete domestic financial market to be better off than an agent with no domestic market, because the former does not incur shipping costs to trade. This is not the case, however: as financial markets are moved away from one country, the relative prices of the *available* securities change so as to exactly compensate for the shipping costs now incurred. A comparison of Economies I and I* clarifies this point. Take two states $\omega^E, \omega^I \in \Omega$ where country 1, respectively, exports and imports at some future time t'. The relative prices of the available Arrow-Debreu securities paying in these states can be expressed in Economy I (where the available securities pay-off in country 1) as $\xi(\omega^E, t')dP(\omega^E|\mathcal{F}_t)/\xi(\omega^I, t')dP(\omega^I|\mathcal{F}_t)$ and in Economy I* (where they pay-off in country 2) as $\xi^*(\omega^E, t')dP(\omega^E|\mathcal{F}_t)/\xi^*(\omega^I, t')dP(\omega^I|\mathcal{F}_t)$, which from (4.10) and (4.12) equals the ratio in Economy I multiplied by k^2. This change in price ratio benefits country 2, which is also the country hurt by the move in financial markets, demonstrating how agents use security prices to share the shipping costs in an efficient way.

(ii) *Higher-dimensional uncertainty*. Our approach could be easily adapted to the case where the underlying Brownian motion has a dimension higher than one. This would allow for sources of uncertainty that are different in the two countries and securities that are not perfectly correlated across countries, which is intuitively appealing. Then, additional risky securities would be needed to complete markets. However, the above results and the corresponding intuition would not be affected, as long as the assumption of market completeness at the international level is maintained (meaning that more risky securities would be needed). This is because the proofs for the above results only require the existence of a unique state-price density ξ that prices all securities (once their prices are expressed in the same numeraire, the country-1 good).[8] The existence of ξ is guaranteed by the assumptions that markets are internationally complete and that trading in securities is unconstrained. The proofs leading to our results (see Appendix) never actually rely on the assumption that the underlying uncertainty is one-dimensional. If we were to assume that the underlying Brownian-motion is multi-dimensional, the characterizations of economic quantities (to be provided in Section 5) would become more cumbersome (with, in particular, multi-dimensional market prices of risk), but our main point on the role of additional securities (over those needed for international market completeness) and financial shipping would not be affected.[9]

(iii) *Tariffs*. Our "shipping costs" would only require to be returned to the economy, as endowments to the agents, to be able to be interpreted as tariffs. The solution and characterization of equilibrium would be similar to the case examined in the paper, with each agent's endowment being modified to account for the redistribution of tariff proceeds.

(iv) *Positive net supply risky securities*. Assuming the agents to be endowed, rather than the endowment processes ϵ_1, ϵ_2, ϵ_3 with positive net supply securities paying (in country 1 and 2 respectively) the dividend processes δ and δ^* would entail no significant changes in the characterization of equilibrium, with endowments replaced by dividends in the expressions, and the agents' static budget constraints ((3.5), (3.14)) adjusted appropriately. For example, agent 1's (assumed to be endowed with e_1 unit(s) of the country 1 security and e_1^* unit(s) of the country

[8]This does not contradict the fact that agents may face individual-specific state-price densities, once the shipping costs are taken into account.

[9]Adopting multi-dimensional uncertainty (with possibly country-specific shocks) would be necessary to study some important issues, such as international portfolio choice and financial flows as, in our case, domestic and foreign securities are equivalent from the viewpoint of an investor; they provide exposure to the same factor of risk.

2 security; $e_1^* = 0$ if no country 2 security is available) static budget constraint would become:

$$E\left[\int_0^T \xi(t)c_1(t)dt\right] \leq e_1 E\left[\int_0^T \xi(t)\delta(t)dt\right] + e_1^* E\left[\int_0^T \xi(t)p(t)\delta^*(t)dt\right]$$

(and analogously for agents 2 and 3). Comparison across economies would become more difficult, however, because modifying the financial market structure would affect the endowments.

5. Further Properties of Equilibrium

This section provides further characterizations of equilibrium quantities (prices, international trade and agents' consumption allocations) in the equilibria described in Section 4. Since most expressions of the equilibrium without financial shipping (Section 4.1) can be adapted from the case with financial shipping (Section 4.2), we follow a different order from the previous section and first consider the equilibrium with financial shipping of Economies II and III.

5.1. Equilibrium with Financial Shipping

This subsection provides characterization of international trade, prices (including the real exchange rate) and consumption dynamics in Economies II and III. We present expressions only for the region of no-trade and that where country 1 imports ($p(t) = k$), as those for the region where country 1 exports are symmetric. We define agent i's absolute risk aversion and prudence coefficients by

$$A_i(t) \equiv -\frac{u_i''(c_i(t))}{u_i'(c_i(t))}, \qquad B_i(t) \equiv -\frac{u_i'''(c_i(t))}{u_i''(c_i(t))}.$$

As demonstrated in Section 4.2, a representative agent can be substituted for the two agents in country 2. The country-2 representative agent has the utility function defined in Proposition 4.2 ((4.13)), an endowment given by $\epsilon_*(t) = \epsilon_2(t) + \epsilon_3(t)$, and risk aversion and prudence coefficients defined as follows:

$$A_*(t) \equiv \frac{1}{\frac{1}{A_2(t)} + \frac{1}{A_3(t)}}, \qquad B_*(t) \equiv \left(\frac{A_*(t)}{A_2(t)}\right)^2 B_2(t) + \left(\frac{A_*(t)}{A_3(t)}\right)^2 B_3(t).$$

This substitution of a representative agent for country 2 entails no strong assumption on preferences (unlike, e.g., in the example given by Obstfeld and Rogoff (1996, Section 5.2.2), where constant relative risk aversion preferences are assumed). Relatively mild (in light of the recent evolution of actual financial markets) requirements on financial structure are sufficient: the presence of securities with a positive value in each country and the absence of restrictions to international capital flows (leading to the availability of financial shipping).

For convenience, we also introduce a "world representative agent" with utility U defined by

$$U(C;p) \equiv \max_{c_1 + pc_* = C} \frac{1}{y_1} u_1(c_1) + \frac{1}{y_2} u_*(c_*). \tag{5.1}$$

Identifying C with $\epsilon_1(t) + p(t)\epsilon_*(t)$ and p with $p(t)$, and deriving the first-order conditions for the problem in (5.1) reveals the representative agent problem's solution to coincide with the equilibrium allocation of Proposition 4.2 ((4.17)).[10] The equilibrium thus effectively coincides with one involving a single (world) representative agent consuming the aggregate world consumption and endowed with the aggregate world endowment, in units of country 1 good. The world representative agent's absolute risk aversion and prudence coefficients satisfy

$$A(t) \equiv -\frac{U''(\epsilon_1(t) + p(t)\epsilon_*(t); p(t))}{U'(\epsilon_1(t) + p(t)\epsilon_*(t); p(t))} = \frac{1}{\frac{1}{A_1(t)} + p(t)\frac{1}{A_*(t)}}; \tag{5.2}$$

$$B(t) \equiv -\frac{U'''(\epsilon_1(t) + p(t)\epsilon_*(t); p(t))}{U''(\epsilon_1(t) + p(t)\epsilon_*(t); p(t))} = \left(\frac{A(t)}{A_1(t)}\right)^2 B_1(t) + p(t)\left(\frac{A(t)}{A_*(t)}\right)^2 B_*(t). \tag{5.3}$$

For comparison, we also introduce the benchmark economies a, without shipping ($k = 0$), and b, with zero shipping costs ($k = 1$).

Proposition 5.1 characterizes the dynamics of international trade in the region where country 1 imports.

Proposition 5.1. *When country 1 imports, the dynamics of the flow of goods arriving in country 1 are given by:* $dx^I(t) = \mu_{x^I}(t)dt + \sigma_{x^I}(t)dW(t)$, *where*

$$\mu_{x^I}(t) = \frac{kA(t)}{A_1(t)A_*(t)}\left(A_*(t)\mu_{\epsilon_*}(t) - A_1(t)\mu_{\epsilon_1}(t)\right) - \frac{1}{2}kA(t)^2\left(\frac{B_*(t)}{A_*(t)} - \frac{B_1(t)}{A_1(t)}\right)\left(\sigma_{\epsilon_1}(t) + k\sigma_{\epsilon_*}(t)\right)^2;$$

$$\sigma_{x^I}(t) = \frac{kA(t)}{A_1(t)A_*(t)}\left(A_*(t)\sigma_{\epsilon_*}(t) - A_1(t)\sigma_{\epsilon_1}(t)\right).$$

The volatility in international trade is given by the (risk-tolerance-weighted) difference between the volatilities of the countries' endowments. Similarly, the growth in international trade is driven by the weighted difference in expected growth rates. The growth and volatility of international trade are impacted more by the endowment growth and volatility of the more risk averse agent; consumption smoothing is more valuable to him so flows of goods have to track the fluctuations in his endowment more closely. In a similar fashion, whether exports are pro- or counter-cyclical (i.e., $\sigma_{x^I} > 0$ or < 0) depends on which country is more affected by endowment risk (i.e., has the higher $A_i\sigma_{\epsilon_i}$):[11] if this is the case of the importing country, then its exports are countercyclical because trade is driven primarily by the need to counterbalance adverse fluctuations in its endowment. The above discussion is equally valid in the economy with no shipping costs (benchmark b). What we observe different here is that: growth and volatility of trade are weighed by k to reflect the reduction of trade due to the shipping cost; and when aggregating countries' endowment volatilities, the exporting country's (country 2's) volatility is scaled down

[10] Unlike in frictionless economies (e.g., Karatzas, Lehoczky and Shreve (1990)), solution of this world representative agent's problem is not sufficient for the determination of equilibrium because of the presence of the endogenous $p(t)$. Clearing in the aggregate world good (whose supply is given by $\epsilon_1 + p\epsilon_*$) is a necessary but not sufficient condition for good clearing in the individual countries.

[11] This assumes that countries' endowments are positively correlated, i.e., $\sigma_{\epsilon_1}\sigma_{\epsilon_*} > 0$.

by the factor k. This captures the fact that in this region marginal changes in country 2 endowment are worth less (by a factor of k) in country 1 (the country that is receiving the flow of goods x^I) terms, since such goods need to be shipped to country 1.

Proposition 5.2 provides expressions for the country-specific price parameters and consumption volatilities.[12]

Proposition 5.2. *The equilibrium country-specific interest rate and market prices of risk are: when country 1 imports:*

$$r_1(t) \;=\; r^*(t) \;=\; A(t)\left(\mu_{\epsilon_1}(t) + k\mu_{\epsilon_*}(t)\right) - \frac{A(t)B(t)}{2}\left(\sigma_{\epsilon_1}(t) + k\sigma_{\epsilon_*}(t)\right)^2, \tag{5.4}$$

$$\theta_1(t) \;=\; \theta^*(t) \;=\; A(t)\left(\sigma_{\epsilon_1}(t) + k\sigma_{\epsilon_*}(t)\right); \tag{5.5}$$

when no trade occurs across countries:

$$r_i(t) \;=\; A_i(t)\mu_{\epsilon_i}(t) - \frac{A_i(t)B_i(t)}{2}\sigma_{\epsilon_i}(t)^2, \qquad i \in \{1, *\}, \tag{5.6}$$

$$\theta_i(t) \;=\; A_i(t)\sigma_{\epsilon_i}(t), \qquad i \in \{1, *\}. \tag{5.7}$$

The equilibrium consumption dynamics are: $dc_i(t) = \mu_{c_i}(t)dt + \sigma_{c_i}(t)dW(t)$*;* $\sigma_{c_i}(t) = \theta_i(t)/A_i(t)$*.*

In the trade regions, interest rates and market prices of risk are equated across countries. The interest rate is positively related to the growth in endowment of the world representative agent, normalized by the world representative agent's absolute risk tolerance. Also the interest rate is negatively related (for decreasing absolute risk aversion agents) to the world representative agent's endowment risk, weighted by his risk aversion and prudence. The market price of risk is given by the world representative agent's endowment risk weighted by his risk aversion. The economy then indeed behaves as if populated by a single world agent endowed with the two goods, with a relative price equal to k. Since agents face identical market prices of risk, the trade allows consumption risk to be shared in the same proportions as in the frictionless economy (benchmark b). The expressions reveal the endowment of the exporting country to have a smaller impact on equilibrium prices (an effect more pronounced when shipping costs are higher); the contribution of country 2's risk tolerance is scaled up by k and that of his endowment volatility is scaled down by k. Expressions in the trade region are otherwise identical to the economy (b) with no shipping costs. All the expressions of the no-trade region in our economy are naturally identical to those in the no-trade benchmark a. The individual countries' interest rates and market prices of risk are not equated and are driven by the individual country's endowment, risk aversion and prudence.

Corollary 5.1 presents the real exchange rate dynamics.

Corollary 5.1. *The exchange rate dynamics are as follows.*
When country 1 imports, $p(t) = k$ *and* $\mu_p(t) = \sigma_p(t) = 0$*.*

[12] These shadow prices coincide with the prices of the corresponding securities when these are available.

When no trade occurs across countries, $p(t) = y_1 u'_* (\epsilon_*(t)) / y_2 u'_1 (\epsilon_1(t))$ *and:*

$$
\begin{aligned}
\mu_p(t) &= (A_1(t)\mu_{\epsilon_1}(t) - A_*(t)\mu_{\epsilon_*}(t)) - \frac{1}{2}\left(A_1(t)B_1(t)\sigma_{\epsilon_1}(t)^2 - A_*(t)B_*(t)\sigma_{\epsilon_*}(t)^2\right) \\
&\quad + A_1(t)^2\sigma_{\epsilon_1}(t)^2 - A_1(t)A_*(t)\sigma_{\epsilon_1}(t)\sigma_{\epsilon_*}(t), \qquad (5.8) \\
\sigma_p(t) &= A_1(t)\sigma_{\epsilon_1}(t) - A_*(t)\sigma_{\epsilon_*}(t). \qquad (5.9)
\end{aligned}
$$

The real exchange rate reflects the imbalance in marginal utility across countries. It is thus driven primarily by the differential in endowment growth parameters, normalized by the risk aversions, between the two countries. The first line in (5.8) equals the interest rate differential $r_1 - r^*$, which is thus established to be a biased predictor of the evolution of the exchange rate. These two quantities are among the main objects of interest in Dumas (1992). Comparisons with his work are difficult, because (in a production economy) he employs a more specialized model and thus obtains sharper results. Nevertheless, to ease comparison with his work (equation (23)), we rewrite the expected (log-) growth of the real exchange rate as

$$\frac{E\left[d\ln p(t)\right]}{dt} = (r_1(t) - r^*(t)) + \frac{1}{2}\left(A_1(t)\sigma_{c_1}(t) + A_*(t)\sigma_{c_*}(t)\right)\sigma_p(t).$$

The difference between the expected log-growth of the exchange rate and the interest rate differential is revealed to be proportional to the exchange rate volatility and the "aggregate" consumption risk times risk aversion, and can thus be interpreted as a foreign exchange risk premium.

5.2. Equilibrium absent Financial Shipping

We now consider Economy I, where a representative agent cannot be substituted for country 2 due to imperfect risk-sharing between agents 2 and 3 (who have neither a complete domestic market nor financial shipping available to trade with each other.) In spite of the properties of equilibrium being quite different, in most situations the characterization of equilibrium can be easily adapted from Section 5.1. In regions SS and BB, the allocation within country 2 is Pareto optimal, so we can substitute country 2 as a single agent as in Section 5.1. In SN, NS, BN or NB, the expressions for the agent not trading are as those for an agent not trading in Section 5.1, while the expressions for the trading agent are as those for the country-2 representative agent. In NN, the expressions for all agents are those provided for autarky in Section 5.1.

The only regions where the expressions are substantially new in the three agent model are SB and BS. Proposition 5.3 reports the dynamics of international trade and the individual specific price parameters and consumption volatilities in region SB. We may introduce a "world representative agent" with utility defined by

$$U(C; p_2, p_3) \equiv \max_{c_1 + p_2 c_2 + p_3 c_3 = C} \frac{1}{y_1}u_1(c_1) + \frac{1}{y_2}u_2(c_2) + \frac{1}{y_3}u_3(c_3). \qquad (5.10)$$

Identifying C with $\epsilon_1(t) + p_2(t)\epsilon_2(t) + p_3(t)\epsilon_3(t)$ and p_2 and p_3 with $p_2(t)$ and $p_3(t)$ respectively, we may verify that the allocations from the representative agent problem (5.10) coincide with the equilibrium ones. The world representative agent's absolute risk aversion and prudence coefficients satisfy

$$\begin{aligned}
A(t) &\equiv -\frac{U''(\epsilon_1(t)+p_2(t)\epsilon_2(t)+p_3(t)\epsilon_3(t);p_2(t),p_3(t))}{U'(\epsilon_1(t)+p_2(t)\epsilon_2(t)+p_3(t)\epsilon_3(t);p_2(t),p_3(t))} = \frac{1}{\frac{1}{A_1(t)}+\frac{p_2(t)}{A_2(t)}+\frac{p_3(t)}{A_3(t)}};\\
B(t) &\equiv -\frac{U'''(\epsilon_1(t)+p_2(t)\epsilon_2(t)+p_3(t)\epsilon_3(t);p_2(t),p_3(t))}{U''(\epsilon_1(t)+p_2(t)\epsilon_2(t)+p_3(t)\epsilon_3(t);p_2(t),p_3(t))}\\
&= \left(\frac{A(t)}{A_1(t)}\right)^2 B_1(t) + p_2(t)\left(\frac{A(t)}{A_2(t)}\right)^2 B_2(t) + p_3(t)\left(\frac{A(t)}{A_3(t)}\right)^2 B_3(t).
\end{aligned}$$

Proposition 5.3. *If equilibrium exists and region SB occurs, the flow of goods arriving in country 1 (from agent 2), and leaving from country 1 (to agent 3), respectively, follow dynamics:* $dx_2^I(t) = \mu_{x_2^I}(t)dt + \sigma_{x_2^I}(t)dW(t)$ *and* $dx_3^E(t) = \mu_{x_3^E}(t)dt + \sigma_{x_3^E}(t)dW(t)$*, where*

$$\begin{aligned}
\mu_{x_2^I}(t) &= \frac{A(t)}{A_1(t)A_2(t)A_3(t)}\Bigg\{kA_3(t)\left[A_2(t)\mu_{\epsilon_2}(t)-A_1(t)\mu_{\epsilon_1}(t)\right]+A_1(t)\left[A_2(t)\mu_{\epsilon_2}(t)-A_3(t)\mu_{\epsilon_3}(t)\right]\\
&\quad -\frac{1}{2}A(t)^2\left[kA_3(t)\left(\frac{B_2(t)}{A_2(t)}-\frac{B_1(t)}{A_1(t)}\right)+A_1(t)\left(\frac{B_2(t)}{A_2(t)}-\frac{B_3(t)}{A_3(t)}\right)\right]\left(\sigma_{\epsilon_1}(t)+k\sigma_{\epsilon_2}(t)+\frac{1}{k}\sigma_{\epsilon_3}(t)\right)^2\Bigg\},\\
\sigma_{x_2^I}(t) &= \frac{A(t)}{A_1(t)A_2(t)A_3(t)}\Bigg\{kA_3(t)\left[A_2(t)\sigma_{\epsilon_2}(t)-A_1(t)\sigma_{\epsilon_1}(t)\right]+A_1(t)\left[A_2(t)\sigma_{\epsilon_2}(t)-A_3(t)\sigma_{\epsilon_3}(t)\right]\Bigg\},\\
\mu_{x_3^E}(t) &= \frac{A(t)}{A_1(t)A_2(t)A_3(t)}\Bigg\{\frac{A_2(t)}{k}\left[A_1(t)\mu_{\epsilon_1}(t)-A_3(t)\mu_{\epsilon_3}(t)\right]+A_1(t)\left[A_2(t)\mu_{\epsilon_2}(t)-A_3(t)\mu_{\epsilon_3}(t)\right]\\
&\quad -\frac{1}{2}A(t)^2\left[\frac{A_2(t)}{k}\left(\frac{B_1(t)}{A_1(t)}-\frac{B_3(t)}{A_3(t)}\right)+A_1(t)\left(\frac{B_2(t)}{A_2(t)}-\frac{B_3(t)}{A_3(t)}\right)\right]\left(\sigma_{\epsilon_1}(t)+k\sigma_{\epsilon_2}(t)+\frac{1}{k}\sigma_{\epsilon_3}(t)\right)^2\Bigg\},\\
\sigma_{x_3^E}(t) &= \frac{A(t)}{A_1(t)A_2(t)A_3(t)}\Bigg\{\frac{A_2(t)}{k}\left[A_1(t)\sigma_{\epsilon_1}(t)-A_3(t)\sigma_{\epsilon_3}(t)\right]+A_1(t)\left[A_2(t)\sigma_{\epsilon_2}(t)-A_3(t)\sigma_{\epsilon_3}(t)\right]\Bigg\}
\end{aligned}$$

The individual-specific interest rates and market prices of risk are:

$$\begin{aligned}
\theta_1(t)=\theta_2(t)=\theta_3(t) &= A(t)\left(\sigma_{\epsilon_1}(t)+k\sigma_{\epsilon_2}(t)+\frac{1}{k}\sigma_{\epsilon_3}(t)\right);\\
r_1(t)=r_2(t)=r_3(t) &= A(t)\left(\mu_{\epsilon_1}(t)+k\mu_{\epsilon_2}(t)+\frac{1}{k}\mu_{\epsilon_3}(t)\right)-\frac{1}{2}A(t)B(t)\left(\sigma_{\epsilon_1}(t)+k\sigma_{\epsilon_2}(t)+\frac{1}{k}\sigma_{\epsilon_3}(t)\right)^2.
\end{aligned}$$

The equilibrium consumption dynamics are: $dc_i(t) = \mu_{c_i}(t)dt + \sigma_{c_i}(t)dW(t)$; $\sigma_{c_i}(t) = \theta_i(t)/A_i(t)$.

The shipping dynamics still reflect, as in the intranational homogeneity case, the imbalance in endowment across agents. The expressions are more complicated than those of Proposition 5.1, however, because shipments are needed not only for trade between countries, but also within country 2. Accordingly, the shipment dynamics are driven by the (risk-aversion weighted) differences in endowment growth and volatility between each of agents 2 and 3 individually and agent 1 (international trade) on the one hand, and between agents 2 and 3 (country-2 intranational trade) on the other hand. Each of agents 2 and 3's trade with agent 1 is determined separately, because country 2 cannot be aggregated into a single agent. Price parameter expressions, however, are still as in the "world" representative agent economy.

6. Conclusion

This paper examines a number of economies with an internationally segmented good market, distinguished by their assumptions on the structure of financial markets. It is shown how new securities, even though they do not enlarge the space of attainable payoffs, may alleviate the burden on risk-sharing imposed by the segmentation. This is of importance given the recent evolution of the international financial system, with financial markets becoming more complete and international financial flows more liberalized while markets for goods remained quite segmented. Interestingly, the new securities are used not only for risk-sharing, but also to transfer goods across countries (by "financial shipping"), allowing some circumvention of the shipping costs that generate the good market segmentation. This in turn allows for improved risk-sharing within countries.

A single-period modeling setup does not capture this dual role of securities. Accordingly, the requirements (in terms of financial market structure) for perfect risk-sharing within countries are much less stringent in a multiperiod setup than in a single-period framework. Our work thus provides support for an important part of the recent international finance literature, that substitutes a representative agent for each country, effectively assuming perfect risk-sharing within countries, as the conditions for this are a reasonable fit to the stylized facts of the current international financial system. We can also offer some partial insight on why in the past few decades the growth in international financial flows far outpaced the increase in good trading, since we show that in an economy with liberalized capital flows, these may have replaced some good trading for risk-sharing purposes. Our work may also shed some insight as to why studies on benefits of international financial integration found these to be so small, because they neglected the benefits of integration for risk-sharing between heterogeneous agents from the same country. We thus demonstrate heterogeneity within a country to be a possibly important fact to incorporate into international finance models, that might help explain other puzzles.

Appendix: Proofs

Proof of Proposition 3.1: From the definition of e_i, agent i's consumption is given by $f(e_i(t), t) \equiv \epsilon_i(t) + (e_i(t))^+/k - k(e_i(t))^-$. Agent i's dynamic budget constraint (3.3) implies (together with $X_i(T) \geq 0$) that e_i satisfies the static budget constraint $E\left[\int_0^T \xi(t)e_i(t)dt\right] \leq 0$. Hence agent i effectively solves the problem

$$\max_{e_i} \quad E\left[\int_0^T u_i\left(f(e_i(t), t)\right) dt\right] \quad \text{subject to} \quad E\left[\int_0^T \xi(t)e_i(t)dt\right] \leq 0,$$

whose first-order conditions are (3.4)-(3.5). *Q.E.D.*

Proof of Proposition 3.2: For any given $\Phi_i(t)$, agent i chooses the security holdings that maximize the value of the drift term in (3.7) or (3.9) (if he did not do so, he could increase his consumption at no extra cost or additional risk-taking). A necessary condition for this maximization problem to have a solution is (3.11). If it holds, substitution into (3.7) or (3.9) reveals that all holdings verifying (3.10) lead to the same dynamics for X_i. *Q.E.D.*

Proof of Proposition 3.3: Agent 1 faces a complete, perfect market problem and so the solution to his problem obtains using standard techniques (Cox and Huang (1989), Karatzas, Lehoczky and Shreve (1987)). For $i \in \{2, 3\}$, agent i's consumption "expenditures" (in units of country 1 good) can be defined as $e_i(t) \equiv p(t)(c_i(t) - \epsilon_i(t))$, in the sense that this is the cash-flow he needs to replicate to finance the consumption plan c_i. He effectively solves the problem

$$\max_{e_i} \quad E\left[\int_0^T u_i\left(\epsilon_i(t) + \frac{e_i(t)}{p(t)}\right) dt\right] \quad \text{subject to} \quad E\left[\int_0^T \xi(t)e_i(t)dt\right] \leq 0,$$

whose first-order conditions are (3.13)-(3.14). *Q.E.D.*

Proof of Proposition 4.1: This is an extension of Step 2 of the proof of Proposition 4.2 to the case of three agents. *Q.E.D.*

Proof of Proposition 4.2: We proceed in two steps. In Step 1, we show that the allocation *within* country 2 coincides with the representative agent's problem in (4.13). In Step 2, we consider the equilibrium allocation *across* countries (given that the representative agent of Step 1 has been substituted for country 2) and show that it coincides with that in the Proposition.

Step 1. If an equilibrium exists, from Section 3.5, c_2 and c_3 are such that $u_2'(c_2(t))/y_2 = u_3'(c_3(t))/y_3$ $(= p(t)\xi(t))$. Additionally using good market clearing (4.1), $\forall t$, agents 2 and 3's consumption allocations solve the problem

$$\max_{c_2(t)+c_3(t)=\epsilon_2(t)+\epsilon_3(t)+kx^E(t)-x^I(t)/k} u_2(c_2(t)) + \frac{y_2}{y_3}u_3(c_3(t)),$$

i.e., the allocations solve the representative agent problem (4.13). Step 2 below then yields the equilibrium expressions (4.10)-(4.17). (The additional budget constraint (4.15), necessary to pin down the allocation within country 2, is agent 2's static budget constraint.)

Conversely, if (4.10)-(4.16) hold, from Step 2 there is an equilibrium involving agent 1 on the one hand, and the country-2 representative agent, with utility u_* defined by (4.13) and endowed with the process $\epsilon_2 + \epsilon_3$, on the other hand. All that remains to be checked is that agents 2 and 3 are content with the allocations from the representative agent problem (4.13). These are such that

$$\frac{u_2'(c_2(t))}{y_2} = \frac{u_3'(c_3(t))}{y_3} = u_*'\left(\epsilon_2(t) + \epsilon_3(t) + kx^E(t) - \frac{x^I(t)}{k}\right) = p(t)\xi(t),$$

where the last equality follows from the country-2 representative agent optimality. This implies (together with satisfaction of the individual budget constraints (4.14)-(4.15)) that these consumption allocations are optimal for agents 2 and 3.

Step 2. Assume that equilibrium exists. Clearing in the consumption good and Step 1 above imply (4.17), while (4.16) follows from agent 1's optimality and the definition of p, and (4.14) obtains by substitution of the expression for ξ given in (4.16) and algebraic manipulation of agent 1's static budget constraint. It is rational for country 1 to import only if $p(t) = k$, hence (4.10). The existence of $x^I(t) > 0$ satisfying (4.10) requires $y_1 u_*'(\epsilon_*(t))/y_2 u_1'(\epsilon_1(t)) < k$, hence the condition for country 1 importing. The case of country 1 exporting is symmetric. Whenever none of the conditions for country 1 importing or exporting holds (i.e., $k \leq y_1 u_*'(\epsilon_*(t))/y_2 u_1'(\epsilon_1(t)) \leq 1/k$), there does not exist any amount of international trade $x^I(t) > 0$ or $x^E(t) > 0$ that is rational, and so countries are in autarky. The individual-specific state price densities must be such that each agent is content to consume his own endowment, hence (4.11).

Conversely, assume that (4.10)-(4.16) hold. From (4.14) and the first equality in (4.16), country 1 finds it optimal to consume as in the first equality of (4.17), while from the second equality of (4.16), and the values of $p(t)$ in (4.10), (4.11) and (4.12) country 2 optimally consumes as in the second equality of (4.17). Hence, the good market clears (given that, additionally, the conditions for the three possible cases cover the whole space of possible values for $u_*'(\epsilon_*(t))/u_1'(\epsilon_1(t))$).

It remains to verify that the corresponding portfolio holdings clear the financial markets. Take the case of Economy I. At any time t, standard arguments imply that agents' wealths verify

$$X_1(t) = \frac{1}{\xi(t)}E\left[\int_t^T \xi(s)(c_1(s) - \epsilon_1(s))ds|\mathcal{F}_t\right] = \frac{1}{\xi(t)}E\left[\int_t^T \xi(s)\left(x^I(s) - x^E(s)\right)ds|\mathcal{F}_t\right],$$

$$X_*(t) = \frac{1}{\xi(t)}E\left[\int_t^T \xi(s)(e_*(s))ds|\mathcal{F}_t\right] = \frac{1}{\xi(t)}E\left[\int_t^T \xi(s)\left\{\frac{1}{k}(\epsilon_*(s) - c_*(s))^- - k(\epsilon_*(s) - c_*(s))^+\right\}ds|\mathcal{F}_t\right].$$

Hence, clearing in the good market implies that $X_1(t) + X_*(t) = 0$. On the other hand, using standard arguments and the martingale representation theorem, agents' holdings in the risky security P are given by:

$$\pi_{Pi} = \frac{\mu(t) - r(t)}{\sigma(t)^2}X_i(t) + \frac{\kappa_i(t)}{\sigma(t)\xi(t)}, \qquad i \in \{1, *\},$$

where κ_1, κ_* satisfy

$$\begin{aligned} \xi(t)X_1(t) + \int_0^t \xi(s)(c_1(s) - \epsilon_1(s))ds &= \int_0^t \kappa_1(s)dW(s), \\ \xi(t)X_*(t) + \int_0^t \xi(s)e_*(s)ds &= \int_0^t \kappa_*(s)dW(s). \end{aligned}$$

Clearing in the good market thus implies that, $\forall t$, $\int_0^t(\kappa_1(s) + \kappa_*(s))dW(s) = 0$, so that $\kappa_1(t) + \kappa_*(t) = 0$, $\forall t$, and $\pi_{P1}(t) + \pi_{P*}(t) = 0$: clearing in the "stock" (P) market. $X_1(t) + X_*(t) = 0$ then implies clearing in the "bond" (M). In Economies II and III, due to the presence of securities that equivalent for the purposes of risk exposure, there exist a continuum of portfolio strategies consistent with agents' consumption choices, among which the agents are indifferent by Proposition 3.2. In particular, the agents are satisfied by holdings in M and P as in Economy I above (and zero holdings in M^* and P^*), so there exist security holdings that allow them to implement their consumption choices and clear markets. *Q.E.D.*

Proof of Corollary 4.1: For all regions, for any t, the equilibrium allocations of Proposition 4.2 can readily be checked to coincide with the solution of (4.18). *Q.E.D.*

Proof of Proposition 5.1: Applying the implicit function theorem to (4.10) yields the derivatives of x^I with respect to ϵ_1 and ϵ_*. Applying Itô's lemma then yields, after substitution of (5.2), the expressions in the Proposition. *Q.E.D.*

Proof of Proposition 5.2: Agents' consumption dynamics follows from Proposition 5.1. Applying Itô's lemma to both sides of their first-order condition and identifying drifts and diffusions then yields, after substitution of (5.2) and (5.3), (5.4)-(5.7). *Q.E.D.*

Proof of Corollary 5.1: (5.8)-(5.9) follow from applying Itô's lemma to the right hand side of $p(t) = y_1 u_*' \left(\epsilon_*(t)\right) / y_2 u_1' \left(\epsilon_1(t)\right)$ and identifying drifts and diffusions. *Q.E.D.*

Proof of Proposition 5.3: This is an extension of the proofs of Propositions 5.1-5.2 to the case of three agents. *Q.E.D.*

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