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ABSTRACT

Efficiency Wages and Unemployment in Cities: The Case of High Relocation Costs*

We develop an urban model in which all jobs are located in the Central Business District (CBD) and workers, who have high relocation costs, optimally choose their residence between the CBD and the city-fringe. We consider two cases. In the first one, firms can pay different wages according to residential location (this is referred to as the unconstrained equilibrium) while in the second case, there is a legal constraint that prevents firms to wage-discriminate on the basis of residential location (this is referred to as the constrained equilibrium). We show that in the unconstrained equilibrium, the efficiency wage in fact increases with distance to jobs. We also demonstrate that both workers and firms are better off in the unconstrained equilibrium. Finally, we show that a policy that reduces the unemployment benefit decreases unemployment but, interestingly, has an ambiguous effect on utilities in both equilibria. Moreover, this policy has no impact on the land rent in the unconstrained equilibrium but increases the competition in the land market in the constrained equilibrium.

JEL Classification: J41 and R14
Keywords: location-dependent wages, relocation costs and urban labour markets

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1 Introduction

It is commonly observed that workers do not change location as soon as they change their employment status. This is because homes are less mobile than jobs. For example, Manning (2003) argues that approximately 20% of workers have job tenure less than a year compared to 10% with residential tenure of less than a year.

Surprisingly, the issue of relocation costs as been largely ignored by the urban economic theory literature (see e.g. Fujita, 1989), in particular in connection with labor economics.[1] To the best of our knowledge, there are only two papers that deal with this issue.[2] Brueckner and Zenou (2003) develop an efficiency wage model in which workers have extremely high mobility costs so that, once located, they are unable to change their residential location. They make the restrictive assumption that the commuting cost of the employed and the unemployed workers is the same, which implies that the time spent in job-search activities is equivalent to the time spent in commuting to work. Wasmer and Zenou (2003) develop a search-matching model in which workers incur a fixed cost of moving. They show that, in equilibrium, mobile workers live close or far away from jobs whereas the immobile workers reside in between the mobile workers.

The aim of this paper is twofold. First, we follow the first approach by introducing high relocation costs in an efficiency wage model but relax the restrictive assumption on the commuting costs. Second, we study and compare two models that differ by the fact that firms are constrained or not in their wage setting.

To be more precise, we develop an urban model in which all jobs are located in the Central Business District (CBD hereafter) and workers, who have high relocation costs, optimally choose their residence between the CBD and the city-fringe. Because of high relocation costs, workers do not change residence.

[1] It is has to be acknowledged that the urban labor economic theory is quite small. See the survey by Zenou (2000).

[2] It has to be clear that we are focusing on the urban labor economic theory literature, i.e. papers that explicitely model both the land/housing market (i.e. the location of workers as well as the land/housing price are endogenous) and the labor market (i.e. the wage setting and the unemployment formation are endogenous) and their interaction. There are obviously regional models (see the migration or the economic geography literature) that deal with interregional mobility costs in a labor market. This is not what we are interested in. Here, we are focusing on the impact of intra-mobility costs within a city on the labor market.

Because we assume perfect capital markets with a zero interest rate, workers are able to smooth their income over time as they enter and leave unemployment: workers save while employed and draw down on their savings when out of work. As a result, workers only care about their average income over time. We consider two cases. In the first one, firms can pay different wages according to residential location (this is referred to as the unconstrained equilibrium) while in the second case, there is a legal constraint that prevents firms to wage-discriminate on the basis of residential location (this is referred to as the constrained equilibrium). We show that in the unconstrained equilibrium, the efficiency wage is increasing with the distance to jobs. This result is obtained because of relocation costs and the fact that the employed workers have higher commuting costs than the unemployed. Indeed, firms set efficiency wages not only to prevent shirking but also to compensate for long commutes. So, because workers are stuck in their location, the ones who are further away need to be compensated more than those who live closer to the CBD. This is no longer true if either workers are perfectly mobile or commuting costs are the same for the employed and the unemployed workers.

We then study the properties of these two equilibria and compare them. We first demonstrate that both workers and firms are better off in the unconstrained equilibrium. For the workers, even though they all have a lower wage in the unconstrained equilibrium, they are better off because the unemployment rate is higher and the competition in the land market is fiercer. We then show that the interaction between the land and the labor market is quite different in the two equilibria. Indeed, in the unconstrained equilibrium, the land rent is only affected by spatial variables, i.e. the commuting costs and the relative number of CBD-trips of the unemployed whereas in the constrained equilibrium, it is affected by both the spatial variables and the non-spatial ones, such as for example the unemployment benefit or the job destruction rate. This has interesting policy implications. Consider for example a policy that consists in reducing the unemployment benefit. This will reduce unemployment in both equilibria but will have an ambiguous effect on utilities. This result is due to the spatial aspects of the model, and in particular to the interaction between land and labor markets, since in a standard non-spatial model (Shapiro and Stiglitz, 1984), the effect will always be positive. Furthermore, a reduction in the unemployment benefit has no impact on the land rent in the unconstrained equilibrium but increases the competition in the land market in the constrained equilibrium.

2 The model

Consider a continuum of workers uniformly distributed along a linear, closed and monocentric city. Their mass is N. Among the N workers, there are L employed and U unemployed so that $N = L + U$. The density of workers in each location is taken to be 1. All land is owned by absentee landlords and all firms are exogenously located in the CBD. There a continuum of M identical firms. All workers are assumed to be infinitely lived and risk neutral. Workers endogenously decide their optimal place of residence between the CBD and the city fringe.

Each individual is identified with one unit of labor. As in the standard efficiency wage model (Shapiro and Stiglitz, 1984), there are only two possible levels of effort: either the worker shirks, has zero effort, $e = 0$ and contributes to zero production or he/she does not shirk, provides full effort, $e > 0$ and contributes to 1 unit of production. Each employed worker goes to the CBD to work and incurs a fixed weekly commuting cost t per unit of distance. He/she also pays a land rent $R(x)$, consumes 1 unity of land and earns a weekly wage w (that will be determined in the labor market equilibrium) so that the instantaneous (indirect) utilities of a non-shirker and a shirker residing at a distance x from the CBD are respectively given by:[3]

$$V_1^{NS} = w - e - t\,x - R(x) \tag{1}$$

$$V_1^{S} = w - t\,x - R(x) \tag{2}$$

Concerning the unemployed, they commute less often to the CBD since they mainly go there to search for jobs so that they incur a fixed commuting cost $s\,t$ per unit of distance, with $0 < s < 1$. For example, $s = 1/2$ implies that the unemployed make only half as many CBD-trips as the employed workers. The unemployed workers earn a fixed unemployment benefit $b > 0$, pay a land rent $R(x)$ and consume 1 unit of land. In this context, the instantaneous (indirect) utility of an unemployed worker is equal to:

$$V_0 = b - s\,t\,x - R(x) \tag{3}$$

At any moment, workers can either be employed or unemployed. We assume that changes in the employment status (employment versus unemployment) are governed by a continuous-time Markov process. Firms cannot perfectly monitor workers so that there is a probability of detected shirking, denoted by θ. If a worker is caught shirking, he/she is automatically fired. Job

[3]The subscripts 0 and 1 refer to the unemployed and the employed workers respectively.

contacts (that is the transition rate from unemployment to employment) randomly occur at an endogenous rate a while the exogenous job separation rate is δ. In this context, the expected duration of employment is given by $1/\delta$ for non-shirkers and $1/(\delta+\theta)$ for shirkers whereas the expected duration of unemployment amounts to $1/a$. It then follows that a worker spends a fraction $a/(a+\delta)$ if non-shirker and $a/(a+\delta+\theta)$ if shirker of his/her lifetime employed and a fraction $\delta/(a+\delta)$ if non-shirker and $(\delta+\theta)/(a+\delta+\theta)$ if shirker of his/her lifetime unemployed. In steady state, flows into and out of unemployment are equal. Therefore, for the unemployment rate of non-shirkers is given by:

$$u \equiv u^{NS} = \frac{\delta}{a+\delta} \tag{4}$$

while for the one of shirkers, we have

$$u^{S} = \frac{\delta+\theta}{a+\delta+\theta} \tag{5}$$

with $u^{S} > u^{NS}$, $\forall a, \delta, \theta > 0$.

We assume that mobility costs are so high (which is the case for most low-income households, especially if they live in council houses or housing projects) that once someone is located somewhere he/she never moves. As a result, people stay in the same location when they change their employment status.

We assume perfect capital markets with a zero interest rate,[4] which enable workers to smooth their income over time as they enter and leave unemployment: workers save while employed and draw down on their savings when out of work. Because workers are able to smooth their income over time, *a worker's residential location remains fixed as he/she enters and leaves unemployment.*

We will thus consider the average expected utility of a worker rather than the lifetime expected utilities of employed and unemployed workers. At any moment of time, the disposable income (since workers are risk neutral utilities are equal to income) of a worker is thus equal to that worker's average income over the job cycle. The intuition is straightforward. Since workers do not discount, only their average net revenue (or equivalently their utility) matters. Indeed, over their lifetime, workers are totally indifferent between being employed today for T periods and then become unemployed for T' periods, and

[4] When there is a zero interest rate, workers have no intrinsic preference for the present so that they only care about the fraction of time they spend employed and unemployed. Therefore, the expected utilities are not state dependent.

being unemployed today for T periods and then become employed for T' periods (which is not true if $r > 0$ since the present and the future are discounted differently). In other words, the only thing that matters for them is the fraction of their lifetime they are employed and the fraction of their lifetime they are unemployed and not the order of their employment career. This is why the average net revenue is the same whatever the present employment status of the worker (employed or unemployed).

Since workers have a zero discount rate, they only care about the average income over time. For a non shirker located at a distance x from the CBD, it is equal to:

$$\begin{aligned} EV^{NS}(x) &= \left(1-u^{NS}\right)\left[w-e-tx-R(x)\right]+u^{NS}\left[b-stx-R(x)\right] \\ &= \frac{a}{\delta+a}(w-e)+\frac{\delta}{\delta+a}b-\left(\frac{a+s\delta}{\delta+a}\right)tx-R(x) \end{aligned} \tag{6}$$

whereas for a shirker residing at a distance x from the CBD, it is given by:

$$\begin{aligned} EV^{S}(x) &= \left(1-u^{S}\right)\left[w-tx-R(x)\right]+u^{S}\left[b-stx-R(x)\right] \\ &= \frac{a}{\delta+\theta+a}w+\frac{\delta+\theta}{\delta+\theta+a}b-\left[\frac{a+s\left(\delta+\theta\right)}{\delta+\theta+a}\right]tx-R(x) \end{aligned} \tag{7}$$

Let us solve the equilibrium of the economy. A steady-state equilibrium requires solving *simultaneously* two problems:

(i) a location and rental price outcome (referred to as an urban land use equilibrium).

(ii) a (steady state) labor market equilibrium with determines wages and unemployment (referred to as a labor market equilibrium).

We will now consider two different cases and thus two different equilibria. In the first one, firms can pay different wages according to residential location (this is referred to as the unconstrained equilibrium). In the second one, there is a legal constraint that prevents firms to wage-discriminate on the basis of residential location (this is referred to as the constrained equilibrium). Of course, in both cases, all workers will have in equilibrium the same expected utility.

3 The unconstrained equilibrium

In the unconstrained equilibrium, all the endogenous variables have the superscript *uc*. Let us calculate the efficiency wage. Firms know that workers have a zero discount rate so they solve $EV^{NS}(x) = EV^{S}(x)$ at each x, i.e. the average income over time of a non shirker is equal to the one of a shirker. By using (6) and (7), we easily obtain:[5]

$$w^{uc}(x) = b + e + \frac{e}{\theta}\left(\delta + a^{uc}\right) + (1 - s)\, tx \tag{8}$$

Since, at the efficiency wage, no worker shirks, we can use the value of a in (4) and plug in (8) to obtain:

$$w^{uc}(x) = b + e + \frac{e}{\theta}\frac{\delta}{u^{uc}} + (1 - s)\, tx \tag{9}$$

This efficiency wage has the standard effects of both non-spatial (Shapiro and Stiglitz, 1984) and spatial models (Zenou and Smith, 1995). Indeed, when b, e, or δ increases or θ or u^{uc} decreases (these are the non-spatial effects), the efficiency wage has to increase in order to prevent shirking. When t increases or s decreases (these are the spatial effects), the efficiency wage has to increase in order to compensate for spatial costs. This implies (as in Zenou and Smith, 1995) that the efficiency wage has two roles: to prevent shirking and to compensate workers for commuting costs.

Now, the main difference here is that mobility costs are very high so that workers always stay in the same location. So when a worker makes his/her decision to shirk or not, he/she trades off longer spells of unemployment but lower effort and lower commuting (if he/she decides to shirk) with longer spells of employment but higher effort and higher commuting costs (if he/she decides not to shirk). As a result, at each location x, the setting of the efficiency wage needs to include these two elements (incentive and spatial compensation). For the spatial compensation element (which varies with x), it has to be that, at each x, the compensation is equal to $tx\left(1 - u^{NS} + su^{NS}\right) - tx\left(1 - u^{S} + su^{S}\right)$, i.e. the average commuting cost when non shirking minus the average commuting cost when shirking, which, using (4) and (5), is exactly equal to $(1 - s)\, tx$.

It is interesting to compare this efficiency wage with the one obtained in the case of no-relocation costs (Zenou and Smith, 1995) and the case of high relocation costs (like here) but same commuting costs for employed and unemployed workers, i.e. $s = 1$ (Brueckner and Zenou, 2003). In both these models, the equilibrium is unconstrained in terms of wage policy. In the former, the efficiency wage is equal to:

$$w^{ZS} = b + e + \frac{e}{\theta}\frac{\delta}{u} + (1 - s)\, t\,(1 - u)\, N \tag{10}$$

[5] See the Appendix for a more rigorous demonstration of the derivation of the efficiency wage.

whereas, in the latter, it is given by:[6]

$$w^{BZ} = b + e + \frac{e}{\theta}\frac{\delta}{u} \tag{11}$$

The main difference between (9) and (10) is that in Zenou and Smith (1995), workers are perfectly mobile, thus change location as soon as they change their employment status, and thus employed and unemployed have different lifetime utilities. As a result, because of perfect mobility, all the employed workers are indifferent to reside between 0 and $(1-u)\,N$ (this is the area of the employed) and firms set an efficiency wage at $(1-u)\,N$ since, at this location, the employed and the unemployed pay exactly the same land rent but do not have the same commuting costs, the difference being precisely $(1-s)\,t\,(1-u)\,N$. This is the spatial compensation role of the efficiency wage highlighted above. The main difference between (9) and (11) is that, in Brueckner and Zenou (2003), because employed and unemployed workers bear exactly the same commuting costs, the efficiency wage do not need anymore to compensate for spatial costs and its only role is to prevent shirking. This is why they obtain exactly the same wage as in Shapiro and Stiglitz (1984).

Both hypotheses, no mobility costs in Zenou and Smith (1995) and same commuting costs for the employed and unemployed in Brueckner and Zenou (2003) do not seem to be supported by the data. For instance, Layard *et al.* (1991) showed that low-income households are not very mobile and the time spent in job search activities is quite low compared to the commuting time of the employed.

Finally, it is interesting to observe that, in an unconstrained equilibrium, the only case when the efficiency wage is location-dependent is when both mobility costs are high *and* the commuting cost of the employed and the unemployed are not the same.

The fact that wages increase with distance to jobs (or equivalently with commuting time) is a well-established empirical fact. For example, Manning (2003) using British data (the Labour force Survey for 1993-2001 and the British Household Panel Survey for 1991-2000) shows that an extra hour of commuting each day is associated, on average, with an increase in wages of 27 log points. This is even more true for highly educated workers since those with more education and in the higher-status occupations are more likely to have both high wages and a long commute. These results are consistent with the ones found in the US. For instance, Madden (1985) uses the PSID to investigate

[6] To obtain (10), it suffices to set $s = 1$ in (9).

how wages vary with distance to the CBD. She finds that, for all workers who changed job, there is a positive relationship between wage change and change in commute. Zax (1991), who uses data from a single company and regresses wages on commutes, also finds a positive relationship.[7]

Let us now solve the urban land use equilibrium. We are able to calculate the bid rent[8] of all workers in the city (at the efficiency wage, nobody will shirk in equilibrium). By plugging (9) in (6), we obtain the following expected utility of a (non-shirker) worker located at x:[9]

$$\begin{aligned} EV^{uc}(x) &= \frac{a^{uc}}{\delta + a^{uc}}\left[w^{uc}(x) - e\right] + \frac{\delta}{\delta + a^{uc}}b - \left(\frac{a^{uc} + s\delta}{\delta + a^{uc}}\right)tx - R^{uc}(x) \\ &= b + \frac{e}{\theta}\frac{\delta\left(1 - u^{uc}\right)}{u^{uc}} - stx - R^{uc}(x) \end{aligned}$$

If we denote that I^{uc} the (expected) utility reached by all workers in the city in the unconstrained equilibrium, then the bid rent is equal to

$$\begin{aligned} \Psi(x, I^{uc}) &= \frac{a^{uc}}{\delta + a^{uc}}w^{uc}(x) - \left(\frac{a^{uc} + s\delta}{\delta + a^{uc}}\right)tx + \frac{\delta}{\delta + a^{uc}}b - \frac{a^{uc}}{\delta + a^{uc}}e - I^{uc} \\ &= b + \frac{e}{\theta}\frac{\delta\left(1 - u^{uc}\right)}{u^{uc}} - stx - I^{uc} \end{aligned} \tag{12}$$

with

$$\begin{aligned} \frac{\partial\Psi(x, I^{uc})}{\partial x} &= \frac{a^{uc}}{\delta + a^{uc}}\frac{\partial w^{uc}(x)}{\partial x} - \left(\frac{a^{uc} + s\delta}{\delta + a^{uc}}\right)t \\ &= -st < 0 \end{aligned}$$

The first line of this equation highlights the role of the land rent in the unconstrained equilibrium, which is to compensate workers for commuting costs and wages. Indeed, workers living further away obtain higher (efficiency) wages but pay higher commuting costs whereas those living closer to the CBD have the reverse trade-off.

The utility I^{uc} is pinned down by the following condition (the bid rent at the city-fringe N is equal to the agricultural rent normalized to zero):

$$\Psi(N, I^{uc}) = b + \frac{e}{\theta}\frac{\delta\left(1 - u^{uc}\right)}{u^{uc}} - stN - I^{uc} = 0$$

[7] For more evidence, see Small (1992) and White (1999).

[8] The bid rent is a standard concept in urban economics (see e.g. Fujita, 1989). It indicates the maximum land rent that a worker located at a distance x from the CBD is ready to pay in order to achieve a utility level.

[9] Recall that $u = \delta/(\delta + a)$ or equivalently $a = \delta(1 - u)/u$.

which implies that

$$I^{uc} = b + \frac{e}{\theta}\frac{\delta\,(1-u^{uc})}{u^{uc}} - stN \tag{13}$$

Furthermore, by plugging (13) in (12), we obtain the equilibrium land rent, which is given by:

$$R^{uc}(x) = st\,(N-x) \tag{14}$$

We can now determine the labor market equilibrium. As stated above, there are M identical firms ($j = 1, ..., M$) in the economy. All firms produce the same composite good and sell it at a fixed market price p (this good is taken as the numeraire so that its price p is set to 1). All workers whatever their location contribute to one unit of production. The production function of each firm is denoted by $f(l_j)$ and it is assumed that $f(\cdot)$ is twice differentiable with $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$, and it satisfies the Inada conditions, i.e. $f'(0) = +\infty$ and $f'(+\infty) = 0$.

We further assume that, when a job is vacant, a firm is always willing to hire a worker whatever his/her location. This means that, once a firm has a vacant job, it is always more profitable to hire the first worker that 'knocks at its door' rather than to wait for the next worker. This is true for all workers, even for the one located at N, i.e. the worker who obtains the highest wage and lives the furthest away from firms.[10] Interestingly, Zax (1991) found that wages are higher for workers who commute more and interprets this result as being evidence of an explicit compensating differential paid by employers to workers with long journey-to-work times. Since, in the present model, workers contribute to the same level of production whatever their location but wages increase with distance to jobs, the alternative assumption would have been, as in Zenou (2002), that firms draw a red line beyond which they do not hire workers because they are too costly. Because workers have high relocation costs and thus do not change residence, the latter assumption is not at all realistic since it implies that some workers will stay all their life unemployed.

A consequence of our assumption is that each position within a firm will be filled totally randomly by a worker residing between 0 and N. Because of the law of large numbers, after a sufficient long period of time (i.e. at the steady state), each position within a firm will be paid at the average-distance wage, i.e. $w^{uc}(N/2)$. Moreover, because all firms are identical, the employment level in each firm j is equal to: $l_j = l = L/M$. As a result, each firm adjusts

[10] Formally, it suffices to assume that the cost of waiting is sufficiently high compared to the profit that a firm makes on a worker located at N.

employment until the marginal product of an additional worker equals the average efficiency wage so that we have

$$w^{uc}(N/2) = f'(l) \tag{15}$$

This determines the labor demand in each firm. Since there are M firms in the economy, the aggregate production function $F(L) = M\, f(L/M)$ and the total labor demand in the economy is equal to $L = M\, l$. The aggregate equivalent of (15) is thus given by:[11]

$$w^{uc}(N/2) = F'(L)$$

Since $L = (1-u)N$, using (9), this is equivalent to:

$$b + e + \frac{e}{\theta}\frac{\delta}{u^{uc}} + (1-s)\,\frac{tN}{2} = F'((1-u^{uc})N) \tag{16}$$

Proposition 1 (Unconstrained Equilibrium) *There exists a unique unconstrained equilibrium* $(u^{uc*}, I^{uc*}, R^{uc*}(x))$*, with* $0 < u^{uc*} < 1, I^{uc*} > 0$ *and* $R^{uc*}(x) > 0$*,* $\forall x \in [0, N[$*.*

Proof. It is easy to see that the left-hand side of (16) is decreasing in u^{uc}, reaching $+\infty$ at zero, whereas the right hand side is increasing in u^{uc}. Thus, equation (16) has a unique solution denoted by u^{uc*}. We know that this solution is strictly positive but we have to check that it is strictly less than 1. Denote $\Omega(u^{uc}) \equiv b + e + \frac{e}{\theta}\frac{\delta}{u^{uc}} + (1-s)\,\frac{tN}{2}$ and $\Phi(u^{uc}) \equiv F'((1-u^{uc})N)$. We need to show that $\Omega(u^{uc} = 1) < \Phi(u^{uc} = 1)$, which is equivalent to: $b + e + \frac{e\delta}{\theta} + (1-s)\,\frac{tN}{2} < F'(0)$. This is always true because of the Inada conditions assumed on the production function. There is thus a unique u^{uc*}, which is strictly between 0 and 1, i.e. $0 < u^{uc*} < 1$. Now, plugging this u^{uc*} in (13) gives a unique I^{uc*}. ■

4 The constrained equilibrium

Let us calculate the efficiency wage under the constrained equilibrium, i.e. firms cannot pay a wage that is location-dependent. In the constrained equilibrium, all the endogenous variables have the superscript c. Since firms know

[11] Observe that

$$F'(L) = Mf'(L/M)(1/M) = f'(l)$$

from (9) that wages are increasing with x the distance from the CBD, they will set the highest possible wage to prevent shirking, i.e.

$$w^c = b + e + \frac{e}{\theta}\frac{\delta}{u^c} + (1 - s)\, tN \tag{17}$$

Now wages are not location-dependent and land rents will only compensate for commuting costs. Let us solve the urban land use equilibrium. By plugging (17) in (6), we obtain the following expected utility of a (non-shirker) worker located at a distance x from the CBD:

$$EV^c(x) = b + \frac{e}{\theta}\frac{\delta\,(1 - u^c)}{u^c} + (1 - u^c)\,(1 - s)\, tN - (1 - (1 - s)\, u^c)\, tx - R^c(x)$$

If we denote that I^c the (expected) utility reached by all workers in the city in the unconstrained equilibrium, then the bid rent is equal to

$$\Psi(x, I^c) = b + \frac{\delta\,(1 - u^c)}{u^c} + (1 - u^c)\,(1 - s)\, tN - (1 - (1 - s)\, u^c)\, tx - I^c \tag{18}$$

The role of the land rent in the constrained equilibrium is to compensate workers only for commuting costs. We have indeed:

$$\frac{\partial \Psi(x, I^c)}{\partial x} = -\,(1 - (1 - s)\, u^c)\, t < 0$$

As above, the utility I^c is determined by the fact that the bid rent at the city-fringe is equal to zero. We obtain:

$$I^c = b + \frac{e}{\theta}\frac{\delta\,(1 - u^c)}{u^c} - stN \tag{19}$$

Furthermore, plugging (19) in (18), we obtain the equilibrium land rent, which is given by:

$$R^c(x) = (1 - (1 - s)\, u^c)\, t\,(N - x) \tag{20}$$

We can now determine the labor market equilibrium. Each firm adjusts employment until the marginal product of an additional worker equals the efficiency wage (17) (and not the *average* efficiency wage as in the unconstrained equilibrium) so that

$$b + e + \frac{e}{\theta}\frac{\delta}{u^c} + (1 - s)\, tN = F'((1 - u^c)N) \tag{21}$$

Proposition 2 (Constrained Equilibrium) *There exists a unique constrained equilibrium* $(u^{c*}, I^{c*}, R^{c*}(x))$*, with* $0 < u^{c*} < 1, I^{c*} > 0$ *and* $R^{c*}(x) > 0$*,* $\forall x \in [0, N[$.

Proof. Observe that, contrary to the unconstrained equilibrium, $R^c(x)$ is now a function of u^c. We can however solve the steady-state equilibrium recursively. First, (16) and (21) are nearly the same equations (the only difference is that the last expression of the left-hand side of (16) is $(1-s)\,\frac{tN}{2}$ whereas for (21), it is $(1-s)\,tN$). As a result, using exactly the same argument as in the proof of Proposition 1, we easily obtain the fact that there exists a unique solution to (21) denoted by u^{c*}, with $0 < u^{c*} < 1$. Second, by plugging this value of u^{c*} in (20) and in (19), we obtain a unique $R^{c*}(x) > 0$ and a unique $I^{c*} > 0$. ■

5 Comparison of the two equilibria

Let us compared these two equilibria (constrained and unconstrained). We have the following result.

Proposition 3 (Comparison of Equilibria)

(i) *The unemployment rate in the unconstrained equilibrium is lower than in the constrained equilibrium, i.e.* $0 < u^{uc*} < u^{c*} < 1$*;*

(ii) *Even though wages are lower, workers' expected utility in the unconstrained equilibrium is higher than in the constrained equilibrium, i.e.* $I^{uc*} > I^{c*} > 0$*;*

(iii) *The equilibrium land rent in the constrained equilibrium is always higher than in the unconstrained equilibrium, i.e.* $R^{c*}(x) > R^{uc*}(x)$*,* $\forall x \in [0, N[$.

Proof.

(i) If we compare (16) and (21), then it is easy to see that the left-hand side of these equations are both a downward sloping curve in u, with the one for the constrained equilibrium being above the one for the unconstrained equilibrium. Since the right-hand side of both (16) and (21) are exactly the same upward-sloping functions, the result follows.

(ii) The result is immediate by comparing (13) and (19) using (i).

(iii) First, observe that since $s < 1$, then $(1 - (1-s)\,u^c) > s$, $\forall u^c \in \,]0, 1[$. As a result, using (14) and (20), it is easy to see that $R^c(x = 0) > R^{uc}(x = 0)$

and $R^c(x = N) = R^{uc}(x = N) = 0$. Moreover, since both land rents are linear and since

$$\left|\frac{\partial R^c(x)}{\partial x}\right| = (1 - (1 - s)\, u^c)\, t > st = \left|\frac{\partial R^{uc}(x)}{\partial x}\right|$$

then the result follows. ■

This proposition shows that the constrained equilibrium is quite inefficient since both workers, who obtain lower expected utility, and firms, who pay higher wages and thus obtain lower profits, are worse off than in the unconstrained equilibrium. For firms, this is not surprising. However, for workers, the result is less obvious. Indeed, the fact that firms are constrained to pay higher wages to all their workers (this is the implication of not allowing location-dependent wages) has two main consequences: Compared to the unconstrained equilibrium, (i) there will be more competition in the land market and thus higher land prices for all workers, and (ii) the unemployment rate will be higher and thus workers will experience longer unemployment spells. Since these two negative effects dominate the positive effect on wages, the (expected) utility of workers will be lower.[12] We have here an interesting result that provides a link between the labor market and the land/housing marking.

Let us now examine the properties of these two equilibria (the proof of this proposition is straightforward).

Proposition 4 (Comparative Statics)

(i) *The comparative-statics effects of the unemployment rates and the expected utilities are the same in the two equilibria and are as follows. An increase in the unemployment benefit b, the effort e, the job-destruction rate, δ or a decrease in the monitoring rate θ, the relative number of CBD-trips for the unemployed s, increases the unemployment rate but has an ambiguous effect on the expected utility in both equilibria while an increase in the commuting cost t rises unemployment but reduces the expected utility in both equilibria.*

(ii) *The comparative statics effects of the land rent are different. Indeed, in the unconstrained equilibrium, an increase in s or t increases $R(x)$. In the constrained equilibrium, because $R(x)$ is affected by the unemployment*

[12] Observe that, if we defined a welfare function as the sum of utilities of workers, firms *and* landlords, then it is not obvious that the unconstrained equilibrium Pareto dominates the constrained equilibrium because of the increase in the land rent that benefits landlords in the constrained equilibrium.

rate, an increase in b, e, δ or a decrease in θ, s, reduces the land rent while t has an ambiguous effect.

The following comments are in order. First, consider the common effects in both equilibria of the *non-spatial exogenous variables*, b, e, δ and θ on the *non-spatial endogenous variables,* the unemployment rate and the expected utility. When b, e or δ increases or θ decreases, the efficiency wage is augmented in order to prevent shirking and thus the unemployment rate increases. However, the effect on the expected utility is ambiguous because there are two opposite forces at work. On the one hand, there is a direct positive effect since wages increase. But, on the other, there is an indirect negative effect since it raises unemployment, which implies that workers will experience more unemployment spells. It is clearly ambiguous because, over their lifetime, workers are richer when employed but spend more time unemployed.

Second, consider the effects of the *spatial exogenous variables*, t and s on all the endogenous variables, i.e. the unemployment rate, the land rent and the expected utility. In both equilibria, when the commuting cost t increases (the relative number of CBD-trips for the unemployed s decreases), firms raise (reduce) the efficiency wage to compensate more (less) for spatial costs. As a result, the unemployment rate increases (decreases). Concerning the land rent, the effects are different between equilibria. In the unconstrained equilibrium, the land rent only depends on s and t and the relationship is positive because an increase in either s or t rises the competition for land since the access to the CBD becomes more costly. Thus, in the unconstrained equilibrium, when t increases, wages are higher but both the land rent and the unemployment increase. Since these two negative effects dominate the positive effect on wages, the expected utility decreases. For s the effect on expected utility is ambiguous because a rise in s decreases wages and increases land rents but reduces unemployment. Concerning the constrained equilibrium, the effects of t and s are more subtle. Indeed, inspection of (20) shows that the land rent is both a direct and indirect (via the unemployment rate) function of t and s. So, when t increases, there is a direct effect, which leads to more competition and thus a higher land rent at each location but there is also an indirect effect since it also implies higher wages and thus higher unemployment, which reduces the capacity to bid for land. The net effect is thus ambiguous. However, the effect of t on the expected utility is negative because the negative unemployment effect dominates the ambiguous land rent effect. For s the two effects on $R(x)$ go in the same direction. Indeed, an increase in s leads to more competition and thus higher land rent, but also lower wage and lower unemployment and thus a higher capacity to bid for land rents. However, the effect of s on the expected utility is ambiguous. Indeed, a rise in s has a negative effect via the increase in land rent but has a positive effect via the decrease in unemployment.

Finally, consider the effects of the *non-spatial exogenous variables*, b, e, δ and θ on the *spatial endogenous variable*, the land rent $R(x)$. They have no impact in the unconstrained equilibrium whereas they strongly affect $R(x)$ in the constrained equilibrium via the unemployment rate. Indeed, in the latter, when b, e or δ increases or θ decreases, the unemployment rate increases. Because workers anticipate that they will experience longer unemployment spells, their capacity to bid for land is lower. There is thus less competition in the land market and, as a result, the land rent at each location decreases.

The main message of this proposition is that a measure that constrains firms' wage policy has a key impact on the housing market that we should not be ignored. This has interesting policy implications since it says that labor market policies can have strong effects on the land/housing market and thus could generate surprising results. It also says that spatial policies can have interesting impact on labor market variables.

Consider for example the standard debate on the unemployment benefit policy. In the present paper, if one reduces the unemployment benefit b, it will decrease unemployment in both equilibria but interestingly will have an ambiguous effect on utilities. This result is due to the spatial aspects of the model, and in particular to the interaction between land and labor markets, since in a standard non-spatial model (Shapiro and Stiglitz, 1984), the effect will always be positive. Furthermore, a reduction in b has no impact on the land rent in the unconstrained equilibrium but increases the land rent in the constrained equilibrium.

Consider another policy that subsidizes the commuting costs of workers, i.e. reduces t in this model. In the unconstrained equilibrium, it will reduces unemployment and land rents and thus increases utilities. However, in the constrained equilibrium, it will have the same impact on the unemployment rate and the utility but interestingly will have an ambiguous effect on land rents.

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APPENDIX

Let us now calculate the efficiency wage (8) using the Bellman equations. When $r > 0$, the standard (steady-state) Bellman equations for the non-shirkers, the shirkers and the unemployed are respectively given by:

$$r\, I_1^{NS} = V_1^{NS} - \delta\,(I_1^{NS} - I_0) \tag{22}$$

$$r\, I_1^{S} = V_1^{S} - (\delta + \theta)\,(I_1^{S} - I_0) \tag{23}$$

$$r\, I_0 = V_0 + a(I_1^{NS} - I_0) \tag{24}$$

where r is the discount rate, I_1^{NS}, I_1^{S} and I_0 represent respectively the expected lifetime utility of a non-shirker, a shirker and an unemployed worker, and V_1^{NS}, V_1^{NS} and V_U are given by (1), (2) and (3).

First, observe that when $r \to 0$, workers are infinitely patient and the value functions of I_1^{NS}, I_1^{S} and I_0 become infinite since there is no more discounting. However, when $r \to 0$, rI_1^{NS} and rI_0 have finite values and in fact $rI_1^{NS} = rI_0$. Indeed, by combining (22) and (24), we obtain:

$$rI_1^{NS} = \frac{(r+a)(w-e) + \delta\, b - (r + \delta\, s + a)tx}{r + \delta + a} - R(x) \tag{25}$$

$$rI_0 = \frac{a(w-e) + (r+\delta)\, b - (r\, s + \delta\, s + a)tx}{r + \delta + a} - R(x) \tag{26}$$

so that

$$\lim_{r\to 0} rI_1^{NS} = \lim_{r\to 0} rI_0 = \frac{a(w-e) + \delta\, b - (\delta\, s + a)tx}{\delta + a} - R(x) \tag{27}$$

which has obviously a finite value. This is very logical since, when $r \to 0$, what matters is not the current employment status but the fraction of time one spends in each state.

The efficiency wage is set as follows. Firms set a wage such that the lifetime discounted expected utility of non-shirking is equal to that of shirking, i.e. $rI_1^{NS} = rI_1^{S} = rI_1$. Let us first derive rI_1^{S}. By combining (23) and (24), we have:

$$rI_1^{S} = \frac{(r+a)w + (\delta+\theta)\, b - (r + \delta\, s + a + \theta\, s)tx}{r + \delta + a + \theta} - R(x) \tag{28}$$

with

$$\lim_{r\to 0} rI_1^{S} = \frac{aw + (\delta+\theta)\, b - (\delta\, s + a + \theta\, s)tx}{\delta + a + \theta} - R(x) \tag{29}$$

In our context, rI_1^{NS} and rI_1^{S} are well-defined functions since when $r \to 0$ they have finite values. By combining (27) and (29), we obtain the following efficiency wage:

$$w(x) = b + e + \frac{e}{\theta}(\delta + a) + (1 - s)\, tx$$

which is exactly (8). ■