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ABSTRACT

Durable Goods with Qualtiy Differentiation*

What is the optimal strategy of a durable-goods monopolist that can offer goods in different qualities? This Paper provides an answer for the case where the market is segmented into low- and high-income buyers. If the monopolist can change its product and price policy sufficiently rapidly, which reduces its commitment power, we find that the whole market is served immediately. Low-quality goods may be sold below costs. These results are strikingly different to those obtained with non-durable goods and to those obtained if the durable good comes only in a single quality. In an extension we further employ our results to discuss how policies of restricted versioning fare differently with non-durable and durable goods.

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1 Introduction

Many firms sells goods in different quality versions. An extreme example are the 'damaged goods' analyzed in Deneckere and McAfee (1996). In their paper, the authors provide a range of examples where firms sell products both in a regular up-market version and in a 'damaged' low-quality version. This strategy of second-degree price discrimination allows firms to capture a larger market while reducing price pressure on their up-market product (Mussa and Rosen 1978, Maskin and Riley 1984). With non-durable goods this strategy to capture a larger market is only profitable if the firm breaks even with each product in its quality range, i.e., in particular, also with the low-quality or 'damaged' version.[1] One key insight of this paper is that selling low-quality versions even below costs can be optimal in the case of durable goods, once we fully spell out the dynamic problem of the firm.

We consider a market containing low- and high-valuation consumers. Each period a monopolistic firm may offer a range of qualities. We are mainly interested in the case where the firm becomes relatively flexible in adjusting its products and prices. (Formally, this is captured by reducing the time between two consecutive periods in the model.) If the firm becomes sufficiently flexible, we find that it immediately serves the whole market. The optimal price for low-quality goods may be so low that it does not allow the firm to recoup costs.

Our findings are different to those obtained if the durable good comes only in a single quality. This problem originates from Coase (1972) and has received considerable attention in the literature (see, e.g., Stokey 1981; Bulow 1982; Fudenberg, Levine, and Tirole 1995; Gul, Sonnenschein, and Wilson 1986). Without access to quality as a means for price discrimination, the firm can only discriminate over time. This exploits the fact that consumers discount future payoffs, implying that high-valuation consumers have more to lose if they delay their purchase. However, if the firm can change its price more rapidly, this erodes its commitment power. This does, however, not mean that the firm no longer *tries* to use time for screening. Quite to the opposite, one typically finds that the number of periods (or price changes) it takes until the market is cleared *increases* as the time between two consecutive periods decreases, i.e., as the power to commit becomes lower. (On the other hand, the *real time* it takes to clear the market decreases.)

In contrast, in our case where the firm has access to a second instrument for price-discrimination, quality, we find that the firm serves the whole market immediately, i.e.,

[1] In fact, put in the terminology of mechanism design, each additional (low-quality) product must generate a positive 'virtual surplus', which encompasses the information rent of up-market buyers.

in the very first period, if it can change its products and prices sufficiently fast.

The introduction of quality as an additional means for price discrimination makes the firm's time inconsistency problem much more severe. Or, putting it differently, the presence of quality as an alternative means to price discriminate further erodes the value of delay to screen among buyers with different valuations. Consequently, the monopolist optimally serves the whole market at the first instance. Low-valuation consumers receive a low-quality good, which ensures that the information rent left to high-valuation consumers is not too high. If the firm, instead, delayed sales to low-valuation buyers, while first serving some or all of the high-valuation buyers, consumers would rationally anticipate that low-valuation buyers would later obtain a good of intermediate quality. This reduces the maximum price that can be extracted from high-valuation buyers, who are supposed to buy earlier.

By clearing the whole market immediately, the firm thus 'commits' not to make a more attractive offer to low-valuation buyers in the future. This in turn allows to extract a higher price from high-valuation buyers.

Related Literature

Our twin predictions that the whole market is served immediately and that goods may be sold below costs are at best a hypothetical benchmark. Often, the monopolist may enjoy some commitment power due to, e.g., limited capacity (formalized in Kahn 1986) or reputational concerns (formalized in Ausubel and Deneckere 1989). This may help to explain why markets for durable goods of different qualities are often penetrated more slowly.[2] Moreover, many instances of below-cost pricing may have different explanations, such as the presence of network externalities (Farrell and Saloner 1986, Shapiro and Varian 1999), consumers with switching costs (see Klemperer 1995 for an overview on switching costs), or price wars (see Marx and Shaffer 2000 for a recent contribution). In light of these alternative explanations, our analysis may provide a useful benchmark: How far can we already go to explain below-cost pricing without appealing to interdependent demand or competition for market share?

The durable-goods monopolist's problem is formally equivalent to that of a seller making repeated offers to a single buyer who is privately informed about his valuation. Wang (1998) has recently introduced menu offers. However, Wang's paper does *not* address the key issues of this paper. In Wang's paper there does *not* exist a commitment problem. Under the particular choice of payoff functions in Wang (1998), even a firm with full commitment power would choose to sell immediately. (Hence, the virtual surplus is strictly positive for *all* types.) In contrast, we are mainly interested in the case where a firm would prefer to commit not to sell to all types, implying that there

[2] Take, for instance, the market for hard-cover and soft-cover editions of books.

exists a true commitment problem.

Bagnoli, Saland, and Swierzbinski (1995) also consider a monopolist who can offer a range of qualities over time. As they assume a finite number of buyers, the firm can essentially price discriminate between individual buyers and, thereby, extract all surplus. Durable goods with different qualities also co-exist in the following models. Takeyama (1997) allows for pirate copying, which creates an imperfect substitute of lower quality.[3] A small recent literature considers the case of successive product generations or upgrades (see, e.g., Fudenberg and Tirole 1998 and Villas-Boas 1999). Typically, the market operates for only two periods. In the second period, the monopolist may provide an enhanced version of the original product. Hence, it is only in the second and final period that the firm has a choice between two different levels of quality.[4]

The remainder of this paper is organized as follows. Section 2 develops the model. In Section 3 we summarize results for the benchmark cases of non-durable goods and durable goods with a single quality. Section 4 derives our main result. In Section 5 we turn to a normative issue and discuss the welfare implications of restricted versioning. Section 6 concludes. All proofs are relegated to the appendix.

2 The Model

We consider a market for an infinitely durable and indivisible good. There is a continuum of mass one of infinitely-lived consumers. Consumers come in two types, $t \in T = \{1, 2\}$ with respective masses $q_t^0 > 0$. We call consumers of type $t = 2$ the 'high-valuation' segment and consumers of type $t = 1$ the 'low-valuation' segment. Consumers want to buy at most a single good. The market is served by a monopolistic firm, which chooses prices and qualities. Quality is measured by a real-valued variable x. If a good's quality equals x and is sold at the price p a buyer of type t realizes utility $U_t(x, p) := V_t(x) - p$, while the firm realizes profits $W(x, p) := p - C(x)$. Consumers have a reservation value of zero. As utility is transferable, we may call $S_t(x) := V_t(x) - C(x)$ the surplus realized with type t. We assume that $V_t(x)$ and $C(x)$ are three times differentiable and invoke the following standard assumptions.

[3] Her model is restricted to two periods. We know from the standard durable goods case that this is problematic. With two periods the seller's payoff is often U-shaped in the discount factor. Moreover, his optimal strategy may be not to sell at all in the first period. This is never optimal with an open time horizon.

[4] Other papers analyze planned obsolence or durability of a single good (see Hendel and Lizzeri 1999 and the references therein) or its repeated replacement (see Fishman and Rob 2000 and the references therein).

Assumption 1: $d^2S_t/dx^2 < 0$ is bounded away from zero; $V_2(x) > V_1(x)$; $dV_2/dx > dV_1/dx$; $d^2V_2/dx^2 \geq d^2V_1/dx^2$.

Total surplus is concave in the quality x. Both a buyer's absolute valuation and his marginal valuation for a higher quality increase with his type. For instance, the type may be an indicator of consumers' income or wealth. The last part in Assumption 1 is a standard technical assumption in the literature on price discrimination. Total surplus $S_t(x)$ is maximized by a unique value, x_t^f. Denote the realized first-best surplus by $S_t^f := S_t(x_t^f)$ for $t \in T$.

Assumption 1: $S_1^f > 0$.

We consider discrete periods of length $z > 0$, which are numbered consecutively by $n \in \{0, 1, ...\}$. We assume that all consumers and the monopolist apply the discount factor $\delta = e^{-rz}$, where $r > 0$. The objective of the firm is to maximize discounted profits.

Each period the monopolist may offer a range of products. A product is fully characterized by a price p and a quality x. Any (remaining) buyer can decide to buy one of the products or to wait for a better deal. Though we do not a priori restrict the number of different quality levels, we specify that each period at most two different levels are offered.[5] Our equilibrium concept is that of subgame perfect equilibrium, with pure strategies for consumers. An equilibrium path can therefore be described by a sequence of offers $\{(x_1^n, p_1^n), (x_2^n, p_2^n)\}$ and a sequence of residual masses $\{q_1^n, q_2^n\}$ for the two types of consumers. It is convenient to denote the fraction of high-valuation buyers in the residual market by $v^n := q_2^n/q^n$, where $q^n := q_1^n + q_2^n$. Note that $v^0 = q_2^0$. The firm cannot directly discriminate between the two market segments, i.e., we rule out third-degree price discrimination. Moreover, to rule out first-degree price discrimination we make the standard assumption that the firm cannot condition its strategy on the actions taken by individual (or mass zero) consumers.[6]

Before proceeding to the analysis, we discuss some of the limitations of our model. Note first that a given consumer can purchase at most once. In particular, we want to rule out the case where a consumer trades in his previous purchase for a good with a possibly higher quality. While we now take this assumption as given, we state below a condition on the costs of production that makes it endogenous. In this case, it is important that the quality is physically embodied in the product, i.e., supplying a consumer with a good of higher quality requires the production of an entirely new product. This assumption

[5]This turns out to be without loss of generality as we focus below on the case of low values of z, for which we find a unique equilibrium.

[6]See Bagnoli, Salant, and Swierzbinski (1989) for the relevance of this assumption. The formal description of strategies and equilibrium requirements is standard and, therefore, omitted for the sake of brevity.

clearly limits the scope of our analysis. For instance, our analysis is less appropriate if there is the possibility of almost costlessly up-grading low-quality versions of the product as in the case of software. On the other side, many cases of consumer electric equipment or information technology may fit our assumptions.

Finally and in sharp contrast to the case with non-durable goods, our results do not extend to price-discrimination with non-linear tariffs and quantity discounts, where additional units could be supplied easily at any future point in time.[7]

3 Benchmarks

Durable-Goods with a Single Quality

If the durable good comes in a single quality and if production is strictly profitable for both consumer types, we obtain the following results (see, e.g., Hart 1989 for the two-type case). The monopolist serves the whole market in finite time by posting a sequence of decreasing prices. If it takes more than one period to clear the whole market, which is the case if the fraction of high-valuation consumers is sufficiently large, low-valuation consumers purchase in the last period, while at least some high-valuation consumers accept a higher price in earlier periods.

As the real time z between two periods shrinks, the number of periods it takes to clear the market typically increases, while the real time to do so decreases. Decreasing z reduces the commitment power of the firm. Sustaining the price in future periods and thereby delaying the purchase of low-valuation consumers is not credible if a large fraction of high-valuation consumers has already made a purchase. Expecting a sharp reduction in prices tomorrow, high-valuation consumers will prefer to hold out as well. As the firm can change its price more and more rapidly, it loses virtually all commitment power and is finally forced to sell at prices that only marginally exceed the lowest valuation.

[7] Assumption 1 also requires that $V_2(x) > V_1(x)$ holds for *all* feasible quality levels x. Thus, we do not allow for the case where $V_2(x) = V_1(x)$ holds for some (minimum) quality level, e.g., for $x = 0$. (Again, this would be the case in a model with non-linear tariffs where x represents quantity.) Then, offering $x = 0$ to low-valuation consumers would allow to extract all rents from high-valuation consumers. If we specified additionally $C(x) = 0$ for $x = 0$, selling a good with quality $x = 0$ to low-valuation consumers would be equivalent to committing not to sell to them at all. Instead, we focus on the polar case where such a commitment is not feasible. However, it can be shown that our results continue to hold if there *does* exist some value x where $V_1(x) = V_2(x)$, provided x is sufficiently low such that the loss associated with selling quality x to low-valuation consumers is sufficiently high.

Price Discrimination with Non-Durable Goods

If the good is non-durable, there is essentially a single period in which it can be supplied. If only the upper segment of the market is served, the firm offers the quality $x = x_2^f$ at the price $p = V_2(x_2^f)$, extracting all surplus from high-type consumers. If the firm serves the whole market, it offers the product in two quality levels, x_t^c, with respective prices, p_t^c. If v denotes the fraction of high-valuation consumers in the (residual) market, qualities and prices are determined as follows (see, e.g., Laffont and Tirole 1993 for the two-type case):

$$\begin{aligned} x_2^c &= x_2^f, \\ \left.\frac{dS_1(x)}{dx}\right|_{x=x_1^c} &= v \left.\frac{dS_2(x)}{dx}\right|_{x=x_1^c}, \\ p_2^c &= V_2(x_2^c) - \left[V_2(x_1^c) - V_1(x_1^c)\right], \\ p_1^c &= V_1(x_1^c). \end{aligned} \tag{1}$$

The bottom-range quality is determined by the trade-off between maximizing surplus and minimizing the information rent obtained by high-valuation consumers. Note that high-valuation consumers will walk away with the utility $V_2(x_1^c) - V_1(x_1^c)$, which is strictly positive by Assumption 1. We write the bottom-range quality as a function $x_1^c(v)$ of the fraction of high-valuation consumers. By (1) $x_1^c(v)$ is continuous and strictly decreasing in v.

Comparing payoffs, the whole market is served if the low-valuation consumers' virtual surplus, which is given by

$$(1-v)S_1(x_1^c(v)) - v\left[V_2(x_1^c(v)) - V_1(x_1^c(v))\right], \tag{2}$$

is positive. By Assumptions 1-2 and the properties of $x_1^c(v)$, this expression is strictly decreasing in v, it becomes strictly negative as $v \to 1$, and it is strictly positive at $v = 0$. This implies the following useful result.

Lemma 0. *If the firm sells a non-durable good, there exists a threshold value $0 < \bar{v} < 1$ for the fraction of high-valuation consumers such that the following holds.*

If the fraction of high-valuation consumers falls below $\bar{v}$, the firm serves the whole market with the range of qualities and prices $\{(x_t^c, p_t^c)\}_{t \in T}$, as characterized in (1). If the fraction of high-valuation consumers exceeds $\bar{v}$, the firm only serves the high-valuation segment.

Price Discrimination with Durable Goods and Commitment

What if the durable-goods monopolist could commit to a price and product policy for all future periods? It is straightforward to show that this problem is analogous to the non-durable goods case. In particular, the firm does not use delay for price discrimination. The whole market is either served immediately or low-valuation consumers will never buy.[8]

From now on we will, therefore, refer to the products and prices characterized in (1) as the 'commitment offer'.

4 Analysis

We state next our main result.

Proposition 1. *There exists a unique equilibrium in which the durable-goods monopolist serves the whole market in the first period by offering the range of qualities and prices* $\{(x_t^c, p_t^c)\}_{t \in T}$ *characterized in (1) if either of the following conditions holds:*

i) the initial fraction of high-valuation consumers falls below the threshold $\bar{v}$*,*

ii) or, regardless of the composition of the market, the real time between two consecutive periods, z*, becomes sufficiently small.*

If the fraction of high-valuation consumers falls below the threshold $\bar{v}$, which is used in Lemma 0, the prediction for the durable goods case is identical to that for the non-durable goods case. This is intuitive as clearing the market immediately with the commitment offer is also would the firm would optimally do in case it could commit.[9] This result has already been obtained by Wang (1998) for the formally equivalent case of two-party bargaining with one-sided offers and menus. As noted in our introduction, Wang assumes a surplus function that ensures that the low type's virtual surplus as defined in (2) is positive *for any* distribution of types.

More interesting is, however, the case where the firm would *not* want to serve the market if it could commit to do so, that is the case where $v^0 > \bar{v}$. Only in this case does the firm face a true commitment (or time-inconsistency) problem, which is at the heart of our analysis. For this case we obtain the following result. If the firm can change its policy sufficiently fast, then the unique equilibrium is as in the case where $v^0 < \bar{v}$, i.e., the whole market is served immediately with the offers determined in (1). While it is again intuitive that the firm makes these offers *provided* that it clears the market immediately, it is surprising that the firm does not make use of delay as an additional instrument for price discrimination. What is even more surprising is that the firm may

[8] This statement is only precise for the case $v \neq \bar{v}$, for which the firm's problem has a unique solution in the non-durable goods case. The fact that the firm does not use delay as an additional sorting variable is stated formally, e.g., in Riley and Zeckhauser (1983).

[9] It is straightforward that this is also what the firm *can* do. On this see Step 1 in the description of the proof of Proposition 1 further below at the end of this section.

be willing to sell its bottom-range product below costs. This is the case whenever it holds that $S_1(x_1^c(v^0)) < 0$. Using the concavity of the surplus function and (1), this yields the following corollary to Proposition 1.

Corollary. *For sufficiently low values of z, the durable-goods monopolist sells to low-valuation consumers below costs if, holding everything else constant, either the fraction of high-valuation consumers is sufficiently large, the first-best surplus level S_1^f realized with low-valuation consumers is sufficiently low, or the 'difference' between high- and low-valuation consumers is sufficiently large.*[10]

We next provide some intuition for Proposition 1. For this it is helpful to recall how the quality for low-valuation buyers is determined in the non-durable goods case. Adding a low-quality version to its product range, the firm creates an alternative also for high-valuation consumers. The more attractive the bottom-range product is, the lower becomes the price that can be commanded by its up-market product. The quality bought by low-valuation consumers is determined by the resulting trade-off between maximizing surplus and reducing the 'information rent' left to high-valuation consumers. Intuitively, if the fraction of low-valuation buyers increases, the firm puts less weight on the reduction of information rent and increases the optimal quality for low-valuation buyers. Suppose now the durable-goods monopolist serves some high-valuation consumers without immediately clearing the whole market. This necessarily decreases the fraction of high-valuation consumers in tomorrow's (residual) market. Applying the logic from the non-durable goods case, a decrease in the fraction of high-valuation consumers will induce the firm to offer low-valuation buyers a more attractive product. As this is now rationally anticipated by all consumers, it reduces the price high-valuation consumers are willing to pay today. In other words, if the monopolist does not serve the whole market immediately, quality as a means for price discrimination cannot be used to its full effect.

What drives our result of immediate market clearing is, therefore, the *interaction* of the commitment (or time-inconsistency) problem in the two dimensions of the firm's strategy: price and quality. Neither can the monopolist commit that he will stick to his current prices nor can he commit that he will stick to his current product range. If the firm can offer a range of qualities, its commitment problem becomes more severe: neither can the firm commit not to offer a (much) lower price in the following period(s), nor can it commit not to offer a (much) higher quality also at the lower end of its product range.

Delaying market clearing would erode the effectiveness of quality as a means for price

[10] Admittedly, we have not introduced a (continuous) parameter capturing buyers' types. Recall that x_1^c is determined, for given v, by the requirement $\left.\frac{dV_1(x)}{dx}\right|_{x=x_1^c} = v \left.\frac{dV_2(x)}{dx}\right|_{x=x_1^c}$. The difference in types is then captured by the difference in the marginal valuations $dV_t(x)/dx$.

discrimination. Our result clearly hinges on the lopsided treatment of quality and time as sorting variables. While the firm cannot commit not to serve a segment of the market in the future, it can commit not to approach again consumers who have previously bought a low-quality variant. One justification for the latter assumption could be that buyers would have to incur sufficiently high switching costs when replacing their initially bought good. For instance, if the product is some household good, e.g., a washing machine or a fridge, delivery and installation costs may make a trade-in unprofitable. Moreover, it may simply not be profitable to produce a new good for low-valuation consumers given that they already own one, albeit one of an inferior quality. Here, it is sufficient to assume that the additionally generated consumption value created by exchanging the 'damaged' version x_1^c with the first-best quality x_1^f, i.e., $V_1(x_1^f) - V_1(x_1^c)$, does not exceed the costs $C(x_1^f)$.

In the rest of this section we provide a short, more technical, description of the proof of Proposition 1.

Description of the Proof of Proposition 1

Step 1: The first step is to note that the firm can always end the game by making the commitment offer. This follows from the following argument. It is intuitive that low-valuation buyers do not expect to obtain a surplus in the market. Hence, when making an offer that allows them to realize an arbitrarily small surplus, the firm can expect that all low-valuation buyers purchase immediately. But this implies that also all high-valuation consumers must optimally buy in that period, as the firm would otherwise be sure that only high-valuation buyers are left and would, consequently, try to extract all of their surplus.

Step 2: The firm will only postpone selling to low-valuation consumers (for one more period) if this allows to extract more rent from high-valuation consumers. Intuitively, such a strategy is only profitable if a sufficiently large fraction of high-valuation consumers buys now. Or, putting it differently, delay is only profitable if the difference $v^{n-1} - v^n$ is sufficiently large in future periods.

Step 3: Suppose now it takes more than one period to clear the market and consider the firm's strategy over the last two periods, $N-1$ and N. If the firm deviates and clears the market already in period $N-1$ with the commitment offer, the respective quality offered to low-valuation buyers is $x_1^c(v^{N-1})$. This leaves high-valuation buyers with the (information) rent $V_2(x_1^c(v^{N-1})) - V_1(x_1^c(v^{N-1}))$. If the firm, however, clears the market only in period N, it will optimally offer the quality $x_1^c(v^N)$ in period N. In this case incentive compatibility implies that high-valuation buyers realize at least the utility $\delta\left[V_2(x_1^c(v^N)) - V_1(x_1^c(v^N))\right]$. Given that $v^{N-1} - v^N > 0$ is bounded away from zero by Step 2, also the difference $x_1^c(v^N) - x_1^c(v^N) > 0$ is bounded away from zero. But this implies that, for high δ and thus low z, the minimum information rent that must be left to high-valuation buyers when clearing the market over two more periods, i.e., $\delta\left[V_2(x_1^c(v^N)) - V_1(x_1^c(v^N))\right]$, must exceed the information rent if the market was already cleared in period $N-1$, i.e., $V_2(x_1^c(v^{N-1})) - V_1(x_1^c(v^{N-1}))$.

Summing up the argument, it is only beneficial to delay market clearing (for one more period) if a sufficiently large fraction of high-valuation consumers is served right now. But this implies that the firm will find it optimal in the next period to increase by some non-marginal amount the quality offered to low-valuation consumers. This again substantially reduces the maximum price that can be demanded from high-valuation consumers right now.

While we think that the arguments underlying Proposition 1 are quite general, our current methodology does not extend to more than two types. We argue next why this is the case. When proving Proposition 1, we allow for any possible way how the firm could clear the market over the last two periods $N-1$ and N. In particular, we also allow for the possibility that both types buy in both periods $N-1$ and N. If the durable good came only in a single quality, this would not be possible, i.e., the equilibrium would have to be 'skimming': high-valuation consumers optimally buy before low-valuation consumers. If the firm can offer a range of qualities in each period, the skimming property is no longer immediately implied by buyer optimality.

An interesting avenue for future work is to study whether the skimming property must still follow from optimality of the firm. Currently, we have only shown uniqueness for high δ (Proposition 1). For low δ the proof of Proposition 1 shows existence of a skimming equilibrium, where the market is cleared over time, but not uniqueness.

5 Restricted Versioning

Consider once again price discrimination with *non-durable* goods. There, the firm will either supplies only the up-market segment or, in case of covering the whole market, it sells goods of inferior quality to low-valuation consumers. This finding suggests that welfare could be improved by restricting the firm's scope for offering downgraded (or 'damaged') versions of the product. Unfortunately, if it is only possible to prescribe that the firm offers a single variant (restricted versioning), the welfare implications are generally ambiguous (Deneckere and McAfee 1996, Varian 1997).

One of the few clear-cut results one obtains for restricted versioning with non-durable goods is the following. Suppose that, originally, the firm offered two qualities of the non-durable good, thus serving the whole market, albeit with an inefficiently low quality at the bottom of the range. If restricted versioning induces the firm to serve only the up-market segment, total welfare is reduced.

In what follows, we present a brief analysis of restricted versioning for the case of *durable goods*. Our main insights are as follows. We first argue that, even if restricted versioning reduces market coverage, this can increase welfare in the case of durable goods. Moreover, again in contrast to the case of non-durable goods, restricted versioning may induce the durable goods monopolist to offer a single good with an excessively high quality.

As the following arguments rely on standard results of the durable-goods literature (with one quality level), we can be relatively brief. In what follows, we also restrict ourselves to the (limit) case where $z \to 0$. Denote the single quality level that is offered under restricted versioning by $\bar{x}$. This quality level is chosen optimally by the firm in the first period. In all future periods, the firm is restricted to offer only goods of this quality.

We can clearly restrict ourselves to cases where $V_2(\bar{x}) > C(\bar{x})$. If $V_1(\bar{x}) \leq C(\bar{x})$ holds, it is credible that the firm sells only to high-valuation buyers and realizes $q_2^0[V_2(\bar{x}) - C(\bar{x})]$.[11] Denote the, by Assumption 1 unique, (threshold) quality level satisfying $x > x_1^f$ and $V_1(x) = C(x)$ by x^T, implying $V_1(x) < C(x)$ for $x > x^T$ and $V_1(x) > C(x)$ for $x \in [x_1^f, x^T)$. If the firm chooses to only serve high-valuation consumers, it follows from concavity of S_2 (Assumption 1) that it optimally chooses the quality $\bar{x}^U := \max\left\{x_2^f, x^T\right\}$. In contrast, if the firm chooses a quality level satisfying $V_1(\bar{x}) > C(\bar{x})$, it will serve the whole market. In this case, it follows from the Coase Conjecture that the firm's profits converge to $V_1(\bar{x}) - C(\bar{x})$ as $z \to 0$. By optimality, the firm will then choose $\bar{x} = x_1^f$. Comparing payoffs, the firm will choose to cover the whole market only if

$$V_1(x_1^f) - C(x_1^f) \geq q_2^0[V_2(\bar{x}^U) - C(\bar{x}^U)]. \tag{3}$$

We have the following results.

Proposition 2. *With restricted versioning and for $z \to 0$, the firm optimally chooses the quality level $\bar{x} = x_1^f$ if it intends to cover the whole market. Otherwise, it chooses $\bar{x} = \bar{x}^U$. Serving the whole market is only optimal if (3) is satisfied.*

Note that, for condition (3) to be satisfied, the fraction of high types in the market must not be too large, while the valuation of consumers at the lower end of the market must not be too low.

We use now Proposition 2 to analyze the welfare implications of restricted versioning. With durable goods, we know from Proposition 1 that, for all sufficiently low values z,

[11] In case $V_1(\bar{x}) = C(\bar{x})$ there also exist equilibria where the firm will sell to low-income buyers. As these equilibria no longer exist when increasing quality by an arbitrarily small amount and as $\bar{x}$ is chosen optimally by the firm in the first place, we can ignore these (continuation) equilibria.

the market is immediately cleared with the commitment quality range.

Suppose now that the commitment offer leads to below-cost selling, while with restricted versioning the firm chooses $\bar{x} = \bar{x}^U$, where it holds additionally that $\bar{x} = x_2^f$. Intuitively, this will be the case if the maximum surplus with low types is small and if the difference in types is large. In this case, restricted versioning leads to a reduction in market coverage, but this strictly increases welfare. Instead of clearing the market with a version offered to low-valuation consumers that is produced below costs, the restricted versioning policy allows the firm to commit not to offer any product to these consumers. Instead, it will only offer x_2^f to high-valuation consumers. Note, however, that while welfare is increased, consumer surplus is strictly lower as high-valuation consumers no longer earn an information rent.

For a final observation on the case with restricted versioning, observe that with *non-durable* goods the firm will never offer a quality exceeding x_2^f. This may, however, be optimal with *durable* goods. Under restricted versioning, the durable-goods monopolist may offer a quality $\bar{x}^U = x^T > x_2^f$ to commit that it will not sell later to low-valuation buyers. Intuitively, this is the case if the fraction of high-valuation consumers is sufficiently large, making it optimal to only serve this segment, while $x^T > x_2^f$ holds if the difference between consumer types is not too large. Again, this result stands in sharp contrast to the case with non-durable goods.

Unfortunately, as in the case with non-durable goods, it depends on the particular circumstances whether restricted versioning will increase or reduce welfare. The following proposition summarizes our previous observations, which document the important differences to the case with non-durable goods.

Proposition 3. *In the durable-goods case, welfare may even increase under restricted versioning if total market coverage is reduced. Moreover, restricted versioning can induce the firm to offer an excessively high quality, which exceeds the first-best quality level for high-valuation buyers* x_2^f.

6 Conclusion

This paper brings together two formerly unrelated strands of the literature on price discrimination by a monopolist. In a game with an open time horizon the monopolist may each period decide to offer a range of products varying in quality and price. This allows to use both quality and delay to extract more surplus from buyers with a higher income, who value the product more in absolute terms and also attach more value to a marginal increase in quality. Hence, our model adds to the standard durable-goods case the possibility to use an additional variable to price discriminate, while it abandons the assumption made in the literature on static price discrimination (Mussa and Rosen (1978)) that the firm can commit to a fixed menu of offers. Solving the model for the two-type case, we show that the monopolist serves the whole market in the first period if players become sufficiently patient. This may even imply that the monopolist sells to the lower segment of the market below costs.

The analysis of this paper is restricted to the two-type case. We conjecture that the intuition for our main result carries over to more general distributions satisfying the 'gap' assumption where trade is strictly profitable with all buyers. The analysis should also be extended to the so-called 'no-gap' case where trade is not profitable with some buyers. For the standard durable-goods case without an additional instrument to price discriminate, Ausubel and Deneckere (1989) showed that the Coase Conjecture need not hold without a gap. As the game continues forever without a gap, a plethora of offer sequences can be sustained by threatening to switch to an equilibrium where the Coase Conjecture applies and the monopolist's future payoff reduces to (almost) zero. If the monopolist has access to an additional instrument for price discrimination, the payoff which the monopolist can extract from a given (residual) market remains always bounded away from zero, even if the market is cleared immediately. It would be interesting to know to what extent this feature rules out the type of equilibria studied in Ausubel and Deneckere (1989).

7 Appendix: Proof of Proposition 1

We first state a series of auxiliary results. Proofs are heavily abbreviated or omitted if they relate to standard results from the durable goods case with a single quality level.

Lemma 1. *Consider some residual market with distribution v. For $v < 1$ all remaining low-valuation consumers realize zero utility. If $v = 1$ holds, then all remaining consumers, who all have a high valuation, realize zero utility.*

The proof of Lemma 1, which is a standard result, is omitted. Using Lemma 1 and the argument provided in the main text, we obtain the following results.

Lemma 2. *If the market is fully served in some final period N, then this is done by the commitment offer $\{(x_t^c, p_t^c)\}_{t \in T}$. If the fraction of high-valuation consumers drops below $\bar{v}$ in some period n, then the whole residual market is served in n. Finally, at any point of time, the profit realized under the commitment offer represents a lower boundary for the payoff that can be extracted from the residual market.*

For Lemma 2 note that the definition of the commitment offer in (1) naturally extends to the boundary cases $v = 0$ and $v = 1$ if we specify that only a single (first-best) offer is made in this case. Lemma 2 already proves the assertion in Proposition 1 relating to the case $v^0 < \bar{v}$. We show next that the game ends in finite time.

Lemma 3. *The game ends in finite time.*

Proof. If $q_1^n < q_1^0$ holds for some finite n, this implies by optimality of high-valuation consumers and by Assumption 1 that $q_2^{n'} = 0$ must hold for some sufficiently large n'. In this case the game ends no later than in period $n' + 1$. It therefore remains to rule out the case where low-valuation consumers are never served. In this case the market distribution v^n forms a nonincreasing sequence. If it drops below $\bar{v}$, we already know from Lemma 2 that the game must end in that period. Using the arguments of Fudenberg, Levine, and Tirole (1985) it is straightforward to show that this follows from the firm's optimality. **Q.E.D.**

Before proceeding to prove Proposition 1, we must show existence. We do so by constructing an equilibrium where the all low-valuation consumers are served in some final period N.

Lemma 4. *An equilibrium exists for all values of* z.

Proof. We specify that *all* low-valuation consumers accept the first offer that generates non-negative utility. By Lemma 1 this is optimal. Regarding high-valuation buyers, we specify that they follow a 'reservation-utility' strategy, which is in the spirit of Gul, Sonnenschein, and Wilson (1986) and Ausubel and Deneckere (1989). For this purpose we order consumers a long a line of length one such that those occupying positions $q \in [0, q_2^0]$ have high valuation. We specify that a consumer $q \leq q_1^0$ buys if the best current offer (x, p) realizes the utility $V_2(x) - p \geq U^R(q)$, where $U^R(q)$ is a nondecreasing and left-continuous function. We now introduce the following auxiliary notation, which will also be useful for the final steps in the proof of Proposition 1:

$$
\begin{aligned}
\Lambda(x) &= V_2(x) - V_1(x), \\
\Omega(v, x) &= (1 - v)S_1(x) - v\Lambda(x), \\
\Pi(v, x) &= vS_2^f + \Omega(v, x).
\end{aligned}
$$

Given the specification of consumers' strategies, the firm solves a dynamic programming problem. Each period it can either continue to serve a fraction of high-valuation consumes, or it can clear the whole market by including an offer acceptable also to low-valuation consumers. By the specification of consumers' strategies the residual market is fully characterized by the threshold q of high-valuation consumers who have already purchased. Given q the residual market has the distribution $v(q) = (q_2^0 - q)/(1 - q)$. Moreover, the firm's continuation payoff depends only on q. If we denote it by $R(q)$, it is given by

$$R(q) = max \begin{cases} max_{y \in [q, q_2^0]} \left[(y-q)(S_2^f - U^R(y)) + \delta R(y) \right] \\ (1-q)\Pi(v(q), x_1^c(v(q))) \end{cases},$$

where we have already used that it is optimal to clear the residual market with the commitment offer.

We are now in a position to apply standard arguments as, e.g., in Ausubel and Deneckere (1989). For all $q \in (\bar{q}, q_2^0]$, where $\bar{q} = (q_b - \bar{v})/(1-\bar{v})$ ($\bar{v}$ was defined in Lemma 0), we know already from Lemma 1 that $R(q) = (1-q)\Pi(v(q), x_1^c(v(q)))$, implying by optimality for high-valuation consumers $U^R(q) = \delta\Lambda(x_1^c(v(q))$. By induction one can then extend $R(q)$ and $U^R(q)$ to lower values of q. **Q.E.D.**

As noted in the main text, the proof of the assertion in Proposition 1 relating to low values of z proceeds by considering the two last periods $N-1$ and N in case the market is not served in $n = 0$. By Lemma 3 such a final period N always exists. We analyze first the case where the residual market in $N-1$ is served by a skimming policy. Subsequently, we rule out all other non-skimming cases. We need the following intuitive auxiliary result.

Lemma 5. *There exists a value $\tilde{v} < 1$ such that for all z and in any equilibrium the fraction of high-valuation consumers in the final period satisfies $v^N < \tilde{v}$.*

Proof. Suppose this was not the case, implying existence of a sequence of equilibria where it holds in the respective final periods N_α that $v^{N_\alpha} \to 1$. By Lemma 2 and the construction of the commitment offer this implies for the respective utilities of high-valuation consumers, which we denote by U_α, that $U_\alpha \to 0$. By Assumption 2 and optimality for high-valuation consumers this gives us a sequence of thresholds $\bar{x}_\alpha$ with $\bar{x}_\alpha \to -\infty$ such that *all* goods sold to low-valuation consumers in an equilibrium indexed by α must have a quality not above $\bar{x}_\alpha$. To complete the argument we can now make use of the fact that surplus is strictly concave, implying that selling to low-valuation consumers becomes increasingly loss making as $\bar{x}_\alpha$ decreases. It is straightforward to show that this cannot constitute an optimal strategy for high values of α. **Q.E.D.**

Recall for the next claim that a skimming sequence of offers and purchases over the last periods $N-1$ and N implies that no low-valuation consumer purchases strictly before some high-valuation consumers, i.e., that $q_1^N < q_1^{N-1}$ implies $q_2^N = 0$.

Lemma 6. *There exists $\bar{z} > 0$ such that for all $z < \bar{z}$ there is no equilibrium where the market is served over more than one periods and where the firm follows a skimming policy over the last two periods $N-1$ and N.*

Proof. We argue to a contradiction. By Lemma 2 the payoff from serving the whole market already in $N-1$ is bounded from below by the commitment offers. Denote this payoff by

$$W_1 = q^{N-1}\Pi(x_1^c(v^{N-1}), v^{N-1}). \tag{4}$$

Suppose now first that no low-valuation consumer buys in $N-1$, implying $q_1^{N-1} = q_1^N$. By Lemma 2 the firm makes the respective commitment offer in N. By incentive compatibility for high-valuation consumers, who must therefore realize at least the utility $\delta\Lambda(x_1^c(v^N))$, the firm's payoff from serving the market over two more periods is then bounded from above by

$$W_2 = S_2^f\left[q_2^{N-1} - (1-\delta)q_2^N\right] + \delta q_1^{N-1} S_1(x_1^c(v_N)) - \delta q_2^{N-1}\Lambda(x_1^c(v^N)). \tag{5}$$

From (4)-(5) we obtain

$$\begin{aligned}\frac{W_2 - W_1}{q^{N-1}} &= \delta\left[\frac{(1-v^{N-1})v^N}{1-v^N}S_2^f + \Omega(v^{N-1}, x_1^c(v^N))\right] \\ &\quad - \left[\frac{(1-v^{N-1})v^N}{1-v^N}S_2^f + \Omega(v^{N-1}, x_1^c(v^{N-1}))\right],\end{aligned} \tag{6}$$

where we used $q_2^N/q^{N-1} = (1-v^{N-1})v^N/(1-v^N)$. We show that (6) becomes strictly negative for high values of δ. As $\Omega(v^{N-1}, x_1^c(v^{N-1}))$ strictly exceeds $\Omega(v^{N-1}, x_1^c(v^N))$ for all $v^{N-1} \neq v^N$ due the construction of x_1^c and as $\delta < 1$, we only have to discuss the case where

$$\frac{(1-v^{N-1})v^N}{1-v^N}S_2^f + \Omega(v^{N-1}, x_1^c(v^{N-1})) < 0. \tag{7}$$

We argue first that (7) implies $v^{N-1} - v^N > \Delta v$ for some threshold $\Delta v > 0$. To see this, note that by Lemma 2 and Lemma 5 we have $v^{N-1} \in [\bar{v}, \tilde{v}]$, where $\bar{v} > 0$ and $\tilde{v} < 1$, while the skimming policy implies $v^N \leq v^{N-1}$. At $v^{N-1} = v^N$ the left-hand side of (7) is equal to $\Pi(v^{N-1}, x_1^c(v^{N-1}))$, which exceeds S_1^f for all choices of v^{N-1}. Recall also that $S_1^f > 0$ by Assumption 2. Using continuity in v^{N-1} and v^N as well as the restriction on v^{N-1}, this implies existence of the threshold Δv. Using the definition of x^c this immediately implies that the difference $x_1^c(v^N) - x_1^c(v^N - 1) > 0$ is bounded away from zero over all feasible choices for v^{N-1} and v^N, and that finally the same holds for the difference $\Omega(v^{N-1}, x_1^c(v^{N-1})) - \Omega(v^{N-1}, x_1^c(v^N)) > 0$. From this it finally follows that (6) must become negative for all sufficiently high δ and thus for all values z falling below some threshold $\bar{z}_1 > 0$.

It remains to discuss the second kind of skimming policy according to which low-valuation consumers are also served in $N-1$. As the previous expressions do not fully apply to this case, we treat it separately, though the same logic applies. By definition this implies $q_2^N = 0$. Using $v^N = 0$, we know from Lemma 2 that the firm offers low-valuation consumers the first-best quality x_1^f in the last period. High-valuation consumers must therefore receive over the last two periods at least the utility $\delta\Lambda(x_1^f)$. This obtains as an upper boundary for the firm's payoff from serving the residual market over periods $N-1$ and N

$$W_2' = q_2^{N-1}\left[S_2^f - \delta\Lambda(x_1^f)\right] + (q_1^{N-1} - q_1^N)S_1(x_1^f) + \delta q_1^N S_1^f. \tag{8}$$

Using (4) and (8) we can again derive $(W_2 - W_1')/q^{N-1}$ in analogy to (6). At $\delta = 1$ this becomes $\Omega(v^{N-1}, x_1^f) - \Omega(v^{N-1}, x_1^c(v^{N-1}))$, which is strictly negative by $v^{N-1} \geq \bar{v} > 0$ and by definition of x^c. Using continuity of $(W_2 - W_1')/q^{N-1}$ in δ and the restriction $v^{N-1} \geq \bar{v}$ thus obtains a second threshold $\bar{z}_2 > 0$ such that for $z < \bar{z}_2$ it is strictly less profitable to serve the market in two more periods in case all high-valuation consumers by in $N-1$. Choosing $\bar{z} = \min\{\bar{z}_1, \bar{z}_2\}$ proves the claim. **Q.E.D.**

Lemma 7. *The firm can not obtain higher profits from serving the residual market over two more periods $N-1$ and N if it applies a non-skimming policy.*

Proof. Suppose first that only low-valuation consumers buy in $N-1$. Denote the purchased quality by x_1^{N-1}. From Lemma 2 we know that the firm makes the commitment offer in the final period N. By the high-valuation consumers' incentive compatibility constraint this implies the requirement $\Lambda(x_1^{N-1}) \leq \delta\Lambda(x_1^c(v^N))$. By concavity of S_1 an upper boundary for the firm's payoff is obtained by choosing x_1^{N-1} such that $\Lambda(x_1^{N-1}) = \delta\Lambda(x_1^c(v^N))$ holds, implying also $x_1^{N-1} < x_1^c(v^N)$. This obtains the payoff

$$(q_1^{N-1} - q_1^N)S_1(x_1^{N-1}) + \delta\left[q_1^N S_1(x_1^c(v^N)) + q_2^{N-1}\left[S_2^f - \Lambda(x_1^c(v_N))\right]\right]. \tag{9}$$

It is now easy to see that (9) is strictly smaller than the payoff from making the commitment offer in $N-1$, which is given by $q^{N-1}\Pi(x_1^c(v^{N-1}), v^{N-1})$. To see this, note first that $S_1(x_1^{N-1}) < S_1(x_1^c(v^N))$, while the payoff in the last period exceeds $q^N S_1^f > 0$, which implies a loss from delay. Hence, (9) is strictly smaller than $q^{N-1}\Pi(x_1^c(v^N), v^{N-1})$, which by construction of x_1^c is strictly smaller than $q^{N-1}\Pi(x_1^c(v^{N-1}), v^{N-1})$.

It remains to discuss the non-skimming policy where consumers of either type buy in both periods. By the previous reasoning the firm's payoff is bounded from above by

$$\begin{aligned}&(q_1^{N-1} - q_1^N)S_1(x_1^{N-1}) + (q_2^{N-1} - q_2^N)\left[S_2^f - \Lambda(x_1^{N-1})\right] + \\ &\delta\left[q_1^N S_1(x_1^N) + q_2^N\left[S_2^f - \Lambda(x_1^c(v^N))\right]\right],\end{aligned} \tag{10}$$

where x_1^{N-1} is purchased by low-valuation consumers in $N-1$. We can further determine x_1^{N-1} by the requirement $\Lambda(x_1^{N-1}) = \delta\Lambda(x_1^c(v^N))$. Using $0 < v^N < 1$ and $0 < v^{N-1} < 1$ we next analyze how the firm's payoff changes if we shift more purchases into period $N-1$ or period N, while keeping the distribution in the final period v^N constant, which ensures that the bottom-range product is offered at the same quality $x_1^c(v^N)$ in the final period. From (10) it follows that decreasing the residual market in period N in this way strictly benefits the firm if it holds that

$$v^N S_2^f + (1 - v^N) S_1(x_1^{N-1}) > \delta \left[v^N S_2^f + (1 - v^N) S_1(x_1^N) \right]. \quad (11)$$

Hence, if (11) holds, the firm would be better off if the offer in $N - 1$ was accepted by all remaining consumers. In this case the firm would also realize more profits from the commitment offer in $N - 1$. Suppose next that (11) does not hold. We distinguish between three cases. First, for $v^N = v^{N-1}$ we can shift masses between $N - 1$ and N without affecting the distributions, implying that the firm's payoff does not exceed $\delta q^{N-1} \Pi(x_1^c(v^{N-1}), v^{N-1})$, which by $\delta < 1$ makes it optimal to serve the market in $N - 1$. Second, if $v^{N-1} < v^N$ holds, (10) does not exceed the payoff received under a non-skimming sequence where all remaining high-valuation consumers purchase in N. This case was already discussed above. Third, if $v^{N-1} > v^N$ holds, (10) is not larger than the payoff under a skimming sequence where the offer x_1^{N-1} is withdrawn, ensuring that low-valuation consumers do not accept in $N - 1$. This completes the proof. **Q.E.D.**

The assertion in Proposition 1 regarding low values of z follows now from combining Lemmas 6-7.

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