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Hans Gersbach

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Hans Gersbach, Universität Heidelberg and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

The Optimal Capital Structure of an Economy*

We examine the optimal allocation of equity and debt across banks and industrial firms when both are faced with incentive problems and firms borrow from banks. Increasing bank equity mitigates the bank-level moral hazard but may exacerbate the firm-level moral hazard due to the dilution of firm equity. Competition among banks does not result in a socially efficient level of equity. Imposing capital requirements on banks leads to the socially-optimal capital structure of the economy in the sense of maximizing aggregate output. Such capital regulation is second-best and must balance three costs: excessive risk-taking of banks, credit restrictions banks impose on firms with low equity, and credit restrictions due to high loan interest rates.

JEL Classification: D41, E40 and G20
Keywords: bank capital, banking regulation, capital structure of the economy, double incentive problems and financial intermediation

Hans Gersbach
Alfred-Weber-Institut
Universität Heidelberg
Grabengasse 14
69117 Heidelberg
GERMANY
Tel: (49 622) 1543 173
Fax: (49 622) 1543 578
Email: gersbach@uni-hd.de

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1 Introduction

Why should we be concerned with the level of bank capital? Why is regulatory intervention needed to ensure an optimal level of bank capital? These questions have occupied economists, regulators, and bank managers over the past decades.

In this paper, we provide a general equilibrium or macroeconomic perspective on the issues raised above. We highlight the costs and benefits of bank capital. Benefits arise when equity acts as a buffer against losses in the presence of macroeconomic risks or reduces excessive risk-taking of banks. Costs arise because banks compete with industrial firms for equity. Higher bank capital can reduce the amount of equity supplied to industrial firms, thereby increasing moral hazard problems and credit constraints within the industrial firms which the banks are supposed to reduce. This lowers aggregate income since credit-constrained industrial firms have higher marginal products than investments in frictionless production. The socially optimal capital structure of an economy, or to put it differently, the optimal debt/equity ratios for financial intermediaries *and* for industrial firms, balance the costs and benefits of bank capital and will therefore maximize aggregate output. The considerations in the paper indicate that the cost of bank capital corresponds to the marginal returns on equity of credit-constrained firms in an economy.

We also show that without regulatory intervention, banks will not and can not raise a socially efficient level of equity. An equilibrium with socially desirable equity levels cannot exist because banks are unable to refrain from risk-taking and offer sufficiently high equity returns simultaneously. Due to competition of other firms for scarce equity, banks need to offer sufficiently high returns to equity holders. If they attracted a sufficient amount of equity, banks would have no incentive to take excessive risks. However, this creates insufficient returns compared to equity channeled into credit-constrained firms. As a result, bank equity is lower than the socially optimal equity levels, which induces banks to gamble and enables them to offer sufficiently high equity returns in order to attract equity in the market.

Regulatory capital requirements can eliminate gambling incentives for banks and can induce the socially preferable capital structure. Although we can provide a general equilibrium rationale for regulatory capital requirements, such regulations cannot achieve a first-best allocation. Using regulatory capital requirements, banks are forced to hold a certain amount of equity. Thus, they must increase loan interest rates in order to generate returns that can attract equity to the extent required. Accordingly, capital requirements generate market power. Higher loan interest rates, however, reduce loan sizes for highly productive yet credit-constrained firms, which in turn lowers aggregate output. Therefore, capital adequacy rates must carefully balance three costs: gambling of banks, credit constraints on firms with low equity and credit constraints from high loan interest rates. In our model, the second-best capital requirement rule in the sense of maximizing aggregate output, prescribes an equity level that minimizes the remaining costs as long as gambling of banks is avoided.

Our model is consistent with the following observations: (i) banks absorb a significant share of equity in an economy. (Gorton and Winton (2000))[1] (ii) banks face substantial costs of issuing equity (Calomiris and Wilson (1998)), (iii) without capital requirements, banks are reluctant to freely raise additional capital (Blum and Hellwig (1995), Gorton and Winton (2000)) and (iv) bank loans and equity are the main source of funding of start-up firms (Petersen and Rajan (1994, 1995)) as we assume in this paper. While it is clear that our investigation is only a first step towards a complete understanding of the optimal capital structure for an economy,[2] the tradeoffs introduced in this paper are likely to be significant.

Our model tackles incentive problems at both the bank and firm level from a macroeconomic perspective. There is a large amount of literature on bank capital focusing on the incentive problem for banks alone, which is a building block in our model.[3] At the macroeconomic level, Blum and Hellwig (1995) have shown that strict capital adequacy rules may reinforce macroeconomic fluctuations. None of the existing work, however, addresses the optimal capital structure for an economy in the pres-

1 The market capitalisation of banks, for instance, amounts to 14 percent in the UK and is over 20 percent in Germany.

2 For instance, we leave out the co-existence of bank lending and bond financing developed by Besanko and Kanatas (1993), Hoshi, Kashyap, and Scharfstein (1993), Chemmanur and Fulghieri (1994), Boot and Thakor (1997), Holmström and Tirole (1997), and Repullo and Suarez (2000) and recently extended by Bolton and Freixas (2000) by introducing outside finance. In particular, these models combined with the model in this paper would further enhance our understanding of an optimal capital structure for an economy.

3 Thorough summaries and discussions of banking regulation can be found in Dewatripont and Tirole (1994), Freixas and Rochet (1997), and Bhattacharya, Boot and Thakor (1998).

ence of multiple incentive problems, as this paper hopes to achieve.

A recent paper by Covitz and Heitfield (2000) examines the overlapping moral hazard problems between borrowers and banks, and between banks and a government guarantor in a partial setting focusing on different issues. They establish that the relationship between market power and loan interest rates (or bank risk) depends on the relative strength of the underlying moral hazard problems. In our paper, we focus on the allocation of equity and debt across banks and firms in a macroeconomic setting to mitigate multiple incentive problems in an economy.[4]

We introduce an optimal capital structure for the economy and endogenize the cost of bank capital, which is equal to the return on equity in credit constrained firms. Two recent papers have provided alternative perspectives on bank capital. Gorton and Winton (2000) have provided an interesting endogenization of the cost of bank capital. In their model, higher bank capital reduces the aggregate amount of bank deposits forcing consumers to hold more information-sensitive bank equity which, however, is a poor liquidity hedge. Since our approach is complementary to Gorton and Winton, one might expect to find that the actual costs of bank capital are even higher than those suggested by both papers in isolation. Diamond and Rajan (2000) have developed another plausible theory of bank capital where higher bank capital reduces the bank's liquidity creation yet increases its chances of survival.

The paper is organized as follows. In the next section we introduce the model. In the third section, we characterize the first-best allocations. We examine financial intermediation without regulatory intervention in the fourth section. In section five, we examine regulatory intervention and the socially optimal capital structure. Section six presents our conclusions.

2 Model

We consider a simple two-period model with one physical good that can be used either for consumption or investment. Time is indexed by t ($t = 1, 2$). Agents live for both periods. The economy consists of a continuum of agents indexed by $[0, 1]$. There are two classes of agents. A fraction η of individuals consists of potential

4 In our context, regulatory capital requirements generate market power because banks must offer competitive equity returns.

entrepreneurs. The rest $1-\eta$ of the population are consumers. The parameter η captures the relationship between the supply of deposits and that of equity and will allow for comparative statics.

Potential entrepreneurs and consumers differ in that only the former have access to investment technologies. Moreover, a number of banks exist that gather equity capital from entrepreneurs and deposits from consumers in $t=1$. They invest their funds in a portfolio of production technologies (described below) and re-pay their claim-holders in $t=2$. Furthermore, each individual is endowed with one unit of labor when young ($t=1$) and none when old ($t=2$).

2.1 Endowment and Technologies

We assume that each individual will obtain an endowment $e>0$ of a physical good in the first period.[5] The economy has three production technologies that can convert time-1 goods into time-2 goods, which we shall describe in the following paragraphs.

Each entrepreneur has access to an investment project that converts time-1 goods into time-2 goods. The required funds for such a project are at least $M>e$. M is the minimal amount of capital needed to obtain output. Hence, an entrepreneur must borrow at least $M-e$ units of the good in order to undertake the investment project. If an entrepreneur obtains additional resources and is able to invest an amount $I \geq M$, he realizes investment returns in the next period amounting to $q_M I$ ($q_M > 1$). q_M is the indicator of the productivity of investment projects. This technology is called the *moral hazard technology*, since we assume that outsiders cannot verify whether or not an entrepreneur will invest. Potential financiers thus face a standard moral hazard problem since entrepreneurs may choose to consume the resources granted to them rather than invest. The non-verifiability of the investment decision is a standard scenario. Often, projects require specific human capital or may need the design of blueprints for machinery, buildings or logistics. Furthermore, an entrepreneur may require a lot of time to read in order to design the blueprints. Whether the efforts are directed towards the project or whether blueprints are competently drafted is unlikely to be observed by outsiders. Even if

5 Such endowments can be justified by a short-term constant-return technology that converts one unit of labor into e units of the physical good in period $t=1$. In this environment, each individual will obtain an endowment e when he supplies labor inelastically.

it becomes clear for financiers ex post whether the entrepreneur has invested or not, investment decisions are not verifiable in court.

The second possibility is a *gambling technology* (**GT**) in which banks can invest[6] and which also converts time-1 goods into time-2 goods. If an amount of G is invested in this technology, the output is given by $q_G zG$ where $q_G > 1$ and z is uniformly distributed over $[0, 1]$. The possibility of gambling will introduce moral hazard problems for financial intermediaries. Gambling technologies may represent investments in particular high-risk sectors in an economy or, in an international context, may be investments in emerging markets.

The third technology is that of a standard constant return technology that converts time-t goods into time-$t+1$ goods. The gross return per unit of investment is given by $q_F > 1$. We call this technology *frictionless technology* (**FT**). We assume that entrepreneurs and banks have access to the frictionless technology.

We summarize the assumptions concerning the different long-term production technologies as follows:

Assumption 1

$q_G > q_M > q_F > q_G/2$

Thus, gambling promises the highest return on investment provided the shock turns out to be favorable. The moral hazard technology is better than the frictionless technology, while the latter dominates the expected return of the gambling technology. It is obvious from our assumption (1), that the gambling technology should never be used in a first-best world. The moral hazard technology and the frictionless technology can absorb an unlimited amount of resources and yield higher expected returns. The constant return assumption of the technologies is solely made for tractability.[7] Note that assumption 1 implies that entrepreneurs will never invest in the frictionless technology.

6 In addition, we might assume that entrepreneurs can use the gambling technology as well, which will tend to increase the cost of bank capital.

7 Interesting extensions of our model concern decreasing returns of the moral hazard and frictionless technology such that in a first-best world all technologies are used. This would allow for an additional kind of credit crunch when bank lending rates rise as a consequence of higher capital requirements.

2.2 Agents

For simplicity, we assume that potential entrepreneurs are risk-neutral and are only concerned with consumption in their old age, i.e. they do not consume in their youth. Consumers consume in both periods. They have utility functions $u(c^1, c^2)$ defined over consumption in two periods, where c^1 and c^2 ($c^1 \geq 0, c^2 \geq 0$) represent the consumption of the consumer when young and old, respectively. For simplicity, we assume that consumers are extremely risk-averse. Therefore only the lowest possible consumption in the second period enters utility. That is, the utility of consumers can be written as $u(c^1, \min\{c^2\})$ when there is uncertainty as to the second-period consumption level. There is never uncertainty about consumption in the first period. Additionally, consumers wish to spread consumption over their lifetime. If a consumer can transfer wealth with certainty between the two periods at a real interest rate, denoted by r ($r > 1$), the solution of the household problem generates the saving function, denoted by $S(r)$ ($S(r) \geq 0$). We follow the standard assumptions in the overlapping generation literature. Namely, that the substitution effect (weakly) dominates the income effect, i.e. savings are a weakly increasing function of the interest rate.[8]

2.3 Financial Intermediation

In order to alleviate the moral hazard problem faced by the entrepreneurs, financial intermediators can act as delegated monitors as introduced by Diamond (1984). This monitoring function justifies their existence. Moreover, if $M - e > e$, intermediaries may be motivated by the fact that more than one investor is required to fund a project.

We assume that there are n banks, indexed by i, that finance entrepreneurs and act as delegated monitors. In all of our arguments, it will be sufficient for two banks to exist and compete.

As delegated monitors, banks are assumed to have access to a monitoring technology. If they have granted a loan with face value l ($l \geq 0$) to an entrepreneur who does not

8 Note that the stark differences between entrepreneurs and consumers are mainly made for tractability. Other useful extensions of the model could allow for portfolio decisions of entrepreneurs with respect to deposits and equity as well as for equity investments of depositors.

invest, this technology secures a repayment of βl ($0 \leq \beta \leq 1$) upon payment of a resource cost. If banks do not monitor entrepreneurs, we assume that the repayment from non-investing entrepreneurs will be zero since entrepreneurs simply consume the funds. Monitoring may take many forms. For instance, banks can collateralize parts of the credit or may release the funds sequentially to the entrepreneur, depending on his investment behavior. Efforts such as these can reduce the private benefits of entrepreneurs choosing not to invest. Hence, a non-investing entrepreneur with a loan of face value l only obtains $(1 - \beta)l$ if the bank monitors. We assume that monitoring costs are sufficiently small, such that banks will always decide to monitor debtors when granting loans. For simplicity of exposition, we shall neglect the monitoring outlays in the following discussion. Moreover, our results do not depend on whether β is positive. Even if banks simply collect funds from many consumers to fund few entrepreneurs, the results still apply by setting $\beta = 0$. Finally, we assume that entrepreneurs can only obtain a loan from one bank[9] and that entrepreneurs, when indifferent to the choice between investing and consuming funds, will choose to invest.

When an entrepreneur has invested, we assume that a verification of output conditional on investment is possible. The assumption is most easily justified if the final products of an entrepreneur's project are physical goods such as houses or machines, so that lenders can secure repayment conditional on investment at a very low cost.[10]

We now discuss the strategic variables that a bank can use in $t = 1$. In order to fund investments, the bank offers deposit contracts to consumers at deposit rates denoted by d_i ($d_i \geq 0$). Due to the extreme risk-aversion of consumers, we assume that they choose only to invest in deposits.[11] Banks then receive deposits denoted by D_i ($i = 1, .., n$) ($D_i \geq 0$). Thus, they promise depositors a repayment $d_i D_i$. The second source of funding is equity contracts offered by potential entrepreneurs. Using such contracts, the holder will obtain the right to participate in the dividend

9 Implicitly, we assume that the monitoring activities of banks include checking whether an entrepreneur applies for loans at other banks.

10 Note that it is possible to detect whether or not the entrepreneur has invested the funds, but the absence of output usually cannot be used as evidence for verifying investment decisions. Alternatively, one can also start from a small amount of uncertainty of investment returns and that the entrepreneur cannot be forced to pay anything back without costly state verification.

11 However, with appropriate capital regulation equity contracts are no longer risky and in such circumstances consumers may want to hold equity as well. Adding arbitrarily small idiosyncratic risk would, however, again induce consumers to invest only in deposits.

payments in the next period depending on his share in the overall equity issued by the bank. We use E_i ($E_i \geq 0$) to denote the amount of equity capital that bank i receives in $t = 1$. Equity holders are the owners and thus residual claimants of banks and will therefore wish to maximize the return on equity. In practice, this task is delegated to managers, who, we assume, act in the interest of shareholders.

Finally, we outline the investment opportunities of banks.

First, banks can offer loan contracts (l_i, R_i^l) to entrepreneurs where l_i ($l_i \geq 0$) denotes the size of the loan made to a single entrepreneur and R_i^l ($R_i^l \geq 1$) the required return.[12] The fraction of entrepreneurs applying for a loan from bank i is denoted by λ_i ($0 \leq \lambda_i \leq 1$). Hence, the bank grants loans amounting to $L_i = \lambda_i \eta l_i$.

Second, the bank can invest an amount of F_i in FT and an amount of G_i in GT. Thus the overall budget restriction of each bank in $t = 1$ is given by

$$E_i + D_i = L_i + F_i + G_i \quad (i = 1, ..., n).$$

To complete our description of the model we assume

Assumption 2

$(1 - \eta)S(q_F) > \eta(M - e)$

Assumption 2 indicates that funds from entrepreneurs and consumers suffice to provide capital for all projects of the moral hazard technology to operate at minimal scale provided consumers earn the return rate q_F. If assumption 2 is violated, only a subset of entrepreneurs can be funded, which complicates the analysis without providing further insights.

3 First-Best Allocation

We first examine the first-best allocation when no incentive problems are present. We assume that there exists a perfect capital market in which consumers can directly invest in different types of technologies. In particular, consumers can offer resources to entrepreneurs who, in turn, can contract upon their investment decisions and can

12 Note that since entrepreneurs are identical, we can w.l.o.g. assume that loan sizes and the required rates of return do not differ for individual entrepreneurs.

credibly promise repayments. We obtain:

Proposition 1

If assumption 1 and 2 hold, the first-best allocation is characterized by

(i) No resources flowing into the gambling or into the frictionless technology.

(ii) One (risk-less) equilibrium return on savings given by $\bar{R}^{FB} = q_M$.

(iii) All consumers saving $S(q_M)$ *and each entrepreneur investing an amount of*

$$e + \frac{1-\eta}{\eta} S(q_M).$$

(iv) An aggregate second-period output Y^{FB} *given by*

$$Y^{FB} = q_M \Big[\eta e + (1-\eta) S(q_M) \Big].$$

Proposition 1 follows immediately from the description of the model. Returns on savings must be equal to the marginal returns of capital q_M. With such savings returns, consumers offer $S(q_M)$ in terms of savings, whereas entrepreneurs use their own funds and borrowings to run their projects. An important characteristic of the first-best allocation is that neither the frictionless technology nor the gambling technology are funded in equilibrium. This is due to the fact that the moral hazard technology dominates the other production possibilities in terms of marginal and average returns. Note that in a first-best world, the capital structure is irrelevant because there is no difference between equity and debt contracts as investors face no risk.

4 Intermediation

In this section, we examine an economy with the incentive problems described in section 2. We first outline the complete sequence of events in the intermediation game.

Period 1

1. Banks offer equity contracts and entrepreneurs decide how much to invest in bank equity.

2. Banks offer loan contracts (l_i, R_i^l) to a fraction of entrepreneurs.

3. Banks offer deposit contracts (d_i) to consumers.

4. Consumers and entrepreneurs decide which contracts (equity or deposits) to accept. Resources are exchanged. Bank i receives a measure D_i of deposits and receives a measure E_i of equity capital.

5. Entrepreneurs decide which loan contract to accept. Each bank grants the loans to each entrepreneur who has accepted. The overall size of loans granted by bank i is denoted by L_i.

6. Banks decide how much they want to invest in the gambling and frictionless technologies G_i and F_i. Entrepreneurs decide whether to invest.

Period 2

7. Entrepreneurs pay off the loans. Returns from the gambling and frictionless technology are realized. Banks pay back depositors. The remaining dividends are distributed among shareholders.

Note that banks cannot credibly commit to their investment decisions when they offer contracts. The risk associated with deposits depends on the amount of gambling investments and whether or not entrepreneurs re-pay the debt. To solve for the overall equilibrium, we shall proceed in steps, as this helps to simplify the presentation of the arguments. In each step some of the variables are kept constant. At the end of the exercise, we shall show that the solution obtained by this procedure is indeed a subgame-perfect equilibrium of the whole game. Throughout the paper, it is assumed that entrepreneurs and consumers follow symmetric strategies, e.g. all entrepreneurs obtain the same loan contracts and offer the same equity contracts. Moreover, we only investigate symmetric equilibria with respect to the banks' strategy choices and we omit the index for banks whenever there is no resulting confusion.

Note that the sequence of events implies that banks offer loan contracts before deposit contracts. This both simplifies the examination of possible deviations from equilibria and is important for the existence of equilibria.

4.1 Loan Contracts

We first discuss the contracting problem between banks and entrepreneurs after entrepreneurs have given the banks equity of $e^+ := \sum_{i=1}^{n} e_i$ and the banks have received deposits. Hence, the remaining equity resources of an entrepreneur for his own investment projects are given by $e^- := e - e^+$. For the examination of loan contracts, we assume that banks have a centralized information system that guarantees that the entrepreneur can only have one loan contract.[13] Given a loan contract (l_i, R_i^l) offered by a bank, the entrepreneur can either invest and obtains

$$q_M e^- + (q_M - R_i^l) l_i$$

or he can simply consume his funds and obtain

$$e^- + (1-\beta) l_i$$

since the bank can recover βl_i as the collateral. Hence, entrepreneurs will decide to invest if and only if

$$l_i \leq l^*(R^l, e^-) := \frac{e^-(q_M - 1)}{(1-\beta) - (q_M - R^l)}. \tag{1}$$

Note that $l^*(R^e, e^-)$ is monotonically decreasing in R^l. In order for entrepreneurs to continue investing at a higher loan rate, a smaller loan size is required. As we shall see, capital requirements raise R^e and thus impose tighter credit restrictions on entrepreneurs. Proceeding, we introduce the following assumption, which we assume to hold throughout the paper:

Assumption 3

$1 - \beta > q_M - q_F$

Assumption 3 guarantees that $l^*(R^l, e^-)$ is not negative. Note that according to assumption 3 the constraint (1) is binding for all loan rates $R_i^l \geq q_F$ since $1 - \beta - (q_M - R^l) > 0$. Assumption 3 also implies that for fixed e^-, the functions $l^*(R^l, e^-)$

13 Our analysis also holds mutatis mutandis, if the entrepreneur can obtain loan offers from all banks. In this case, each bank will only offer $1/n$ of the credit volume such that the entrepreneur is still motivated to invest. Note also that banks anticipate that $R^e \geq q_M$ would lead entrepreneurs to consume all borrowed sums and not invest at all. Hence, banks would never offer $R^e \geq q_M$.

as well as $R^l l^*(R^l, e^-)$ are monotonically decreasing in R^l. In the following, for ease of presentation, we assume a tie-breaking rule: If banks are indifferent between offering loans or investments in the frictionless technology they choose the former. We obtain:

Proposition 2

Suppose that, in an equilibrium, banks invest a positive amount in the frictionless technology and that the entrepreneurs' inside capital is given by e^-. *We define* $l^*(e^-) := l^*(q_F, e^-)$ *and obtain:*

(i) If $e^- + l^*(e^-) < M$, *then no loans are granted.*

(ii) If $e^- + l^*(e^-) \geq M$, *then there exists a unique equilibrium* $R_i^l = q_F$ *and* $l_i = l^*(e^-)$ $(i = 1, .., n)$.

The proof is given in the appendix. In proposition 2, two cases can occur. In the first case, the maximal loan size that can guarantee a repayment at rateq_F, together with the entrepreneur's equity cannot cover the minimal project size. In the second case, equity and the maximal loan at rate q_F can cover M. Hence, banks offer exactly $l^*(e^-)$ and expect a sure repayment. Proposition 2 implies that banks offer loan contracts at rate q_F provided the equity of entrepreneurs is sufficient to motivate them to invest and to pay back. In this situation, equilibrium interest rates on loans are equal to the refinancing costs and thus intermediation margins are zero.

4.2 Gambling and Frictionless Investments

We now discuss the remaining investment decisions of banks. They are derived using the assumption that banks only grant loans if they (correctly) anticipate that entrepreneurs will invest and honor their repayment promises. Positive profits for the bank then only occur if

$$\Pi = q_G z G + q_F(E + D - G - L) + R^l L - dD \geq 0$$

or, equivalently if

$$z \geq \frac{1}{q_G}\left(q_F - \frac{H}{G}\right) := \underline{z}(G) \tag{2}$$

where

$$H := q_F(E + D - L) + R^l L - dD. \tag{3}$$

Note that we assume that banks maximize expected returns on equity and thus the profits are the dividends shareholders receive. Due to limited liability, only positive realizations need to be considered. Since equity is given at the investment stage we can directly work with the absolute amount of dividends accruing to shareholders as the object that banks maximize. We need to differentiate between two cases: First, $\underline{z}(G) \leq 0$. In this case, expected profits are given by

$$\pi^{(1)}(G) = \int_0^1 \Pi \, dz = \left(\frac{1}{2}q_G - q_F\right) G + H.$$

Whereas, in the other case, they amount to

$$\pi^{(2)}(G) = \int_{\underline{z}(G)}^1 \Pi \, dz = \frac{1}{2} q_G G \left(1 - (\underline{z}(G))^2\right) + \left(1 - \underline{z}(G)\right)(H - q_F G). \tag{4}$$

In order to derive the optimal investment G in the gambling technology, we observe that $\pi^{(1)}(G)$ monotonically decreases in G, because $\frac{1}{2}q_G < q_F$. Therefore, in the case of $\pi^{(1)}(G)$, the optimal choice is $G = 0$. In the case of $\pi^{(2)}(G)$, we take the derivatives of the profit function using equation (2), equation (3) and equation (4):

$$\begin{aligned}
\frac{d\pi^{(2)}(G)}{dG} &= \frac{d}{dG}\left(\frac{1}{2}q_G G\left(1 - \frac{1}{q_G^2}\left(q_F - \frac{H}{G}\right)^2\right) + \left(1 - \frac{1}{q_G}\left(q_F - \frac{H}{G}\right)\right)(H - q_F G)\right) \\
&= \frac{d}{dG}\left(\frac{1}{2}q_G G + H - q_F G + \frac{G}{2q_G}\left(q_F - \frac{H}{G}\right)^2\right) \\
&= \frac{1}{2}q_G - q_F + \frac{1}{2q_G}\left(q_F - \frac{H}{G}\right)^2 + \frac{1}{q_G}\left(q_F - \frac{H}{G}\right)\frac{H}{G} \\
&= \frac{1}{2}q_G - q_F + \frac{q_F^2}{2q_G} - \frac{q_F}{q_G}\frac{H}{G} + \frac{1}{2q_G}\frac{H^2}{G^2} + \frac{q_F}{q_G}\frac{H}{G} - \frac{1}{q_G}\frac{H^2}{G^2} \\
&= \frac{1}{2q_G G^2}\left((q_G - q_F)^2 G^2 - H^2\right)
\end{aligned}$$

Since

$$\frac{d^2\pi^{(2)}(G)}{dG^2} = \frac{H^2}{q_G G^3} \tag{5}$$

is negative only for negative G, the maximum of $\pi^{(2)}(G)$ is given by choosing $G = E + D - L$ (complete gambling) or $G = 0$ (no gambling). Proceeding, we introduce the following assumption, which we assume to hold throughout the paper:

Assumption 4

$$\frac{1-\eta}{\eta}\frac{S(q_F)}{e} > \frac{q_M - 1}{1-\beta-(q_M-q_F)}$$

Assumption 4 implies that loans L are smaller than deposits D for any two possible interest rates d and R^l. Since $R^l \geq d$, in any potential equilibrium, it follows that $R^l L - dD < 0$. This implies that $\underline{z}(G)|_{G=E+D-L} > 0$ in the gambling case. In the non-gambling case we have $\lim_{G\to 0} \underline{z}(G) = -\infty$. Thus, profits in the non-gambling and in the gambling case are given by:

$$\begin{aligned}
\pi_F &= \pi^{(1)}(G)|_{G=0} \\
&= H \\
&= q_F(E+D-L) + R^l L - dD \\
\pi_G &= \pi^{(2)}(G)|_{G=E+D-L} \\
&= \frac{1}{2}q_G(E+D-L)\left(1-\left(\frac{R^l L - dD}{q_G(E+D-L)}\right)^2\right) \\
&\qquad + \left(1+\frac{R^l L - dD}{q_G(E+D-L)}\right)(R^l L - dD) \\
&= \frac{1}{2}q_G(E+D-L)\left(1+\frac{R^l L - dD}{q_G(E+D-L)}\right)^2
\end{aligned}$$

We summarize these observations in the following proposition:

Proposition 3

Suppose that assumptions 1 to 4 hold and a bank has received equity E, deposits D at deposit rate d, and has granted loans L at loan interest rate R^l. Then the bank

(i) does not gamble ($G = 0$) if $\pi_F \geq \pi_G$, i.e.

$$q_F \geq \frac{1}{2}q_G\left(1+\left(\frac{R^l L - dD}{q_G(E+D-L)}\right)^2\right) \tag{6}$$

(ii) gambles completely ($G = E + D - L$) if $\pi_F < \pi_G$, i.e.

$$q_F < \frac{1}{2}q_G\left(1+\left(\frac{R^l L - dD}{q_G(E+D-L)}\right)^2\right) \tag{7}$$

By solving (7) for E and recalling that due to assumption 4, $R^l L - dD < 0$ holds, we obtain the following corollary:

Corollary 1

There exists a unique value of E, E^*, *such that banks gamble completely if and only if*

$$E \leq E^* := \frac{-(R^l L - dD)}{\sqrt{q_G(2q_F - q_G)}} - (D - L) \tag{8}$$

Corollary 1 implies that banks gamble if the level of equity is small and below a critical level. For $q_G(2q_F - q_G) \leq 1$, the critical equity level is *ceteris paribus* increasing in D and decreasing in L. At a given level of E, a greater amount of deposits leads to a higher incentive to gamble. This in turn requires E to be at a higher threshold in order to cease gambling. The more funds are invested in loans, the smaller the incentive to gamble.

In order to derive the overall equilibrium, we proceed sequentially. In the next section, we assume that banks offer deposit contracts where $d_i = q_F$. Whether or not $d_i = q_F$ holds in equilibrium will be discussed when we establish the overall equilibrium.

4.3 Deposit and Equity Contracts

We now derive the amount of equity that banks obtain in equilibrium. We first calculate marginal returns on equity. We use $R^{eb}(E)$ to denote the marginal return on equity invested in a bank. Similarly, we use $R^{ef}(e^-)$ to denote the marginal return on equity invested in the entrepreneur's project. Let us first calculate $R^{ef}(e^-)$. The payoff for entrepreneurs from investing in the project in equilibrium is denoted by g and given by

$$g = q_M(e^- + l^*(R^e, e^-)) - l^*(R^e, e^-)R^l \tag{9}$$

If an entrepreneur increases the amount of equity allocated to his project, he could obtain a larger loan. Inserting (9) into (1) and differentiating yields the equilibrium returns on equity in firms R^{ef}:

$$\begin{aligned}
R^{ef}(e^-) &= \frac{\partial g}{\partial e^-} \\
&= \frac{\partial}{\partial e^-}\left(q_M(e^- + l^*(R^e, e^-)) - l^*(R^e, e^-)R^l\right) \\
&= \frac{\partial}{\partial e^-}\left(q_M\left(e^- + \frac{e^-(q_M-1)}{(1-\beta)-(q_M-R^l)}\right) - \frac{e^-(q_M-1)}{(1-\beta)-(q_M-R^l)}R^l\right) \\
&= q_M + \frac{q_M-1}{(1-\beta)-(q_M-R^l)}(q_M-R^l) \\
&= \frac{R^l-\beta q_M}{1-\beta-(q_M-R^l)} > q_M
\end{aligned} \tag{10}$$

Therefore, return on equity is larger[14] than q_M since entrepreneurs can enlarge their debt capacity by reserving more equity for themselves. In equilibrium, the return on equity offered by banks must be at least equal to R^{ef}, otherwise entrepreneurs would not be willing to accept bank equity contracts. The payoff from investing in bank equity for entrepreneurs is given by:

$$b_k = \frac{\pi_k}{E}e^+ \tag{11}$$

k is an indicator variable which represents either the gambling or the non-gambling case ($k = G, F$). $\frac{\pi^k}{E}$ does not depend on e^+, as a single entrepreneur can only marginally change the total amount of equity a bank receives. In order to compare the return on equity in firms and banks, we assume for the moment that$R^l = d = q_F$. Accordingly, the return on bank equity is given as:

- non-gambling case:

$$R^{eb} = \frac{\partial b_F}{\partial e^+} = q_F \tag{12}$$

- gambling case:

$$R^{eb} = \frac{\partial b_G}{\partial e^+} = \frac{\pi^G}{E} \tag{13}$$

Proposition 4

Assume assumptions 1 to 4 hold. Then, there is gambling in any subgame-perfect equilibrium with $R^l = d = q_F$.

14 Note that for $R^l \geq q_F$ and from $1-\beta > q_M - R^l$ it directly follows that $R^l > \beta q_M$.

Proof of proposition 4

Suppose there exists an equilibrium without gambling. According to the non-gambling case, $R^{ef} > q_M$ and $R^{eb} = q_F$ which, in turn, implies $E = 0$. From (8), $D > L$ and $q_G > q_F$, it follows that $E^* > 0$ and thus $E < E^*$ holds. According to corollary 1 the bank gambles completely which is a contradiction. Therefore, no equilibrium without gambling can exist.

□

Proposition 4 has important implications. If banks attracted a sufficient amount of equity, they would have no incentive to gamble as this would yield a low return on equity. This, in turn, makes it impossible for banks to attract a high level of equity from the beginning. Since banks cannot simultaneously refrain from gambling and offer sufficiently high equity returns, no subgame-perfect equilibrium with non-gambling can exist.

We now examine the gambling case. If $R^{ef} = R^{eb}$ in the gambling case, investing in a project or investing in equity contracts is equally attractive to the entrepreneur. Thus, we can calculate the level of equity at the bank.

From $R^{ef} = R^{eb}$ and thus $R^{ef}E = \pi^G$ we obtain for $R^l = d = q_F$:

$$ER^{ef} = \frac{1}{2} q_G \left(E + D - L\right) \left(1 + \frac{q_F(L-D)}{q_G(E+D-L)}\right)^2$$

After rearranging terms and adding $2q_G R^{ef}(D-L)(E+D-L)$ on both sides, we obtain:

$$\begin{aligned} 2R^{ef} q_G (E+D-L)^2 &= \left(q_G(E+D-L) - q_F(D-L)\right)^2 \\ &+ 2q_G R^{ef}(D-L)(E+D-L) \end{aligned}$$

Setting $X = E + D - L$ yields:

$$X^2 q_G(2R^{ef} - q_G) - 2Xq_G(R^{ef} - q_F)(D-L) - q_F^2(D-L)^2 = 0$$

Since $R^{ef} > q_F > 0$ and $D \geq L$, the quadratic equation has a positive solution only if $2R^{ef} > q_G$. If $2R^{ef} > q_G$, there exist two real solutions, one negative and one positive. The positive root yields the solution:

$$X = \frac{D-L}{2R^{ef}-q_G}\left(R^{ef}-q_F+\sqrt{R^{ef}\left(R^{ef}-2q_F\left(1-\frac{q_F}{q_G}\right)\right)}\right)$$

or

$$E^{*G} = \frac{D-L}{2R^{ef}-q_G}\left(q_G-q_F-R^{ef}+\sqrt{R^{ef}\left(R^{ef}-2q_F\left(1-\frac{q_F}{q_G}\right)\right)}\right) \tag{14}$$

We obtain:

Proposition 5

If assumptions 1 to 4 hold, then:

$$E^{*G} > 0$$

The proof of proposition 5 is given in the appendix. Proposition 5 shows that the level of equity banks attract in the gambling case is indeed positive. Equation (14) shows how the level of bank equity E^{*G} in the gambling case is related to the amount of deposit and loans and to the return on firm equity (R^{ef}), which itself depends on the loan interest rate R^l.

4.4 Overall Equilibrium

We are now ready to characterize the overall subgame-perfect equilibrium. Before we turn to the equilibrium, we need to address the issue of bank defaults. We shall proceed in two steps. In the first step and in the following proposition, we assume that deposits are insured up to the maximal return q_F. If a bank fails to pay back depositors, depositors will be protected by government transfers, which are assumed to be raised by lump sum taxation in period 2. Therefore, deposits are safe. Moreover, lump sum taxation in period 2 will have no distortionary effects on the behavior in period 1. In the second step, we discuss the results when there is no deposit insurance. If deposits are insured we obtain

Proposition 6

If assumptions 1 to 4 hold and deposits are insured, then a subgame-perfect equilibrium exists:

$$\begin{aligned} R^l &= d = q_F \\ D &= \tfrac{1-\eta}{n} S(q_F) \\ E &= E^{*G} \text{ (as defined by equation (14))} \\ L &= \tfrac{\eta}{n} l^*(q_F, e^-) \text{ where } e^- = e - \tfrac{n}{\eta} E^{*G} \\ G &= E + D - L \\ F &= 0 \end{aligned}$$

Proof of proposition 6

Let us first consider the equilibrium choice $d = q_F$. A deviation $d_i < q_F$ would leave a bank without deposits. A deviation $d_i > q_F$ cannot credibly promise higher returns on deposits since returns on loans are q_F and the expected returns on gambling are smaller than q_F. A deviation to higher loan rates, $R_i^l > q_F$, would not attract any entrepreneurs and therefore is also not profitable. From proposition 2, we must have $R^l = d$ and therefore $R^l = d = q_F$. Thus, D is given by $\frac{1-\eta}{n} S(q_F)$. Proposition 4 implies that an equilibrium can only exist if the bank gambles completely.

□

Proposition 6 indicates that, with certain parameter restrictions, an equilibrium with complete gambling, loan contracts and zero intermediation margins exists. It is obvious that the allocation is inefficient since gambling is undesirable. We will show that stipulating a larger bank capital requirement can improve welfare, but at the same time moral hazard problems at firms will be less mitigated.

We now discuss the case when deposits are not insured. Proposition 6 requires an important adjustment. Given that banks gamble and depositors are assumed to be extremely risk averse, the savings decisions of depositors must be based on the lowest return yielded by the gambling investment, i.e. $z = 0$. If we assume that the liquidation value is distributed evenly among the depositors in the case of a bank default, the saving decisions of depositors are based on the relationship between $R^l L$ and D. Therefore, $S = S\left(\frac{R^l L}{D}\right)$ will enter into the equilibrium considerations. This additional endogeneity leads to the same qualitative features of the equilibrium. However, without a specific savings function S, the equilibrium can no longer be solved explicitly.

5 Optimal Capital Structure

In the previous section, we saw that although banks obtain equity, a general equilibrium implies gambling and inefficient allocation. In this section, we include a regulation which forces banks to adopt a particular capital structure. A key consideration is that banks can only fulfill capital requirements in the non-gambling case if they make sufficiently high profits. This means they must set R^l higher than deposit rates in order to attract equity and be allowed to operate. Thus, a capital requirement generates market power. Moreover, a regulator must set the amount of equity E for banks sufficiently high in order that banks are able to offer loan contracts with a return $R^l \in (q_F, q_M)$[15] such that R^{eb} equals R^{ef} in the non-gambling case. We use E^R to denote the equity level imposed by regulatory requirements. The return equalization in the equity market yields

$$R^{ef} = R^{eb}$$
$$\frac{R^l - \beta q_M}{1 - \beta - q_M + R^l} = \frac{q_F(E + D - L) + R^l L - dD}{E}$$

where we have applied (10), $R^{eb} = \frac{\pi}{E}$, and $\pi = q_F(E + D - L) + R^l L - dD$. Using $d = q_F$ and $L = \frac{\eta}{n}\left(e - \frac{n}{\eta}E\right)\frac{q_M - 1}{1 - \beta - q_M + R^l}$ and solving for E yields:

$$E^R(R^l) = \left(\frac{e\eta}{n}\right)\left(\frac{q_M - 1}{q_M - q_F}\right)\left(\frac{R^l - q_F}{R^l - \beta}\right) \tag{15}$$

Note that $E^R(R^l)$ monotonically increases and is concave for $R^l \in (q_F, q_M)$ with $E^R(q_F) = 0$ and $E^R(q_M) = \left(\frac{e\eta}{n}\right)\left(\frac{q_M - 1}{q_M - \beta}\right)$. On the other hand, equity E for banks must exceed E^* in order to encourage them to refrain from gambling. In order to discuss $E^* = E(L(E, R^l), R^l)$, we require some further evaluations of E^*. Recall from equation (8) that E^* is given by:

$$E^* = \frac{-R^l L + dD}{\sqrt{q_G(2q_F - q_G)}} - (D - L)$$

15 Since the return on an entrepreneur's project is q_M, he would not accept loan contracts with $R^l \geq q_M$.

Inserting $L = \frac{\eta}{n}\left(e - \frac{n}{\eta}E^*\right)\frac{q_M-1}{1-\beta-q_M+R^l}$, $d = q_F$ and using the abbreviations $r_M = q_M - 1$ and $\tilde{r} = \sqrt{q_G(2q_F - q_G)}$, we obtain:

$$E^* = \frac{-R^l\left(\frac{\eta}{n}e - E^*\right)\frac{r_M}{R^l-\beta-r_M} + q_F D}{\tilde{r}} - D + \left(\frac{\eta}{n}e - E^*\right)\frac{r_M}{R^l - \beta - r_M}$$

Inserting $D = \frac{1-\eta}{n}S(q_F)$, $E^*(R^l)$ is then given as:

$$E^*(R^l) = \frac{1}{n}\frac{(q_F - \tilde{r})(1-\eta)S(q_F) - \frac{r_M(R^l-\tilde{r})}{R^l-r_M-\beta}\eta e}{\tilde{r} - \frac{r_M(R^l-\tilde{r})}{R^l-r_M-\beta}} \tag{16}$$

5.1 Optimal Capital Structure with $\beta = 0$

We now compute the level of R^l that equalizes $E^*(R^l)$ and $E^R(R^l)$. For simplicity, we assume $\beta = 0$, i.e. the recovery rate to be zero. We shall later generalize our results for $\beta \neq 0$. $E^*(R^l) = E^R(R^l)$ then yields:

$$\begin{aligned}
\left(\frac{r_M}{r_M - r_F}\right)\left(\frac{R^l - 1 - r_F}{R^l}\right) &= \frac{s - \frac{r_M(R^l-\tilde{r})}{R^l-r_M}}{\tilde{r} - \frac{r_M(R^l-\tilde{r})}{R^l-r_M}} \\
\left(\frac{r_M}{r_M - r_F}\right)\left(R^l - 1 - r_F\right) &= \frac{s(R^l - r_M) - r_M(R^l - \tilde{r})}{\tilde{r} - r_M}
\end{aligned}$$

where we have used the definition $s := (q_F - \tilde{r})\frac{1-\eta}{\eta}\frac{S(q_F)}{e}$ and $r_F := q_F - 1$. This equation is linear in R^l. Solving for R^l yields:

$$\hat{R}^l := r_M\left(1 + \frac{r_M - \tilde{r}}{r_M(r_F - \tilde{r}) + s(r_M - r_F)}\right)$$

Inserting this expression for $\hat{R}^l$ into (15) yields the equity level:

$$E^R(\hat{R}^l) = \frac{s(r_M - r_F) + r_M(1 + r_F - \tilde{r}) - s}{s(r_M - r_F) + r_M(1 + r_F - \tilde{r}) - \tilde{r}}\left(\frac{e\eta}{n}\right)$$

It remains to be shown whether $E^*(R^l)$ and $E^R(R^l)$ are equal for $R^l \in [q_F; q_M]$. Note that $E^R(R^l = q_F) = 0$ and $E^R(R^l = q_M) = \frac{e\eta}{n}\frac{r_M}{1+r_M}$. From assumption 4, we obtain that $E^*(R^l = q_F) > 0$. $E^*(R^l = q_M)$ is given by:

$$E^*(R^l = q_M) = \frac{e\eta}{n}\frac{s - r_M(q_M - \tilde{r})}{q_M(\tilde{r} - r_M)}$$

The following condition guarantees that $E^*(R^l = q_M) < E^R(R^l = q_M)$:

$$\frac{s - r_M(q_M - \tilde{r})}{\tilde{r} - r_M} < r_M$$

By multiplying with $(\tilde{r} - r_M)^2$ and rearranging terms, this can be shown to be equivalent to:

$$(s - r_M)(\tilde{r} - r_M) < 0$$

Proceeding, we introduce the following assumption which we assume to hold throughout the paper:

Assumption 5

$(s - r_M)(\tilde{r} - r_M) < 0$

If assumption 5 holds, we have a unique solution for $R^l \in [q_F, q_M]$ which equalizes $E^*(R^l)$ and $E^R(R^l)$. We obtain:

Proposition 7

If assumptions 1 to 5 hold and $\beta = 0$, then a unique regulatory equity level $\hat{E}^R$, a loan rate $\hat{R}^l \in [q_F, q_M]$ and an associated unique subgame-perfect equilibrium exists with:

$$\begin{aligned}
R^l &= \hat{R}^l \\
D &= \tfrac{1-\eta}{n}\, S(q_F) \\
E &= \hat{E}^R \\
L &= \tfrac{\eta}{n}\, l^*(q_F, \hat{e}^-) \text{ where } e^- = e - \tfrac{n}{\eta}\, \hat{E}^R \\
G &= 0 \\
F &= E + D - L
\end{aligned}$$

The proof of proposition 7 follows the same reasoning as the earlier propositions with one important variation. Banks have no incentive to undercut the loan interest rate $\hat{R}^l$, since their loan capacity is constrained by the capital requirement.

Proposition 7 shows that the regulator can stipulate an equity level such that banks choose to cease gambling. However, since the solution R^l is in the interval $[q_F, q_M]$ and $E^*(R^l = q_F) \neq E^R(R^l = q_F)$, R^l will exceed q_F and thus the moral hazard problem becomes more acute at the firm level. The intuition and the consequences of proposition 7 are straightforward. In order to attract the required equity, banks must be able to offer returns as high as R^{ef}, which requires larger profit margins $R^l - q_F$. As a consequence, since l^* monotonically decreases in R^l, loan sizes shrink and thus the moral hazard problem is more acute in the sense that aggregate investment in the most profitable (moral hazard) technology declines.

5.2 Optimal Capital Structure with $\beta \geq 0$

Finally, we generalize our results for $\beta \neq 0$. In this case, however, explicit solutions are very tedious to obtain. Nevertheless, the main result holds and in the appendix we show:

Proposition 8

If assumptions 1 to 5 hold and $\beta \geq 0$, a unique second-best regulatory equity level $\hat{E}^R$ exists such that banks do not gamble.

5.3 Output

Using regulatory intervention $\hat{E}^R$, we calculate aggregate output again assuming $\beta = 0$. Aggregate output can be determined as the sum of gross returns for consumers $(1-\eta)S(q_F)q_F$ and entrepreneurs. The latter is composed of the entrepreneurs' project ($\eta q_M e^-$) and returns on bank equity ($\eta R^{eb} e^+$). Using

$$\begin{aligned}
R^{eb} &= R^{ef} = \frac{\hat{R}^l}{\hat{R}^l - r_M} = 1 + \frac{r_M(r_F - \tilde{r}) + s(r_M - r_F)}{r_M - \tilde{r}} \\
e^+ &= \frac{n}{\eta} E^R(\hat{R}^l) = \frac{s(r_M - r_F) + r_M(1 + r_F - \tilde{r}) - s}{s(r_M - r_F) + r_M(1 + r_F - \tilde{r}) - \tilde{r}} e \\
e^- &= e - e^+ = \frac{s - \tilde{r}}{s(r_M - r_F) + r_M(1 + r_F - \tilde{r}) - \tilde{r}} e
\end{aligned}$$

we obtain the second-best aggregate output

$$\begin{aligned}
Y^{SB} &= (1-\eta)S(q_F)q_F + \eta q_M \left(e - \frac{n}{\eta} E^R(\hat{R}^l) \right) + \eta R^{eb} \frac{n}{\eta} E^R(\hat{R}^l) \\
&= \eta e \left(\frac{1-\eta}{\eta} \frac{S}{e} q_F + q_M + \frac{n}{\eta e} E^R(\hat{R}^l)(R^{ef} - q_M) \right)
\end{aligned} \tag{17}$$

It is intuitive that Y^{SB} is indeed the second-best aggregate output. If capital requirements were only slightly lower than $\hat{E}^R$, banks would gamble completely, which would lower aggregate output. This discontinuity in bank investment behavior indicates that Y^{SB} also decreases when capital requirements are lower than $\hat{E}^R$. If capital requirements were higher than $\hat{E}^R$, aggregate investment in the profitable moral hazard would decline which also lowers aggregate output.[16]

The second-best output depends on parameters and allows for interesting comparative statics. For instance, inspecting equation (17) yields:

Corollary 2

The second-best output is monotonically increasing in the share of entrepreneurs, i.e., $\frac{\partial Y^{SB}}{\partial \eta} > 0$

The intuition for corollary 2 is straightforward. A higher share of entrepreneurs alleviates the scarcity of equity, thereby reducing returns on equity. This allows for larger loans per entrepreneur. Aggregate investment in the best technology increases as a result of higher loans and a larger fraction of entrepreneurs.

6 An Example

Due to the complexity of assumption 5, we illustrate the determination of $\hat{R}^l$, $E^R(\hat{R}^l)$ and Y^{SB} with an example.

$$\beta = 0 \qquad S = 2 \qquad q_F = 1 \qquad q_M = \frac{7}{5}$$
$$q_G = \frac{8}{5} \qquad e = 2 \qquad \eta = \frac{1}{2}$$

Note that $s = \frac{1}{5}$. It is easy to verify that all assumptions are fulfilled and proposition 7 holds. Evaluating the corresponding expressions yields:

$$\tilde{r} = \frac{4}{5} \qquad \hat{R}^l = \frac{16}{15} \qquad R^{eb} = R^{ef} = \frac{8}{5}$$
$$E^R(\hat{R}^l) = \frac{1}{16}\frac{1}{n} \qquad Y^{SB} = \frac{193}{80} \qquad Y^{FB} = \frac{14}{5}$$

The example illustrates that the cost of bank capital can become very high and may exceed the marginal return on the investment projects of entrepreneurs (q_M) by a substantial margin.

16 The intuitive statement can be proved rigorously by tedious comparisons which we omit. The proof is available from the author.

7 Ramifications and Conclusion

Our paper has highlighted the fact that an optimal capital structure in an economy must balance multiple incentive problems and that bank capital requirements must take into account the impact on firms' equity. Our analysis is only a first step towards a thorough understanding of capital structures for economies. A variety of issues deserve further research. In particular, there are a number of other potential general equilibrium links which may further justify regulatory intervention (or the absence of it). For instance, higher bank capital and thus higher bank safety may increase loan demand at a given lending rate and may decrease deposit funding costs which increase bank capital through retentions over time. Such an endogenous link between bank capital today and in the future still requires capital requirements initially, but they become less binding over time. A dynamic analysis, such as this one, might explain why banks often have more capital than required.

The current analysis might also be complemented by allowing for uninsured debt to curb bank risk taking as shown by Calomiris and Kahn (1991). In principle, our analysis can be repeated for uninsured deposits as well, by specifying savings decisions of depositors depending on the return assessments of each individual bank. Without a specific savings function, the equilibrium cannot be solved anymore explicitly. In specific examples[17] it turns out that bank capital is still below the socially optimal level. At the highest level of importance two further issues await more definite answers. First, all considerations of optimal capital structures for economies rely on the empirical plausibility of the assumptions regarding which incentive problems are most important at the bank and at the firm level. Second, how important is bank capital in buffering macroeconomic shocks?[18] The necessary macroeconomic perspective on these issues promises to generate new guidelines for banking regulation.

17 This occurs, for instance, when depositors are extremely risk averse. Details are available upon request.

18 Although Gersbach and Wenzelburger (2000) provide a clear-cut answer, this well-known problem is far from being solved.

8 Appendix

Proof of Proposition 2:

First, note that no bank would offer a loan contract with $R_i^l < q_F$ since it is always more profitable to invest in FT. Therefore, we can assume that $R_i^l \geq q_F$ for all banks. Second, no bank would offer a loan contract (l_i, R_i^l) with $l_i > l^*(R_i^l, e^-)$ or a loan contract with $l_i < M - e^-$ since the firm would never invest in these cases and the bank would receive a lower return than by investing in FT. Statement (i) is a direct consequence of these two observations.

Now suppose that $l^*(e^-) + e^- \geq M$ and that in a symmetric equilibrium all banks have offered loan contracts (l_i, R_i^l) with $R_i^l > q_F$. At this stage, we know that $l_i \leq l^*(R_i^l, e^-)$. Consider the case where $e^- + l_i \geq M$. Here, bank loans amount to ηl_i and each bank grants $1/n$ of this amount. However, this implies that banks have an incentive to deviate from the equilibrium strategy by a contract $(\tilde{l}_i, \tilde{R}_i^l)$ with $\tilde{R}_i^l$ slightly lower than R_i^l and with $\tilde{l}_i = l^*(\tilde{R}_i^l, e^-)$. If this contract is offered to a τ fraction of entrepreneurs with $\tau\eta\tilde{l}_i = F + \eta l_i/n$, where F is the amount invested in FT, those entrepreneurs will apply for loans and the deviating bank invests all its funds in loans.[19]

This implies that returns on F are now given by $\tilde{R}_i^l$ instead of q_F. This compensates for slightly lower returns on $\eta l_i/n$ if $(R_i^l - \tilde{R}_i^l)$ is sufficiently small. In the second case, where $e^- + l_i < M$, no loans are granted and by the same logic as above and using the tie-breaking rule, deviation from a contract $(\tilde{l}_i, \tilde{R}_i^l)$ with $\tilde{R}_i^l = q_F$ and $\tilde{l}_i = l^*(e^-)$ is profitable.

Thus, we have shown that in any equilibrium $R_i^l = q_F$. Therefore, we know that $l_i \leq l^*(e^-)$. Using our tie-breaking rule, l_i cannot be lower than l^*, since, in this case, a bank would have an incentive to reallocate resources from FT to loans by offering a contract with $l_i = l^*(e^-)$. Finally, by the same reasoning as above, deviations from equilibrium (ii) are not profitable.

□

19 This fact can be derived by the following reasoning: As long as $R_i^l \leq q_M$, returns for entrepreneurs are increasing in the offered loan size and decreasing in the loan rate. But $l^*(\cdot, e^-)$ is decreasing in R_i^l and hence $\tilde{R}_i^l < R_i^l$ and $\tilde{l}_i > l_i$.

Proof of Proposition 5:

We define $\theta = \frac{D-L}{2R^{ef}-q_G}$, which is positive by assumption.

Moreover, let $\gamma = q_F\left(1-\frac{q_F}{q_G}\right) > 0$.

Then

$$E^{*G} = \left(q_G - q_F - R^{ef} + \sqrt{R^{ef}\left(R^{ef}-2\gamma\right)}\right)\theta$$

We first examine

$$\frac{\partial E^{*G}}{\partial R^{ef}} = \left(\frac{R^{ef}-\gamma}{\sqrt{R^{ef}(R^{ef}-2\gamma)}} - 1\right)\theta$$

$\frac{\partial E^{*G}}{\partial R^{ef}} > 0$ if and only if

$$(R^{ef}-\gamma)^2 > R^{ef}(R^{ef}-2\gamma)$$

which is true since $\gamma^2 > 0$. E^{*G} is monotonically increasing in R^{ef} and $R^{ef} > q_F$, it is sufficient to show that $E^{*G}(q_F) > 0$. Setting $R^{ef} = q_F$, we obtain

$$\begin{aligned} E^{*G}(q_F) &= q_G - 2q_F + \sqrt{q_F\left(q_F - 2q_F\left(1-\frac{q_F}{q_G}\right)\right)} \\ &= q_G\left(1-\frac{q_F}{q_G}\left(2-\sqrt{2\frac{q_F}{q_G}-1}\right)\right) \end{aligned}$$

Setting $\alpha = \frac{q_F}{q_G}$, $E^{*G}(q_F) > 0$ is equivalent to:

$$\begin{aligned} 1-2\alpha+\alpha\sqrt{2\alpha-1} &> 0 \\ \Leftrightarrow \quad \sqrt{2\alpha-1}\left(-\sqrt{2\alpha-1}+\alpha\right) &> 0 \end{aligned}$$

It remains to be shown that $-\sqrt{2\alpha-1}+\alpha > 0$, which is equivalent to $\alpha^2-2\alpha+1 > 0$ or $(\alpha-1)^2 > 0$, which always holds for $\alpha \neq 1$, i.e. $q_F \neq q_G$. Therefore $E^{*G} > 0$.

□

Proof of Proposition 8

Equation (16) can be written as:

$$E^*(R^l) = a\frac{R^l - b}{R^l - c}$$

where we use $J = \frac{1-n}{\eta}\frac{S}{e}$ and the constants a, b, c are given by:

$$\begin{aligned} a &= \Big(\frac{e\eta}{n}\Big)\Big(\frac{J(q_F - \tilde{r}) - r_M}{\tilde{r} - r_M}\Big) \\ b &= \frac{J(q_F - \tilde{r})(q_M + \beta) - \tilde{r}\, r_M}{J(q_F - \tilde{r}) - r_M} \\ c &= \frac{\beta\tilde{r}}{\tilde{r} - r_M} \end{aligned}$$

From assumption 5, it follows that $c < q_F$. Since, a, b, c are constants, we observe that

$$\begin{aligned} \frac{\partial E^*}{\partial R^l} &= a\frac{b - c}{(R^l - c)^2} \\ \frac{\partial^2 E^*}{(\partial R^l)^2} &= \frac{2a(b - c)}{(R^l - c)^3} \end{aligned}$$

Hence, $E^*(R^l)$ is monotonically increasing or decreasing and is either concave or convex on $[q_F, q_M]$. Hence, the equation $E^*(R^l) = E^R(R^l)$ has exactly one solution in $[q_F, q_M]$ since $E^R(R^l)$ is monotonically increasing and concave and $E^*(q_F) > E^R(q_F)$ as well as $E^*(q_M) < E^R(q_M)$.

□

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