Centre for Economic Policy Research

AN EVALUATION OF ALTERNATIVE INDICATOR REGIMES FOR MONETARY POLICY

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ABSTRACT

This paper evaluates alternative designs of indicator regimes for monetary policy in a small stochastic macromodel of the small open economy with rational expectations. The paper first derives methods for deriving optimal control rules and optimal simple feedback rules in rational expectations models, taking account of the Lucas critique. Use of the money supply, exchange rate, nominal income and price level as alternative indicators for monetary policy are then considered, with the objective of minimising a linear combination of price and output fluctuations. The price level indicator regime is found to be robust in the face of different types of disturbance and parameter changes. Moreover, this regime generally dominates the other regimes, and exhibits only a slight increase in loss relative to the full optimal rule.

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SUMMARY

An Evaluation of Alternative Indicator Regimes for Monetary Policy

The aim of this paper is to examine the usefulness of alternative variables as indicators for the conduct of monetary policy.

The widespread adoption of monetary targets over the past decade in a number of OECD countries has meant that movements in the money supply (appropriately defined) are taken as a signal to the monetary authorities to adjust interest rates so as to correct any deviation of the money supply from its target level or target range. Thus an excessive growth in the money supply will trigger a rise in interest rates designed to dampen this growth. If targeting is adopted wholeheartedly, the money supply represents an **intermediate target** of policy, being intermediate between the final objectives of policy (such as output and prices) and the instruments of policy (such as interest rates and other monetary control variables). However, the monetary authorities may not be willing to accept the degree of variability in interest rates and other instruments required for strict short-term control of the money supply, and may prefer to trade off variability in the money supply against variations in interest rates. In this case, responses to deviations of the money supply from target will be more relaxed, and the money supply acts as an **indicator** of policy.

The money supply is not the only variable that can act as an intermediate target or indicator variable. The exchange rate represented an intermediate target throughout the Bretton Woods era, and continues to do so within the European Monetary System

and in many third world countries; for many other countries, it constitutes an important indicator variable for monetary policy. The use of nominal income as an intermediate target has been advocated for the US (by, for example, Tobin) and for the UK (by, for example, Meade), as well as for other countries. A price index or an index of nominal wages might similarly be used as an indicator variable for policy.

An important aspect of the argument for monetary targets is that they offer some assurance concerning the future course of inflation. However, this is a general argument for targeting a nominal variable rather than the money supply in particular, since targeting other nominal variables offers similar assurance. Each of the indicator variables discussed above appears to be on an equal footing in this regard. A possible exception is the exchange rate where an adjustment to the target is required for prospective inflation in the rest of the world. This adjustment is, in principle, straightforward for the small open economy; but is complicated for the large economy, such as the US, because inflation in the rest of the world may not be independent of domestic developments.

More generally, a good indicator variable will provide reasonably reliable forward information concerning movements in target variables (such as prices and output), so that policy may respond in time. Indicator variables rather close to final objectives (such as nominal income or a price index itself) will provide accurate information, but possibly without early warning. Variables further from final objectives (such as the money supply or the exchange rate) may act as leading indicators, but possibly at the cost of providing a less reliable signal.

In the light of this, it is clear that the evaluation of indicator regimes requires a stochastic analysis in the context of a fully specified macromodel. In this paper, a comparison is made of the relative stabilisation properties of the indicator regimes discussed above (using alternatively the money supply, the exchange rate, nominal income and the price level) for the small open economy. The comparison is made for a developed stochastic analytical model, with dynamics arising from lags in the expenditure and money demand functions, from the wage/price sector, from asset accumulation via the government budget and the balance of payments, from exchange rate behaviour, and from persistence in some of the exogenous stochastic disturbances to the system. Parameters are assigned on the basis of plausibility and consistency with available econometric evidence. In addition, considerable emphasis is placed on testing the robustness of the results in the face of changes in parameter values. The indicator regimes are evaluated on the basis of an objective function that weights price and output variability equally, but these weights are varied to test robustness. The evaluation is carried out for five types of stochastic disturbance: an aggregate demand shock; a money demand shock; an aggregate supply shock; a foreign price level shock; and a foreign interest rate shock.

A major problem in evaluating such regimes is the fact that the mechanism whereby the private sector forms its expectations of future variables is unlikely to be independent of the regime in force. Thus, for example, the adoption of the exchange rate as an indicator will generate quite different dynamic paths for the exchange rate than, say, the use of the money supply. This difference is likely to be reflected in private sector expectations, at least once the regime has been in force for some time. Because of speculative influences and other mechanisms, different expectations about the future path of the exchange rate will lead to different values for the current exchange rate, and therefore generate quite different time paths for all the endogenous variables of the system.

This problem is best handled by assuming consistent or rational expectations, so that all model simulations are carried out on the assumption that the private sector makes no systematic errors in prediction. This provides a useful test-bed for policy regimes, since a policy which performs badly under consistent expectations could perform well only by virtue of systematic forecasting errors by the private sector, and this provides an ill-founded and inherently unstable basis for policy.

The comparison of alternative indicator regimes is best carried out in an optimal control framework, so that the rule linking interest rates to the indicator can be chosen optimally. However, the adoption of consistent expectations poses important problems for standard control theory, which assumes the model structure to be independent of the policy rule in force. The first part of the paper is therefore devoted to developing the theory of stochastic optimal control in models of rational expectations, thereby allowing for the dependence of expectations on policy regime. This theory is then applied to the model described above.

The results indicate that the use of the exchange rate as an indicator for monetary policy generates rather high variability in output and prices, and is therefore unsatisfactory. The use of the money supply performs moderately if the nature of the shock to the system is known, but rather badly if the nature of the shock is uncertain. The use of nominal income typically performs better than both these regimes, and shows some robustness in the face of uncertainty about the source of shock. However, the use of the price level as an indicator dominates all the other three regimes in stabilising the system in the face of both known and unknown disturbances. Moreover, it is shown that the performance of the price rule is only marginally inferior to the fully optimal rule, where policy responds to all variables in the system. These results hold true when the parameters of the model are varied widely, indicating that the price level may provide a rather robust indicator for monetary policy.

Further work will pursue this line of inquiry for more developed models, including interdependent economies, and will also examine the use of fiscal policy in conjunction with monetary policy.

1. Introduction

In this paper we are concerned with the evaluation of alternative types of indicator regimes for monetary policy in the context of a fairly developed stochastic macromodel of the small open economy. There are in the literature, two strands of work concerned with assessing such regimes. The first, springing from the work of Poole (1970), assesses alternative regimes in the context of analytical models. (See, for example, Parkin (1978), Artis and Currie (1981).) A weakness of this literature is that analytical tractability necessarily limits the model deployed to have, at best, rather rudimentary dynamics. The second strand is concerned with alternative regimes in the context of the large estimated macroeconometric models. (See, for example, Artis and Karakitsos (1982), Currie and Karakitsos (1983).) In assessing these results, advantages of using realistic estimated models must be set against the highly model-specific character of the results and the susceptibility of such models to the Lucas (1976) critique.

In this paper, we steer an intermediate course between these two strands of analysis. We analyse a model with developed money/price/wage output interactions, which also takes account of the wealth interactions arising from the government budget and the current account of the balance of payments. In this respect, the analysis represents a step towards a greater degree of realism of model. At the same time, we are able to apply control theory (developed in Section 2 of the paper) which takes account of the interdependence between policy rules and private sector expectations mechanisms and behaviour. Thus, we take account of the Lucas critique in assessing policy regimes.

In this paper, we examine three alternative types of policy. The first is the fully optimal policy, obtained by minimising the welfare loss subject to the constraints imposed by the model and by the assumption that private sector expectations are formed rationally. As we indicate in section 2, such optimal policy cannot be properly defined by a linear, time-invariant feedback rule in terms of the state variables of the model. The second type of policy is where we optimise in a similar fashion, but subject to the additional constraint that the policy should be properly defined by a linear, time-invariant feedback rule. By comparing the increase in the loss function as we move from the first to the second policy, we are able to assess the cost associated with the insistence on a linear, time-invariant rule. The third type of policy is where we limit the feedback rule still further, to one expressable in terms of a linear, time-invariant feedback rule on only certain variables of the system. Such quasi-optimal rules, as we may call them, may correspond to intermediate target regimes if the variables in question are suitably chosen. Thus, we may consider as indicators the use of the money supply, nominal income, the exchange rate and the price level, where the instruments of policy are constrained to feedback only on the relevant variable. Again, the cost of further restricting policy to these simple rules may be assessed by examining the increase in the loss function.

It should be noted that, on this interpretation, the indicator regime encompasses the special case of intermediate targets where instruments are used to keep the intermediate variable on a predetermined path. Thus, with interest rates as the instrument, a monetary target regime corresponds to the case where the interest rate is adjusted to avoid fluctuations in the money supply. But since in general the optimal degree of response may be more muted, under the indicator regime the money supply need not follow a predetermined path. The money supply acts as an indicator of policy, but is not necessarily exogenously fixed by policy.

The first class of policy, fully optimal rules, exhibits certainty equivalence, so that the design of policy is independent of the type of stochastic shocks. As we show in Section 2, this is no longer so once we insist on defining policy in terms of a linear time-invariant feedback rule (i.e. for the second and third types of policy). This complicates the comparison of the alternative policies. It also increases the costs associated with time-invariance, since a policy designed for one particular type of shock may perform very badly in the face of other disturbances. This suggests that emphasis should be placed on robust rules that work tolerably well in the face of all possible disturbances. These disadvantages associated with time-invariance have to be set against the advantages of simplicity in policy design. In particular, we would note that the likely gain in understanding of policy by the private sector makes the assumption of rational expectations (which underlies our methods of policy evaluation) more tenable.

Our analysis is conducted in terms of a continuous time stochastic model. As Phillips (1954, 1957) noted, the use of continuous time gives an unduly optimistic view of the efficacy of stabilisation policy. A future extension of this work will be to examine the discrete time case. Other extensions are to examine the use of monetary and fiscal policy in conjunction, and to relax the assumption of perfect capital mobility.

Section 2 develops the control theory needed for our subsequent analysis. Section 3 reports results from a simpler model without wealth effects and with a rudimentary price sector. Section 4 sets out the model with wealth effects, whilst Section 5 presents the results and conclusions.

2. The Solution Procedure

We consider the following general model:

$$\begin{bmatrix} dz \\ dx^e \end{bmatrix} = A \begin{bmatrix} z \\ x \end{bmatrix} dt + Bwdt + dv \tag{2.1}$$

where $z(t)$ is an $(n-m) \times 1$ vector of variables predetermined at time t, $x(t)$ is an $m \times 1$ vector of non-predetermined or 'free' variables, $dx^e = x^e(t + dt, t) - x(t)$ where $x^e(t,\tau)$ denotes the expectation of $x(t)$ formed at time τ, $w(t)$ is an $r \times 1$ vector of instruments, A and B are $n \times n$ and $n \times r$ matrices respectively with time-invariant coefficients and dv is an $n \times 1$ vector of white noise disturbances independently distributed with distribution $N(0,\Sigma dt)$ where Σ is a positive definite matrix with time-invariant coefficients. Variables z, x and w are all measured as deviations about the long-run equilibrium.

We seek a linear time-invariant feedback rule,

$$w = D \begin{bmatrix} z \\ x \end{bmatrix} \tag{2.2}$$

where D is an $r \times n$ matrix with time-invariant coefficients which minimises the asymptotic expected value, asyE(W), where

$$W = [z^T x^T] Q \begin{bmatrix} z \\ x \end{bmatrix} + w^T R w \tag{2.3}$$

is a quadratic cost function and Q and R are $n \times n$ and $r \times r$ time-invariant positive definite matrices respectively.(1) By appropriate restrictions on the coefficients of D, (2.2) can represent a simple feedback rule on only some variables of the system.

The first step of the solution procedure is to obtain the rational expectations solution to (2.1) for a given feedback rule (2.2) which is assumed to be known by economic agents along with the model and the current endogenous variables. The following is an outline of the solution presented in more detail in Currie and Levine (1982) which, in turn is a stochastic extension of the solution of Dixit (1980). Substituting (2.2) into (2.3), we obtain

$$\begin{bmatrix} dz \\ dx^e \end{bmatrix} = [A + BD] \begin{bmatrix} z \\ x \end{bmatrix} dt + dv \tag{2.4}$$

We are concerned only with solutions to (2.4) which have the saddle-point property that the number of eigenvalues of A + BD with positive part equals M, the remaining n-m eigenvalues having negative real parts. We shall assume that the pair (A,B) is '<u>stabilizable in the rational expectations sense</u>'. By this we mean there exists at least one D such that A + BD has the saddle-point property. Then <u>stochastic stability</u> follows since we only have additive disturbances (Turnovsky (1977)). We now require the immediate response of the non-predetermined variables x to the feedback rule (2.2). This is found by first forming the matrix of left eigenvectors of A + BD, M say, with rows ordered so that the first n-m are the eigenvectors associated with the stable eigenvectors. We then partition so that

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{2.5}$$

where M_{11} is an (n-m) x (n-m) matrix and M_{22} is m x m. Then, provided that A + BD has the saddle-point property, the rational expectations solution places the trajectory on the unique saddle path

$$x = -M_{22}^{-1} M_{21} z = -N z \tag{2.6}$$

The feedback rule (2.2) now becomes

$$w = (D_1 - D_2 N)z = \tilde{D} z \tag{2.7}$$

where $D = [D_1 \; D_2]$ with D_1 of dimension $r \times (n-m)$ and D_2 of dimension $r \times m$. We note that (2.7) is a feedback rule only on the predetermined variables. Let A be partitioned as for M, $B = \begin{bmatrix} B^1 \\ B^2 \end{bmatrix}$ where B^1 is $(n-m) \times r$, B^2 is $m \times r$ and $dv = \begin{bmatrix} dv^1 \\ dv^2 \end{bmatrix}$ where dv^1 is $(n-m) \times 1$ and dv^2 $m \times 1$. Then substituting (2.6) and (2.7) into (2.1), we have from the first (n-m) rows that

$$dz = [A_{11} - A_{12}N + B^1 \tilde{D}]\, z\, dt + dv^1. \tag{2.8}$$

The solution to (2.8) is

$$z(t) = \int_0^t e^{C(t-s)}\, dv^1(s) + e^{Ct} z(o) \tag{2.9}$$

where $C = [A_{11} - A_{12}N + B^1 \tilde{D}]$ (see Currie and Levine (1982), p. 25). Equations (2.6) and (2.9) constitute the rational expectations solution to (2.1) for a given feedback rule (2.2).

The second step of the control problem is to optimise with respect to D. Before proceeding to this, it is useful to conceptualize the private sector's response to a government feedback rule as a 'response function'. Rational expectations imply that private sector behaviour (conceptualised as a mapping from variables parametric to the private sector to its decision variables) is dependent upon choice of policy rule. In terms of our general specification in (2.1), this dependence or response of private sector behaviour to changes in the government policy rule is summarised by the relationship (2.6) between free and predetermined variables, given by the matrix $N = -M_{22}^{-1} M_{21}$. We note that M and N are functions of the matrix D defining the government rule. We write $N = f(D)$ and term f the 'private sector's response function'.

Consider for the time being the optimisation problem to be one of selecting the optimal value of $\tilde{D}$ rather than D with $N = f(\tilde{D})$ (i.e. put $D = [\tilde{D}, 0]$). The government's rule is found by using its knowledge of the private sector's response function to minimise asyE(W), where substituting (2.2) into (2.3), we have

$$E(W) = tr(E(W)) = tr((Q + D^T RD)Y) \tag{2.10}$$

where $Y = \text{asy cov}\binom{z}{x}$ and we have used the result $tr(ABC) = tr(CAB)$

The asymptotic covariance matrix $z = \text{asy}E(z^T z)$ satisfies

$$ZC^T + CZ + \Sigma_{11} = 0 \tag{2.11}$$

where we recall that $C = A_{11} - A_{12}N + B^1\tilde{D}$ and $\Sigma_{11}dt = cov(dv^1)$ (see, for example, Currie and Levine (1982), p. 25). Then combining (2.6) and (2.10) we obtain

$$\text{asyE}(W) = \text{tr}(\tilde{Q} Z) \tag{2.12}$$

where $\tilde{Q} = Q_{11} - 2N^T Q_{21} + N^T Q_{22} N + \tilde{D}^T R \tilde{D}$. The welfare loss asyE(W) can now be minimised with respect to $\tilde{D}$ by a standard numerical gradient method. The minimisation problem is not unconstrained because $A + BD = A + B^1 \tilde{D}$ must have the saddle-point property. Formally, if for each $\tilde{D}$ the number of eigenvalues of $A + B^1 \tilde{D}$ with positive real part is $P(\tilde{D})$ then the constraint is $P(\tilde{D}) = m$.

Two points are worth noting about our solution to the control problem. First, the optimal value of $\tilde{D}$, $\tilde{D}^*$ say, may not be unique within a range of feasible controls, i.e. there may be more than one solution to the global minimisation problem (though this is unlikely). Let us suppose that it is unique. Then the control rule $w^* = \tilde{D}^* z$ is unique only within a class of rules acting only on the predetermined variables. It corresponds to an infinite number of control rules acting on all variables (i.e. of the form (2.2)0 which from (2.7) satisfy

$$D_1^* - D_2^* N^* = \tilde{D}^* \tag{2.13}$$

where $N^* = f(\tilde{D}^*)$

The second point is that the optimal control rule is in general dependent upon the covariance matrix of the disturbance Σ (or, more precisely, on Σ_{11}). This is an important point because for stochastic models it means that the controller must know Σ_{11} in order to calculate the appropriate feedback rule. How a controller could handle a situation where Σ_{11} is, in fact, unknown is discussed in Sections 3 and 5.

We have considered, up to this point, optimal rules within the class of linear time-invariant feedback rules. We now turn to the full optimal control rule. The following is a solution procedure, employing Pontryagin's maximum principle, first proposedby Calvo (1978) and later developed by Driffill (1982) and Miller and Salmon (1983). We consider first the deterministic finite time-horizon problem with objective function

$$W = \int_0^\tau (y^T Qy + w^T Rw)dt \tag{2.14}$$

and $y^T = [z\ x]$. Then on introducing the costate row vector $\lambda(t)$, by the maximum principle we minimise

$$J = W + \int_0^\tau \lambda(Ay + Bw - \dot{y})dt \tag{2.15}$$

with respect to w, y and λ. Define the Hamiltonian

$$H = (y^T Qy + w^T Rw) + \lambda (Ay + Bw) \tag{2.16}$$

Then

$$J = \int_0^\tau (H - \lambda\dot{y})dt \tag{2.17}$$

Hence, considering arbitrary variations in λ, $\delta J = 0$ if and only if

$$\frac{\partial H}{\partial \lambda} = \dot{y} \tag{2.18}$$

which, from (2.16, is simply the model (2.1) in the deterministic case.

Now consider variations in J due to independent variations in w and y. Integrating (2.17) by parts, we have

$$J = -\lambda(\tau)y(\tau) + \lambda(0)y(0) + \int_0^\tau (H + y\dot{\lambda})\,dt \tag{2.19}$$

Differentiating (2.19),

$$\delta J = -\lambda(\tau)\delta y(\tau) + \lambda(0)\,\delta y(0) + \int_0^\tau \left[\left(\frac{\partial H}{\partial y} + \dot{\lambda}\right)\delta y + \frac{\partial H}{\partial w}\,\delta w \right] dt \tag{2.20}$$

Partition $\lambda(0) = [\lambda_1(0),\ \lambda_2(0)]$ where λ_1 is $1 \times (n-m)$ and λ_2 is $1 \times m$. Then $\lambda(0)\delta y(0) = \lambda_1(0)\delta z(0) + \lambda_2(0)\delta x(0) = \lambda_2(0)\delta x(0)$ since z(t) is predetermined (i.e. z(0) is given). It follows that $\delta J = 0$ for arbitrary changes $\delta y(\tau)$, $\delta x(0)$, δy and δw if and only if

$$\lambda(\tau) = 0 \tag{2.21}$$

$$\lambda_2(0) = 0 \tag{2.22}$$

$$\frac{\partial H}{\partial w} = 0 \tag{2.23}$$

and

$$\dot{\lambda} = -\frac{\partial H}{\partial y} \tag{2.24}$$

The condition (2.22) can also be obtained (Driffill (1982)) by using the standard result that $\frac{\partial W}{\partial y(0)} = \lambda(0)$ at the optimal point. Hence $\frac{\partial W}{\partial x(0)} = \lambda_2(0)$. But the welfare loss must be insensitive to changes in the initial values of the non-predetermined variables $x(0)$. Thus $\frac{\partial W}{\partial x(0)} = 0$ and the result follows.

From (2.23) and (2.24) with H defined by (2.16), we obtain

$$w = -\frac{1}{2} R^{-1} B^T \lambda^T \tag{2.25}$$

and

$$\dot{\lambda} = -(2y^T Q + \lambda A) \tag{2.26}$$

Define $p = \frac{1}{2}\lambda^T$. Then the optimal rule for the deterministic control problem is given by

$$w = -R^{-1} B^T p \tag{2.27}$$

where

$$\begin{bmatrix} \dot{y} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} y \\ p \end{bmatrix} = H \begin{bmatrix} y \\ p \end{bmatrix} \tag{2.28}$$

and z, p satisfy the boundary conditions that $z(0)$ is given, $p_2(0) = 0$ and $p(\tau) = 0$.

It is a standard result in control theory (see, for example, Kwakernaak and Sivan , p 147) that provided H has 2n distinct eigenvalues, n of these associated with predetermined variables $[z, p_2]^T$ will be stable and n associated with non-predetermined variables $[x, p_1]^T$ will be unstable where $p^T = [p_1^T \ , \ p_2^T]$.

Then rearranging (2.28) we have

$$\begin{bmatrix} \dot{z} \\ \dot{p}_2 \\ \dot{p}_1 \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A_{11} & -J_{12} & -J_{11} & A_{12} \\ -Q_{21} & -A_{22}^T & -A_{12}^T & -Q_{22} \\ -Q_{11} & -A_{21}^T & -A_{11}^T & -Q_{12} \\ A_{21} & -J_{22} & -J_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z \\ p_2 \\ p_1 \\ x \end{bmatrix} = \tilde{H} \begin{bmatrix} z \\ p_2 \\ p_1 \\ x \end{bmatrix} \tag{2.29}$$

where $J = BR^{-1} B^T$ and matrices A, J and Q are partitioned as before. Equation (2.29) expresses the model in a form analogous to the standard deterministic rational expectations model. The case of the infinite time horizon, $\tau \to \infty$, is analytically tractable. For in this case (using an argument analagous to that leading to (2.6)) the rational expectations assumption that $y(t) \to 0$ as $t \to \infty$ (i.e. the model is stable) together with condition (2.21) which implies $p(t) \to 0$ as $t \to \infty$, imposes the relationship

$$\begin{bmatrix} p_1 \\ x \end{bmatrix} = - \tilde{M}_{22}^{-1}\tilde{M}_{21} \begin{bmatrix} z \\ p_2 \end{bmatrix} = - \tilde{N} \begin{bmatrix} z \\ p_2 \end{bmatrix} \tag{2.30}$$

where $\tilde{M}$ is the matrix of left-eigenvectors of H formed and partitioned as for M in (2.6) except that this time we have n stable and n unstable roots.

The feedback rule (2.27) now becomes

$$w = - R^{-1} B^{T} [- \tilde{N}_{11} z - \tilde{N}_{12} p_2 , p_2]^{T} = - R^{-1} S \begin{bmatrix} z \\ p_2 \end{bmatrix} \tag{2.31}$$

where $S = [- B_1 \tilde{N}_{11}, \; B_2 - B_1 \tilde{N}_{12}]$ where $B^T = [B_1, B_2]$ and $\begin{bmatrix} z \\ p_2 \end{bmatrix}$ is given by

$$\begin{matrix} \dot{z} \\ \dot{p}_2 \end{matrix} = [\tilde{H}_{11} - \tilde{H}_{12} \tilde{N}] \begin{bmatrix} z \\ p_2 \end{bmatrix} = \tilde{C} \begin{bmatrix} z \\ p_2 \end{bmatrix} \tag{2.32}$$

and $\tilde{H}$ is partitioned into four $n \times n$ blocks. The solution to (2.32) is

$$\begin{bmatrix} z \\ p_2 \end{bmatrix} = e^{\tilde{C}t} \begin{bmatrix} z(0) \\ p_2(0) \end{bmatrix} = e^{\tilde{C}t} \begin{bmatrix} z(0) \\ 0 \end{bmatrix} \tag{2.33}$$

which completes the optimal control solution in an 'open-loop' form.

The optimal rule (2.33) is then in the form of a feedback rule on the vector $[z \; p_2]^T$. It can be compared with rules of the form (2.7) as follows:

put $E(t) = e^{\tilde{C}t}$ and partition E as for A and M. Then from (2.35),

$p_2(t) = E_{21}(t) \; z(0) = E_{21}(t) E_{11}^{-1}(t) \; z(t)$ for all t for which $E_{11}^{-1}(t)$ exists.

Hence (2.33) may be written as

$$w = R^{-1} \; [S_1 + S_2 \; E_{21}(t) E_{11}^{-1}(t)] \, z \tag{2.34}$$

partitioning $S = [S_1, S_2]$ where S_1 is $r \times n{-}m$ and S_2 is $r \times m$. Thus the full optimal feedback rule on z has time-varying coefficients.

However the optimal rule can be expressed in terms of a linear time-invariant rule on the vector $[z \; x]^T$. Thus from the lower block of (2.30), we have that

$$p_2 = - \tilde{N}_{22}^{-1} (x + \tilde{N}_{21} \; z) \tag{2.35}$$

provided that $\tilde{N}_{22}$ is non-singular. Substituting from (2.35) for p_2 in (2.31), we obtain

$$w = - R^{-1} \; [\; S_1 - S_2 \tilde{N}_{22}^{-1} \;\; \tilde{N}_{21}, \; - \; S_2 \tilde{N}_{22}^{-1} \;] \quad \begin{bmatrix} z \\ x \end{bmatrix} \tag{2.36}$$

which represents a linear time-invariant feedback rule for the instrument in terms of the original endogenous variables of the system, z and x.

We now have an apparent paradox. Earlier we showed that a linear time-invariant feedback rule defined on $[\; z \;\; x \;]^T$ is equivalent to one defined on z alone. (See the discussion before (2.13) above.) Yet we have just demonstrated that the optimal rule is expressible in terms of a linear time-invariant feedback rule on $[\; z \;\; x \;]^T$, but only in terms of a linear time-varying rule on z.

The resolution of this paradox is follows. The optimal policy is defined by (2.31) and (2.32) above. Given this policy, private sector expectations formation under rational expectations ensures that the system lies on the unique stable trajectory of (2.29), so that (2.30) is satisfied. Under these circumstances, the variables of the system move such that (2.36) is satisfied, so that w and $[\; z \;\; x \;]^T$ are related by a linear relationship. But since the dynamics of the system under the optimal rule are of order n, z and x are not linearly dependent, so there is no linear relationship between w and z alone.

Now suppose that the authorities do not announce (2.31) and (2.32), but instead announce the linear rule defined by (2.36). Let the matrix product in (2.36) be D. Then private agents can substitute this rule into (2.1) to obtain (2.4). Applying the standard solution technique for rational expectations, we obtain a relationship of the form (2.6) and the feedback rule reduces to (2.7). The dynamics of the system will be entirely different from those under the optimal policy. Indeed the dynamics will be of order (n-m) instead of n; and, moreover, it may be the case that the matrix (A+BD) does not have the unique saddle point condition so that a unique stable solution need not exist.

The upshot of this is that, although under the optimal policy the instruments are linearly related to the endogenous variables of the system, this linear relationship does not define the optimal policy. The government cannot announce a linear feedback rule and thereby implement the optimal policy. It must pursue the rule defined by (2.31) and (2.32). This is in marked contrast with optimal control without rational expectations, and appears to have gone unremarked in the literature hitherto. For reasons outlined in the introduction, we are interested in simple rules that are defined by a linear time invariant feedback rule on observable variables. What we have shown is that the optimal rule does not fall into this class of rules, so that the insistence on even this limited degree of simplicity is not without cost.

To explore further the cost of constraining policy to be defineable in terms of simple rules, we consider the stochastic control problem. It is a standard result of control theory that "certainty equivalence" applies as between the deterministic and stochastic optimisation problems. This enables us to calculate the loss under the optimal policy for the stochastic case. For the deterministic case we have that $\frac{dW}{dz(0)} = 2\,p_1(0)$

But from (2.30) we have $p_1(0) = -\tilde{N}_{11}z(0)$. Hence

$$W = -\, z^T(0)\,\tilde{N}_{11}\; z(0) \tag{2.35}$$

provided that $\tilde{N}_{11}$ is symmetric (which can be shown to be the case). The corresponding welfare loss in the stochastic case is the same provided we put $z(0)\,z^T(0) = \Sigma_{11}$ where $\mathrm{cov}(dv^1) = \Sigma_{11}dt$. (See Appendix A of Currie and Levine (1983).) Thus given that (2.31) and (2.32) define the optimal rule for the deterministic problem, they also define the optimal rule for the stochastic counterpart quite independently of the covariance matrix of the disturbances perturbing the system. However, this principle is violated once we restrict policy to the class of simple rules defined by (2.7). Hence the optimal choice of linear feedback rule is not independent of the covariance matrix of disturbances. This greatly complicates policy design, since it requires some estimates of the shocks likely to hit the system, as well as some consideration of the consequences if such estimates are in error. These complications represent some of the costs accompanying the restriction of simplicity in policy design.

3. A Basic Model of Money/Output/Price Interactions in an Open Economy

In this section we review an optimal control exercise considered in more detail in Currie and Levine (1983). The model consists of the following four dynamic relationships:

$$dy = \psi_1 [\alpha_1 (e - p) - \alpha_2 (r - \dot{p}^e) - y] dt + dv_1 \tag{3.1}$$

$$dp = [\beta_1 y + \beta_2 (e - p)] dt + dv_2 \tag{3.2}$$

$$dm = \psi_2 [\gamma_1 y - \gamma_2 r + p - m] dt + dv_3 \tag{3.3}$$

$$de^e = r dt + dv_4 \tag{3.4}$$

where m, e and r are defined as before, y and p are logarithms of real output and the price level respectively, $\dot{p}^e$ is the agent's current expectation of current inflation, dv is a white noise process as before and ψ_1, ψ_2, α_1, α_2, γ_1, γ_2, β_1 and β_2 are non-negative parameters. Again all variables are measured about their equilibrium values.

Equation (3.1) represents the IS curve, with output adjusting sluggishly to competitiveness and the real interest rate with a mean lag of ψ_1^{-1}. Equation (3.2) is a Phillips curve with prices adjusting sluggishly to changes in output and import costs, the latter with a mean lag of β_2^{-1}. Equation (3.3) represents the LM curve. The money supply is assumed to be demand determined for any given level of interest rates, and money demand adjusts sluggishly to output and interest rate changes with a mean lag of ψ_2^{-1} Equation (3.4) models the exchange rate as asset market determined under conditions of perfect capital mobility. The expected rate of depreciation of the exchange rate in an interval dt (denoted by $de^e = e^e(t + dt,t) - e(t)$ where $e^e(\tau, t)$ is the expected exchange rate at time τ, formed at time t) exactly offsets the interest rate differential in favour of the home currency. (Note that $r = 0$ in equilibrium corresponds to the domestic and foreign interest rates being equal.) Unlike the money stock, which adjusts slowly and is a predetermined variable, the exchange rate is non-predetermined and can make discrete jumps in response to changes in exogenous variables or policy rules. We consider $dv = (dv_1 \ dv_2 \ dv_3 \ dv_4)^T$ to be a vector made up of an aggregate demand disturbance, a supply disturbance, a money demand disturbance and a foreign exchange rate disturbance respectively.

By the assumption of rational expectations, $\dot{p}^e$ is obtained from the model, i.e. $\dot{p}^e = \beta_1 y + \beta_2 (e - p)$ from (3.2). Substituting in (3.1) the system becomes, in matrix form,

$$\begin{bmatrix} dy \\ dp \\ dm \\ de^e \end{bmatrix} = \begin{bmatrix} -\psi_1(1-\beta_1\alpha_2) & -\psi_1(\alpha_1+\alpha_2\beta_2) & 0 & \psi_1(\alpha_1+\alpha_2\beta_2) \\ \beta_1 & -\beta_2 & 0 & \beta_2 \\ \psi_2\gamma_1 & \psi_2 & -\psi_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ p \\ m \\ e \end{bmatrix} dt + \begin{bmatrix} -\psi_1\alpha_2 \\ 0 \\ -\psi_2\gamma_2 \\ 1 \end{bmatrix} rdt \tag{3.5}$$

The model is now in the form (2.1) with y, p, m the predetermined variables, e the non-predetermined variable and r the instrument. We shall confine ourselves to a simple form of objective function without covariance so that

$$Q = \begin{bmatrix} a & & & 0 \\ & b & & \\ & & 0 & \\ 0 & & & 0 \end{bmatrix} \tag{3.6}$$

and R = 1. Plausible parameter values for the model are $\alpha_1 = 0.3$, $\alpha_2 = 0.1$, $\gamma_1 = \gamma_2 = 1$, $\beta_1 = 0.15$, $\beta = 0.25$ and $\psi_1 = \psi_2 = 0.5$.

FORTRAN programs have been developed to implement the solution procedure of Section 2 using, in the case of the general time-invariant rule, a NAG minimisation subroutine. We also consider four more restricted feedback rules (i.e. indicator regimes) – an exchange rate target ($r = \beta(y + p)$), a monetary target ($r = \beta m$) and a price level target ($r = \beta p$) where β is a non-negative parameter and the targets are the long-run equilibrium values of the relevant variables. For each of these rules, the same minimisation procedure as for the general rule is used to obtain quasi-optimal time- invariant feedback rules.

An important feature of the general and quasi- optimal time-invariant rules is that, with the exception of the exchange rate rule, they are dependent upon the expected disturbance (or combination of disturbances). For an economy being continuously subjected to shocks, their nature may be unknown and this raises a problem for thepolicymaker. One approach is to attach probabilities to each of the disturbances and then to minimise the welfare loss using an estimated covariance matrix Σ say. An alternative approach, adopted here, is to assume that the economy is subjected to only one major disturbance at one time and to assess the correct strategy from a 'pay-off matrix' in which the i j th element is the welfare loss when disturbance dv_i is expected, but dv_j occurs. For our model, money is passive and a money demand disturbance does not affect the welfare loss since money does not enter the objective function. For this reason, we only consider expected disturbances dv_1 and dv_2. For all three actual disturbances, the results of the general and quasi-optimal time invariant rules are compared with those obtained using the full optimal control rule. The results are displayed in Tables 1 and 2 for two objective functions, the rules being in order of preference (decreasing welfare loss) for the expected disturbance equal to the actual disturbance (i.e. along the diagonals). The disturbances all have variances equal to unity.

We may draw the following conclusions from these results. First, consider the results where the disturbance is known or, alternatively, where the aim is to return the economy to equilibrium from a given, known displacement of either output or the price level. In this case, the welfare loss associated with using time-invariant optimal as opposed to the full optimal control rule is very small (in fact, undetectable to 2 decimal places in most cases). The welfare loss associated with adopting the best quasi-optimal as opposed to the optimal time-invariant rule is very slight for a dv_1 disturbance and in the region of 12% for a dv_2 disturbance. Within the class of quasi-optimal time invariant rules, the money rule performs best for a dv_1 disturbance and a price level rule for a dv_2 disturbance.

Second, consider the economy near or at equilibrium, subject to future unknown disturbances. The choice of policy within the class of time invariant rules is now far more

Expected Disturbance (unit variance)	Control Rule*	Actual Disturbance (unit variance) dv_1	dv_2	dv_3
	Full Optimal	4.04	3.02	0.00
	General ($r = 0.15y-0.86p+1.04m$)	4.04	4.26	0.71
	Monetary ($r = 1.96m$)	4.07	3.22	1.29
dv_1	Nominal Income ($r = 0.59(y+p)$)	4.16	3.44	0.00
	Price Level ($r = 40p$)	4.23	253.00	0.00
	Exchange Rate	5.00	4.16	0.00
	Full Optimal	4.04	3.02	0.00
	General ($r = -0.95y-0.07p+0.36m$)	5.90	3.02	0.17
	Price Level ($r = 0.46p$)	4.56	3.19	0.00
dv_2	Monetary ($r = 1.50m$)	4.07	3.22	0.91
	Nominal Income ($r = 0.55(y+p)$)	4.16	3.44	0.00
	Exchange Rate	5.00	4.16	0.00

Table 1: Welfare Loss for Loss Function asy$E(5y^2 + 2p^2 + r^2)$

Expected Disturbance (unit variance)	Control Rule*	Actual Disturbance (unit variance) dv_1	dv_2	dv_3
	Full Optimal	1.70	2.31	0.00
	General ($r = 0.06y-0.1p+0.5m$)	1.71	2.98	0.22
	Monetary ($r = 0.61m$)	1.71	2.79	0.27
dv_1	Price Level ($r = 19.5p$)	1.73	84.60	0.00
	Nominal Income ($r = 0.34(y+p)$)	1.75	2.94	0.00
	Exchange Rate	2.12	3.80	0.00
	Full Optimal	1.70	2.31	0.00
	General ($r = -1.03y-0.02p+0.21m$)	3.60	2.31	0.07
	Price Level ($r = 0.56p$)	1.84	2.59	0.00
dv_2	Monetary ($r = 1.72m$)	1.75	2.69	1.07
	Nominal Income	1.84	2.94	0.00
	Exchange Rate	2.12	3.80	0.00

Table 2: Welfare Loss for Loss Function asy$E(2y^2 + 2p^2 + r^2)$

* All rules, except the full optimal rule, are linear time-invariant feedback rules.

complex. It is now necessary to consider the full pay-off matrix for each policy option. Let us focus on one objective function asy$E(5y^2 + 2p^2 + r^2)$ (Table 1). The general time-invariant rule is now no longer clearly superior to simple rules. If the policy-maker makes plans for a dv_1 disturbance but, in fact, a dv_2 occurs, then the general rule turns out to be worse than optimal monetary, nominal income and exchange rate rules. Planning for a dv_2 disturbance, using a general rule, is equally disastrous if a dv_1 disturbance occurs. For the simple time-invariant rules, several regimes are 'dominated strategies', i.e. out- performed by some other regime for any disturbance. These are, for an expected disturbance dv_1, the price level and exchange rate rules and, for an expected disturbance dv_2, the latter regime. Furthermore, the monetary rule for dv_1 ($r = 1.96m$) is dominated by that for dv_2 ($r = 1.50m$). This leaves us with four candidates for the best time-invariant rule - the general rule, a nominal income rule between $r = 0.55(y + p)$ to $r = 0.59(y + p)$, a monetary rule $r = 1.50m$ and a price level rule $r = 0.46p$.

To narrow the choice further, one needs estimates for the variances and the probability of occurrence for each disturbance. Proceeding in this way, the overall conclusion arrived at in Currie and Levine (1982) is that unless money demand shocks are relatively insignificant compared with aggregate demand and supply shocks, then under conditions of uncertainty, the policy-maker, confined to time-invariant rules, should choose a nominal income target $r = \beta(y + p)$ where β is in the region (0.55,0.59). This conclusion holds for an objective function asy$E(5y^2 + 2p^2 + r^2)$; for the alternative objective function asy$E(2y^2 + 2p^2 + r^2)$ a similar analysis reveals a preference for a price target regime $r = 0.56p$.

4. The Model with Wealth Interaction

The model analysed in this and subsequent sections extends that of Section 3 in three main ways. First, it incorporates the wealth interactions arising from the government budget and the current account of the balance of payments, allowing for wealth effects in both the goods market and the financial market. Hence, it incorporates the effects now familiar from the extensive literature on the government budget constraint. Second, it develops the price sector by modelling wages explicitly; by allowing for higher order dynamics in the price/wage mechanism; and by allowing for the dependence of the general price index directly on import prices and hence the exchange rate. Third, we allow for autoregressive foreign wage level and foreign interest rate disturbances to the system. These changes are substantive, resulting in significantly higher order dynamic processes; but they do not alter in any way the general method of analysis outlined in Section 2 and applied in Section 3.

The model is represented by the following set of equations:

$$dy = \psi_1[\alpha_1 q - \alpha_2(r - \dot{p}^e) + \alpha_3 v - y]\,dt + dv_1 \tag{4.1}$$

$$dm = \psi_2[\gamma_1 y - \gamma_2 r + p + \gamma_3 v - m]\,dt + dv_2 \tag{4.2}$$

$$dv = [-\phi_1 y - \phi_2 q]dt - dp \tag{4.3}$$

$$d\dot{w} = \psi_3[\beta_1 y + \dot{p}^e - \dot{w}]\,dt + dv_3 \tag{4.4}$$

$$\bar{p}^d = c_1 w + (1 - c_1)(w^* + e) \tag{4.5}$$

$$\bar{p} = \Theta\bar{p}^d + (1 - \Theta)(w^* + e) \tag{4.6}$$

$$dp = \psi_4(\bar{p} - p)\,dt \tag{4.7}$$

$$de^e = (r - r^*)\,dt \tag{4.8}$$

$$dw^* = -\mu_1 w^* dt + dv_4 \tag{4.9}$$

$$dr^* = -\mu_2 r^* dt + dv_5 \tag{4.10}$$

$$q = w^* + e - w \tag{4.11}$$

where the following notation is used:

y real output

q competitiveness

r domestic nominal rate of interest

v real net financial wealth of the private sector

m nominal money supply

p general price index

p^d price index of domestic output

w nominal wages

w^* nominal wages overseas

r^* foreign nominal rate of interest

dv_i white noise disturbance

All variables are measured in terms of deviations of their logarithm from equilibrium, except for interest rates which are measured as deviations of proportions. All parameters are defined to be positive.

Equation (4.1) generalises (3.1), allowing for wealth to influence aggregate demand. (4.2) generalises (3.3) by allowing for a wealth effect on money demand. (4.3) determines the change in real wealth from the determinants of the sum of the government budget deficit and the current account of the balance of payments. Neglecting interest payments and approximating this relationship log-linearly, this makes the change in real wealth depend positively on competitiveness and negatively on output and inflation. (4.4) determines the rate of change of nominal wages. Long-run wage inflation is determined by an expectations-augmented Phillips curve; but actual wage inflation adjusts sluggishly towards this long-run relationship. The sluggishness of wage adjustment generates fluctuations in real output in the face of demand disturbances, even under rational expectations. (see, for example, Buiter (1980).) Equation (4.5) is a partial equilibrium relationship giving the price index of domestic output as a weighted average of domestic and foreign wages (the influence of the latter variable working partly through a mark-up on costs and partly through competitive pricing effects). The corresponding general price index in partial equilibrium is given from (4.6) as a weighted average of domestic prices and foreign wages. Actual prices adjust quickly but not instantly according to (4.7) where ψ_4 is large. (4.8) corresponds to (3.4), with foreign interest rates entering explicitly. Equations (4.9) and (4.10) specify exogenous first order autoregressive processes for foreign wages and foreign interest rates respectively. Some persistence in these disturbances is required if they are to have any impact on domestic variables, and this process involves the minimum additional complication. Finally, (4.11) defines competitiveness, q.

We treat $(y, m, v, \dot{w}, w, p, w^*, r^*, e)^T$ as our state vector and r as the instrument, eliminating q, $\bar{p}$ and $\bar{p}^d$.

Then

$$dp = \psi_4(\Theta c_1 w + (1 - \Theta c_1)(w^* + e) - p)\,dt \tag{4.11}$$

and hence

$$p^e = \psi_4(\Theta c_1 w + (1 - \Theta c_1)(w^* + e) - p).$$

Using (4.11) and (4.10), we may therefore write the system as:

$$\begin{bmatrix} dy \\ dm \\ dv \\ d\dot{w} \\ dw \\ dp \\ dw^* \\ dr^* \\ de^e \end{bmatrix} = A \begin{bmatrix} y \\ m \\ v \\ \dot{w} \\ w \\ p \\ w^* \\ r^* \\ e \end{bmatrix} dt + Brdt + \begin{bmatrix} dv_1 \\ dv_2 \\ -(1 - c_1)dv_4 \\ dv_3 \\ 0 \\ 0 \\ dv_4 \\ dv_5 \\ 0 \end{bmatrix} \tag{4.12}$$

where

$$A=\begin{bmatrix} -\psi_1 & 0 & \psi_1\gamma_3 & 0 & \psi_1(\psi_4\alpha_2\delta-\alpha) & -\psi_1\psi_4\alpha_2 & \psi_1(\psi_4\alpha_2(1-\delta)+\alpha_1) & 0 & \psi_1(\psi_4\alpha_2(1-\delta)+\alpha_1) \\ \psi_2\gamma_1 & -\psi_2 & \psi_2\alpha_3 & 0 & 0 & \psi_2 & 0 & 0 & 0 \\ -\phi_1 & 0 & 0 & 0 & -(\phi_2+\psi_4\delta) & \psi_4 & \phi_2-\psi_4(1-\delta) & 0 & \phi_2-\psi_4(1-\delta) \\ \psi_3\beta_1 & 0 & 0 & -\psi_3 & \psi_3\psi_4\delta & -\psi_3\psi_4 & \psi_3\psi_4(1-\delta) & 0 & \psi_3\psi_4(1-\delta) \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_4\delta & -\psi_4 & \psi_4(1-\delta) & 0 & \psi_4(1-\delta) \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \tag{4.13}$$

$$B = \begin{bmatrix} -\psi_1\alpha_2\delta_1 \\ -\psi_2\gamma_2 \\ -(1-\delta) \\ \psi_3(1-\delta) \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{4.14}$$

and we have put $\delta = \Theta c_1$. Equation (4.12) has the form of (2.1), so that the analysis of Section 2 applies directly.

The set of assumed parameter values are presented in Table 3. We assume the same objective function (with identical parameters) as in Section 3 so that the Q matrix is of the form

$$Q = \begin{bmatrix} a & & & & & & & & 0 \\ & 0 & & & & & & & \\ & & 0 & & & & & & \\ & & & 0 & & & & & \\ & & & & 0 & & & & \\ & & & & & b & & & \\ & & & & & & 0 & & \\ & & & & & & & 0 & \\ 0 & & & & & & & & 0 \end{bmatrix} \tag{4.15}$$

As in the previous section, we consider three classes of policy rules: the optimal rule, the optimal general linear time-invariant feedback rule; and the optimal linear restricted feedback rules defined alternatively with respect to the exchange rate, nominal income, the money supply and the price level.

5. Results and Conclusions

Tables 4 to 11 present the results from the model of the previous section and are comparable to tables 1 and 2. Money demand disturbances (dv_2) have no impact under any of the regimes, and so have no tabulated results. (This may appear paradoxical for the case where the money supply is the intermediate target, until one notes that a sufficiently relaxed application of this regime makes the effects of such disturbance negligible.) For exchange rate targets, the loss is invariant with respect to the parameter of the feedback rule except in the face of foreign interest rate disturbances. Hence, the parameter of the feedback rule is indeterminate except for dv_5 shocks. Apart from this difference, the dv_4 and dv_5 shocks have identical effects.

First consider central parameter values and loss function $asyE(2y^2 + 2p^2 + r^2)$. Comparing the optimal rule and the general linear time-invariant feedback rule (2), we note that their performance is similar for dv_1 and dv_3 shocks. However, the general rule designed to cope with dv_4 and dv_5 shocks is not robust in the face of other shocks notably the dv_3 shock. It would seem preferable to design policy on the assumption of either dv_1 or dv_3 shocks: if other shocks then materialise, the costs (over and above those of the optimal rule) are not very large. Provided that this is done, the costs of imposing time-invariance in the feedback rule is not very significant.

Turning now to the simple rules, we note that the exchange rate rule performs very badly indeed in the face of dv_3 (supply) shocks, and is clearly not a serious contender. (This result contrasts markedly with that of Artis and Currie (1981), and illustrates the importance of dynamics in the assessment of policy.) Nominal income and monetary targets perform badly in the face of dv_3 (supply) shocks if designed with dv_1 (demand) shocks in mind. (This contrasts with the case of the general rule.) But if designed for other shocks, these regimes are fairly robust, though in some cases the variance is two to four times that of the optimal rule.

The simple rule that totally dominates the others is the price rule. It is superior in every case when the shock is known and vastly supperior to the nearest contenders ($r=0.12m$ or $r = 2.26\ (y+p)$) in conditions of uncertainty.

Three other features of the price rule are also striking. Its performance is rather insensitive to the choice of feedback parameter, so that there is rather little difficulty in choosing an appropriate value for this parameter which performs well in the face of all disturbances. Thus for the price rule, the absence of certainty equivalence is not a problem. Moreover, the additional loss over the fully optimal rule is not great. Provided the price rule is adopted, the costs of simplicity are not high. Finally, as table 5 shows, the preference for a price rule holds for both objective functions. This contrasts with our simpler model where the more 'Keynesian' objective function asyB($5y^2 + 2p^2 + r^2$) leads to the superior performance of a nominal income rule.

In tables 6 - 11, results are presented for alternative parameter values, all evaluated under the objective function which weights output and price movements equally. Table 6 examines the effect of increased openness of the economy as measured by the parameters θ and c_1. In Table 7, the influence of demand on inflation, as measured by the parameter β_1, is increased. In Table 8, the wealth effect on expenditures, as measured by α_3, is substantially augmented. Table 9 examines the consequences of an increased direct channel for monetary policy, as measured by the interest semi-elasticity of expenditures, α_2. Table 10 examines the consequences of slowing down the response of prices to costs, as measured by the adjustment parameter ψ_4. Finally, in Table 11, the wealth effects on both expenditures and money demand (as measured by α_3 and γ_3 respectively) are substantially reduced. The striking result is that the superiority of the price rule is not greatly influenced by variations in these key parameters. The price rule appears to exhibit a high degree of robustness, in contrast to the other rules, at least within the context of the model of this paper.

We conclude by mentioning extensions of this work currently being undertaken by the authors. First, fiscal policy using as instrument real government spending and/or tax rates will be similarly analysed both on its own and jointly with monetary policy. Second, sensitivity analysis with respect to specification (e.g. the wage equation) will be a major extension of this work. Finally, an important development of the model is the addition of capital immobility as a special case. These developments will then throw light on the dependence of policy design, involving both fiscal and monetary policy, on different aspects of model specification and estimation.

Footnotes

1. We ignore time discounting and we adopt an infinite time horizon. Then ½ asyE(W) with W given by (2.3) is the appropriate objective function. (see, for example, Kwakernaak and Sivan (1972), p 264.)

2. The general linear rule is computationally expensive to obtain and we have omitted this rule for tables 5 onwards.

3. The welfare loss is invariant with respect to the parameter β for the dv_4 disturbance but not for the dv_5 disturbance. The reported optimal feedback rule and associated welfare loss is for the dv_5 disturbance only .

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Parameter	Low	Central	High
ψ_1		0.5	
ψ_2		0.5	
ψ_3		0.5	
ψ_4	5.0	10.0	
α_1		0.3	
α_2		0.1	0.5
α_3	0.1	1.0	2.0
γ_1		1.0	
γ_2		1.0	
γ_3	0.1	1.0	
β_1		0.3	2.0
ϕ_1		1.3	
ϕ_2		0.1	
c_1	0.5	0.7	
θ	0.5	0.7	
μ_1		0.5	
μ_1		0.5	

<u>Table 3: Parameter Values</u>

Expected Disturbance (unit variance)	Actual Disturbance (unit variance) / Control Rule*	dv_1	dv_2	dv_3	dv_4	dv_5
	Full Optimal	2.28	0.00	2.78	0.09	0.38
	General ($r=0.27y - 0.59m + 0.03v - 0.05\dot{w} + 0.09w + 0.59p$)	2.28	0.042	3.10	0.16	0.65
dv_1	Price Level ($r = 10p$	2.31	0.00	3.12	0.19	0.75
	Monetary ($r = 0.00m$)	2.44	0.00	64.19	0.96	3.83
	Nominal Income ($r = 0.00(y+p)$)	2.44	0.00	64.44	0.96	3.83
	Exchange Rate (Indeterminate)	2.44	0.00	64.80	0.96	Indeterminate
	Full Optimal	2.28	0.00	2.78	0.09	0.38
	General ($r = -1.1y + 0.67m + 0.41v + 0.09\dot{w} - 0.51w + 0.47p + 0.01p^*$)	3.57	0.41	2.78	0.27	1.10
dv_3	Price Level ($r = 2.15p$)	2.32	0.00	2.97	0.15	0.61
	Monetary ($r = 0.12m$)	2.67	0.04	3.36	0.55	2.19
	Nominal Income ($r = 1.02(y+p)$)	3.90	0.00	5.16	0.19	0.77
	Exchange Rate (Indeterminate)	2.44	0.00	64.80	0.96	Indeterminate
	Full Optimal	2.28	0.00	2.78	0.09	0.38
dv_4 and	General ($r=2.20y - 1.03m - 0.31v - 2.40\dot{w} + 2.50w + 0.69p + 0.01p^* - 0.11r^*$)	6.30	1.28	63.94	0.09	0.40
dv_5	Price Level ($r=2.33p$)	2.32	0.00	2.97	0.15	0.61
	Nominal Income ($r=2.26\ (y+p)$)	6.07	0.00	5.44	0.16	0.66
	Monetary ($r=10m$)	6.73	9.13	8.68	0.17	0.68
	Exchange Rate(3) ($r=1.91e$)	2.44	0.00	64.80	0.96	0.80

Table 4: Welfare Loss for Loss Function asy$(2y^2 + 2p^2 + r^2)$
Parameter Values: Central

Expected Disturbance (unit variance)	Control Rule / Actual Disturbance (unit variance)	dv_1	dv_2	dv_3	dv_4	dv_5
	Full Optimal	5.55	0.00	6.47	0.15	0.40
	Price Level ($r = 10p$)	5.73	0.00	6.81	0.19	0.77
dv_1	Monetary ($r = 0.00m$)	6.02	0.00	68.66	1.03	4.11
	Nominal Income($r = 0.00(y + p)$)	6.02	0.00	69.10	1.03	4.11
	Exchange Rate (Indeterminate)	6.02	0.00	69.47	1.03	Indeterminate
	Full Optimal	5.55	0.00	6.47	0.15	0.40
	Price Level ($r = 2.48p$)	5.78	0.00	6.69	0.16	0.64
dv_3	Monetary ($r = 0.12m$)	6.37	0.04	7.20	0.59	2.38
	Nominal Income($r = 0.94(y + p)$)	7.67	0.00	9.17	0.22	0.88
	Exchange Rate (Indeterminate)	6.02	0.00	69.47	1.03	Indeterminate
	Full Optimal	5.55	0.00	6.47	0.15	0.40
dv_4	Price Level ($r = 2.54p$)	5.77	0.00	6.69	0.16	0.64
and	Nominal Income($r = 2.50(y + p)$)	10.76	0.00	9.64	0.18	0.71
dv_5	Monetary ($r = 10m$)	10.22	9.14	12.16	0.18	0.74
	Exchange Rate(3) ($r = 2.02e$)	6.02	0.00	69.47	1.03	0.91

Table 5: Welfare Loss for Loss Function asy($5y^2 + 2p^2 + r^2$)
Parameter Values: Central

Expected Disturbance (unit variance)	Actual Disturbance (unit variance) / Control Rule	dv_1	dv_2	dv_3	dv_4	dv_5
dv_1	Full Optimal	2.11	0.00	1.61	0.12	0.47
	Price Level (r = 10p)	2.18	0.00	1.64	0.20	0.82
	Monetary (r = 0.00m)	2.25	0.00	14.19	1.53	6.11
	Nominal Income(r = 0.00(y + p))	2.25	0.00	14.31	1.53	6.11
	Exchange Rate (Indeterminate)	2.25	0.00	14.39	1.53	Indeterminate
dv_3	Full Optimal	2.11	0.00	1.61	0.12	0.47
	Price Level (r = 4.46p)	2.19	0.00	1.64	0.19	0.76
	Monetary (r = 0.04m)	2.32	0.01	1.88	1.19	4.76
	Nominal Income(r = 0.55(y + p))	3.17	0.00	3.12	0.43	1.70
	Exchange Rate (Indeterminate)	2.25	0.00	14.39	1.53	Indeterminate
dv_4 and dv_5	Full Optimal	2.11	0.00	1.61	0.12	0.47
	Price Level (r = 3.29p)	2.19	0.00	1.64	0.19	0.75
	Nominal Income(r = 5.23(y + p))	11.54	0.00	3.87	0.23	0.90
	Monetary (r = 10m)	6.69	9.22	6.18	0.27	1.06
	Exchange Rate(3) (r = 3.14e)	2.25	0.00	14.39	1.53	0.87

Table 6: Welfare Loss for Loss Function $\Sigma y (2y^2 + 2p^2 + r^2)$
Parameter Values: Central except $c_1 = \theta = 0.5$

Expected Disturbance (unit variance	Control Rule	Actual Disturbance (unit variance) dv_1	dv_2	dv_3	dv_4	dv_5
	Full Optimal	13.15	0.00	2.98	0.12	0.47
	Price Level (r = 10p)	23.67	0.00	5.30	0.22	0.88
dv_1	Monetary	44.22	9.27	7.10	0.39	1.54
	Nominal Income (r =ß(y + p))	unstable (3 unstable				
	Exchange Rate (r =ße)	roots for all ß ε[0,10])				
	Full Optimal	13.15	0.00	2.98	0.12	0.47
	Price Level (r = 10p)	23.67	0.00	5.30	0.22	0.88
dv_3	Monetary (r = 10m)	44.22	9.27	7.10	0.39	1.54
	Nominal Income	unstable				
	Exchange Rate	as above				
	Full Optimal	13.15	0.00	2.98	0.12	0.47
dv_4	Price Level (r = 7.25p)	26.52	0.00	5.84	0.22	0.87
and	Monetary (r = 10m)	44.22	9.27	7.10	0.39	1.54
dv_5	Nominal Income	unstable				
	Exchange Rate	as above				

Table 7: Welfare Loss for Loss Function asy $(2y^2 + 2p^2 + r^2)$
Parameter Values: Central except $\beta_1 = 2.00$

Expected Disturbance (unit variance)	Control Rule	Actual Disturbance (unit variance) dv_1	dv_2	dv_3	dv_4	dv_5
	Full Optimal	2.06	0.00	2.61	0.10	0.41
	Price Level ($r = 10p$)	2.20	0.00	2.88	0.19	0.76
dv_1	Nominal Income ($r = 0.00(y+p)$)	2.34	0.00	64.44	1.06	4.23
	Monetary ($r = 0.00m$)	2.34	0.00	66.36	1.06	4.23
	Exchange Rate (Indeterminate)	2.34	0.00	66.85	1.06	Indeterminate
	Full Optimal	2.06	0.00	2.61	0.10	0.41
	Price Level ($r = 3.32p$)	2.23	0.00	2.81	0.16	0.65
dv_3	Monetary ($r = 1.19(y + p)$)	3.86	9.13	3.03	0.19	0.78
	Nominal Income ($r = 1.19(y + p)$)	4.91	0.00	4.93	0.21	0.84
	Exchange Rate (Indeterminate)	2.34	0.00	66.85	1.06	Indeterminate
	Full Optimal	2.06	0.00	2.61	0.10	0.41
dv_4	Price Level ($r = 2.73p$)	2.24	0.00	2.81	0.16	0.64
and	Monetary ($r = 10m$)	3.86	9.13	3.03	0.19	0.78
dv_5	Nominal Income ($r = 1.54(y + p)$)	6.48	0.00	4.98	0.20	0.81
	Exchange Rate (3) ($r = 2.17e$)	2.34	0.00	66.85	1.06	0.82

Table 8: Welfare Loss for Loss Function asy $(2y^2 + 2p^2 + r^2)$
Parameter Values: Central except $\alpha_3 = 2.0$

Expected Disturbance (unit variance)	Control Rule / Actual Disturbance (unit variance)	dv_1	dv_2	dv_3	dv_4	dv_5
	Full Optimal	2.13	0.00	2.82	0.12	0.47
	Price Level (r = 1.36p)	2.38	0.00	3.13	0.19	0.77
dv_1	Nominal Income(r = 0.09(y + p))	2.46	0.00	12.37	0.69	2.76
	Monetary (r = 0.00m)	2.47	0.00	65.04	1.02	4.08
	Exchange Rate (Indeterminate)	2.47	0.00	65.04	1.02	Indeterminate
	Full Optimal	2.13	0.00	2.82	0.12	0.47
	Price Level (r = 1.48p)	2.38	0.00	3.13	0.19	0.76
dv_3	Monetary (r = 0.11m)	2.74	0.04	3.28	0.59	2.38
	Nominal Income(r = 1.01(y + p))	2.95	0.00	5.19	0.24	0.98
	Exchange Rate (Indeterminate)	2.47	0.00	2.82	1.02	Indeterminate
	Full Optimal	2.13	0.00	2.82	0.12	0.47
dv_4	Price Level (r = 2.29p)	2.38	0.00	3.16	0.18	0.72
and	Monetary (r = 10m)	8.43	10.31	9.06	0.21	0.84
dv_5	Nominal Income(r = 1.76(y + p))	3.44	0.00	5.42	0.23	0.91
	Exchange Rate (r = 1.78e)	2.47	0.00	65.04	1.02	0.90

Table 9: Welfare Loss for Loss Function asy $(2y^2 + 2p^2 + r^2)$

Parameter Values: Central except $\alpha_2 = 0.5$

Expected Disturbance (unit variance)	Control Rule / Actual Disturbance (unit variance	dv1	dv2	dv3	dv4	dv5
	Full Optimal	2.27	0.00	2.75	0.09	0.36
	Price Level ($r = 10p$)	2.30	0.00	3.06	0.18	0.71
dv_1	Monetary ($r = 0.00m$)	2.43	0.00	64.14	0.92	3.67
	Nominal Income($r = 0.00(y + p)$)	2.43	0.00	64.38	0.92	3.67
	Exchange Rate (Indeterminate)	2.43	0.00	64.75	0.92	Indeterminate
	Full Optimal	2.27	0.00	2.75	0.09	0.36
	Price Level ($r = 2.02p$)	2.31	0.00	2.90	0.14	0.56
dv_3	Monetary ($r = 0.12m$)	2.69	0.03	3.24	0.52	2.07
	Nominal Income($r = 1.02(y + p)$)	3.82	0.00	5.11	0.18	1.07
	Exchange Rate (Indeterminate)	2.43	0.00	64.75	0.92	Indeterminate
	Full Optimal	2.27	0.00	2.75	0.09	0.36
dv_4	Price Level ($r = 2.31p$)	2.31	0.00	2.90	0.14	0.56
and	Nominal Income($r = 2.30(y + p)$)	5.97	0.00	5.83	0.15	0.60
dv_5	Monetary ($r = 10m$)	7.29	9.11	8.63	0.16	0.65
	Exchange Rate(3) ($r = 1.83e$)	2.43	0.00	64.75	0.92	0.79

Table 10: Welfare Loss for Loss Function asy $(2y^2 + 2p^2 + r^2)$

Parameter Values: Central except $\psi_4 = 5.0$

Expected Disturbance (unit variance	Control Rule	Actual Disturbance (unit variance) dv1	dv2	dv3	dv4	dv5
	Full Optimal	2.39	0.00	6.23	0.10	0.38
	Price Level ($r = 0.94p$)	2.49	0.00	6.39	0.18	0.73
dv_1	Nominal Income($r = 0.71(y + p)$)	3.37	0.00	13.09	0.24	0.96
	Monetary ($r = 0.00m$)	3.74	0.00	49.67	1.14	4.58
	Exchange Rate (Indeterminate)	3.74	0.00	50.31	1.15	Indeterminate
	Full Optimal	2.39	0.00	6.23	0.10	0.38
	Price Level ($r = 1.13p$)	2.49	0.00	6.38	0.17	0.67
dv_3	Monetary ($r = 0.07m$)	4.03	0.02	11.17	0.79	3.16
	Nominal Income($r = 2.47(y + p)$)	3.87	0.00	12.84	0.15	0.65
	Exchange Rate (Indeterminate)	3.74	0.00	50.31	1.15	Indeterminate
	Full Optimal	2.39	0.00	6.23	0.10	0.38
dv_4	Price Level ($r = 2.22p$)	2.52	0.00	6.48	0.15	0.59
and	Nominal Income($r = 2.32(y + p)$)	3.81	0.00	12.84	0.16	0.64
dv_5	Monetary ($r = 10m$)	4.75	8.76	17.06	0.21	0.84
	Exchange Rate(3) ($r = 2.07e$)	3.74	0.00	50.31	1.15	0.80

Table 11: Welfare Loss for Loss Function say $(2y^2 + 2p^2 + r^2)$
Parameter Values: Central except $\alpha_3 = 0.1$, $\gamma_3 = 0.1$